國 立 交 通 大 學

電機工程學系

博 士 論 文

合作式放大傳遞多輸入多輸出中繼系統之線性與非線性傳收機設計

Linear and Nonlinear Transceiver Designs in Amplify-and-Forward MIMO Relay Systems

研 究 生：曾凡碩

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摘要

三節點放大傳遞多輸入多輸出中繼系統之傳收機設計包含兩種通訊鏈結 - 直接鏈結(direct link)與中繼鏈結(relay link)，及兩種前置編碼器 – 來源端前置編碼(source precoder)與中繼端前置編碼(relay precoder)。在這種系統中，大部分的傳收機設計都只設計中繼前置編碼器，有些設計甚至只考量中繼鏈結以簡化最佳化過程。在本論文中，我們提出新穎的線性與非線性傳收機設計，其中來源端前置編碼與中繼端前置編碼是根據直接鏈結與中繼鏈結的通道資訊，合併最佳化設計。在本論文所探討的傳收機中，中繼端前置編碼為線性，傳送端前置編碼與接收機可為線性或非線性。具體而言，我們考量四種傳收機設計，第一種為線性來源端前置編碼、線性中繼端前置編碼與最小均方錯誤(minimum mean-squared error)接收機。第二種考量非線性來源端前置編碼、線性中繼端前置編碼與最小均方錯誤接收機之傳收設計，第三種為線性來源端前置編碼、線性中繼端前置編碼與非線性 QR 干擾消除接收機(successive-interference-cancellation)傳收設計，最

後一種為線性來源端前置編碼、線性中繼端前置編碼與非線性最小均方錯誤干擾消除接收機傳收設計。在所有考量的傳收機設計中，不管是線性或非線性，困難點在於其最佳化的成本函數為來源端前置編碼與中繼端前置編碼的非線性函數，並且為非凹曲線最佳化(convex optimization)，為了克服這個設計上的困難，我們提出不同以往的前置編碼結構與設計方法，使得原本的傳收機設計可轉換為凹曲線最佳化問題，因此可推導出解析解。最後，本論文進一步探討以服務品質(quality-of-service)觀點的線性來源端前置編碼、線性中繼端前置編碼與線性最小均方錯誤接收機之傳收設計，並延伸前述所提及之設計方式，提出此問題的解析解，由模擬結果得知，相較於其他現有方法，所提出的傳收機架構有較好的效能表現。

Linear and Nonlinear Transceiver Designs in Amplify-and-Forward MIMO Relay Systems

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Abstract

The transceiver design in three-node amplify-and-forward (AF) multiple-input multiple-output (MIMO) relay systems involve two links, the direct and relay links, and two precoders, the source and relay precoders. Most existing methods only consider the design with a relay precoder, and some even ignore the direct link. In this dissertation, we propose new linear/nonlinear transceiver design methods taking the direct and relay links into account, and jointly optimizing the source and relay precoders. In our designs, the relay precoder is linear, and the source precoder and the receiver can be linear or nonlinear. Specifically, four scenarios are considered. The first is the design with a linear source precoder and a liner minimum-mean-square-error (MMSE) receiver, the second a nonlinear Tomlinson-Harashima source precoder and a linear MMSE receiver, the third a linear source precoder and a nonlinear QR successive-interference-cancelation (QR-SIC)

receiver, and the fourth a linear source precoder and a nonlinear MMSE-SIC receiver. All the designs, either the linear or nonlinear precoded systems, are difficult since the cost functions to be optimized are highly nonlinear functions of the source and relay precoders. Yet, the corresponding optimization problems are not convex. To overcome the difficulties, we propose new precoder architectures and methods such that the design problems can be translated into scalar-valued and convex optimization problems. And, the closed-form solutions can be obtained by the corresponding Karush-Kuhn Tucker (KKT) conditions. Finally, we consider the precoders design with quality-of-service (QoS) constraints. In the scenario, the linear precoder is used at the source and the MMSE receiver at the destination. Again, this problem is difficult and the optimization problem is not convex. We then extend the method proposed for the systems mentioned above to derive a closed-form solution. Simulation results show that the performance of the proposed transceiver design methods is significantly better than that of existing methods.

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Chapter 1

Introduction

Diversity is a commonly used technique to overcome the multipath channel fading effect in wireless communications. Existing diversity schemes include time diversity, frequency diversity, and spatial diversity. Among these schemes, the spatial diversity is particularly attractive. This is because it can combine with the other two diversity techniques with no time or bandwidth expansion [1]- [3]. The conventional way to obtain the spatial diversity is the use of multiple transmit or multiple receive antennas. When both multiple transmit and receive antennas are used, the system is referred to as a multiple-input multiple-output (MIMO) system [1]- [21]. The MIMO system has been widely studied in the literature since it can enhance the diversity or spectral efficiency in an efficient way, [5]- [21]. However, due to shadowing, multipath fading, interference, and distance-dependent path losses, the link quality between the source and the destination in a wireless network may not be always good enough for reliable communication. The fundamentally linking problem greatly affects the transmission in wireless systems.

Recently, cooperative communication has been garnered great interest. In cooperative systems, relays at some strong shadowing areas are deployed such that the signal from the source can be transmitted to the destination by the source-to-destination link (direct link) and the source-relay-destination links (relay links). With the additional relay links, the channel quality can be effectively improved, and the spatial diversity is implemented in a distributed way, referred to as distributed spatial diversity [22]- [49]. Various relay protocols have been proposed including amplify-and-forward (AF), decode-and-forward (DF), and compress-and-forward (CF) [22] [37]- [38]. In AF, the relays receive the signal from the source and retransmit it to the destination with signal amplification only. Such a system is also called a non-regenerate cooperative system [41]- [43], [45]. In DF, the relays decode the received signals, re-encode the information bits, and then retransmit the resultant signals to the destination. The system is also called a regenerative cooperative system. One problem associated with the DF is that the decision errors can occur in the relays. The CF is a compromise structure between AF and DF where the received signals at the relays are estimated and compressed, and then re-transmitted to the destination. It is simple to see that the DF protocol requires a higher computational complexity and a larger processing delay at the relay nodes. In this dissertation, we only consider the AF-based cooperative system.

Recently, the MIMO technique was introduced to cooperative systems as a means for further performance enhancement. With the multiple antennas equipped at each node, a MIMO relay system is constructed [39]- [46]. Capacity bounds for a single-relay MIMO channel was first addressed in [39]. Similar to conventional MIMO systems, the precoding operation can be conducted in a MIMO relay system. For the MIMO relay systems, the relay precoder with AF protocol was first designed in [41]- [42] to enhance overall channel capacity. In most of those approaches, only the relay link is considered. It was shown that the capacity can be further increased if the direct link is taken into account [42]. Apart from the capacity, the link quality is another criterion has been considered. As well known, the precoder design is a transceiver design problem which means a specific precoder is designed for a specific receiver. In [43]- [44], a relay precoder was designed for a minimum mean-square error (MMSE) receiver. Precoding in multiple-relay MIMO systems was investigated in [44]. Note that the above works all address the spatial multiplexing scenario. Recently, the design for the transmission of a single data stream, referred to as beamforming, was also considered. For example, [46] derived the optimum source and relay beamformers using a maximum signal-to-noise-ratio (SNR) criterion. In the work, the optimum solution was derived for the relay-link-only system. In addition to beamforming, antenna selection in MIMO relay systems was also studied. With the MMSE criterion, an optimum selection scheme was developed in [45]. In this approach, only one antenna is selected at the source and the relay, respectively, for signal transmission.

As mentioned, in the precoders design for spatial multiplexing AF MIMO relay systems, the existing works only consider the precoder at the relay. Also, the direct link is frequently ignored [41]- [44]. In this dissertation, we consider the transceiver designs in three-node AF MIMO relay systems taking the direct and relay links into consideration, and jointly optimizing the source and relay precoders. Since the relay only amplifies its receive signal, a linear precoder is used in our study. We first consider the linear transceiver design where the linear precoder is used at the source and the MMSE receiver at the destination. The MMSE criterion is also used in [43]- [44]; however, only the relay precoder at the relay link is considered. With our formulation, it is found that the MMSE is a complicated function of precoding matrices, and a direct minimization is almost not possible to conduct. To overcome the difficulty, we pose some structural constraints on the precoders so as to diagonalize the MSE matrix in the cost function. With the precoders, we can then derive an MSE upper bound. Minimization with this upper bound, instead of the original MMSE, then becomes feasible. The proposed precoders can finally be computed via an iterative water-filling technique. Note that the MSE criterion to minimize is the total MSE of the multiplexed signal streams. With the specially designed structure, the proposed precoders can make the individual MSEs of all signal streams equal, indicating that the bit-error-rate (BER) of the proposed precoded system will be the minimal among all precoded systems with the same minimum total MSE [8].

To enhance the performance of the precoded system, we then use the nonlinear Tomlinson-Harashima precoder (THP) at the source and a MMSE receiver at the destination. As that in the linearly precoded MMSE system, the cost function is a highly nonlinear function of the source and relay precoders. Since the nonlinear THP is involved, the optimization problem becomes more difficult. Even with the numerical method [51], finding the optimum solution is not a simple task. To overcome the problem, we propose to cascade a unitary precoder with the THP. The unitary precoder can not only simplify the optimization problem but also improve the MMSE performance. With the specially designed unitary precoder at the source, the primal decomposition approach [51], decomposing the original optimization problem into a master and a subproblem optimization problems, can be applied. The optimum source precoder in the subproblem can thus be derived as a function of the relay precoder. The problem is then similar to the precoding design in conventional MIMO systems [16] and the solution is readily obtained. The focus then becomes how to solve the master problem, in which the cost function is a function of the relay precoder only. Due to the nonlinear cost function, the relay precoder in the master optimization cannot be solved. We then propose an relay precoder structure and translate the relay optimization problem into a standard scalar-valued concave optimization problem. Using the Karush-Kuhn-Tucker (KKT) conditions, we then obtain a closed-form solution for the relay and source precoders.

An alternative to enhance system performance is to use a nonlinear receiver at the destination. We then consider the system with a linear precoder at the source and a nonlinear QR successive-interference-cancelation (QR-SIC) or the MMSE-SIC receiver at the destination. In the transceiver design, the most desirable criterion to minimize is the BER. However, it is generally acknowledged that a design minimizing BER is difficult to obtain. As an alternative, in this dissertation we propose to use the block-error-rate (BLER) instead of the BER as the design criterion. For a MIMO system with a QR-SIC receiver, the precoder which can minimize the BLER has been solved by the geometric mean decomposition (GMD) technique [6],[7]. We then extend its use in AF MIMO relay systems to design the source and relay precoders, jointly. Although the AF MIMO relay system can be formulated as a general MIMO system and the GMD criterion can be easily applied, the cost function is a highly nonlinear function of the source and relay precoders. A direct optimization of such a function turns out to be infeasible. Fortunately, the GMD method allows us to express the source precoder as the function of the relay precoder. As a result, the two-precoder design problem can be reduced to a single-precoder problem. Similar to the approach used in the THP precoded system, we apply the primal decomposition to translate the problem to a standard scalar convex optimization problem. The closed-form solutions for the source and relay precoders can then be obtained. It is noted here that GMD can also maximize a lower bound of channel's free distance [12]. As known, the free distance is the metric used in the maximum likelihood detector (MLD). So, it is expected that the proposed precoders can also improve the performance of the MLD [1]. As well known, a precoded MIMO system with the MMSE-SIC receiver outperforms that with QR-SIC receiver. We can assert that the same result can be obtained for MIMO relay systems. For MIMO systems, the precoder with the MMSE-SIC receiver can be solved by the uniform channel decomposition (UCD) method. However, the UCD is not directly applicable in AF MIMO relay systems. We show that if the source precoder is constrained to be unitary, the problem can be easily overcome. Using the UCD, we can jointly design the source and relay precoders such that the signal-to-interference-plus-noise ratio (SINR) for each layer is equal and maximized. As a result, the BLER can be minimized.

So far, all precoded systems we described are designed to improve the link performance [41]- [46]. The constraints posed on the designs are the source and relay power. In many applications, however, quality-of-service (QoS) may be more critical. For instance, a multimedia system providing high quality video service may require constraints on BER and processing delay. Precoded AF MIMO relay systems with QoS constraints have been investigated in the literature [48]- [49]. In [48], the precoders were designed to asymptotically satisfy the QoS constraints. In such a system, the direct link was ignored and only the relay precoders were considered. Alternatively, [49] addresses the similar problem in the multi-user scenario in which each user is equipped with one antenna and the direct link was ignored. As far as we

[1]The GMD method is asymptotically optimal for high SNR [11], in terms of both channel throughput and BER performance. The optimal design here means that the precoder design does not need tradeoffs between the throughput and BER

known, the joint source and relay precoders design in the AF MIMO relay system satisfying QoS constraints has not been studied before [2]. In the last part of this dissertation, we aim to study the problem. As that in the previous parts, we take the both the source and relay precoders into consideration. We use a linear precoder at the source and a linear MMSE receiver at the destination. Since there is a one-to-one mapping between the BER and the MSE, we use the MSEs of signal streams as our QoS constraints. Similar to previous cases, the optimum solution here is difficult to derive. To overcome the problem, we first consider the two-hop system and propose the precoder structures that can simplify the design problem and lead to a closed-form solution. In general MIMO relay systems, the problem becomes much more involved. The precoder structures, however, enable us to derive an MSE upper bound. Using the upper bound as the constraint function, we can translate the original matrix-valued optimization problem into a standard scalar-valued optimization problem. A solution can then be solved by the primal decomposition [51] and corresponding KKT conditions. From the solution, we further provide a sufficient condition to determine if the system is proper to be operated in the cooperative mode or not.

This dissertation is organized as follows. In Chapter 2, we consider the system with a linear precoder at the source, a liner precoder at the relay, and an MMSE transceiver at the destination. In Chapter 3, we consider the same system except that the linear precoder at the source is replaced by the THP. In Chapter 4, we consider the system with a linear precoder at the source, a linear precoder at the relay, and a nonlinear QR-SIC receiver at the destination. In Chapter 5, we consider the same system as that in Chapter 4 except that the QR-SIC receiver is replaced by the MMSE-SIC receiver. In Chapter 6, we consider a precoded system with QoS constraints. The linear precoders are used at the source and the relay, and the MMSE receiver is adopted at the destination. Finally, we draw conclusions in Chapter 7.

[2] Note that the relay precoder in [48] is designed to asymptotically satisfy its QoS constraints. The multi-user system in [49] only uses one antenna for each user, and thus the equalization is not required at the receiver. The cooperative systems are basically different from those we consider.

Chapter 2

Joint MMSE Transceiver Design with Linear Source and Relay Precoders

In this chapter, we consider a precoded AF MIMO relay system in which linear precoders are used at the source and the relay, and an MMSE receiver at the destination. In Section 2.1, we build the system model and derive the MMSE solution. It is found that the design problem is essentially an optimization problem, and the cost function, the MSE, is a complicated function of the source and the relay precoders. In Section 2.2, we propose a new method to solve the problem. The main idea of our method is to pose a structural constraint on the precoders so as to diagonalize the MSE matrix in the cost function. With the precoders, we can then derive an MSE upper bound. Minimization with this upper bound, instead of the original MSE, is much simpler. The proposed precoders can finally be computed via an iterative water-filling technique. In Section 2.3, we give some application examples demonstrating the effectiveness of the proposed method.

§ 2.1 System Model and Problem Formulation

§ 2.1.1 *MMSE Receier with Linear Source and Relay Precoders*

We consider a typical three-node half-duplex cooperative AF MIMO relay system where multiple antennas are placed at each node. Under this scenario, signals can be transmitted from the source to the destination, and from the source to the relay and then to the destination. To avoid the interference between the direct and relay links, we consider the time-division-duplexing scheme [41]- [44] used in a typical two-phase transmission mentioned above (See Fig. 2.1).

Let N, R, and M denote the number of antennas at the source, the relay, and the destination, and assume that all channels are flat-fading. For the first phase, the received signals at the destination and the relay can be expressed as

$$\mathbf{y}_{D,1} = \mathbf{H}_{SD}\mathbf{F}_S\mathbf{s} + \mathbf{n}_{D,1} \tag{2.1}$$

and

$$\mathbf{y}_R = \mathbf{H}_{SR}\mathbf{F}_S\mathbf{s} + \mathbf{n}_R, \tag{2.2}$$

respectively, where $\mathbf{s} \in \mathbb{C}^{L\times 1}$ is the transmitted signal vector with L being the number of the substreams, $\mathbf{F}_S \in \mathbb{C}^{N\times L}$ is the precoding matrix at the source, $\mathbf{H}_{SR} \in \mathbb{C}^{R\times N}$ and $\mathbf{H}_{SD} \in \mathbb{C}^{M\times N}$ are the channel matrices corresponding to the source-to-relay and source-to-destination channels, respectively; $\mathbf{n}_{D,1} \in \mathbb{C}^{M\times 1}$ is the first-phase received noise vector at the destination, and $\mathbf{n}_R \in \mathbb{C}^{R\times 1}$ is the received noise vector at the relay. Here, we assume that $L \leq min\{N, M\}$ to provide sufficient degrees of freedom for signal detection.

In the second phase of the transmission, the relay retransmits the received signal with another precoding matrix. Thus, the received signals at the destination can be expressed as

$$\mathbf{y}_{D,2} = \mathbf{H}_{RD}\mathbf{F}_R\mathbf{y}_R + \mathbf{n}_{D,2} = \mathbf{H}_{RD}\mathbf{F}_R\mathbf{H}_{SR}\mathbf{F}_S\mathbf{s} + \left(\mathbf{H}_{RD}\mathbf{F}_R\mathbf{n}_R + \mathbf{n}_{D,2}\right), \tag{2.3}$$

where $\mathbf{F}_R \in \mathbb{C}^{R\times R}$ is the precoding matrix at the relay, $\mathbf{H}_{RD} \in \mathbb{C}^{M\times R}$ is the channel matrix corresponding to the relay-to-destination channel, and $\mathbf{n}_{D,2} \in \mathbb{C}^{M\times 1}$ is the second-phase received noise vector at the destination. Here, we assume that each element in $\mathbf{n}_{D,1}$ has a zero-mean circularly symmetric Gaussian distribution, and all the elements are independent identically distributed (i.i.d.). The same assumption is applied for $\mathbf{n}_{D,2}$ and $\mathbf{n}_R$. As a result, the received signal vectors $\mathbf{y}_{D,1}$ and $\mathbf{y}_{D,2}$ for the two phases can be combined into a single vector, denoted as $\mathbf{y}_D \in \mathbb{C}^{2M\times 1}$. Consequently, we have

$$\mathbf{y}_D := \begin{bmatrix} \mathbf{y}_{D,1} \\ \mathbf{y}_{D,2} \end{bmatrix} = \mathbf{H}\mathbf{F}_S\mathbf{s} + \mathbf{n}, \tag{2.4}$$

where

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_{SD} \\ \mathbf{H}_{RD}\mathbf{F}_R\mathbf{H}_{SR} \end{bmatrix}, \quad \text{and} \quad \mathbf{n} = \begin{bmatrix} \mathbf{n}_{D,1} \\ \mathbf{H}_{RD}\mathbf{F}_R\mathbf{n}_R + \mathbf{n}_{D,2} \end{bmatrix}. \tag{2.5}$$

Here, $\mathbf{H}$ is the equivalent channel matrix with rank $(\mathbf{H}) = N$, and $\mathbf{n}$ is the equivalent noise vector at the destination. It is noteworthy that the noise received at the relay is amplified by the relay precoder and the relay-to-destination channel. Also, the equivalent channel matrix in (2.4) is a function of the relay precoder $\mathbf{F}_R$. This is quite different from the scenario considered in conventional MIMO systems. The precoders design problem actually is a joint transceiver design problem. In other words, the optimum precoders not only depends on the channels, but also the receiver. Similar to previous works, we will consider the linear MMSE receiver in our design [43], [44].

§ 2.1.2 *MMSE Receiver and Related MSE Matrix*

Let $\mathbf{R}_{\mathbf{n}_{D,1}} = E[\mathbf{n}_{D,1}\mathbf{n}_{D,1}^H] = \sigma_{n,d}^2\mathbf{I}_M$, $\mathbf{R}_{\mathbf{n}_{D,2}} = E[\mathbf{n}_{D,2}\mathbf{n}_{D,2}^H] = \sigma_{n,d}^2\mathbf{I}_M$, and $\mathbf{R}_R = E[\mathbf{n}_R\mathbf{n}_R^H] = \sigma_{n,r}^2\mathbf{I}_R$, where $\sigma_{n,d}^2$ and $\sigma_{n,r}^2$ are the noise variances at the destination and the relay, respectively. Also, the elements of the transmitted symbols are i.i.d. with zero-mean and a covariance matrix $\mathbf{R}_s = \sigma_s^2\mathbf{I}_L$, where σ_s^2 is the transmitted symbol power.

Using the setting, we can have the covariance matrix of the equivalent noise vector as

$$\mathbf{R}_n = E\left[\mathbf{n}\mathbf{n}^H\right] = \begin{bmatrix} \sigma_{n,d}^2\mathbf{I}_M & \mathbf{0} \\ \mathbf{0} & \sigma_{n,r}^2\mathbf{H}_{RD}\mathbf{F}_R\mathbf{F}_R^H\mathbf{H}_{RD}^H + \sigma_{n,d}^2\mathbf{I}_M \end{bmatrix}. \tag{2.6}$$

Let $\mathbf{G}$ be the equalization matrix in the receiver. Then, the MSE for recovering $\mathbf{s}$, denoted as J, is given by

$$J = E\left\{\|\mathbf{G}\mathbf{y}_D - \mathbf{s}\|^2\right\}. \tag{2.7}$$

Minimization of (2.7) leads to the optimal equalization matrix [7] as

$$\mathbf{G}_{opt} = \sigma_s^2 \mathbf{F}_S^H \mathbf{H}^H \left(\sigma_s^2 \mathbf{H}\mathbf{F}_S\mathbf{F}_S^H\mathbf{H}^H + \mathbf{R}_n\right)^{-1}, \tag{2.8}$$

Substituting (2.8) into (2.7) and invoking the matrix inversion lemma [50], we can then have the MMSE, denoted by J_{min}, as

$$J_{min} = tr\left\{\mathbf{E}\right\}, \tag{2.9}$$

where

$$\mathbf{E} = \left(\sigma_s^{-2}\mathbf{I}_L + \mathbf{E}_S + \mathbf{E}_R\right)^{-1}. \tag{2.10}$$

In (2.10),

$$\mathbf{E}_S = \sigma_{n,d}^{-2}\mathbf{F}_S^H\mathbf{H}_{SD}^H\mathbf{H}_{SD}\mathbf{F}_S \tag{2.11}$$

and

$$\mathbf{E}_R = \mathbf{F}_S^H\mathbf{H}_{SR}^H\mathbf{F}_R^H\mathbf{H}_{RD}^H\left(\sigma_{n,r}^2\mathbf{H}_{RD}\mathbf{F}_R\mathbf{F}_R^H\mathbf{H}_{RD}^H + \sigma_{n,d}^2\mathbf{I}_M\right)^{-1}\mathbf{H}_{RD}\mathbf{F}_R\mathbf{H}_{SR}\mathbf{F}_S. \tag{2.12}$$

As we can see from (2.10), the MMSE is a function of $\mathbf{F}_S$ and $\mathbf{F}_R$. It is also simple to see that $\mathbf{E}_S$ accounts for the MMSE contributed in the direct link and $\mathbf{E}_R$ for that in the relay link. If we ignore the direct link and only consider the relay precoder, the problem will be degenerated to the case considered in [43].

§ 2.1.3 *Problem Formulation*

As shown in (2.9), the MMSE is a function of the two precoding matrices, $\mathbf{F}_S$ and $\mathbf{F}_R$. Our task here is to design these two matrices such that the MSE in (2.9) can be minimized. The optimization problem can then be formulated as below.

$$
\begin{aligned}
&\min_{\mathbf{F}_S,\mathbf{F}_R} tr\{\mathbf{E}\} = \sum_{i=1}^{L} \mathbf{E}_{(i,i)} \\
&s.t. \\
&\mathbf{E} = \Bigg(\sigma_s^{-2}\mathbf{I}_L + \underbrace{\sigma_{n,d}^{-2}\mathbf{F}_S^H\mathbf{H}_{SD}^H\mathbf{H}_{SD}\mathbf{F}_S}_{:=\mathbf{E}_S} + \\
&\underbrace{\mathbf{F}_S^H\mathbf{H}_{SR}^H\mathbf{F}_R^H\mathbf{H}_{RD}^H\left(\sigma_{n,r}^2\mathbf{H}_{RD}\mathbf{F}_R\mathbf{F}_R^H\mathbf{H}_{RD}^H + \sigma_{n,d}^2\mathbf{I}_M\right)^{-1}\mathbf{H}_{RD}\mathbf{F}_R\mathbf{H}_{SR}\mathbf{F}_S}_{:=\mathbf{E}_R} \Bigg)^{-1} \\
&tr\left\{E\left[\mathbf{F}_R\mathbf{y}_R\mathbf{y}_R^H\mathbf{F}_R^H\right]\right\} = tr\left\{\mathbf{F}_R\left(\sigma_{n,r}^2\mathbf{I}_R + \sigma_s^2\mathbf{H}_{SR}\mathbf{F}_S\mathbf{F}_S^H\mathbf{H}_{SR}^H\right)\mathbf{F}_R^H\right\} \le P_{R,T} \\
&tr\left\{\mathbf{F}_S E\left[\mathbf{s}\mathbf{s}^H\right]\mathbf{F}_S^H\right\} = \sigma_s^2 tr\left\{\mathbf{F}_S\mathbf{F}_S^H\right\} \le P_{S,T}.
\end{aligned}
\tag{2.13}
$$

The inequalities in (2.13) indicate that the precoders have to satisfy the transmit power constraints both at the source and the relay where $P_{S,T}$ and $P_{R,T}$ denote the maximal available transmit power at the source and the relay, respectively.

From (2.13), we can readily find that (2.13) is not a convex optimization. Also, the cost function involves a series of matrix multiplications and inversions, it is a complicated and non-linear function of $\mathbf{F}_S$ and $\mathbf{F}_R$. The cost function may have many local minimums, and the optimal solution, even with numerical methods [51], is difficult to derive. We will propose a method, described below, to solve these problems.

§ 2.2 Joint Source/Relay Precoders Design

As mentioned above, the optimum solution for (2.13) is difficult to derive. In this subsection, we then propose a method to seek for a suboptimum solution. One difficulty in (2.13) is that the number of unknown parameters in $\mathbf{F}_R$ and $\mathbf{F}_S$ can be large. The first idea of our approach is to use a constrained precoder structure such that the number of unknowns can be effectively reduced. The other difficulty in (2.13) is that the formulae are too complicated to work with.

Our second idea is to derive an MMSE upper bound having a simple expression, and conduct minimization with this upper bound. Even though the cost function can be simplified dramatically with the proposed method, a closed-form solution is still difficult to obtain. We then use an iterative water-filling method to solve the problem.

§ 2.2.1 *Proposed Method*

When the direct link is ignored and only a relay precoder is considered, the optimum MMSE precoder can be analytically obtained through a MSE matrix diagonalization procedure [43]. Motivated by this fact, we propose to conduct a similar matrix diagonalization in our design. Indeed, if the error matrix $\mathbf{E}$ in (2.13) can be diagonalized, the trace operation can be easily conducted, and the whole problem can be greatly simplified. To do that, we firstly consider the following singular value decomposition (SVD) for the channel matrices in all links:

$$\mathbf{H}_{SD} = \mathbf{U}_{sd}\mathbf{\Sigma}_{sd}\mathbf{V}_{sd}^{H}; \tag{2.14}$$

$$\mathbf{H}_{SR} = \mathbf{U}_{sr}\mathbf{\Sigma}_{sr}\mathbf{V}_{sr}^{H}; \tag{2.15}$$

$$\mathbf{H}_{RD} = \mathbf{U}_{rd}\mathbf{\Sigma}_{rd}\mathbf{V}_{rd}^{H}, \tag{2.16}$$

where $\mathbf{U}_{sd} \in \mathbb{C}^{M\times M}$, $\mathbf{U}_{sr} \in \mathbb{C}^{R\times R}$, and $\mathbf{U}_{rd} \in \mathbb{C}^{M\times M}$ are the left singular matrices of $\mathbf{H}_{SD}$, $\mathbf{H}_{SR}$, and $\mathbf{H}_{RD}$, respectively; $\mathbf{\Sigma}_{sd} \in \mathbb{R}^{M\times N}$, $\mathbf{\Sigma}_{sr} \in \mathbb{R}^{R\times N}$, and $\mathbf{\Sigma}_{rd} \in \mathbb{R}^{M\times R}$, are diagonal singular-value matrices of $\mathbf{H}_{SD}$, $\mathbf{H}_{SR}$, and $\mathbf{H}_{RD}$, respectively; $\mathbf{V}_{sd}^{H} \in \mathbb{C}^{N\times N}$, $\mathbf{V}_{sr}^{H} \in \mathbb{C}^{N\times N}$, and $\mathbf{V}_{rd}^{H} \in \mathbb{C}^{R\times R}$ are the right singular matrices of $\mathbf{H}_{SD}$, $\mathbf{H}_{SR}$, and $\mathbf{H}_{RD}$, respectively.

Observing (2.13), we will readily find that a complete diagonalization of $\mathbf{E}$ will be difficult. We then first consider the diagonalization of $\left(\sigma_{n,r}^2\mathbf{H}_{RD}\mathbf{F}_R\mathbf{F}_R^H\ \mathbf{H}_{RD}^H + \sigma_{n,d}^2\mathbf{I}_M\right)^{-1}$ using $\mathbf{F}_R$ so that the inverse operation can be easily tackled. Such an approach, though suboptimal, will considerably simplify our derivation. It also allows us to derive an MSE upper bound, and then obtain a scalar-valued optimization problem. With the SVD in (2.16), an immediate choice for $\mathbf{F}_R$ to diagonalize $\left(\sigma_{n,r}^2\mathbf{H}_{RD}\mathbf{F}_R\mathbf{F}_R^H\mathbf{H}_{RD}^H + \sigma_{n,d}^2\mathbf{I}_M\right)$ is

$$\mathbf{F}_R = \mathbf{V}_{rd}\mathbf{\Sigma}_r\mathbf{U}_r, \tag{2.17}$$

where $\mathbf{\Sigma}_r \in \mathbb{R}^{R\times R}$ is a diagonal matrix and $\mathbf{U}_r \in \mathbb{C}^{R\times R}$ is a unitary matrix to be determined. With (2.17), we have

$$\left(\sigma_{n,r}^2 \mathbf{H}_{RD}\mathbf{F}_R\mathbf{F}_R^H\mathbf{H}_{RD}^H + \sigma_{r,d}^2\mathbf{I}_M\right)^{-1} = \mathbf{U}_{rd}\left(\sigma_{n,r}^2\mathbf{\Sigma}_{rd}\mathbf{\Sigma}_r^2\mathbf{\Sigma}_{rd}^H + \sigma_{r,d}^2\mathbf{I}_M\right)^{-1}\mathbf{U}_{rd}^H. \tag{2.18}$$

To further diagonalize $\mathbf{F}_S^H\mathbf{H}_{SR}^H\mathbf{F}_R^H\mathbf{H}_{RD}^H\left(\sigma_{n,r}^2\mathbf{H}_{RD}\mathbf{F}_R\mathbf{F}_R^H\mathbf{H}_{RD}^H + \sigma_{n,d}^2\mathbf{I}_M\right)^{-1}\mathbf{H}_{RD}\mathbf{F}_R\mathbf{H}_{SR}\mathbf{F}_S$, we can select

$$\mathbf{U}_r = \mathbf{U}_{sr}^H, \tag{2.19}$$

and

$$\mathbf{F}_S = \mathbf{V}_{sr}\mathbf{\Sigma}_s\mathbf{U}_s, \tag{2.20}$$

where $\mathbf{\Sigma}_s \in \mathbb{R}^{N\times L}$ is a diagonal matrix and $\mathbf{U}_s \in \mathbb{C}^{L\times L}$ is an unitary matrix yet to be specified. From (2.17) and (2.19), we have

$$\mathbf{F}_R = \mathbf{V}_{rd}\mathbf{\Sigma}_r\mathbf{U}_{sr}^H. \tag{2.21}$$

After some manipulations, we can obtain the MSE in (2.13) as

$$\begin{aligned} tr\{\mathbf{E}\} &= tr\left\{\left(\sigma_s^{-2}\mathbf{I}_L + \mathbf{U}_s^H\mathbf{\Sigma}_s^H\mathbf{\Sigma}_{sr}^H\mathbf{\Sigma}_r^H\mathbf{\Sigma}_{rd}^H\left(\sigma_{n,r}^2\mathbf{\Sigma}_{rd}\mathbf{\Sigma}_r^2\mathbf{\Sigma}_{rd}^H + \sigma_{n,d}^2\mathbf{I}_M\right)^{-1}\mathbf{\Sigma}_{rd}\mathbf{\Sigma}_r\mathbf{\Sigma}_{sr}\mathbf{\Sigma}_s\mathbf{U}_s + \right.\right. \\ &\left.\left. \sigma_{n,d}^{-2}\mathbf{U}_s^H\mathbf{\Sigma}_s^H\mathbf{V}^H\mathbf{\Sigma}_{sd}^H\mathbf{\Sigma}_{sd}\mathbf{V}\mathbf{\Sigma}_s\mathbf{U}_s\right)^{-1}\right\} \\ &= tr\left\{\left(\sigma_s^{-2}\mathbf{I}_L + \underbrace{\mathbf{\Sigma}_s^H\mathbf{\Sigma}_{sr}^H\mathbf{\Sigma}_r^H\mathbf{\Sigma}_{rd}^H\left(\sigma_{n,r}^2\mathbf{\Sigma}_{rd}\mathbf{\Sigma}_r^2\mathbf{\Sigma}_{rd}^H + \sigma_{n,d}^2\mathbf{I}_M\right)^{-1}\mathbf{\Sigma}_{rd}\mathbf{\Sigma}_r\mathbf{\Sigma}_{sr}\mathbf{\Sigma}_s}_{:=\mathbf{E}_R} + \right.\right. \\ &\left.\left. \underbrace{\sigma_{n,d}^{-2}\mathbf{\Sigma}_s^H\mathbf{V}^H\mathbf{\Sigma}_{sd}^H\mathbf{\Sigma}_{sd}\mathbf{V}\mathbf{\Sigma}_s}_{:=\mathbf{E}_S}\right)^{-1}\right\} \end{aligned} \tag{2.22}$$

where

$$\mathbf{V} = \mathbf{V}_{sd}^H\mathbf{V}_{sr} \tag{2.23}$$

is a constant matrix related to the channels. Note that the inclusion of the unitary matrix $\mathbf{U}_s$ in (2.22) will not change the cost function at all. However, by an appropriate design of $\mathbf{U}_s$, we can make the diagonal components of $\mathbf{E}$ equal. It has been shown that under a fixed MSE, i.e., $tr\{\mathbf{E}\}$, the receiver that make the MSEs of the MIMO components equal has the lowest BER performance [8]. From (2.22), we now have some observations in order. First, we see that (2.22) is obtained with the constrained structure of the precoding matrices specified in (2.21) and (2.20). The minimum MSE obtained with the precoders can serve as an upper bound of the true minimum MSE. Second, the unknown matrices become $\boldsymbol{\Sigma}_r$ and $\boldsymbol{\Sigma}_s$, which are diagonal and the whole problem is easier to handle. Finally, the matrix $\mathbf{E}_S$ cannot be diagonalized. However, starting from (2.22) and exploiting the diagonal nature of $\mathbf{E}_R$, we can further derive an MSE upper bound and use it to diagonalize $\mathbf{E}_S$.

To proceed, let us use the matrix inverse lemma to rewrite (2.22) as:

$$
\begin{aligned}
tr(\mathbf{E}) &= tr\left(\left[\underbrace{\left(\sigma_s^{-2}\mathbf{I}_L + \mathbf{E}_R\right)}_{:=\mathbf{A}} + \boldsymbol{\Sigma}_s^H \underbrace{\left(\sigma_{n,d}^{-2}\mathbf{V}^H\boldsymbol{\Sigma}_{sd}^H\boldsymbol{\Sigma}_{sd}\mathbf{V}\right)}_{:=\mathbf{B}} \boldsymbol{\Sigma}_s\right]^{-1}\right) \\
&= tr\left(\mathbf{A}^{-1}\right) - tr\left(\mathbf{A}^{-1}\boldsymbol{\Sigma}_s^H\left(\mathbf{B}^{-1} + \boldsymbol{\Sigma}_s\mathbf{A}^{-1}\boldsymbol{\Sigma}_s^H\right)^{-1}\boldsymbol{\Sigma}_s\mathbf{A}^{-1}\right). \qquad (2.24)
\end{aligned}
$$

It is note here that to make sure the inverse of $\mathbf{B}$ exists, $\mathbf{B}$ should be positive definite. To achieve that, we assume $N \leq M$. Based on (2.24), the desired MSE upper bound can be obtained by the aid of the next lemma.

Lemma 2.1: Let $\mathbf{D}_1$ and $\mathbf{D}_2$ be diagonal matrices, with the diagonal entries of $\mathbf{D}_2$ being positive. Then for any positive definite matrix $\mathbf{X}$, we have

$$
tr\left(\mathbf{D}_1^H\left(\mathbf{X} + \mathbf{D}_2\right)^{-1}\mathbf{D}_1\right) \geq tr\left(\mathbf{D}_1^H\left(\mathrm{diag}(\mathbf{X}) + \mathbf{D}_2\right)^{-1}\mathbf{D}_1\right), \qquad (2.25)
$$

where $\mathrm{diag}(\mathbf{X})$ is obtained from $\mathbf{X}$ by setting its off-diagonal entries to zero. The equality in (2.25) holds if $\mathbf{X}$ is diagonal.

Proof: See Appendix A.1.

By the lemma, it follows that

$$
\begin{aligned}
& tr\left(\mathbf{A}^{-1}\boldsymbol{\Sigma}_s^H\left(\mathbf{B}^{-1} + \boldsymbol{\Sigma}_s\mathbf{A}^{-1}\boldsymbol{\Sigma}_s^H\right)^{-1}\boldsymbol{\Sigma}_s\mathbf{A}^{-1}\right) \geq \\
& tr\left(\mathbf{A}^{-1}\boldsymbol{\Sigma}_s^H\left(\mathrm{diag}\left(\mathbf{B}^{-1}\right) + \boldsymbol{\Sigma}_s\mathbf{A}^{-1}\boldsymbol{\Sigma}_s^H\right)^{-1}\boldsymbol{\Sigma}_s\mathbf{A}^{-1}\right). \qquad (2.26)
\end{aligned}
$$

Using (2.24) and (2.26), we can have the following key result.

$$tr(\mathbf{E}) \le tr\left(\mathbf{A}^{-1}\right) - tr\left(\mathbf{A}^{-1}\mathbf{\Sigma}_s^H\left(diag\left(\mathbf{B}^{-1}\right) + \mathbf{\Sigma}_s\mathbf{A}^{-1}\mathbf{\Sigma}_s^H\right)^{-1}\mathbf{\Sigma}_s\mathbf{A}^{-1}\right)$$
$$= \sum_{i=1}^{L} \frac{1}{\sigma_s^{-2} + \frac{\sigma_{s,i}^2\sigma_{r,i}^2\sigma_{sr,i}^2\sigma_{rd,i}^2}{\sigma_{n,r}^2\sigma_{r,i}^2\sigma_{rd,i}^2+\sigma_{n,d}^2} + \sigma_{s,i}^2\left(\mathbf{B}^{-1}(i,i)\right)^{-1}}. \tag{2.27}$$

Compared with the original MSE function (2.13), the upper bound in (2.27) admits a much simpler form and is analytically tractable. Hence, we propose to design the precoder by minimizing the upper bound in (2.27). For convenience, let $p_{s,i} = \sigma_{s,i}^2$ and $p_{r,i} = \sigma_{r,i}^2$ in (2.27). The optimization can finally be formulated as:

$$\min_{p_{s,i},p_{r,i},i=1,\cdots,L} \sum_{i=1}^{L} \frac{1}{\sigma_s^{-2} + \frac{p_{s,i}p_{r,i}\sigma_{sr,i}^2\sigma_{rd,i}^2}{\sigma_{n,r}^2 p_{r,i}\sigma_{rd,i}^2+\sigma_{n,d}^2} + p_{s,i}\left(\mathbf{B}^{-1}(i,i)\right)^{-1}}$$
$$s.t.$$
$$tr\left\{\mathbf{\Sigma}_r\left(\sigma_{n,r}^2\mathbf{I}_R + \sigma_s^2\mathbf{\Sigma}_{sr}\mathbf{\Sigma}_s\mathbf{\Sigma}_s^H\mathbf{\Sigma}_{sr}^H\right)\mathbf{\Sigma}_r^H\right\} = \sum_{i=1}^{L} p_{r,i}\left(\sigma_{n,r}^2 + \sigma_s^2 p_{s,i}\sigma_{sr,i}^2\right) \le P_{R,T}$$
$$\sigma_s^2 tr\{\mathbf{\Sigma}_s\mathbf{\Sigma}_s^H\} = \sigma_s^2\sum_{i=1}^{L} p_{s,i} \le P_{S,T},\ p_{s,i} \ge 0,\ p_{r,i} \ge 0,\ \forall i. \tag{2.28}$$

It is simple to see that the problem in (2.28) is not a convex optimization problem either, and the optimum solution is still difficult to find. However, note that if one of $p_{r,i}$ and $p_{s,i}$ is given, (2.28) will become a convex optimization problem. This suggests a method, referred to as the iterative water-filling method [17], [52], [53], to find a suboptimum solution. For a given $p_{s,i}$, the optimum $p_{r,i}$ can be expressed as (See Appendix A.2):

$$p_{r,i} = \left[\frac{\mu_r\sigma_{n,d}\sqrt{p_{s,i}}\sigma_{sr,i}\sigma_{rd,i}\left(\sigma_s^2 p_{s,i}\sigma_{sr,i}^2 + \sigma_{n,r}^2\right)^{-1/2} - \sigma_{n,d}^2\left(\sigma_s^{-2} + p_{s,i}(\mathbf{B}^{-1}(i,i))^{-1}\right)}{\sigma_{rd,i}^2\left(\sigma_{n,r}^2\left(\sigma_s^{-2} + p_{s,i}(\mathbf{B}^{-1}(i,i))^{-1}\right) + p_{s,i}\sigma_{sr,i}^2\right)}\right]^+, \tag{2.29}$$

where $[y]^+ = max[0,y]$, and μ_r is the water level chosen to satisfy the power constraint at the relay, i.e., $\sum_{i=1}^{L} p_{r,i}\left(\sigma_{n,r}^2 + \sigma_s^2 p_{s,i}\sigma_{sr,i}^2\right) = P_{R,T}$. With $p_{r,i} = \sigma_{r,i}^2$ in (2.29), the relay precoder can be obtained by (2.21). For a given $p_{r,i}$, the optimum $p_{s,i}$ can be expressed as

$$p_{s,i} = \left[\frac{\mu_s\sqrt{\beta_i} - \sigma_s^{-2}(\sigma_{n,d}^2 + p_{r,i}\sigma_{n,r}^2\sigma_{rd,i}^2)}{((\mathbf{B}^{-1}(i,i))^{-1}(\sigma_{n,d}^2 + p_{r,i}\sigma_{n,r}^2\sigma_{rd,i}^2) + p_{r,i}\sigma_{sr,i}^2\sigma_{rd,i}^2)}\right]^+, \tag{2.30}$$

where μ_s is the water level chosen to meet the power constraint at the source, i.e., $\sum_{i=1}^{L} p_{s,i} = P_{S,T}$, and

$$\beta_i = \left(\sigma_{n,d}^2 + p_{r,i}\sigma_{n,r}^2\sigma_{rd,i}^2\right)\left((\mathbf{B}^{-1}(i,i))^{-1}\left(\sigma_{n,d}^2 + p_{r,i}\sigma_{n,r}^2\sigma_{rd,i}^2\right) + p_{r,i}\sigma_{sr,i}^2\sigma_{rd,i}^2\right). \tag{2.31}$$

Thus, we can use (2.29) and (2.30) to solve $p_{r,i}$ and $p_{s,i}$ iteratively. To determine the $\mathbf{U}_s$, we first substitute (2.29) and (2.30) into (2.21) and (2.20), respectively, and express the error matrix in (2.10) as

$$\mathbf{E} = \left(\sigma_s^{-2}\mathbf{I}_L + \mathbf{U}_s^H\tilde{\mathbf{E}}\mathbf{U}_s\right)^{-1} \tag{2.32}$$

where

$$\begin{aligned}\tilde{\mathbf{E}} = {} & \mathbf{\Sigma}_s^H\mathbf{\Sigma}_{sr}^H\mathbf{\Sigma}_r^H\mathbf{\Sigma}_{rd}^H\left(\sigma_{n,r}^2\mathbf{\Sigma}_{rd}\mathbf{\Sigma}_r^2\mathbf{\Sigma}_{rd}^H + \sigma_{n,d}^2\mathbf{I}_M\right)^{-1}\mathbf{\Sigma}_{rd}\mathbf{\Sigma}_r\mathbf{\Sigma}_{sr}\mathbf{\Sigma}_s + \\ & \sigma_{n,d}^{-2}\mathbf{\Sigma}_s^H\mathbf{V}^H\mathbf{\Sigma}_{sd}^H\mathbf{\Sigma}_{sd}\mathbf{V}\mathbf{\Sigma}_s.\end{aligned} \tag{2.33}$$

Our task now is to design $\mathbf{U}_s$ such that (2.32) has equal diagonal MSE values. To do that, we consider the following eigen-decomposition

$$\tilde{\mathbf{E}} = \mathbf{V}_{\tilde{\mathbf{E}}}\mathbf{D}_{\tilde{\mathbf{E}}}\mathbf{V}_{\tilde{\mathbf{E}}}^H \tag{2.34}$$

where $\mathbf{V}_{\tilde{\mathbf{E}}} \in \mathbb{C}^{L\times L}$ is a matrix with the eigenvectors of $\tilde{\mathbf{E}}$ as its columns, and $\mathbf{D}_{\tilde{\mathbf{E}}} \in \mathbb{R}^{L\times L}$ is a diagonal matrix with the eigenvalues of $\tilde{\mathbf{E}}$ as its diagonal components. Therefore, if we let

$$\mathbf{U}_s = \mathbf{V}_{\tilde{\mathbf{E}}}\mathbf{F}_L, \tag{2.35}$$

where $\mathbf{F}_L$ is the L-points DFT matrix, (2.32) can be re-expressed as

$$\mathbf{E} = \mathbf{F}_L^H\left(\sigma_s^2\mathbf{I}_L + \mathbf{D}_{\tilde{\mathbf{E}}}\right)^{-1}\mathbf{F}_L \tag{2.36}$$

which reveals that $\mathbf{E}$ is a circulant matrix with equal diagonal elements. It is simple to check the unitary property that $\mathbf{U}_s\mathbf{U}_s^H = \mathbf{U}_s^H\mathbf{U}_s = \mathbf{I}_L$.

The proposed scheme mainly involves the operations of the SVD in (2.14)-(2.16), (2.34) and the inversion of the matrix $\mathbf{B}$ in (2.27). The computational complexity of the proposed scheme, measured in terms of floating-point operations (FLOPs), is summarized in Table 2.1.

§ 2.2.2 *Special Case: Cooperative Beamforming*

In this subsection, we consider the cooperative beamforming in a two-hop cooperative system. This is a special case of our precoding problem in which $L = 1$ and the direct link is not considered (i.e., $\mathbf{H}_{SD} = \mathbf{0}$).

For a given source beamforming vector $\mathbf{f}_S$, the optimal relay precoder can be derived by [43]

$$\mathbf{F}_R = \mathbf{V}_{rd}\boldsymbol{\Sigma}_r\mathbf{U}_{sr}^H, \tag{2.37}$$

where $\boldsymbol{\Sigma}_r = \text{diag}\{\sigma_{r,1}, \cdots, \sigma_{r,R}\}$ with $\sigma_{r,1} \geq \cdots \geq \sigma_{r,R}$.

Let $\mathbf{f}_S = \sqrt{\alpha_s}\mathbf{v}_S \in \mathbb{C}^{N\times 1}$ where $\mathbf{v}_S$ is an unit vector and $\mathbf{f}_S$ satisfies the transmit power constraint, i.e., $\sigma_s^2\alpha_s\|\mathbf{v}_S\|^2 \leq P_{S,T}$. Substituting the beamformer and (2.37) into (2.9) with $\mathbf{H}_{SD} = \mathbf{0}$, we have

$$\begin{aligned}
J_{min} &= tr\left\{\left(\sigma_s^{-2} + \mathbf{f}_S^H\mathbf{V}_{sr}\boldsymbol{\Sigma}_{sr}^H\boldsymbol{\Sigma}_r^H\boldsymbol{\Sigma}_{rd}^H\left(\sigma_{n,d}^2\mathbf{I}_M + \sigma_{n,r}^2\boldsymbol{\Sigma}_{rd}\boldsymbol{\Sigma}_r\boldsymbol{\Sigma}_r^H\boldsymbol{\Sigma}_{rd}^H\right)^{-1}\boldsymbol{\Sigma}_{rd}\boldsymbol{\Sigma}_r\boldsymbol{\Sigma}_{sr}\mathbf{V}_{sr}^H\mathbf{f}_S\right)^{-1}\right\} \\
&= tr\left\{\left(\sigma_s^{-2} + \alpha_s\underbrace{\mathbf{v}_S^H\mathbf{V}_{sr}}_{:=\mathbf{w}_{sr}^H}\boldsymbol{\Sigma}_{sr}^H\boldsymbol{\Sigma}_r^H\boldsymbol{\Sigma}_{rd}^H\left(\sigma_{n,d}^2\mathbf{I}_M + \sigma_{n,r}^2\boldsymbol{\Sigma}_{rd}\boldsymbol{\Sigma}_r\boldsymbol{\Sigma}_r^H\boldsymbol{\Sigma}_{rd}^H\right)^{-1}\right.\right. \\
&\qquad \left.\left.\boldsymbol{\Sigma}_{rd}\boldsymbol{\Sigma}_r\boldsymbol{\Sigma}_{sr}\underbrace{\mathbf{V}_{sr}^H\mathbf{v}_S}_{:=\mathbf{w}_{sr}}\right)^{-1}\right\} \\
&= \frac{1}{\sigma_s^{-2} + \alpha_s\sum_{i=1}^{min\{N,M,R\}}|w_{sr,i}|^2\frac{\sigma_{r,i}^2\sigma_{sr,i}^2\sigma_{rd,i}^2}{\sigma_{n,d}^2+\sigma_{n,r}^2\sigma_{r,i}^2\sigma_{rd,i}^2}}
\end{aligned} \tag{2.38}$$

where $\mathbf{w}_{sr} = \mathbf{V}_{sr}^H\mathbf{v}_S = [w_{sr,1}, \cdots, w_{sr,N}]^T$ and $\|\mathbf{w}_{sr}\|^2 = 1$, $\boldsymbol{\Sigma}_{sr}$, $\boldsymbol{\Sigma}_{rd}$, $\boldsymbol{\Sigma}_r$ are diagonal matrices with their diagonal elements arranged in a decreasing order. The beamforming problem can then be formulated as

$$\min_{\alpha_s, w_{sr,i}, \sigma_{r,i}, \forall i} \frac{1}{\sigma_s^{-2} + \alpha_s \sum_{i=1}^{min\{N,M,R\}} |w_{sr,i}|^2 \frac{\sigma_{r,i}^2 \sigma_{sr,i}^2 \sigma_{rd,i}^2}{\sigma_{n,d}^2 + \sigma_{n,r}^2 \sigma_{r,i}^2 \sigma_{rd,i}^2}}$$
$$s.t.$$
$$\sigma_s^2 \|\mathbf{f}_S\|^2 = \sigma_s^2 \alpha_s \|\mathbf{v}_S\|^2 \leq P_{S,T}$$
$$\sum_{i=1}^{N} |w_{sr,i}|^2 = 1$$
$$tr\left\{\mathbf{\Sigma}_r \left(\sigma_{n,r}^2 \mathbf{I}_R + \sigma_s^2 \mathbf{\Sigma}_{sr} \mathbf{V}_{sr}^H \mathbf{f}_S \mathbf{f}_S^H \mathbf{V}_{sr} \mathbf{\Sigma}_{sr}^H\right) \mathbf{\Sigma}_r^H\right\} \leq P_{R,T}. \tag{2.39}$$

Theorem 2.1: The optimal beamforming vector denoted by $\mathbf{f}_S^*$ and the optimal relay precoder denoted by $\mathbf{F}_R^*$ for (2.39) are $\sqrt{\frac{P_{S,T}}{\sigma_s^2}}\mathbf{V}_{sr}(:,1)$ and $\sqrt{\frac{P_{R,T}}{(\sigma_{n,r}^2 + P_{S,T}\sigma_{sr,1}^2)}}\mathbf{V}_{rd}(:,1)[\mathbf{U}_{sr}(:,1)]^H$, where $\mathbf{V}_{rd}(:,i)$ and $\mathbf{U}_{sr}(:,i)$ denote the ith column of $\mathbf{V}_{rd}$ and $\mathbf{U}_{sr}$, respectively.

Proof: We first derive the optimal $\mathbf{w}_{sr}$ for given α_s and $\sigma_{r,i}$, $i = 1, \cdots, R$. From (2.39), it is simple to see that the optimal $\mathbf{w}_{sr}$ can be derived by the following equivalent problem

$$\max_{\mathbf{w}_{sr}} \sum_{i=1}^{min\{N,M,R\}} |w_{sr,i}|^2 \frac{\sigma_{r,i}^2 \sigma_{sr,i}^2 \sigma_{rd,i}^2}{\sigma_{n,d}^2 + \sigma_{n,r}^2 \sigma_{r,i}^2 \sigma_{rd,i}^2}$$
$$\sum_{i=1}^{N} |w_{sr,i}| = 1. \tag{2.40}$$

From (2.40), it is obvious that optimum $\mathbf{w}_{sr}$ is $[1, 0, \cdots, 0]^T$. This can be easily checked by

$$\frac{\sigma_{r,i}^2 \sigma_{sr,i}^2 \sigma_{rd,i}^2}{\sigma_{n,d}^2 + \sigma_{n,r}^2 \sigma_{r,i}^2 \sigma_{rd,i}^2} \geq \frac{\sigma_{r,j}^2 \sigma_{sr,j}^2 \sigma_{rd,j}^2}{\sigma_{n,d}^2 + \sigma_{n,r}^2 \sigma_{r,j}^2 \sigma_{rd,j}^2}, \quad i \geq j. \tag{2.41}$$

The solution implies that the optimum $\mathbf{v}_S$, denoted by $\mathbf{v}_S^*$, is

$$\mathbf{v}_S^* = \mathbf{V}_{sr}(:,1). \tag{2.42}$$

As a result, the design problem can therefore be expressed as

$$
\begin{aligned}
&\max_{\alpha_s, \sigma_{r,i}, \forall i} \alpha_s \frac{\sigma_{r,1}^2 \sigma_{sr,1}^2 \sigma_{rd,1}^2}{\sigma_{n,d}^2 + \sigma_{n,r}^2 \sigma_{r,1}^2 \sigma_{rd,1}^2} \\
&s.t. \\
&0 \leq \alpha_s \leq \frac{P_{S,T}}{\sigma_s^2} \\
&tr\left\{\mathbf{\Sigma}_r \left(\sigma_{n,r}^2 \mathbf{I}_R + \alpha_s \sigma_s^2 \mathbf{\Sigma}_{sr} \mathbf{V}_{sr}^H \mathbf{v}_S \mathbf{v}_S^H \mathbf{V}_{sr} \mathbf{\Sigma}_{sr}^H\right) \mathbf{\Sigma}_r^H\right\} = \\
&\left(\sigma_{n,r}^2 \sum_{i=1}^{R} \sigma_{r,i}^2\right) + \alpha_s \sigma_s^2 \sigma_{r,1}^2 \sigma_{sr,1}^2 \leq P_{R,T}.
\end{aligned} \tag{2.43}
$$

Taking a close look at (2.43), we first find that the cost function is only related to α_s and $\sigma_{r,1}^2$. Then, we can have the following observation:

$$
\alpha_s \frac{\sigma_{r,1}^2 \sigma_{sr,1}^2 \sigma_{rd,1}^2}{\sigma_{n,d}^2 + \sigma_{n,r}^2 \sigma_{r,1}^2 \sigma_{rd,1}^2} \text{ is monotonous in } \sigma_{r,1}^2 \text{ and } \alpha_s. \tag{2.44}
$$

From (2.44) and (2.43), we can rewrite the power constraint for the relay as

$$
\sigma_{r,1}^2 \left(\sigma_{n,r}^2 + \alpha_s^2 \sigma_s^2 \sigma_{sr,1}^2\right) \leq P_{R,T}, \tag{2.45}
$$

and consequently have the following relationship

$$
\sigma_{r,1}^2 \leq \frac{P_{R,T}}{\left(\sigma_{n,r}^2 + \alpha_s^2 \sigma_s^2 \sigma_{sr,1}^2\right)}. \tag{2.46}
$$

It is noteworthy that (2.45) also implies that the optimal $\mathbf{\Sigma}_r$, denoted by $\mathbf{\Sigma}_r^*$, is

$$
\mathbf{\Sigma}_r^* = \begin{bmatrix} \sigma_{r,1} & 0 & \cdots & 0 \\ 0 & \sigma_{r,2} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \sigma_{r,R} \end{bmatrix} = \begin{bmatrix} \sigma_{r,1} & 0 & \cdots & 0 \\ 0 & 0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & 0 \end{bmatrix}. \tag{2.47}
$$

Substituting (2.46) into the cost function in (2.43), we have

$$
\alpha_s \frac{\sigma_{r,1}^2 \sigma_{sr,1}^2 \sigma_{rd,1}^2}{\sigma_{n,d}^2 + \sigma_{n,r}^2 \sigma_{r,1}^2 \sigma_{rd,1}^2} \leq \frac{\alpha_s \sigma_{sr,1}^2 \sigma_{rd,1}^2}{\sigma_{n,d}^2 \frac{\left(\sigma_{n,r}^2 + \alpha_s \sigma_s^2 \sigma_{sr,1}^2\right)}{P_{R,T}} + \sigma_{n,r}^2 \sigma_{rd,1}^2}, \tag{2.48}
$$

where the upper bound of the cost function can be achieved if

$$\sigma_{r,1}^2 = \frac{P_{R,T}}{\left(\sigma_{n,r}^2 + \alpha_s \sigma_s^2 \sigma_{sr,1}^2\right)}. \tag{2.49}$$

Therefore, via (2.49), the problem can finally be expressed as the minimization of a function of α_s given by

$$\begin{aligned} &\max_{\alpha_s} \frac{\sigma_{sr,1}^2 \sigma_{rd,1}^2 \alpha_s}{\frac{\sigma_{n,d}^2 \sigma_s^2 \sigma_{sr,1}^2}{P_{R,T}} \alpha_s + \sigma_{n,r}^2 \sigma_{rd,1}^2 + \frac{\sigma_{n,d}^2 \sigma_{n,r}^2}{P_{R,T}}} \\ &s.t. \quad 0 \leq \alpha_s \leq \frac{P_{S,T}}{\sigma_s^2}. \end{aligned} \tag{2.50}$$

Since the cost function in (2.50) is monotonically increasing in α_s, it is clear that the optimum α_s, denoted by α_s^*, is

$$\alpha_s^* = \frac{P_{S,T}}{\sigma_s^2}. \tag{2.51}$$

Combining (2.51) and (2.42), we finally obtain the optimum beamforming vector as:

$$\mathbf{f}_s^* = \sqrt{\frac{P_{S,T}}{\sigma_s^2}} \mathbf{V}_{sr}(:,1). \tag{2.52}$$

Substituting (2.51) into (2.49) and combining (2.47) and (2.37), we have the optimum relay precoder as

$$\sqrt{\frac{P_{R,T}}{\left(\sigma_{n,r}^2 + P_{S,T}\sigma_{sr,1}^2\right)}} \mathbf{V}_{rd}(:,1)[\mathbf{U}_{sr}(:,1)]^H. \tag{2.53}$$

It is noteworthy that the result of Theorem 2.1 is the same as that in [46], in which the criterion for the beamformer design is the maximization of the received SNR. Here, we use the MMSE criterion and obtain the same solution.

§ 2.3 Applications

The proposed linear source and relay precoded scheme can be used in many scenarios. In this subsection, we conduct simulations to evaluate the performance of the linear source and relay precoded scheme in three different applications, namely a single-input-single-output (SISO) orthogonal-frequency-division-multiplexing (OFDM), a two-hop MIMO relay system (where only the relay link is considered), and a general MIMO relay system. Assume that all channel state information (CSI) of all the links are available at all nodes, and perfect synchronization can be achieved. For the first case, the channel is assumed to be frequency-selective-fading, and for the rest of the cases, the channel is assumed to be flat fading. Also, the modulation scheme is QPSK.

§ 2.3.1 *SISO OFDM Relay System*

Assume that the cyclic prefix (CP) length is longer than the overall channel delay spread such that inter-symbol interference will not occur. Also, the channel is assumed to be quasi-static, meaning that its response remains constant during each OFDM symbol. Note that each node only has one antenna. As a result, the equivalent frequency domain channel matrices of all links are diagonal. The linear source and relay precoders in (2.20) and (2.21), therefore, become $\mathbf{F}_S = \mathbf{F}_L^H \mathbf{\Sigma}_s \mathbf{F}_L$ and $\mathbf{F}_R = \mathbf{\Sigma}_r$ where the relay precoder becomes a subcarrier power allocation (PA) problem. Let $h_{sr}(l)$, $h_{rd}(l)$, and $h_{sd}(l)$ be the channel impulse responses for the source-to-relay, relay-to-destination, and source-to-destination channels, respectively. The channel taps, $h_{sr}(l)$, $h_{rd}(l)$, and $h_{sd}(l)$, $0 \leq l \leq 5$, are generated from i.i.d. complex Gaussian random variables with zero mean and variance $1/6$, such that $E\left\{\sum_{l=0}^{5} |h_{sr}(l)|^2\right\}$ = $E\left\{\sum_{l=0}^{5} |h_{sd}(l)|^2\right\}$ = $E\left\{\sum_{l=0}^{5} |h_{rd}(l)|^2\right\}$ = 1. Also, let $N = 64$ and the total available powers at the source and the relay be equal, and SNR$_{sr}$, SNR$_{rd}$ and SNR$_{sd}$ be defined as the received SNR at the source-to-relay, relay-to-destination, and source-to-destination links. Here, we let SNR$_{sr}$ = SNR$_{rd}$ = SNR$_{sd}$ = SNR. Fig. 2.2 and Fig. 2.3 show the MSE and BER comparisons for the un-precoded and linear source and relay precoded systems, respectively. As shown in the figures, the linear source and relay precoded system significantly outperforms the un-precoded system. This is because the linear source and relay system considers all the link resources and allocates the power properly.

§ 2.3.2 *Two-Hop MIMO Relay System*

In this scenario, the channel condition in the direct link is poor such that the destination only receives the signal from the relay link. Here, we first consider the case that $N = R = M = L = 4$. Let the elements of each channel matrix be i.i.d. complex Gaussian random variables with zero mean and unity variance. Let SNR_{sr} and SNR_{rd} denote, respectively, the SNR per receive antenna of the source-to-relay and relay-to-destination links. Here, we set SNR_{sr} = $20\ dB$ and vary SNR_{rd}. Fig. 2.4 and Fig. 2.5 show the MSE and the BER comparisons, respectively for (a) an un-precoded system with ZF receiver, (b) an un-precoded system with MMSE receiver, (c) the optimal relay precoder with MMSE receiver [43], and (d) the linear source and relay precoded system. From those figures we can see that the linear source and relay precoded system outperforms not only the un-precoded system, but also the relay precoded system in [43]. This is because the linear source and relay precoded system incorporates the additional source precoder such that the performance can be enhanced even the direct link is not considered.

We also report the simulation result for cooperative beamforming, i.e., $L = 1$. As discussed in Theorem 2.1, our design for this case is optimal. We let $N = R = M = 4$ and $\text{SNR}_{sr} = 5\ dB$. Fig. 2.6 shows the BER comparison for the antennas selection method in [45] and the linear source and relay precoded method. From the figure, we can see that the linear source and relay precoded method is superior to the antenna selection. This is expected since our design here is optimal.

§ 2.3.3 *General MIMO Relay Channel*

In this scenario, we consider a symmetric MIMO relay system, i.e., $N = M = R = L = 4$. As the previous case, each element of the channel matrices is assumed to be i.i.d. complex Gaussian random variables with zero mean and same variance. We let SNR_{sr}, SNR_{rd} be the same as those defined in Section 2.3.2, and SNR_{sd} as the SNR per receive antenna for the source-to-destination link. Here, we set SNR_{sr} = $15\ dB$, SNR_{rd} = $10\ dB$ and vary SNR_{sd}. Fig.

Table 2.1: Complexity of linear source and relay precoders (MMSE receiver).

Operation	FLOPs
SVD, (2.14)-(2.16)	$(14MN^2+8N^3)+(14RN^2+8N^3)+(14MR^2+8R^3)$
$\mathbf{B}^{-1}$, (2.24)	$2MN^2 + 2MN + 2N^3 + 13/4N^2 + N^2$
$p_{s,i}$ and $p_{r,i}$, (2.29)-(2.30)	$(21LI_r+14LI_s)I_i$
$\tilde{\mathbf{E}}$, (2.33)	$14L+10M+4NL+2L^2N$
SVD of $\tilde{\mathbf{E}}$, (2.34)	$22L^3$
$\mathbf{U}_S$, (2.35)	$2L^3$
$\mathbf{F}_S$ and $\mathbf{F}_R$, (2.20)-(2.21)	$(2NL+2NL^2)+(2R^2+2R^3)$

N: number of transmit antennas

R: number of relay antennas

M: number of receive antennas

L: number of transmitted symbol streams

I_r: number of iteration for computing $p_{r,i}$

I_s: number of iteration for computing $p_{s,i}$

I_i: number of iteration of the water-filling process

2.7 and Fig. 2.8 show the MSE and BER comparisons, respectively, for the linear source and relay precoded system and other systems described in Section 2.3.2. Note that the optimal relay precoder in [43] only considers the two-hop relay system. For fair comparison, we include the direct link at the destination when implementing the MMSE receiver. As expected, the linear source and relay precoded method outperforms all other systems.

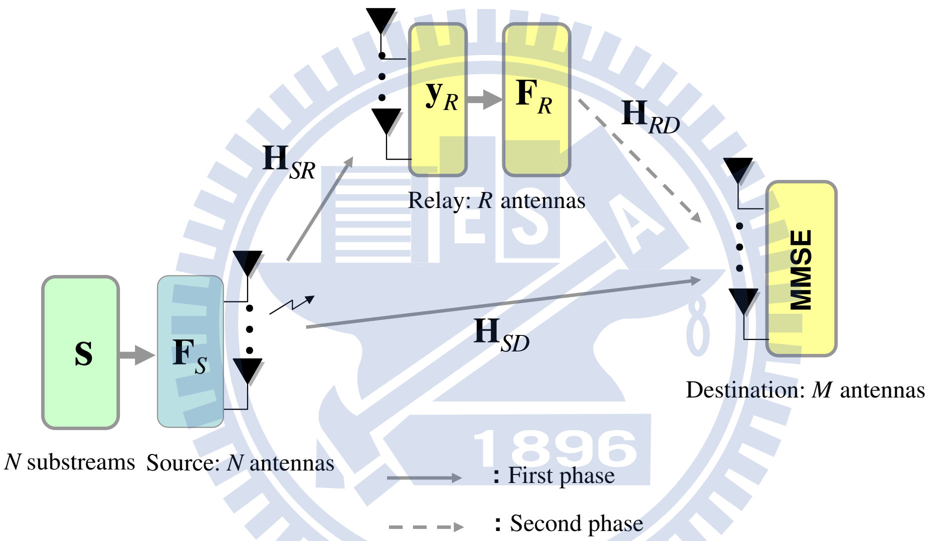


Figure 2.1: Linear source and relay precoded AF MIMO relay system with MMSE receiver.

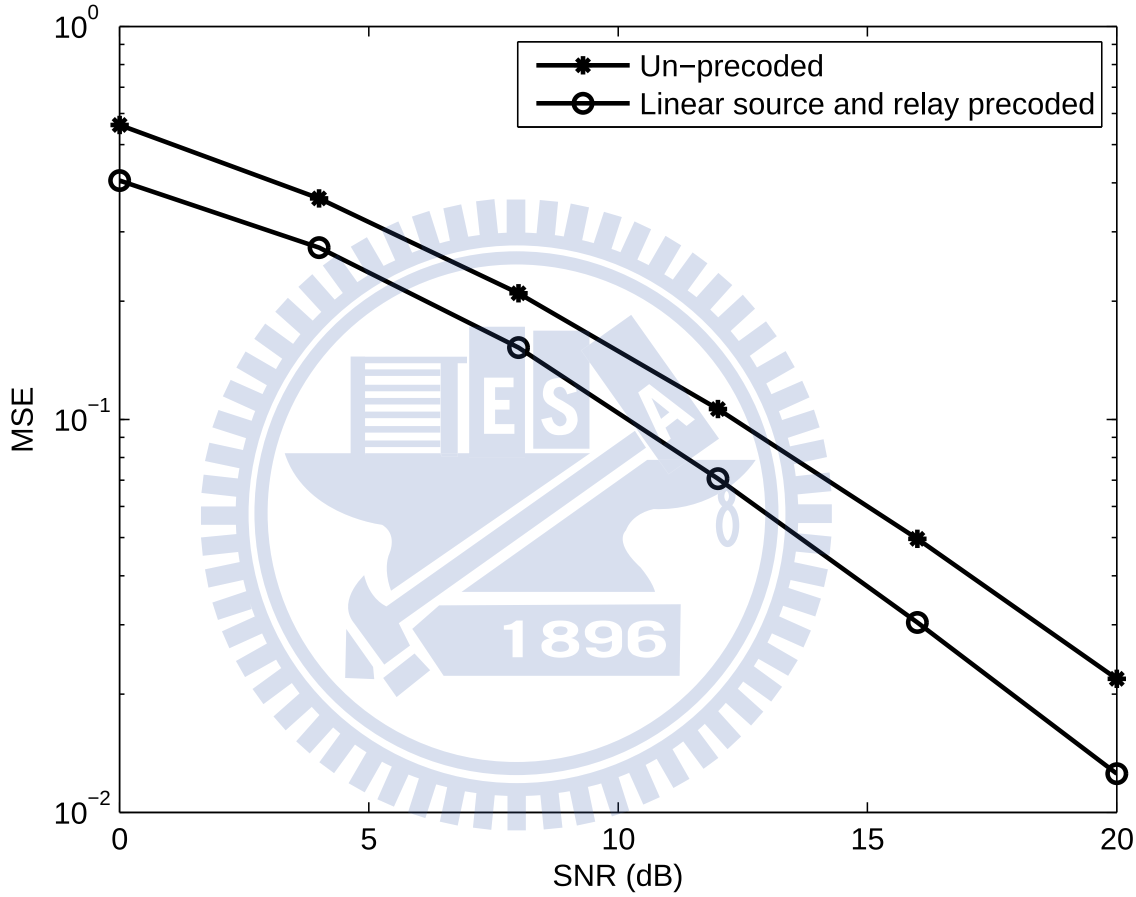


Figure 2.2: MSE performance comparison for un-precoded and linear source and relay precoded AF SISO-OFDM cooperative systems.

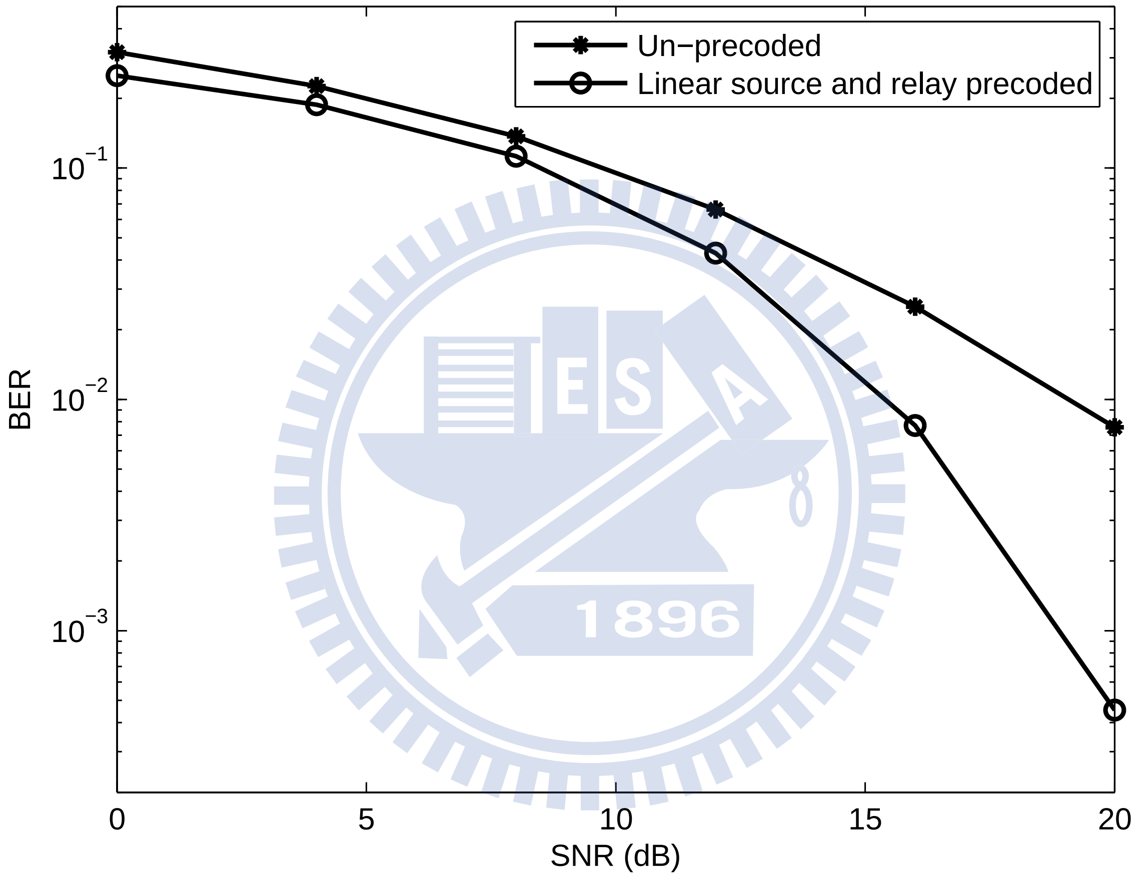


Figure 2.3: BER performance comparison for un-precoded and linear source and relay precoded AF SISO-OFDM cooperative systems.

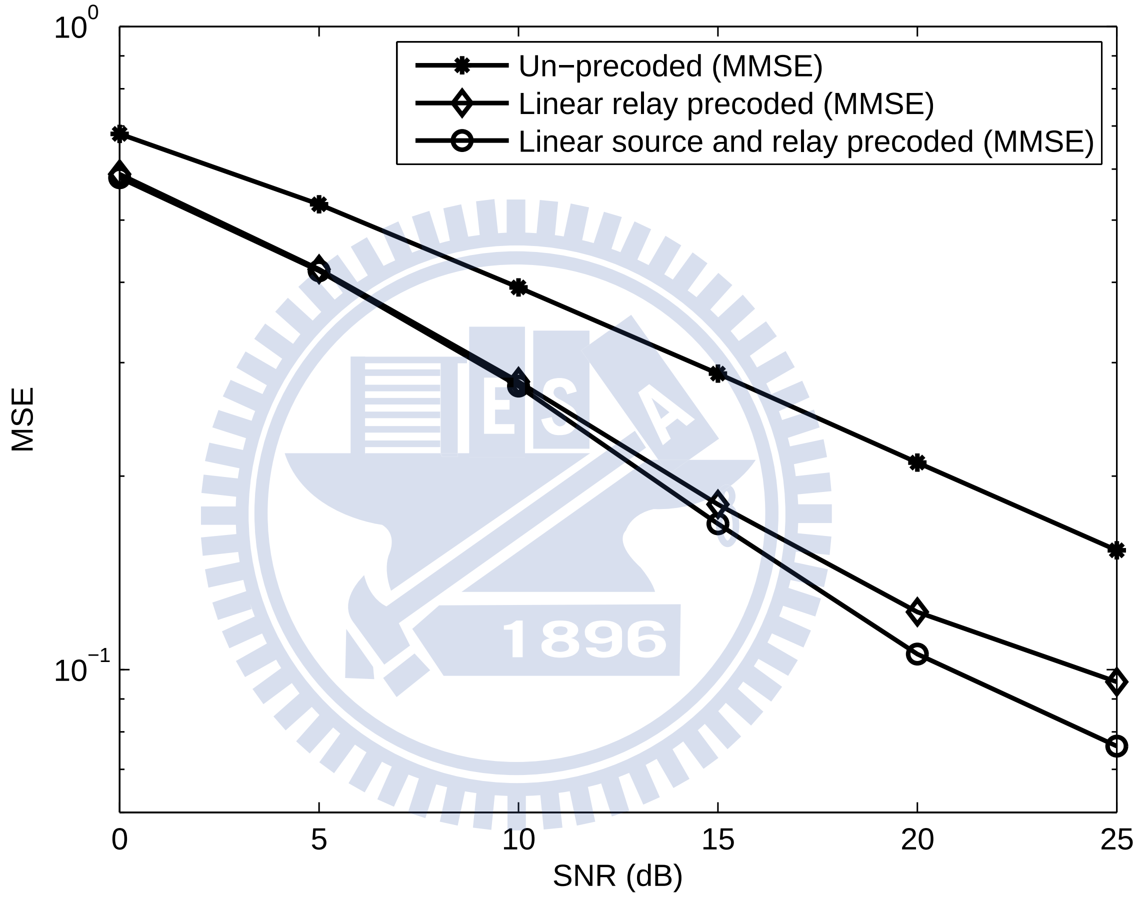


Figure 2.4: MSE performance comparison for existing un-precoded/precoded and linear source and relay precoded AF two-hop MIMO relay systems.

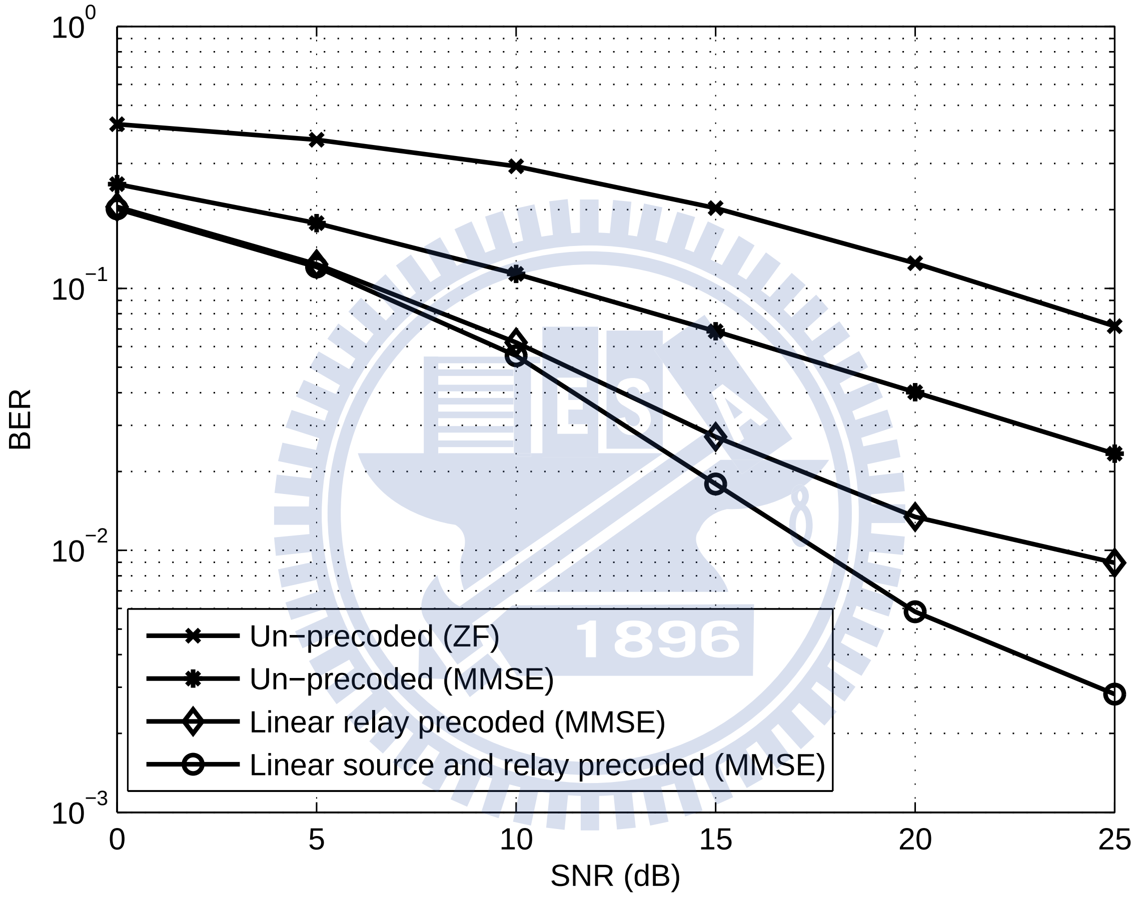


Figure 2.5: BER performance comparison for existing un-precoded/precoded and linear source and relay precoded AF two-hop MIMO relay systems.

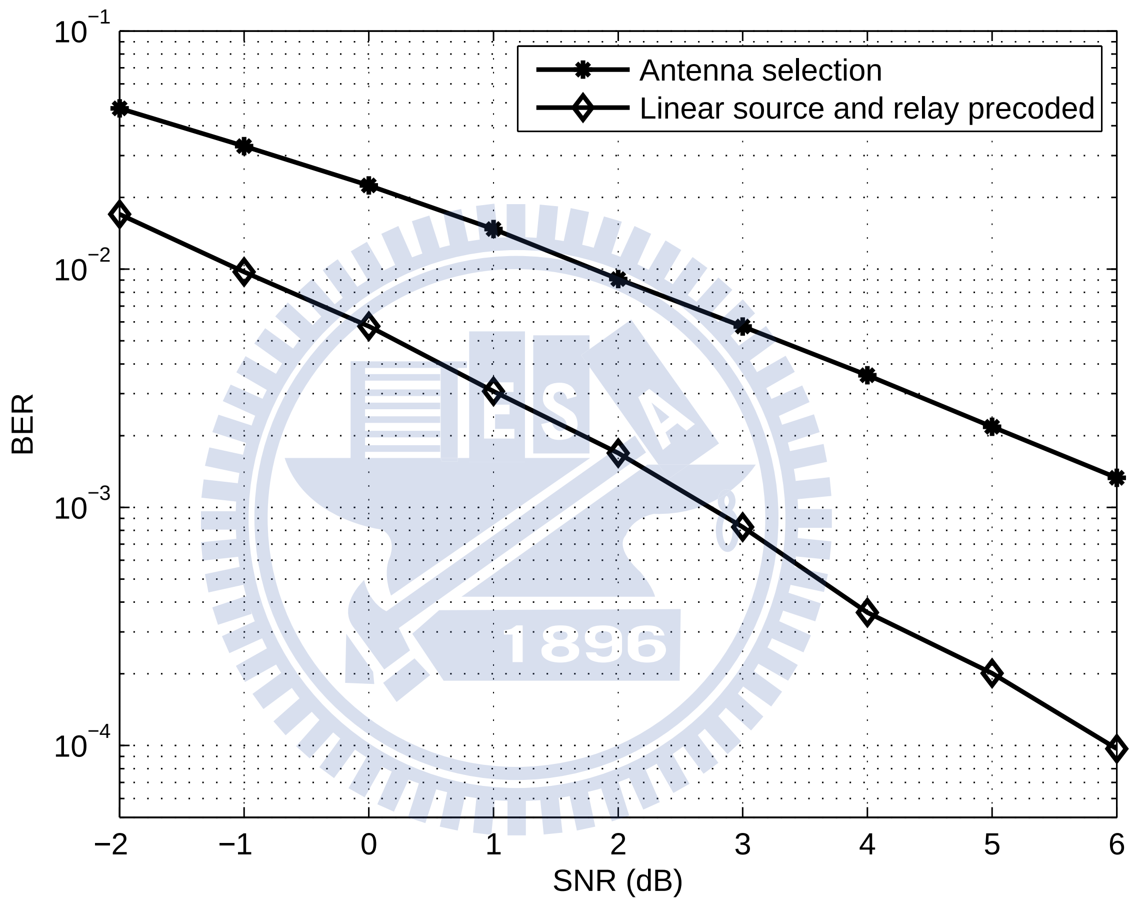


Figure 2.6: BER performance comparison for antenna selection [45] and linear source and relay precoded AF two-hop MIMO relay systems ($L = 1$ and $N = R = M = 4$).

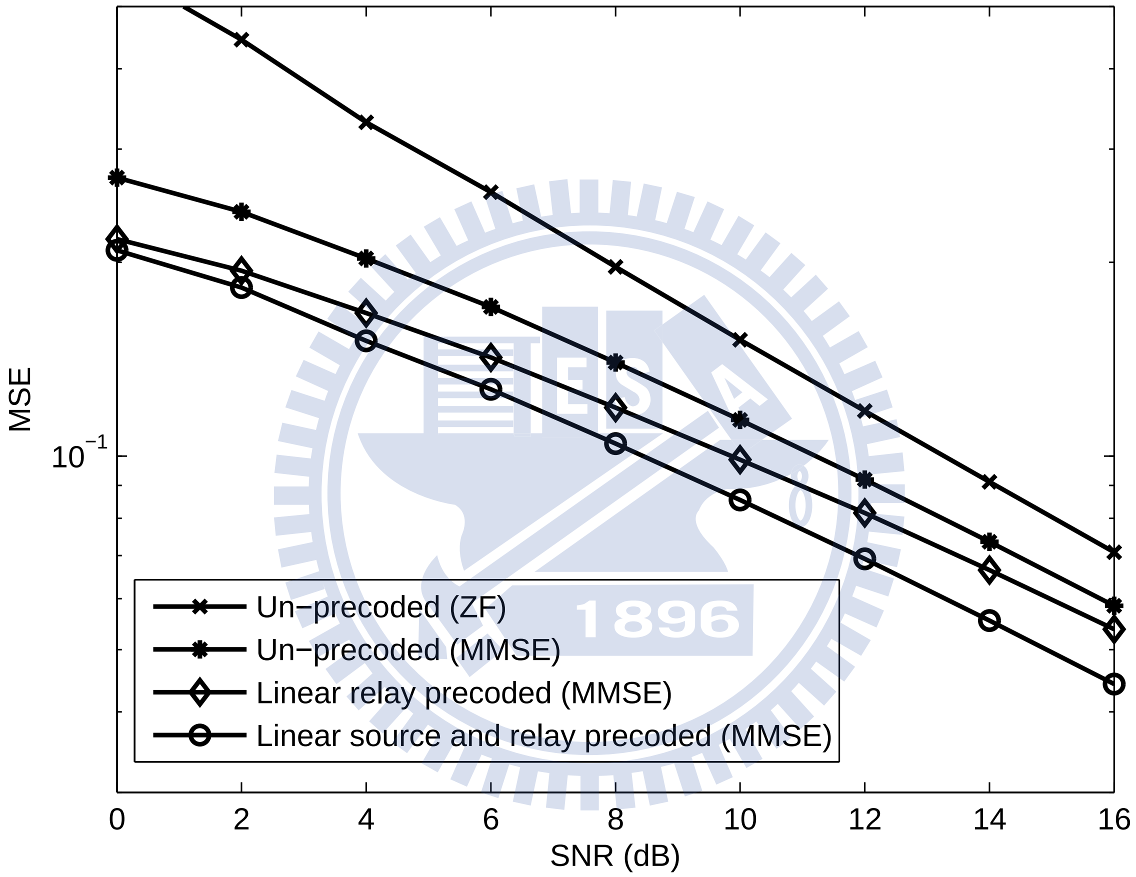


Figure 2.7: MSE performance comparison for existing un-precoded/precoded and linear source and relay precoded AF MIMO relay systems.

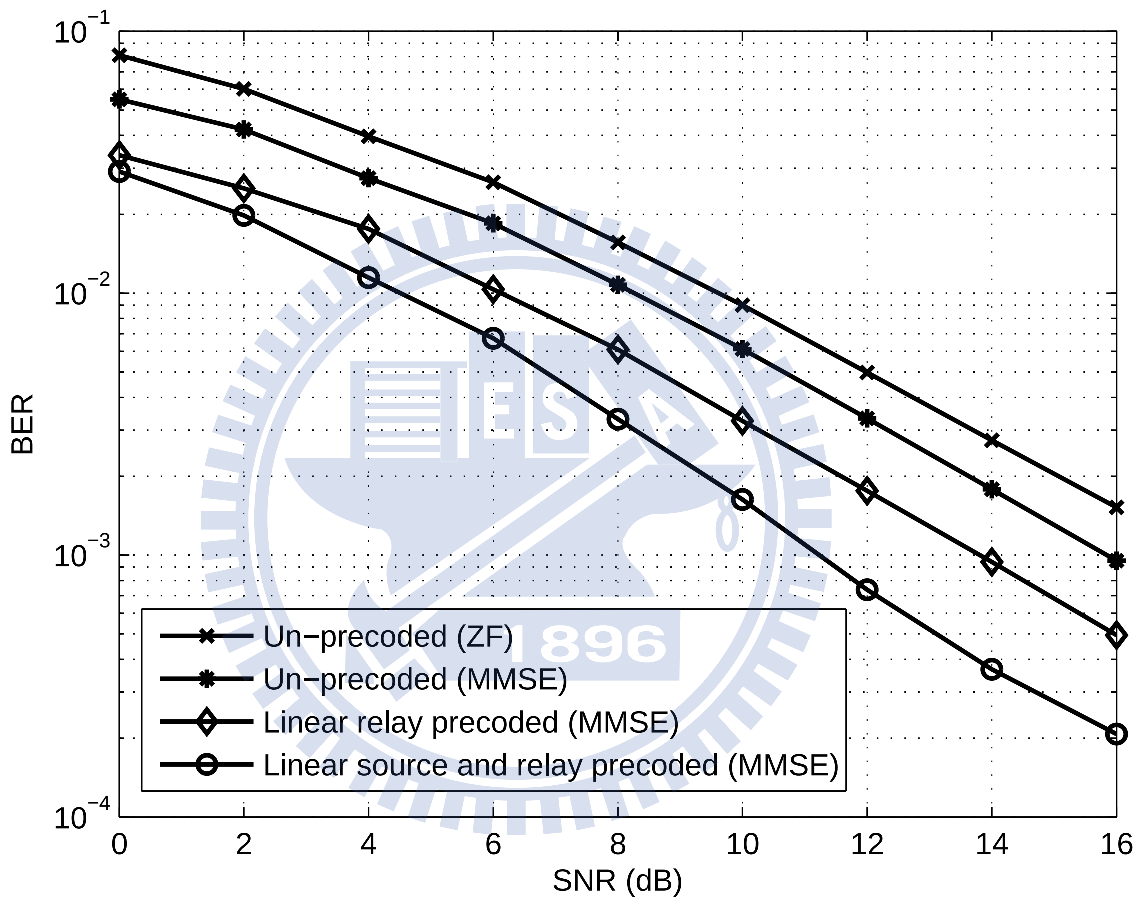


Figure 2.8: BER performance comparison for existing un-precoded/precoded and linear source and relay precoded AF MIMO relay systems.

Chapter 3

Joint MMSE Transceiver Design with Tomlinson-Harashima Source and Linear Relay Precoders

In this chapter, we address the problem of the MMSE transceiver design with a nonlinear THP. In Section 3.1, we first formulate the precoded system model in which a THP cascaded with a linear precoder are used at the source, a linear precoder at the relay, and the MMSE receiver at the destination. As that in the previous section, we found that the MSE is a complicated function of the source and the relay precoders, and the corresponding optimization is difficult to conduct. In Section 3.2, we then propose a new method to overcome the problem. The main idea is to use the primal decomposition such that the two-precoder design problem can be translated into a single-relay precoder problem. With the proposed method, the optimization problem can be further expressed as a convex optimization problem, and the closed-form solution can be obtained by KKT conditions. Finally, we evaluate the performance of the proposed method in Section 3.3.

§ 3.1 System Model and Problem Formulation

§ 3.1.1 *MMSE Receiver with Tomlinson-Harashima Source and Linear Relay Precoders*

We consider the precoded three-node AF MIMO relay precoding system as shown in Fig. 3.1, where we include two precoders - a THP source precoder and a linear relay precoder $\mathbf{F}_R$, and a linear MMSE receiver, $\mathbf{G}$, is applied at the destination. Here, we also consider the general two-phase transmission protocol [41]- [45]. In the first phase, the source signal $\mathbf{s} \in \mathbb{C}^{N\times 1}$ is fed into the nonlinear THP in which a successive cancellation operation characterized by a backward squared matrix $\mathbf{B}$ and a modulo operation $\text{MOD}_m(\cdot)$. The source signals $\mathbf{s} = [s_1, \cdots, s_N]^T$ are modulated by m-QAM where the real and image parts of s_k as the set $\{\pm 1, \cdots, \pm(\sqrt{m} - 1)\}$. The feedback matrix $\mathbf{B}$ has a lower triangular structure and the diagonal elements are all zeros. The modulo operation acts over the real and image parts of the inputs, respectively, is expressed as follows:

$$\text{MOD}_m(x) = x - 2\sqrt{m}\lfloor \frac{x + \sqrt{m}}{2\sqrt{m}} \rfloor. \tag{3.1}$$

It is clear that the transmitted signal $\mathbf{x}$ is bounded between $-\sqrt{m}$ and $\sqrt{m}$. With $\mathbf{B}$ and the operation in (3.1), the elements of $\mathbf{x}$ can be recursively expressed as [16]

$$x_k = s_k - \sum_{l=1}^{k-1} \mathbf{B}(k,l)x_l + \mathbf{e}_k \tag{3.2}$$

where x_k is the kth elements of vector $\mathbf{x}$ and $\mathbf{B}(k,l)$ is the (k,l) element of matrix $\mathbf{B}$; $\mathbf{e} = [e_1, \ldots, e_N]^T$ denotes the errors of the modulo operation (the difference of the input and the output). From (3.2), we can reformulate the transmitted signal $\mathbf{x}$ after THP with the following matrix form

$$\mathbf{x} = \mathbf{C}^{-1}\mathbf{v} \tag{3.3}$$

where $\mathbf{C} = \mathbf{B} + \mathbf{I}_N$ is a lower triangular with ones in its diagonal, and $\mathbf{v} = \mathbf{s} + \mathbf{e}$. The THP precoded $\mathbf{x}$ is then passed through a unitary precoder matrix $\mathbf{F}_S$ and subsequently sent to the relay and the destination simultaneously. The unitary precoder, as we will see, can greatly facilitate the joint precoders design and improve the BER performance.

In the second phase, the received signal at the relay is multiplied the relay precoder and then is transmitted to the destination. Therefore, the signal received at the destination in the two consecutive phases can be expressed as a vector form as

$$\mathbf{y}_D := \underbrace{\begin{bmatrix} \mathbf{H}_{SD} \\ \mathbf{H}_{RD}\mathbf{F}_R\mathbf{H}_{SR} \end{bmatrix}}_{:=\mathbf{H}} \mathbf{F}_S\mathbf{x} + \underbrace{\begin{bmatrix} \mathbf{n}_{D,1} \\ \mathbf{H}_{RD}\mathbf{F}_R\mathbf{n}_R + \mathbf{n}_{D,2} \end{bmatrix}}_{:=\mathbf{n}} \tag{3.4}$$

where $\mathbf{H}$ and $\mathbf{n}$ denote the equivalent channel matrix and the equivalent noise vector, respectively, as the same definition in (2.4). In (3.4), $\mathbf{x} \in \mathbb{C}^{N\times 1}$ is the THP precoded signal vector (3.3); $\mathbf{y}_D \in \mathbb{C}^{2M\times 1}$ is the received signal vector at the destination. Note that if $\mathbf{v}$ can be estimated at the destination, $\mathbf{s}$ can then be recovered by the modulo operation in (3.1). Thus, the optimum $\mathbf{G} \in \mathbb{C}^{2M\times N}$ can be found by minimizing the MSE defined as

$$J = E\left\{\|\mathbf{G}\mathbf{y}_D - \mathbf{v}\|^2\right\}. \tag{3.5}$$

To solve the problem in (3.5), we assume that the precoded signal $\mathbf{x}_k$'s are statistically independent and they have the zero-mean and the same variance. Let the variance of each element in $\mathbf{s}$ be denoted as σ_s^2. We then have $E\left[\mathbf{x}\mathbf{x}^H\right] = \sigma_s^2\mathbf{I}_N$ and $E\left[\mathbf{v}\mathbf{v}^H\right] = \sigma_s^2\mathbf{C}\mathbf{C}^H$. It is noted that the assumption is valid when the QAM size is large ($m \geq 16$) [15], [16]. Then, the optimum solution of (3.5) can be obtained as [16]

$$\mathbf{G}_{opt} = \sigma_s^2\mathbf{C}\mathbf{F}_S^H\mathbf{H}^H\left(\sigma_s^2\mathbf{H}\mathbf{F}_S\mathbf{F}_S^H\mathbf{H}^H + \mathbf{R}_n\right)^{-1}, \tag{3.6}$$

where $\mathbf{G}_{opt}$ is the optimum $\mathbf{G}$; $\mathbf{R}_n = E[\mathbf{n}\mathbf{n}^H]$ is the covariance matrix of the equivalent noise vector $\mathbf{n}$, as also defined in (2.6). Considering the noise components $\sigma_{n,d}^2$ and $\sigma_{n,r}^2$ in (3.6) and substituting (3.6) in (3.5), we can have the MSE matrix

$$\begin{aligned} \mathbf{E} &= \mathbf{C}\left(\sigma_s^2\mathbf{I}_N + \mathbf{F}_S^H\mathbf{H}^H\mathbf{R}_n^{-1}\mathbf{H}\mathbf{F}_S\right)^{-1}\mathbf{C}^H \\ &= \mathbf{C}\left(\sigma_s^2\mathbf{I}_N + \mathbf{F}_S^H\widetilde{\mathbf{H}}^H\widetilde{\mathbf{H}}\mathbf{F}_S\right)^{-1}\mathbf{C}^H \end{aligned} \tag{3.7}$$

and

$$J_{min} = tr\{\mathbf{E}\} \tag{3.8}$$

where

$$\begin{aligned} \widetilde{\mathbf{H}} &= \mathbf{R}_n^{-\frac{1}{2}}\mathbf{H} \\ &= \begin{bmatrix} \sigma_{n,d}^{-1}\mathbf{H}_{SD} \\ \left(\sigma_{n,r}^2\mathbf{H}_{RD}\mathbf{F}_R\mathbf{F}_R^H\mathbf{H}_{RD}^H + \sigma_{n,d}^2\mathbf{I}_M\right)^{-\frac{1}{2}}\mathbf{H}_{RD}\mathbf{F}_R\mathbf{H}_{SR} \end{bmatrix} \end{aligned} \tag{3.9}$$

is defined as the equivalent channel matrix after noise whitening. Note that the MSE is contributed by both the direct and relay links. By ignoring the direct link and adopting a single precoder at the relay, the problem is reduced those considered in [43] and [44]. Here, we incorporate the THP as the source precoder and take the direct link into consideration. A significant performance enhancement can then be expected.

§ 3.1.2 *Problem Formulation*

From the MMSE criterion in (3.5)-(3.9), we now can formulate our joint design problem as:

$$\begin{aligned} &\min_{\mathbf{C},\mathbf{F}_S,\mathbf{F}_R} tr\left\{\underbrace{\mathbf{C}\left(\sigma_s^{-2}\mathbf{I}_N + \mathbf{F}_S^H\widetilde{\mathbf{H}}^H\widetilde{\mathbf{H}}\mathbf{F}_S\right)^{-1}\mathbf{C}^H}_{:=\mathbf{E}}\right\} \\ &s.t. \\ &\widetilde{\mathbf{H}}^H\widetilde{\mathbf{H}} = \sigma_{n,d}^{-2}\mathbf{H}_{SD}^H\mathbf{H}_{SD} + \\ &\mathbf{H}_{SR}^H\mathbf{F}_R^H\mathbf{H}_{RD}^H\left(\sigma_{n,r}^2\mathbf{H}_{RD}\mathbf{F}_R\mathbf{F}_R^H\mathbf{H}_{RD}^H + \sigma_{n,d}^2\mathbf{I}_M\right)^{-1}\mathbf{H}_{RD}\mathbf{F}_R\mathbf{H}_{SR} \\ &\mathbf{F}_S = \alpha\mathbf{U}_S \\ &tr\left\{E\left[\mathbf{F}_S\mathbf{x}\mathbf{x}^H\mathbf{F}_S^H\right]\right\} = \sigma_s^2 tr\left\{\mathbf{F}_S\mathbf{F}_S^H\right\} \le P_{S,T} \\ &tr\left\{\mathbf{F}_R\left(\sigma_{n,r}^2\mathbf{I}_R + \sigma_s^2\mathbf{H}_{SR}\mathbf{F}_S\mathbf{F}_S^H\mathbf{H}_{SR}^H\right)\mathbf{F}_R^H\right\} \le P_{R,T}, \end{aligned} \tag{3.10}$$

where the inequalities in (3.10) indicate the transmitted power constraints at source and relay (the maximal available power is $P_{S,T}$ and $P_{R,T}$, respectively). Here, we force $\mathbf{F}_S = \alpha\mathbf{U}_S$

in which α is a scalar and $\mathbf{U}_S$ is an unitary matrix. Taking a close look at (3.10), we can observe that the cost function and the power constraints are nonlinear functions of $\mathbf{F}_S$ and $\mathbf{F}_R$. Moreover, (3.10) is not a convex optimization problem. As a result, it is difficult to solve the problem, directly. In the next subsection, we propose a new approach to seek for a solution.

§ 3.2 Joint Source/Relay Precoders Design

We resort to the primal decomposition approach [51] translating (3.10) into a subproblem and a master problem. The subproblem is first optimized for the source precoder, and subsequently the master problem is optimized for the relay precoder. To proceed, we reformulate (3.10) as

$$\begin{aligned}
&\min_{\mathbf{C},\mathbf{F}_S,\mathbf{F}_R} tr\left\{E\right\} = \min_{\mathbf{F}_R}\min_{\mathbf{C},\mathbf{F}_S} tr\left\{E\right\}\\
&s.t.\\
&\mathbf{E} = \mathbf{C}\left(\sigma_s^{-2}\mathbf{I}_N + \alpha^2\mathbf{F}_S^H\widetilde{\mathbf{H}}^H\widetilde{\mathbf{H}}\mathbf{F}_S\right)^{-1}\mathbf{C}^H\\
&\widetilde{\mathbf{H}}^H\widetilde{\mathbf{H}} \text{ in } (3.10),\\
&\mathbf{F}_S = \alpha\mathbf{U}_S\\
&\sigma_s^2 tr\left\{\mathbf{F}_S\mathbf{F}_S^H\right\} \le P_{S,T}\\
&tr\left\{\mathbf{F}_R\left(\sigma_{n,r}^2\mathbf{I}_R + \alpha^2\sigma_s^2\mathbf{H}_{SR}\mathbf{H}_{SR}^H\right)\mathbf{F}_R^H\right\} \le P_{R,T}.
\end{aligned} \tag{3.11}$$

In the subproblem, the relay precoder $\mathbf{F}_R$ is assumed to be given. Then, the optimum $\mathbf{C}$ and $\mathbf{F}_S$ can first be derived as a function of $\mathbf{F}_R$. Therefore, the joint precoders design is reduced to the master optimization problem in which the optimum relay precoder remains to be determined.

Since the unitary $\mathbf{F}_S = \alpha\mathbf{U}_S$, the subproblem thus becomes optimizing α, $\mathbf{U}_S$ and $\mathbf{C}$, given as

$$\begin{aligned}
&\min_{\mathbf{C}(\mathbf{F}_R),\alpha,\mathbf{U}_S(\mathbf{F}_R)} tr\left\{\mathbf{E}\right\} \\
&s.t. \\
&\mathbf{E} = \mathbf{C}\left(\sigma_s^{-2}\mathbf{I}_N + \alpha^2\mathbf{U}_S^H\widetilde{\mathbf{H}}^H\widetilde{\mathbf{H}}\mathbf{U}_S\right)^{-1}\mathbf{C}^H \\
&N\sigma_s^2\alpha^2 \leq P_{S,T} \\
&tr\left\{\mathbf{F}_R\left(\sigma_{n,r}^2\mathbf{I}_R + \alpha^2\sigma_s^2\mathbf{H}_{SR}\mathbf{H}_{SR}^H\right)\mathbf{F}_R^H\right\} \leq P_{R,T}.
\end{aligned} \tag{3.12}$$

To find the solutions in (3.12), we first fixed $\mathbf{U}_S$ and $\mathbf{C}$, finding optimum α, denoted α_{opt}. The solution can be easily obtain as

$$\alpha_{opt} = \sqrt{\frac{P_{S,T}}{N\sigma_s^2}}. \tag{3.13}$$

This is because the cost function is a strictly decreasing function in α, we enlarge α with satisfying the source power constraint. In this manner, α_{opt} can also maximize the SINR at the relay, reducing the noise enhancement at the relay node and thus minimizing the MSE value. The subproblem thus becomes

$$\min_{\mathbf{C}(\mathbf{F}_R),\mathbf{U}_S(\mathbf{F}_R)} tr\left(\mathbf{C}\left(\sigma_s^{-2}\mathbf{I}_N + \frac{P_{S,T}}{N\sigma_s^2}\mathbf{U}_S^H\widetilde{\mathbf{H}}^H\widetilde{\mathbf{H}}\mathbf{U}_S\right)^{-1}\mathbf{C}^H\right). \tag{3.14}$$

The resultant relay power constraint $tr\left\{\mathbf{F}_R\left(\sigma_{n,r}^2\mathbf{I}_R + \frac{P_{S,T}}{N}\mathbf{H}_{SR}\mathbf{H}_{SR}^H\right)\mathbf{F}_R^H\right\} \leq P_{R,T}$, is removed to the master problem since it is only the function of the relay precoder.

Fortunately, without considering the relay precoder, the problem in (3.14) has been addressed in non-cooperative MIMO system [15], [16], and the optimum solutions can be expressed as

$$\mathbf{C}_{opt} = \mathbf{D}\mathbf{L}^{-1}, \tag{3.15}$$

$$\mathbf{F}_{S,opt} = \sqrt{\frac{P_{S,T}}{N\sigma_s^2}}\mathbf{V}_{\widetilde{\mathbf{H}}}\mathbf{U}'_S, \tag{3.16}$$

where

$$\mathbf{L}\mathbf{L}^H = \left(\sigma_s^{-2}\mathbf{I}_N + \frac{P_{S,T}}{N\sigma_s^2}\left(\mathbf{V}_{\widetilde{\mathbf{H}}}\mathbf{U}'_S\right)^H\widetilde{\mathbf{H}}^H\widetilde{\mathbf{H}}\mathbf{V}_{\widetilde{\mathbf{H}}}\mathbf{U}'_S\right)^{-1} \tag{3.17}$$

is the Cholesky factorization of $\left(\sigma_s^{-2}\mathbf{I}_N + \frac{P_{S,T}}{N\sigma_s^2}\left(\mathbf{V}_{\widetilde{\mathbf{H}}}\mathbf{U}'_S\right)^H \widetilde{\mathbf{H}}^H\widetilde{\mathbf{H}}\mathbf{V}_{\widetilde{\mathbf{H}}}\mathbf{U}'_S\right)^{-1}$; $\mathbf{D}$ is a diagonal matrix that scales the elements on the diagonal of $\mathbf{C}$ to unity; $\mathbf{V}_{\widetilde{\mathbf{H}}} \in \mathbb{C}^{N\times N}$ is the left singular matrices of $\widetilde{\mathbf{H}}$; $\mathbf{U}'_S \in \mathbb{C}^{N\times N}$ is an unitary matrix that needs to be further specified. Substituting (3.16) into (3.17), we have

$$\mathbf{L}\mathbf{L}^H = \mathbf{U}'^H_S \underbrace{\left(\sigma_s^{-2}\mathbf{I}_N + \frac{P_{S,T}}{N\sigma_s^2}\mathbf{\Lambda}\right)^{-1}}_{:=\widetilde{\mathbf{D}}} \mathbf{U}'_S \tag{3.18}$$

where $\mathbf{\Lambda} = diag\left\{\lambda_{\widetilde{\mathbf{H}},1}, \ldots, \lambda_{\widetilde{\mathbf{H}},N}\right\}$ is the eigenvalues of $\widetilde{\mathbf{H}}^H\widetilde{\mathbf{H}}$. It is simple to see that that $\widetilde{\mathbf{D}}$ is diagonal matrix here. Applying GMD on $\widetilde{\mathbf{D}}$, we can express $\widetilde{\mathbf{D}}$ as

$$\widetilde{\mathbf{D}}^{1/2} = \mathbf{Q}\mathbf{R}\mathbf{P}^H, \tag{3.19}$$

where $\mathbf{Q}$, $\mathbf{P}$ are unitary matrix and $\mathbf{R}$ is upper triangular matrix with equal diagonal elements. Letting $\mathbf{U}'_S = \mathbf{P}$ and substituting (3.15), (3.16) in (3.8), we then have the resultant MSE as

$$J_{\min} = \sum_{k=1}^{N} \mathbf{L}(k,k)^2 = N\prod_{k=1}^{N}\left(\frac{1}{\lambda_{\widetilde{\mathbf{H}},k}\frac{P_{S,T}}{N\sigma_s^2} + \sigma_s^{-2}}\right)^{1/N}. \tag{3.20}$$

Now, the problem becomes the minimization of (3.20) in the master problem. From (3.15)-(3.19), we note that the original THP precoding does not include the unitary $\mathbf{F}_S$ [15]. Here the including unitary $\mathbf{F}_S$ has two main concerns: *(i)* The additional unitary precoder can facilitate the relay power constraint, as described in (3.14), in solving the optimization. *(ii)* By adequately designing $\mathbf{U}_S$, we can make $\mathbf{L}(i,i) = \mathbf{L}(j,j), \forall i \neq j$ in (3.20). In this manner, the minimum MSE can be expressed as (3.20) and, most importantly, as we will see, optimizations with (3.20) are more tractable for optimization.

Now, our residual problem is to solve the master problem. To proceed, let us first see the following equivalence:

$$\min_{\mathbf{F}_R} N\prod_{k=1}^{N}\left(\frac{1}{\lambda_{\widetilde{\mathbf{H}},k}\frac{P_{S,T}}{N\sigma_s^2} + \sigma_s^{-2}}\right)^{1/N} = \max_{\mathbf{F}_R}\left(\sigma_s^{-2}\frac{P_{S,T}}{N}\right)^N \det\left(\left(\frac{N}{P_{S,T}}\mathbf{I}_N + \widetilde{\mathbf{H}}^H\widetilde{\mathbf{H}}\right)\right). \tag{3.21}$$

proof: The result can be easily obtained since

$$\det\left(\sigma_s^{-2}\mathbf{I}_N + \frac{P_{S,T}}{N\sigma_s^2}\widetilde{\mathbf{H}}^H\widetilde{\mathbf{H}}\right) = \prod_{k=1}^{N}\left(\frac{P_{S,T}}{N\sigma_s^2}\lambda_{\widetilde{\mathbf{H}},k} + \sigma_s^{-2}\right). \tag{3.22}$$

Considering (3.21) and ignoring $\left(\sigma_s^{-2}\frac{P_{S,T}}{N}\right)^N$ in the master problem, we then reformulate the optimizations as

$$\begin{aligned}
&\max_{\mathbf{F}_R} \det\left(\left(\frac{N}{P_{S,T}}\mathbf{I}_N + \widetilde{\mathbf{H}}^H\widetilde{\mathbf{H}}\right)\right)\\
&s.t\\
&\widetilde{\mathbf{H}}^H\widetilde{\mathbf{H}} \text{ in } (3.10)\\
&tr\left\{\mathbf{F}_R\left(\sigma_{n,r}^2\mathbf{I}_R + \frac{P_{S,T}}{N}\mathbf{H}_{SR}\mathbf{H}_{SR}^H\right)\mathbf{F}_R^H\right\} \le P_{R,T}.
\end{aligned} \tag{3.23}$$

To solve (3.23), we use the Hardamard inequality, described in the following Lemma.

Lemma 3.1 [50]: Let $\mathbf{M} \in \mathbb{C}^{N\times N}$ be a positive definite matrix, then

$$\det(\mathbf{M}) \le \prod_{i=1}^{N}\mathbf{M}(i,i), \tag{3.24}$$

where $\mathbf{M}(i,i)$ denotes the ith diagonal element of $\mathbf{M}$. The equality in (3.24) holds when $\mathbf{M}$ is a diagonal matrix. If we let $\mathbf{M} = \widetilde{\mathbf{H}}^H\widetilde{\mathbf{H}}$, it turns out that when $\mathbf{M}$ is diagonalized, the cost function in (3.23) is maximized. Unfortunately, from (3.10) we can see that $\widetilde{\mathbf{H}}^H\widetilde{\mathbf{H}}$ is a summation of two separated matrices and one of them dose not depend on $\mathbf{F}_R$, and the diagonalization cannot be directly conducted. The following lemma suggests a feasible way to overcome the problem.

Lemma 3.2: Let $\mathbf{A} \in \mathbb{C}^{N\times N}$ be a positive matrix and $\mathbf{B} \in \mathbb{C}^{N\times N}$, then

$$\det(\mathbf{A}+\mathbf{B}) = \det(\mathbf{A})\det\left(\mathbf{I}_N + \mathbf{A}^{-1/2}\mathbf{B}\mathbf{A}^{-1/2}\right). \tag{3.25}$$

Proof: See Appendix A.3.

Form (3.25), we let $\mathbf{B} = \mathbf{H}_{SR}^H\mathbf{F}_R^H\mathbf{H}_{RD}^H\left(\sigma_{n,r}^2\mathbf{H}_{RD}\mathbf{F}_R\mathbf{F}_R^H\mathbf{H}_{RD}^H + \sigma_{n,d}^2\mathbf{I}_M\right)^{-1}\mathbf{H}_{RD}\mathbf{F}_R\mathbf{H}_{SR}$

and $\mathbf{A} = \frac{N}{P_{S,T}}\mathbf{I}_N + \sigma_{n,d}^{-2}\mathbf{H}_{SD}^H\mathbf{H}_{SD}$, we have the following equivalence

$$\begin{aligned}&\arg\max_{\mathbf{F}_R} \det\left(\frac{N}{P_{S,T}}\mathbf{I}_N + \widetilde{\mathbf{H}}^H\widetilde{\mathbf{H}}\right)\\&= \arg\max_{\mathbf{F}_R} \det\left(\mathbf{I}_N + \mathbf{H}_{SR}^{\prime H}\mathbf{F}_R^H\mathbf{H}_{RD}^H\right.\\&\left.\left(\sigma_{n,r}^2\mathbf{H}_{RD}\mathbf{F}_R\mathbf{F}_R^H\mathbf{H}_{RD}^H + \sigma_{n,d}^2\mathbf{I}_M\right)^{-1}\mathbf{H}_{RD}\mathbf{F}_R\mathbf{H}_{SR}^{\prime}\right)\end{aligned} \tag{3.26}$$

where $\mathbf{H}_{SR}^{\prime} = \mathbf{H}_{SR}\left(N/P_{S,T}\mathbf{I}_N + \sigma_{n,d}^{-2}\mathbf{H}_{SD}^H\mathbf{H}_{SD}\right)^{-\frac{1}{2}}$ and $\det\mathbf{A}$ are ignored since they are not functions of $\mathbf{F}_R$. Equation (3.26) provides a feasible way to diagonalize the cost function. The optimization problem in (3.23) can now be reformulated as

$$\begin{aligned}&\max_{\mathbf{F}_R} \det(\mathbf{M})\\&s.t.\\&\mathbf{M}^{\prime} = \left(\mathbf{I}_N + \mathbf{H}_{SR}^{\prime H}\mathbf{F}_R^H\mathbf{H}_{RD}^H\left(\sigma_{n,r}^2\mathbf{H}_{RD}\mathbf{F}_R\mathbf{F}_R^H\mathbf{H}_{RD}^H + \sigma_{n,d}^2\mathbf{I}_M\right)^{-1}\mathbf{H}_{RD}\mathbf{F}_R\mathbf{H}_{SR}^{\prime}\right)\\&\mathbf{M}^{\prime}\ \text{ is diagonal}\\&\sigma_{n,r}^2\|\mathbf{F}_R\|_2^2 + \frac{P_{S,T}}{N}\|\mathbf{F}_R\mathbf{H}_{SR}\|_2^2 \le P_{R,T}.\end{aligned} \tag{3.27}$$

There exists certain structure for the relay precoder such that the diagonalization can be achieved. Consider following SVD:

$$\mathbf{H}_{RD} = \mathbf{U}_{rd}\boldsymbol{\Sigma}_{rd}\mathbf{V}_{rd}^H \tag{3.28}$$

$$\mathbf{H}^{\prime}_{SR} = \mathbf{U}^{\prime}_{sr}\boldsymbol{\Sigma}^{\prime}_{sr}\mathbf{V}^{\prime H}_{sr} \tag{3.29}$$

where $\mathbf{U}_{rd} \in \mathbb{C}^{M\times M}$ and $\mathbf{U}_{sr}^{\prime} \in \mathbb{C}^{R\times R}$ are left singular matrices of $\mathbf{H}_{RD}$ and $\mathbf{H}_{SR}^{\prime}$, respectively; $\boldsymbol{\Sigma}_{rd} \in \mathbb{R}^{M\times R}$ and $\boldsymbol{\Sigma}_{sr}^{\prime} \in \mathbb{R}^{R\times N}$ are the diagonal singular value matrices of $\mathbf{H}_{RD}$ and $\mathbf{H}_{SR}^{\prime}$, respectively; $\mathbf{V}_{rd}^H \in \mathbb{C}^{R\times R}$ and $\mathbf{V}_{sr}^{\prime H} \in \mathbb{C}^{N\times N}$ are the right singular matrices of $\mathbf{H}_{RD}$ and $\mathbf{H}_{SR}^{\prime}$, respectively; We found that if the optimal $\mathbf{F}_R$ have the following structure, a full diagonalization of $\mathbf{M}^{\prime}$ can be achieved:

$$\mathbf{F}_{R,opt} = \mathbf{V}_{rd}\boldsymbol{\Sigma}_r\mathbf{U}_{sr}^{\prime H}, \tag{3.30}$$

where $\boldsymbol{\Sigma}_r$ is a diagonal matrix with ith diagonal element $\sigma_{r,i}$, $i = 1, \cdots, \kappa$, yet to be determined. Here, $\kappa = min\{N, R\}$. Let $\sigma_{rd,i}$ and $\sigma_{sr,i}^{\prime}$ be the ith diagonal element of $\boldsymbol{\Sigma}_{rd}$ and $\boldsymbol{\Sigma}_{sr}^{\prime}$, respectively. Substituting (3.28), (3.29) and (3.30) into (3.27) and taking the ln operation to the cost function, we can then rewrite (3.27) as:

$$\begin{aligned}
&\max_{p_{r,i}, 1 \le i \le \kappa} \sum_{i=1}^{\kappa} \ln\left(1 + \frac{p_{r,i}\sigma_{n,d}^2 \sigma_{rd,i}^2 \sigma_{sr,i}'^2}{p_{r,i}\sigma_{n,r}^2 \sigma_{rd,i}^2 + \sigma_{n,d}^2}\right) \\
&s.t. \\
&\sum_{i=1}^{\kappa} p_{r,i}\left(\frac{P_{S,T}}{N}\sigma_{sr,i}'^2 \mathbf{D}'_{sr}(i,i) + \sigma_{n,r}^2\right) \le P_{R,T}, p_{r,i} \ge 0,
\end{aligned} \tag{3.31}$$

where $p_{r,i} = \sigma_{r,i}^2$ and $\mathbf{D}'_{sr} = \mathbf{V}'^H_{sr}\left(N/P_{S,T}\mathbf{I}_N + \sigma_{n,d}^{-2}\mathbf{H}_{SD}^H \mathbf{H}_{SD}\right)\mathbf{V}'_{sr}$ with $\mathbf{D}'_{sr}(i,i)$ being the ith diagonal element of $\mathbf{D}'_{sr}$. The cost function now is simplified to a function of scalar parameters. Since the cost function and the inequalities are all concave for $p_{r,i} \ge 0$ [51], (3.31) is a standard concave optimization problem. As a result, the optimal solutions $p_{r,i}$, $i = 1, \ldots, \kappa$, can be solved by means of KKT conditions given by

$$\begin{aligned}
p_{r,i} \;=\; &\left[\sqrt{\frac{\mu}{\sigma_{rd,i}^2\left(\frac{P_{S,T}}{N}\sigma'^2_{sr,i}\mathbf{D}'_{sr}(i,i) + \sigma_n^2\right)\left(\sigma_{n,r}^2\sigma_{n,d}^{-2}\sigma'^{-2}_{sr,i} + 1\right)} + \frac{\frac{\sigma_{n,d}^4}{4\sigma_{n,r}^4}}{\sigma_{rd,i}^4\left(\frac{\sigma_{n,r}^2}{\sigma_{n,d}^2\sigma'^2_{sr,i}} + 1\right)^2}}\right. \\
&\left. - \frac{1 + \frac{\sigma_{n,d}^2\sigma'^2_{sr,i}}{2\sigma_{n,r}^2}}{\sigma_{rd,i}^2\left(\frac{\sigma_{n,r}^2}{\sigma_{n,d}^2} + \sigma'^2_{sr,i}\right)}\right]^+,
\end{aligned} \tag{3.32}$$

where μ is chosen to satisfy the power constraint in (3.31). We have also proposed a water-filling algorithm to solve (3.32). The detailed derivations of (3.32) and the water-filling algorithm are given in Appendix A.4 and A.5. Substituting (3.32) into (3.30), we can finally obtain the optimum relay precoder. With the relay precoder, $\widetilde{\mathbf{H}}$ in (3.9) can be obtained. Subsequently, the unitary source prefilter can be derived by substituting (3.19) into (3.16) and $\mathbf{C}$ can be obtained by (3.15). The computations of the THP source and linear relay precoders mainly involve SVD, GMD, and matrix inversion operations. The overall computational complexity, measured in terms of FLOPs, is summarized in Table 3.1.

Table 3.1: Computational Complexity of THP source and linear relay precoders (MMSE receiver).

Step	Operation	FLOPs
1	$\mathbf{H}'_{SR}$ (3.27)	$O(N^3 + RN^2)$
2	SVD $\mathbf{H}_{RD} = \mathbf{U}_{rd}\mathbf{\Sigma}_{rd}\mathbf{V}_{rd}^H$ (3.28)	$O(MR^2 + R^3)$
3	SVD $\mathbf{H}'_{SR} = \mathbf{U}'_{sr}\mathbf{\Sigma}'_{sr}\mathbf{V}'^H_{sr}$ (3.29)	$O(RN^2 + N^3)$
4	$\mathbf{\Sigma}_r$ (3.32)	$O(\kappa I_r)$
5	$\mathbf{F}_R$ (3.30)	$O(R^3)$
6	SVD $\widetilde{\mathbf{H}}$	$O(MN^2 + N^3)$
7	GMD $\widetilde{\mathbf{D}}^{1/2} = \mathbf{QRP}^H$ (3.19)	$O(N^3)$
8	$\mathbf{L}$ (3.18)	$O(N^3)$
9	$\mathbf{C}_{opt}$ (3.15)	$O(N^3)$
10	$\mathbf{F}_{S,opt}$ (3.16)	$O(N^3)$

I_r is denoted as the iteration number of the water-filling process in (3.32).

§ 3.3 Simulations

We consider an AF MIMO relay system with N=R=M=4. The elements of each channel matrix are assumed to be i.i.d. complex Gaussian random variables with zero-mean and unity variance. Here, we let SNR$_{sr}$=15 dB, SNR$_{rd}$=15 dB and vary SNR$_{sd}$=15. Also, we use 16-QAM for each transmitted symbols. Fig. 3.2 and Fig. 3.3 show the MSE and BER performances comparison, respectively, for (a) an un-precoded system with the MMSE receiver, (b) the optimum relay precoded system with MMSE receiver [43], (c) the linear source and relay precoded with MMSE receiver in Chapter 2, and (d) the THP source and linear relay precoded with MMSE receiver in Chapter 3. Note that optimum relay precoder in [43] only considers the relay link. For better performance, we further include the direct link when implementing the MMSE receiver. As we can see, the proposed THP precoded system significantly outperforms other methods. Although two precoders are used in Chapter 2, the performance is limited. This is because both precoders are linear.

Table 3.2: Proposed water-filling algorithm solving (3.32)

$\mu_M = \mu_{M,0}$, $\mu_L = \mu_{L,0}$, δ_μ

while $\delta_\mu > \epsilon$

  $\mu = \frac{\mu_M + \mu_L}{2}$

  if $\sum_{i=1}^{\kappa} \left[\sqrt{a_i \left(\mu + \frac{b_i}{a_i} \right)} - c_i \right]^+ d_i \leq P_{R,T}$

   $\mu_L = \mu$

  else

   $\mu_M = \mu$

  end

  $\mu' = \frac{\mu_M + \mu_L}{2}$, $\delta_\mu = |\mu' - \mu|$

end

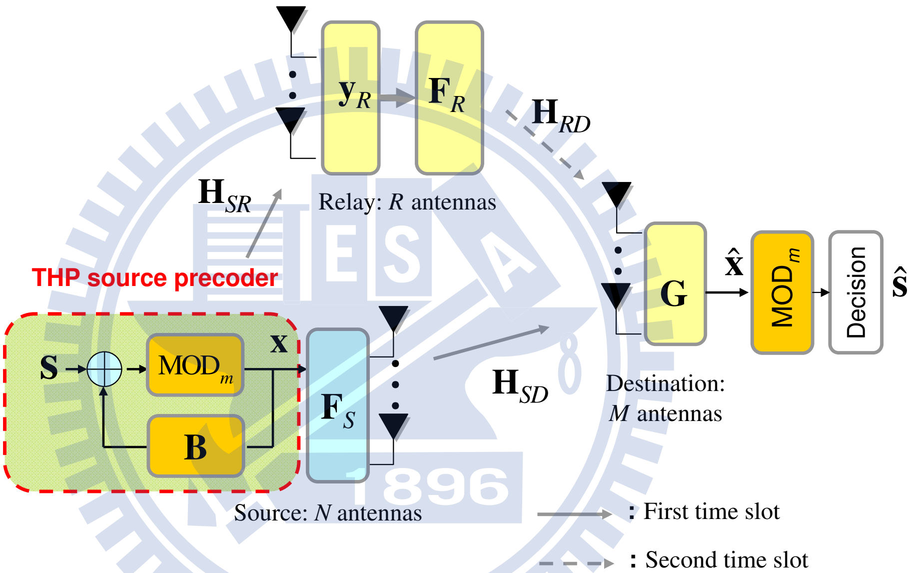


Figure 3.1: THP source and linear relay precoded AF MIMO relay system with MMSE receiver.

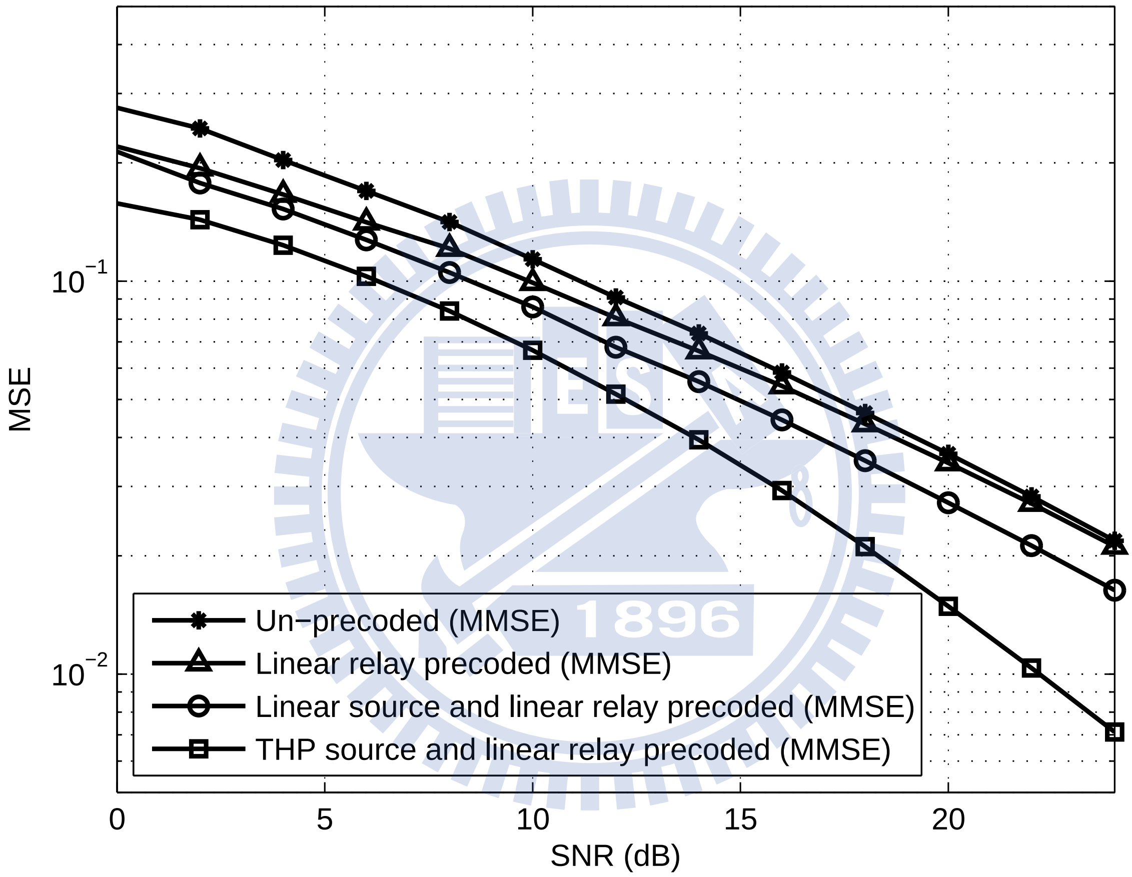


Figure 3.2: MSE performance comparison for existing precoded systems and THP source and linear relay precoded system with MMSE receiver.

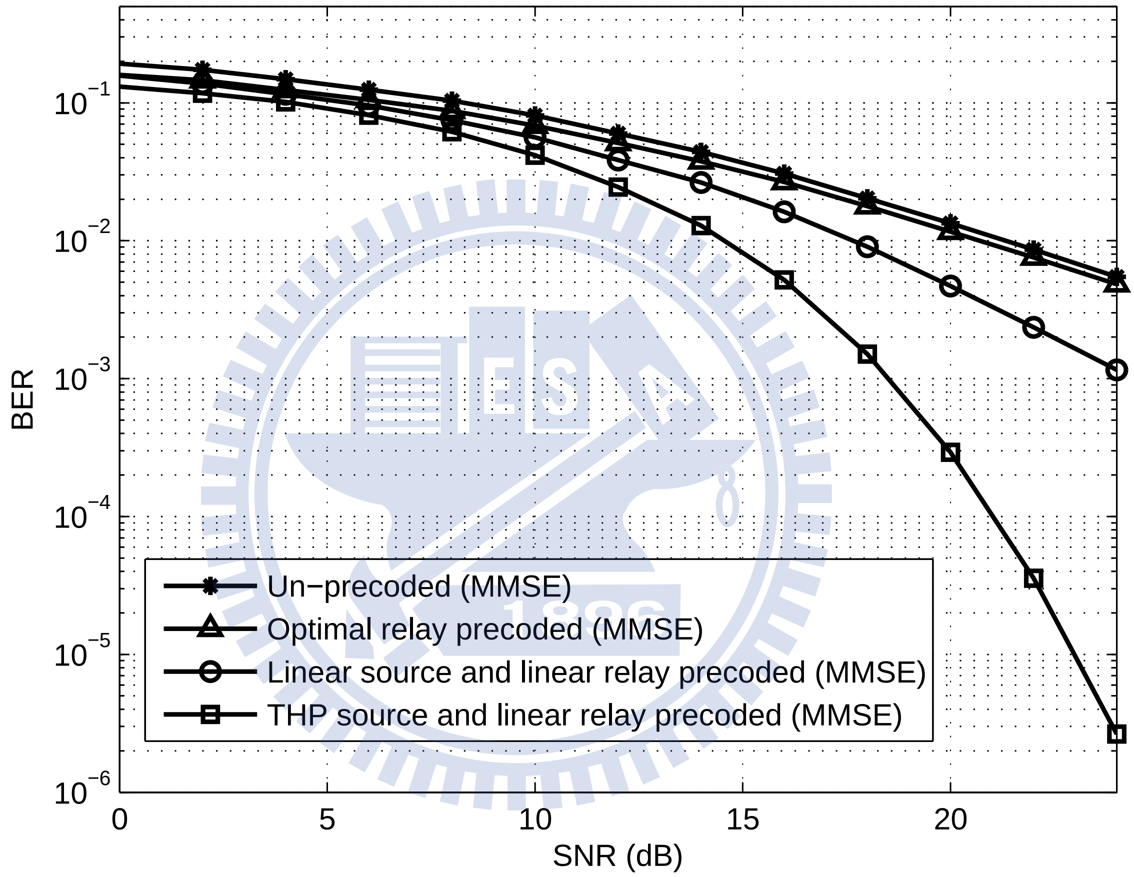


Figure 3.3: BER performance comparison for existing precoded systems and THP source and linear relay precoded system with MMSE receiver.

Chapter 4

Joint QR-SIC Transceiver Design with Linear Source and Relay Precoders

We have addressed the precoded AF MIMO relay system with the linear MMSE receiver in the previous two chapters. In this chapter, we study the precoded system with a nonlinear receiver. In general, nonlinear receivers require higher computational complexity. One exception is the QR-SIC receiver. It is known that the QR-SIC receiver has good performance while its computational complexity is low. Therefore, we consider the precoded system with the linear precoders at the source and the relay, and the QR-SIC receiver at the destination. In Section 4.1, we give the system model accommodating the QR-SIC receiver. In Section 4.2, we propose a GMD related method to derive the source and relay precoders. Finally, we report simulation results in Section 4.3 to evaluate the performance of the proposed method.

§ 4.1 System Model and Problem Formulation

§ 4.1.1 *QR-SIC Receiver with Linear Source and Relay Precoders*

Recall that the received signals with linear source and relay precoders are expressed as (2.4)

$$\mathbf{y}_D = \mathbf{H}\mathbf{F}_S\mathbf{s} + \mathbf{n}, \tag{4.1}$$

where

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_{SD} \\ \mathbf{H}_{RD}\mathbf{F}_R\mathbf{H}_{SR} \end{bmatrix}, \ \mathbf{n} = \begin{bmatrix} \mathbf{n}_{D,1} \\ \mathbf{H}_{RD}\mathbf{F}_R\mathbf{n}_R + \mathbf{n}_{D,2} \end{bmatrix}. \tag{4.2}$$

Here, particularly, we assume that $L = N \leq M$. This assumption can guarantee the existence of the solution of the proposed method (see Lemma 3.1). For the case of $L < N$, we can apply the antenna selection technique, to be discussed in Section 4.2.2. As previous mentioned, the equivalent channel matrix $\mathbf{H}$ does not include the source precoder.

By the same statistical assumptions in (2.6), we can also find that the equivalent noise vector is not white. To facilitate later analysis of the QR-SIC receiver, we first apply a whitening operation to the equivalent receive vector. Let $\mathbf{W}$ be a whitening matrix. Multiplying (4.1) with $\mathbf{W}$, we can have

$$\tilde{\mathbf{y}}_D := \mathbf{W}\mathbf{y}_D = \tilde{\mathbf{H}}\mathbf{F}_S\mathbf{s} + \tilde{\mathbf{n}}, \tag{4.3}$$

where $\tilde{\mathbf{H}} = \mathbf{W}\mathbf{H}$ and $\tilde{\mathbf{n}} = \mathbf{W}\mathbf{n}$. By the whitening, we have $E\left[\tilde{\mathbf{n}}\tilde{\mathbf{n}}^H\right] = E\left[\mathbf{W}\mathbf{n}\mathbf{n}^H\mathbf{W}^H\right] = \mathbf{I}_{2M}$. From (2.6) and (4.3), we can then obtain the whitening matrix as

$$\mathbf{W} = \begin{bmatrix} \sigma_{n,d}^{-1}\mathbf{I}_M & \mathbf{0} \\ \mathbf{0} & \left(\sigma_{n,r}^2\mathbf{H}_{RD}\mathbf{F}_R\mathbf{F}_R^H\mathbf{H}_{RD}^H + \sigma_{n,d}^2\mathbf{I}_M\right)^{-1/2} \end{bmatrix}. \tag{4.4}$$

The equivalent channel matrix after the whitening process can then be written as

$$\tilde{\mathbf{H}} = \mathbf{W}\mathbf{H} = \begin{bmatrix} \sigma_{n,d}^{-1}\mathbf{H}_{SD} \\ \left(\sigma_{n,r}^2\mathbf{H}_{RD}\mathbf{F}_R\mathbf{F}_R^H\mathbf{H}_{RD}^H + \sigma_{n,d}^2\mathbf{I}_M\right)^{-1/2}\mathbf{H}_{RD}\mathbf{F}_R\mathbf{H}_{SR} \end{bmatrix}. \tag{4.5}$$

From (4.3), we can see that an AF MIMO relay system can be regarded as a MIMO system with the channel matrix shown in (4.5). However, note that the channel in (4.5) is a function of the relay precoder, and this is quite different from the scenario considered in conventional MIMO systems. Since $\mathbf{F}_R$ is unknown, existing design methods in MIMO systems cannot directly be applied.

It is well-known that nonlinear MIMO receivers perform better than linear receivers though their complexity may be higher. In this chapter, we consider a computationally efficient non-linear receiver, the QR-SIC receiver. In the receiver, the equivalent channel of the precoded system is first factorized by the QR method, i.e. $\tilde{\mathbf{H}}\mathbf{F}_S = \mathbf{QR}$, where $\mathbf{Q}$ is a $2M \times 2M$ orthogonal matrix, and $\mathbf{R}$ is a $2M \times N$ upper triangular matrix. Equation (4.3) can then be rewritten as

$$\begin{aligned} \hat{\mathbf{y}}_D &= \mathbf{Q}^H\tilde{\mathbf{y}}_\mathbf{D} = \mathbf{Q}^H\mathbf{QRs} + \mathbf{Q}^H\tilde{\mathbf{n}} = \mathbf{Rs} + \hat{\mathbf{n}} \\ &= \underbrace{\begin{bmatrix} r_{1,1} & r_{1,2} & \cdots & r_{1,N} \\ 0 & r_{2,2} & \cdots & r_{2,N} \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & r_{N,N} \\ 0 & \cdots & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & \cdots & 0 \end{bmatrix}}_{:=\mathbf{R}} \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_N \end{bmatrix} + \underbrace{\begin{bmatrix} \hat{n}_1 \\ \hat{n}_2 \\ \vdots \\ \hat{n}_N \\ \hat{n}_{N+1} \\ \vdots \\ \hat{n}_{2M} \end{bmatrix}}_{:=\hat{\mathbf{n}}}, \end{aligned} \tag{4.6}$$

where $\hat{\mathbf{n}} = \mathbf{Q}^H\tilde{\mathbf{n}}$ and $E\left[\hat{\mathbf{n}}\hat{\mathbf{n}}^H\right] = \mathbf{I}_{2M}$. Note that the equivalent channel for QR factorization here includes the source precoder. Thus, signal detection of a QR-SIC receiver can then be conducted as:

$$\begin{aligned} &\text{for } i = N : -1 : 1 \\ &\hat{s}_i = Dec\left[\left(\hat{y}_i - \sum_{j=i+1}^{N} r_{i,j}\hat{s}_j\right)/r_{i,i}\right], \\ &\text{end} \end{aligned} \tag{4.7}$$

where $Dec[\cdot]$ denotes the decision operation, $\hat{y}_i$ the ith element of $\hat{\mathbf{y}}_D$, and $\hat{s}_j$ the estimation of the jth transmitted symbol.

§ 4.1.2 *Problem Formulation*

In the transceiver design, the most desirable criterion we want to use is the minimum BER. However, for the QR-SIC receiver, the precoders which can minimize the BER is very difficult to design. Fortunately, for a MIMO system with CSI available at the transmitter, [11] and [12] propose a well-known precoder design method, called GMD. In this approach, the precoded MIMO channel, is first QR factorized. Then, the precoding matrix is designed such that the diagonal elements of $\mathbf{R}$ in (4.6) are made equal and maximized. It has been proven that GMD can minimize the BLER, and maximize the lower bound of channel's free distance [12]. As known, the free distance is the metric used in the MLD. This implies the GMD method can also improve the performance of the MLD.

So, in this section, we adopt the GMD design criterion to solve the precoding problem in AF MIMO relay systems. By this manner, we have the following advantages: (*i*) The BLER is minimized. (*ii*) As we will see, the GMD can facilitate the optimizations. Specifically, we can transfer the joint source and relay precoders optimization to the relay-only optimization problem where the source recoder becomes the function of the relay precoder. We can then pose the similar optimization processes described in Section 3.2 to seek the closed-form solutions. We give the following demonstration for detailed.

With GMD, the source and relay precoders are derived such that the diagonal elements of $\mathbf{R}$ in (4.6) is equal and maximized. Note that the problem is much more involved than that in conventional MIMO systems, where only a source precoder is considered. Let the channel matrix $\tilde{\mathbf{H}}$ in (4.5) have a rank of N. Treating the channel matrix $\tilde{\mathbf{H}}$ in (4.5) as a conventional MIMO channel matrix, we can apply the GMD method and obtain the following factorization [11], [12]:

$$\tilde{\mathbf{H}} = \tilde{\mathbf{Q}}\tilde{\mathbf{R}}\tilde{\mathbf{P}}^H, \tag{4.8}$$

where $\tilde{\mathbf{Q}} \in \mathbb{C}^{2M\times 2M}$ and $\tilde{\mathbf{P}} \in \mathbb{C}^{N\times N}$ are unitary matrices, and $\tilde{\mathbf{R}} \in \mathbb{C}^{2M\times N}$ is an upper triangular matrix having identical diagonal elements given by

$$\tilde{r}_{i,i} = \left(\prod_{k=1}^{N} |r_{k,k}|\right)^{1/N} = \left(\prod_{k=1}^{N} \sigma_{\tilde{\mathbf{H}},k}\right)^{1/N}, \text{ for all } i = 1, \cdots, N. \tag{4.9}$$

Here, $\tilde{r}_{i,i}$ is the ith diagonal element in $\tilde{\mathbf{R}}$, and $\sigma_{\tilde{\mathbf{H}},k} > 0$ is the kth nonzero singular value of $\tilde{\mathbf{H}}$. The source precoder can then be determined as

$$\mathbf{F}_S = \alpha\tilde{\mathbf{P}}, \tag{4.10}$$

where α is a scalar chosen to satisfy the power constraint at the source, i.e., $tr\left(\mathbf{F}_S E\left[\mathbf{s}\mathbf{s}^H\right]\mathbf{F}_S^H\right)$ = $\sigma_s^2 N\alpha^2 \leq P_{S,T}$ or equivalently $\alpha \leq \sqrt{P_{S,T}/(N\sigma_s^2)}$. Here, $P_{S,T}$ is the maximal available power at the source. From (4.6) and (4.9), we can see that the equivalent channel of the precoded system is then $\alpha\tilde{\mathbf{Q}}\tilde{\mathbf{R}}$, and that the larger the $\tilde{r}_{i,i}$, the larger SNR in the receiver we can obtain. Based on the GMD approach and the above observation, we can then formulate our design problem as

$$\begin{aligned}
&\max_{\mathbf{F}_S,\,\mathbf{F}_R} \alpha\tilde{r}_{i,i} = \alpha\left(\prod_{k=1}^{N}\sigma_{\tilde{\mathbf{H}},k}\right)^{1/N}\\
&\text{s.t. } \mathbf{F}_S = \alpha\tilde{\mathbf{P}},\\
&tr\left(\sigma_s^2\mathbf{F}_S\mathbf{F}_S^H\right) \leq P_{S,T},\\
&tr\left(\mathbf{F}_R\left(\sigma_s^2\mathbf{H}_{SR}\mathbf{F}_S\mathbf{F}_S^H\mathbf{H}_{SR}^H + \sigma_{n,r}^2\mathbf{I}_R\right)\mathbf{F}_R^H\right) \leq P_{R,T}.
\end{aligned} \tag{4.11}$$

As will be shown later, the optimum $\mathbf{F}_S$ is simple to obtain (in terms of $\mathbf{F}_R$). However, the cost function in (4.11) involves the singular values of the channel matrix $\tilde{\mathbf{H}}$, which is a complicated nonlinear function of the relay precoder $\mathbf{F}_R$, as shown in (4.5). A direct maximization of (4.11) to solve $\mathbf{F}_R$ is then difficult. In the next subsection, we will propose the same method described in Section 3.2 to overcome the problem.

§ 4.2 Joint Source/Relay Precoders Design

§ 4.2.1 *Proposed Method*

Taking a close look into (4.11), we readily see that the optimum α is easy to obtain. From the first two constraints, we can obtain the optimum source precoder, denoted by $\mathbf{F}_{S,opt}$, as

$$\mathbf{F}_{S,opt} = \sqrt{\frac{P_{S,T}}{\sigma_s^2 N}}\tilde{\mathbf{P}}. \tag{4.12}$$

From $\tilde{\mathbf{P}}$ in (4.8) and $\tilde{\mathbf{H}}$ in (4.5), we see that $\tilde{\mathbf{P}}$ is a function of $\mathbf{F}_R$; therefore, $\mathbf{F}_{S,opt}$ is a function of $\mathbf{F}_R$. Substituting (4.12) into (4.11), we can simplify the joint precoders optimization problem as a relay precoder design problem, as shown below:

$$\begin{aligned}
&\max_{\mathbf{F}_R} \sqrt{\frac{P_{S,T}}{\sigma_s^2 N}} \left(\prod_{k=1}^{N} \sigma_{\tilde{\mathbf{H}},k}\right)^{1/N} \\
&\text{s.t.} \\
&tr\left(\mathbf{F}_R \left(\frac{P_{S,T}}{N}\mathbf{H}_{SR}\mathbf{H}_{SR}^H + \sigma_{n,r}^2\mathbf{I}_R\right)\mathbf{F}_R^H\right) \le P_{R,T}.
\end{aligned} \tag{4.13}$$

As mentioned, the singular values of $\tilde{\mathbf{H}}$ are involved in (4.13), a direct maximization in (4.13) is difficult. To overcome the problem, we first propose to maximize an alternative cost function, having the same optimum precoder $\mathbf{F}_{R,opt}$ as (4.13), as

$$\mathbf{F}_{R,opt} = \arg\max_{\mathbf{F}_R} \sqrt{\frac{P_{S,T}}{\sigma_s^2 N}} \left(\prod_{k=1}^{N} \sigma_{\tilde{\mathbf{H}},k}\right)^{1/N} \tag{4.14}$$

$$= \arg\max_{\mathbf{F}_R} \left(\prod_{k=1}^{N} \sigma_{\tilde{\mathbf{H}},k}\right)^{2} \tag{4.15}$$

$$= \arg\max_{\mathbf{F}_R} det\left(\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}\right), \tag{4.16}$$

where

$$\tilde{\mathbf{H}}^H\tilde{\mathbf{H}} = \left[\sigma_{n,d}^{-2}\mathbf{H}_{SD}^H\mathbf{H}_{SD} + \mathbf{H}_{SR}^H\mathbf{F}_R^H\mathbf{H}_{RD}^H\left(\sigma_{n,r}^2\mathbf{H}_{RD}\mathbf{F}_R\mathbf{F}_R^H\mathbf{H}_{RD}^H + \sigma_{n,d}^2\mathbf{I}_M\right)^{-1}\mathbf{H}_{RD}\mathbf{F}_R\mathbf{H}_{SR}\right]. \tag{4.17}$$

The equality in (4.15) is due to the fact that $\left(\prod_{k=1}^{N} \sigma_{\tilde{\mathbf{H}},k}\right) > 0$, and the cost function is monotonically increasing functions of $\left(\prod_{k=1}^{N} \sigma_{\tilde{\mathbf{H}},k}\right)$. The equality in (4.16) is due to the following property.

$$\left(\prod_{k=1}^{N} \sigma_{\tilde{\mathbf{H}},k}\right)^2 = \prod_{k=1}^{N} \lambda_{\tilde{\mathbf{H}}^H\tilde{\mathbf{H}},k} = det\left(\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}\right), \tag{4.18}$$

where $\lambda_{\tilde{\mathbf{H}}^H\tilde{\mathbf{H}},i}$ is the ith eigenvalue of $\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}$. With the cost function in (4.16), the solution becomes much easier to work with. To solve the maximization of the determinate problem in (4.16), we can resort to the techniques proposed in Section 3.2 where the optimization process mainly follows the Hardamard inequality and Lemma 3.2. As a result, by setting $\mathbf{A} = \sigma_{n,d}^{-2}\mathbf{H}_{SD}^H\mathbf{H}_{SD}$ and $\mathbf{B} = \mathbf{H}_{SR}^H \mathbf{F}_R^H\mathbf{H}_{RD}^H \left(\sigma_{n,r}^2\mathbf{H}_{RD}\mathbf{F}_R\mathbf{F}_R^H\mathbf{H}_{RD}^H + \sigma_{n,d}^2\mathbf{I}_M\right)^{-1} \mathbf{H}_{RD}\mathbf{F}_R\mathbf{H}_{SR}$ in Lemma 3.2, we have the following equivalences

$$\begin{aligned} \mathbf{F}_{R,opt} &= \arg\max_{\mathbf{F}_R} det\left(\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}\right) \\ &= \arg\max_{\mathbf{F}_R} det\left(\mathbf{A}\right) det\left(\mathbf{I}_N + \mathbf{A}^{-1/2}\mathbf{B}\mathbf{A}^{-1/2}\right) \\ &= \arg\max_{\mathbf{F}_R} det\left(\mathbf{I}_N + \mathbf{A}^{-1/2}\mathbf{B}\mathbf{A}^{-1/2}\right), \end{aligned} \tag{4.19}$$

where $det(\mathbf{A})$ in (4.19) is ignored since it is not a function of $\mathbf{F}_R$. To make sure the existence of $\mathbf{A}^{-1}$, we assume that $N \leq M$. As a result, the rank of $\mathbf{A}$ is N and that of $\mathbf{B}$ is $min\{N, R\}$. Using the result, similar to the optimization addressed in Section 3.2, the relay precoder optimization can be expressed as

$$\begin{aligned} &\max_{\mathbf{F}_R} det(\mathbf{M}) \\ &\text{s.t.} \\ &\mathbf{M} = \mathbf{I}_N + \sigma_{n,d}^2\left(\mathbf{H}_{SD}^H\mathbf{H}_{SD}\right)^{-1/2}\mathbf{H}_{SR}^H\mathbf{F}_R^H\mathbf{H}_{RD}^H \\ &\left(\sigma_{n,r}^2\mathbf{H}_{RD}\mathbf{F}_R\mathbf{F}_R^H\mathbf{H}_{RD}^H + \sigma_{n,d}^2\mathbf{I}_M\right)^{-1}\mathbf{H}_{RD}\mathbf{F}_R\mathbf{H}_{SR}\left(\mathbf{H}_{SD}^H\mathbf{H}_{SD}\right)^{-1/2} \\ &\mathbf{M} \text{ is diagonal, and} \\ &tr\left(\mathbf{F}_R\left(\frac{P_{S,T}}{N}\mathbf{H}_{SR}\mathbf{H}_{SR}^H + \sigma_{n,r}^2\mathbf{I}_R\right)\mathbf{F}_R^H\right) \leq P_{R,T}. \end{aligned} \tag{4.20}$$

So, we can let $\mathbf{H}''_{SR} = \sigma_{n,d}\mathbf{H}_{SR}\left(\mathbf{H}_{SD}^H\mathbf{H}_{SD}\right)^{-1/2}$ and consider the following SVD:

$$\mathbf{H}''_{SR} = \mathbf{U}''_{sr}\mathbf{\Sigma}''_{sr}\mathbf{V}''^H_{sr}, \tag{4.21}$$

where $\mathbf{U}''_{sr} \in \mathbb{C}^{R\times R}$ are left singular matrices of $\mathbf{H}''_{SR}$; $\mathbf{\Sigma}''_{sr} \in \mathbb{R}^{R\times N}$ are the diagonal singular-value matrices of $\mathbf{H}''_{SR}$; $\mathbf{V}''_{sr} \in \mathbb{C}^{N\times N}$ are the right singular matrices of $\mathbf{H}''_{SR}$. To have a full diagonalization of $\mathbf{M}$, we let the precoder $\mathbf{F}_R$ have the following structure:

$$\mathbf{F}_R = \mathbf{V}_{rd}\mathbf{\Sigma}_r\mathbf{U}''^H_{sr}, \tag{4.22}$$

where $\mathbf{\Sigma}_r$ is a diagonal matrix with its ith diagonal element defined as $\sigma_{r,i}$, $i = 1, \cdots, \kappa$, yet to be determined. Here, $\kappa = min\{N, R\}$. Let $\sigma_{rd,i}$ and $\sigma''_{sr,i}$ be the ith diagonal element of $\mathbf{\Sigma}_{rd}$ and $\mathbf{\Sigma}''_{sr}$, respectively. Substituting (3.28), (4.21) and (4.22) into (4.20) and taking the natural log operation to the cost function, we can rewrite (4.20) as

$$\begin{aligned}&\max_{p_{r,i},1\le i\le\kappa}\sum_{i=1}^{\kappa} ln\left(1+\frac{p_{r,i}\sigma_{n,d}^2\sigma_{rd,i}^2\sigma''^2_{sr,i}}{p_{r,i}\sigma_{n,r}^2\sigma_{rd,i}^2+\sigma_{n,d}^2}\right)\\ &\text{s.t.}\\ &\sum_{i=1}^{\kappa} p_{r,i}\left(\frac{P_{S,T}}{N}\sigma''^2_{sr,i}\mathbf{D}''_{sr}(i,i)+\sigma_{n,r}^2\right)\le P_{R,T},\ p_{r,i}\ge 0,\ \text{for all } i,\end{aligned} \tag{4.23}$$

where $p_{r,i} = \sigma_{r,i}^2$ and $\mathbf{D}''_{sr} = \sigma_{n,d}^{-2}\mathbf{V}'^H_{sr}\left(\mathbf{H}_{SD}^H\mathbf{H}_{SD}\right)\mathbf{V}'_{sr}$ with $\mathbf{D}''_{sr}(i,i)$ is the ith diagonal element of $\mathbf{D}''_{sr}$. The cost function now is reduced to a function of scalars. As wee can see, the optimization in (4.23) is the same with (3.31) except the definitions of $\mathbf{D}'_{sr}$ and $\mathbf{H}'_{sr}$ where $\mathbf{D}'_{sr} = \mathbf{V}'^H_{sr}\left(N/P_{S,T}\mathbf{I}_N + \sigma_{n,d}^{-2}\mathbf{H}_{SD}^H\mathbf{H}_{SD}\right)\mathbf{V}'_{sr}$ and $\mathbf{H}'_{SR} = \mathbf{H}_{SR}\left(N/P_{S,T}\mathbf{I}_N + \sigma_{n,d}^{-2}\mathbf{H}_{SD}^H\mathbf{H}_{SD}\right)^{-\frac{1}{2}}$ in Section 3.2. As a result, the optimal solutions $p_{r,i}$, $i = 1, \cdots, \kappa$, can be solved as

$$\begin{aligned}p_{r,i} \;=\; &\left[\sqrt{\frac{\mu}{\sigma_{rd,i}^2\left(\frac{P_{S,T}}{N}\sigma''^2_{sr,i}\mathbf{D}''_{sr}(i,i)+\sigma_{n,r}^2\right)\left(\sigma_{n,r}^2\sigma_{n,d}^{-2}\sigma''^{-2}_{sr,i}+1\right)}+\frac{\frac{\sigma_{n,d}^4}{4\sigma_{n,r}^4}}{\sigma_{rd,i}^4\left(\frac{\sigma_{n,r}^2}{\sigma_{n,d}^2\sigma''^2_{sr,i}}+1\right)^2}}\right.\\ &\left.-\frac{1+\frac{\sigma_{n,d}^2\sigma''^2_{sr,i}}{2\sigma_{n,r}^2}}{\sigma_{rd,i}^2\left(\frac{\sigma_{n,r}^2}{\sigma_{n,d}^2}+\sigma''^2_{sr,i}\right)}\right]^+,\end{aligned} \tag{4.24}$$

Table 4.1: Computational Complexity of linear source and relay precoders (QR-SIC receiver).

Step	Operation	FLOPs
1	$\mathbf{H}''_{SR}$ (4.21)	$O(N^3 + RN^2)$
2	SVD $\mathbf{H}_{RD} = \mathbf{U}_{rd}\mathbf{\Sigma}_{rd}\mathbf{V}_{rd}^H$	$O(MR^2 + R^3)$
3	SVD $\mathbf{H}''_{SR} = \mathbf{U}''_{sr}\mathbf{\Sigma}''_{sr}\mathbf{V}''^H_{sr}$ (4.21)	$O(RN^2 + N^3)$
4	$\mathbf{\Sigma}_r$ (4.24)	$O(\kappa I_r)$
5	$\mathbf{F}_R$ (4.22)	$O(R^3)$
6	GMD $\tilde{\mathbf{H}} = \tilde{\mathbf{Q}}\tilde{\mathbf{R}}\tilde{\mathbf{P}}^H$ (4.8)	$O(MN^2 + N^3)$
7	$\mathbf{F}_{S,opt}$ (4.12)	$O(N)$

I_r is denoted as the iteration number of the water-filling process in (4.24).

where μ is chosen to satisfy the power constraint in (4.20). Finally, substituting (4.22) into (4.5) and conducting the decomposition in (4.8), we then obtain the optimum source precoder via (4.12). The computational complexity of the proposed source and relay precoders in terms of FLOPs is summarized in Table 4.1.

§ 4.2.2 *Antenna Selection*

The algorithm developed above assumes that $L = N \leq M$. If $L < N \leq M$, the precoder matrix is not square and the GMD method cannot be applied. A simple remedy to this problem is to use the antenna selection method. Using the method, we can select L antennas from the N antennas ($L \leq N, R, M$) such that the geometric mean of the equivalent channel in (4.13) is maximized. Thus, the problem can be formulated as

$$\max_{\mathbf{P}_i \in \mathcal{P}} \max_{\mathbf{F}_R} \sqrt{\frac{P_{S,T}}{\sigma_s^2 L}} \left(\prod_{k=1}^{L} \sigma_{\tilde{\mathbf{H}}\mathbf{P}_i,k} \right)^{1/L}$$
$$\text{s.t.}$$
$$tr\left(\mathbf{F}_R \left(\frac{P_{S,T}}{L} \mathbf{H}_{SR} \mathbf{P}_i \mathbf{P}_i^H \mathbf{H}_{SR}^H + \sigma_{n,r}^2 \mathbf{I}_R \right) \mathbf{F}_R^H \right) \leq P_{R,T}, \tag{4.25}$$

where $\sigma_{\tilde{\mathbf{H}}\mathbf{P}_i,k}$ denotes the kth singular value of $\tilde{\mathbf{H}}\mathbf{P}_i$, $\mathbf{P}_i$ the ith $N \times L$ antenna selection matrix, and $\mathcal{P}$ the set of all possible $\mathbf{P}_i$. It is simple to see that the size of $\mathcal{P}$ is $N!/L!(N-L)!$. Note that each column of $\mathbf{P}_i$ contain only one nonzero element (with the value of one) indicating the antenna selected. For example, for a 3×2 system, we have

$$\mathcal{P} = \left\{ \mathbf{P}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, \mathbf{P}_2 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, \mathbf{P}_3 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \right\}. \tag{4.26}$$

After the optimum antenna selection matrix is determined, the optimum source precoder can then obtained by (4.12) and the proposed method can be applied accordingly.

§ 4.3 Simulations

We consider a single relay AF MIMO relay system, and assume that CSI of all links are known at all nodes. Furthermore, the elements of each channel matrix are i.i.d. complex Gaussian random variables with zero-mean and same variance. Without of loss generality, we let the modulation scheme be 4-QAM and use the BER as the performance measure. For the first set of simulations, we let $N = L = R = M = 4$, SNR$_{sr}$=SNR$_{rd}$ $= 15\ dB$, and SNR$_{sd}$ be varied. Seven systems are compared, namely (a) the un-precoded system with the zero forcing (ZF) receiver, (b) the un-precoded system with the MMSE receiver, (c) the un-precoded system with the QR-SIC receiver, (d) the un-precoded system with the MMSE-ordered SIC (OSIC) receiver [13], (e) the linear relay precoded system with MMSE receiver [43], (f) the precoded system with the GMD source precoder [11], [12], and (g) the linear source and relay precoded system with QR-SIC receiver. It is noteworthy that the simulation conducted in [43] does not consider the signal received from the direct link. For better performance, we further take the signal into consideration when implementing the MMSE receiver. Fig. 4.2 shows the BER performance comparison. As we can see, the performance of the un-precoded systems is limited. The best un-precoded system is the one with the MMSE-OSIC receiver. Although it is better than the linear relay precoded system in the high SNR region, it is much worse than the GMD source and the linear source and relay precoded systems. The linear source and relay precoded system significantly outperforms the GMD source precoded system; SNR is improved by 4 dB when BER is 10^{-3}. This is because the linear source and relay precoded system takes two precoders into consideration, yielding a higher received SNR at the destination. We then consider another scenario where $\text{SNR}_{sr} = 15\ dB$ and $\text{SNR}_{rd} = 0\ dB$. Now, the link between the relay and the destination becomes poorer. Theoretically, the relay precoder will become less critical in this scenario. Fig. 4.3 shows the performance comparison for all systems. As expected, the performance of the relay-only precoded system is seriously degraded. The linear source and relay precoded system, however, still has the superior performance. The performance gap between the linear source and relay precoded and the GMD source precoded system becomes somewhat smaller. This is also expected since the role of the relay precoder is less critical, as mentioned.

As discussed, the GMD criterion can maximize the lower bound of the channel free distance [12], and the performance of an ML detector is directly related to the free distance. We can expect that the linear source and relay precoders can also improve the system performance if an ML receiver is used at the destination. Fig. 4.4 shows the performance comparison for the uncoded, the GMD source precoded, and the linear source and relay precoded systems with the ML receiver applied at the destination. Here, SNR_{sr} = $\text{SNR}_{rd} = 10\ dB$ and $L = N = R = M = 2$. As shown in this figure, the performance of the linear source and relay precoded system is better than that of the the un-precoded and the GMD source precoded systems. The

GMD source precoder has better performance than un-precoded system since the channel's free distance is improved by the source precoder. The performance gap between the linear source and relay precoded method and GMD source precoder is due to the additional relay precoder, further enlarging the lower bound of the channel's free distance.

In conventional (non-cooperative) MIMO systems, spatial multiplexing cannot be applied when $L = N > M$. However, in a cooperative system, with the aid of the relay, the degree of freedom of the overall system is increased. In other words, even when $L = N > M$, spatial multiplexing can still be used in cooperative systems. The final set of simulations is to compare the performance of the un-precoded and the linear source and relay precoded systems in this scenario. For the un-precoded schemes, we let $L = N = 4$, $R = M = 2$ and the modulation scheme be 4-QAM. Since $L = N > M$, the proposed system has to conduct antenna selection. Here, we let $L = 2$, $N = 4$, $R = M = 2$ and the modulation scheme be 16-QAM. With the setting, the transmission rates of the un-precoded and precoded systems are the same (8 bits/channel usage). Let SNR_{sr}=$\text{SNR}_{rd} = 15\ dB$ and SNR_{sd} be varied. Fig. 4.5 shows the performance of the un-precoded and linear source and relay precoded systems. As we can see, the linear source and relay precoded system significantly outperforms the un-precoded systems. There exists error floors for the un-precoded systems since the noise at the relay link tends to dominate the overall performance when SNR_{sd} is high. Due to the precoding operation, we do not observe the error floor phenomenon in the linear source and relay precoded system.

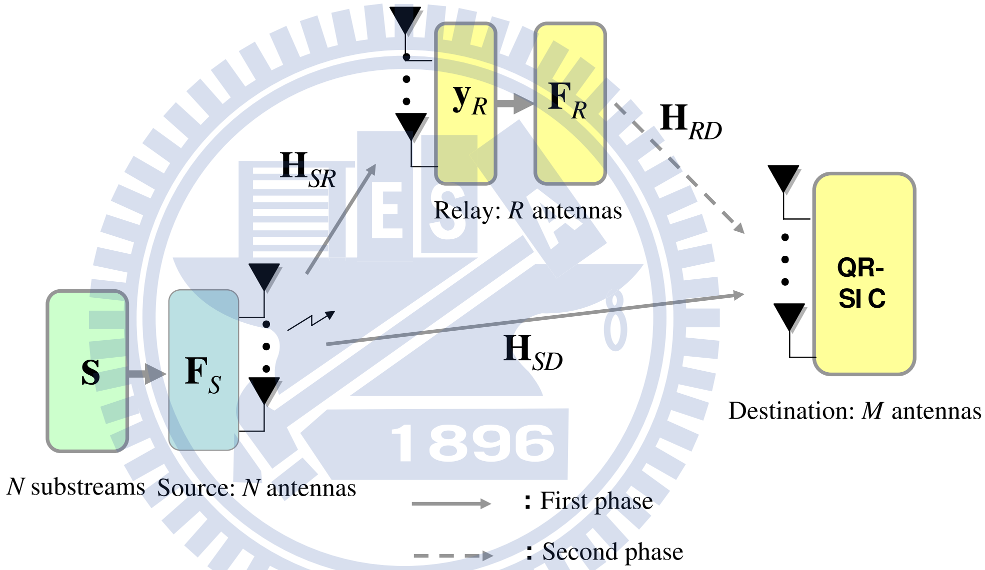


Figure 4.1: Linear source and relay precoded AF MIMO relay system with QR-SIC receiver.

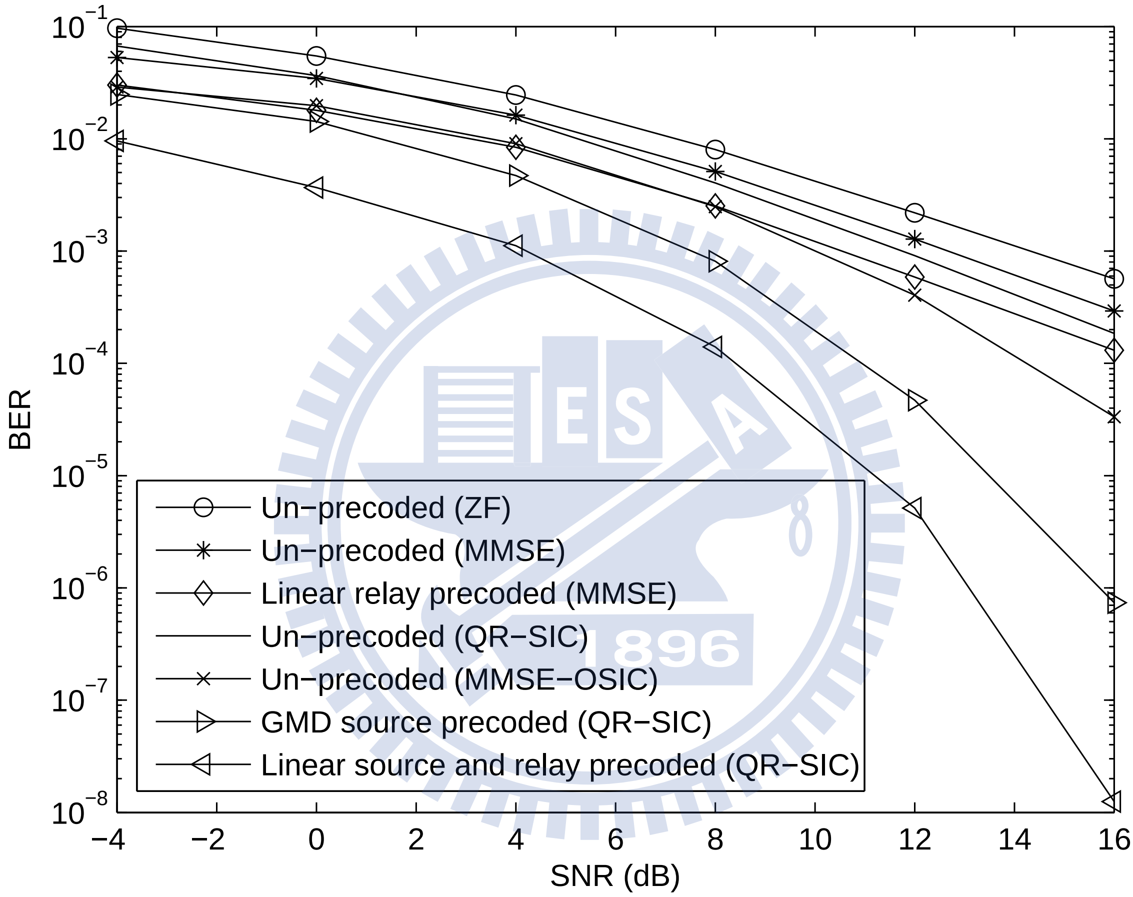


Figure 4.2: BER performance comparison for linear source and relay precoded system with QR-SIC receiver and existing precoded systems ($L = N = R = M = 4$, SNR_{sr}=$\text{SNR}_{rd} = 15$ dB).

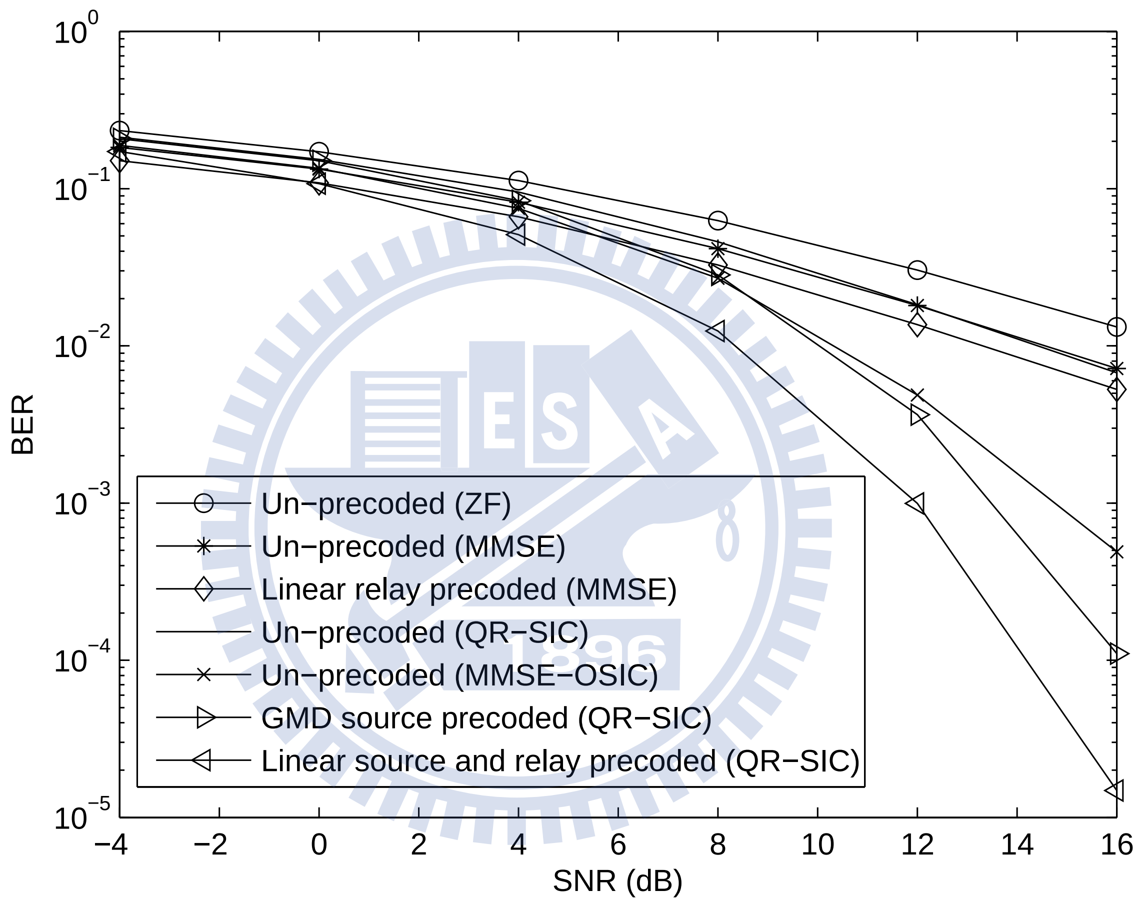


Figure 4.3: BER performance comparison for linear source and relay precoded system with QR-SIC receiver and existing precoded systems ($L = N = R = M = 4$, SNR$_{sr}$=15, SNR$_{rd}$=0 dB).

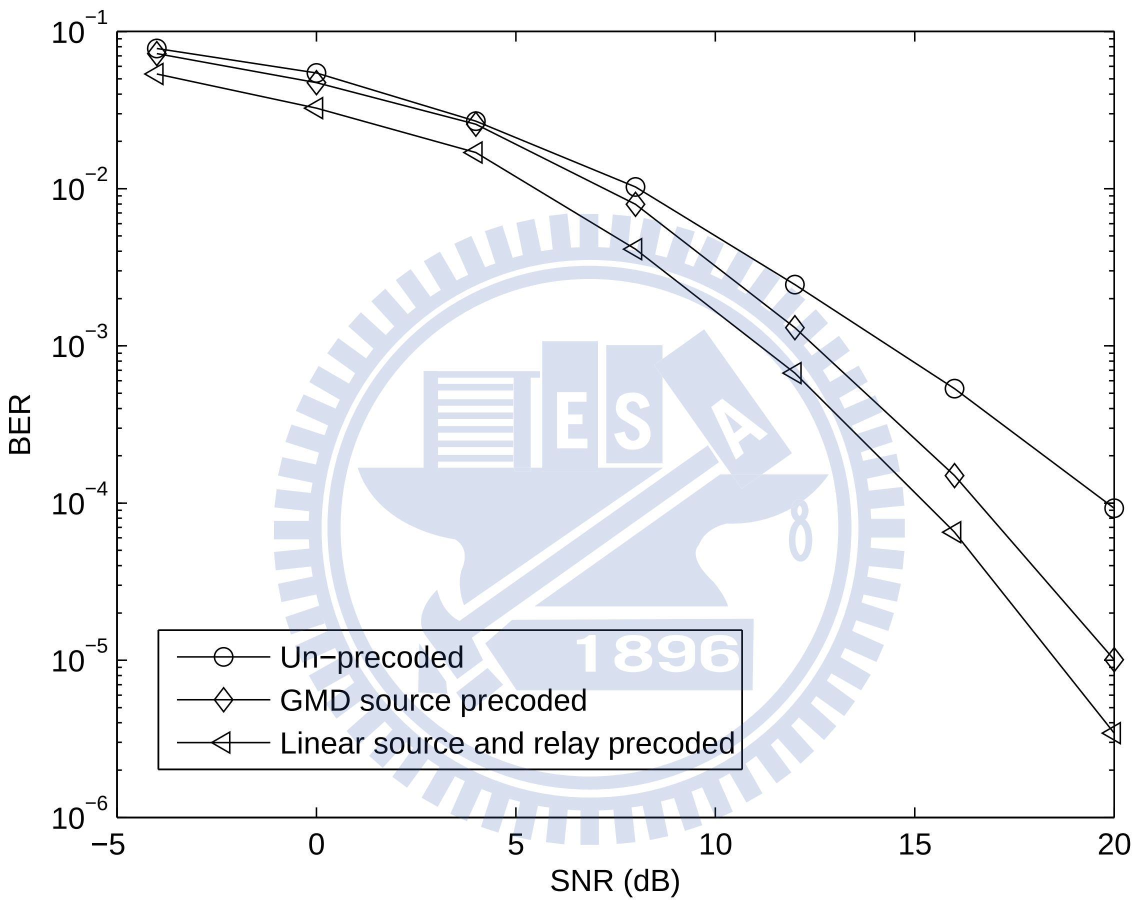


Figure 4.4: BER performance comparison for un-precoded, GMD source precoded, and linear source and relay precoded systems with ML receiver at the destination ($L = N = R = M = 2$, SNR_{sr}=SNR_{rd}=10 dB).

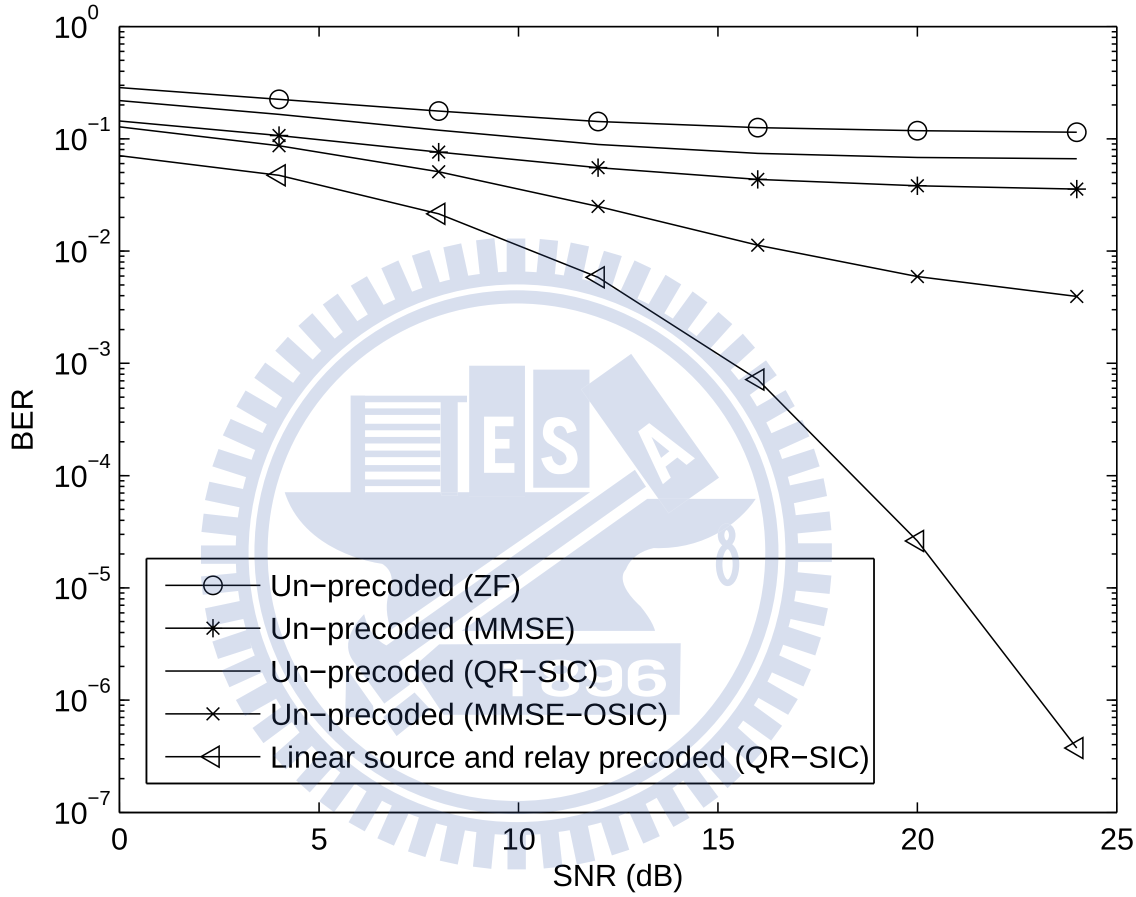


Figure 4.5: BER performance comparison for linear source and relay precoded system with QR-SIC receiver and un-precoded systems ($L = N = 4$, $R = M = 2$ and 4-QAM is used for un-precoded systems; $N = 4$, $L = R = M = 2$ and 16-QAM is used for linear source and relay precoded system with QR-SIC receiver).

Chapter 5

Joint MMSE-SIC Transceiver Design with Linear Source and Relay Precoders

As well known in the precoded MIMO systems, the MMSE-SIC receiver outperforms the QR-SIC receiver. It is reasonable to assert that the same result can be obtained for MIMO relay systems. For MIMO systems, the precoder with the MMSE-SIC receiver can be solved by the UCD method. In this chapter, we consider the system where linear precoders are used at the source and the relay, and an MMSE-SIC receiver at the destination. We show that the UCD is not directly applicable in AF MIMO relay systems. However, if the source precoder is constrained to be unitary, the problem can be solved. In Section 5.1, we give the system model for the MMSE-SIC receiver. In Section 5.2, we propose a modified UCD method to derive the source and relay precoders. Finally, we report simulation results in Section 5.3 to conform the effectiveness of the proposed method.

§ 5.1 System Model and Problem Formulation

§ 5.1.1 *MMSE-SIC Receiver with Linear Source and Relay Precoders*

We consider the same received signal model as that in (4.3) and rewrite it as

$$\widetilde{\mathbf{y}}_D := \widetilde{\mathbf{H}}\mathbf{F}_S\mathbf{s} + \widetilde{\mathbf{n}} = \widehat{\mathbf{H}}\mathbf{s} + \widetilde{\mathbf{n}}. \tag{5.1}$$

With the MMSE-SIC applied at the receiver, the symbol streams can be detected by the successive vector suppression process. Specifically, the ith layer symbol is detected after an inner product operation is conducted with the suppression vector ν_i. And, its signal component is then subtracted from the received signal. By (5.1), the suppressing vector ν_i for ith layer can be expressed as [13],

$$\nu_i = \left(\sum_{j=1}^{i} \widehat{\mathbf{h}}_j \widehat{\mathbf{h}}_j^H + \sigma_s^{-2}\mathbf{I}\right)^{-1}, \quad i = 1, \ldots, N, \tag{5.2}$$

where $\widehat{\mathbf{H}} := \widetilde{\mathbf{H}}\mathbf{F}_S = \left[\widehat{\mathbf{h}}_1, \ldots, \widehat{\mathbf{h}}_N\right]$. In [13], the suppression vector is alternatively found via QR-decomposition,

$$\begin{bmatrix} \widehat{\mathbf{H}} \\ \sigma_s^{-1}\mathbf{I}_N \end{bmatrix} = \mathbf{Q}_1\mathbf{R}_1 = \begin{bmatrix} \mathbf{Q}_1^u \\ \mathbf{Q}_1^l \end{bmatrix} \mathbf{R}_1, \tag{5.3}$$

where $\mathbf{R}_1 \in \mathbb{C}^{(2M+N)\times N}$ is an upper triangular matrix with positive diagonal elements, and $\mathbf{Q}_1 \in \mathbb{C}^{(2M+N)\times(2M+N)}$ is an unitary matrix. Note that $\mathbf{Q}_1^u \in \mathbb{C}^{2M\times(2M+N)}$ and $\mathbf{Q}_1^u \in \mathbb{C}^{N\times(2M+N)}$ are not unitary matrices. The suppression vector in (5.2) can be then expressed as

$$\nu_i = \mathbf{R}_1^{-1}(i,i)\mathbf{Q}_1^u(:,i), \quad i = 1, \ldots, N, \tag{5.4}$$

where $\mathbf{R}_1(i,i)$ denotes the ith diagonal element of $\mathbf{R}_1$ and $\mathbf{Q}_1^u(:,i)$ the ith column of $\mathbf{Q}_1^u$. If the error propagation effect is ignored, the following equivalence holds for each i:

$$\sigma_s^{-2}\left(1 + \text{SINR}_i\right) = \mathbf{R}_1^2(i,i), \quad i = 1, \ldots, N, \tag{5.5}$$

where

$$\text{SINR}_i = \widehat{\mathbf{h}}_i^H \left(\sum_{j=1}^{i-1} \widehat{\mathbf{h}}_j \widehat{\mathbf{h}}_j^H + \sigma_s^{-2} \mathbf{I}_{2M} \right)^{-1} \widehat{\mathbf{h}}_i, \quad i = 1, \dots, N, \tag{5.6}$$

denotes the SINR of the ith layer recovered signal in the MMSE-SIC receiver.

§ 5.1.2 *Problem Formulation*

In MIMO systems, the relationship between the received SINR and the averaged BER is complicated, and the precoders design minimizing the averaged BER is difficult to obtain. The UCD method, developed for conventional MIMO systems [13], can derive the precoder such that the SINR of each layer is the same and the BLER in the MMSE-SIC receiver is minimized. Motivated by the idea, we formulate our problem as

$$\begin{aligned}
&\max_{\mathbf{F}_S, \mathbf{F}_R} \text{SINR}_i \\
&s.t. \\
&\sigma_s^{-2} (1 + \text{SINR}_i) = \mathbf{R}_1^2(i,i) \\
&\text{SINR}_i \text{ are equal}, \ \forall i, \\
&\sigma_s^2 tr \left\{ \mathbf{F}_S \mathbf{F}_S^H \right\} \le P_{S,T} \\
&tr \left\{ \mathbf{F}_R \left(\sigma_{n,r}^2 \mathbf{I}_R + \sigma_s^2 \mathbf{H}_{SR} \mathbf{F}_S \mathbf{F}_S^H \mathbf{H}_{SR}^H \right) \mathbf{F}_R^H \right\} \le P_{R,T}.
\end{aligned} \tag{5.7}$$

The inequalities in (5.7) indicate that the transmit power at the source and at the relay has to satisfy the maximal power constraints $P_{S,T}$ and $P_{R,T}$, respectively. As we can see from (5.7), the SINR_i is a complicated function of $\mathbf{F}_S$ and $\mathbf{F}_R$, and the optimum solution for this problem is very difficult to obtain. In the next subsection, we propose a new approach to solve the problem.

§ 5.2 Joint Source/Relay Precoders Design

§ 5.2.1 *Proposed Method*

In (5.7), we have seen that SINR_i is a complicated function of $\mathbf{F}_S$ and $\mathbf{F}_R$. To facilitate the optimization, we first seek for an alternative cost function for maximization.

Proposition: The following optimizations are equivalent.

$$\begin{aligned} &\max_{\mathbf{F}_S,\mathbf{F}_R,\ \text{SINR}_i \ \text{are equal},\ \forall i} \text{SINR}_i \\ &\max_{\mathbf{F}_S,\mathbf{F}_R,\ \text{SINR}_i \ \text{are equal},\ \forall i} \ln\det\left(\sigma_s^{-2}\mathbf{I}_N + \mathbf{F}_S^H\widetilde{\mathbf{H}}^H\widetilde{\mathbf{H}}\mathbf{F}_S\right). \end{aligned} \tag{5.8}$$

Proof: The derivation can be directly obtained from [13].

The cost function with the determinate operation in (5.8) is easier to work with than the original cost function. Similar to previous approaches, we propose to use the primal decomposition [51] to decompose our problem into a subproblem and a master problem. The subproblem is first optimized for the source precoder only, and subsequently the master problem is optimized for the relay precoder. From (5.8), we see that two precoders are involved and the UCD method, developed for MIMO systems, cannot be applied. Here, we pose a unitary constraint for the source precoder. As a result, (5.7) can be reformulated as

$$\begin{aligned} &\max_{\mathbf{F}_R}\max_{\mathbf{F}_S(\mathbf{F}_R)} \ln\det\left(\sigma_s^{-2}\mathbf{I}_N + \mathbf{F}_S^H\widetilde{\mathbf{H}}^H\widetilde{\mathbf{H}}\mathbf{F}_S\right) \\ &s.t. \\ &\sigma_s^{-2}\left(1+\text{SINR}_i\right) = \mathbf{R}_1^2(i,i) \\ &\text{SINR}_i \ \text{are equal},\ \forall i \\ &\mathbf{F}_S = \alpha\mathbf{U}_S \\ &\sigma_s^2 tr\left\{\mathbf{F}_S\mathbf{F}_S^H\right\} \le P_{S,T} \\ &tr\left\{\mathbf{F}_R\left(\sigma_{n,r}^2\mathbf{I}_R + \sigma_s^2\mathbf{H}_{SR}\mathbf{F}_S\mathbf{F}_S^H\mathbf{H}_{SR}^H\right)\mathbf{F}_R^H\right\} \le P_{R,T}, \end{aligned} \tag{5.9}$$

where $\mathbf{U}_S$ is an unitary matrix and α is a scalar, and both are to be further determined. The unitary constraint for $\mathbf{F}_S$, as we will see, can greatly facilitate the optimization in the subproblem and the master problem. Note that the original UCD precoder in the MIMO system is not restricted to have the unitary structure.

§ 5.2.2 *Proposed Subproblem Optimization*

Let $\mathbf{F}_R$ be given. From (5.9), the subproblem can be expressed ($\mathbf{F}_S$ is unitary):

$$\begin{aligned} &\max_{\mathbf{F}_S(\mathbf{F}_R)} \ln\det\left(\sigma_s^{-2}\mathbf{I}_N + \alpha^2\widetilde{\mathbf{H}}^H\widetilde{\mathbf{H}}\right) \\ &\sigma_s^{-2}(1+\text{SINR}_i) = \mathbf{R}_{1(i,i)}^2 \\ &\text{SINR}_i \text{ are equal, } \forall i \\ &\alpha^2\sigma_s^2 N \leq P_{S,T}, \end{aligned} \tag{5.10}$$

where $\mathbf{F}_S(\mathbf{F}_R)$ denotes that $\mathbf{F}_S$ is a function of $\mathbf{F}_R$. With the unitary source precoder structure, the relay power constraint is not a function of the source precoder and then it is not necessary to include it here. Also, $\mathbf{U}_S$ does not affect the cost function in (5.9), and the cost function becomes the function of α only. It is clear that to maximize the cost function, we can select the optimum α, denoted as α^*, as

$$\alpha^* = \sqrt{\frac{P_{S,T}}{\sigma_s^2 N}}. \tag{5.11}$$

To achieve the equal SINR_i constraint, $\mathbf{U}_S$ can be designed using the method proposed in [13]. By the SVD, we can have $\widetilde{\mathbf{H}} = \mathbf{U}_{\widetilde{\mathbf{H}}}\mathbf{\Sigma}_{\widetilde{\mathbf{H}}}\mathbf{V}_{\widetilde{\mathbf{H}}}^H$. Let $\mathbf{F}_S = \alpha^*\mathbf{U}_S = \alpha^*\mathbf{V}_{\widetilde{\mathbf{H}}}\mathbf{U}'_S$, where $\mathbf{U}'_S$ is also an unitary matrix to be determined. Substituting the results into (5.10), we can rewrite it as

$$\begin{bmatrix} \widetilde{\mathbf{H}}\mathbf{F}_S \\ \sigma_s^{-1}\mathbf{I}_N \end{bmatrix} = \begin{bmatrix} \mathbf{U}_{\widetilde{\mathbf{H}}}\mathbf{\Sigma}_{\widetilde{\mathbf{H}}}\alpha^*\mathbf{U}'_S \\ \sigma_s^{-1}\mathbf{I}_N \end{bmatrix} = \begin{bmatrix} \mathbf{U}_{\widetilde{\mathbf{H}}} & \mathbf{0} \\ \mathbf{0} & \mathbf{U}'^H_S \end{bmatrix} \begin{bmatrix} \alpha^*\mathbf{\Sigma}_{\widetilde{\mathbf{H}}} \\ \sigma_s^{-1}\mathbf{I}_N \end{bmatrix} \mathbf{U}'_S. \tag{5.12}$$

Applying the GMD, [11], [12], on $\begin{bmatrix} \alpha^*\mathbf{\Sigma}_{\widetilde{\mathbf{H}}} \\ \sigma_s^{-1}\mathbf{I}_N \end{bmatrix}$, we have

$$\begin{bmatrix} \alpha^*\mathbf{\Sigma}_{\widetilde{\mathbf{H}}} \\ \sigma_s^{-1}\mathbf{I}_N \end{bmatrix} = \mathbf{Q}_2\mathbf{R}_2\mathbf{P}_2^H, \tag{5.13}$$

where $\mathbf{Q}_2 \in \mathbb{C}^{(2M+N)\times(2M+N)}$, $\mathbf{P}_2 \in \mathbb{C}^{N\times N}$ are unitary matrices, and $\mathbf{R}_2 \in \mathbb{C}^{(2M+N)\times N}$ is the upper triangular matrix with equal diagonal elements. Substituting (5.13) into (5.12) and using (5.3), we then have

$$\begin{bmatrix} \widehat{\mathbf{H}} \\ \sigma_s^{-1}\mathbf{I}_N \end{bmatrix} = \begin{bmatrix} \mathbf{U}_{\widetilde{\mathbf{H}}}\alpha^*\mathbf{\Sigma}_{\widetilde{\mathbf{H}}}\mathbf{U}'_S \\ \sigma_s^{-1}\mathbf{I}_N \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{U}_{\widetilde{\mathbf{H}}} & \mathbf{0} \\ \mathbf{0} & \mathbf{U}'^H_S \end{bmatrix}\mathbf{Q}_2}_{:=\mathbf{Q}_1} \underbrace{\mathbf{R}_2\mathbf{P}_2^H\mathbf{U}'_S}_{:=\mathbf{R}_1}. \tag{5.14}$$

So, if we let $\mathbf{U}'_S = \mathbf{P}_2$, $\mathbf{R}_1$ will be equal to $\mathbf{R}_2$. In this way, the diagonal elements in $\mathbf{R}_1$ become equal. This makes the SINR of each layer the same, which can be checked with (5.5). The optimum source precoder can be expressed as

$$\mathbf{F}_S = \sqrt{\frac{P_{S,T}}{\sigma_s^2 N}}\mathbf{V}_{\widetilde{\mathbf{H}}}\mathbf{P}_2. \tag{5.15}$$

Substituting (5.15) in (5.9), we can then solve the master problem, as shown in the next.

§ 5.2.3 *Proposed Master Problem Optimization*

With (5.15), the master problem thus becomes

$$\begin{aligned}
&\max_{\mathbf{F}_R} \ln\left(\left(\frac{P_{S,T}}{\sigma_s^2 N}\right)^N \cdot \det\left(\frac{N}{P_{S,T}}\mathbf{I}_N + \widetilde{\mathbf{H}}^H\widetilde{\mathbf{H}}\right)\right) \\
&s.t. \\
&\widetilde{\mathbf{H}}^H\widetilde{\mathbf{H}} = \sigma_{n,d}^{-2}\mathbf{H}_{SD}^H\mathbf{H}_{SD} + \\
&\mathbf{H}_{SR}^H\mathbf{F}_R^H\mathbf{H}_{RD}^H\left(\sigma_{n,r}^2\mathbf{H}_{RD}\mathbf{F}_R\mathbf{F}_R^H\mathbf{H}_{RD}^H + \sigma_{n,d}^2\mathbf{I}_M\right)^{-1}\mathbf{H}_{RD}\mathbf{F}_R\mathbf{H}_{SR} \\
&tr\left\{\mathbf{F}_R\left(\sigma_{n,r}^2\mathbf{I}_R + \frac{P_{S,T}}{N}\mathbf{H}_{SR}\mathbf{H}_{SR}^H\right)\mathbf{F}_R^H\right\} \le P_{R,T}.
\end{aligned} \tag{5.16}$$

As we can see from (5.16), $\widetilde{\mathbf{H}}^H\widetilde{\mathbf{H}}$ is also a complicated function of $\mathbf{F}_R$, so the relay precoder is difficult to solve. However, we can resort to the same diagonalization in Chapter 3 and 4 to find the optimum relay precoder. Let $\mathbf{A} = \frac{N}{P_{S,T}}\mathbf{I}_N + \sigma_{n,d}^{-2}\mathbf{H}_{SD}^H\mathbf{H}_{SD}$ and $\mathbf{B} = \mathbf{H}_{SR}^H\mathbf{F}_R^H\mathbf{H}_{RD}^H\left(\sigma_{n,r}^2\mathbf{H}_{RD}\mathbf{F}_R\mathbf{F}_R^H\mathbf{H}_{RD}^H + \sigma_{n,d}^2\mathbf{I}_M\right)^{-1}\mathbf{H}_{RD}\mathbf{F}_R\mathbf{H}_{SR}$. We then have the same optimization in (3.26). The details are then omitted here. As a result, the optimum relay precoder can be found by substituting (3.32) into (3.30). After the optimum relay precoder is determined, the source precoder can then be obtained via (5.15).

We summarize the computational complexity of the linear source and relay precoders in Table 5.1. Comparing to Table 4.1, we can find that the procedures to compute the precoders proposed in this chapter and Chapter 4 are similar except for the GMD in (5.13) and (4.8). Since the THP and unitary precoders are involved in (3.15) and (3.16), the computational complexity of the precoders proposed in Chapter 3 is higher, as shown in the additional steps 7-9 of Table 3.1.

Finally, we summarize and compare the nonlinear transceivers proposed in this dissertation in Table 5.2. As shown in this table, $\mathbf{F}_S$ of each structure is expressed as an unitary matrix. It is seen that the relay precoders for the THP source precoded system and the linear source precoded system (with MMSE-SIC receiver) are the same. We also summarize physical implications of the precoders optimization in Table 5.3. From the table, we see that the unitary $\mathbf{F}_S$ is designed to equalize either the MSE or the SNR/SINR of each data substream in the subproblem problem. The relay precoders decouple the effective channels and allocate the power for each parallel channel to either maximize SNR/SINR or to minimize the MSE.

§ 5.3 Simulations

In this section, we evaluate the performance of the proposed precoded systems studied in this dissertation. As previously, we assume that the CSIs of all links are known at all nodes. The elements of each channel matrix are i.i.d. complex Gaussian random variables with zero-mean and same variance. For the first set of simulations, we let $N = R = M = 4$, SNR$_{sr}$ = 20 dB, SNR$_{sd}$ = 5 dB, and vary SNR$_{rd}$. Eight systems are compared, namely (a) un-precoded system with MMSE receiver, (b) linear relay precoded system with MMSE receiver [43], [44], (c) un-precoded system with QR-SIC receiver, (d) linear source and relay precoded system with MMSE receiver (Chapter 2), (e) un-precoded system with MMSE-OSIC receiver, (f) THP

Table 5.1: Complexity of linear source and relay precoders (MMSE-SIC receiver).

Step	Operation	FLOPs
1	$\mathbf{H}'_{SR}$ (3.27)	$O(N^3 + RN^2)$
2	SVD $\mathbf{H}_{RD} = \mathbf{U}_{rd}\mathbf{\Sigma}_{rd}\mathbf{V}_{rd}^H$ (3.28)	$O(MR^2 + R^3)$
3	SVD $\mathbf{H}'_{SR} = \mathbf{U}'_{sr}\mathbf{\Sigma}'_{sr}\mathbf{V}'^H_{sr}$ (3.29)	$O(RN^2 + N^3)$
4	$\mathbf{\Sigma}_r$ (3.32)	$O(\kappa I_r)$
5	$\mathbf{F}_R$ (3.30)	$O(R^3)$
6	SVD $\widetilde{\mathbf{H}} = \mathbf{U}_{\widetilde{\mathbf{H}}}\mathbf{\Sigma}_{\widetilde{\mathbf{H}}}\mathbf{V}_{\widetilde{\mathbf{H}}}^H$	$O(MN^2 + N^3)$
7	GMD $\begin{bmatrix} \sqrt{\frac{P_{S,T}}{\sigma_s^2 N}}\mathbf{\Sigma}_{\widetilde{\mathbf{H}}} \\ \sigma_s^{-1}\mathbf{I}_N \end{bmatrix}$ (5.13)	$O\left((2M+N)N^2 + N^3\right)$
8	$\mathbf{F}_{S,opt}$ (5.15)	$O(N^3)$

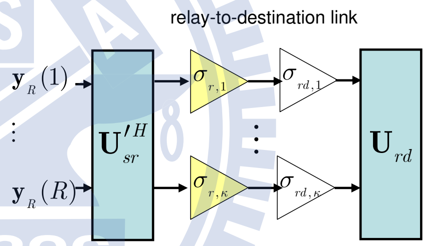
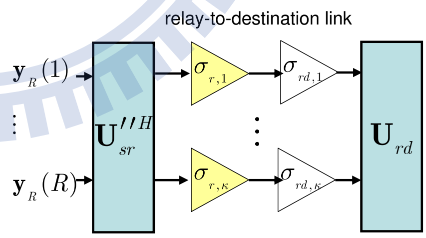
I_r is denoted as the iteration number of the water-filling process (3.32).

source and linear relay precoded system with MMSE receiver (Chapter 3), (g) linear source and relay precoded system with QR-SIC receiver (Chapter 4), and (h) linear source and relay precoded system with MMSE-SIC receiver (Chapter 5). Since the THP source and linear relay precoded system is considered, we adopt the 16-QAM modulation scheme. Fig, 5.1 and Fig. 5.2 show the simulated BLER and BER for the systems mentioned above, respectively. From Fig. 5.1, we can observe that the performance of the linear receivers are limited. The performance of un-precoded system can be improved by the linear relay precoder and can be further enhanced by the linear source and relay precoders. When the SNR of the relay-to-destination link is sufficiently high, the significance of the relay precoder is reduced. This indicates that the relay precoder is not critical. So, the performance of un-precoded and the relay precoded systems is close. Also, since nonlinear receivers can provide higher diversity gain [13], they perform well

Table 5.2: Source and relay precoders in the proposed nonlinear transceivers.

Structure	THP source and linear relay precoded (MMSE)	Linear source and relay precoded (MMSE-SIC)	Linear source and relay precoded (QR-SIC)
Source Precoder	$\mathbf{C}_{opt} = \mathbf{D}\mathbf{L}^{-1}$ (3.15) $\mathbf{F}_S = \sqrt{\frac{P_{S,T}}{N\sigma_s^2}}\mathbf{V}_{\tilde{\mathbf{H}}}\mathbf{P}$ (3.16)	$\sqrt{\frac{P_{S,T}}{\sigma_s^2 N}}\mathbf{V}_{\tilde{\mathbf{H}}}\mathbf{P}_2$ (5.15)	$\sqrt{\frac{P_{S,T}}{\sigma_s^2 N}}\tilde{\mathbf{P}}$ (4.12)
Relay Precoder	$\mathbf{V}_{rd}\Sigma_r\mathbf{U}'^H_{sr}$ (3.30)	$\mathbf{V}_{rd}\Sigma_r\mathbf{U}'^H_{sr}$ (3.30)	$\mathbf{V}_{rd}\Sigma_r\mathbf{U}''^H_{sr}$ (4.22)

Table 5.3: Optimizations of the source and relay precoders in the proposed nonlinear transceivers.

Joint source/relay precoders optimizations		
Structure	Subproblem	Master problem
THP source and linear relay precoded (MMSE)	$\mathbf{C}+\mathbf{F}_S$ (3.15)-(3.16) to equalize MSE_i	
Linear source and relay precoded (MMSE-SIC)	$\mathbf{F}_S$ (5.15) to equalize SINR_i	
Linear source and relay precoded (QR-SIC)	$\mathbf{F}_S$ (4.12) to equalize SNR_i	

in the high SNR regions, even for the un-precoded systems. As expected, the linear source and relay precoded system with the nonlinear receivers are better than the un-precoded systems. Due to the fact that the MMSE-SIC receiver has larger diversity gain, the precoded system with the MMSE-SIC receiver outperforms the precoded system with the QR-SIC receiver. Although the THP source and linear relay precoded system is also a nonlinear system, its BLER performance is inferior to that of the precoded systems with the QR-SIC and MMSE-SIC receivers. This is because the former system is designed by the MMSE criterion, while the latter system by the BLER criterion. However, in terms of the BER, the THP source and linear relay precoded system performs slightly better than the linear source and relay precoded system with the QR-SIC/MMSE-SIC receivers, as shown in Fig. 5.2.

Fig. 5.3 and 5.4 show the BLER and BER performances of the aforementioned transceivers with using 4-QAM modulation for each substream. As we can see, the THP source and linear relay precoded system with MMSE receiver is much worse. This is because it only workable for higher order modulation ($m \geq 16$), as Section 3.1.1 described.

For the second set of simulations, we still compare the performance of aforementioned precoded systems. However, we let SNR$_{sd}$ = 5 dB, SNR$_{rd}$ = 20 dB, and vary SNR$_{sr}$. Fig 5.5 and Fig. 5.6 show the simulation results for the BLER and the BER, respectively. As we can see, the relay precoded system with the MMSE receiver outperforms the un-precoded systems with the linear and nonlinear QR-SIC receivers along the increase of the SNR. This is because the performance is dominated by the links of the source-to-destination and the relay-to-destination when SNR$_{sr}$ is high. As a result, the additional relay precoder can improve the overall link quality. Unlike the previous case, the performance of the un-precoded system with the MMSE-OSIC receiver is inferior to the source and relay precoded system with the MMSE receiver. This is because when the SNR of the source-to-relay link is sufficiently high, the MIMO relay system is degenerated to the MIMO system. As a result, the significance of the relay precoder is increased. Also, as expected, the precoded systems with the nonlinear source precoder or with the nonlinear receivers outperform the un-precoded systems and the precoded systems with linear precoders and linear receivers.

The MLD receiver is known to be optimal. It is then interesting to know its performance in MIMO relay sytsems. For the third set of simulations, we compare the performance of the un-precoded system with the ML receiver and that of precoded systems. Since the computational complexity of the ML receiver is high, we only consider the system with $N = R = M = 2$. Let SNR$_{sd}$ = 5 dB, SNR$_{rd}$ = 20 dB, and vary SNR$_{sr}$. Fig. 5.7 shows the BLER comparison. As shown in this figure, the un-precoded system with ML receiver outperforms other un-precoded systems. However, it is poorer than all the precoded systems we proposed.

In real-world applications, CSIs have to be transmitted to the location where the precoders are calculated. Thus, quantization and transmission errors may arise. We refer this phenomenon as imperfect CSI. In the final set of simulations, we compares the performance of all un-precoded/precoded systems when CSIs are not perfect. As that in the previous works [54], the imperfect channel $\hat{\mathbf{H}}$ is related to the true channel $\mathbf{H}$ via the equation of $\hat{\mathbf{H}} = \sqrt{1-\rho}\mathbf{H} + \sqrt{\rho}\Delta\mathbf{H}$, where $\Delta\mathbf{H}$ models the channel error and the coefficient ρ characterizes the magnitude of the error. The elements of $\Delta\mathbf{H}$ are modeled as i.i.d. Gaussian distributions with zero mean and same variance. Fig. 5.8, shows the simulation results. Here, we let SNR$_{sd}$ = 5 dB, SNR$_{sr}$ = 25 dB, SNR$_{rd}$ = 20 dB, and ρ be varied. As we can see, the performance of the precoded systems degrades as ρ increases, especially for the THP source precoded system with MMSE receiver, and the linear source precoded system with MMSE-SIC receiver. The linear source precoded system with QR-SIC is less affected. Note that the CSIs are assumed perfectly known at the destination for un-precoded systems, and their BLERs are not affected by the value of ρ.

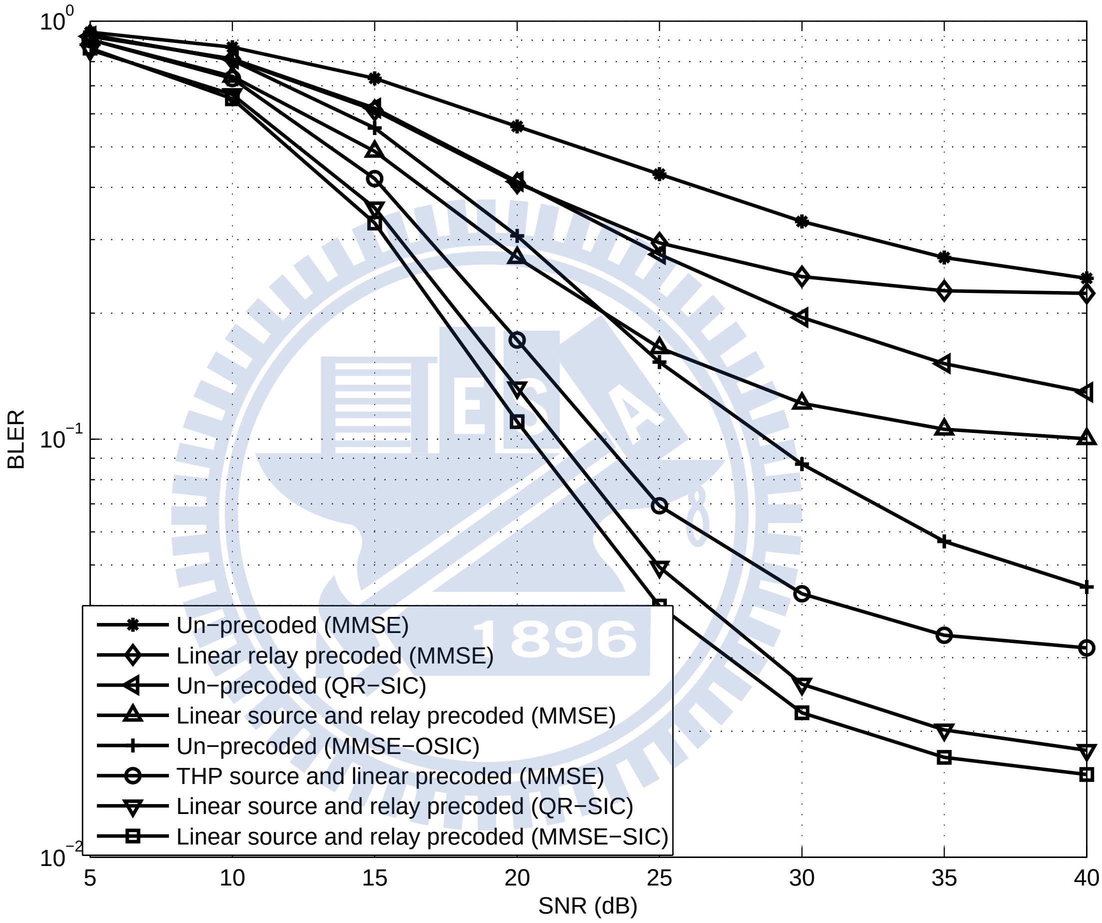


Figure 5.1: BLER performance comparison for un-precoded and precoded systems (16QAM, $N = R = M = 4$, SNR$_{sr}$=20, SNR$_{sd}$=5 dB).

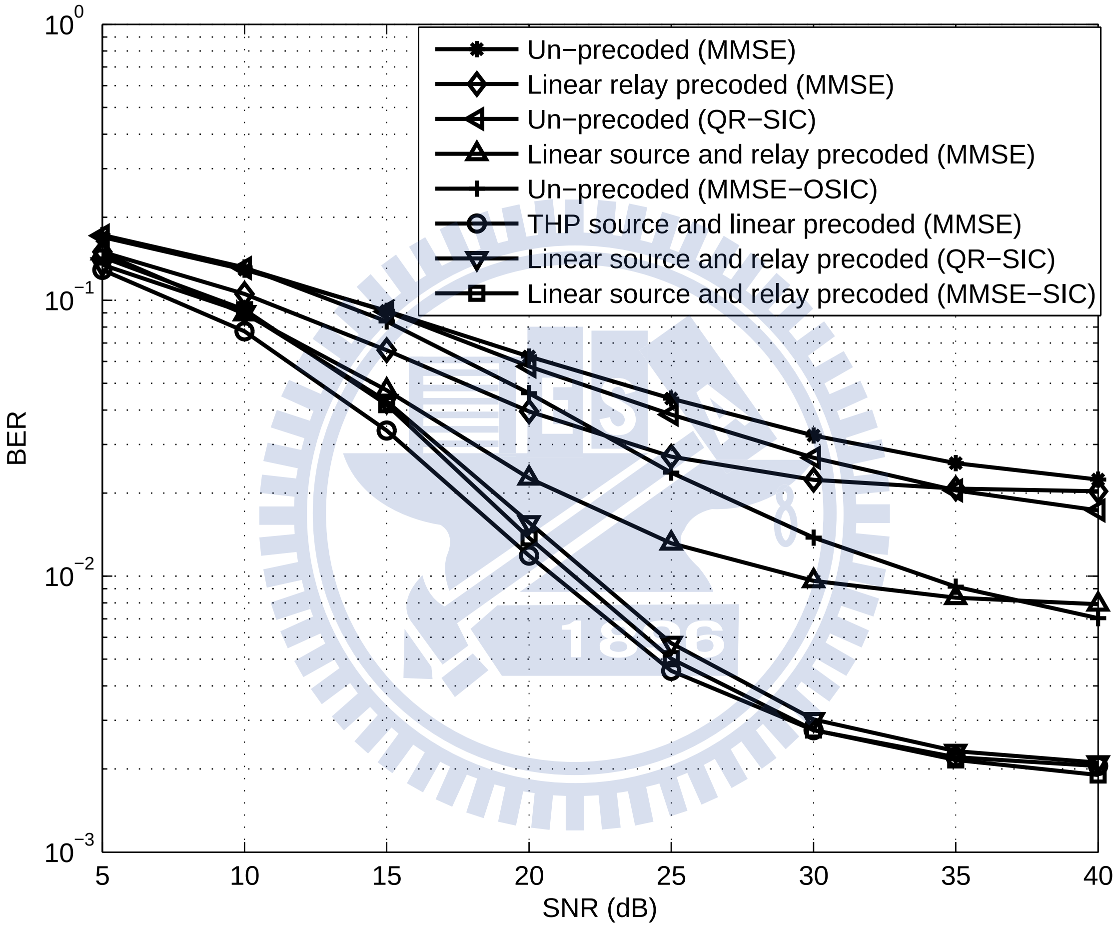


Figure 5.2: BER performance comparison for un-precoded and precoded systems (16QAM, $N = R = M = 4$, SNR$_{sr}$=20, SNR$_{sd}$=5 dB).

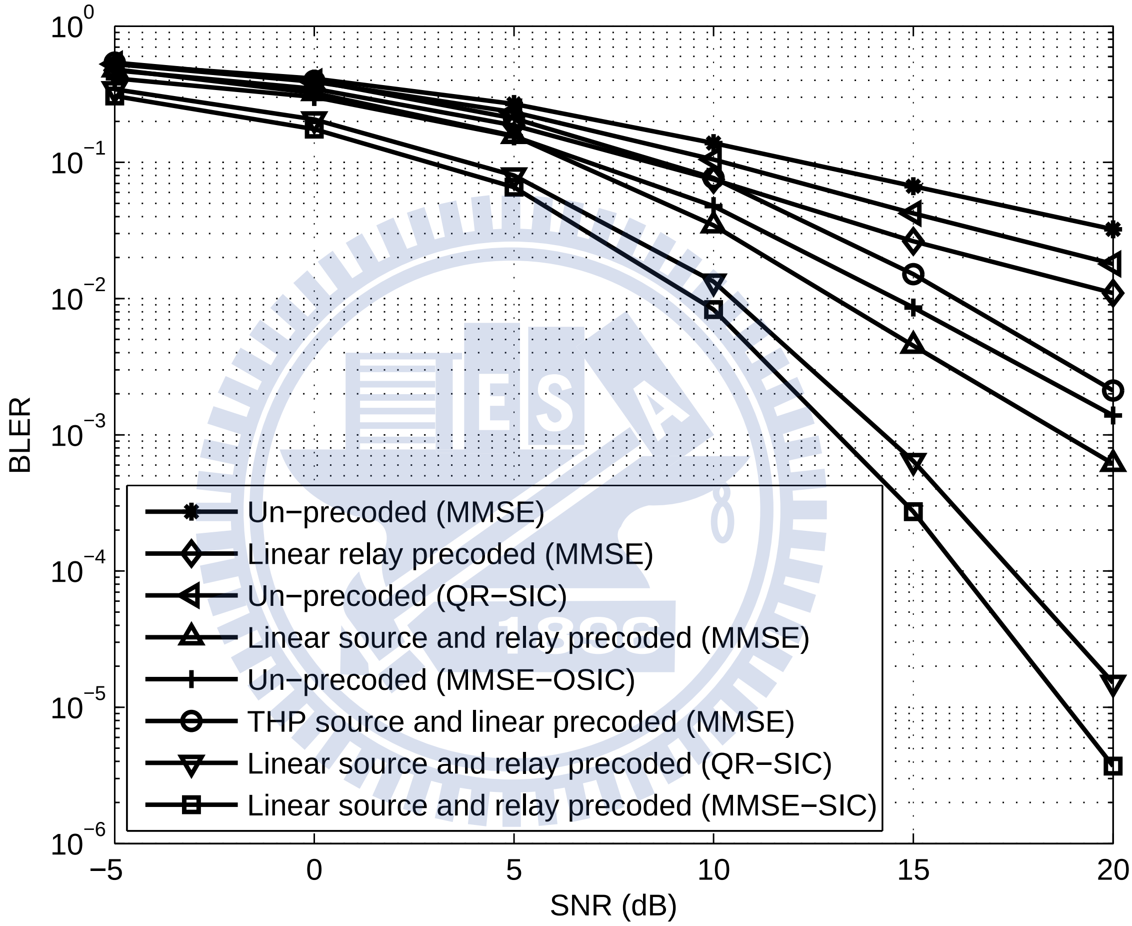


Figure 5.3: BLER performance comparison for un-precoded and precoded systems (4-QAM, $N = R = M = 4$, SNR$_{sr}$=20, SNR$_{sd}$=5 dB).

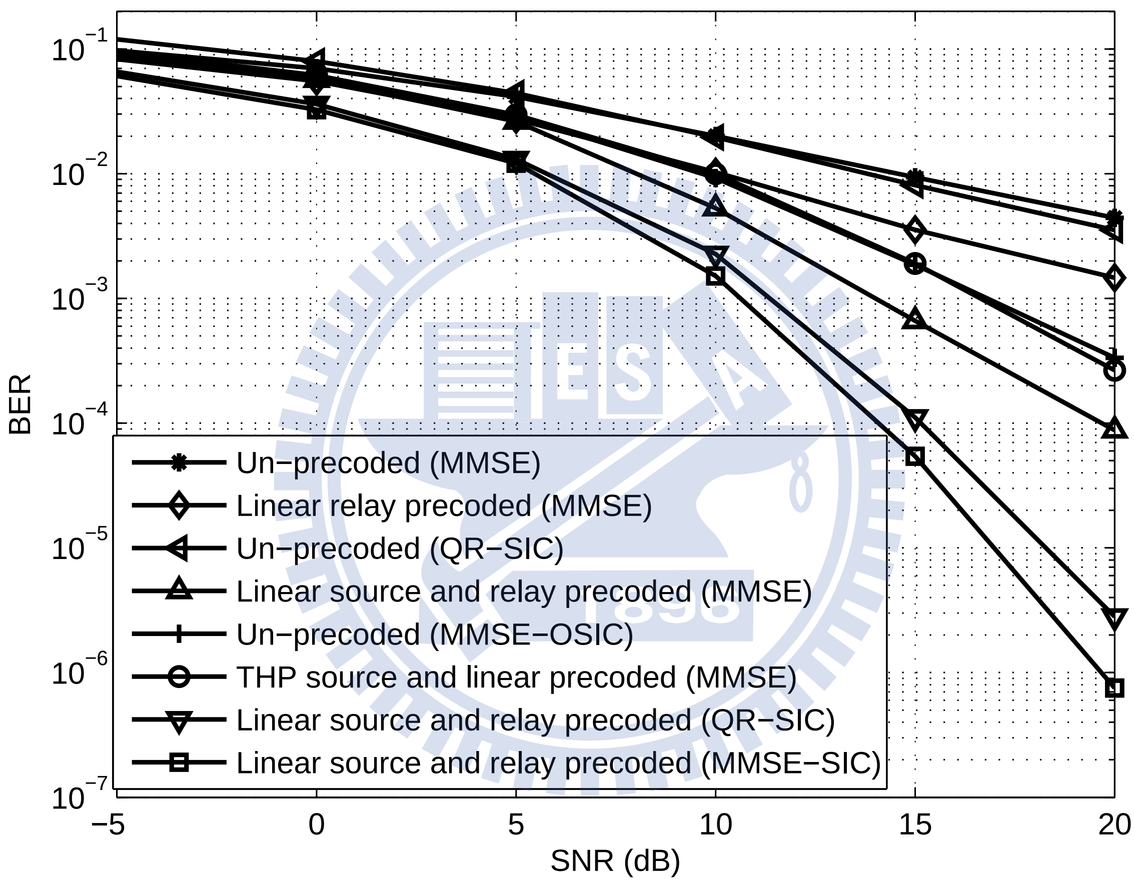


Figure 5.4: BER performance comparison for un-precoded and precoded systems (4-QAM, $N = R = M = 4$, SNR$_{sr}$=20, SNR$_{sd}$=5 dB).

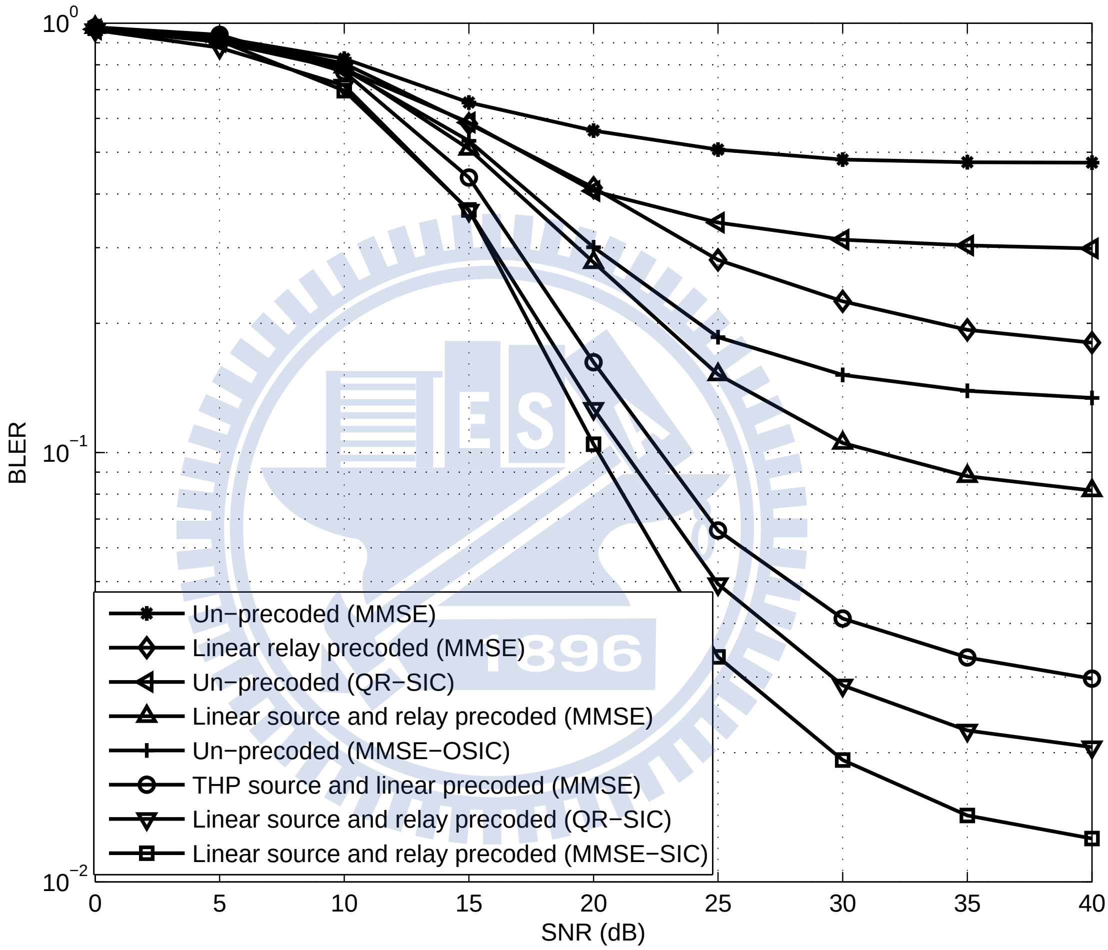


Figure 5.5: BLER performance comparison for un-precoded and precoded systems (16QAM, $N = R = M = 4$, SNR_{sd}=5, SNR_{rd}=20 dB).

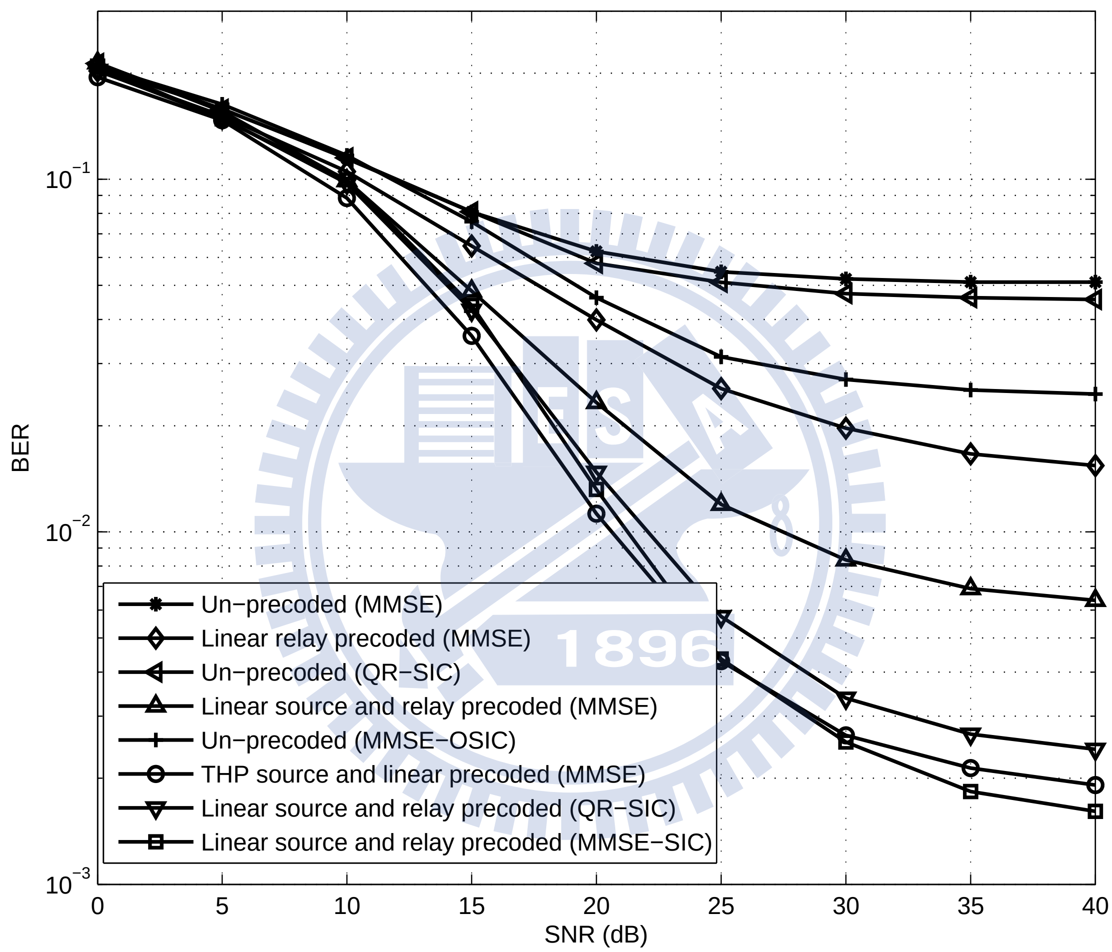


Figure 5.6: BER performance comparison for un-precoded and precoded systems (16QAM, $N = R = M = 4$, SNR$_{sd}$=5, SNR$_{rd}$=20 dB).

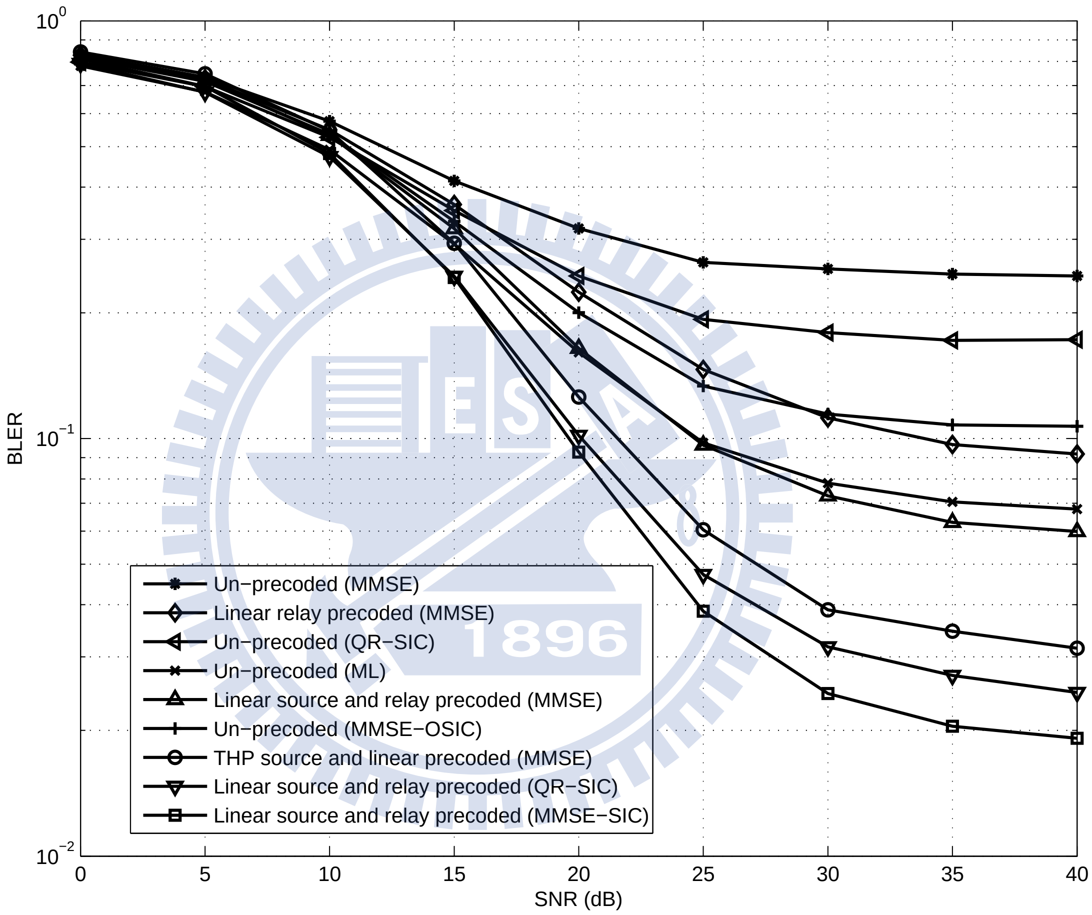


Figure 5.7: BLER performance comparison for un-precoded and precoded systems (16QAM, $N = R = M = 2$, SNR$_{sd}$=5, SNR$_{rd}$=20 dB).

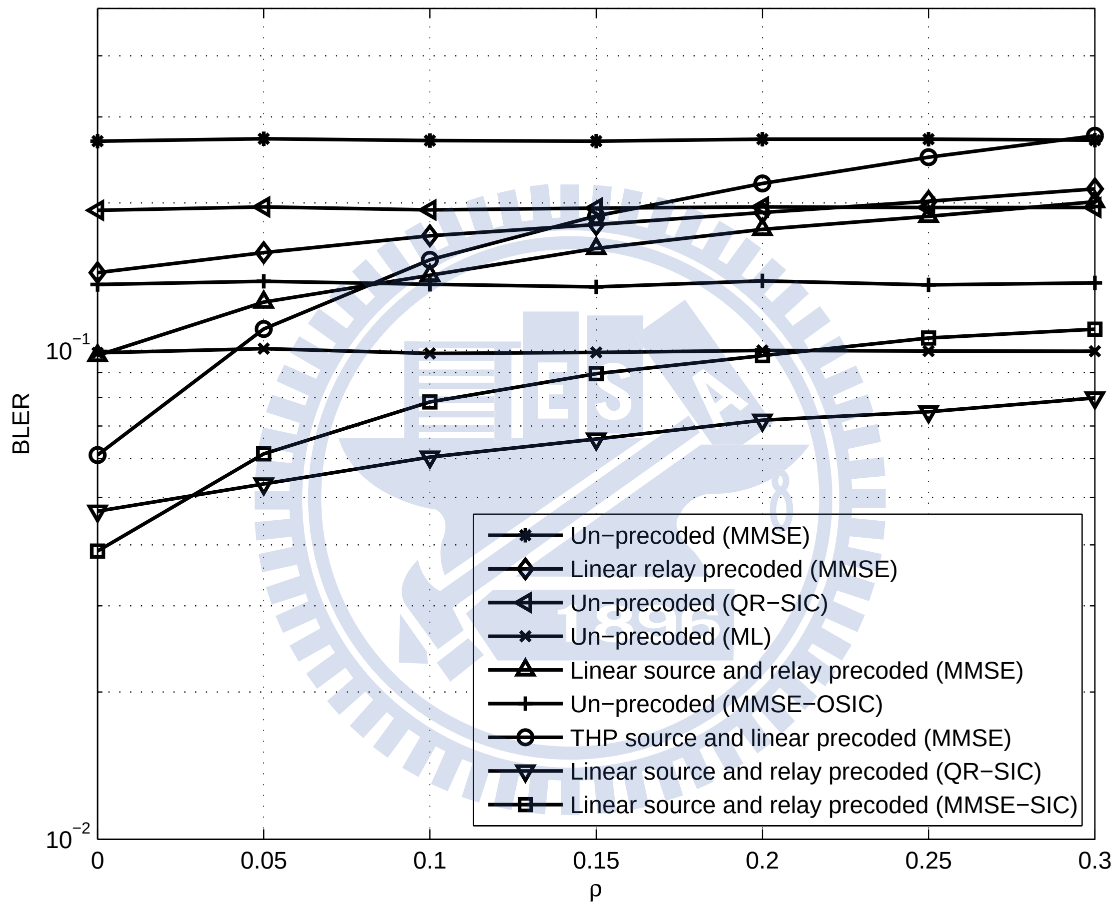


Figure 5.8: BLER performance comparison for un-precoded and precoded systems with imperfect CSIs (16QAM, $N = R = M = 2$, SNR$_{sd}$=5, SNR$_{sr}$=25 dB, SNR$_{rd}$=20 dB).

Chapter 6

Joint MMSE Transceiver Design with Quality-of-Service (QoS) Constraints

In previous chapters, we address the transceiver designs in MIMO relay systems maximizing the system performance under the power constraints. In this chapter, we consider the transceiver design minimizing the transmission power under QoS constraints. The transceiver structure considered here is the same as that proposed in Chapter 2, in which the linear precoder is used at the source (and the relay), and the linear MMSE receiver at the destination. Since there is a one-to-one mapping between the BER and the MSE, we use the MSEs of signal streams as our QoS constraints. We first consider the precoders design in two-hop systems and then general MIMO relay systems. Our formulation leads to an optimization problem that the constraint function is a highly nonlinear function of the precoders, either in the two-hop or general MIMO relay systems. To overcome the problem, we first propose new precoder structures which can simplify the optimization in the two-hop system. The proposed structures can translate the matrix-valued optimization problem into a scalar-valued one, facilitating the derivation of the optimum solution. For general MIMO relay systems, the problem becomes more involved since the direct link is included. Based on the proposed precoder structure, however, we can derive an MSE upper bound. Using the upper bound as the constraint function, the original optimization problem can be greatly simplified, and the solution can be obtained by the primal decomposition approach. In Section 6.1, we first give the system model and the related optimization problem. After that, we derive the source and relay precoders in the two-hop MIMO relay and then the general MIMO relay systems, respectively, in Section 6.2 and 6.3. Finally, we evaluate the performance of the proposed precoders in Section 6.4.

§ 6.1 System Model and Problem Formulation

§ 6.1.1 *MMSE Receiver with Linear Source and Relay Precoders*

We consider the same transceiver in Chapter 2. Recall (2.4), the received signal can thus be expressed as

$$\mathbf{y}_D := \begin{bmatrix} \mathbf{y}_{D,1} \\ \mathbf{y}_{D,2} \end{bmatrix} = \mathbf{H}\mathbf{F}_S\mathbf{s} + \mathbf{n}, \tag{6.1}$$

where

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_{SD} \\ \mathbf{H}_{RD}\mathbf{F}_R\mathbf{H}_{SR} \end{bmatrix}, \quad \text{and} \quad \mathbf{n} = \begin{bmatrix} \mathbf{n}_{D,1} \\ \mathbf{H}_{RD}\mathbf{F}_R\mathbf{n}_R + \mathbf{n}_{D,2} \end{bmatrix}. \tag{6.2}$$

Based on (6.1), the MMSE equalization matrix $\mathbf{G}_{opt}$, as shown in (2.8), can be expressed as

$$\mathbf{G}_{opt} = \sigma_s^2\mathbf{F}_S^H\mathbf{H}^H\left(\sigma_s^2\mathbf{H}\mathbf{F}_S\mathbf{F}_S^H\mathbf{H}^H + \mathbf{R}_n\right)^{-1}. \tag{6.3}$$

The resultant minimal MSE and MSE matrix can then be expressed as

$$J_{min} = tr\left\{\left(\sigma_s^{-2}\mathbf{I}_L + \mathbf{E}_D + \mathbf{E}_R\right)^{-1}\right\}, \tag{6.4}$$

and

$$\mathbf{E} = \left(\sigma_s^{-2}\mathbf{I}_L + \mathbf{E}_D + \mathbf{E}_R\right)^{-1}, \tag{6.5}$$

respectively, where

$$\mathbf{E}_D = \sigma_{n,d}^{-2}\mathbf{F}_S^H\mathbf{H}_{SD}^H\mathbf{H}_{SD}\mathbf{F}_S \tag{6.6}$$

and

$$\mathbf{E}_R = \mathbf{F}_S^H \mathbf{H}_{SR}^H \mathbf{F}_R^H \mathbf{H}_{RD}^H \left(\sigma_{n,r}^2 \mathbf{H}_{RD} \mathbf{F}_R \mathbf{F}_R^H \mathbf{H}_{RD}^H + \sigma_{n,d}^2 \mathbf{I}_M\right)^{-1} \mathbf{H}_{RD} \mathbf{F}_R \mathbf{H}_{SR} \mathbf{F}_S. \tag{6.7}$$

As we can see from (6.4)-(6.5), the MMSE and MSE matrix are the functions of $\mathbf{F}_S$ and $\mathbf{F}_R$. Also, $\mathbf{E}_D$ denotes the MSE component due to the direct link and $\mathbf{E}_R$ is contributed by the relay link. If only the relay link is considered (which is also known as the two-hop MIMO relay system), we can set $\mathbf{E}_D = \mathbf{0}$ [43]- [44].

§ 6.1.2 *Problem Formulation*

To start with, we first let the QoS constraints be defined in terms of the MSEs at the receiver, i.e.,

$$\mathbf{E}(i,i) := E\left[|s_i - \hat{s}_i|^2\right] \leq \rho_i, \quad 1 \leq i \leq L, \tag{6.8}$$

where $\mathbf{E}(i,i)$ is the ith component of $\mathbf{E}$ and ρ_i is the MSE constraint for the ith data stream. Here we note that $0 \leq \rho_i < \sigma_s^2$ since $E\left[|s_i - \hat{s}_i|^2\right] <= E\left[|s_i|^2\right] = \sigma_s^2$.

Our task is to design the source and relay precoders such that the transmission power is minimized and designated MSE constraints are satisfied. To proceed, let us define the power consumption at the source and the relay, respectively, as

$$P_{S,T} = tr\left(E\left[\mathbf{F}_S \mathbf{s}\mathbf{s}^H \mathbf{F}_S^H\right]\right) \tag{6.9}$$

and

$$\begin{aligned} P_{R,T} &= tr\left(E\left[\mathbf{F}_R \left(\mathbf{H}_{SR}\mathbf{F}_S\mathbf{s} + \mathbf{n}_R\right)\left(\mathbf{H}_{SR}\mathbf{F}_S\mathbf{s} + \mathbf{n}_R\right)^H \mathbf{F}_R^H\right]\right) \\ &= tr\left(\mathbf{F}_R\left(\sigma_s^2 \mathbf{H}_{SR}\mathbf{F}_S\mathbf{F}_S^H\mathbf{H}_{SR}^H + \sigma_{n,r}^2\mathbf{I}_R\right)\mathbf{F}_R^H\right). \end{aligned} \tag{6.10}$$

With (6.5)-(6.10), the joint source/relay precoders design problem can be formulated as

$$
\begin{aligned}
&\min_{\mathbf{F}_S,\mathbf{F}_R} tr\left\{\mathbf{F}_R\left(\sigma_s^2\mathbf{H}_{SR}\mathbf{F}_S\mathbf{F}_S^H\mathbf{H}_{SR}^H+\sigma_{n,r}^2\mathbf{I}_R\right)\mathbf{F}_R^H\right\}+\sigma_s^2 tr\left\{\mathbf{F}_S\mathbf{F}_S^H\right\}\\
&s.t.\\
&\mathbf{E}(i,i)\le\rho_i\\
&\mathbf{E}=\left(\sigma_s^{-2}\mathbf{I}_L+\sigma_{n,d}^{-2}\mathbf{F}_S^H\mathbf{H}_{SD}^H\mathbf{H}_{SD}\mathbf{F}_S+\right.\\
&\left.\mathbf{F}_S^H\mathbf{H}_{SR}^H\mathbf{F}_R^H\mathbf{H}_{RD}^H\left(\sigma_{n,r}^2\mathbf{H}_{RD}\mathbf{F}_R\mathbf{F}_R^H\mathbf{H}_{RD}^H+\sigma_{n,d}^2\mathbf{I}_M\right)^{-1}\mathbf{H}_{RD}\mathbf{F}_R\mathbf{H}_{SR}\mathbf{F}_S\right)^{-1}.
\end{aligned} \tag{6.11}
$$

Taking a closer look at (6.11), we readily find that the MSE matrix $\mathbf{E}$ involves a series of matrix multiplications and inversions and is a complicated function of $\mathbf{F}_S$ and $\mathbf{F}_R$. Also, the problem is not a convex optimization problem. Therefore, the exact solution to (6.11) is almost impossible to derive. In the next section, we propose a new method to solve the problem

§ 6.2 Joint Source/Relay Precoders Design for Two-Hop MIMO Relay System

§ 6.2.1 *Proposed Precoder Structures*

In the two-hop MIMO relay system, the optimization problem in (6.11) can be reformulated as

$$
\begin{aligned}
&\min_{\mathbf{F}_S,\mathbf{F}_R} tr\left\{\mathbf{F}_R\left(\sigma_s^2\mathbf{H}_{SR}\mathbf{F}_S\mathbf{F}_S^H\mathbf{H}_{SR}^H+\sigma_{n,r}^2\mathbf{I}_R\right)\mathbf{F}_R^H\right\}+\sigma_s^2 tr\left\{\mathbf{F}_S\mathbf{F}_S^H\right\}\\
&s.t.\\
&\mathbf{E}(i,i)\le\rho_i\\
&\mathbf{E}=\left(\sigma_s^{-2}\mathbf{I}_L+\right.\\
&\left.\mathbf{F}_S^H\mathbf{H}_{SR}^H\mathbf{F}_R^H\mathbf{H}_{RD}^H\left(\sigma_{n,r}^2\mathbf{H}_{RD}\mathbf{F}_R\mathbf{F}_R^H\mathbf{H}_{RD}^H+\sigma_{n,d}^2\mathbf{I}_M\right)^{-1}\mathbf{H}_{RD}\mathbf{F}_R\mathbf{H}_{SR}\mathbf{F}_S\right)^{-1}.
\end{aligned} \tag{6.12}
$$

It is simple to check that the problem in (6.12) is not a convex optimization problem and the optimum solution is still difficult to derive. For simplicity, we first study the scenario that all the MSE constraints are the same. In this case, the problem can be reformulated the following equivalent optimization, similar to that in the conventional MIMO system [9]:

$$\begin{aligned}
&\min_{\mathbf{F}_S,\mathbf{F}_R} tr\left\{\mathbf{F}_R\left(\sigma_s^2\mathbf{H}_{SR}\mathbf{F}_S\mathbf{F}_S^H\mathbf{H}_{SR}^H+\sigma_{n,r}^2\mathbf{I}_R\right)\mathbf{F}_R^H\right\}+\sigma_s^2 tr\left\{\mathbf{F}_S\mathbf{F}_S^H\right\}\\
&s.t.\\
&\mathbf{E}\quad \text{with equal diagonal elements},\\
&tr\{\mathbf{E}\}\le L\rho. \qquad (6.13)
\end{aligned}$$

Equation (6.13) is difficult to solve though it is simplified. In what follows, we will propose a source and a relay precoder structures that can diagonalize the cost function in (6.13) facilitating the derivation of the optimum solution. To proceed, we consider the following SVD:

$$\mathbf{H}_{SR}=\mathbf{U}_{sr}\mathbf{\Sigma}_{sr}\mathbf{V}_{sr}^H, \qquad (6.14)$$

$$\mathbf{H}_{RD}=\mathbf{U}_{rd}\mathbf{\Sigma}_{rd}\mathbf{V}_{rd}^H, \qquad (6.15)$$

where $\mathbf{V}_{sr}\in\mathbb{C}^{N\times N}$ and $\mathbf{V}_{rd}\in\mathbb{C}^{R\times R}$ are the right singular matrices of $\mathbf{H}_{SR}$ and $\mathbf{H}_{RD}$, respectively; $\mathbf{U}_{sr}\in\mathbb{C}^{R\times R}$ and $\mathbf{U}_{rd}\in\mathbb{C}^{M\times M}$ are the left singular matrices of $\mathbf{H}_{SR}$ and $\mathbf{H}_{RD}$, respectively; $\mathbf{\Sigma}_{sr}$ and $\mathbf{\Sigma}_{rd}$ are the diagonal matrices where $\sigma_{sr,i}$ and $\sigma_{rd,i}$ are the ith diagonal element of $\mathbf{\Sigma}_{sr}$ and $\mathbf{\Sigma}_{rd}$, respectively. Now, for given any $\mathbf{F}_S$ and $\mathbf{F}_R$, we can always express $\mathbf{F}_S$ and $\mathbf{F}_R$ as

$$\mathbf{F}_S=\mathbf{V}_{sr}\mathbf{\Phi}_s\mathbf{U}_S, \qquad (6.16)$$

$$\mathbf{F}_R=\mathbf{V}_{rd}\mathbf{\Phi}_r\mathbf{U}_{sr}^H, \qquad (6.17)$$

where $\mathbf{U}_S\in\mathbb{C}^{L\times L}$ is an unitary matrix to be further decided; $\mathbf{\Phi}_s\in\mathbb{C}^{N\times L}$ and $\mathbf{\Phi}_r\in\mathbb{C}^{R\times R}$ are matrices not restricted to be diagonal here [1]. Substituting (6.16) and (6.17) into (6.13), we then have

$$\mathbf{E}=\mathbf{U}_S^H\left(\sigma_s^{-2}\mathbf{I}_L+\mathbf{\Phi}_s^H\mathbf{\Sigma}_{sr}^H\mathbf{\Phi}_r^H\mathbf{\Sigma}_{rd}^H\left(\sigma_{n,r}^2\mathbf{\Sigma}_{rd}\mathbf{\Phi}_r\mathbf{\Phi}_r^H\sigma_{rd}^H+\sigma_{n,d}^2\mathbf{I}_M\right)^{-1}\mathbf{\Sigma}_{rd}\mathbf{\Phi}_r\mathbf{\Sigma}_{sr}\mathbf{\Phi}_s\right)^{-1}\mathbf{U}_S. \qquad (6.18)$$

[1]The $\mathbf{\Phi}_s$ and $\mathbf{\Phi}_r$ are equivalently as $\mathbf{V}_{sr}^H\mathbf{F}_S\mathbf{U}_S^H$ and $\mathbf{V}_{rd}^H\mathbf{F}_R\mathbf{U}_{sr}$, respectively.

Using the eigenvalue-decomposition (EVD), we can have an expression of

$$\mathbf{\Sigma}_{sr}^H\mathbf{\Phi}_r^H\mathbf{\Sigma}_{rd}^H\left(\sigma_{n,r}^2\mathbf{\Sigma}_{rd}\mathbf{\Phi}_r\mathbf{\Phi}_r^H\sigma_{rd}^H+\sigma_{n,d}^2\mathbf{I}_M\right)^{-1}\mathbf{\Sigma}_{rd}\mathbf{\Phi}_r\mathbf{\Sigma}_{sr}=\mathbf{U}_H\mathbf{D}_H\mathbf{U}_H^H, \tag{6.19}$$

where $\mathbf{U}_H \in \mathbb{C}^{N\times N}$ and $\mathbf{D}_H \in \mathbb{R}^{N\times N}$ are a unitary and a diagonal matrices, respectively; both are functions of $\mathbf{F}_R$. We can then re-express $\mathbf{E}$ in (6.18) as

$$\mathbf{E}=\mathbf{U}_S^H\left(\sigma_s^{-2}\mathbf{I}_L+\mathbf{\Phi}_s^H\mathbf{U}_H\mathbf{D}_H\mathbf{U}_H^H\mathbf{\Phi}_s\right)^{-1}\mathbf{U}_S. \tag{6.20}$$

We can further conduct EVD of $\mathbf{\Phi}_s^H\mathbf{U}_H\mathbf{D}_H\mathbf{U}_H^H\mathbf{\Phi}_s$ and have

$$\mathbf{\Phi}_s^H\mathbf{U}_H\mathbf{D}_H\mathbf{U}_H^H\mathbf{\Phi}_s=\mathbf{U}\mathbf{D}\mathbf{U}^H, \tag{6.21}$$

where $\mathbf{U}\in\mathbb{C}^{L\times L}$ and $\mathbf{D}\in\mathbb{R}^{L\times L}$ are a unitary and a diagonal matrices, respectively. Similarly, both $\mathbf{U}$ and $\mathbf{D}$ are functions of $\mathbf{F}_S$ and $\mathbf{F}_R$. From (6.21), we then have

$$\mathbf{U}^H\mathbf{\Phi}_s^H\mathbf{U}_H\mathbf{D}_H\mathbf{U}_H^H\mathbf{\Phi}_s\mathbf{U}=\mathbf{D}:=\mathbf{D}^{1/2}\mathbf{Q}\mathbf{Q}^H\mathbf{D}^{1/2}, \tag{6.22}$$

where $\mathbf{D}^{1/2}\mathbf{D}^{1/2}=\mathbf{D}$ and $\mathbf{Q}\in\mathbb{C}^{L\times L}$ is an arbitrary unitary matrix. From (6.21) and (6.22), we have

$$\mathbf{\Phi}_s=\mathbf{U}_H\mathbf{D}_H^{-1/2}\mathbf{Q}^H\mathbf{D}^{1/2}\mathbf{U}^H. \tag{6.23}$$

From (6.16) we see that the power consumption at the source can be expressed as

$$\sigma_s^2tr\{\mathbf{F}_S\mathbf{F}_S^H\}=\sigma_s^2tr\{\mathbf{\Phi}_s\mathbf{\Phi}_s^H\}=\sigma_s^2tr\left\{\mathbf{D}^{1/2}\mathbf{D}^{1/2}\mathbf{Q}\mathbf{D}_H^{-1}\mathbf{Q}^H\right\}. \tag{6.24}$$

For any semi-definite Hermitian matrices $\mathbf{A}\in\mathbb{C}^{L\times L}$ and $\mathbf{B}\in\mathbb{C}^{L\times L}$ matrices [50], the following property holds.

$$tr\left(\mathbf{A}\mathbf{B}\right)=\sum_{i=1}^{L}\lambda_{\mathbf{A},i}\lambda_{\mathbf{B},L-i+1}, \tag{6.25}$$

where $\lambda_{\mathbf{A},i}$ and $\lambda_{\mathbf{B},i}$ is the ith eigenvalue of $\mathbf{A}$ and $\mathbf{B}$, respectively (with decreasing order). Letting $\mathbf{A}=\mathbf{B}$ and $\mathbf{B}=\mathbf{Q}\mathbf{D}_H^{-1}\mathbf{Q}^H$ and denote the ith diagonal element of $\mathbf{D}$ and $\mathbf{D}_H$ as $\sigma_{D,i}^2$ and $\sigma_{H,i}^2$, respectively, we can have a lower bound of (6.24) as

$$\sigma_s^2tr\{\mathbf{F}_S\mathbf{F}_S^H\}=\sigma_s^2tr\left\{\mathbf{D}\mathbf{Q}\mathbf{D}_H^{-1}\mathbf{Q}^H\right\}\geq\sum_{i=1}^{L}\frac{\sigma_{D,i}^2}{\sigma_{H,i}^2}, \tag{6.26}$$

where the equality holds when $\mathbf{Q} = \mathbf{I}_L$. Similarly, from (6.16) and (6.17), we can have the power consumption at the relay as

$$\begin{aligned}
&tr\left\{\mathbf{F}_R\left(\sigma_s^2\mathbf{H}_{SR}\mathbf{F}_S\mathbf{F}_S^H\mathbf{H}_{SR}^H + \sigma_{n,r}^2\mathbf{I}_R\right)\mathbf{F}_R^H\right\} \\
&= tr\left\{\mathbf{\Phi}_r\left(\sigma_s^2\mathbf{\Sigma}_{sr}\mathbf{\Phi}_s\mathbf{\Phi}_s^H\mathbf{\Sigma}_{sr}^H + \sigma_{n,r}^2\mathbf{I}_R\right)\mathbf{\Phi}_r^H\right\} \\
&= tr\left\{\mathbf{\Sigma}_{rd}^{-1}\mathbf{\Sigma}_{rd}\mathbf{\Phi}_r\left(\sigma_s^2\mathbf{\Sigma}_{sr}\mathbf{\Phi}_s\mathbf{\Phi}_s^H\mathbf{\Sigma}_{sr}^H + \sigma_{n,r}^2\mathbf{I}_R\right)\mathbf{\Phi}_r^H\mathbf{\Sigma}_{rd}^H\mathbf{\Sigma}_{rd}^{-H}\right\} \\
&= tr\left\{\mathbf{\Sigma}_{rd}^{-1}\mathbf{U}'\mathbf{D}'\mathbf{U}'^H\mathbf{\Sigma}_{rd}^{-H}\right\} \qquad (6.27)
\end{aligned}$$

where

$$\mathbf{\Sigma}_{rd}\mathbf{\Phi}_r\left(\sigma_s^2\mathbf{\Sigma}_{sr}\mathbf{\Phi}_s\mathbf{\Phi}_s^H\mathbf{\Sigma}_{sr}^H + \sigma_{n,r}^2\mathbf{I}_R\right)\mathbf{\Phi}_r^H\mathbf{\Sigma}_{rd}^H := \mathbf{U}'\mathbf{D}'\mathbf{U}'^H. \qquad (6.28)$$

Here, $\mathbf{U}' \in \mathbb{C}^{L\times L}$ and $\mathbf{D}' \in \mathbb{R}^{L\times L}$ are a unitary and a diagonal matrices, respectively. Also, $\mathbf{D}'$ = diag$\{\sigma'^2_{D,1}, \cdots, \sigma'^2_{D,M}\}$ denoting the collection of the eigenvalues ($\sigma'^2_{D,1} \geq \cdots \sigma'^2_{D,M}$). By (6.27), we can have a lower bound of the relay power consumption as

$$\left\{\mathbf{F}_R\left(\sigma_s^2\mathbf{H}_{SR}\mathbf{F}_S\mathbf{F}_S^H\mathbf{H}_{SR}^H + \sigma_{n,r}^2\mathbf{I}_R\right)\mathbf{F}_R^H\right\} = tr\left\{\mathbf{\Sigma}_{rd}^{-1}\mathbf{U}'\mathbf{D}'\mathbf{U}'^H\mathbf{\Sigma}_{rd}^{-H}\right\} \geq \sum_{i=1}^{L}\frac{\sigma'^2_{D,i}}{\sigma^2_{rd,i}}, \quad (6.29)$$

where the equality holds when $\mathbf{U}' = \mathbf{I}_R$.

From (6.26) and (6.29), we have a lower bound of the total power consumption for each feasible set of $(\mathbf{F}_S, \mathbf{F}_R)$ as

$$\sigma_s^2 tr\{\mathbf{F}_S\mathbf{F}_S^H\} + tr\left\{\mathbf{F}_R\left(\sigma_s^2\mathbf{H}_{SR}\mathbf{F}_S\mathbf{F}_S^H\mathbf{H}_{SR}^H + \sigma_{n,r}^2\mathbf{I}_R\right)\mathbf{F}_R^H\right\} \geq \sum_{i=1}^{L}\frac{\sigma^2_{D,i}}{\sigma^2_{H,i}} + \sum_{i=1}^{L}\frac{\sigma'^2_{D,i}}{\sigma^2_{rd,i}}. \quad (6.30)$$

The lower bound can be achieved when $\mathbf{Q} = \mathbf{I}_L$ and $\mathbf{U}' = \mathbf{I}_R$. Note that $\mathbf{Q}$ is a matrix that we can choose, and the condition that $\mathbf{Q} = \mathbf{I}_L$ can be easily satisfied. However, as we can see from (6.22) and (6.28), $\sigma^2_{D,i}$ and $\sigma'^2_{D,i}$ are complicated functions of $\mathbf{\Phi}_s$ and $\mathbf{\Phi}_r$. It is then difficult to find a general set of $(\mathbf{\Phi}_s, \mathbf{\Phi}_r)$ such that the condition $\mathbf{U}' = \mathbf{I}_R$ can be met (see (6.28)). One possible solution is to let $\mathbf{\Phi}_s$ and $\mathbf{\Phi}_r$ be diagonal matrices and then $\mathbf{U}' = \mathbf{I}_R$. Denote the diagonal $\mathbf{\Phi}_s$ and $\mathbf{\Phi}_r$ matrices as $\mathbf{\Sigma}_s$ and $\mathbf{\Sigma}_r$, respectively, and the ith diagonal element of $\mathbf{\Sigma}_s$

and $\mathbf{\Sigma}_r$ as $\sigma_{s,i}$ and $\sigma_{r,i}$, respectively. We then can reformulate (6.16) and (6.17) as

$$\mathbf{F}_S = \mathbf{V}_{sr}\mathbf{\Sigma}_s\mathbf{U}_S, \tag{6.31}$$

$$\mathbf{F}_R = \mathbf{V}_{rd}\mathbf{\Sigma}_r\mathbf{U}_{sr}. \tag{6.32}$$

The MSE in (6.13) thus becomes

$$\begin{aligned} tr(\mathbf{E}) &= tr\Big(\mathbf{U}_S^H\big(\sigma_s^{-2}\mathbf{I}_L + \mathbf{\Sigma}_s^H\mathbf{\Sigma}_{sr}^H\mathbf{\Sigma}_r^H\mathbf{\Sigma}_{rd}^H \\ &\quad \left(\sigma_{n,r}^2\mathbf{\Sigma}_{rd}\mathbf{\Sigma}_r\mathbf{\Sigma}_r^H\mathbf{\Sigma}_{rd}^H + \sigma_{n,d}^2\mathbf{I}_M\right)^{-1}\mathbf{\Sigma}_{rd}\mathbf{\Sigma}_r\mathbf{\Sigma}_{sr}\mathbf{\Sigma}_s\big)^{-1}\mathbf{U}_S\Big) \\ &= \sum_{i=1}^{L}\left(\sigma_s^{-2} + \frac{\sigma_{s,i}^2\sigma_{r,i}^2\sigma_{sr,i}^2\sigma_{rd,i}^2}{\sigma_{n,r}^2\sigma_{r,i}^2\sigma_{rd,i}^2 + \sigma_{n,d}^2}\right)^{-1}. \end{aligned} \tag{6.33}$$

Note that the requirement of equal diagonal elements in (6.13) can be easily achieved by letting $\mathbf{U}_S = \mathbf{F}_L$ which is an L-point DFT matrix. Therefore, substituting (6.31)-(6.33) in (6.13), we can have the optimization problem as

$$\begin{aligned} &\min_{p_{s,i},p_{r,i}} \sum_{i=1}^{L} p_{r,i}\sigma_{n,r}^2 + \sum_{i=1}^{L}\sigma_s^2 p_{s,i}p_{r,i}\sigma_{sr,i}^2 + \sigma_s^2\sum_{i=1}^{L}p_{s,i}^2 \\ &s.t. \\ &\sum_{i=1}^{L}\left(\sigma_s^{-2} + \frac{p_{s,i}p_{r,i}\sigma_{sr,i}^2\sigma_{rd,i}^2}{\sigma_{n,r}^2 p_{r,i}\sigma_{rd,i}^2 + \sigma_{n,d}^2}\right)^{-1} \le L\rho, \\ &p_{s,i} = \sigma_{s,i}^2 \ge 0, \quad p_{r,i} = \sigma_{r,i}^2 \ge 0, \quad \forall i. \end{aligned} \tag{6.34}$$

Here, we define $p_{r,i} = 0$ if $i > R$. As we can see, the matrix-valued optimization problem in (6.13) becomes a scalar-valued optimization problem which is much easier to work with.

For the case of different MSE constraints, it will more difficult to find the optimum precoder structure. However, we can still use the precoder structures described in (6.31) and (6.32) to solve the problem. This method, though suboptimal, can also transfer the matrix operations to a series of scalar-valued operations in (6.12). In this case, however, we have to let $\mathbf{U}_S = \mathbf{I}_L$ in (6.31). From (6.31) and (6.32), we can rewrite the MSE matrix in (6.12) as

$$\mathbf{E} = \left(\sigma_s^{-2}\mathbf{I}_L + \mathbf{\Sigma}_s^H\mathbf{\Sigma}_{sr}^H\mathbf{\Sigma}_r^H\mathbf{\Sigma}_{rd}^H\left(\sigma_{n,r}^2\mathbf{\Sigma}_{rd}\mathbf{\Sigma}_r\mathbf{\Sigma}_r^H\mathbf{\Sigma}_{rd}^H + \sigma_{n,d}^2\mathbf{I}_M\right)^{-1}\mathbf{\Sigma}_{rd}\mathbf{\Sigma}_r\mathbf{\Sigma}_{sr}\mathbf{\Sigma}_s\right)^{-1}. \tag{6.35}$$

As a result, the optimization problem is translated into

$$\begin{aligned}
&\min_{p_{s,i},p_{r,i}} \sum_{i=1}^{L} p_{r,i}\sigma_{n,r}^2 + \sum_{i=1}^{L} \sigma_s^2 p_{s,i} p_{r,i} \sigma_{sr,i}^2 + \sigma_s^2 \sum_{i=1}^{L} p_{s,i} \\
&s.t. \\
&\left(\sigma_s^{-2} + \frac{p_{s,i} p_{r,i} \sigma_{sr,i}^2 \sigma_{rd,i}^2}{\sigma_{n,r}^2 p_{r,i} \sigma_{rd,i}^2 + \sigma_{n,d}^2}\right)^{-1} \leq \rho_i, \\
&p_{s,i} = \sigma_{s,i}^2 \geq 0, \quad p_{r,i} = \sigma_{r,i}^2 \geq 0, \quad \forall i.
\end{aligned} \tag{6.36}$$

§ 6.2.2 *Optimum Solutions in (6.34) and (6.36)*

From (6.34) and (6.36), it is simple to find that the optimization problems are both not convex. However, resorting to the primal decomposition approach [51], we can transfer the original optimization problem into two optimization problems, which are referred to as a subproblem and a master problem. Using the method, we can re-formulate the cost function in (6.34) and (6.36) as

$$\begin{aligned}
&\min_{p_{s,i},p_{r,i}} \sum_{i=1}^{L} p_{r,i}\sigma_{n,r}^2 + \sum_{i=1}^{L} \sigma_s^2 p_{s,i} p_{r,i} \sigma_{sr,i}^2 + \sigma_s^2 \sum_{i=1}^{L} p_{s,i} \\
&= \min_{p_{r,i}} \min_{p_{s,i}(p_{r,i})} \sum_{i=1}^{L} p_{r,i}\sigma_{n,r}^2 + \sum_{i=1}^{L} \sigma_s^2 p_{s,i} p_{r,i} \sigma_{sr,i}^2 + \sigma_s^2 \sum_{i=1}^{L} p_{s,i}^2.
\end{aligned} \tag{6.37}$$

As we will see, both the master problem and the subproblem are scalar-valued convex optimization problems and closed-form solutions can be easily derived. Given $p_{r,i}$, $i = 1, \cdots, L$, the subproblem in (6.36) is given by

$$\begin{aligned}
&\min_{p_{s,i}(p_{r,i})} \sum_{i=1}^{L} p_{r,i}\sigma_{n,r}^2 + \sum_{i=1}^{L} \sigma_s^2 p_{s,i} p_{r,i} \sigma_{sr,i}^2 + \sigma_s^2 \sum_{i=1}^{L} p_{s,i} \\
&s.t. \\
&\left(\sigma_s^{-2} + \frac{p_{s,i} p_{r,i} \sigma_{sr,i}^2 \sigma_{rd,i}^2}{\sigma_{n,r}^2 p_{r,i} \sigma_{rd,i}^2 + \sigma_{n,d}^2}\right)^{-1} \leq \rho_i, \quad p_{s,i} \geq 0, \quad \text{for} \quad i = 1, \cdots, L.
\end{aligned} \tag{6.38}$$

It is simple to see that the subproblem (6.38) is a convex optimization problem. From the KKT conditions [9], the optimum $p_{s,i}$ can be derived as

$$p_{s,i} = \left(\rho_i^{-1} - \sigma_s^{-2}\right)\left(\frac{\sigma_{n,r}^2 p_{r,i}\sigma_{rd,i}^2 + \sigma_{n,d}^2}{p_{r,i}\sigma_{sr,i}^2\sigma_{rd,i}^2}\right). \tag{6.39}$$

Substituting (6.39) into (6.37), we can have the master problem as

$$\begin{aligned}
&\min_{p_{r,i}} \sum_{i=1}^{L} p_{r,i}\left(\sigma_{n,r}^2 + \sigma_s^2\sigma_{sr,i}^2\left(\rho_i^{-1} - \sigma_s^{-2}\right)\left(\frac{\sigma_{n,r}^2 p_{r,i}\sigma_{rd,i}^2 + \sigma_{n,d}^2}{p_{r,i}\sigma_{sr,i}^2\sigma_{rd,i}^2}\right)\right)\\
&+\sigma_s^2\sum_{i=1}^{L}\left(\rho_i^{-1} - \sigma_s^{-2}\right)\left(\frac{\sigma_{n,r}^2 p_{r,i}\sigma_{rd,i}^2 + \sigma_{n,d}^2}{p_{r,i}\sigma_{sr,i}^2\sigma_{rd,i}^2}\right)\\
&s.t.\\
&p_{r,i} \geq 0, \quad \text{for} \quad i = 1, \cdots, L.
\end{aligned} \tag{6.40}$$

The optimization in (6.40) is also a convex optimization problem. The optimum $p_{r,i}$ can therefore be derived by the corresponding KKT conditions as (See Appendix A.6 for the detailed derivation):

$$p_{r,i} = \sqrt{\frac{\sigma_{n,d}^2\sigma_s^2\left(\rho_i^{-1} - \sigma_s^{-2}\right)}{\sigma_{n,r}^2\sigma_{sr,i}^2\sigma_{rd,i}^2\left(1 + \sigma_s^2\left(\rho_i^{-1} - \sigma_s^{-2}\right)\right)}}. \tag{6.41}$$

Using (6.39) and (6.41) in (6.31) and (6.32), we can obtain the closed-form solution for the precoders in the two-hop MIMO relay system. Interestingly, the solution in (6.34) is the same with that of (6.36) by simply setting $\rho_1 = \rho_2 = \cdots = \rho$.

§ 6.3 Joint Source/Relay Precoders Design for General MIMO Relay System

§ 6.3.1 *Problem Formulation in MIMO Relay System*

As we can see, the optimization problem in (6.11) is not a convex problem and the constraint function is a highly nonlinear function of the source and relay precoders. Even with numerical methods [51], it is difficult to obtain the optimum solution. To overcome the problem, we propose to use the precoder structures in (6.31) and (6.32). As that in previous case, we let $\mathbf{U}_S = \mathbf{I}_L$. Invoking the SVD of $\mathbf{H}_{SD}$, we have $\mathbf{H}_{SD} = \mathbf{U}_{sd}\boldsymbol{\Sigma}_{sd}\mathbf{V}_{sd}^H$ where σ_{sd} is a diagonal matrix with the ith diagonal element of $\sigma_{sd,i}$ and $\sigma_{sd,1} \geq \cdots \geq \sigma_{sd,\min\{N,M\}}$. From (6.11), the optimization problem can then be reformulated as

$$\begin{aligned}
&\min_{p_{s,i},p_{r,i}} \sum_{i=1}^{L} p_{r,i}\left(\sigma_{n,r}^2 + \sigma_s^2 p_{s,i}\sigma_{sr,i}^2\right) + \sigma_s^2 \sum_{i=1}^{L} p_{s,i}^2 \\
&s.t. \\
&\mathbf{E}(i,i) \leq \rho_i \\
&\mathbf{E} = \Bigg(\sigma_s^{-2}\mathbf{I}_L + \underbrace{\sigma_{n,d}^{-2}\boldsymbol{\Sigma}_s^H\mathbf{V}^H\boldsymbol{\Sigma}_{sd}^H\boldsymbol{\Sigma}_{sd}\mathbf{V}\boldsymbol{\Sigma}_s}_{=\mathbf{E}_S} + \\
&\underbrace{\boldsymbol{\Sigma}_s^H\boldsymbol{\Sigma}_{sr}^H\boldsymbol{\Sigma}_r^H\boldsymbol{\Sigma}_{rd}^H\left(\sigma_{n,r}^2\boldsymbol{\Sigma}_{rd}\boldsymbol{\Sigma}_r\boldsymbol{\Sigma}_r^H\boldsymbol{\Sigma}_{rd}^H + \sigma_{n,d}^2\mathbf{I}_M\right)^{-1}\boldsymbol{\Sigma}_{rd}\boldsymbol{\Sigma}_r\boldsymbol{\Sigma}_{sr}\boldsymbol{\Sigma}_s}_{=\mathbf{E}_R}\Bigg)^{-1} \\
&p_{s,i} = \sigma_{s,i}^2 \geq 0, \quad p_{r,i} = \sigma_{r,i}^2 \geq 0, \quad \forall i,
\end{aligned} \tag{6.42}$$

where $\mathbf{V} = \mathbf{V}_{sd}^H\mathbf{V}_{sr}$. As we can see, $\mathbf{E}$ in (6.42) is not fully diagonalized and solving the problem is still difficult. To provide a feasible solution, we apply the following lemma.

Lemma 6.1: For $\mathbf{E}$ in (6.42), we have the following MSE upper bound.

$$\begin{aligned}
\mathbf{E}(i,i) &\leq \left(\sigma_s^{-2} + \frac{p_{s,i}p_{r,i}\sigma_{sr,i}^2\sigma_{rd,i}^2}{\sigma_{n,r}^2 p_{r,i}\sigma_{rd,i}^2 + \sigma_{n,d}^2} + p_{s,i}\left(\left(\underbrace{\sigma_{n,d}^{-2}\mathbf{V}^H\boldsymbol{\Sigma}_{sd}^H\boldsymbol{\Sigma}_{sd}\mathbf{V}}_{:=\mathbf{B}}\right)^{-1}(i,i)\right)^{-1}\right)^{-1} \\
&:= \tilde{\mathbf{E}}(i,i),
\end{aligned} \tag{6.43}$$

where the equality holds when $\mathbf{V} = \mathbf{I}$ or $\boldsymbol{\Sigma}_{sd} = \mathbf{0}$ (i.e. the two-hop system).

Proof: See Appendix A.7.

Let $\tilde{\mathbf{E}}$ be a diagonal matrix and its diagonal components be equal to the upper bounds in

(6.43). We can substitute $\mathbf{E}$ with $\tilde{\mathbf{E}}$ in (6.42) and have

$$\begin{aligned}
&\min_{p_{s,i},p_{r,i}} \sum_{i=1}^{L} p_{r,i}\left(\sigma_{n,r}^2 + \sigma_s^2 p_{s,i}\sigma_{sr,i}^2\right) + \sigma_s^2 \sum_{i=1}^{L} p_{s,i}^2 \\
&s.t. \\
&\tilde{\mathbf{E}}(i,i) = \left(\sigma_s^{-2} + \frac{p_{s,i}p_{r,i}\sigma_{sr,i}^2\sigma_{rd,i}^2}{\sigma_{n,r}^2 p_{r,i}\sigma_{rd,i}^2 + \sigma_{n,d}^2} + p_{s,i}\left(\left(\underbrace{\sigma_{n,d}^{-2}\mathbf{V}^H\mathbf{\Sigma}_{sd}^H\mathbf{\Sigma}_{sd}\mathbf{V}}_{:=\mathbf{B}}\right)^{-1}(i,i)\right)^{-1}\right)^{-1} \leq \rho_i, \\
&p_{s,i} = \sigma_{s,i}^2 \geq 0, \quad p_{r,i} = \sigma_{r,i}^2 \geq 0, \quad \forall i. \qquad (6.44)
\end{aligned}$$

Comparing (6.42) and (6.44), we see that the constraints in (6.44) are more stringent. In other words, if the precoders satisfy (6.44), they will also satisfy (6.42). Similar to the problems we have in the two-hop scenario, the optimization in (6.44) cannot be conducted since it is not a convex problem. As Section 6.2 described, this problem can be solved by the primal decomposition method.

§ 6.3.2 *Optimum Solution in (6.44)*

Given $p_{r,i}$, $i = 1, \cdots, L$, the subproblem in (6.44) can be expressed as

$$\begin{aligned}
&\min_{p_{s,i}(p_{r,i})} \sum_{i=1}^{L} p_{r,i}\left(\sigma_{n,r}^2 + \sigma_s^2 p_{s,i}\sigma_{sr,i}^2\right) + \sigma_s^2 \sum_{i=1}^{L} p_{s,i}^2 \\
&s.t. \\
&\tilde{\mathbf{E}}(i,i) = \left(\sigma_s^{-2} + \frac{p_{s,i}p_{r,i}\sigma_{sr,i}^2\sigma_{rd,i}^2}{\sigma_{n,r}^2 p_{r,i}\sigma_{rd,i}^2 + \sigma_{n,d}^2} + p_{s,i}\left(\left(\underbrace{\sigma_{n,d}^{-2}\mathbf{V}^H\mathbf{\Sigma}_{sd}^H\mathbf{\Sigma}_{sd}\mathbf{V}}_{:=\mathbf{B}}\right)^{-1}(i,i)\right)^{-1}\right)^{-1} \leq \rho_i, \\
&p_{s,i} = \sigma_{s,i}^2 \geq 0, \quad p_{r,i} = \sigma_{r,i}^2 \geq 0, \quad \forall i. \qquad (6.45)
\end{aligned}$$

It is straightforward to check that this optimization is a convex optimization problem. So, similar to (6.39), the solution can be easily derived as

$$p_{s,i} = \left(\rho_i^{-1} - \sigma_s^{-2}\right)\left(\frac{p_{r,i}\sigma_{sr,i}^2\sigma_{rd,i}^2}{p_{r,i}\sigma_{n,r}^2\sigma_{rd,i}^2 + \sigma_{n,d}^2} + \left((\mathbf{B}^{-1})(i,i)\right)^{-1}\right)^{-1}. \qquad (6.46)$$

Substituting (6.46) into (6.44), the master problem can then be expressed as

$$\min_{p_{s,i},\forall i}\sum_{i=1}^{L}\left(\sigma_{n,r}^2+\sigma_s^2\sigma_{sr,i}^2\left(\rho_i^{-1}-\sigma_s^{-2}\right)\left(\frac{p_{r,i}\sigma_{sr,i}^2\sigma_{rd,i}^2}{p_{r,i}\sigma_{n,r}^2\sigma_{rd,i}^2+\sigma_{n,d}^2}+\left((\mathbf{B}^{-1})(i,i)\right)^{-1}\right)^{-1}\right)+$$
$$\sum_{i=1}^{L}\left(\rho_i^{-1}-\sigma_s^{-2}\right)\left(\frac{p_{r,i}\sigma_{sr,i}^2\sigma_{rd,i}^2}{p_{r,i}\sigma_{n,r}^2\sigma_{rd,i}^2+\sigma_{n,d}^2}+\left((\mathbf{B}^{-1})(i,i)\right)^{-1}\right)^{-1}$$
$$s.t.$$
$$p_{r,i}\geq 0,\quad \text{for}\quad i=1,\cdots,L. \tag{6.47}$$

Comparing (6.47) and (6.40), we can find that the two problems are very similar. The only difference lies in the additional term $\left(\left(\mathbf{B}^{-1}\right)(i,i)\right)^{-1}$ in (6.47). The solution can be also obtained by the KKT conditions. After some tedious manipulations (see Appendix A.8 for details), we can obtain the result as

$$p_{r,i}=\begin{cases}\frac{-B_i+\sqrt{B_i^2-4A_iC_i}}{2A_i}, & \text{if } C_i<0\\ 0, & \text{if } C_i\geq 0.\end{cases} \tag{6.48}$$

where

$$A_i = \sigma_{rd,i}^4\sigma_{n,r}^2\left(\frac{\left(\mathbf{B}^{-1}(i,i)\right)^{-2}\sigma_{n,r}^4+2\sigma_{n,r}^2\sigma_{sr,i}^2\left(\mathbf{B}^{-1}(i,i)\right)^{-1}+\sigma_{sr,i}^4}{\sigma_{sr,i}^2\left(\sigma_s^2\rho_i^{-1}-1\right)}+\right.$$
$$\left.\sigma_{n,r}^2\left(\mathbf{B}^{-1}(i,i)\right)^{-1}+\sigma_{sr,i}^2\right)>0, \tag{6.49}$$

$$B_i = \left(\mathbf{B}^{-1}(i,i)\right)^{-1}\left(\frac{2\sigma_{n,r}^2\sigma_{n,d}^2\sigma_{rd,i}^2}{\sigma_{sr,i}^2\left(\sigma_s^2\rho_i^{-1}-1\right)}\left(\sigma_{n,r}^2\left(\mathbf{B}^{-1}(i,i)\right)^{-1}+\sigma_{sr,i}^2\right)+2\sigma_{n,r}^2\sigma_{n,d}^2\sigma_{rd,i}^2\right)$$
$$>0, \tag{6.50}$$

and

$$C_i=\sigma_{n,d}^4\left(\frac{\left(\mathbf{B}^{-1}(i,i)\right)^{-2}\sigma_{n,r}^2}{\sigma_{sr,i}^2\left(\sigma_s^2\rho_i^{-1}-1\right)}+\left(\mathbf{B}^{-1}(i,i)\right)^{-1}-\sigma_{n,d}^{-2}\sigma_{rd,i}^2\right). \tag{6.51}$$

Substituting (6.48) into (6.46), we then have a closed-form solution for $p_{s,i}$. Finally, substituting (6.46) and (6.48) into (6.31) and (6.32), we obtain a solution for the precoders. The solution in

Table 6.1: Complexity of linear source and linear relay precoders (QoS constraints).

Step	Operation	FLOPs
1	$\mathbf{H}_{SR} = \mathbf{U}_{sr}\mathbf{\Sigma}_{sr}\mathbf{V}_{sr}$	$O(N^3 + RN^2)$
2	$\mathbf{H}_{RD} = \mathbf{U}_{rd}\mathbf{\Sigma}_{rd}\mathbf{V}_{rd}^H$	$O(MR^2 + R^3)$
3	$\mathbf{H}_{SD} = \mathbf{U}_{sd}\mathbf{\Sigma}_{sd}\mathbf{V}_{sd}^H$	$O(MN^2 + N^3)$
4	$\mathbf{B}^{-1}$	$O(N^3)$
5	$\mathbf{\Sigma}_r$ (6.48)	$O(L)$
6	$\mathbf{F}_R$ (6.17)	$O(R^3)$
7	$\mathbf{\Sigma}_s$ (6.46)	$O(L)$
8	$\mathbf{F}_S$ (6.16)	$O(NL^2)$

(6.48) also implies that the cooperative operation may not always be advantageous. One may ask when is the right time for cooperative communication, and the variable C_i in (6.48) gives the answer. From (6.48), we can observe that if

$$\frac{\sigma_{n,d}^4\sigma_{n,r}^2\left(\mathbf{B}^{-1}(i,i)\right)^{-2}}{\sigma_{sr,i}^2\left(\sigma_s^2\rho_i^{-1}-1\right)} + \left(\mathbf{B}^{-1}(i,i)\right)^{-1} \geq \sigma_{n,d}^2\sigma_{rd,i}^2, \quad \text{for all } \ i, \tag{6.52}$$

the relay link is off since $p_{r,i} = 0$ for all i. In other words, the condition (6.52) provides a "sufficient condition" that can be used to determine if the system should be operated in the cooperative mode or not.

The complexity of the linear source and relay precoders mainly involves the SVD and matrix inversion operations. The overall complexity of producing the precoders can be summarized in Table 6.1. As we can see, compared to Tables 2.1-5.1, $\mathbf{\Sigma}_r$ proposed in this chapter has the lower computational complexity. This is because the closed-form expression is expressed as a simpler formulation.

§ 6.4 Simulations

In this subsection, we conduct simulations to evaluate the performance of the proposed precoding schemes. We assume that perfect synchronization is achieved, and the exact CSIs are available at all nodes. Also, we assume that $L = N = R = M = 4$, the elements of each channel matrix be i.i.d. complex Gaussian random variable with zero mean and same variance, and $\sigma_{n,r}^2 = \sigma_{n,d}^2 = 0.01$. Also, let σ_{sr}^2, σ_{rd}^2, and σ_{sd}^2 denote, respectively, the variance of each channel element for the source-to-relay, the relay-to-destination, and the source-to-destination links.

§ 6.4.1 *Two-hop MIMO Relay System*

In this subsection, we evaluate the performance of the proposed precoding scheme for two-hop MIMO relay systems. We first consider the scenario of equal MSE constraints, i.e., $\rho_1 = \cdots = \rho_4 = \rho$, and simulate two cases. In Case 1, $\sigma_{sr}^2 = 0.0316$ and $\sigma_{rd}^2 = 0.3162$. In this case, $10\log_{10}\left(\sigma_{sr}^2/\sigma_{n,r}^2\right) = 5\ dB$ and $10\log_{10}\left(\sigma_{rd}^2/\sigma_{n,r}^2\right) = 15\ dB$. In Case 2, $\sigma_{sr}^2 = 0.3162$ and $\sigma_{rd}^2 = 0.0316$. Figs. 6.1 and 6.2 show the simulation result for Case 1 and Figs 6.3 and 6.4 for Case 2. Figs 6.1 and 6.3 give the source power consumption (P_s) , the relay power consumption (P_r), and the total power consumption (P_t) , while Figs 6.2 and 6.4 show the resultant MSEs. As we can see, the resultant MSEs are equal to the required MSEs and thus it indeed satisfy the MSE constraints. Comparing these two figures, we can observe that when the source-to-relay link is poor (Case 1), the system allocates more power to the source than to the relay. On the contrary, when the relay-to-destination link is poor, the system allocates more power at the relay. We then consider the scenario of different MSE constraints in which $(\rho_1, \rho_2, \rho_3, \rho_4)$ = $(0.008, 0.009, 0.01, 0.011)$, $\sigma_{rd}^2 = 1$, and σ_{sd}^2 is varied. In this case, $10\log_{10}\left(\sigma_{rd}^2/\sigma_{n,r}^2\right) = 20$ dB. Fig. 6.5 shows the simulation result. In the figure, the solid lines denote the required MSE constraints and the dash lines the resultant MSEs for each data stream. As the figure shows, the solid and dash lines are overlapped indicating that the proposed precoding scheme can precisely satisfy the MSE constraints.

§ 6.4.2 *General MIMO Relay System*

In this subsection, we evaluate the performance of the proposed precoding scheme for general MIMO relay systems. We first consider the scenario of equal MSE constraints where $\rho_1 = \cdots = \rho_4 = \rho$, $\sigma_{sr}^2 = 1$, $\sigma_{rd}^2 = 1$ and $\sigma_{sd}^2 = 0.1$. In this case, $10\log_{10}\left(\sigma_{sr}^2/\sigma_{n,r}^2\right) = 20$ dB. Fig. 6.6 shows the total required transmission power, and Fig. 6.7 the resultant MSE. As shown in this figure, there is a gap between the resultant and the required MSEs and the resultant MSE is lower. This is because the proposed precoders are designed based on the MSE upper bounds in (6.44). We then consider the scenario of different MSE constraints where $(\rho_1, \rho_2, \rho_3, \rho_4) = (0.008, 0.009, 0.01, 0.011)$, $\sigma_{rd}^2 = 1$, $\sigma_{sd}^2 = 0.0316$ and σ_{sr}^2 is varied. Here, $10\log_{10}\left(\sigma_{rd}^2/\sigma_{n,r}^2\right) = 20$ dB, and $10\log_{10}\left(\sigma_{sd}^2/\sigma_{n,r}^2\right) = 20$ dB. Fig. 6.8 shows the resultant MSEs. In the figures, the dash lines denote the resultant MSEs while the solid lines the required MSEs. From the figure, we observe that the behavior of the proposed precoders is similar to that in the previous case, i.e. the resultant MSEs are lower than the required MSEs. However, the gap between the resultant and required MSEs is smaller when the SNR of the source-to-relay relay link is higher. This is because in this case the resultant MSE will be closer to the upper bound (see Lemma 6.1 in (6.43)).

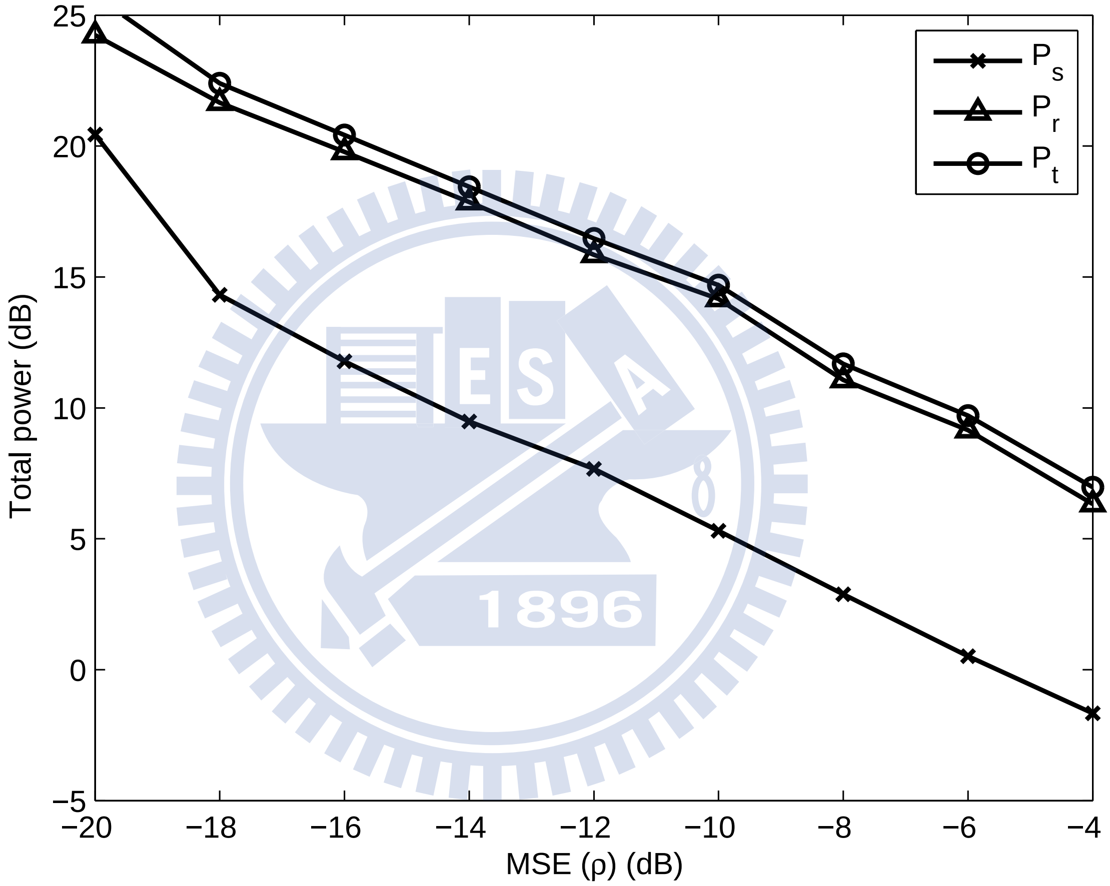


Figure 6.1: Power consumption for proposed joint precoders method in two-hop MIMO relay system with $\sigma_{sr}^2 = 0.0316$ and $\sigma_{rd}^2 = 0.3162$.

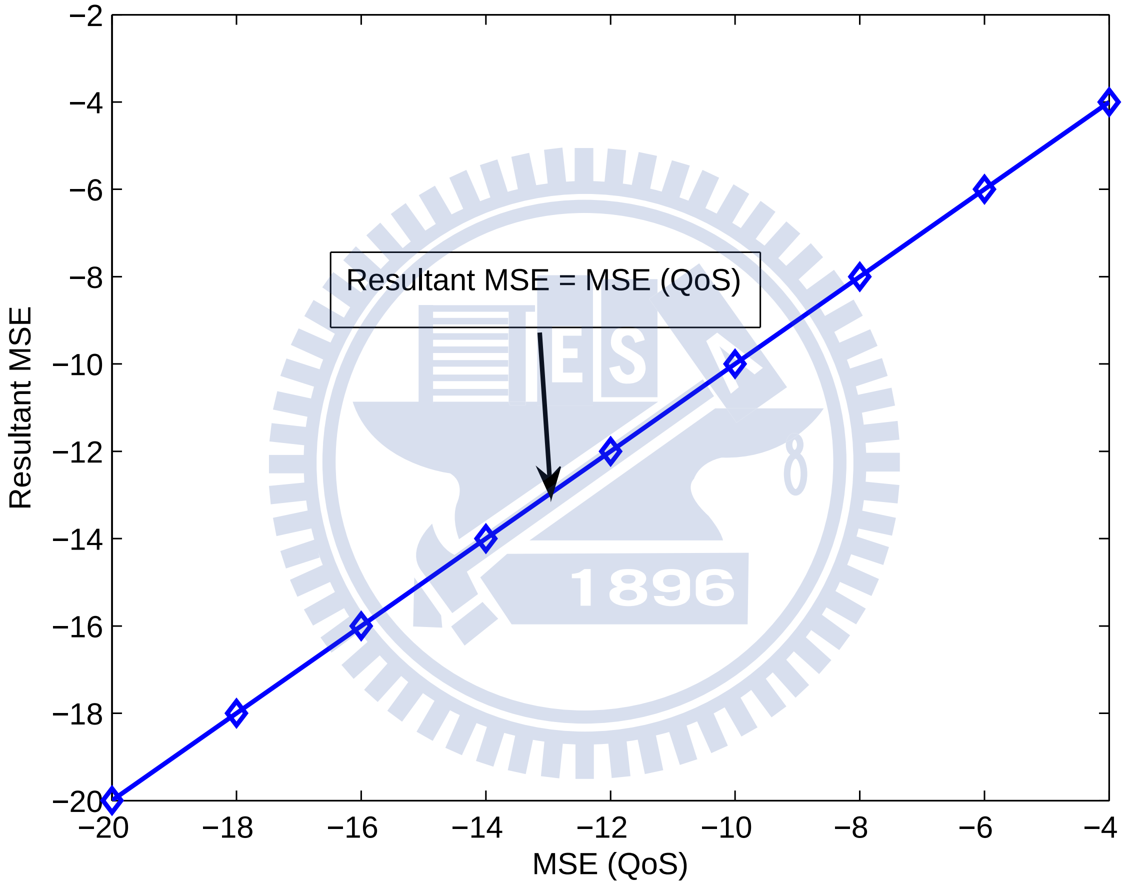


Figure 6.2: Resultant MSE versus QoS with $\sigma_{sr}^2 = 0.0316$ and $\sigma_{rd}^2 = 0.3162$.

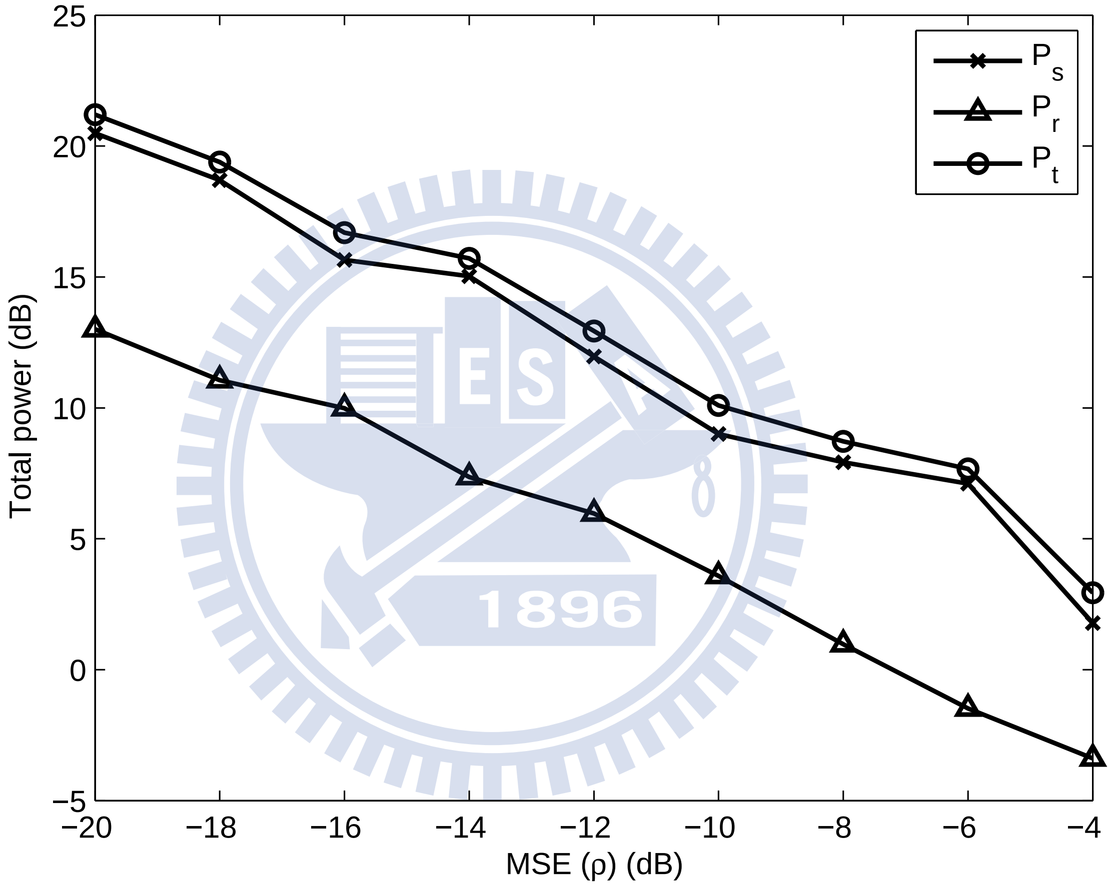


Figure 6.3: Power consumption for proposed joint precoders method in two-hop MIMO relay system with $\sigma_{sr}^2 = 0.3162$ and $\sigma_{rd}^2 = 0.0316$.

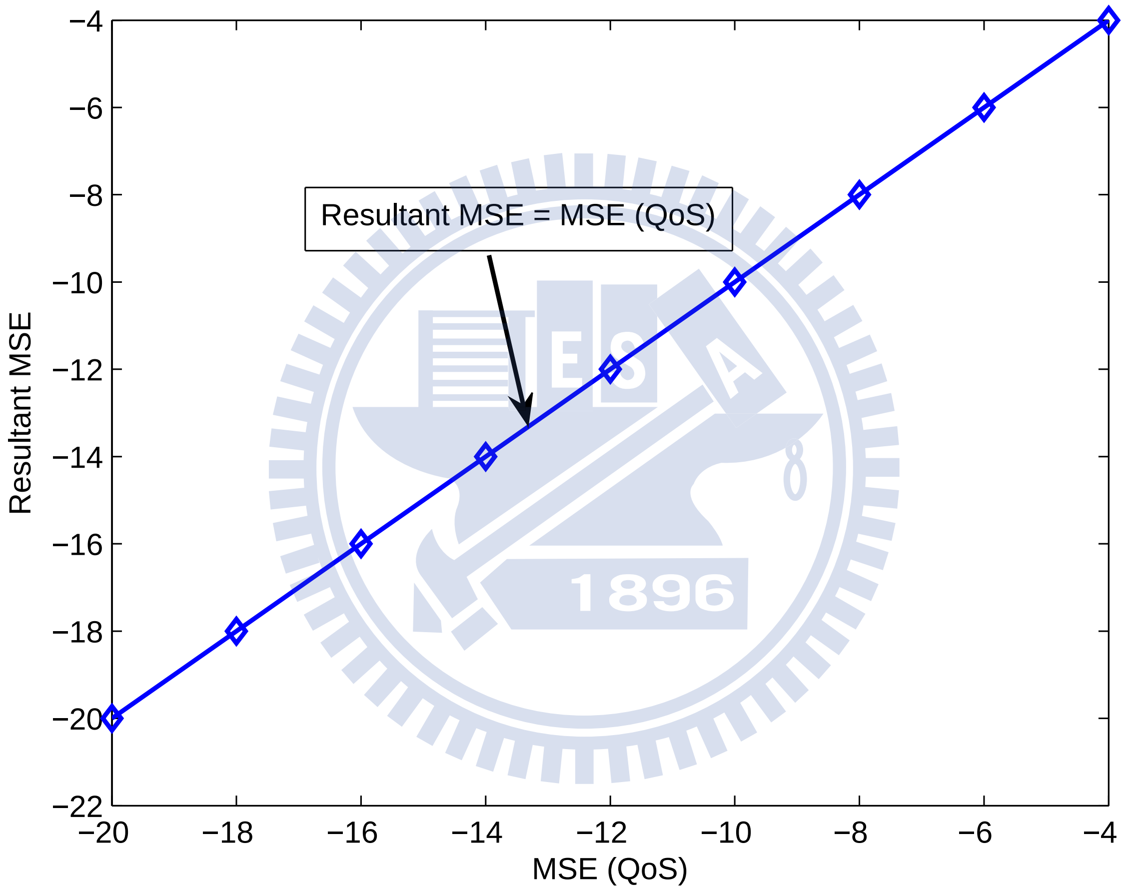


Figure 6.4: Resultant MSE versus QoS with $\sigma_{sr}^2 = 0.3162$ and $\sigma_{rd}^2 = 0.0316$.

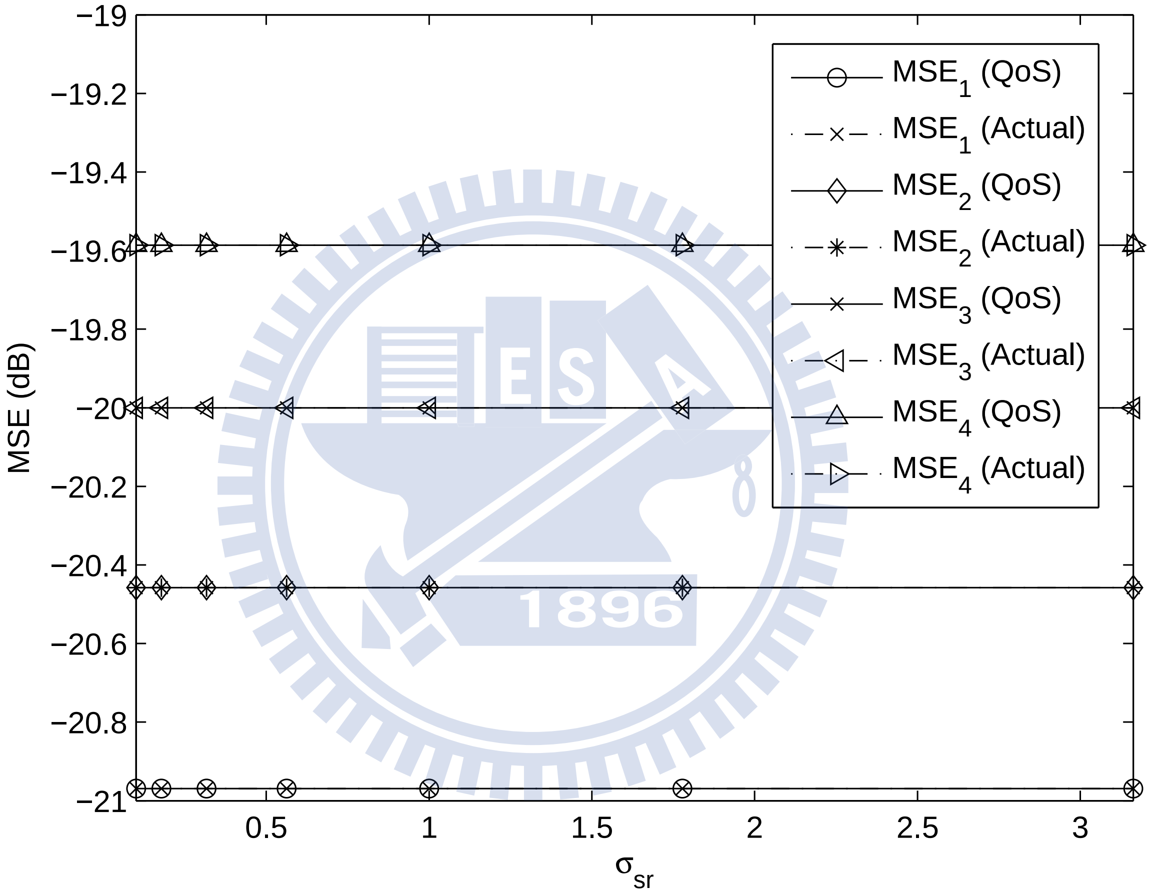


Figure 6.5: Resultant MSE performance of proposed precoders method in two-hop MIMO relay system with $(\rho_1, \rho_2, \rho_3, \rho_4) = (0.008, 0.009, 0.01, 0.011)$.

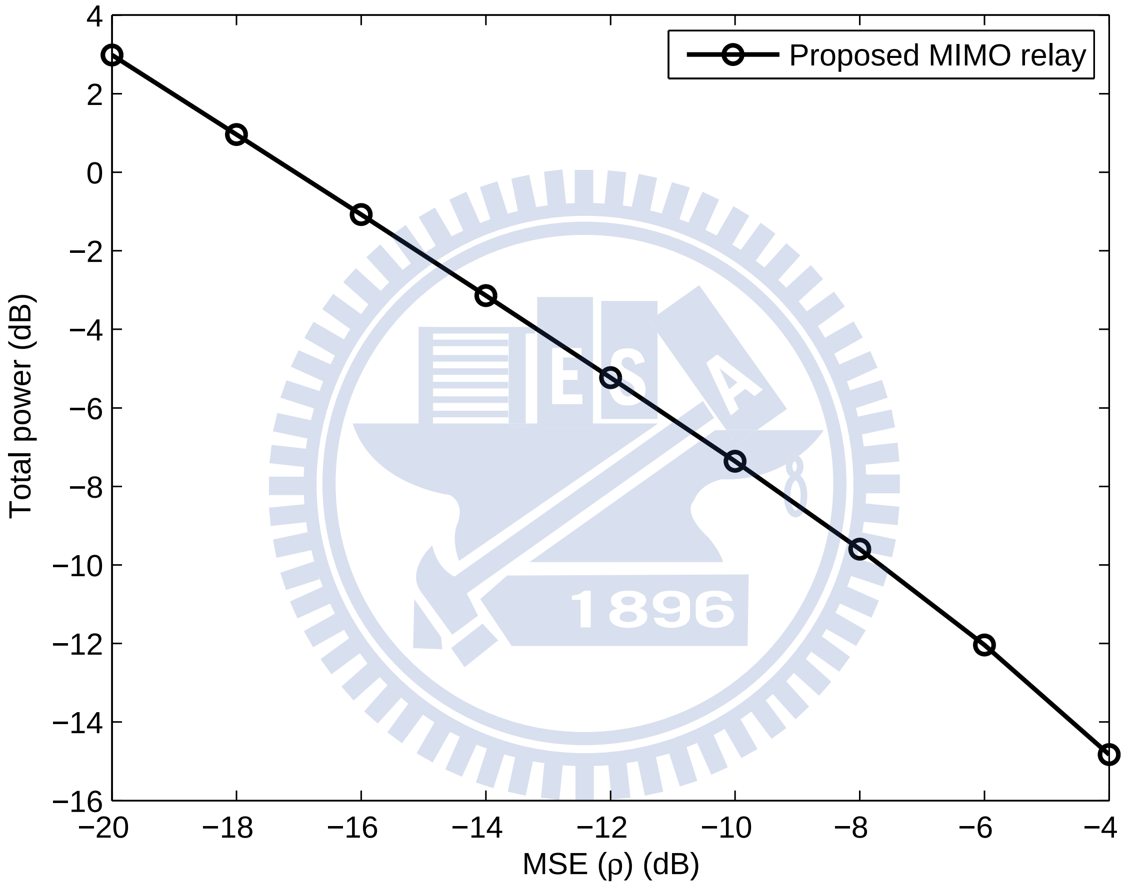


Figure 6.6: Power consumption for proposed joint precoders method in MIMO relay system with $\sigma_{sr}^2 = 1$, $\sigma_{rd}^2 = 1$ and $\sigma_{sd}^2 = 0.1$.

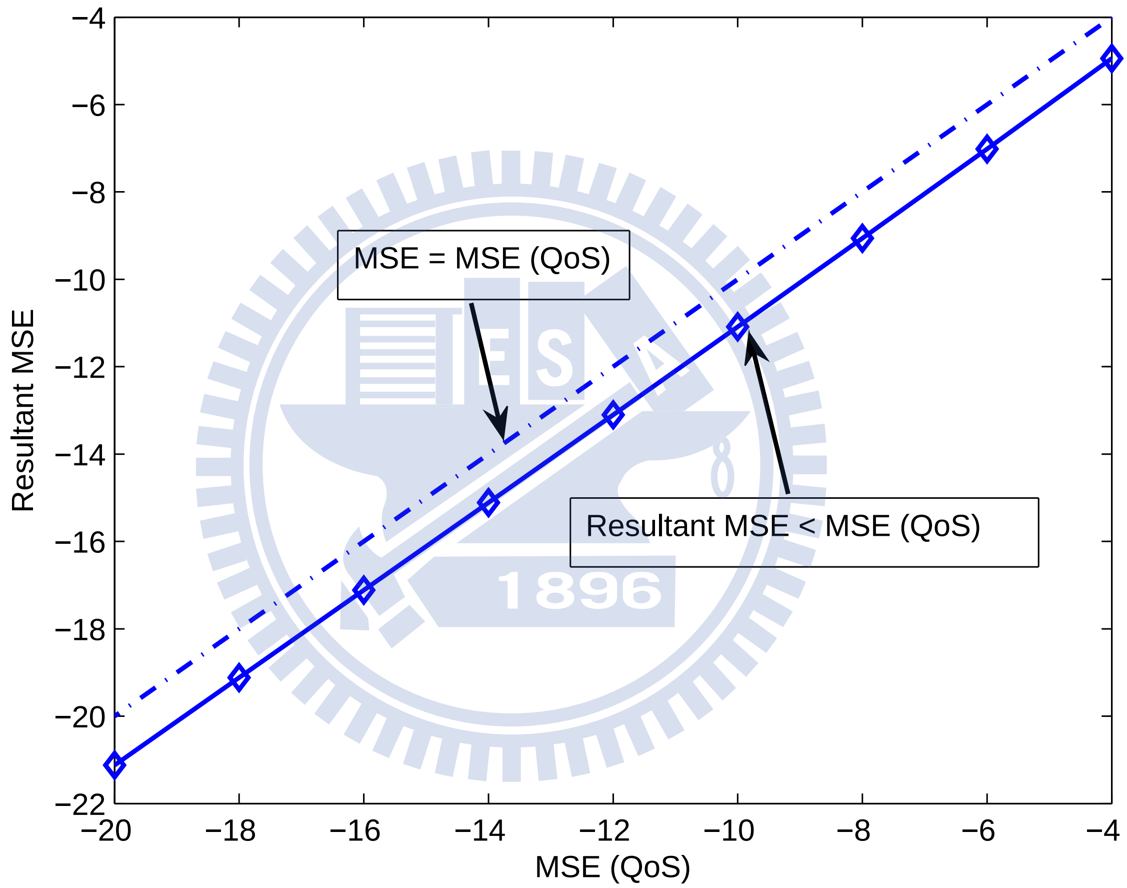


Figure 6.7: Resultant MSE versus QoS with $\sigma_{sr}^2 = 1$, $\sigma_{rd}^2 = 1$ and $\sigma_{sd}^2 = 0.1$

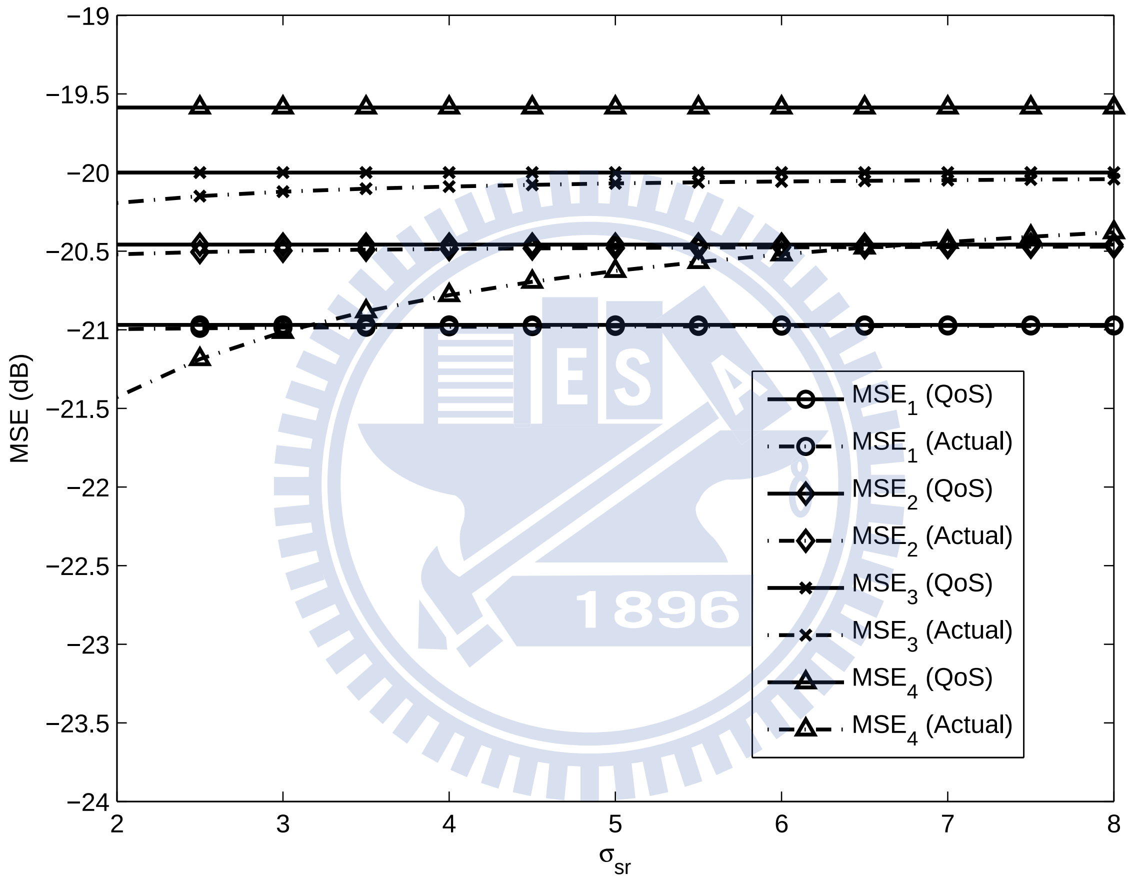


Figure 6.8: Resultant MSE performance of proposed precoders method in MIMO relay system with $(\rho_1, \rho_2, \rho_3, \rho_4) = (0.008, 0.009, 0.01, 0.011)$.

Chapter 7

Conclusions

Conventional transceiver designs in three-node MIMO relay systems only consider the relay precoder and some even ignore the direct link, and the system resource is not fully explored. This motivates us to develop new designs taking the direct and relay links into account, and jointly optimizing the source and relay precoders. In our designs, the relay precoder is linear and the source precoder and the receiver can be linear or nonlinear. We study four transceiver structures and propose new methods for the precoders design. The transceivers we considered are: the MMSE receiver with the linear source precoder, the MMSE receiver with the THP source precoder, the QR-SIC receiver with the linear source precoder, and the MMSE-SIC receiver with the linear source precoder. Although these design problems can be easily formulated as some optimization problems, the cost functions are inherent to be highly nonlinear functions of the precoding matrices, and the optimum solutions are very difficult to derive. To overcome the problem, we seek for suboptimum solutions by constraining the precoders to have some specific structures. With the primal decomposition approach, we can then transfer the optimization problems into convex optimization problems, and the closed-form solutions can then be derived by the KKT conditions. In addition to the designs mentioned above for the enhancement of link quality, we also consider the design satisfying QoS constraints. Simulations show that the proposed source and relay precoded MIMO relay systems significantly outperform the existing precoded systems. In concluding the dissertation, we suggest some possible topics for future research.

1. In MIMO relay systems, the estimation of CSI is less addressed. As known, CSI is required in all the designs. It is then important to design training sequences or pilots for effective channel estimation. Note that the estimation problem here is fundamentally different to that in MIMO systems. For example, the destination may have to estimate the source-to-relay channel which is not directly observable.

2. In this dissertation, the precoders are designed based on the assumption that perfect CSIs are available at all nodes. In practical systems, however, this may not be always possible. How to design robust transceivers is an important issue in real-world applications. Moreover, how to design efficient feedback systems such that CSIs can be fed back to the transmitters also deserves further studies.

3. In practical cooperative system, implementation of an AF linear precoder at the relay node essentially encounters several problems such as analog/digital conversion (ADC), lack of symbol timing/frequency synchronization, physical layer waveform design, and automatic gain control, etc. Especially, unlike the analog waveform repeater, the signals sampled by the relay's ADC are first sampled at buffer in the FPGA. Then the stored signals multiply a linear precoder before sending them to the DAC. The ADC and DAC increase the RF transceiver's setting load. How to solve these practical problems and/or reduce the complexity of implementation at the relay are also the interesting research topics.

4. In this dissertation, we only study a typical three-node MIMO relay system. In the system, there are only one source node, one relay node, and one destination node. In a general relay system, there may be multiple source nodes, multiple relay nodes and multiple destination nodes. In addition, the system may be a multi-hop relay system. The precoders design in such a system is challenging and deserves for further study.

5. The relay precoder we considered is a linear precoder. However, the relay precoder can be nonlinear. How to design a nonlinear source and a nonlinear relay precoders associated with a nonlinear receiver is still an open problem.

6. To the best of our knowledge, the existing transceiver designs in MIMO relay system all assume the AF protocol. The design with the DF protocol is also an open and interesting problem.

7. The OFDM modulation scheme is widely used in real-world communication systems. It is then desirable to consider the precoders design in MIMO-OFDM relay systems. The design problem is obviously much complicated since one extra dimension, the subcarrier power allocation, is added.

Appendix

§ A.1 Proof of (2.25)

Let us first rewrite

$$tr\left(\mathbf{D}_1^H\left(\mathbf{X}+\mathbf{D}_2\right)^{-1}\mathbf{D}_1\right)=\sum_{i=1}^{L}|\mathbf{D}_1(i,i)|^2(\mathbf{X}+\mathbf{D}_2)^{-1}(i,i), \tag{1}$$

where $\mathbf{D}_1 \in \mathbb{R}^{N\times L}$ and $\mathbf{D}_2 \in \mathbb{R}^{N\times N}$, $N \geq L$, are diagonal matrices with the positive elements; $\mathbf{X} \in \mathbb{C}^{N\times N}$ is a Hermitian matrix. So, $(\mathbf{X}+\mathbf{D}_2)$ is a positive definite matrix.

We claim that

$$\left(\mathbf{X}+\mathbf{D}_2\right)^{-1}(i,i) \geq \frac{1}{\left(\mathbf{X}+\mathbf{D}_2\right)(i,i)}, \tag{2}$$

which will be proved in the next paragraph. From (1) and (2) we immediately have

$$\begin{aligned} tr\left(\mathbf{D}_1^H\left(\mathbf{X}+\mathbf{D}_2\right)^{-1}\mathbf{D}_1\right) &\geq \sum_{i=1}^{L}\frac{|\mathbf{D}_1(i,i)|^2}{(\mathbf{X}+\mathbf{D}_2)(i,i)} \\ =\sum_{i=1}^{L}\frac{|\mathbf{D}_1(i,i)|^2}{(diag(\mathbf{X})+\mathbf{D}_2)(i,i)} &= tr\left(\mathbf{D}_1^H\left(diag(\mathbf{X})+\mathbf{D}_2\right)^{-1}\mathbf{D}_1\right) \end{aligned} \tag{3}$$

which proves the lemma.

[Proof of (2)]: Let $\mathbf{Z} := (\mathbf{X}+\mathbf{D}_2) = \mathbf{U}\mathbf{\Sigma}\mathbf{U}^H$ be the eigen-decomposition of the positive-definite matrix $\mathbf{Z}$. Since $1 = \mathbf{e}_i^T\mathbf{I}\mathbf{e}_i = \mathbf{e}_i^T\mathbf{Z}^{1/2}\mathbf{Z}^{-1/2}\mathbf{e}_i$, where $\mathbf{e}_i$ is the ith unit standard vector, we then have

$$1 = \|\mathbf{e}_i^T\mathbf{Z}^{1/2}\mathbf{Z}^{-1/2}\mathbf{e}_i\|_2^2 \leq \|\mathbf{e}_i^T\mathbf{Z}^{1/2}\|_2^2\|\mathbf{Z}^{-1/2}\mathbf{e}_i\|_2^2, \tag{4}$$

where the inequality in (4) follows from the sub-multiplicative property of the matrix norm [50]. Since $\|\mathbf{e}_i^T\mathbf{Z}^{1/2}\|_2^2 = \left(\mathbf{e}_i^T\mathbf{Z}^{1/2}\mathbf{Z}^{1/2}\mathbf{e}_i\right) = \mathbf{Z}(i,i)$ and $\|\mathbf{e}_i^T\mathbf{Z}^{-1/2}\|_2^2 = \left(\mathbf{e}_i^T\mathbf{Z}^{-1/2}\mathbf{Z}^{-1/2}\mathbf{e}_i\right) = \mathbf{Z}^{-1}(i,i)$, the inequality (4) thus leads to $1 \leq \mathbf{Z}(i,i)\mathbf{Z}^{-1}(i,i)$, or equivalenty $\mathbf{Z}^{-1}(i,i) \geq \frac{1}{\mathbf{Z}(i,i)}$.

§ A.2 Derivation of (2.29) and (2.30)

The Lagrangian function with respect to (2.28) can be written as

$$L = \sum_{i=1}^{L} \frac{1}{\sigma_s^{-2} + \frac{p_{s,i}p_{r,i}\sigma_{sr,i}^2\sigma_{rd,i}^2}{\sigma_{n,r}^2 p_{r,i}\sigma_{rd,i}^2 + \sigma_{n,d}^2} + p_{s,i}\left(\mathbf{B}^{-1}(i,i)\right)^{-1}} + \lambda_s\left(\sigma_s^2\sum_{i=1}^{L}p_{s,i} - P_{S,T}\right) +$$

$$\lambda_r\left(\sum_{i=1}^{L}p_{r,i}\left(\sigma_{n,r}^2 + \sigma_s^2 p_{s,i}\sigma_{sr,i}^2\right) - P_{R,T}\right) - \sum_{i=1}^{L}\mu_{s,i}p_{s,i} - \sum_{i=1}^{L}\mu_{r,i}p_{r,i}. \tag{5}$$

As mentioned, if $p_{s,i}$ is given, (2.28) is a convex optimization problem (for $p_{r,i}$). Thus, we can obtain the optimum $p_{r,i}$ using the KKT conditions [51]. The KKT optimality conditions for solving $p_{r,i}$, $1 \leq i \leq L$, are given as follows:

$$-\frac{p_{s,i}\sigma_{n,d}^2\sigma_{sr,i}^2\sigma_{rd,i}^2}{c(p_{s,i},p_{r,i})} + \lambda_r\left(\sigma_{n,r}^2 + \sigma_s^2 p_{s,i}\sigma_{sr,i}^2\right) - \mu_{r,i} = 0, \tag{6}$$

where

$$c\left(p_{s,i},p_{r,i}\right) = \left[\left(\sigma_s^{-2} + p_{s,i}\left(\mathbf{B}^{-1}(i,i)\right)^{-1}\right)\left(p_{r,i}\sigma_{n,r}^2\sigma_{rd,i}^2 + \sigma_{n,d}^2\right) + p_{s,i}p_{r,i}\sigma_{sr,i}^2\sigma_{rd,i}^2\right]^2 \tag{7}$$

$$\mu_{r,i} \geq 0. \tag{8}$$

$$\lambda_r \geq 0. \tag{9}$$

$$\mu_{r,i}p_{r,i} = 0. \tag{10}$$

$$\lambda_r\left(\sum_{i=1}^{L}p_{r,i}\left(\sigma_{n,r}^2 + \sigma_s^2 p_{s,i}\sigma_{sr,i}^2\right) - P_{R,T}\right) = 0 \tag{11}$$

Combining (6) and (8), we have

$$\lambda_r\left(\sigma_{n,r}^2 + \sigma_s^2 p_{s,i}\sigma_{sr,i}^2\right) \geq \frac{p_{s,i}\sigma_{n,d}^2\sigma_{sr,i}^2\sigma_{rd,i}^2}{c(p_{s,i},p_{r,i})}. \tag{12}$$

Substituting (6) into (10) leads to

$$p_{r,i}\left(\lambda_r\left(\sigma_{n,r}^2+\sigma_s^2 p_{s,i}\sigma_{sr,i}^2\right)-\frac{p_{s,i}\sigma_{n,d}^2\sigma_{sr,i}^2\sigma_{rd,i}^2}{c(p_{s,i},p_{r,i})}\right)=0. \tag{13}$$

To satisfy (13), we then have

1. If

$$\lambda_r\left(\sigma_{n,r}^2+\sigma_s^2 p_{s,i}\sigma_{sr,i}^2\right)>\frac{p_{s,i}\sigma_{n,d}^2\sigma_{sr,i}^2\sigma_{rd,i}^2}{c\left(p_{s,i},p_{r,i}\right)},$$

   then $p_{r,i}=0$.

2. If

$$\lambda_r\left(\sigma_{n,r}^2+\sigma_s^2 p_{s,i}\sigma_{sr,i}^2\right)=\frac{p_{s,i}\sigma_{n,d}^2\sigma_{sr,i}^2\sigma_{rd,i}^2}{c\left(p_{s,i},p_{r,i}\right)},$$

   then

$$p_{r,i}=\frac{\frac{\sqrt{p_{s,i}}\sigma_{n,d}\sigma_{sr,i}\sigma_{rd,i}}{\lambda_r^{1/2}\left(\sigma_{n,r}^2+\sigma_s^2 p_{s,i}\sigma_{sr,i}^2\right)^{1/2}}-\sigma_{n,d}^2\left(\sigma_s^{-2}+p_{s,i}(\mathbf{B}^{-1}(i,i))^{-1}\right)}{\sigma_{rd,i}^2\left(\sigma_{n,r}^2\left(\sigma_s^{-2}+p_{s,i}(\mathbf{B}^{-1}(i,i))^{-1}\right)+p_{s,i}\sigma_{sr,i}^2\right)}.$$

Considering 1), 2) and $p_{r,i}\geq 0$, we then find the solution of $p_{r,i}$ as:

$$p_{r,i}=\left[\frac{\mu_r\sigma_{n,d}\sqrt{p_{s,i}}\sigma_{sr,i}\sigma_{rd,i}\left(\sigma_s^2 p_{s,i}\sigma_{sr,i}^2+\sigma_{n,r}^2\right)^{-1/2}-\sigma_{n,d}^2\left(\sigma_s^{-2}+p_{s,i}(\mathbf{B}^{-1}(i,i))^{-1}\right)}{\sigma_{rd,i}^2\left(\sigma_{n,r}^2\left(\sigma_s^{-2}+p_{s,i}(\mathbf{B}^{-1}(i,i))^{-1}\right)+p_{s,i}\sigma_{sr,i}^2\right)}\right]^+, \tag{14}$$

where $[y]^+=max[0,y]$, and $\mu_r=\lambda_r^{-1/2}$ is the water level which should be chosen to satisfy the power constraint at the relay. Similarly, we can obtain the optimum $p_{s,i}$ for a given $p_{r,i}$ as shown in (2.30). The details, however, are omitted.

§ A.3 Proof of Lemma 3.2

Since $\mathbf{A}$ is a positive definite matrix, the related eigenvalue decomposition can be expressed

$$\mathbf{A}=\mathbf{U}_A\mathbf{\Sigma}_A\mathbf{U}_A^H=\mathbf{A}^{1/2}\mathbf{A}^{1/2}, \tag{15}$$

where $\mathbf{\Sigma}_A$ is a diagonal matrix with the positive eigenvalues, $\mathbf{U}_A$ is the related eigenvector matrix, and $\mathbf{A}^{1/2} = \mathbf{U}_A \mathbf{\Sigma}_A^{1/2} \mathbf{U}_A^H$. From (15), we can have the following equation:

$$(\mathbf{A} + \mathbf{B}) = \mathbf{A}^{1/2} \left(\mathbf{I}_N + \mathbf{A}^{-1/2}\mathbf{B}\mathbf{A}^{-1/2}\right) \mathbf{A}^{1/2}. \tag{16}$$

From [50], we see that

$$det\left(\mathbf{CD}\right) = det\left(\mathbf{DC}\right) = det\left(\mathbf{C}\right) det\left(\mathbf{D}\right). \tag{17}$$

Using this property in (16), we then have

$$det\left(\mathbf{AB}\right) = det\left(\mathbf{A}^{1/2}\left(\mathbf{I}_N + \mathbf{A}^{-1/2}\mathbf{B}\mathbf{A}^{-1/2}\right)\mathbf{A}^{1/2}\right) = det\left(\mathbf{A}\right) det\left(\mathbf{I}_N + \mathbf{A}^{-1/2}\mathbf{B}\mathbf{A}^{-1/2}\right). \tag{18}$$

Q.E.D.

§ A.4 Optimum Solution in (3.32)

The Lagrangian function in (3.31) can be expressed as

$$\begin{aligned} L \;=\; & \sum_{i=1}^{\kappa} ln\left(1 + \frac{p_{r,i}\sigma_{n,d}^2\sigma_{rd,i}^2{\sigma'}_{sr,i}^2}{p_{r,i}\sigma_{n,r}^2\sigma_{rd,i}^2 + \sigma_{n,d}^2}\right) + \\ & \lambda\left[\sum_{i=1}^{\kappa} p_{r,i}\left(\frac{P_{S,T}}{N}{\sigma'}_{sr,i}^2\mathbf{D'}_{sr}(i,i) + \sigma_{n,r}^2\right) - P_{R,T}\right] - \sum_{i=1}^{\kappa} v_{r,i}p_{r,i}, \end{aligned} \tag{19}$$

where $\lambda \geq 0$. $v_{r,i} \geq 0$ with $i = 1, \cdots, \kappa$. By the KKT conditions for all i, we have

$$\frac{\partial L}{\partial p_{r,i}} = -\frac{\frac{\sigma_{n,d}^2\sigma_{n,r}^2\sigma_{rd,i}^2{\sigma'}_{sr,i}^2}{\left(p_{r,i}\sigma_{n,r}^2\sigma_{rd,i}^2+\sigma_{n,d}^2\right)^2}}{1 + \frac{p_{r,i}\sigma_{n,d}^2\sigma_{rd,i}^2{\sigma'}_{sr,i}^2}{p_{r,i}\sigma_{n,r}^2\sigma_{rd,i}^2+\sigma_{n,d}^2}} + \lambda\left(\frac{P_{S,T}}{N}{\sigma'}_{sr,i}^2\mathbf{D'}_{sr}(i,i) + \sigma_{n,r}^2\right) - v_{r,i} = 0; \tag{20}$$

$$\lambda,\; v_{r,i},\; p_{r,i} \geq 0; \tag{21}$$

$$v_{r,i}p_{r,i} = 0; \tag{22}$$

$$\lambda\left[\sum_{i=1}^{\kappa} p_{r,i}\left(\frac{P_{S,T}}{N}{\sigma'}_{sr,i}^2\mathbf{D'}_{sr}(i,i) + \sigma_{n,r}^2\right) - P_{R,T}\right] = 0. \tag{23}$$

Substituting (20) into (22) and noting the fact that $p_{r,i} > 0$, we have

$$\frac{1}{\underbrace{p_{r,i}^2\,\sigma_{rd,i}^2\left(\frac{\sigma_{n,r}^2}{\sigma_{n,d}^2\sigma'^2_{sr,i}}+1\right)}_{:=A_i} + 2p_{r,i}\underbrace{\left(\sigma'^{-2}_{sr,i}+\frac{\sigma_{n,d}^2}{2\sigma_{n,r}^2}\right)}_{:=B_i} + \underbrace{\frac{\sigma_{n,d}^2}{\sigma_{n,r}^2\sigma_{rd,i}^2\sigma'^2_{sr,i}}}_{:=C_i}}$$
$$= \lambda\left(\frac{P_{S,T}}{N}\sigma'^2_{sr,i}\mathbf{D}'_{sr}(i,i)+\sigma_{n,r}^2\right), \tag{24}$$

$$P_{r,i} = \sqrt{\frac{1}{\lambda\left(\frac{P_{S,T}}{N}\sigma'^2_{sr,i}\mathbf{D}'_{sr}(i,i)+\sigma_{n,r}^2\right)A_i}+\frac{B_i^2}{A_i^2}-\frac{C_i}{A_i}}-\frac{B_i}{A_i}, \tag{25}$$

$$\frac{B_i^2-A_iC_i}{A_i^2} = \frac{\frac{\sigma_{n,d}^4}{4\sigma_{n,r}^4}}{\sigma_{rd,i}^4\left(\frac{\sigma_{n,r}^2}{\sigma_{n,d}^2\sigma'^2_{sr,i}}+1\right)^2}, \tag{26}$$

$$\frac{B_i}{A_i} = \frac{1+\frac{\sigma_{n,d}^2\sigma'^2_{sr,i}}{2\sigma_{n,r}^2}}{\sigma_{rd,i}^2\left(\frac{\sigma_{n,r}^2}{\sigma_{n,d}^2}+\sigma'^2_{sr,i}\right)}. \tag{27}$$

After some straightforward manipulations and the use of (21), we can have the optimum $p_{r,i}$ as

$$p_{r,i} = \left[\sqrt{\frac{\mu}{\sigma_{rd,i}^2\left(\frac{P_{S,T}}{N}\sigma'^2_{sr,i}\mathbf{D}'_{sr}(i,i)+\sigma_n^2\right)\left(\sigma_{n,r}^2\sigma_{n,d}^{-2}\sigma'^{-2}_{sr,i}+1\right)}+\frac{\frac{\sigma_{n,d}^4}{4\sigma_{n,r}^4}}{\sigma_{rd,i}^4\left(\frac{\sigma_{n,r}^2}{\sigma_{n,d}^2\sigma'^2_{sr,i}}+1\right)^2}}\right.$$
$$\left.-\frac{1+\frac{\sigma_{n,d}^2\sigma'^2_{sr,i}}{2\sigma_{n,r}^2}}{\sigma_{rd,i}^2\left(\frac{\sigma_{n,r}^2}{\sigma_{n,d}^2}+\sigma'^2_{sr,i}\right)}\right]^+, \tag{28}$$

where $\mu = 1/\lambda$ should be chosen to satisfy the power constraint in (3.31).

§ A.5 Water-Filling Algorithm for (3.32)

For convenience, we let

$$a_i = \frac{1}{\sigma^2_{rd,i}\left(\frac{P_{S,T}}{N}{\sigma'}^2_{sr,i}\mathbf{D}'_{sr}\left(i,i\right)+\sigma^2_n\right)\left(\sigma^2_{n,r}\sigma^{-2}_{n,d}{\sigma'}^{-2}_{sr,i}+1\right)}, \tag{29}$$

$$b_i = \frac{\frac{\sigma^4_{n,d}}{4\sigma^4_{n,r}}}{\sigma^4_{rd,i}\left(\frac{\sigma^2_{n,r}}{\sigma^2_{n,d}{\sigma'}^2_{sr,i}}+1\right)^2}, \tag{30}$$

$$c_i = \frac{1+\frac{\sigma^2_{n,d}{\sigma'}^2_{sr,i}}{2\sigma^2_{n,r}}}{\sigma^2_{rd,i}\left(\frac{\sigma^2_{n,r}}{\sigma^2_{n,d}}+{\sigma'}^2_{sr,i}\right)}, \tag{31}$$

$$d_i = \left(\frac{P_{S,T}}{N}{\sigma'}^2_{sr,i}\mathbf{D}'_{sr}(i,i)+\sigma^2_{n,r}\right). \tag{32}$$

We can rewrite (3.32) as a general water-filling form

$$p_{r,i} = \left[\sqrt{a_i\left(\mu+\frac{b_i}{a_i}\right)}-c_i\right]^+. \tag{33}$$

An easy way to solve (33) and at the same time satisfy the power constraint in (3.31) is the bisection method sumarrized in Table 3.2. In the table, $\mu_{M,0}$ and $\mu_{L,0}$ denotes the maximal and minimal initial μ, respectively; ϵ is the tolerate error determining the numbers of iterations. We use a simple method to determine $\mu_{M,0}$ and $\mu_{L,0}$. Let $D_i = min\{b_i/a_i\}$, $i = 1, 2, \cdots, \kappa$. We ignore the operation of $[.]^+$ in (33), replace (b_i/a_i) with D_i in (33), and solve $p_{r,i}$ for $i = 1, 2, \cdots, \kappa$. Using the power constraint, we can then obtain a upper bound of μ which can serve as as $\mu_{M,0}$, and a mathematical expression as

$$\left(\sqrt{\mu+D_i}\right)\left(\sum_{i=1}^{\kappa}\sqrt{a_i}\right)-\sum_{i=1}^{\kappa}c_i \le P_{R,T}. \tag{34}$$

From (34), we have

$$\mu \le \left(\left[\frac{\sum_{i=1}^{\kappa}c_i+P_{R,T}}{\sum_{i=1}^{\kappa}\sqrt{a_i}}\right]^2-D_i\right) := \mu_{M,0}. \tag{35}$$

For $\mu_{L,0}$, we can simply use the minimum μ such that $p_{r,i}$ is nonnegative in (33). For $p_{r,i}$ being nonnegative, we must have

$$\mu \geq \frac{c_i^2 - b_i}{a_i} \geq 0, \; \forall i. \tag{36}$$

From (29), (30), and (31), we can see that $\frac{c_i^2 - b_i}{a_i} \geq 0$. Thus, we let $\mu_{L,0}$ = min$\left\{\frac{c_i^2 - b_i}{a_i}, \; \forall i\right\}$.

§ A.6 Derivation of (6.41)

Considering (6.40), we can observe that the optimization problem is a convex. The Lagrangian function corresponding to (6.40)

$$\begin{aligned} L &= \sigma_s^2 \sum_{i=1}^{L} \frac{\left(\rho_i^{-1} - \sigma_s^{-2}\right)\left(p_{r,i}\sigma_{n,r}^2\sigma_{rd,i}^2 + \sigma_{n,d}^2\right)}{p_{r,i}\sigma_{sr,i}^2\sigma_{rd,i}^2} + \\ & \sum_{i=1}^{L} p_{r,i}\left(\sigma_{n,r}^2 + \sigma_s^2 \frac{\left(\rho_i^{-1} - \sigma_s^{-2}\right)\left(p_{r,i}\sigma_{n,r}^2\sigma_{rd,i}^2 + \sigma_{n,d}^2\right)}{p_{r,i}\sigma_{rd,i}^2}\right) - \sum_{i=1}^{L} \mu_{r,i} p_{r,i}, \end{aligned} \tag{37}$$

where $p_{r,i} \geq 0$ with $i = 1, \cdots, L$. By the KKT conditions, for all i, we have,

$$\frac{\partial L}{\partial p_{r,i}} = -\sigma_s^2\left(\rho_i^{-1} - \sigma_s^{-2}\right)\frac{\left(\sigma_{n,d}^2\sigma_{sr,i}^2\sigma_{rd,i}^2\right)}{\left(p_{r,i}\sigma_{sr,i}^2\sigma_{rd,i}^2\right)^2} + \left(\sigma_{n,r}^2 + \sigma_s^2\sigma_{n,r}^2\left(\rho_i^{-1} - \sigma_s^{-2}\right)\right) - \mu_{r,i} = 0. \tag{38}$$

$$\mu_{r,i} p_{r,i} = 0. \tag{39}$$

$$\mu_{r,i} \geq 0. \tag{40}$$

From (38), we have

$$\mu_{r,i} = -\sigma_s^2\left(\rho_i^{-1} - \sigma_s^{-2}\right)\frac{\left(\sigma_{n,d}^2\sigma_{sr,i}^2\sigma_{rd,i}^2\right)}{\left(p_{r,i}\sigma_{sr,i}^2\sigma_{rd,i}^2\right)^2} + \sigma_{n,r}^2 + \sigma_s^2\sigma_{n,r}^2\left(\rho_i^{-1} - \sigma_s^{-2}\right). \tag{41}$$

With the condition (39) and the assumption of $p_{r,i} > 0$, we have $\mu_{r,i} = 0$ and

$$\frac{\sigma_s^2\sigma_{n,d}^2\left(\rho_i^{-1} - \sigma_s^{-2}\right)}{\sigma_{n,r}^2\sigma_{sr,i}^2\sigma_{rd,i}^2\left(1 + \sigma_s^2\left(\rho_i^{-1} - \sigma_s^{-2}\right)\right)} = p_{r,i}^2. \tag{42}$$

Following (42) and the condition $p_{r,i} \geq 0$, we have

$$p_{r,i} = \sqrt{\frac{\sigma_s^2\sigma_{n,d}^2\left(\rho_i^{-1} - \sigma_s^{-2}\right)}{\sigma_{n,r}^2\sigma_{sr,i}^2\sigma_{rd,i}^2\left(1 + \sigma_s^2\left(\rho_i^{-1} - \sigma_s^{-2}\right)\right)}} \tag{43}$$

Q.E.D.

§ A.7 Derivation of (6.43)

We first rewrite MSE matrix by the matrix inversion lemma as

$$\begin{aligned}\mathbf{E} &= \Bigg(\underbrace{\left(\sigma_s^{-2}\mathbf{I}_L + \mathbf{E}_R\right)}_{:=\mathbf{A}} + \boldsymbol{\Sigma}_s^H \underbrace{\left(\sigma_{n,d}^{-2}\mathbf{V}_{sr}^H\mathbf{V}_{sd}\boldsymbol{\Sigma}_{sd}^H\boldsymbol{\Sigma}_{sd}\mathbf{V}_{sd}^H\mathbf{V}_{sr}\right)}_{:=\mathbf{B}}\boldsymbol{\Sigma}_s\Bigg)^{-1} \\ &= \mathbf{A}^{-1} - \mathbf{A}^{-1}\boldsymbol{\Sigma}_s^H\left(\mathbf{B}^{-1} + \boldsymbol{\Sigma}_s\mathbf{A}^{-1}\boldsymbol{\Sigma}_s^H\right)^{-1}\boldsymbol{\Sigma}_s\mathbf{A}^{-1}, \qquad (44)\end{aligned}$$

where $\mathbf{A}$ and $\mathbf{B}$ are defined as diagonal and non-diagonal matrices, respectively.

Now, we let $\mathbf{Z} := \left(\mathbf{B}^{-1} + \boldsymbol{\Sigma}_s\mathbf{A}^{-1}\boldsymbol{\Sigma}_s^H\right) = \mathbf{U}_z\boldsymbol{\Sigma}_z\mathbf{U}_z^H$ be the eigen-decomposition of the positive-definite matrix $\mathbf{Z}$ where $\mathbf{E}_R \in \mathbb{R}^{L\times L}$ is the diagonal matrix and $\mathbf{E}_S \in \mathbb{C}^{L\times L}$ is a Hermitian matrix defined in (6.42); $\mathbf{U}_z$ and $\boldsymbol{\Sigma}_z$ are the corresponding eigenvectors and eigenvalues, respectively. Let $\mathbf{e}_i$ be the ith unit standard vector. Since $1 = \mathbf{e}_i^T\mathbf{I}_L\mathbf{e}_i = \mathbf{e}_i^T\mathbf{Z}^{1/2}\mathbf{Z}^{-1/2}\mathbf{e}_i$, we then have

$$1 = \|\mathbf{e}_i^T\mathbf{Z}^{1/2}\mathbf{Z}^{-1/2}\mathbf{e}_i\|_2^2 \leq \|\mathbf{e}_i^T\mathbf{Z}^{1/2}\|_2^2\|\mathbf{Z}^{-1/2}\mathbf{e}_i\|_2^2, \qquad (45)$$

where the inequality in (45) follows from the sub-multiplicative property of the matrix norm [50]. Since $\|\mathbf{e}_i^T\mathbf{Z}^{1/2}\|_2^2 = \left(\mathbf{e}_i^T\mathbf{Z}^{1/2}\mathbf{Z}^{1/2}\mathbf{e}_i\right) = \mathbf{Z}(i,i)$ and $\|\mathbf{e}_i^T\mathbf{Z}^{-1/2}\|_2^2 = \left(\mathbf{e}_i^T\mathbf{Z}^{-1/2}\mathbf{Z}^{-1/2}\mathbf{e}_i\right) = \mathbf{Z}^{-1}(i,i)$, the equality (45) thus leads to $1 \leq \mathbf{Z}(i,i)\mathbf{Z}^{-1}(i,i)$, or equivalently $\mathbf{Z}^{-1}(i,i) \geq \frac{1}{\mathbf{Z}(i,i)}$. Therefore, taking $\mathbf{Z}$ in (44), we then have the result in (6.43).

§ A.8 Derivation of (6.48)

Similar to the derivation in Section 6.5.1, we conduct the solutions by the KKT conditions. The Lagrangian respect to (6.47) is

$$
\begin{aligned}
L &= \sigma_s^2 \sum_{i=1}^{L} \frac{\rho_i^{-1} - \sigma_s^{-2}}{\left(\frac{\sigma_{sr,i}^2 \sigma_{rd,i}^2}{\sigma_{n,r}^2 \sigma_{rd,i}^2 + \sigma_{n,d}^2 p_{r,i}^{-1}} + \frac{1}{(\mathbf{B}^{-1})(i,i)}\right)} + \\
& \sum_{i=1}^{L} p_{r,i} \left(\sigma_{n,r}^2 + \frac{\sigma_s^2 \sigma_{sr,i}^2 \left(\rho_i^{-1} - \sigma_s^{-2}\right)}{\left(\frac{\sigma_{sr,i}^2 \sigma_{rd,i}^2}{\sigma_{n,r}^2 \sigma_{rd,i}^2 + \sigma_{n,d}^2 p_{r,i}^{-1}} + \frac{1}{(\mathbf{B}^{-1})(i,i)}\right)} \right) - \sum_{i=1}^{L} \mu_{r,i} p_{r,i},
\end{aligned} \tag{46}
$$

where $p_{r,i} \geq 0$ with $i = 1, \cdots, L$. By the KKT conditions. We have

$$
\begin{aligned}
& \frac{\partial L}{\partial p_{r,i}} = \sigma_{n,r}^2 + \frac{\sigma_{sr,i}^2 \left(\sigma_s^2 \rho_i^{-1} - 1\right)}{\left(\frac{\sigma_{sr,i}^2 \sigma_{rd,i}^2}{\sigma_{n,r}^2 \sigma_{rd,i}^2 + \sigma_{n,d}^2 p_{r,i}^{-1}} + \frac{1}{(\mathbf{B}^{-1})(i,i)}\right)} - \\
& \left(p_{r,i} \sigma_{sr,i}^2 + 1\right) \frac{\sigma_{n,d}^2 \sigma_{sr,i}^2 \sigma_{rd,i}^2 p_{r,i}^{-2} \left(\sigma_s^2 \rho_i^{-1} - 1\right)}{\left(\frac{\sigma_{sr,i}^2 \sigma_{rd,i}^2}{\sigma_{n,r}^2 \sigma_{rd,i}^2 + \sigma_{n,d}^2 p_{r,i}^{-1}} + \frac{1}{(\mathbf{B}^{-1})(i,i)}\right)^2 \left(\sigma_{n,r}^2 \sigma_{rd,i}^2 + \sigma_{n,d}^2 p_{r,i}^{-1}\right)^2} - \mu_{r,i} = 0.
\end{aligned} \tag{47}
$$

$$
\mu_{r,i} p_{r,i} = 0. \tag{48}
$$

$$
\mu_{r,i} \geq 0. \tag{49}
$$

With the condition (46), (48) and the assumption of $p_{r,i} > 0$, then $\mu_{r,i} = 0$ and thus

$$
\begin{aligned}
& \sigma_{n,r}^2 \left(\frac{\sigma_{sr,i}^2 \sigma_{rd,i}^2}{\sigma_{n,r}^2 \sigma_{rd,i}^2 + \sigma_{n,d}^2 p_{r,i}^{-1}} + \frac{1}{\left(\mathbf{B}^{-1}\right)(i,i)} \right)^2 \left(\sigma_{n,r}^2 \sigma_{rd,i}^2 + \sigma_{n,d}^2 p_{r,i}^{-1}\right)^2 + \\
& \sigma_{sr,i}^2 \left(\sigma_s^2 \rho_i^{-1} - 1\right) \left(\frac{\sigma_{sr,i}^2 \sigma_{rd,i}^2}{\sigma_{n,r}^2 \sigma_{rd,i}^2 + \sigma_{n,d}^2 p_{r,i}^{-1}} + \frac{1}{\left(\mathbf{B}^{-1}\right)(i,i)} \right) \left(\sigma_{n,r}^2 \sigma_{rd,i}^2 + \sigma_{n,d}^2 p_{r,i}^{-1}\right)^2 \\
& = \sigma_{n,d}^2 \sigma_{sr,i}^2 \sigma_{rd,i}^2 p_{r,i}^{-2} \left(p_{r,i} \sigma_{sr,i}^2 + 1\right) \left(\sigma_s^2 \rho_i^{-1} - 1\right).
\end{aligned} \tag{50}
$$

Let $\alpha = \sigma_{sr,i}^2\sigma_{rd,i}^2$, $\beta = \left(\sigma_s^2\rho_i^{-1} - 1\right)$, $\gamma = \left(\sigma_{n,r}^2\sigma_{rd,i}^2 + \sigma_{n,d}^2 p_{r,i}^{-1}\right)$, $\delta = \left(\mathbf{B}^{-1}(i,i)\right)^{-1}$, we rewrite (50) as

$$\gamma^2\left(\frac{\delta^2\sigma_{n,r}^2}{\beta\sigma_{sr,i}^2} + \delta - \sigma_{n,d}^{-2}\sigma_{rd,i}^2\right) + \gamma\left(\frac{2\alpha\delta\sigma_{n,r}^2}{\beta\sigma_{sr,i}^2} + \underbrace{\alpha - \sigma_{rd,i}^2\sigma_{sr,i}^2}_{=0} + 2\sigma_{n,d}^{-2}\sigma_{n,r}^2\sigma_{rd,i}^4\right) + \left(\frac{\alpha^2\sigma_{n,r}^2}{\beta\sigma_{sr,i}^2} + \sigma_{n,r}^2\sigma_{sr,i}^2\sigma_{rd,i}^4 - \sigma_{n,d}^{-2}\sigma_{n,r}^4\sigma_{rd,i}^6\right) = 0. \tag{51}$$

After some manipulations, (51) can be reformulated as

$$\underbrace{\left(\sigma_{n,r}^4\sigma_{rd,i}^4\left(\frac{\delta^2\sigma_{n,r}^2}{\beta\sigma_{sr,i}^2} + \delta\right) + \sigma_{n,r}^2\sigma_{rd,i}^2\left(\frac{2\alpha\delta\sigma_{n,r}^2}{\beta\sigma_{sr,i}^2}\right) + \frac{\alpha^2\sigma_{n,r}^2}{\beta\sigma_{sr,i}^2} + \sigma_{n,r}^2\sigma_{sr,i}^2\sigma_{rd,i}^4\right)}_{:=A_i} p_{r,i}^2$$
$$+ \underbrace{\left(2\sigma_{n,r}^2\sigma_{n,d}^2\sigma_{rd,i}^2\left(\frac{\delta^2\sigma_{n,r}^2}{\beta\sigma_{sr,i}^2} + \delta - \sigma_{n,d}^{-2}\sigma_{rd,i}^2\right) + \frac{2\alpha\delta\sigma_{n,r}^2\sigma_{n,d}^2}{\beta\sigma_{sr,i}^2} + 2\sigma_{n,r}^2\sigma_{rd,i}^4\right)}_{:=B_i} p_{r,i}$$
$$+ \underbrace{\sigma_{n,d}^4\left(\frac{\delta^2\sigma_{n,r}^2}{\beta\sigma_{sr,i}^2} + \delta - \sigma_{n,d}^{-2}\sigma_{rd,i}^2\right)}_{:=C_i} = 0 \tag{52}$$

A_i, B_i and C_i can be further expressed as (6.49), (6.50) and (6.51), respectively. Since $A_i, B_i > 0$ and $p_{r,i} \geq 0$, the solution is found to be

$$p_{r,i} = \begin{cases} \frac{-B_i + \sqrt{B_i^2 - 4A_iC_i}}{2A_i}, & \text{if } C_i < 0 \\ 0, & \text{if } C_i \geq 0. \end{cases} \tag{53}$$

Q.E.D.

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