Theory of phase-conjugate oscillators. I

Wun-Shung Lee and Sien Chi

Institute of Electro-Optical Engineering, National Chiao Tung University, Hsinchu, Taiwan, China

Pochi Yeh and Ragini Saxena

Rockwell International Science Center, Thousand Oaks, California 91360

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We have developed a theory for nondegenerate oscillations in optical resonators containing an intracavity phase-conjugate element. The phase-conjugate element consists of a nonlinear transparent medium that is pumped externally by a pair of counterpropagating laser beams of the same frequency and intensity. Phase conjugation of an input beam of slightly different frequency occurs because of nondegenerate four-wave mixing. The theory takes into account linear absorption (or gain) in the medium and is applied to study the threshold behavior of phase-conjugate oscillators. For the special case of no conventional mirrors, the phase-conjugate oscillator reduces to an ordinary phase-conjugate mirror, and our general formulation yields the results of previous studies. Our analysis shows that the parametric gain required for oscillation increases (or decreases) as a result of linear absorption (or gain) in the medium, and oscillation can occur at a frequency different from that of the pump beams in the presence of large linear gain (or loss). The effects of linear absorption (or gain) on the filter operation are also examined.

INTRODUCTION

Optical resonators containing a phase-conjugate element have been a subject of great interest and importance. For correction of intracavity aberration, the phase-conjugate element can be employed as an end mirror of the optical resonator.[1-5] In these resonators, the phase-conjugate element acts as a unique kind of mirror (often called a phase-conjugate mirror) that combines reflection with phase reversal. Sufficiently high reflectivities are necessary for efficient operation.

In addition to their unique property of correcting wavefront aberrations, these phase-conjugate elements can also provide parametric gain and conjugate coupling between the oscillating beams. As a result of the parametric gain, oscillation is possible even without the conventional gain medium. Such oscillations are known as phase-conjugate oscillations.[6] Recent theoretical analysis indicates that the insertion of a phase-conjugate element inside a ring-laser cavity results in a reduction of the lock-in threshold and reduces the imbalance between the amplitudes of the oppositely directed traveling waves in some ring-laser systems.[7] In the extreme case of phase-conjugate oscillation without conventional gain media, it is shown that the lock-in can be completely eliminated.[8,9] The study of these resonators is also important in understanding the stability of laser oscillation in situations when backscattered laser radiation may enter the resonator and undergo parametric four-wave mixing with the oscillating beams.

Although a few special cases of phase-conjugate oscillators have been studied, a general theory that includes nondegenerate oscillations is not available. In this paper the authors develop a general theory of phase-conjugate oscillators by studying the problem of wave propagation along the axis of the resonator. The matrix method introduced in Ref. 6 is now extended to the case of nondegenerate four-wave mixing. The approach is general, so that many of the situations studied previously can be shown to be special cases in this formalism.

FORMULATION OF THE PROBLEM

Referring to Fig. 1, we consider a linear optical resonator that consists of two partially reflecting mirrors and a nonlinear medium that is pumped by a pair of external laser beams of equal intensity. These two laser beams are counterpropagating, and their frequency is ω. The nonlinear medium provides linear gain–absorption as well as parametric gain by means of optical four-wave mixing. We assume that the bandwidth of the linear gain is sufficiently broad. To investigate the general properties of such a resonator, we must treat the problem of wave propagation along the axis of the resonator.

Let the electric field of the waves be written as

$$E = \begin{cases} \{\mathscr{E}_1 \exp[-ik_1(z + \mathrm{a})] + \mathscr{E}_4 \exp[ik_1(z + a)]\} \exp(i\omega_1 t) + \{\mathscr{E}_3 \exp[-ik_2(z + a)] + \mathscr{E}_2 \exp[ik_2(z + a)]\}\exp(i\omega_2 t) & \text{for } z < -a \\ [\mathscr{A}_1(z) \exp(-ik_1 z) + \mathscr{A}_4(z) \exp(ik_1 z)] \exp(i\omega_1 t) + [\mathscr{A}_3(z) \exp(-ik_2 z) + \mathscr{A}_2(z) \exp(ik_2 z)] \exp(i\omega_2 t) & \text{for } 0 < z < l, \\ \{\mathscr{G}_1 \exp[-ik_1(z - l - b)] + \mathscr{G}_4 \exp[ik_1(z - l - b)]\} \exp(i\omega_1 t) + \{\mathscr{G}_3 \exp[-ik_2(z - l - b)] + \mathscr{G}_2 \exp[ik_2(z - l - b)]\} \exp(i\omega_2 t) & \text{for } z > l + b \end{cases}$$

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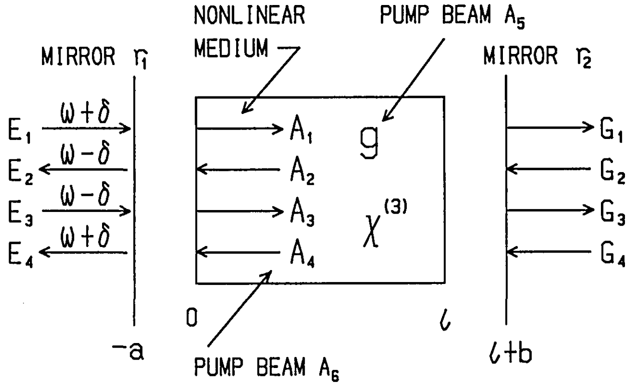

Fig. 1. Basic geometry of linear phase-conjugation oscillation by means of nearly degenerate four-wave mixing. In this case, the incident probe wave, whose frequency $\omega \pm \delta$ is slightly detuned from that of the pump waves (both at frequency ω), will result in a conjugate wave with an inverted frequency $\omega \mp \delta$. g is the linear nonsaturating background (intensity) net gain coefficient.

where $\mathcal{E}_1$, $\mathcal{E}_3$, $\mathcal{A}_1(z)$, $\mathcal{A}_3(z)$, $\mathcal{G}_1$, and $\mathcal{G}_3$ are the complex amplitudes of the plane waves traveling in the $+z$ direction and $\mathcal{E}_2$, $\mathcal{E}_4$, $\mathcal{A}_2(z)$, $\mathcal{G}_2$, and $\mathcal{G}_4$ are those of the plane waves traveling in the $-z$ direction. k_1, k_2, k_3, and k_4 are the wave numbers that correspond to the frequencies ω_1, ω_2, ω_3, and ω_4, respectively, where $\omega_1 = \omega + \delta$, $\omega_2 = \omega - \delta$, $\omega_3 = \omega_2$, and $\omega_4 = \omega_1$. E_i, A_i, and G_i are the plane waves corresponding to the complex amplitudes $\mathcal{E}_i$, $\mathcal{A}_i$, and $\mathcal{G}_i$ (where $i = 1, 2, 3$, and 4); A_5 and A_6 are pump laser beams; l is the length of the four-wave mixing interaction region; $-a$ and $l + b$ are the positions of the mirrors, and $\mathcal{A}_1$, $\mathcal{A}_2$, $\mathcal{A}_3$, and $\mathcal{A}_4$ are functions of z because of the linear absorption–gain and wave coupling owing to four-wave mixing in the nonlinear medium. The problem at hand is to derive expressions for all the wave amplitudes for a given set of boundary conditions.

If the regions between $z = -a$ and $z = 0$ and between $z = l$ and $z = l + b$ are linear dielectric media, then the following linear relationships between the wave amplitudes exist[10,11]:

$$\begin{bmatrix} A_1(0) \\ A_2(0) \\ A_3(0) \\ A_4(0) \end{bmatrix} = M_1 \begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ E_4 \end{bmatrix}, \quad \begin{bmatrix} G_1 \\ G_2 \\ G_3 \\ G_4 \end{bmatrix} = M_2 \begin{bmatrix} A_1(l) \\ A_2(l) \\ A_3(l) \\ A_4(l) \end{bmatrix}, \tag{2}$$

where M_1 and M_2 are 4×4 matrices. If we further assume that there is no Fresnel reflection at the surfaces ($z = 0$ and $z = l$) of the nonlinear medium and lump together all the reflections at $z = -a$ and $z = l + b$, then the matrices can be written as

$$M_1 = S(a)F_1, \tag{3}$$

$$M_2 = F_2 S(b), \tag{4}$$

with

$$F_i = \frac{1}{t_i} \begin{bmatrix} 1 & 0 & 0 & -r_i \\ 0 & 1 & -r_i & 0 \\ 0 & -r_i & 1 & 0 \\ -r_i & 0 & 0 & 1 \end{bmatrix}, \quad i = 1, 2 \tag{5}$$

and

$$S(\phi) = \begin{bmatrix} \exp(-ik_1\phi) & 0 & 0 & 0 \\ 0 & \exp(ik_2\phi) & 0 & 0 \\ 0 & 0 & \exp(-ik_2\phi) & 0 \\ 0 & 0 & 0 & \exp(ik_1\phi) \end{bmatrix}, \tag{6}$$

where $\phi = a, b, l$; r_i and t_i are the amplitude reflection and transmission coefficients, respectively, of the end mirrors from the front surfaces (left sides). The matrices F_1 and F_2 account for the Fresnel reflection and transmission at the mirrors, whereas the matrices $S(a)$ and $S(b)$ account for the propagation through the bulk of the linear regions.

In the nonlinear medium between $z = 0$ and $z = l$, the waves A_1 and A_2 and the pump beams are coupled by optical four-wave mixing. The waves A_3 and A_4 and the pump beams are similarly coupled. If we assume no pump depletion of the waves A_5 and A_6 to describe oscillation near threshold, then the amplitudes $A_1(0)$, $A_2(0)$, $A_3(0)$, $A_4(0)$ and $A_1(l)$, $A_2(l)$, $A_3(l)$, $A_4(l)$ will be shown to be related by

$$\begin{bmatrix} A_1(l) \\ A_2(l) \\ A_3(l) \\ A_4(l) \end{bmatrix} = S(l)K \begin{bmatrix} A_1(0) \\ A_2(0) \\ A_3(0) \\ A_4(0) \end{bmatrix}, \tag{7}$$

where $S(l)$ and K are 4×4 matrices. Using an approach similar to that used in Ref. 6, we now determine the matrix K.

We derive this matrix by solving the coupled-mode equations for the four-wave mixing processes. As a result of four-wave mixing, the input wave $\mathcal{A}_1$ interacts with two external pumping laser beams $\mathcal{A}_5$ and $\mathcal{A}_6$, and a phase-conjugate wave $\mathcal{A}_2$ is generated. These two waves are related by the coupled-mode equation[12–15]

$$\frac{d\mathcal{A}_1^*}{dz} = i\kappa_1 \mathcal{A}_2 \exp(-i\Delta k z) + \frac{g}{2}\mathcal{A}_1^*,$$

$$\frac{d\mathcal{A}_2}{dz} = i\kappa_2^* \mathcal{A}_1^* \exp(i\Delta k z) - \frac{g}{2}\mathcal{A}_2, \tag{8}$$

where the amplitudes of waves 1 and 2 have been redefined in order to account for pump-induced phase modulation. $\kappa_i^* = (\omega_i/2)\sqrt{\mu/\varepsilon}\,\chi^{(3)}\,\mathcal{A}_5\mathcal{A}_6 e^{gl/2}$ is the complex coupling coefficient, and in deriving Eqs. (8) we have assumed that the input wave and its conjugate beam are small compared with the pump beams. Then pump depletion is negligible, and $\mathcal{A}_5$ and $\mathcal{A}_6$ may be regarded as constants, so that our theory describes nondegenerate oscillation near threshold. Note that, if the two pumps have different intensities, then κ_i becomes a function of z owing to the additional phase mismatch introduced by the unequal pumps.[12,13] In what follows we shall assume that the two pumps have equal intensities so that κ_i is independent of z. $\Delta k = k_1 - k_2$ is the phase mismatch, and g is the linear, nonsaturable net gain (or loss) coefficient. In order to solve Eqs. (8) we introduce the new variables a_1 and a_2:

$$\mathcal{A}_1 = a_1 e^{gz/2},$$

$$\mathcal{A}_2 = a_2 e^{-gz/2}. \tag{9}$$

In terms of the new variables, the coupled equations (8) reduce to

$$\frac{d}{dz}a_1^* = i\kappa_1 a_2 \exp[-i(\Delta k - ig)z],$$
$$\frac{d}{dz}a_2 = i\kappa_2^* a_1^* \exp[i(\Delta k - ig)z]. \quad (10)$$

Solving the differential equations (10) in terms of $a_1^*(0)$ and $a_2(l)$, which are specified by boundary conditions, we obtain

$$a_1(l) = \frac{1}{D^*}\{a_1(0)s^* \exp[i(\Delta k + ig)l] - 2i\kappa_1^* a_2^*(l) \times \exp[i(\Delta k + ig)l]\} \sinh[s^* l/2],$$
$$a_2(0) = \frac{1}{D}\{-2i\kappa_2^* a_1^*(0) \sinh[sl/2] + a_2(l)s \times \exp[-i(\Delta k - ig)l]\},$$

where

$$s = [-(\Delta k - ig)^2 - 4\kappa_1\kappa_2^*]^{1/2},$$
$$D = (-g - i\Delta k)\sinh[sl/2] + s\cosh[sl/2]. \quad (12)$$

From Eqs. (9) and (11) we obtain the solutions in terms of the original variables:

$$\mathcal{A}_1(l) = \frac{1}{\alpha^*}\exp(i\Delta kl/2) \times \{[(\alpha^*)^2 - \kappa_1^*\kappa_2(\beta^*)^2]\mathcal{A}_1(0) - i\kappa_2^*\beta^*\mathcal{A}_2^*(0)\},$$
$$\mathcal{A}_2(l) = \frac{1}{\alpha}\exp(i\Delta kl/2)[i\kappa_2^*\beta\mathcal{A}_1^*(0) + \mathcal{A}_2(0)], \quad (13)$$

where

$$\alpha = \frac{1}{D}s,$$
$$\beta = \frac{2}{D}\sinh\left[\frac{sl}{2}\right]. \quad (14)$$

Equations (13) can be rewritten in matrix notation as

$$\begin{bmatrix}\mathcal{A}_1(l)\\ \mathcal{A}_2(l)\end{bmatrix} = \begin{bmatrix} M & PX\\ QX & N\end{bmatrix}\begin{bmatrix}\mathcal{A}_1(0)\\ \mathcal{A}_2(0)\end{bmatrix}, \quad (15)$$

where

$$M = \frac{1}{\alpha^*}\exp(i\Delta kl/2)[(\alpha^*)^2 - \kappa_1^*\kappa_2(\beta^*)^2],$$
$$N = \frac{1}{\alpha}\exp(i\Delta kl/2),$$
$$P = -\frac{i}{\alpha^*}\exp(i\Delta kl/2)\kappa_1^*\beta^*,$$
$$Q = \frac{i}{\alpha}\exp(i\Delta kl/2)\kappa_2^*\beta, \quad (16)$$

and X is the complex-conjugated operator, defined as $XH = H^*$, where H is an arbitrary number.

Similarly, we obtain the following matrix equation for the waves $\mathcal{A}_3$ and $\mathcal{A}_4$:

$$\begin{bmatrix}\mathcal{A}_3(l)\\ \mathcal{A}_4(l)\end{bmatrix} = \begin{bmatrix} M' & P'X\\ Q'X & N'\end{bmatrix}\begin{bmatrix}\mathcal{A}_3(0)\\ \mathcal{A}_4(0)\end{bmatrix}, \quad (17)$$

where

$$M' = \frac{1}{\alpha}\exp(-i\Delta kl/2)[(\alpha)^2 - \kappa_1\kappa_2^*(\beta)^2],$$
$$N' = \frac{1}{\alpha^*}\exp(-i\Delta kl/2),$$
$$P' = -\frac{i}{\alpha}\exp(-i\Delta kl/2)\kappa_2^*\beta,$$
$$Q' = \frac{i}{\alpha^*}\exp(-i\Delta kl/2)\kappa_1^*\beta^*. \quad (18)$$

In arriving at Eqs. (17) and (18), we assumed exactly the same pumping, so that $\kappa_3 = \kappa_2$ and $\kappa_4 = \kappa_1$. By using Eqs. (15) and (17), we can now write the matrix K in Eq. (7):

$$K = \begin{bmatrix} M & PX & 0 & 0\\ QX & N & 0 & 0\\ 0 & 0 & M' & P'X\\ 0 & 0 & Q'X & N'\end{bmatrix}. \quad (19)$$

By using Eqs. (2)–(4) and (7), we can write the complex amplitudes G_1, G_2, G_3, G_4, E_1, E_2, E_3, and E_4:

$$\begin{bmatrix}G_1\\G_2\\G_3\\G_4\end{bmatrix} = F_2 S(l+b)KS(a)F_1\begin{bmatrix}E_1\\E_2\\E_3\\E_4\end{bmatrix}. \quad (20)$$

Equation (20) may now be used to study the reflection and transmission properties of such a resonator. We consider the most general case, where $r_1r_2 \neq 0$, $g \neq 0$, $|\kappa_1\kappa_2^*| \neq 0$, and $\Delta k \neq 0$. Using Eqs. (5), (6), and (19) and carrying out the multiplication in Eq. (20), we obtain

$$\begin{bmatrix}G_1\\G_2\\G_3\\G_4\end{bmatrix} = \frac{1}{t_2 t_1 t_1^*}\begin{bmatrix} F_{11} & F_{12}X & F_{13}X & F_{14}\\ F_{21}X & F_{22} & F_{23} & F_{24}X\\ F_{31}X & F_{32} & F_{33} & F_{34}X\\ F_{41} & F_{42}X & F_{43}X & F_{44}\end{bmatrix}\begin{bmatrix}E_1\\E_2\\E_3\\E_4\end{bmatrix}, \quad (21)$$

where

$$F_{11} = t_1^* M\exp[-ik_1(l+b+a)] + t_1^* r_1 r_2 N'\exp[ik_1(l+b+a)],$$
$$F_{12} = t_1 P\exp[-ik_1(l+b) - ik_2 a] + t_1 r_1^* r_2^* Q'\exp[ik_1(l+b) + ik_2 a],$$
$$F_{13} = -t_1 r_1^* P\exp[-ik_1(l+b) - ik_2 a] - t_1 r_2 Q'\exp[ik_1(l+b) + ik_2 a],$$
$$F_{14} = -t_1^* r_1 M\exp[-ik_1(l+b+a)] - t_1^* r_2 N'\exp[ik_1(l+b+a)],$$

$$
\begin{aligned}
F_{21} &= t_1 Q \exp[ik_2(l+b) + ik_1 a] \\
&\quad + t_1 r_2 r_1^* P' \exp[-ik_2(l+b) - ik_1 a], \\
F_{22} &= t_1^* N \exp[ik_2(l+b+a)] \\
&\quad + t_1^* r_1 r_2 M' \exp[-ik_2(l+b+a)], \\
F_{23} &= -t_1^* r_1 N \exp[ik_2(l+b+a)] \\
&\quad - t_1^* r_2 M' \exp[-ik_2(l+b+a)], \\
F_{24} &= -t_1 r_1^* Q \exp[ik_2(l+b) + ik_1 a] \\
&\quad - t_1 r_2 P' \exp[-ik_2(l+b) - ik_1 a], \\
F_{31} &= -t_1 r_2 Q \exp[ik_2(l+b) + ik_1 a] \\
&\quad - t_1 r_1^* P' \exp[-ik_2(l+b) - ik_1 a], \\
F_{32} &= -t_1^* r_2 N \exp[ik_2(l+b+a)] \\
&\quad - t_1^* r_1 M' \exp[-ik_2(l+b+a)], \\
F_{33} &= t_1^* r_1 r_2 N \exp[ik_2(l+b+a)] \\
&\quad + t_1^* M' \exp[-ik_2(l+b+a)], \\
F_{34} &= t_1 r_1^* r_2 Q \exp[ik_2(l+b) + ik_1 a] \\
&\quad + t_1 P' \exp[-ik_2(l+b) - ik_1 a], \\
F_{41} &= -t_1^* r_2 M \exp[-ik_1(l+b+a)] \\
&\quad - t_1^* r_1 N' \exp[ik_1(l+b+a)], \\
F_{42} &= -t_1 r_2 P \exp[-ik_1(l+b) - ik_2 a] \\
&\quad - t_1 r_1^* Q' \exp[ik_1(l+b) + ik_2 a], \\
F_{43} &= t_1 r_2 r_1^* P \exp[-ik_1(l+b) - ik_2 a] \\
&\quad + t_1 Q' \exp[ik_1(l+b) + ik_2 a], \\
F_{44} &= t_1^* r_2 r_1 M \exp[-ik_1(l+b+a)] \\
&\quad + t_1^* N' \exp[ik_1(l+b+a)], \qquad (22)
\end{aligned}
$$

and we recall that X is the complex-conjugate operator.

If we view E_1, E_3, G_2, and G_4 as the input waves at the two mirrors, then the output waves E_2, E_4, G_1, and G_3 can be solved from Eqs. (21) in terms of E_1, E_3, G_2, and G_4.

At oscillation, a finite solution for output waves E_2, E_4, G_1, and G_3 at the two mirrors may exist even if there are no input waves. By setting $E_1 = E_3 = G_2 = G_4 = 0$ in Eqs. (21), we obtain

$$
\begin{aligned}
G_2 &= 0 = \frac{1}{t_1 t_1^* t_2}[F_{22}E_2 + F_{24}E_4^*], \\
G_4 &= 0 = \frac{1}{t_1 t_1^* t_2}[F_{42}E_2^* + F_{44}E_4]. \qquad (23)
\end{aligned}
$$

For a nontrivial solution for the output waves E_2 and E_4, the determinant of the coefficients in Eq. (23) must vanish, i.e., $F_{24}^* F_{24} - F_{22}^* F_{44} = 0$. From Eqs. (22), this condition can be written as

$$
\begin{aligned}
&\{r_1 Q^* \exp[-ik_2(l+b) - ik_1 a] \\
&\quad + r_2^* P^{*\prime} \exp[ik_2(l+b) + ik_1 a]\} \\
&\quad \times \{r_2 P \exp[-ik_1(l+b) - ik_2 a] \\
&\quad + r_1^* Q' \exp[ik_1(l+b) + ik_2 a]\} \\
&= \{N^* \exp[-ik_2(l+b+a)] \\
&\quad + r_1^* r_2^* M^{*\prime} \exp[ik_2(l+b+a)]\} \\
&\quad \times \{r_2 r_1 M \exp[-ik_1(l+b+a)] \\
&\quad + N' \exp[ik_1(l+b+a)]\}, \qquad (24)
\end{aligned}
$$

where P, Q, P', Q', M, N, M', and N' are given by Eqs. (16) and (18).

The above oscillation condition depends on Δk, g, κ_1, κ_2, a, b, l, r_1, and r_2. In what follows, we investigate the oscillation condition by adjusting these parameters.

We now consider the case when there is only one input wave. For the case of incidence from the left on the mirror at $z = -a$, E_1 may be considered the incident wave, with a frequency of $\omega + \delta$. If this is the only incident wave, then $E_3 = 0$ at this mirror, while G_2 and G_4 are zero at the second mirror. The wave E_2 at $\omega - \delta$ is generated as a result of the optical four-wave mixing. The wave E_4 is produced by reflections off the second mirror at $z = l + b$. The problem at hand is to derive expressions for all the ouput waves E_2, E_4, G_1, and G_3, given an incident wave E_1 at $\omega + \delta$. Using Eqs. (21), we obtain

$$
\begin{aligned}
G_2 &= 0 = F_{21}E_1^* + F_{22}E_2 + F_{24}E_4^*; \\
G_4 &= 0 = F_{41}E_1 + F_{42}E_2^* + F_{44}E_4. \qquad (25)
\end{aligned}
$$

By eliminating E_4 we obtain the following expression for the phase-conjugate reflection coefficient:

$$
r_p = \frac{E_2}{E_1^*} = -\frac{F_{21}F_{44}^* - F_{24}F_{41}^*}{F_{22}F_{44}^* - F_{24}F_{42}^*}, \qquad (26)
$$

while the phase-conjugate power reflectivity is given by $R_p = |r_p|^2$. Similarly, we may obtain the coherent reflection coefficient at $\omega + \delta$ as

$$
r_s = \frac{E_4^*}{E_1^*} = -\frac{F_{21}F_{42}^* - F_{22}F_{41}^*}{F_{24}F_{42}^* - F_{22}F_{44}^*}, \qquad (27)
$$

and the coherent power reflectivity is given by $R_s = |r_s|^2$.

In addition to the two reflected waves, there are also two transmitted waves, as illustrated in Fig. 1. These are the straight-through part of the incident beam G_1 at $\omega + \delta$. Reflection off the second mirror generates another incident beam at $\omega + \delta$. Phase conjugation with frequency flipping at the nonlinear medium generate the beam G_3 at $\omega - \delta$. Using Eq. (21), we obtain

$$
\begin{aligned}
G_1 &= \frac{1}{t_2 t_1 t_1^*}[F_{11}E_1 + F_{12}E_2^* + F_{14}E_4], \\
G_3 &= \frac{1}{t_2 t_1 t_1^*}[F_{31}E_1^* + F_{32}E_2 + E_{34}E_4^*]. \qquad (28)
\end{aligned}
$$

Substituting Eqs. (26) and (27) for E_2 and E_4, respectively, into Eqs. (28), we obtain the expressions for the two transmission coefficients:

$$
\begin{aligned}
t_s &= \frac{G_1}{E_1} = \frac{1}{t_2 t_1 t_1^*}[F_{11} + F_{12}r_p^* + F_{14}r_s^*], \\
t_p &= \frac{G_3}{E_1^*} = \frac{1}{t_2 t_1 t_1^*}[F_{31} + F_{32}r_p + F_{34}r_s], \qquad (29)
\end{aligned}
$$

while the power-transmission coefficients are given by $T_s = |t_s|^2$ and $T_p = |t_p|^2$.

The four reflection and transmission coefficients derived above for one input wave E_1 and $\omega + \delta$ are useful for studying the oscillation conditions for various types of phase-conjugate oscillator. The analysis for a single input wave at $\omega - \delta$ is similar and may be obtained from our general formulation by taking E_3 as the input wave and $E_1 = G_2 = G_4 = 0$.

We are now ready to investigate three special cases of great interest. These are the following:

(i) No conventional mirrors ($r_1 = r_2 = 0$), so that the phase-conjugate oscillator reduces to a phase-conjugate mirror.

(ii) Only one conventional mirror ($r_1 = 0$), so that the phase-conjugate oscillator reduces to a phase-conjugate resonator, i.e., a resonator bounded by a conventional mirror and a phase-conjugate mirror.

(iii) Both conventional mirrors present ($r_1, r_2 \neq 0$), which is the phase-conjugate oscillator.

In each of the cases, we will consider four different operation conditions: (1) $\Delta k = 0$, $g = 0$, $|\kappa_1| = |\kappa_2| = |\kappa| \neq 0$, i.e., degenerate four-wave mixing without linear absorption/gain in the medium. (2) $\Delta k = 0$, $g \neq 0$, $|\kappa_1| = |\kappa_2| = |\kappa| \neq 0$, i.e., degenerate four-wave mixing with linear absorption/gain in the medium. (3) $\Delta k \neq 0$, $g = 0$, $|\kappa_1{}^*\kappa_2| > 0$, i.e., nondegenerate four-wave mixing in the absence of linear absorption/gain in the medium. (4) $\Delta k \neq 0$, $g \neq 0$, $|\kappa_1{}^*\kappa_2| > 0$, i.e., nondegenerate four-wave mixing with linear absorption/gain in the medium. In this paper we discuss only case (i). Cases (ii) and (iii) will be discussed in a subsequent paper.

PHASE-CONJUGATE OSCILLATORS WITHOUT CONVENTIONAL MIRRORS

In this section we set $r_1 = r_2 = 0$. The frequency of the input wave E_1 is $\omega + \delta$. In this case the problem then reduces to the standard nondegenerate four-wave mixing in a transparent medium,[12–14] which is characterized by a linear gain or absorption in addition to the parametric gain. We will show that the general theory developed in this paper yields the results of previous studies.[12–14]

From Eqs. (26), (22), (16), (18), and (12), the amplitudes of the reflected wave at the input plane ($z = 0$) can be written as

$$r_p = -\frac{2ik_2{}^* \sinh \frac{s}{2} l}{(-g - i\Delta k) \sinh \frac{s}{2} l + s \cosh \frac{s}{2} l},$$
$$r_s = 0. \tag{30}$$

Thus, in the absence of the conventional mirrors, there is no coherently reflected wave at $\omega + \delta$, only the phase-conjugated beam at $\omega - \delta$ is reflected by he nonlinear medium. The transmitted waves at output plane ($z = l$) are

$$t_p = 0,$$
$$t_s = \frac{s^*}{(-g + i\Delta k) \sinh \frac{s^*}{2} l + s^* \cosh \frac{s^*}{2} l} \times \exp[-i(k_1 + k_2)l/2]. \tag{31}$$

Thus, in the absence of the conventional mirrors, there is only one transmitted beam at $\omega + \delta$. We note that when $r_1 = r_2 = 0$ and $a = b = 0$, then $E_i = A_i(0)$ and $G_i = A_i(l)$, where $i = 1, 2, 3, 4$. If we define complex amplitude transmission as $t_s' = \mathscr{A}_1(l)/\mathscr{A}_1(0)$, then by Eq. (1) $A_1(l) = \mathscr{A}_1(l)\exp(-ik_1 l)$ and $A_1(0) = \mathscr{A}_1(0)$; hence by Eqs. (29) we obtain $t_s' = t_s \exp(ik_1 l)$, and when this equation is substituted into Eqs. (31) we get

$$t_s' = \frac{s^*}{(-g + i\Delta k) \sinh \frac{s^*}{2} l + s^* \cosh \frac{s^*}{2} l} \exp(i\Delta k l/2). \tag{32}$$

With $r_1 = r_2 = 0$, the oscillation condition [Eq. (24)] becomes $N'N^* = 0$. Substituting Eqs. (16), (18), and (12) into Eq. (24), we obtain the following oscillation condition:

$$D = (-g - i\Delta k)\sinh \frac{s}{2} l + s \cosh \frac{s}{2} l = 0. \tag{33}$$

Note that, at oscillation, r_p and t_s approach infinity according to Eqs. (30), (31), and (33). We now consider the four different operation situations and compare our results with previous studies.

$\Delta k = 0, g = 0, \kappa_1 = \kappa_2 = \kappa$ This is the case of degenerate four-wave mixing in a transparent medium without linear gain or absorption.[14]

Under these conditions the oscillation is provided by the parametric gain. From Eqs. (13), (30), and (31) we obtain the phase-conjugate complex reflection coefficient and coherent transmission coefficient. They are

$$r_p = -i\frac{\kappa^*}{|\kappa|} \tan|\kappa|l,$$
$$t_s = \frac{1}{\cos|\kappa|l} \exp(-ikl). \tag{34}$$

The oscillation condition will now be $|\kappa|l = \pi/2$, $3\pi/2, \ldots$, etc. Similar results have been obtained by others.[14]

$\Delta k = 0, g \neq 0, \kappa_1 = \kappa_2 = \kappa$ This corresponds to degenerate four-wave mixing in a transparent medium that also exhibits linear gain or absorption.[12]

From Eqs. (13) and (32), the phase-conjugate complex reflection coefficient can be written as

$$r_p = -\frac{i\kappa^* \tan[|\kappa|^2 - (g/2)^2]^{1/2} l}{[|\kappa|^2 - (g/2)^2]^{1/2} - (g/2) \tan[|\kappa|^2 - (g/2)^2]^{1/2} l}. \tag{35}$$

According to Eq. (35), oscillation occurs when the following condition is satisfied:

$$\tan\{[|\kappa|^2 - (g/2)^2]^{1/2} l\} = \frac{2[|\kappa| - (g/2)^2]^{1/2}}{g}, \tag{36}$$

where $\kappa = (\omega/2)\sqrt{\mu/\varepsilon}\, \chi^{(3)} A_5{}^* A_6{}^* e^{gl/2} = \kappa' e^{gl/2}$. Equations (35) and (40) agree formally with the results derived in Ref. 12, except that they have considered linear absorption only. Thus, if we replace g with $-\alpha$, we will obtain exactly the same result as in Ref. 12.

Using Eq. (36), in Fig. 2 we plot the parametric gain $|\kappa'|l$ versus linear gain gl at the oscillation conditions to show

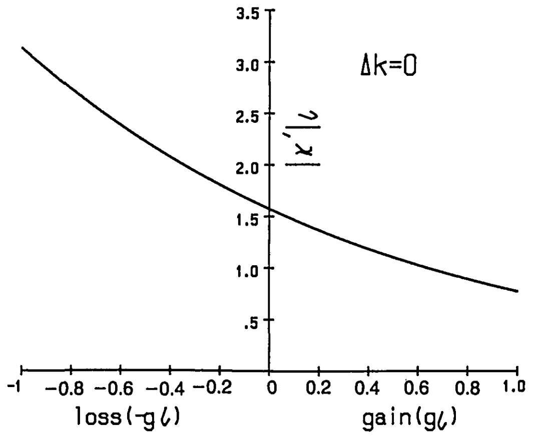

Fig. 2. Parametric gain $|\kappa' l|$ versus linear gain gl at the oscillation condition $\kappa = \kappa' e^{gl/2}$.

the effect of gain (or loss) on the coupling constant $|\kappa'|l$ for degenerate four-wave mixing. The figure shows that the parametric gain required for oscillation is considerably increased (decreased) owing to linear absorption (gain) in the medium.

$\Delta k \neq 0,\ g = 0,\ \kappa_1\kappa_2^* > 0$ This is the case of degenerate four-wave mixing in a transparent medium without linear absorption or gain.[15]

Substituting $g = 0$ into Eqs. (30), we obtain the phase-conjugate reflection coefficient:

$$r_p = \frac{-i\kappa_2^* \tan\{[\kappa_1\kappa_2^* + (\Delta k/2)^2]^{1/2} l\}}{[\kappa_1\kappa_2^* + (\Delta k/2)^2]^{1/2} - i(\Delta k/2)\tan\{[\kappa_1\kappa_2^* + (\Delta k/2)^2]^{1/2} l\}}, \tag{37}$$

which is identical to the result of Ref. 15. According to Eq. (37), oscillation occurs only when $\Delta k = 0$ and $\sqrt{\kappa_1\kappa_2^*}\, l = \pi/2, 3\pi/2, \ldots,$ etc., so that nondegenerate oscillation due to four-wave mixing is not possible in a transparent Kerr medium. Oscillation with no input wave will occur only at the pump frequency.

$\Delta k \neq 0,\ g \neq 0,\ |\kappa_1\kappa_2^*| > 0$ This is the case of degenerate four-wave mixing in a transparent medium that exhibits linear absorption–gain and a parametric gain. This is the first time to our knowledge that the effects of nonsaturable background losses or gain in the transparent medium on phase conjugation by degenerate four-wave mixing have been studied.

By Eqs. (12), (30), and (31), the phase-conjugate complex reflection and coherent transmission coefficients are

$$r_p = \frac{-i\kappa_2^* \tan\{[\kappa_1\kappa_2^* + (\Delta k - ig)^2/4]^{1/2} l\}}{[\kappa_1\kappa_2^* + (\Delta k - ig)^2/4]^{1/2} - \dfrac{i(\Delta k - ig)}{2}\tan\{[\kappa_1\kappa_2^* + (\Delta k - ig)^2/4]^{1/2} l\}},$$

$$t_s = \frac{[\kappa_1^*\kappa_2 + (\Delta k + ig)^2/4]^{1/2} \sec\{[\kappa_1^*\kappa_2 + (\Delta k + ig)^2/4]^{1/2} l\}}{[\kappa_1^*\kappa_2 + (\Delta k + ig)^2/4]^{1/2} + \dfrac{i(\Delta k + ig)}{2}\tan\{[\kappa_1^*\kappa_2 + (\Delta k + ig)^2/4]^{1/2} l\}} \exp[-i(k_1 + k_2)l/2]. \tag{38}$$

The oscillation condition can be obtained by either setting the denominators to zero in Eqs. (38) or simply using Eq. (33). With $s = u + iv$, Eq. (33) can be written as

$$D = \left[-\sinh\frac{u}{2}l\left(g\cos\frac{v}{2}l + v\sin\frac{v}{2}l\right) + \cosh\frac{u}{2}l\left(\Delta k \sin\frac{v}{2}l + u\cos\frac{v}{2}l\right)\right] + i\left[-\cosh\frac{u}{2}l\left(g\sin\frac{v}{2}l - v\cos\frac{v}{2}l\right) + \sinh\frac{u}{2}l\left(u\sin\frac{v}{2}l - \Delta k\cos\frac{v}{2}l\right)\right] = 0. \tag{39}$$

By setting the real and the imaginary parts of the denominator separately equal to zero, we obtain two simultaneous nonlinear equations involving three dimensionless variables: gl, $\kappa' l$, and Δkl, where $\kappa' = \sqrt{\kappa_1'\kappa_2^*}$. If we set Δkl as the independent variable, then, by using Brown's method to solve the two nonlinear equations, we obtain multiple-valued solutions for gl and $\kappa' l$. Note that when $\Delta kl = 0$, the imaginary part of D is equal to zero, and the real part of D reduces to Eq. (36). Using Eq. (36), we find that for oscillation at the pump frequency the parametric gain required is $\kappa' l = 3.13824, 1.57080, 0.76250$ for a linear absorption–gain of $gl = -1, 0, 1$, respectively. Nondegenerate oscillation is not possible for these sets of parameter values. Figures 3 and 4 show the phase-conjugate power reflectivity R_p and the coherent power transmissivity T_s, respectively, versus normalized wavelength detuning Ψ for three values of $gl = 0, \pm 1$ at oscillation condition. By definition, $\Psi = (\Delta\lambda/2)(2nl/\lambda^2)$, which is also equal to the phase mismatch Δkl divided by 2π. The wavelength-detuning parameter $\Delta\lambda/2$ corresponds to the difference in wavelengths of the probe field E_1 relative to the pump fields $A_{5,6}$. These two figures show that linear absorption losses in the medium substantially increase the threshold value of $\kappa' l$ for which oscillation will occur at the pump frequency. If the medium were somehow to exhibit linear gain instead of absorption, then the threshold value of the coupling strength would be correspondingly lowered owing to the additional gain then available from medium.

Using Eqs. (38), we plot in Fig. 5 the power-reflection coefficient R_p versus a normalized wavelength-detuning parameter Ψ for $|\kappa'|l = \pi/2$ and several values of the linear gain gl. For finite g, oscillation ceases to occur at $|\kappa'|l = \pi/2$, but it occurs at higher (lower) values for linear absorption (gain) in the medium.

Figures 6 and 7 are the normalized phase-conjugate power reflectivity and normalized coherent power transmissivity, respectively, versus normalized wavelength detuning Ψ. They show the effects of linear gain or absorption on the wavelength response for the filter application. Several prominent features should be noted. First,

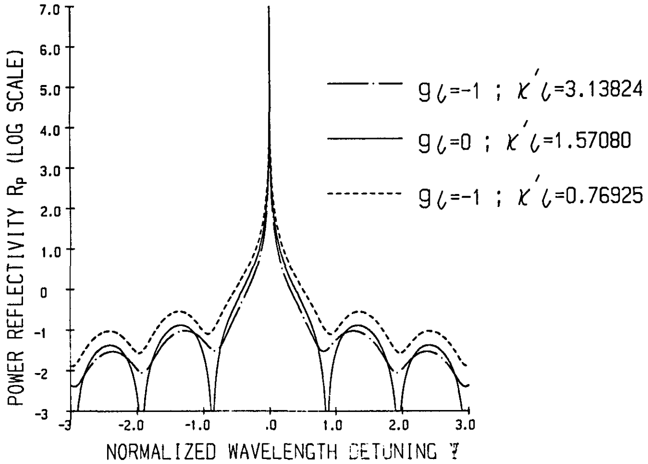

Fig. 3. Phase-conjugate power reflectivity R_p versus normalized wavelength detuning Ψ for several values of linear gain $gl = 0$, ± 1 when $|\kappa' l|$ satisfies the oscillation condition. For the example given in the text, unity along the abscissa corresponds to $\Delta\lambda/2 = 0.0772$ Å, or $\Delta\nu = 9.29$ GHz.

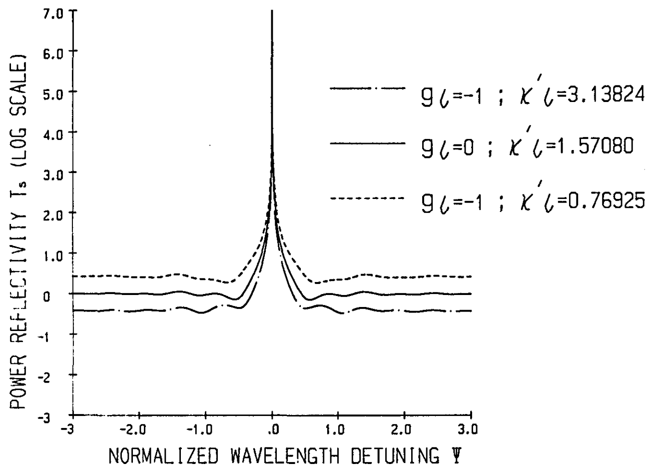

Fig. 4. Coherent power transmissivity R_s versus normalized wavelength detuning Ψ for several values of linear gain $gl = 0$, ± 1 when $|\kappa'|l$ satisfies the oscillation condition. For the example given in the text, unity along the abscissa corresponds to $\Delta\lambda/2 = 0.0772$ Å, or $\Delta\nu = 9.26$ GHz.

when $|gl|$ is less than 0.1, the effect on the filter of gain ($gl > 0$) or loss ($gl < 0$) on the filter characteristic is negligible. Second, larger linear gain (or loss) degrades the filter characteristics. Third, the filter characteristics of the phase-conjugate reflection are better than those of transmission for finite gain (or loss).

In practice, linear gain of the nonlinear medium depends on the pumping source. Using Eqs. (38), we plot in Figs. 8 and 9 the linear gain (or loss) gl versus normalized wavelength detuning Ψ for constant reflectances and transmittances, respectively, when $|\kappa'|l = \pi/2$. We recall that $\kappa = (\omega/2)\sqrt{\mu/\varepsilon}\,\chi^{(3)} A_5{}^* A_6{}^* e^{gl/2} = \kappa' e^{lh/2}$. In these figures, we plot only the minimum absolute linear gain $|g|l$ versus normalized wavelength detuning Ψ, because gl is a multiple-valued function of Δk for constant reflectances or transmittances in Eqs. (38).

If we increase the absolute value of $|gl|$ above 2, then nondegenerate oscillation at a frequency different from that of the pumps becomes possible. Figure 10 shows the solution of $D = 0$ for gl and $\kappa' l$ versus normalized wavelength detuning Ψ. Because the solution is multiple valued, it is possible to have many pairs of gl and $\kappa' l$ values for a particular value of Ψ. In this figure, the curve pair 3 (shown as a dashed curve) for $\kappa' l$ is not shown because it is greater than 1.8. The curve pair 1 (shown the solid curve) shows that, for example, if linear gain gl is increased to 4.32152, then one can decrease the parametric gain $\kappa' l$ to 0.13281 in order to observe nondegenerate

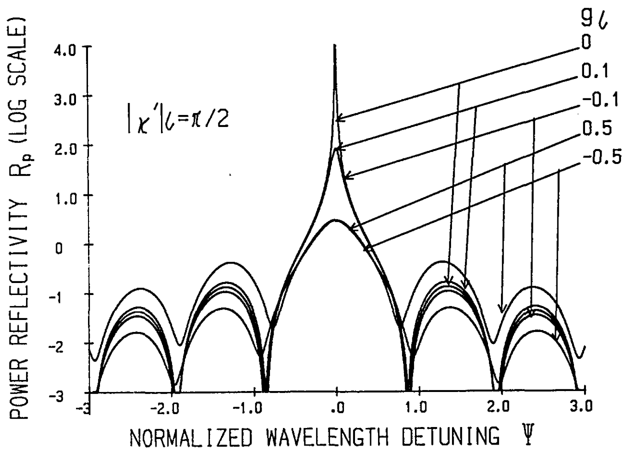

Fig. 5. Power-reflection coefficient R_p versus a normalized wavelength-detuning parameter Ψ for $|\kappa'|l = \pi/2$ and several values of the linear gain gl. For the example given in the text, unity along the abscissa corresponds to $\Delta\lambda/2 = 0.0772$ Å, or $\Delta\nu = 9.26$ GHz.

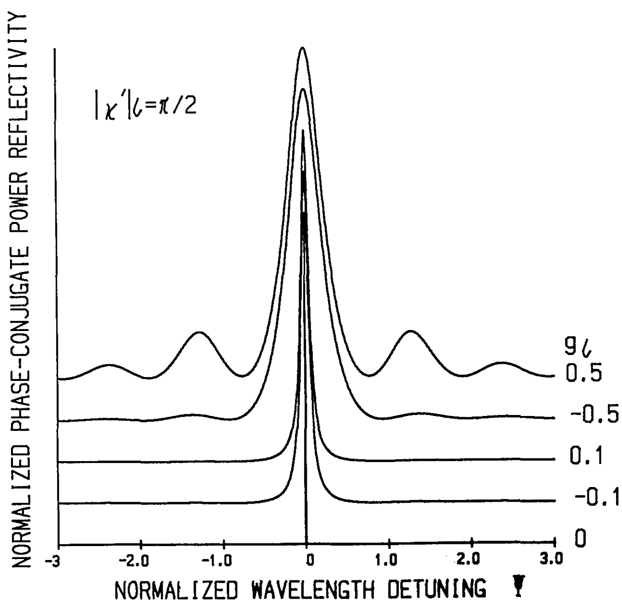

Fig. 6. Normalized phase-conjugate power reflectivity versus normalized wavelength detuning Ψ for parametric gain $|\kappa|l = \pi/2$ and several values of linear gain gl. All curves are normalized to unity power reflectivity to emphasize the frequency bandpass of the interaction.

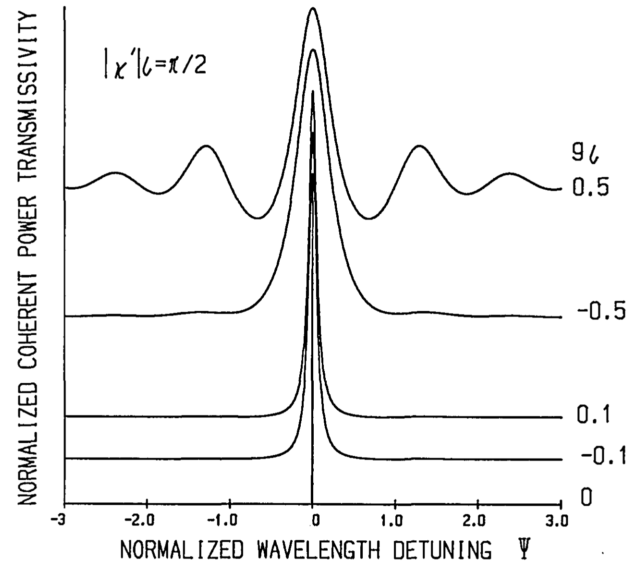

Fig. 7. Normalized coherent power transmissivity versus normalized wavelength detuning Ψ for parametric gain $|\kappa|l = \pi/2$ and several values of linear gain gl. All curves are normalized to unity power transmission to emphasize the frequency bandpass of the interaction.

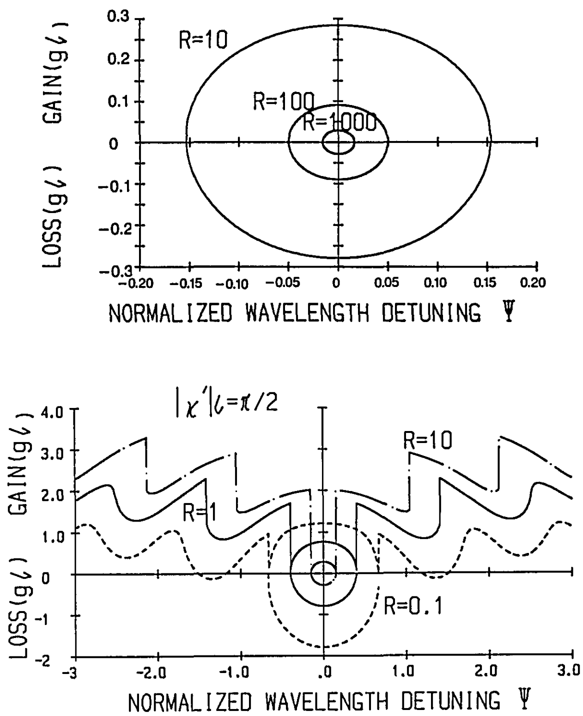

Fig. 8. Contours of equal reflectance for $|\kappa'|l = \pi/2$ on the linear gain (or loss) versus normalized wavelength-detuning plane. For the example given in the text, unity along the abscissa corresponds to $\Delta\lambda/2 = 0.0772$ Å, or $\Delta\nu = 9.26$ GHz.

oscillation at $|\Psi| = 1.31$. Figure 11 is a plot of R_p and T_s versus the normalized wavelength detuning for $gl = 4.32152$; $\kappa'l = 0.13281$, showing the possibility of observing oscillation at a frequency different from that of the pump beam for this set of parameter values.

CONCLUSION

In conclusion, we have treated the generalized theory of the propagation of electromagnetic radiation in phase-conjugate oscillators. Wavelength detuning and linear and parametric gain are all taken into account. Phase-conjugate power reflectivity and transmissivity, coherent power reflectivity and transmissivity, and the oscillation condition are derived. We have studied the special case of no conventional mirrors by using this general theory and compared our results with those of previous studies. Our

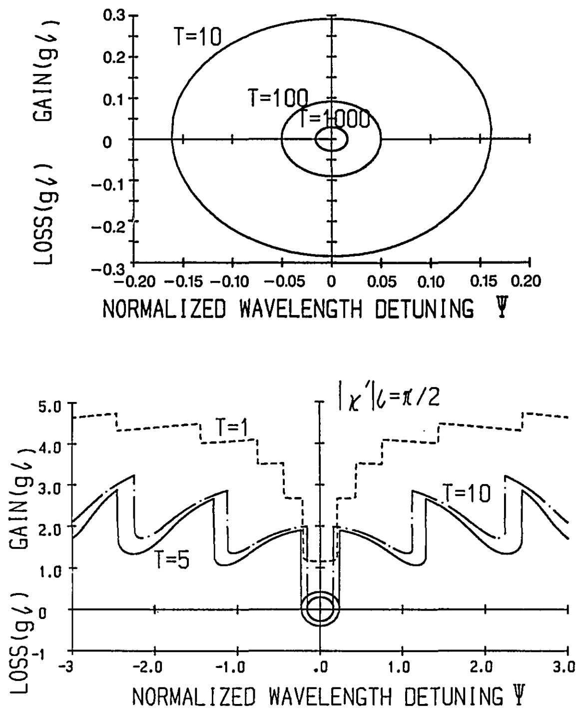

Fig. 9. Contours of equal transmittance for $|\kappa'|l = \pi/2$ on the linear gain (or loss) versus normalized wavelength-detuning plane. For the example given in the text, unity along the abscissa corresponds to $\Delta\lambda/2 = 0.0772$ Å, or $\Delta\nu = 9.26$ GHz.

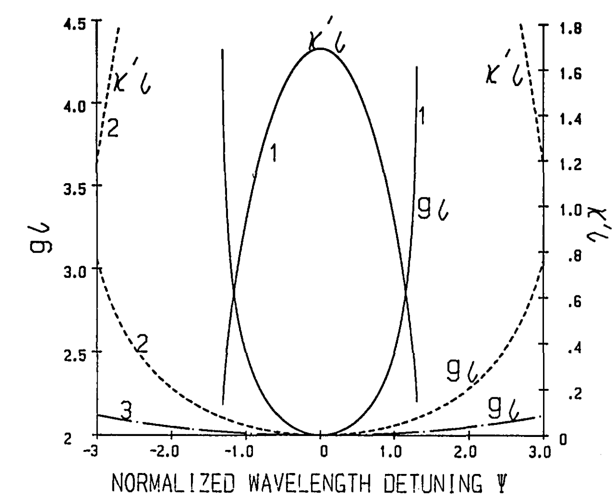

Fig. 10. Parametric gain $\kappa'l$ and linear gain gl versus normalized wavelength detuning at the oscillation condition.

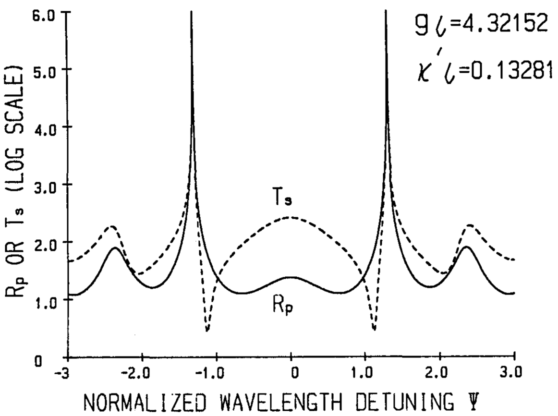

Fig. 11. Phase-conjugate power reflectivity R_p (solid curve) and coherent-power transmissivity T_s (dashed curve) versus normalized wavelength detuning Ψ for $gl = 4.32152$, $\kappa' l = 0.13281$.

results indicate that in the presence of large linear gain (or loss), oscillation can occur at a frequency different from that of the pump beams.

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