國 立 交 通 大 學

資訊管理研究所

碩　士　論　文

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Visualizing & Analyzing Customer's Preferences by Gower Plots

研 究 生： 陳淳怡

指導教授：黎漢林 博士

中華民國　九十三 年 六 月

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研 究 生：陳淳怡　　Student: CY Chen

指導教授：黎漢林 博士　　Advisor: Dr. HanLin Li

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應用高爾圖分析消費者偏好

學生：陳淳怡　　　　　　　　　　指導教授：黎漢林 博士

國立交通大學 資訊管理研究所 碩士班

摘　　要

本論文是應用圖形技術-高爾圖來分析收集自網路問卷調查的消費者偏好，並以此作為擬定行銷策略的依據，進而進行公司資源的分配與產品設計與生產的導向。高爾圖可提供比傳統行銷分析手法更有快速且有效的結論，來掌握市場的狀況，鎖定主要客戶層。

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student：CY Chen Advisors：Dr. HanLin Li

Institute of Information Management
National Chiao Tung University

ABSTRACT

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Chun-Yi Chen

June 25, 2004

Institute of Information Management
National Chiao Tung University
Management Building 2, 1001 Ta Hsueh Road,
Hsinchu, Taiwan 300, ROC

cychenb@tsmc.com.tw
hlli@cc.nctu.edu.tw

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1. Introduction

Companies and their marketing managers frequently encounter difficulties when trying to determine what a consumer really wants about a specific product or service. In past decades, researchers have developed several measuring techniques in the mathematical psychology field, psychometrics and customer behavior patterns that can aid the manager in determining the relative importance of a product's multidimensional attributes. Companies were busy collecting and sifting mountains of data regarding preferences and behaviors, dividing consumers into ever-finer segments, and honing their products, services, and marketing pitches. But they could still not see the complete picture. Few companies have bothered to carefully look at the broader context in which a customer selects, purchases and uses products and services. They have been so focused on fine-tuning their own offerings that they fail to see whether those products and services actually meet their customers' needs. They have spent a great amount of time and money to advertise our products and services, but fail to appreciate the effect of those advertisements.

Consumer preference research is a critical issue in the marketing planning process. Understanding what stimulates customer's intention, how customers process their buying decision and finally take actions to purchase the specific product/service is and should be the most important factor in every aspect of company's marketing plan. Traditionally, in the marketing research area, customer preference is mainly studied by the Conjoint technique, developed in 1960s. This research applies a graphical technique to develop of a marketing planning model. The outcome of the analysis can be used as a guide for marketing plan development when allocate marketing resources and arranging customer manipulation strategies. The purpose of this paper is to describe a graphical technique to develop a global vision of the complex relationship between message extracts from a marketing questionnaire and its analysis.

We combine graphical analysis with Consumer Behavior to show how to visualize respondent's data and to detect contradiction between preferences to provide a more efficient and easy way to monitor, control and even conduct the marketing plan. Genest and Zhang developed the graphical analysis named Gower Plots in 1970s. The graphical method provides an especially valuable diagnostic tool: Ordinal Gower Plot detects the contradiction between one preference and the other; and cardinal Gower Plot perceives the proportional relation of respondent judgments. The overall flow includes gathering data using a questionnaire through on-line questionnaire with Likert scales, computing and analyzing the data using the Gower Plot method to ranking customer preferences in order to define market characteristics and customer behaviors.

In this article, for comparison between two visualizing tools: Conjoint analysis and Gower Plots, we first introduce Conjoint technique, the traditional tool for preference study in marketing research, following, describe Gower Polts and how the graphical method works. Finally, we will provide some empirical cases where we applied the graphical method to show how it is useful to the marketing plan and the management improvement.

2. Literature review

To emphasize that the benefit of Gower Plots, we will introduce the traditional tool for preference study in marketing research- Conjoint analysis.
In past decade, many techniques, such as multidimensional scaling analysis, conjoint analysis and so on, were developed to handle those problems have a common structure that companies and their marketing managers frequently encounter in trying to figure our shat a consumer really wants in a product or service. The most famous technique is so called conjoint analysis. Conjoint analysis was developed by Paul E. Green (July- August 1975) and his collaborators as a method from the field of mathematical psychology and psychometrics that can aid the marketing manager in sorting out the relative importance of a product’s multidimensional attributes.

The word “conjoint” has to do with the notion that the relative values of things considered jointly can be measured when they might not be measurable if taken one at a time. Quite often respondents are asked to express the relative value to them of various alternatives by ordering the alternatives form most desirable to least desirable. The attempt in a conjoint analysis solution is to assign value to the levels of each of the attributes so that the resulting values or utilities are as monotonic as possible with the input rank-order judgments.

Conjoint measurement starts with the consumer's overall or global judgments about a set of complex alternative. It then performs the job of decomposing respondent's original evaluations into separate and compatible utility scales by which the original global judgments can be reconstituted.

The stimuli in conjoint analysis represent some predetermined combinations of attributes (ex: products or brands), and respondents are asked to make judgments about their preference for these various arrtibute combinations. The basic aim is to determine the features respondents most prefer. Respondents might use, for example, such attributes as package design, brand name, price, money-back guarantee, and so on in making judgments about which carpet cleaner they most prefer. If asked to so directly, many respondents might find it very difficult to state which attributes they were using and how they were combining them to form overall judgments. Conjoint analysis attempts to handle this problem by estimating how much each of the attributes is valued on the basis of the choice respondents make along product concepts that are varied in systematic ways. In essence respondents' value systems are inferred form their behaviors as reflected in their choices rather than form self reports about how import each of the various attributes are to them.

The procedure for determining the individual's utilities for each of several product attributes followed in conjoint analysis is quite similar to that followed in multidimensional scaling analysis. The technique is quite dependent on the availability of a high-speed computer. Just as in multidimensional scaling, the computer program emphasis is on generating an initial solution and on modifying that solution through a series of iteration to improve the goodness-of-fit. More specifically, given a set of input judgments, the computer program will:

1. Assign arbitrary utilities to each level of each attribute;
2. Calculate the utilities for each alternative by somehow combining (most typically adding) the individual utility values;
3. Calculate the goodness of fit between the ranking of the alternatives using these derived utility values and the original ordering of the input judgments;

4. Modify the utility values in a systematic way until the derived utility produce evaluations that, when ordered, correspond as closely as possible to the order of the input judgments.

The following case will be taken as an example to see how conjoint measurement work.

A company was interested in marketing in a new spot remover for carpets and upholstery. The technology staff has developed a new product that is designed to handle spot. Management interest centers on five attributes or factors that it expects will influence consumer preference: an applicator-type package design, brand name, price, a Good Housekeeping seal of endorsement, and a money-back guarantee.

As below Table 1, the result comes from respondent's evaluation. There are three brand names: K2R, Glory, and Bissell. Three alternative prices being considered are $1.19, $1.39, and $1.59. Since they are three alternatives for each of these factors, they are called three-level factors. The Good Housekeeping seal and money-back guarantee are two-level factors, since each is either present or not. A total of 3*3*3*2*2=108 alternatives would have to be tested if the research were to array all possible combinations of the five attributes. As an alternative, however, the research can take advantage of a special experimental design, called an orthogonal array, in which the test combinations are selected so that the independent contributions of all five factors are balanced. In this way each factor's weight is kept separate and is not confused with those of the other factors. The table shows an orthogonal array that involves only 18 of 108 possible combinations that the company wishes to test in this case. The last column of the experimental table shows one respondent's actual ranking of the 18 cards; rank number 1 denotes her highest evaluated concept. Note particularly that only ranked data need to be obtained and that only 18 (out of 108) combinations are evaluated.

Various computer programs carry out computation of the utility scales of each attribute, which determine how influential each is in the consumers' evaluations. The ranked data of a single respondent (or the composite ranks of a group of respondents) are entered in the program. The computer then searches for a set of scale values for each factor in the experimental design. The scale values for each level of each factor are chosen so that when they are added together the total utility of each combination will correspond to the original ranks as closely as possible.

There are two problems need to be involved here. First, as mentioned previously, the experimental design shows only 18 of 108 combinations. Second, only rank-order data are supplied to the algorithms. This means that the data themselves do not determine how much more influential one attribute is than another in the consumers' choices. However, despite these limitations, the algorithms are able to find a numerical representation of the utilities, thus providing an indication of each factor's relative importance.
As can be observed in Table 2, the technique obtains a utility function for each level of each factor. For example, the total utility of combination 18 is 3.1 (0.6+0.5+1.0+0.3+0.7), which is the respondent's highest evaluation of all 18 combinations listed.

By focusing attention on only the package design, the company's marketing researches can see from Figure 1 that Design B displays highest utility. Moreover, all utility scales are expressed in a common unit (although their zero points are arbitrary). This means that we can compare utility ranges from factor to factor so as to get some idea of their relative importance.

In the case of the spot remover, as shown in Figure 1, the utility ranges are:

Package design (1.0-0.1=0.9)
Brand name (0.5-0.2=0.3)
Price (1.0-0.1=0.9)
Good Housekeeping seal (0.3-0.2=0.1)
Money-back guarantee (0.7-0.2=0.5)

The lower portion of Figure 1 shows the relative size of the utility ranges express in histogram form. As noted, package design and price are the most important factors, and together they account for about two thirds of the total range in utility.

The relative importance of a factor depends on the levels that are included in the design. As a crude indication of what factors to concentrate on, factor importance calculations provide a useful by-product of the main analysis regardless of such limitation.

Limitations of the conjoint analysis are as the following:

1. The techniques suspect that in many instances the simpler (additive) model

represents a very good approximation of reality. That is, what is often called "interactions" in traditional ANOVA applications may be the result of the failure to measure the effects of independent variables on the correct scales.

2. The respondent may not behave unidimensionally toward the prespecified criterion. This could be reflected in the failure of the paired comparisons to generate a complete rank order (necessitating the use of nearest adjoining order techniques) or the failure of a one-dimensional scaling solution to accommodate the data.

3. In dealing with large-size problems, the ranking task becomes formidable. Moreover, some comparisons could be omitted, inasmuch the programs tolerate missing data.

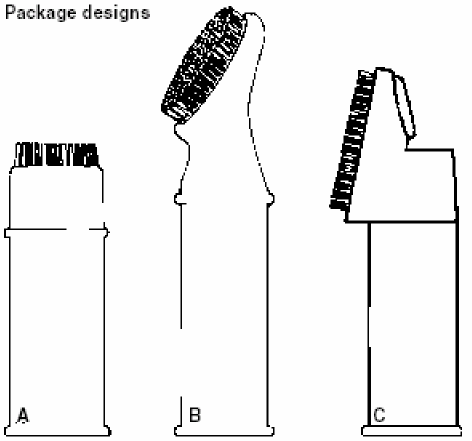
4. As in nonnumeric scaling, conjoint solutions are susceptible to certain types of degeneracy.

5. Conjoint analysis is still a moot point as to whether direct numerical estimation procedures would lead to results comparable to those arrived at through ranking followed by conjoint measurement.

Table 1- Experimental design for evaluation of carpet cleaner

Experimental design for evaluation of a carpet cleaner

Package designs



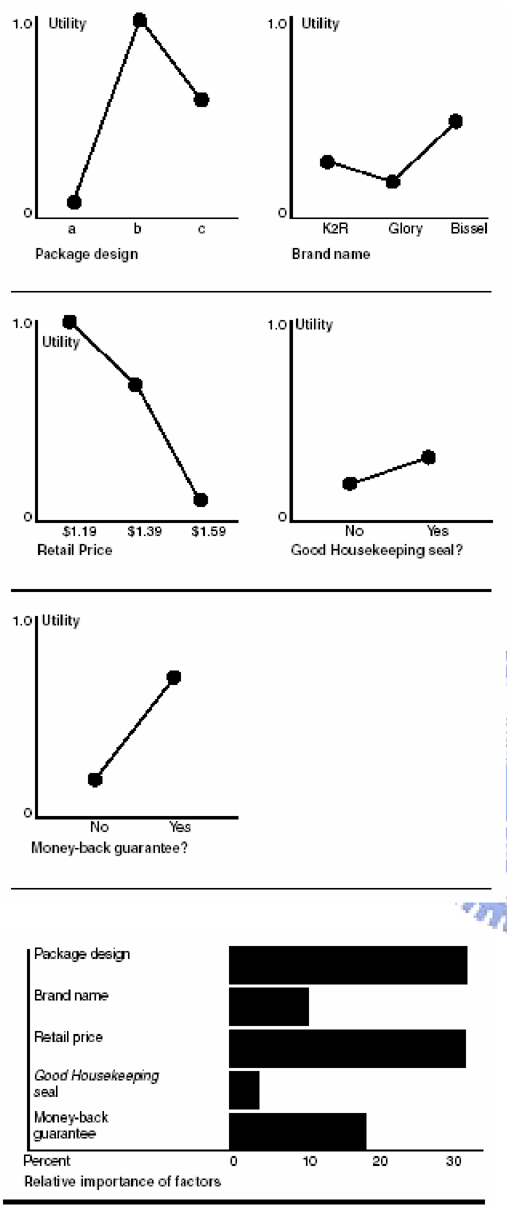
Package design	Brand name	Price	Good House-keeping seal?	Money-back guarantee?	Respondent's evaluation (rank number)
1 A	K2R	$1.19	No	No	13
2 A	Glory	1.39	No	Yes	11
3 A	Bissel	1.59	Yes	No	17
4 B	K2R	1.39	Yes	Yes	2
5 B	Glory	1.59	No	No	14
6 B	Bissel	1.19	No	No	3
7 C	K2R	1.59	No	Yes	12
8 C	Glory	1.19	Yes	No	7
9 C	Bissel	1.39	No	No	9
10 A	K2R	1.59	Yes	No	18
11 A	Glory	1.19	No	Yes	8
12 A	Bissel	1.39	No	No	15
13 B	K2R	1.19	No	No	4
14 B	Glory	1.39	Yes	No	6
15 B	Bissel	1.59	No	Yes	5
16 C	K2R	1.39	No	No	10
17 C	Glory	1.59	No	No	16
18 C	Bissel	1.19	Yes	Yes	1*

*Highest ranked

Table 2- Utility of factor

Package Design		Brand Name		Retail Price		Good Housekeeping seal		Money-back guarantee	
Factor	Utility	Factor	Utility	Factor	Utility	Factor	Utility	Factor	Utility
A	0.1	K2R	0.3	$1.19	1.0	No	0.2	No	0.2
B	1.0	Glory	0.2	$1.39	0.3	Yes	0.3	Yes	0.7
C	0.6	Bissel	0.5	$1.59	0.1				

Figure 1- Results of computer analysis of experimental data of Table 4



3. Gower Plot

The proposed method, based on the Analytic Hierarchy Process (Saaty, 1977) methodology and Gower plots (Gower, 1977), is close in spirit to the statistical technique known as multidimensional scaling, and is also a spatial representation of the result. AHP provides the alternative derived from a complete set of paired comparisons expressed as a ratio scale. Gower plots provide an especially valuable diagnostic tool to visualize such data and to detect cardinal and ordinal inconsistencies in the respondent preferences.

3.1. Analytic Hierarchy Process (AHP)

AHP is a ratio scaled assessment of an agent's preferences between pairs of alternatives. One of the basic ingredients of this procedure is the evaluation of the strength of individual preference through pairwise comparison of alternatives at each level of the hierarchy. When asked to compare alternatives j and k from a collection of size $n \geqq 2$ with respect to a single criterion, the respondent must elicit the ratio $\frac{\omega_j}{\omega_k}$ measuring the relative dominance of item j over item k in terms of the underlying priority weights $\omega_1 > 0, \ldots \omega_n > 0$, taken to sum up to one by convention. This give rise to $n(n-1)/2$ data points of the form

$$r_{jk} = \frac{\omega_j}{\omega_k} \varepsilon_{jk}, \qquad (1)$$

where ε_{jk} is a multiplicative term accounting for inconsistencies in judgment that are usually observed in practice. Following Saaty (1977), it will be convenient to store these observations in an $n \times n$ response matrix $R=(r_{jk})$, with the convention that

$r_{kj} = \frac{1}{r_{jk}}$ for all $1 \leqq j, k \leqq n$.

3.2. Gower Plots

The interpretation of Gower plots constructed from the skew-symmetric matrix $S=(s_{jk})$ of size n with entries $s_{jk} = \log(r_{jk})$. By definition, the singular value of a matrix A of rank n are the positive square roots of the eigenvalues of the symmetric matrix $A'A$, where' denotes transposition. In the special case where A is skew-symmetric, that is when $A'=-A$, the singular values of the matrix A are also equal to the norm of its pure imaginary eigenvalues. As $-\lambda$ must then be an eigenvalue of A whenever λ is, the matrix typically has $n/2$ singular values of multiplicity 2 when n is even, and an additional singular equal to 0 when n is odd. Let $\lambda_1 \geq ... \geq \lambda_m \geq 0$ (and $\lambda_{m+1} = 0$ if required) denote these values, with m standing for the integer part of $n/2$. For $j = 1,...,m$, let also U_{2j-1} and U_{2j} be orthonormal eigenvectors of $A'A$ corresponding to λ_j^2. It is then possible to write A in the form

$$\sum_{j=1}^{m} \lambda_j (u_{2j-1}u_{2j}' - u_{2j}u_{2j-1}'), \qquad (2)$$

as a sum of m elementary rank 2, skew-symmetric matrix. This is a special case of the singular value decomposition theorem for arbitrary complex matrices, as described in Ch7 of the book by Horn and Johnson (1985).

From a classical theorem of Wckart and Young (1936), the first l terms of Equation (2) give the best least-squares fit of rank $2l$ to A. In particular, the matrix

$$A^* = \lambda_1 (UV' - VU'),$$

with $U = U_1$ and $V = U_2$ provides the best approximation of rank 2. Plotting the vectors $U = (u_1,...,u_n)'$ and $V = (v_1,...,v_n)'$ as n points $P_j = (u_j, v_j)$ in the plane should thus provide a reasonable two-dimensional representation of A. Such a graphical display, unique up to a rotation if $\lambda_1 > \lambda_2$, is referred to herein as a Gower plot (Gower 1977, Constantine and Gower 1978). Although independent constructions that are essentially to his abound in the literature (cf., e.g., Escoufier and Grorud 1980, Chino 1990, Harshman and Lundy 1990). As pointed out by many of these authors, an adequate measure of the faithfulness of the graphical representation of A is then provided by the proportion of the variability in A that is captured by A^*, *viz*.,

$$v = \frac{\|A^*\|}{\|A\|} = \frac{\lambda_1^2}{\sum_{j=1}^{m} \lambda_j^2}, \qquad (3)$$

3.2.1 Cardinal Gower Plot

General guidelines for the interpretation of Gower plots can find in the paper of Gower (1977) or in the follow-up article by Constantine and Gower (1978). As mentioned by these authors, the (*j*,*k*)th entry a^*_{jk} of A^* is proportional to the signed area $u_j v_k - v_j u_k$ of the parallelogram subtended at the origin by points P_j and P_k on the graphical display. Indeed if θ_{jk} represents the directed angle between P_j and P_k, one has

$$a^*_{jk} = \lambda_1 (u_j v_k - v_j u_k) = \lambda_1 |P_j| |P_k| \sin(\theta_{jk}) \qquad (4)$$

This relationship can be exploited to show that the angle θ_{jk} between two points P_j and P_k of a cardinal Gower plot is indicative of item *j* and *k*'s ordinal ranking, while the area between the points reveals something about the response's degree of preference for one item over the other. The conclusions derived from the graph will be reliable, provided that the plot explains a large proportion of the variability contained in the data. The degree of faithfulness of the graphical representation will be measured by the quantity *v* defined in (3) with an appropriate choice of skew-symmetric matrix *A*.

When the agent's responses are cardinally consistent, the construction yields a set of points $P_1 = (u_1, v_1), \ldots, P_n = (u_n, v_n)$ on a straight line that does not cross the origin. Lack of collinearity easily can be spotted and provides a means of detecting delinquent comparison, provided the data were ordinally consistent to begin with.

A cardinal inconsistency is said to have occurred if a set of a set of distinct alternative $1 \leqq j,k,l \leqq n$ can be found for which the identity $r_{jl} = r_{jk} \times r_{kl}$ do not hold.

3.2.2 Ordinal Gower Plot

While angles and areas in the Gower plot of *S* respectively contain information about the ordinal and cardinal behavior of a respondent's set of judgment, it is also possible to exploit Gower's method to concentrate exclusively on the ordinal structure of a response matrix. Applying his technique to the tournament matrix T derived from R through Equation (1) can do this.

To check more specifically for ordinal consistency, one can draw a Gower plot of the

$n \times n$ tournament matrix (1 denotes a victory and 0stands for a defeat) $T=(t_{jk})$ defined by

$$t_{jk} = \begin{cases} 1 \text{ if } r_{jk} > 1, \\ 0 \text{ if } r_{jk} = 1, \\ -1 \text{ if } r_{jk} < 1, \end{cases}$$

Ideally, the points $P_1,....,P_n$ should be equidistant from the origin, arranged counterclockwise in order of preference with in a 180-defree arc. This is what happens when the tournament matrix T (or its parent R) is ordinally consistent. At the other extreme, the points $P_1,....,P_n$ will correspond to the vertices of a regular polygon inscribed in a circle centered at the origin in the case where T is a maximally intransitive tournament.

An ordinal inconsistency occurs if, in addition, the implication

$$r_{jk} > 1 \text{ and } r_{kl} > 1 \Rightarrow r_{jl} > 1$$

is violated for some $1 \leqq j,k,l \leqq n$

3.3. Applying Gower's technique to AHP

The above methodology is not directly applicable to the AHP context, where a respondent's judgments r_{jk} are coded on a ratio scale. However, a natural skew-symmetric to which Gower's technique can be applied is obtained when the responses are linearized via the transformation $s_{jk} = \log(r_{jk})$. The Gower plot of $S=(s_{jk})$, hereafter referred to as a cardinal Gower plot, provides much insight into the structure of a set of responses. However, such plots are naturally subject to the strong interplay between ordinal and cardinal aspects of individual judgments in AHP that was underscored in the work of Genest etal. (1993). To focus on the ordinal structure of the responses, one may thus wish to apply Gower's technique to *tournament matrix* associated with *R*. The resulting graph can be regarded as a complementary tool to the graphical display developed by Genest and his collaborators in their paper.

3.4. Cardinal / Ordinal Gower Plot Measure

3.4.1 Cardinal Gower Plot Measure

Let $R=(r_{jk})$ be an *n x n* response matrix with entries $r_{jk}>0$ satisfying $r_{kj}=1/r_{jk}$ for all $1\le j,k\le n$. Define the skew-symmetric *S*=(s_{jk}) with entries $s_{jk}=\log(r_{jk})$, $1\le j,k\le n$, and let $S^*=(s^*_{jk})=\lambda_1(u_j v_k - v_j u_k)$ be its best least-squares approximation of rank2. Let also $P_1=(u_1,v_1),\ldots,P_n=(u_n,v_n)$ be the points appearing on the Gower *S*.

(a) If $P_1,\ldots.,P_n$ lie within a 180 degree arc, then S* is ordinally consistent in the sense that

$$S^*_{jk}>0 \ \text{ and } \ S^*_{kl}>0 \Rightarrow S^*_{jl}>0, \qquad (5)$$

for all possible choices of indices $1\le j,k,l\le n$. In that case, one has $\theta_{jk}>0$ if and only if P_j is strictly preferred to P_k in the ordinal ranking implied by *S**.

(b) If $P_1,\ldots.,P_n$ are collinear, then *S** is cardinally consistent, in the sense that

$$s^*_{jk}+s^*_{kl}=s^*_{jl} \qquad (6)$$

for all possible choices of indices $1\le j,k,l\le n$.

(c) The response matrix *R* is cardinally consistent if and only if $P_1,\ldots.,P_n$ are collinear and $v=\|S^*\|/\|S\|=1$

In the view of part (a) of the above proposition, the fact that a point P_k lies within 180 degrees counterclockwise from another point P_j may be inferred to imply that item j is preferred to item k by the respondent, insofar as S^* approximates S well. Thus, if R is ordinally transitive, and if the least-squares approximation is sufficiently good that

$$s_{jk>0} \Leftrightarrow s^*_{jk} > 0 \qquad (7)$$

P_k would lie within 180 degrees counterclockwise of P_j on the Gower plot of R. In practice, it may be that equation (7) does not hold, or that S^* be ordinally consistent while R is not. In fact, the preference relation induced by the θ_{jk}'s will not necessarily be transitive, because S^* is not always ordinally consistent. As a result, there may sometime exist point P_j ,P_k and P_l such that $\theta_{jk} < 180$, $\theta_{kl} < 180$ and $\theta_{jl} < 180$.

Outside the conditions of part (b) of the proposition, the signed areas of the parallelograms subtended by pairs of points (P_l , P_k) and (P_k , P_l) would not generally add up to the area of the parallelogram corresponding to (P_j , P_l). This lack of transitivity, imply by a lack of collinearity in the display, may thus be taken as a rough indication of cardinal inconsistency in the respondent's judgments. It should be emphasized that the ordinal or cardinal consistency of S^* does not necessarily extend to R, unless $v = 1$.

3.4.2 Ordinal Gower Plot Measure

3.4.2.1 Tournament Matrix

Let $B = (b_{jk})$ be an $n \times n$ skew-symmetric matrix with entries $b_{jk} = 1$ for all $j < k$. For n odd, let also $C = (c_{jk})$ be another skew-symmetric matrix with entries $c_{jk} - (-1)^{j+k+1}$ for arbitrary $j < k$.

(a) A tournament matrix T is said to be ordinally consistent if there exists a permutation matrix P such that $T = PBP'$.

(b) A tournament matrix T is said to be maximally intransitive if there exists a permutation matrix P such that $T = PCP'$

3.4.2.2 Basic Features of Ordinal Gower Plots

Let $R=(r_{jk})$ be an n x n response matrix with entries $r_{jk}>0$ satisfying $r_{kj}=1/r_{jk}$ for all $1\leq j,k\leq n$. Let also $P_1=(u_1,v_1),...,P_n=(u_n,v_n)$ be the points appearing on the Gower plot of the skew-symmetric matrix T defined as per equation (1).

(a) If R is ordinally consistent, the points $P_1,....,P_n$ are then located on a circle centered at the origin and arranged counterclockwise in order of preference; the angle between two consecutive points is then equal to $180/n$ degrees, so that all the $P_j^{'}s$ lie in a half-plane determined by a line going through the origin.

(b) If R is maximally intransitive, the points $P_1,....,P_n$ then correspond to the vertices of a regular polygon inscribed in a circle of radius $\sqrt{2/n}$ centered at the origin.

(c) The index $v=\|T^*\|/\|T\|=1$ is bounded above by $\cot^2(\pi/2n/\|T\|$, which reduces to $2\cot^2(\pi/2n)/n(n-1)$; when there are no ties, i.e., when $t_{jk}\neq 0$ for all $l\leq j\neq k\leq n$; the latter bound is achieved in the two extreme cases where R is either ordinally consistent or maximally intransitive.

(d) If $t_{jk}\neq 0$ for all $j\neq k$, the index v is bounded below by $2/n$; this bound is achieved whenever $(T+I_n)'(T+I_n)=nI_n$, where I_n stand for the identity matrix of size n.

In view of the above proposition, the angles between the points on the Gower plot of a tournament matrix are again indicative of the items' preference ranking, provided that the proportion $v=\|T^*\|/\|T\|$ of the variability in T explained by the graphical display is sufficiently large. In particular, it follows from part (a) of the proposition that the relation

$$\theta_{jk}+\theta_{kl}=\theta_{jl}, \qquad (8)$$

holds for all indices j,k,l when the tournament matrix is ordinally transitive, while part (b) of the result implies that the same relation is violated to an extreme degree when the tournament matrix is maximally intransitive. However, the distance from a point to the origin can also be regarded as a measure of the influence of that item on the ordering of the alternatives and, to a large extent, on the appearance of the plot.

Matrices $H=T+I_n$ that meet the condition stated in part (d) are called skew

Hadamard. Such matrices are known not to exist unless n equals 1,2, or a multiple of 4. Explicit examples of skew Hadamard matrices exist for small multiple of 4 (e.g., $n \le 4 \times 40$), but their existence for large multiples of 4 remains in doubt at present.

The lower bound for v is reached when T is skew Hadamard of size $4n$, in which case

$$\lambda_1 = \cdots = \lambda_m = \sqrt{4n-1}\,.$$

However, this bound is clearly not the best possible under the assumption $\lambda_1 > \lambda_2$ required for Gower plots to be uniquely defined. It should be noted that condition $v > \frac{1}{2}$ is sufficient but not necessary to insure that the definition of the display is unambiguous. The determination of the best lower bound for v under the assumption $\lambda_1 > \lambda_2$ would seem to be a difficult problem.

3.5 Data collection

The questionnaire is designed based on the Likert method.
Attitude is one of the most pervasive notions in all of marketing. It plays a pivotal role in the major models describing consumer behavior, as well as in many, if not most, investigations of consumer behavior that do not rely on a formal integrated model. This research we selected the factor category base on EKB model.
3.5.1. The Likert method
The Likert method of summated ratings overcomes the previous criticisms about scoring and allowing an express of intensity of feeling. The method is both constructed and used in a slightly different way than equal-appearing intervals.
The basic format of the scale for the summated ratings method is the same in both construction and use. Subjects are asked to indicate their degree of agreement or disagreement with each and every statement in a series by checking the appropriate cell. The researcher attempts to develop great many statements that reflect qualities of things about the object that possible influence a person's attitude toward it. The method is quiet different, though, in terms of the judgment sample and what is asked of the subjects. A total attitude score can be calculated for each subject using the same scoring procedure. The procedure, known as item analysis, rests on the proposition that there should be consistency in the response pattern of any individual. If the individual has a very favorable attitude toward the object, the individual should basically agree with the favorable statements and disagree with the unfavorable ones

and vice versa.

3.5.2. Attitude Measurement - Scales of Measurement

To properly address the subject of attitude measure, it is necessary to define measurement and to briefly review the types of scales that can be used in measure. We summarize some of the more important features of these scales, which are briefing in the following.

1. Nominal Scale

   One of the simplest properties of the scale of number is identity. These numbers simply identify the individual assigned the number. With a nominal scale, the only permissible operation is counting. Thus, the “Mode” is the only legitimate measure of central tendency.

2. Ordinal Scale

   A second property of the scale of numbers is that of order. Note that the ordinal scale implies identity, since the same number would be used for all objects that are the same. For example, this assignment would still indicate the class level of each person and the relative standing of two persons when compared in terms of who is further along in the academic program. The difference in rank says nothing about the difference in academic achievement between two ranks.

   We can transform an ordinal scale in any way that we wish as long as we maintain the basic ordering of the object. The ordinal scale is thus said to allow any monotonic positive transformation of the assigned numerals, because the differences in numerals are void of meaning other than order.

   With ordinal scales, both the “Median” and “Mode” are permissible or meaningful measures of average.

3. Interval Scale

   A third property of the scale of numbers is that the intervals between the numbers are meaning full in the sense that the numbers tell us how far apart the objects are with respect to the attribute. This means that the differences can be compared. One classic example of an interval scale is the temperature scale. Suppose that the low temperature for the day was 40^0F and the high was 80^0F. We cannot say that the high temperature was twice as hot as the low temperature. Thus, we cannot compare the absolute magnitude of numbers when measurement is made on the basis of an interval scale.

   The comparison of intervals is legitimate with an interval scale because the relationships among the differences hold regardless of the particular constants chosen for a and b when transforming an interval set of numbers. With an interval scale, the “Mean”, “Median”, and “Mode” are all meaningful measures of average.

4. Ratio Scale

   The ratio scale differs from an interval scale in that it possesses a natural or absolute zero, one for which there is universal agreement about its location. Height and weight are examples. With a ratio scale, the comparison of the absolute magnitude of the number is legitimate. This means that with a ratio scale we can compare intervals, rank objects according to magnitude, or use the numbers to identify the objects. Ratio scales only allow the proportionate transformation of the scale value and not the addition of an arbitrary constant as do interval scales. A proportionate transformation is of the form y=bx, where x again represents the original values and y the transformed values and b is some positive constant.

   The “Geometric mean” as well as the more usual arithmetic mean, median, and mode are meaningful measures of average when attributes are measured on a ratio scale.

Table 3- Scales of Measurement

Scale	**Basic Comparison**	**Typical Example**	**Measures of Average**
Nominal	**Indentity**	**Male-Female**	**Mode**
Ordinal	**Order**	**Preference for brands**	**Median**
Interval	**Comparison of absolute magnitude**	**Units sold number of purchasers**	**Geometric mean**

4. Case Study I - Customer Preferences for Car Purchase

For experiment on and explanation of the graphical technique, we design a survey of the customer preference for car through an online questionnaire to obtain the sample data (questionnaire refers to appendix). With those response data, we can carry out the data sheet and then result of the response matrix to apply ordinal and cardinal plot.

Seven attributes being considered are Price, Color, Shape, Function, Disposition, Brand and Purpose. We perform the attributes as comparison between one attribute to the other in order to minimize the size of combinations. That is, we won't ask respondents to weight each attribute respectively. Therefore, the $n(n-1)/2$ data points will become a ratio of the form $r_{jk} = \frac{\omega_j}{\omega_k}$. As we mentioned previous, the questionnaire was designed with the Likert method. We predefine the weight of each response category under the consideration of attitude score ranking calculation. Since the data points are performed as a ratio, we use the "Geometric mean" as a meaningful measures of average when attributes are measured on a ratio scale.

Result can be observed as the following data sheet, response matrix and Gower plots.

4.1 Response data sheet (Overall condition)

The upper portion of Table 4 shows the response category, predefined weight, data point and comparison attribute. The left portion of Table 4 shows the result of the calculation. The last column of Table 4 shows the sample size. Originally, we asked 60 respondents for answer 21 questions through the web. Then, we can only get 40 effective copies, because there is no supervisor to monitor those respondents to exactly complete the online questionnaire. Although sample size is only 40, we still can get enough data points to process the experiment. And, you will find the technique works well to carry out the results.

Notice that Table 4 is under overall condition, that is, we did not put any criterion to classify the data points such as age, sex and so on.

Table 4-- Response data sheet of Cardinal Gower plot

Response Category / Given Weight / Data Point / Comparison Attribute	Strongly disagree:	Disagree:	Neither agree nor disagree:	Agree:	Strongly Agree:	Cardinal		
	0.25	0.50	1.00	2.00	4.00	Geomean	Tournament	Size
Price is important than Color	0	5	1	14	20	2.338	1	40
Price is important than Shape	1	16	2	16	5	1.149	1	40
Price is important than Function	4	24	2	8	2	0.707	-1	40
Price is important than Disposition	0	20	1	14	5	1.072	-1	40
Price is important than Brand	3	19	4	7	7	0.933	-1	40
Price is important than Purpose	4	29	1	4	2	0.605	-1	40
Color is important than Shape	2	25	4	7	2	0.732	-1	40
Color is important than Function	15	20	1	4	0	0.451	-1	40
Color is important than Disposition	8	21	3	6	2	0.626	-1	40
Color is important than Brand	14	20	1	5	0	0.475	-1	40
Color is important than Purpose	9	26	1	4	0	0.500	-1	40
Shape is important than Function	6	22	2	8	2	0.683	-1	40
Shape is important than Disposition	2	18	5	14	1	0.901	-1	40
Shape is important than Brand	5	21	4	7	3	0.732	-1	40
Shape is important than Purpose	8	27	1	3	1	0.518	-1	40
Function is important than Disposition	0	6	1	14	19	2.219	1	40
Function is important than Brand	0	6	2	13	19	2.181	1	40
Function is important than Purpose	0	13	4	10	13	1.490	1	40
Disposition is important than Brand	2	15	4	12	7	1.129	1	40
Disposition is important than Purpose	4	21	3	9	3	0.785	-1	40
Brand is important than Purpose	9	13	5	13	0	0.732	-1	40

The data in Table 5 was converted from the column named Tournament in Table 4. This is nonstandard use of the term "tournament matrix." The latter terminology usually refers to matrix in which "1" denotes a victory and "-1" stands for a defeat. For example, to find the value of Price vs. Color in Table 5, we can read off the value of " Price is important than Color" of Table 4. The value of " Price is important than Color" is 1 (because we take the result of Strongly Agree(20) + Agree(14) > Strongly Disagree(0) + Disagree(5) as Price is more important than Color). The rule of calculation for Tournament is that when the sum of tendency to agree (includes Strongly Agree and Agree) for the comparison attribute is greater than the sum of tendency to disagree (includes Strongly Disagree and Disagree), we will put "1" to the category of tendency to agree. Contrary to the case that category of tendency to disagree put "-1" to the category of tendency to disagree. We take the response category of Neither agree nor disagree as "neural" and put "0" to this category.

Table 5--Response Matrix for Ordinal Gower Plot

	Price	Color	Shape	Function	Disposition	Brand	Purpose
Price	0	1	1	-1	-1	-1	-1
Color	-1	0	-1	-1	-1	-1	-1
Shape	-1	1	0	-1	-1	-1	-1
Function	1	1	1	0	1	1	1
Disposition	1	1	1	-1	0	1	-1
Brand	1	1	1	-1	-1	0	-1
Purpose	1	1	1	-1	1	1	0

The following we will describe how to obtain the response matrix for Cardinal Gower plot. As the mentioned previously, we calculate the data of cardinal plot by using "Geometric mean" as a meaningful measures of average because the attributes are measured on a ratio scale. We take the value of " Price is important than Color" in Table 6 as the example. The value is 2.338 ($\sqrt[40]{0.25^0 \times 0.5^5 \times 1^1 \times 2^{14} \times 4^{20}}$ =2.34).

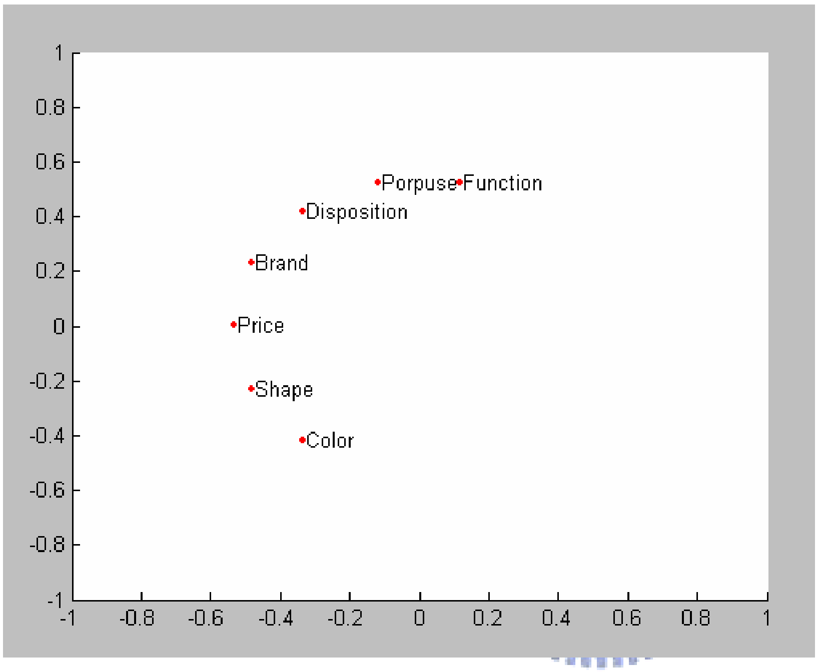
Notice that the value of Price vs. Price in Table 5 is 1, because this is skew-symmetric matrix. The Response Matrix for Cardinal Gower Plot is an $n \times n$ response matrix with entries $r_{jk} > 0$ satisfying $r_{kj} = 1/r_{jk}$ for all $1 \leq j,k \leq n$.

Table 6 -- Response Matrix for Cardinal Gower Plot

	Price	Color	Shape	Function	Disposition	Brand	Purpose
Price	1	2.34	1.15	0.71	1.07	0.93	0.60
Color	0.43	1	0.73	0.45	0.63	0.47	0.50
Shape	0.87	1.37	1	0.68	0.90	0.73	0.52
Function	1.41	2.22	1.46	1	2.22	2.18	1.49
Disposition	0.93	1.60	1.11	0.45	1	1.13	0.78
Brand	1.07	2.11	1.37	0.46	0.89	1	0.73
Purpose	1.65	2.00	1.93	0.67	1.27	1.37	1

The Ordinal Gower Plot of Overall condition displays as Figure 2. We can inferred the ranking can from the ordinal Gower plot of Figure 2, the preference ordering would thus seem to be Function> Purpose > Disposition > Brand > Price > Shape > Color. That is, customers will focus their attention on Function of car. Marketing people would propose their marketing plan according to this preference ranking and develop a vehicle, which focuses on function improvement or innovation. On the other hand, marketing people can also find the market position of their own product and draft a proper marketing plan for product promotion.

Figure 2-- Ordinal Gower Plot of Overall condition



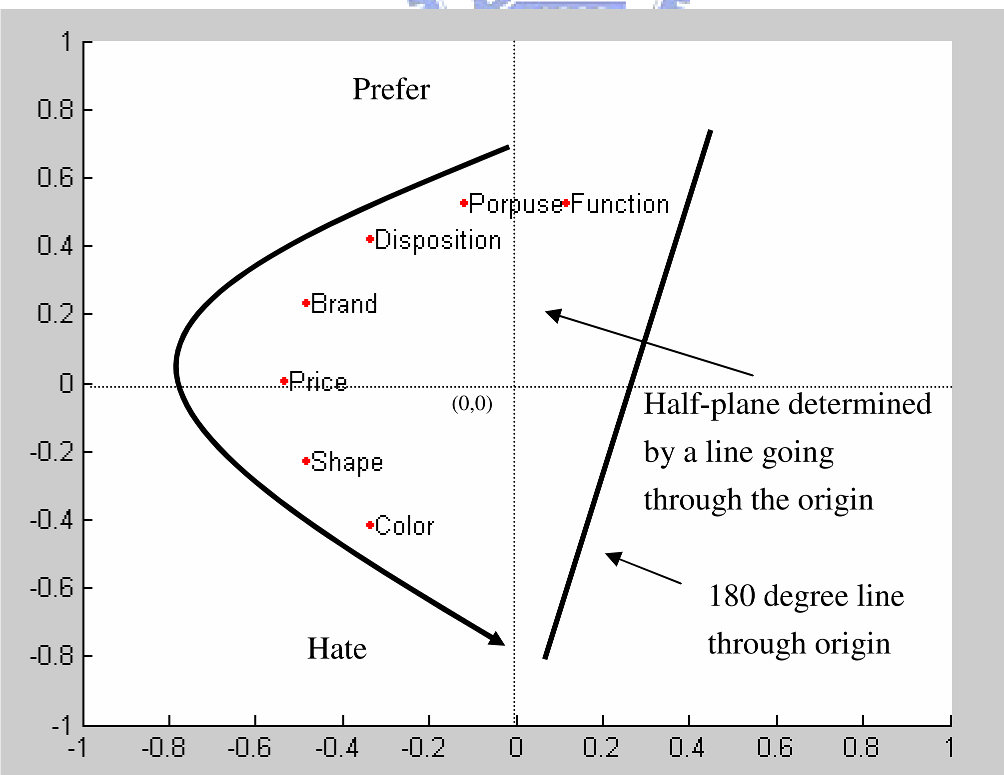
As the result of Figure 2-- Ordinal Gower Plot of Overall condition, we know that the preference ordering would thus seem to be Function> Purpose > Disposition > Brand > Price > Shape > Color. However, while we check the cardinal ranking of Brand, Price and Disposition in Figure 4-Cardinal Gower plot of overall condition, we can find that the intensity of those three preferences turn into a ambiguous status. As we have mentioned in the previous article, the data will lack of collinearity in the display while cardinal inconsistency is not hold for which the identity $r_{jl} = r_{jk} \times r_{kl}$. We can take this as a rough indication of cardinal inconsistency in the respondent's judgment. Although, it's lack of cardinal consistency, we still can take the result of ordinal Gower plot. In this case, we have found that ordinal Gower Plot has a great shape to support the result of preference ranking of the market: Function> Purpose > Disposition > Brand > Price > Shape > Color. Then, we can learn the specific meaning of those preferences in ambiguous status from cardinal Gower Plot. That is, there is only few data points to support the preference ranking of Disposition > Brand

> Price. It means that the preference (ex: Disposition win Brand) win by a narrow edge in Tournament. We can brief that lack of cardinal consistency is because the data points of one attribute would not exactly to be proportional to the data points of the other attribute.

The result of lack of cardinal consistency did mean something to marketing but not much. As everyone knows, a company designs the product or provides the service is to satisfy the great majority of customers in the market. There is always exception. Therefore, ordinal Gower plot provides the general information of the market and cardinal Gower plot provides the specific information to check the detailing. Market people could make the judgment for their marketing plan while they have learned the information. They can focus on those preferences (ex: Function and Purpose) that have high degree of intention and ignore those preferences that are in ambiguous status for product design. Or, they can pay attention to the ambiguous preference but not much and make some change in their marketing plan.

Whatever the actions the marketing people take, the actions are according to the market status. Ordinal and cardinal Gower plots did provide the solid result of the market status for sure.

Figure 3- Diagrammatic explanation to Ordinal Gower plot of overall condition



As we describe Figure3, the points are located on a circle centered at the origin and arranged counterclockwise in order of preference; the angle between two consecutive points is then equal to $180/n$ degrees, so that all the points lie in a half-plane determined by a line going through the origin.

The following we will display how we turn Table5 Matrix into Figure 2 Ordinal Gower Plot:

```
r=[ 0    1    1   -1   -1   -1   -1
 -1      0   -1   -1   -1   -1   -1
    -1   1    0   -1   -1   -1   -1
     1   1    1    0    1    1    1
     1   1    1   -1    0    1   -1
     1    1   1   -1   -1    0   -1
     1   1    1   -1    1    1    0]
a=r
[u,s,v]=svd(a);
```

The “svd” is singular value decomposition.

The data for X-axis from [u]=U

[-0.5345
-0.3333
-0.4816
0.1189
-0.3333
-0.4816
-0.1189]

The data for Y-axis from [v]=V

[0.0000
-0.4179
-0.2319
0.5211
0.4179
0.2319
0.5211]

Figure 4-Cardinal Gower plot of overall condition

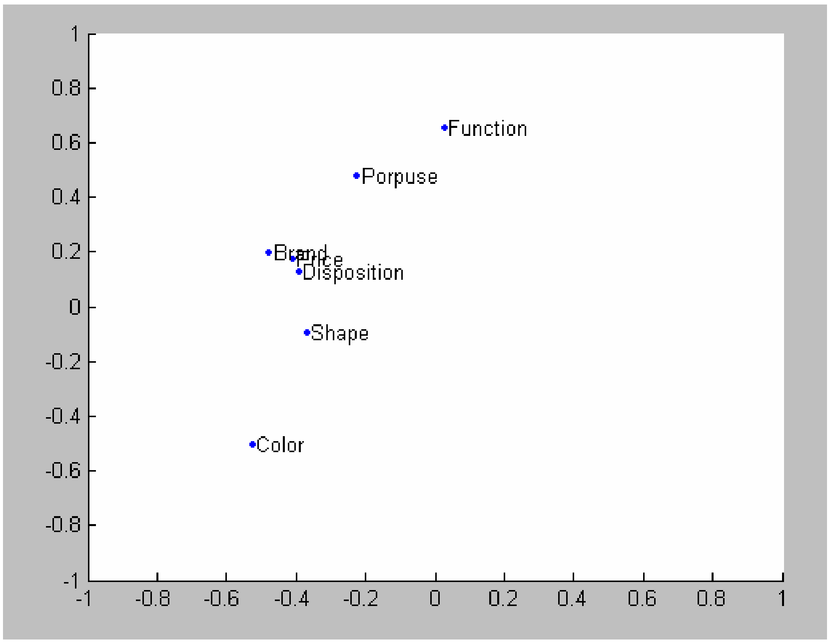
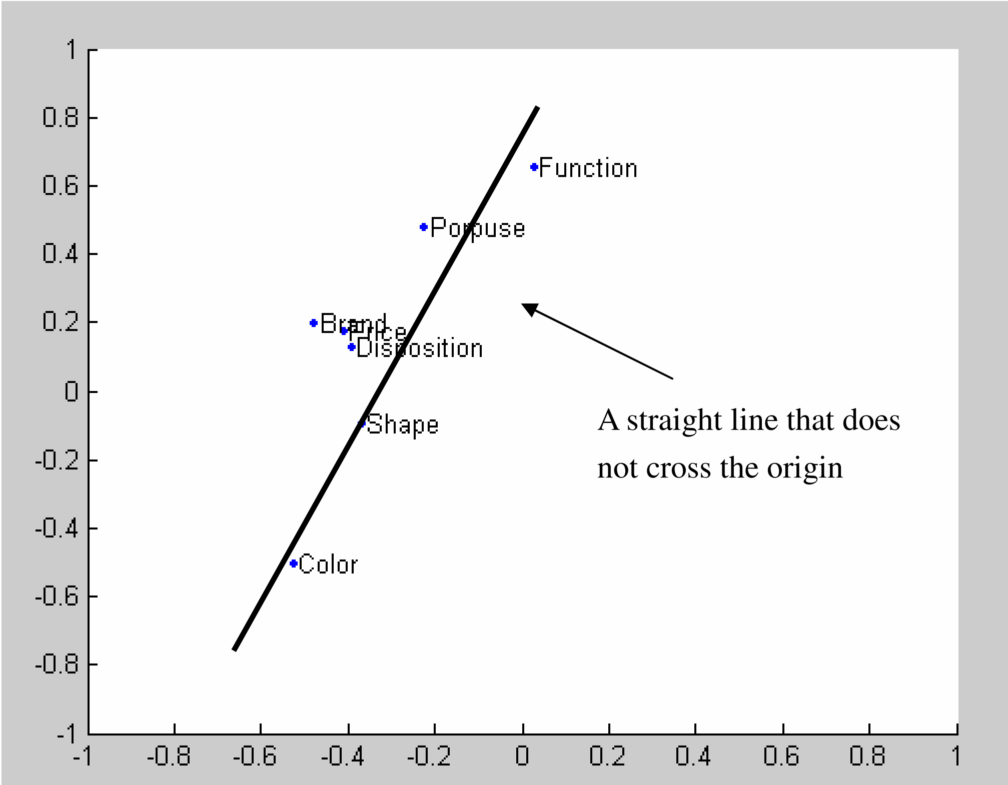


Figure 5 is a diagrammatic explanation that is base on the previous description of the method to judge if the cardinal Gower plot is cardinal consistency. When the agent's responses are cardinally consistent, the construction yields a set of points on a straight line that does not cross the origin.

Figure 5- Diagrammatic explanation to Cardinal Gower plot of overall condition



The following we will display how we turn Table6 Matrix into Figure 4 Cardinal Gower Plot:

r=[1	2.34	1.15	0.71	1.07	0.93	0.60;
0.43	1	0.73	0.45	0.63	0.47	0.50;
0.87	1.37	1	0.68	0.90	0.73	0.52;
1.41	2.22	1.46	1	2.22	2.18	1.49;
0.93	1.60	1.11	0.45	1	1.13	0.78;
1.07	2.11	1.37	0.46	0.89	1	0.73;
1.65	2.00	1.93	0.67	1.27	1.37	1;]

a=log(r)

[u,s,v]=svd(a); The "svd" is singular value decomposition.

The data for X-axis from [u]=U

[-0.4074

-0.5223

-0.3656

0.0280

-0.3899
-0.4751
-0.2213]

The data for Y-axis from [v]=V
[0.1723
-0.5037
-0.0970
0.6534
0.1235
0.1976
0.4753]

4.2 Response data sheet (Breakdown in certain criteria)

The following we will introduce the Gower plot by data breakdown in certain criteria: Age, Sex and intention to buy a car within 2 years.

In previous paragraph, we show the power of Gower plot in overall market status analysis. We hereby demonstrate the explanation of Gower plot for some certain group of customers.

The calculation method is same as §4.1. The difference between §4.1 and §4.2 is that we should classify the data with the certain criteria and then calculate the classified data to come out the ordinal and cardinal Gower plots.

1. Table 7- Response data sheet of Age below 20

Reponse Category	Strongly disagree:	Disagree:	Neither agree nor disagree:	Agree:	Strongly Agree:	Cardinal	Ordinal	Sample
Point	0.25	0.50	1.00	2.00	4.00	Geomean	Tournament	Size
Price is important than Color	0	0	0	1	0	2.000	1	1
Price is important than Shape	0	0	0	0	1	4.000	1	1
Price is important than Function	0	0	0	0	1	4.000	1	1
Price is important than Disposition	0	1	0	0	0	0.500	-1	1
Price is important than Brand	1	0	0	0	0	0.250	-1	1
Price is important than Purpose	0	0	0	0	1	4.000	1	1
Color is important than Shape	0	0	0	1	0	2.000	1	1
Color is important than Fuction	0	0	0	0	1	4.000	1	1
Color is important than Disposition	0	0	0	0	1	4.000	1	1
Color is important than Brand	0	1	0	0	0	0.500	-1	1
Color is important than Purpose	0	0	0	0	1	4.000	1	1
Shape is important than Function	0	0	0	1	0	2.000	1	1
Shape is important than Disposition	0	0	0	1	0	2.000	1	1
Shape is important than Brand	0	1	0	0	0	0.500	-1	1
Shape is important than Purpose	0	0	0	1	0	2.000	1	1
Function is important than Disposition	1	0	0	0	0	0.250	-1	1
Function is important than Brand	1	0	0	0	0	0.250	-1	1
Function is important than Purpose	1	0	0	0	0	0.250	-1	1
Disposition is important than Brand	0	1	0	0	0	0.500	-1	1
Disposition is important than Purpose	0	1	0	0	0	0.500	-1	1
Brand is important than Purpose	0	0	0	1	0	2.000	1	1

2. Table 8- Response data sheet of Age between 21-30

Reponse Category	Strongly disagree:	Disagree:	Neither agree nor disagree:	Agree:	Strongly Agree:	Cardinal	Ordinal	Sample
Point	0.25	0.50	1.00	2.00	4.00	Geomean	Tournament	Size
Price is important than Color	5	6	0	0	0	0.365	-1	11
Price is important than Shape	0	5	2	4	0	0.939	-1	11
Price is important than Function	0	3	1	7	0	1.287	1	11
Price is important than Disposition	1	7	0	3	0	0.685	-1	11
Price is important than Brand	2	3	2	4	0	0.828	-1	11
Price is important than Purpose	1	1	0	9	0	1.459	1	11
Color is important than Shape	0	3	0	8	0	1.370	1	11
Color is important than Fuction	0	2	0	6	3	1.878	1	11
Color is important than Disposition	0	3	1	6	1	1.370	1	11
Color is important than Brand	0	2	1	5	3	1.763	1	11
Color is important than Purpose	0	3	0	5	3	1.656	1	11
Shape is important than Function	1	4	1	4	1	1.000	0	11
Shape is important than Disposition	1	5	1	4	0	0.828	-1	11
Shape is important than Brand	0	1	3	6	1	1.554	1	11
Shape is important than Purpose	1	1	0	6	3	1.763	1	11
Function is important than Disposition	5	5	0	1	0	0.414	-1	11
Function is important than Brand	4	4	1	2	0	0.533	-1	11
Function is important than Purpose	4	2	2	3	0	0.643	-1	11
Disposition is important than Brand	1	4	1	5	0	0.939	0	11
Disposition is important than Purpose	0	3	2	6	0	1.208	1	11
Brand is important than Purpose	0	4	2	3	2	1.208	1	11

3. Table 9- Response data sheet of Age between 31-40

Reponse Category	Strongly disagree:	Disagree:	Neither agree nor disagree:	Agree:	Strongly Agree:	Cardinal	Ordinal	Sample
Point	0.25	0.50	1.00	2.00	4.00	Geomean	Tournament	Size
Price is important than Color	13	7	1	3	0	0.420	-1	24
Price is important than Shape	4	9	0	11	0	0.841	-1	24
Price is important than Function	0	5	1	15	3	1.587	1	24
Price is important than Disposition	3	5	1	15	0	1.122	1	24
Price is important than Brand	3	4	2	13	2	1.224	1	24
Price is important than Purpose	1	3	1	17	2	1.587	1	24
Color is important than Shape	2	4	4	13	1	1.224	1	24
Color is important than Fuction	0	2	1	14	7	2.119	1	24
Color is important than Disposition	1	3	1	14	5	1.731	1	24
Color is important than Brand	0	2	0	13	9	2.311	1	24
Color is important than Purpose	0	1	1	20	2	1.943	1	24
Shape is important than Function	0	4	1	16	3	1.682	1	24
Shape is important than Disposition	0	8	3	12	1	1.189	1	24
Shape is important than Brand	2	4	1	14	3	1.414	1	24
Shape is important than Purpose	0	2	1	19	2	1.834	1	24
Function is important than Disposition	10	8	1	5	0	0.515	-1	24
Function is important than Brand	10	9	1	4	0	0.486	-1	24
Function is important than Purpose	8	8	2	6	0	0.595	-1	24
Disposition is important than Brand	4	6	3	9	2	0.972	1	24
Disposition is important than Purpose	2	5	1	13	3	1.335	1	24
Brand is important than Purpose	0	9	3	7	5	1.260	1	24

4. Table 10- Response data sheet of Age between 41-50

Reponse Category	Strongly disagree:	Disagree:	Neither agree nor disagree:	Agree:	Strongly Agree:	Cardinal	Ordinal	Sample
Point	0.25	0.50	1.00	2.00	4.00	Geomean	Tournament	Size
Price is important than Color	2	1	0	1	0	0.500	-1	4
Price is important than Shape	1	2	0	1	0	0.595	-1	4
Price is important than Function	2	0	0	2	0	0.707	0	4
Price is important than Disposition	1	1	0	2	0	0.841	0	4
Price is important than Brand	1	0	0	2	1	1.414	1	4
Price is important than Purpose	0	0	0	3	1	2.378	1	4
Color is important than Shape	0	0	0	3	1	2.378	1	4
Color is important than Fuction	0	0	0	0	4	4.000	1	4
Color is important than Disposition	1	0	1	1	1	1.189	1	4
Color is important than Brand	0	0	0	2	2	2.828	1	4
Color is important than Purpose	0	0	0	1	3	3.364	1	4
Shape is important than Function	1	0	0	1	2	1.682	1	4
Shape is important than Disposition	0	1	1	1	1	1.414	1	4
Shape is important than Brand	1	1	0	1	1	1.000	0	4
Shape is important than Purpose	0	0	0	1	3	3.364	1	4
Function is important than Disposition	3	1	0	0	0	0.297	-1	4
Function is important than Brand	4	0	0	0	0	0.250	-1	4
Function is important than Purpose	0	0	0	4	0	2.000	1	4
Disposition is important than Brand	2	1	0	1	0	0.500	-1	4
Disposition is important than Purpose	1	0	0	2	1	1.414	1	4
Brand is important than Purpose	0	0	0	2	2	2.828	1	4

5. Table 11- Response data sheet of Female

Reponse Category	Strongly disagree:	Disagree:	Neither agree nor disagree:	Agree:	Strongly Agree:	Cardinal	Ordinal	Sample
Point	0.25	0.50	1.00	2.00	4.00	Geomean	Tournament	Size
Price is important than Color	11	5	0	1	0	0.346	-1	17
Price is important than Shape	3	6	1	7	0	0.816	-1	17
Price is important than Function	2	3	1	9	2	1.277	1	17
Price is important than Disposition	3	5	0	9	0	0.922	1	17
Price is important than Brand	3	3	2	7	2	1.085	1	17
Price is important than Purpose	1	2	0	13	1	1.566	1	17
Color is important than Shape	1	3	2	10	1	1.330	1	17
Color is important than Fuction	0	1	1	8	7	2.354	1	17
Color is important than Disposition	2	3	0	7	5	1.503	1	17
Color is important than Brand	0	3	0	6	8	2.170	1	17
Color is important than Purpose	0	2	0	12	3	1.920	1	17
Shape is important than Function	2	3	1	8	3	1.330	1	17
Shape is important than Disposition	1	4	4	8	0	1.085	1	17
Shape is important than Brand	2	4	2	6	3	1.177	1	17
Shape is important than Purpose	1	1	0	12	3	1.843	1	17
Function is important than Disposition	10	6	1	0	0	0.346	-1	17
Function is important than Brand	8	7	1	1	0	0.408	-1	17
Function is important than Purpose	6	5	3	3	0	0.565	-1	17
Disposition is important than Brand	3	4	3	6	1	0.922	0	17
Disposition is important than Purpose	0	3	2	10	2	1.566	1	17
Brand is important than Purpose	0	3	5	6	3	1.443	1	17

6. Table 12- Response data sheet of Male

Reponse Category	Strongly disagree:	Disagree:	Neither agree nor disagree:	Agree:	Strongly Agree:	Cardinal	Ordinal	Sample
Point	0.25	0.50	1.00	2.00	4.00	Geomean	Tournament	Size
Price is important than Color	9	9	1	4	0	0.500	-1	23
Price is important than Shape	2	10	1	9	1	0.914	-1	23
Price is important than Function	0	5	1	15	2	1.525	1	23
Price is important than Disposition	2	9	1	11	0	0.942	0	23
Price is important than Brand	4	4	2	12	1	1.062	1	23
Price is important than Purpose	1	2	1	16	3	1.720	1	23
Color is important than Shape	1	4	2	15	1	1.393	1	23
Color is important than Fuction	0	3	0	12	8	2.124	1	23
Color is important than Disposition	0	3	3	14	3	1.669	1	23
Color is important than Brand	0	2	1	14	6	2.061	1	23
Color is important than Purpose	0	2	1	14	6	2.061	1	23
Shape is important than Function	0	5	1	14	3	1.572	1	23
Shape is important than Disposition	0	10	1	10	2	1.128	1	23
Shape is important than Brand	1	3	2	15	2	1.525	1	23
Shape is important than Purpose	0	2	1	15	5	2.000	1	23
Function is important than Disposition	9	8	0	6	0	0.547	-1	23
Function is important than Brand	11	6	1	5	0	0.500	-1	23
Function is important than Purpose	7	5	1	10	0	0.762	-1	23
Disposition is important than Brand	4	8	1	9	1	0.860	-1	23
Disposition is important than Purpose	3	6	1	11	2	1.095	1	23
Brand is important than Purpose	0	10	0	7	6	1.312	1	23

7. Table 13- Response data sheet of Plan to buy a car within 2 year

Reponse Category	Strongly disagree:	Disagree:	Neither agree nor disagree:	Agree:	Strongly Agree:	Cardinal	Ordinal	Sample
Point	0.25	0.50	1.00	2.00	4.00	Geomean	Tournament	Size
Price is important than Color	10	10	1	3	0	0.459	-1	24
Price is important than Shape	2	10	1	11	0	0.917	-1	24
Price is important than Function	1	5	1	15	2	1.414	1	24
Price is important than Disposition	2	6	1	15	0	1.155	1	24
Price is important than Brand	3	6	1	11	3	1.155	1	24
Price is important than Purpose	2	2	1	16	3	1.587	1	24
Color is important than Shape	1	4	4	14	1	1.335	1	24
Color is important than Fuction	0	4	1	11	8	1.943	1	24
Color is important than Disposition	2	4	2	14	2	1.335	1	24
Color is important than Brand	0	2	0	13	9	2.311	1	24
Color is important than Purpose	0	3	1	16	4	1.834	1	24
Shape is important than Function	1	6	2	12	3	1.335	1	24
Shape is important than Disposition	0	12	0	10	2	1.059	0	24
Shape is important than Brand	2	3	2	14	3	1.456	1	24
Shape is important than Purpose	0	2	1	17	4	1.943	1	24
Function is important than Disposition	10	10	1	3	0	0.459	-1	24
Function is important than Brand	10	8	1	5	0	0.515	-1	24
Function is important than Purpose	7	6	1	10	0	0.749	-1	24
Disposition is important than Brand	6	4	4	9	1	0.866	0	24
Disposition is important than Purpose	3	4	1	14	2	1.260	1	24
Brand is important than Purpose	0	8	4	7	5	1.297	1	24

8. Table 14- Response data sheet of No plan to buy a car within 2 year

Reponse Category	Strongly disagree:	Disagree:	Neither agree nor disagree:	Agree:	Strongly Agree:	Cardinal	Ordinal	Sample
Point	0.25	0.50	1.00	2.00	4.00	Geomean	Tournament	Size
Price is important than Color	10	4	0	2	0	0.386	-1	16
Price is important than Shape	3	6	1	5	1	0.805	-1	16
Price is important than Function	1	3	1	9	2	1.414	1	16
Price is important than Disposition	3	8	0	5	0	0.677	-1	16
Price is important than Brand	4	1	3	8	0	0.958	1	16
Price is important than Purpose	0	2	0	13	1	1.756	1	16
Color is important than Shape	1	3	0	11	1	1.414	1	16
Color is important than Fuction	0	0	0	9	7	2.709	1	16
Color is important than Disposition	0	2	1	7	6	2.089	1	16
Color is important than Brand	0	3	1	7	5	1.834	1	16
Color is important than Purpose	0	1	0	10	5	2.278	1	16
Shape is important than Function	1	2	0	10	3	1.682	1	16
Shape is important than Disposition	1	2	5	8	0	1.189	1	16
Shape is important than Brand	1	4	2	7	2	1.242	1	16
Shape is important than Purpose	1	1	0	10	4	1.915	1	16
Function is important than Disposition	9	4	0	3	0	0.439	-1	16
Function is important than Brand	9	5	1	1	0	0.386	-1	16
Function is important than Purpose	6	4	3	3	0	0.569	-1	16
Disposition is important than Brand	1	8	0	6	1	0.917	-1	16
Disposition is important than Purpose	0	5	2	7	2	1.297	1	16
Brand is important than Purpose	0	5	1	6	4	1.477	1	16

Response Matrix for Ordinal Gower Plot

1. Table15- Response Matrix for Ordinal Gower Plot of Age below 20

	Price	Color	Shape	Function	Disposition	Brand	Purpose
Price	0	1	1	1	-1	-1	1
Color	-1	0	1	1	1	-1	1
Shape	-1	-1	0	1	1	-1	1
Function	-1	-1	-1	0	-1	-1	-1
Disposition	1	-1	-1	1	0	-1	-1
Brand	1	1	1	1	1	0	1
Purpose	-1	-1	-1	1	1	-1	0

2. Table16- Response Matrix for Ordinal Gower Plot of Age between 21-30

	Price	Color	Shape	Function	Disposition	Brand	Purpose
Price	0	-1	-1	1	-1	-1	1
Color	1	0	1	1	1	1	1
Shape	1	-1	0	0	-1	1	1
Function	-1	-1	0	0	-1	-1	-1
Disposition	1	-1	1	1	0	0	1
Brand	1	-1	-1	1	0	0	1
Purpose	-1	-1	-1	1	-1	-1	0

3. Table17- Response Matrix for Ordinal Gower Plot of Age between 31-40

	Price	Color	Shape	Function	Disposition	Brand	Purpose
Price	0	-1	-1	1	1	1	1
Color	1	0	1	1	1	1	1
Shape	1	-1	0	1	1	1	1
Function	-1	-1	-1	0	-1	-1	-1
Disposition	-1	-1	-1	1	0	1	1
Brand	-1	-1	-1	1	-1	0	1
Purpose	-1	-1	-1	1	-1	-1	0

4. Table18- Response Matrix for Ordinal Gower Plot of Age between 41-50

	Price	Color	Shape	Function	Disposition	Brand	Purpose
Price	0	-1	-1	0	0	1	1
Color	1	0	1	1	1	1	1
Shape	1	-1	0	1	1	0	1
Function	0	-1	-1	0	-1	-1	1
Disposition	0	-1	-1	1	0	-1	1
Brand	-1	-1	0	1	1	0	1
Purpose	-1	-1	-1	-1	-1	-1	0

5. Table19- Response Matrix for Ordinal Gower Plot of Female

	Price	Color	Shape	Function	Disposition	Brand	Purpose
Price	0	-1	-1	1	1	1	1
Color	1	0	1	1	1	1	1
Shape	1	-1	0	1	1	1	1
Function	-1	-1	-1	0	-1	-1	-1
Disposition	-1	-1	-1	1	0	0	1
Brand	-1	-1	-1	1	0	0	1
Purpose	-1	-1	-1	1	-1	-1	0

6. Table 20- Response Matrix for Ordinal Gower Plot of Male

	Price	Color	Shape	Function	Disposition	Brand	Purpose
Price	0	-1	-1	1	0	1	1
Color	1	0	1	1	1	1	1
Shape	1	-1	0	1	1	1	1
Function	-1	-1	-1	0	-1	-1	-1
Disposition	0	-1	-1	1	0	-1	1
Brand	-1	-1	-1	1	1	0	1
Purpose	-1	-1	-1	1	-1	-1	0

7. Table 21- Response Matrix for Ordinal Gower Plot of Plan to buy a car within 2 year

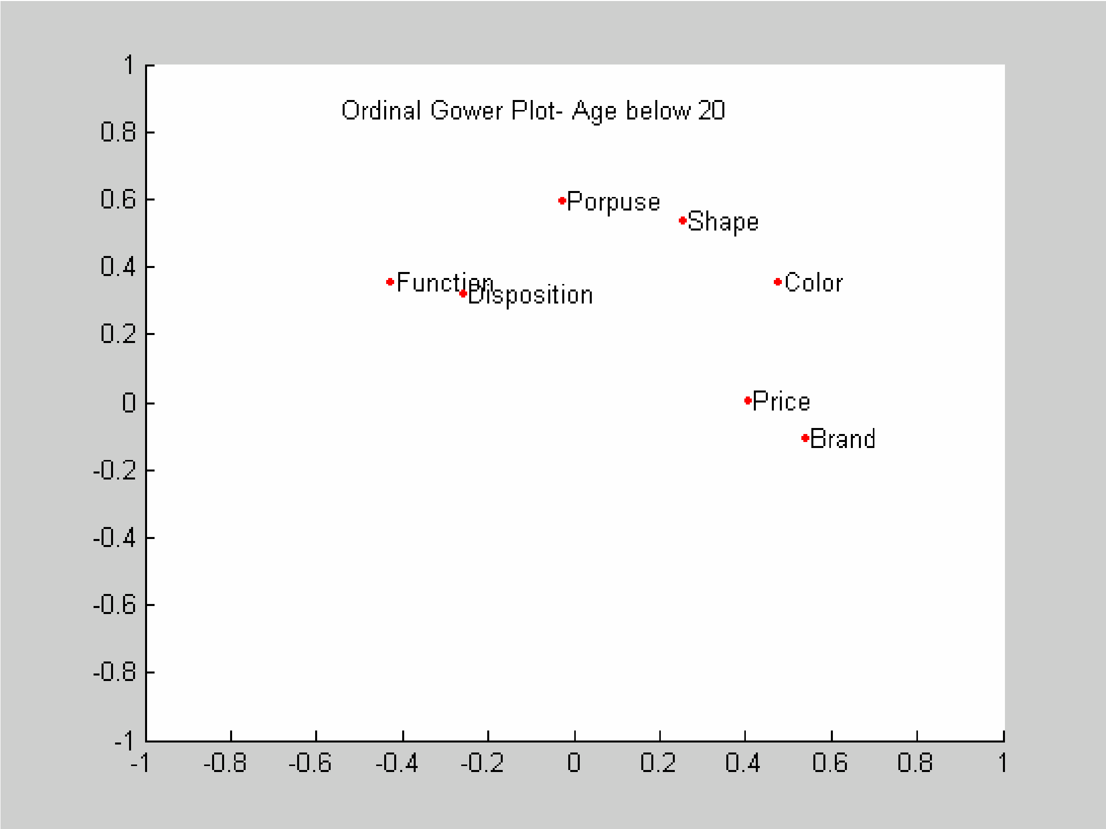
	Price	Color	Shape	Function	Disposition	Brand	Purpose
Price	0	-1	-1	1	1	1	1
Color	1	0	1	1	1	1	1
Shape	1	-1	0	1	0	1	1
Function	-1	-1	-1	0	-1	-1	-1
Disposition	-1	-1	0	1	0	0	1
Brand	-1	-1	-1	1	0	0	1
Purpose	-1	-1	-1	1	-1	-1	0

8. Table 22- Response Matrix for Ordinal Gower Plot of No plan to buy a car within 2 year

	Price	Color	Shape	Function	Disposition	Brand	Purpose
Price	0	-1	-1	1	-1	1	1
Color	1	0	1	1	1	1	1
Shape	1	-1	0	1	1	1	1
Function	-1	-1	-1	0	-1	-1	-1
Disposition	1	-1	-1	1	0	-1	1
Brand	-1	-1	-1	1	1	0	1
Purpose	-1	-1	-1	1	-1	-1	0

Ordinal Gower Plot

Figure 6- Ordinal Gower Plot of Age below 20



Comparing with the preference ranking of overall market status: Function> Purpose > Disposition > Brand > Price > Shape > Color, you may find easily the diversity of the preference ranking of the targeted respondent for age below 20: Function> Disposition > Purpose > Shape > Color > Price > Brand. You also may find that the data point of age below 20 is only one sample data. This is resulted from online-questionnaire. Since we use online-questionnaire as the medium of collecting data points, we hardly control the position distribution of respondents. In statistical sampling, one sample data will not provide any meaning of statistical inference to population though we still can come out its Gower plots.

We summarize the preference ranking of ordinal Gower plot as below:

Age 21-30: Color> Disposition > Shape > Brand > Price> Purpose > Function.

Age 31-40: Function> Purpose > Brand > Disposition > Price > Shape> Color.

Age 41-50: Purpose > Function> Disposition > Price> Brand > Shape> Color.

Female: Function> Purpose > Brand = Disposition > Price > Shape> Color.

Male: Function> Purpose > Disposition > Brand > Price > Shape> Color.

Buy a car in 2 years: Color> Shape> Price >Disposition> Brand >Purpose> Function.

Not buy a car in 2 years: Function > Purpose > Brand = Disposition = Price > Shape

> Color.

As the result of preference ranking above, we have found an interesting issue. Those customers who plan to buy a car within 2 years will pay more attention to color of cat. This situation is surprised us due to it differs from the result of overall situation, but it also tell us a very important information regarding the customer intention of car purchasing. That is, targeted customers do care about color when they do want to buy a car. We also dig out the some information that the young (under 30 years old) treats color as an important factor more than the elder.

Therefore, a vehicle company should focus the color of car when they would like to target their customers to the young.

Figure 7- Ordinal Gower Plot of Age 21-30

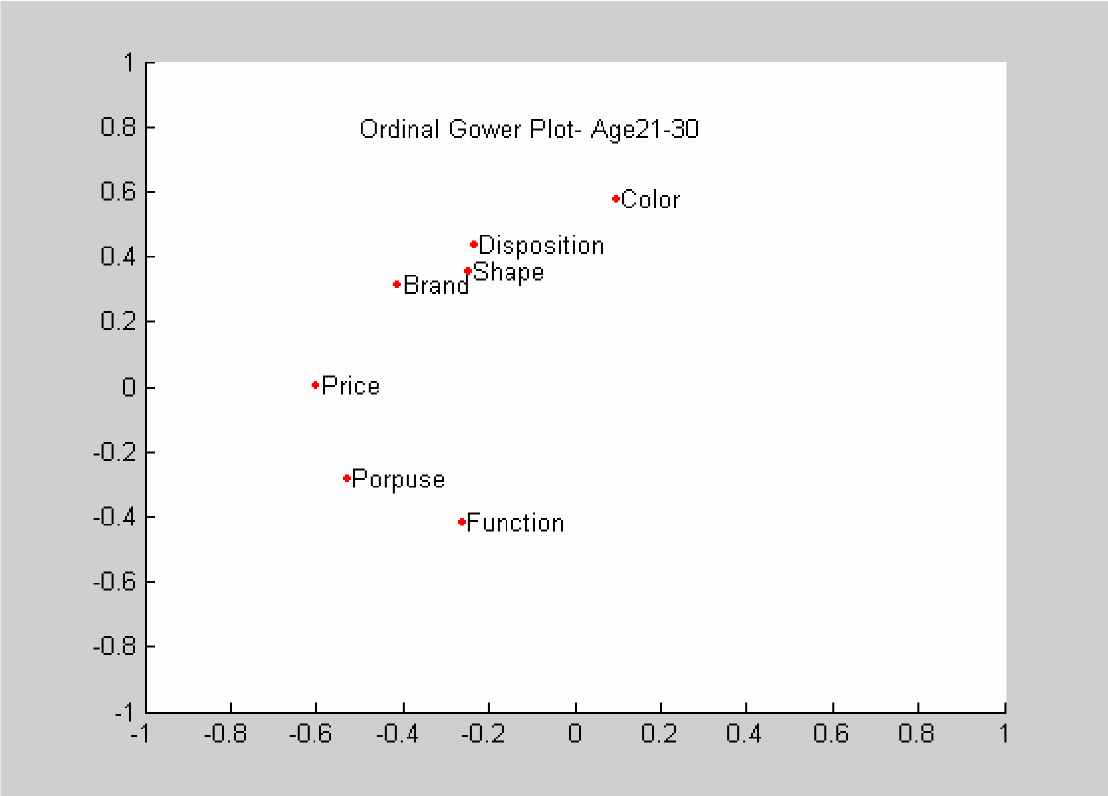


Figure 8- Ordinal Gower Plot of Age 31-40

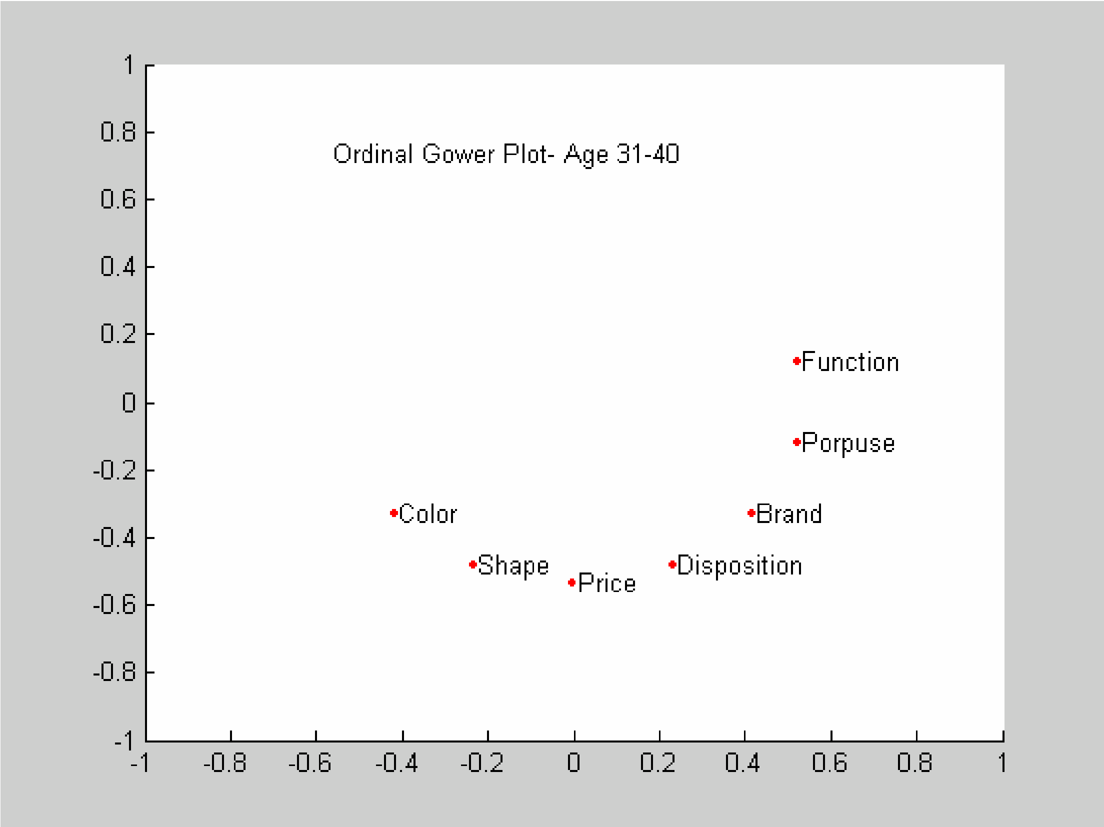


Figure 9- Ordinal Gower Plot of Age 41-50

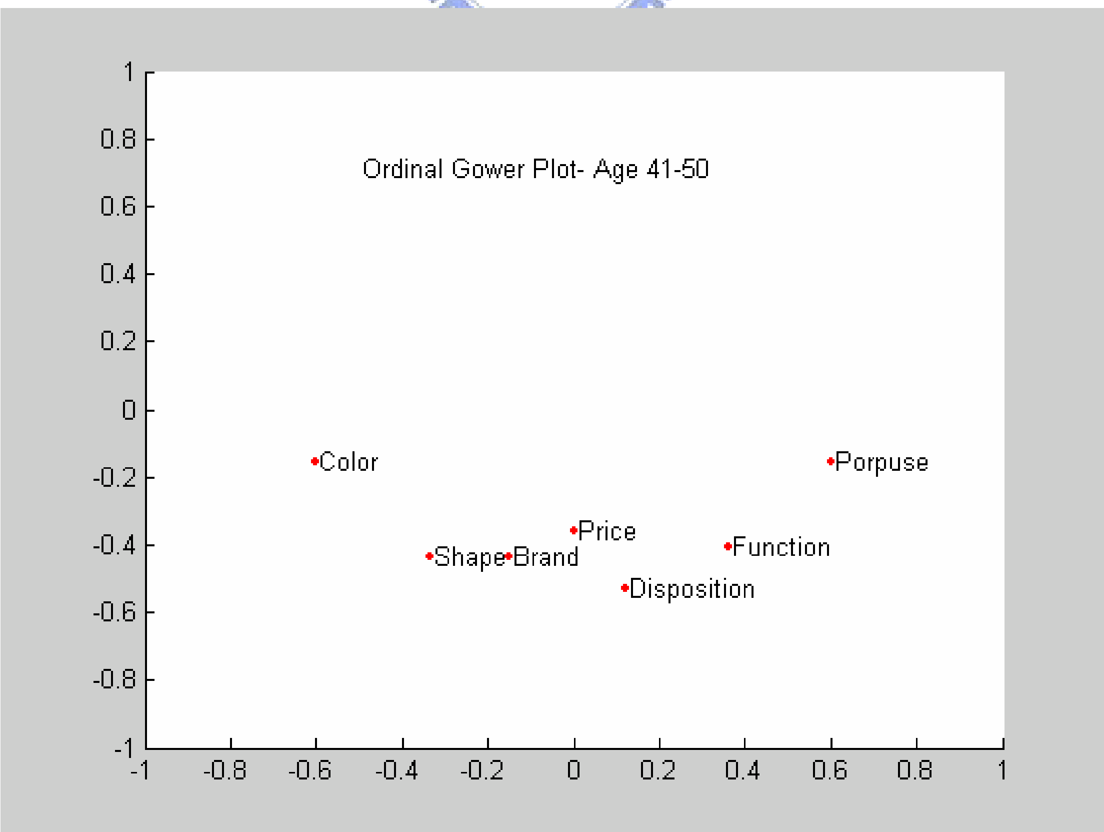


Figure 10- Ordinal Gower Plot of Female

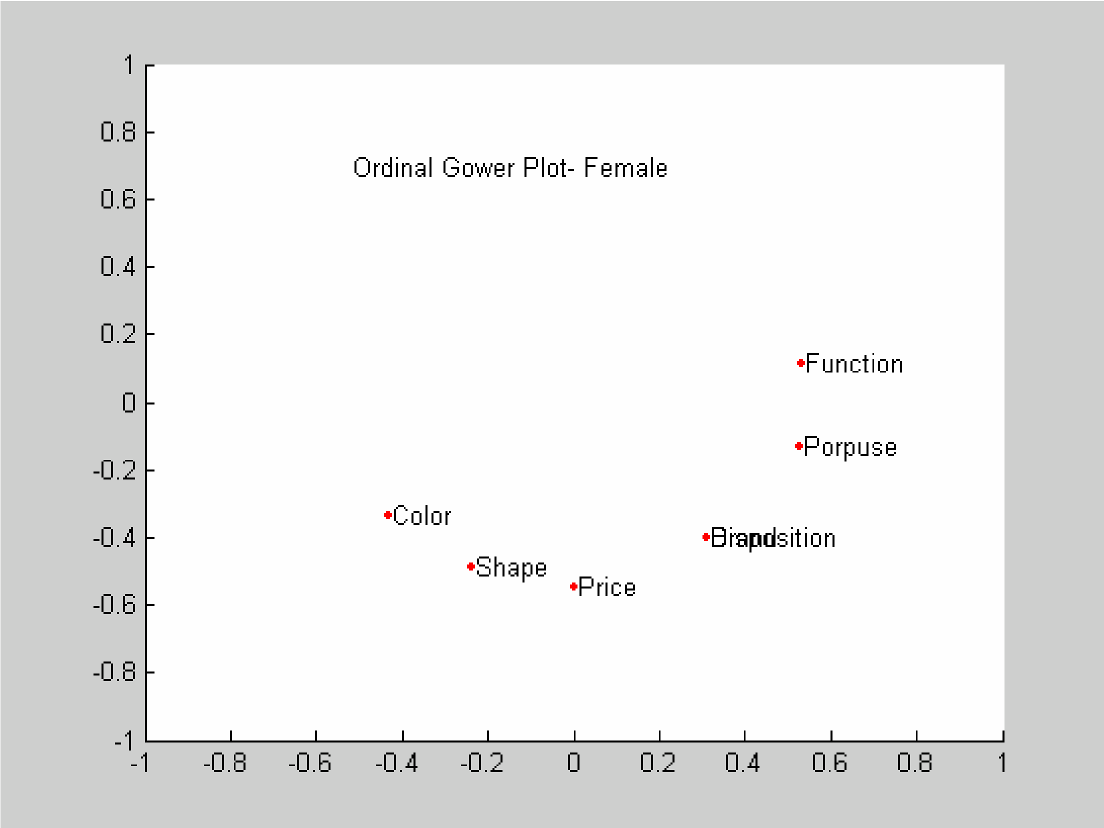


Figure 11- Ordinal Gower Plot of Male

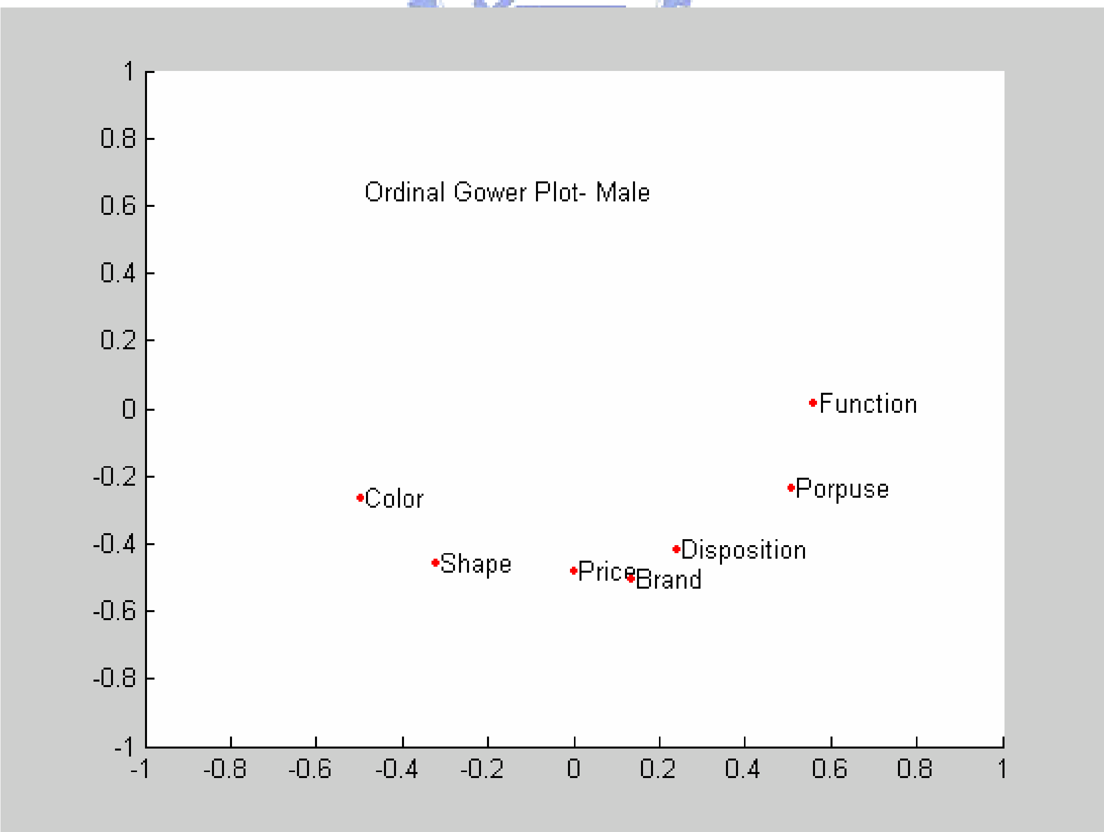


Figure 12- Ordinal Gower Plot of plan to buy a car within 2 years

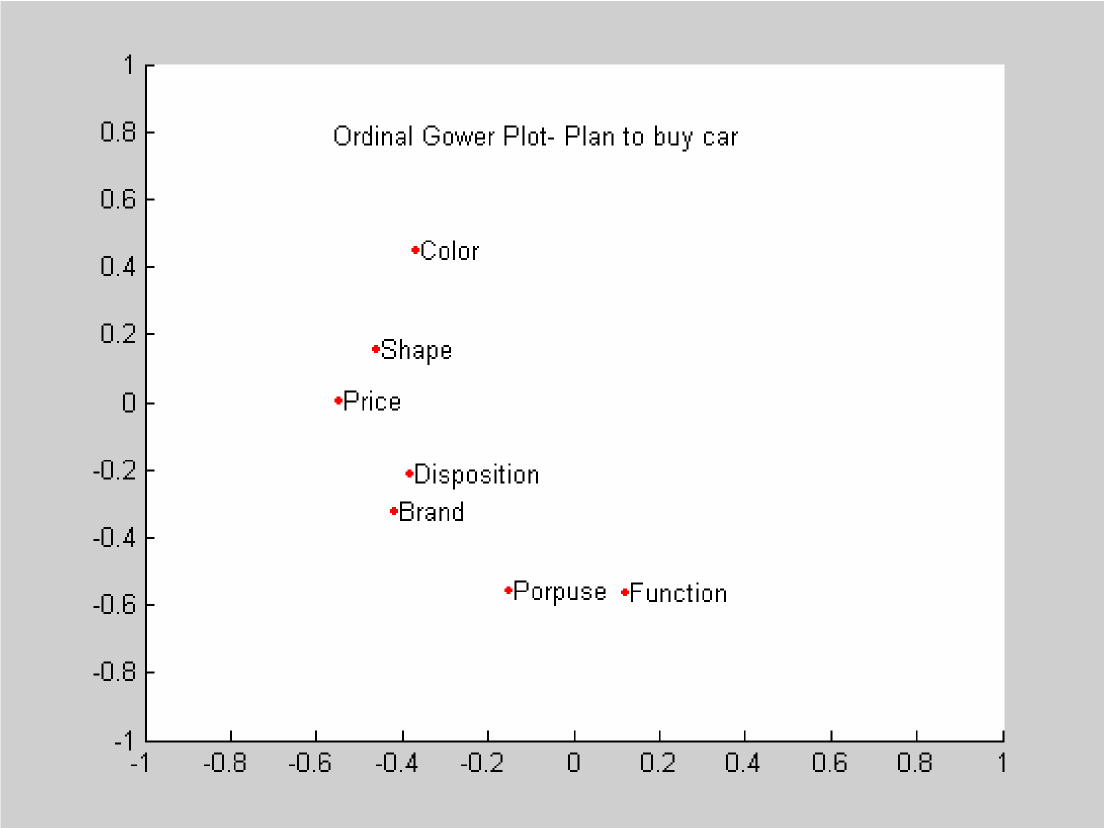
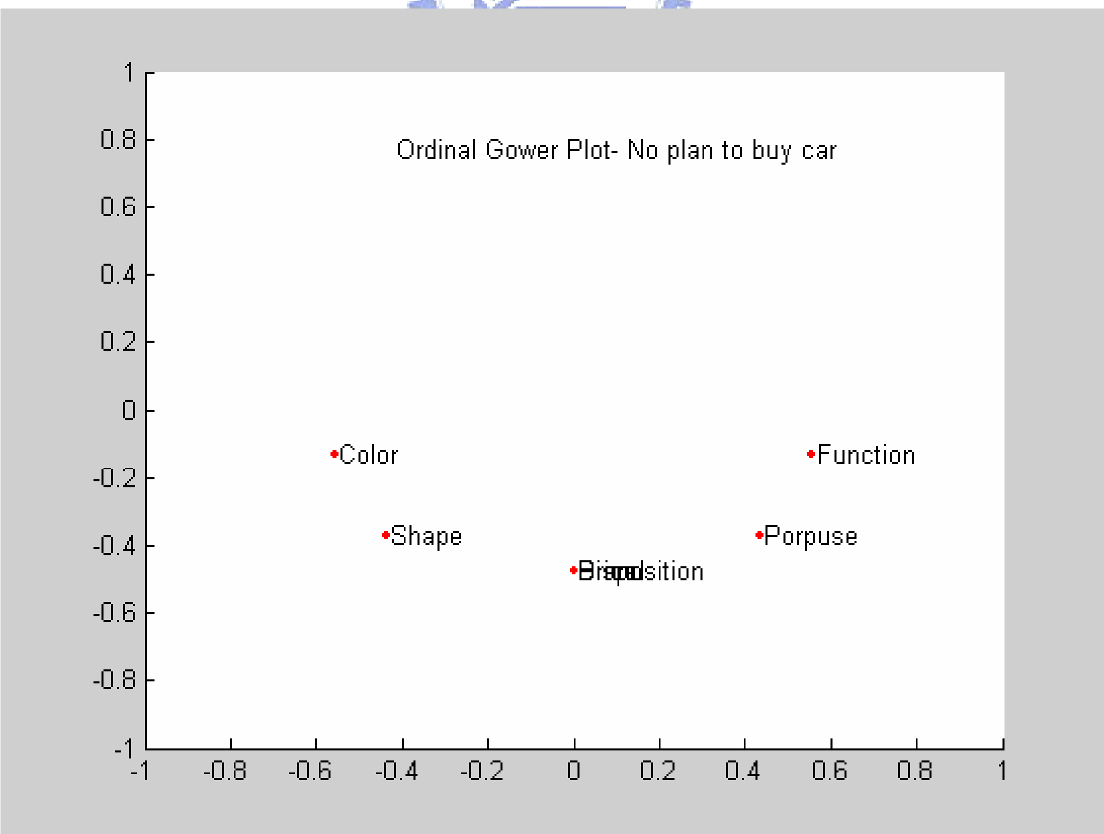


Figure 13- Ordinal Gower Plot of no plan to buy a car within 2 years



Response Matrix for Cardinal Gower Plot

1. Table 23- Response Matrix for Cardinal Gower Plot of Age below 20

	Price	Color	Shape	Function	Disposition	Brand	Purpose
Price	1	2.00	4.00	4.00	0.50	0.25	4.00
Color	0.50	1	2.00	4.00	4.00	0.50	4.00
Shape	0.25	0.50	1	2.00	2.00	0.50	2.00
Function	0.25	0.25	0.50	1	0.25	0.25	0.25
Disposition	2.00	0.25	0.50	4.00	1	0.50	0.50
Brand	4.00	2.00	2.00	4.00	2.00	1	2.00
Purpose	0.25	0.25	0.50	4.00	2.00	0.50	1

2. Table 24- Response Matrix for Cardinal Gower Plot of Age between 21-30

	Price	Color	Shape	Function	Disposition	Brand	Purpose
Price	1	0.36	0.94	1.29	0.69	0.83	1.46
Color	2.74	1	1.37	1.88	1.37	1.76	1.66
Shape	1.07	0.73	1	1.00	0.83	1.55	1.76
Function	0.78	0.53	1.00	1	0.41	0.53	0.64
Disposition	1.46	0.73	1.21	2.42	1	0.94	1.21
Brand	1.21	0.57	0.64	1.88	1.07	1	1.21
Purpose	0.69	0.60	0.57	1.55	0.83	0.83	1

3. Table 25- Response Matrix for Cardinal Gower Plot of Age between 31-40

	Price	Color	Shape	Function	Disposition	Brand	Purpose
Price	1	0.42	0.84	1.59	1.12	1.22	1.59
Color	2.38	1	1.22	2.12	1.73	2.31	1.94
Shape	1.19	0.82	1	1.68	1.19	1.41	1.83
Function	0.63	0.47	0.59	1	0.51	0.49	0.59
Disposition	0.89	0.58	0.84	1.94	1	0.97	1.33
Brand	0.82	0.43	0.71	2.06	1.03	1	1.26
Purpose	0.63	0.51	0.55	1.68	0.75	0.79	1

3. Table 26- Response Matrix for Cardinal Gower Plot of Age between 41-50

	Price	Color	Shape	Function	Disposition	Brand	Purpose
Price	1	0.50	0.59	0.71	0.84	1.41	2.38
Color	2.00	1	2.38	4.00	1.19	2.83	3.36
Shape	1.68	0.42	1	1.68	1.41	1.00	3.36
Function	1.41	0.25	0.59	1	0.30	0.25	2.00
Disposition	1.19	0.84	0.71	3.36	1	0.50	1.41
Brand	0.71	0.35	1.00	4.00	2.00	1	2.83
Purpose	0.42	0.30	0.30	0.50	0.71	0.35	1

5. Table 27- Response Matrix for Cardinal Gower Plot of Female

	Price	Color	Shape	Function	Disposition	Brand	Purpose
Price	1	0.35	0.82	1.28	0.92	1.08	1.57
Color	2.89	1	1.33	2.35	1.50	2.17	1.92
Shape	1.23	0.75	1	1.33	1.08	1.18	1.84
Function	0.78	0.42	0.75	1	0.35	0.41	0.57
Disposition	1.08	0.67	0.92	2.89	1	0.92	1.57
Brand	0.92	0.46	0.85	2.45	1.08	1	1.44
Purpose	0.64	0.52	0.54	1.77	0.64	0.69	1

6. Table 28- Response Matrix for Cardinal Gower Plot of Male

	Price	Color	Shape	Function	Disposition	Brand	Purpose
Price	1	0.50	0.91	1.52	0.94	1.06	1.72
Color	2.00	1	1.39	2.12	1.67	2.06	2.06
Shape	1.09	0.72	1	1.57	1.13	1.52	2.00
Function	0.66	0.47	0.64	1	0.55	0.50	0.76
Disposition	1.06	0.60	0.89	1.83	1	0.86	1.09
Brand	0.94	0.49	0.66	2.00	1.16	1	1.31
Purpose	0.58	0.49	0.50	1.31	0.91	0.76	1

7. Table 29- Response Matrix for Cardinal Gower Plot of Plan to buy a car within 2 year

	Price	Color	Shape	Function	Disposition	Brand	Purpose
Price	1	0.46	0.92	1.41	1.16	1.16	1.59
Color	2.18	1	1.33	1.94	1.33	2.31	1.83
Shape	1.09	0.75	1	1.33	1.06	1.46	1.94
Function	0.71	0.51	0.75	1	0.46	0.51	0.75
Disposition	0.87	0.75	0.94	2.18	1	0.87	1.26
Brand	0.87	0.43	0.69	1.94	1.16	1	1.30
Purpose	0.63	0.55	0.51	1.33	0.79	0.77	1

8. Table 30- Response Matrix for Cardinal Gower Plot of No plan to buy a car within 2 year

	Price	Color	Shape	Function	Disposition	Brand	Purpose
Price	1	0.39	0.81	1.41	0.68	0.96	1.76
Color	2.59	1	1.41	2.71	2.09	1.83	2.28
Shape	1.24	0.71	1	1.68	1.19	1.24	1.92
Function	0.71	0.37	0.59	1	0.44	0.39	0.57
Disposition	1.48	0.48	0.84	2.28	1	0.92	1.30
Brand	1.04	0.55	0.81	2.59	1.09	1	1.48
Purpose	0.57	0.44	0.52	1.76	0.77	0.68	1

Cardinal Gower Plot

Figure 14- Cardinal Gower Plot of Age below 20

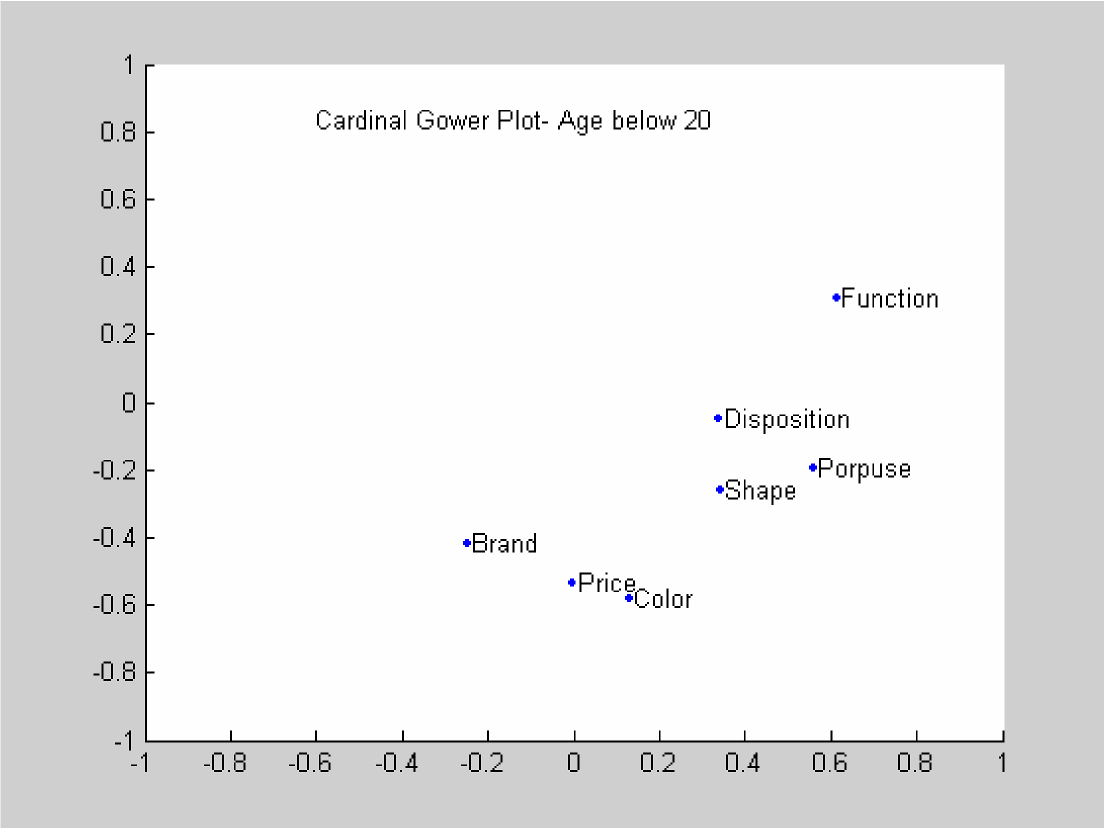


Figure 15- Cardinal Gower Plot of Age21-30

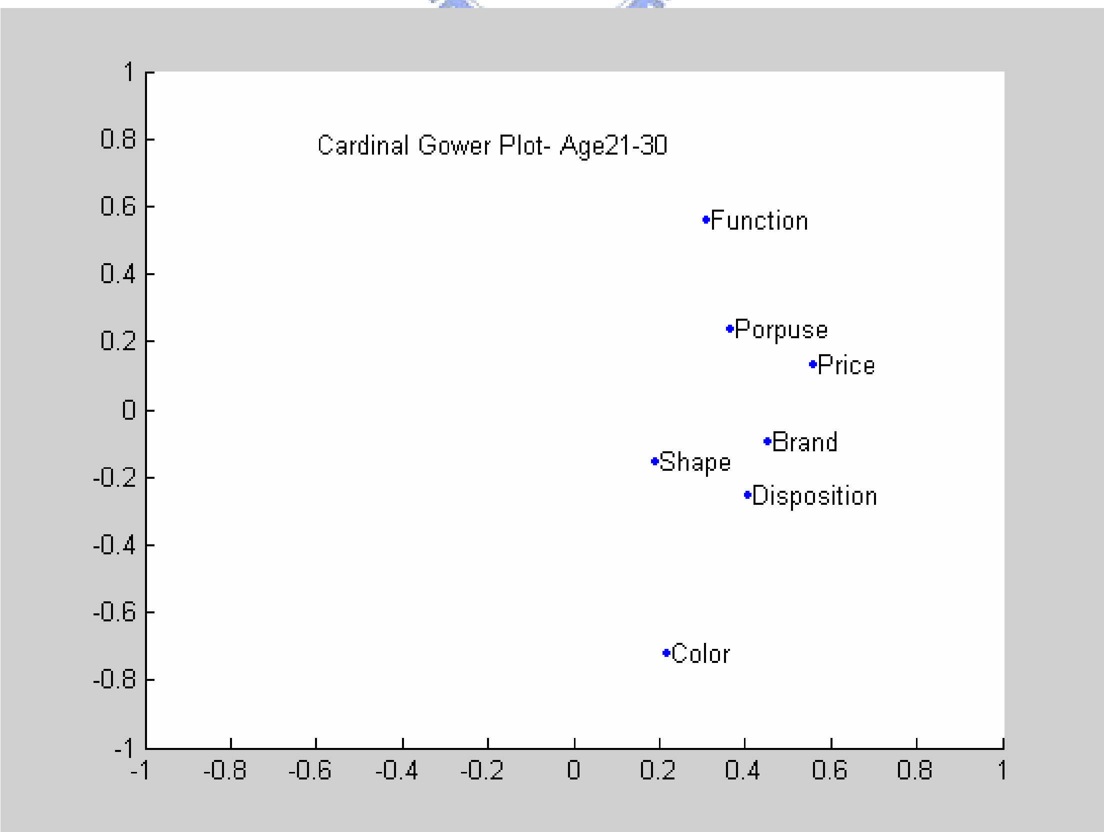


Figure 16- Cardinal Gower Plot of Age 31-40

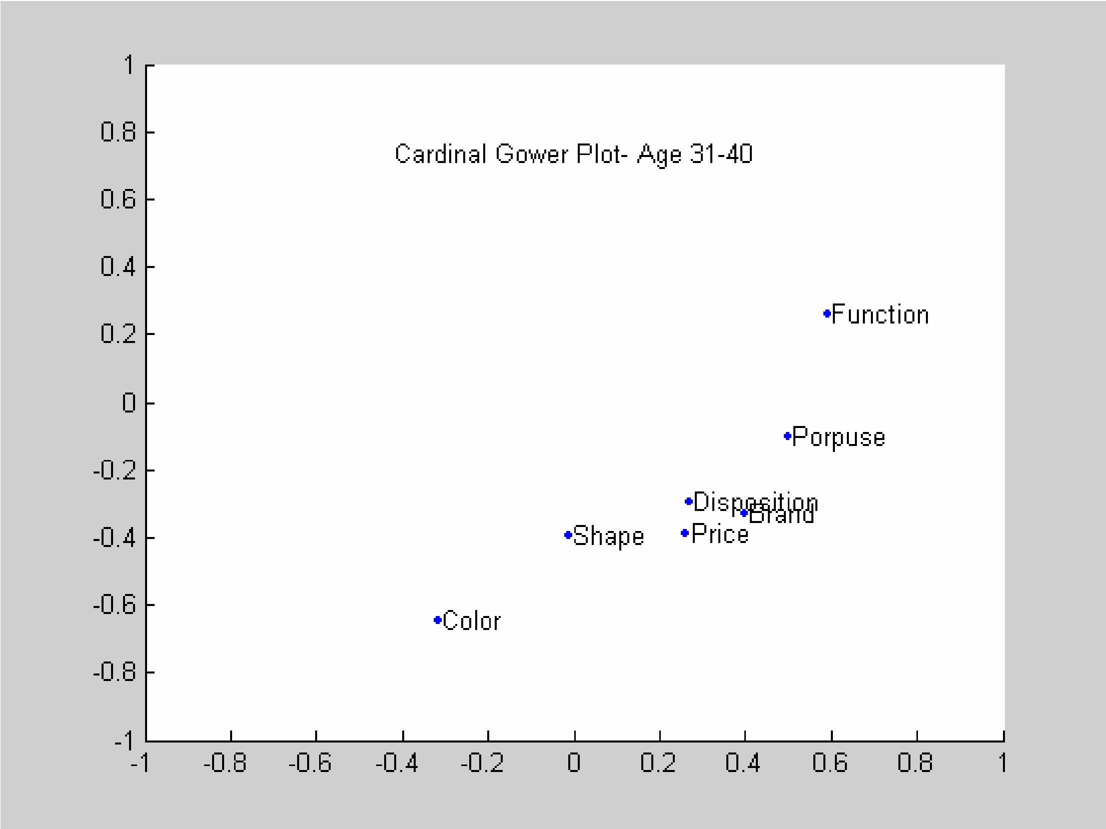


Figure 17- Cardinal Gower Plot of Age41-50

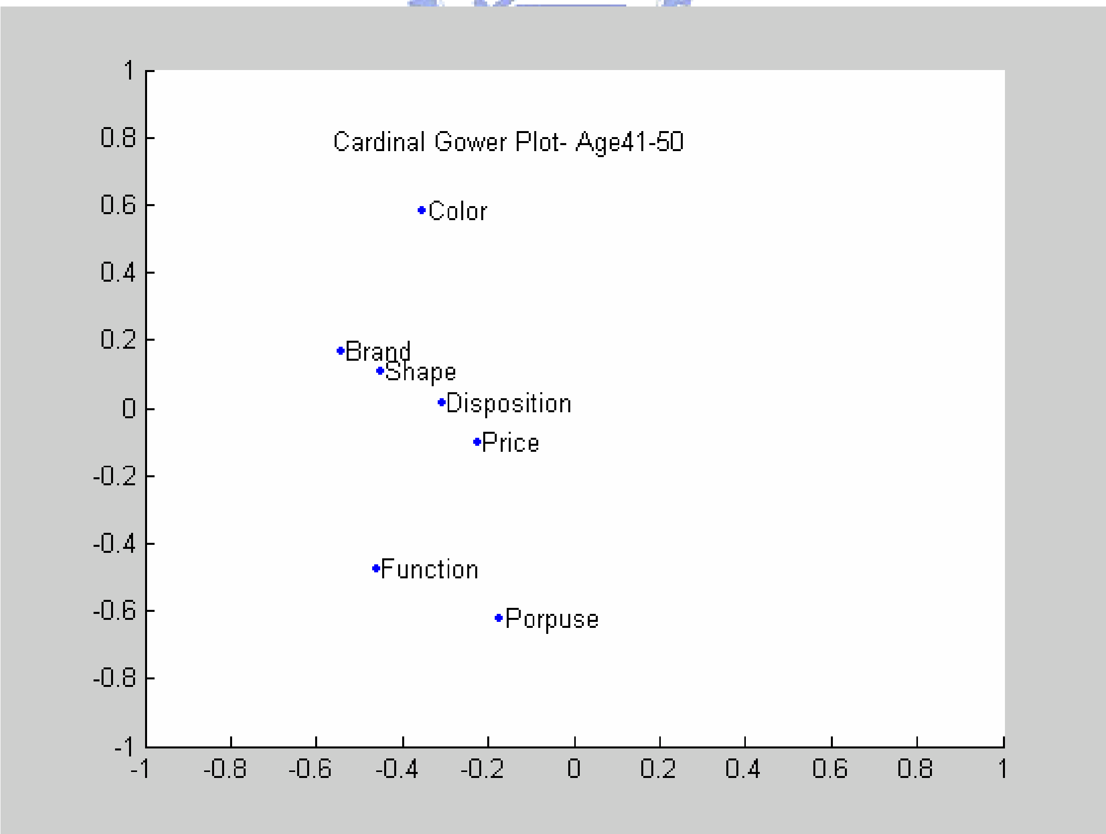


Figure 18- Cardinal Gower Plot of Female

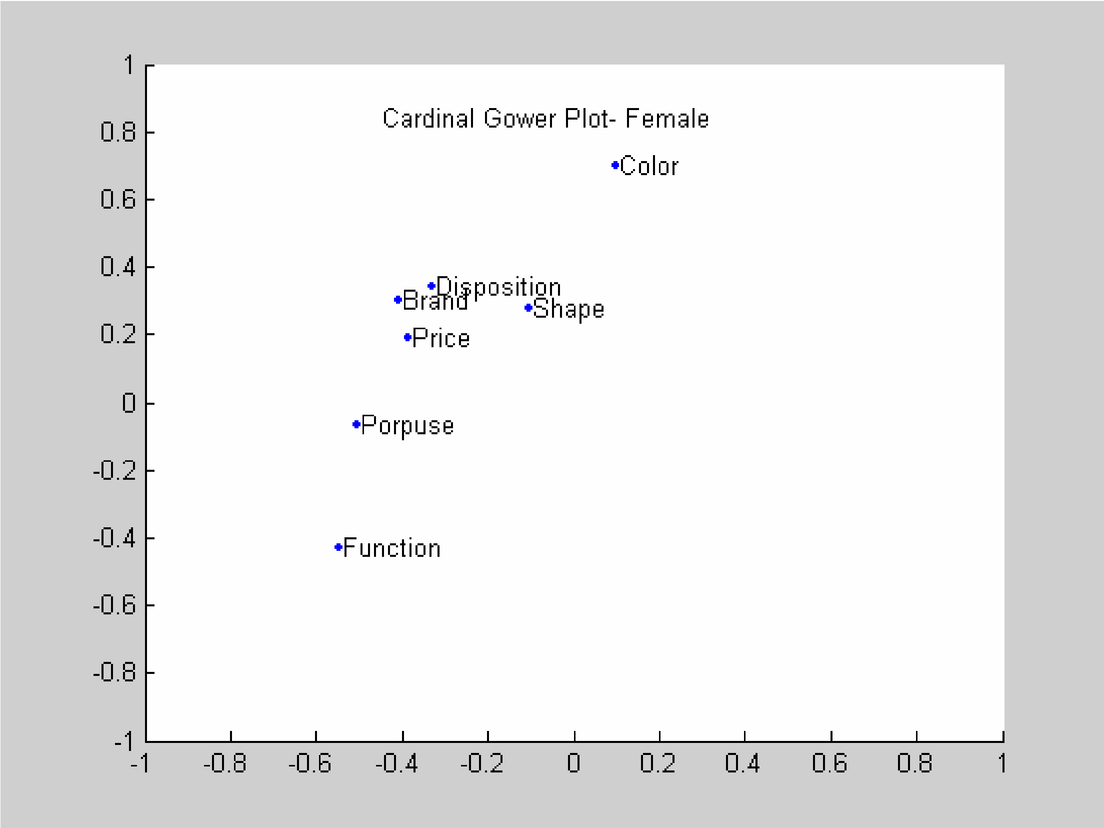


Figure 19- Cardinal Gower Plot of Male

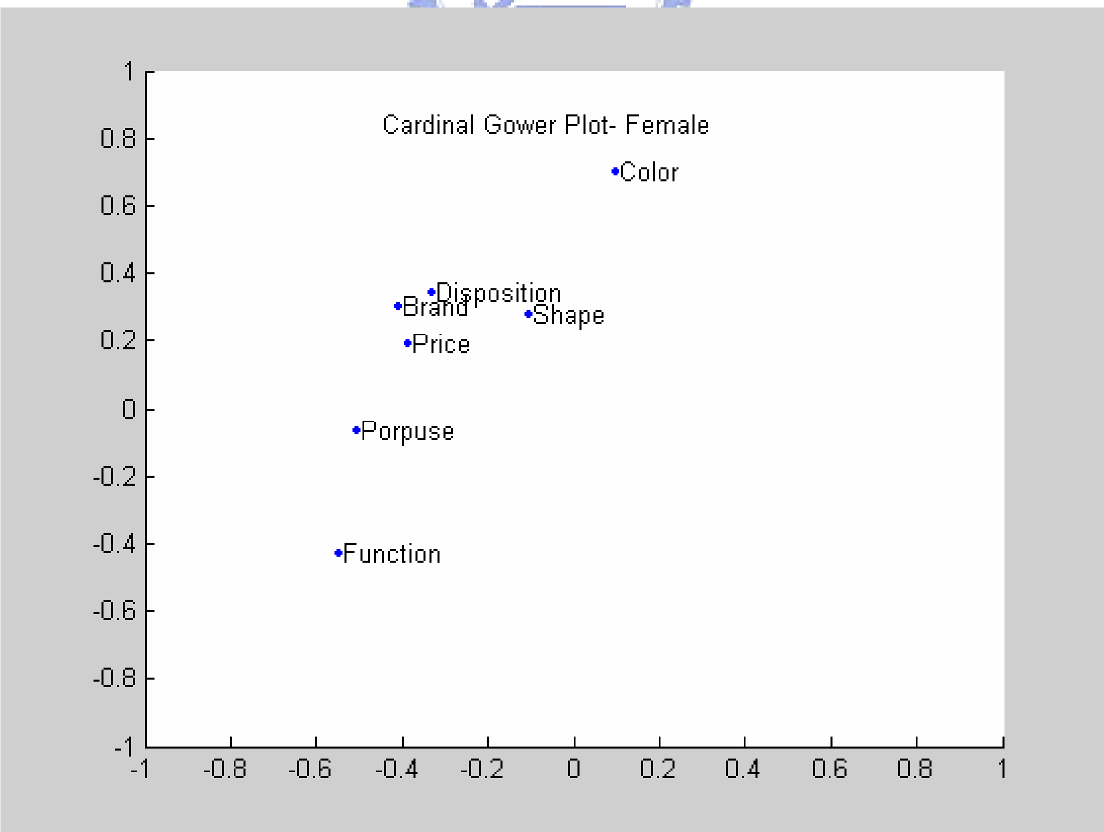


Figure 20- Cardinal Gower Plot of Plan to buy a car

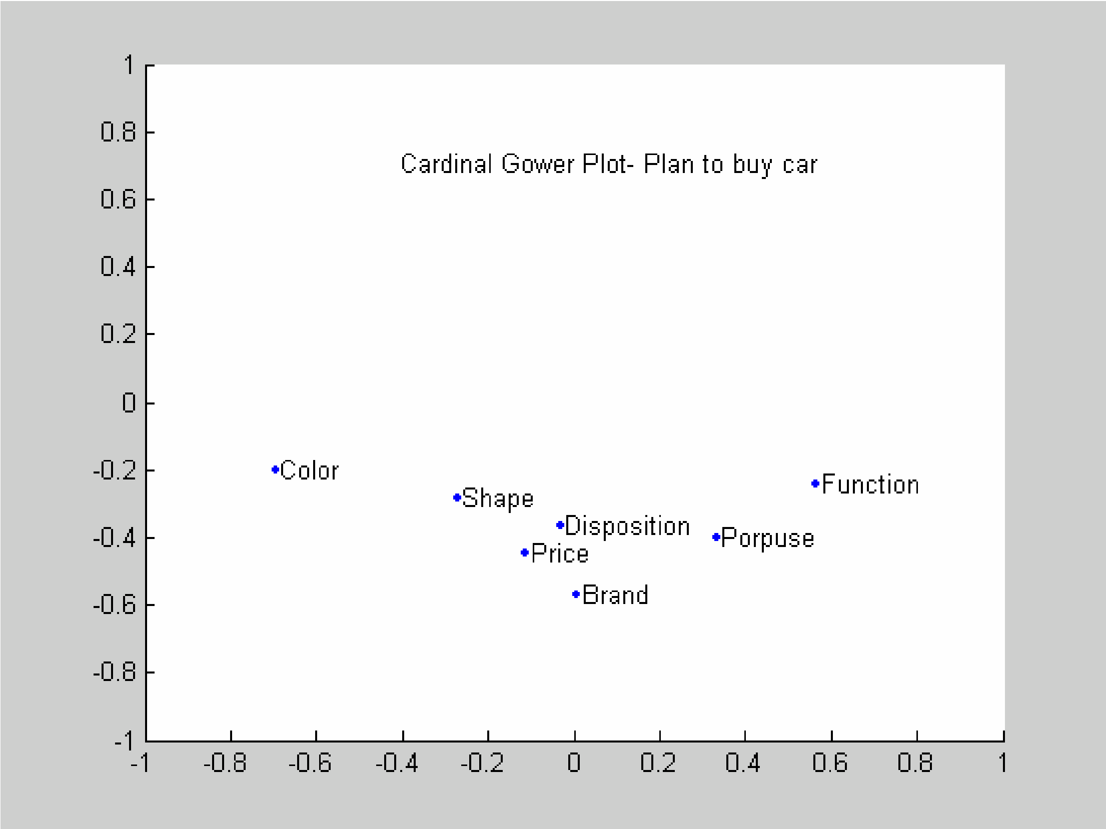
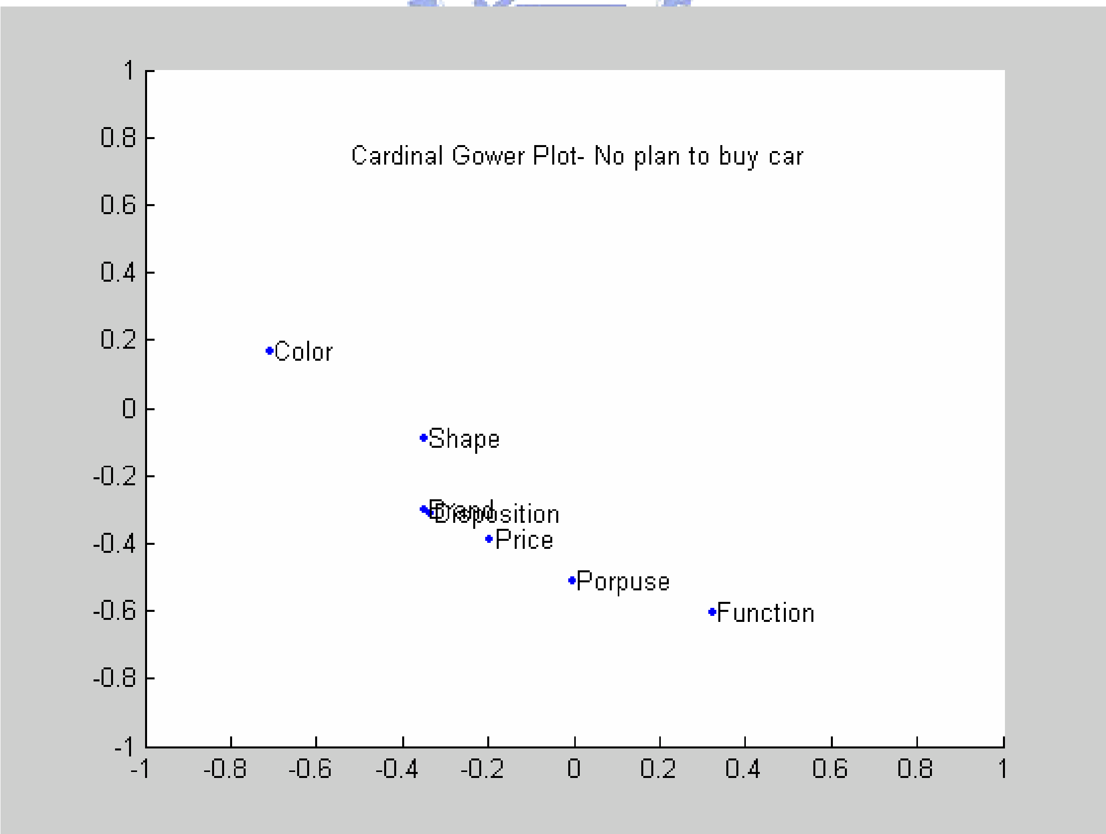


Figure 21- Cardinal Gower Plot of No plan to buy a car



5. Case Studies- i2 users satisfaction survey of Semiconductor Company

Supply chain management is a big issue for semiconductor industry for its business flow is too complex to trace and define. We conduct a survey for a semiconductor company to find out the real concern of end users.

The following we brief the result of the satisfaction survey of supply chain system that was developed by i2:
The most unsatisfied of i2 is user interface comfort
The second unsatisfied of i2 is system efficiency
The third unsatisfied of i2 is system compatibility
The overall satisfied of i2 is unsatisfied.

The result as above is the result that can come out by any kind of survey based on simple statistical method. Since the resource for IT is limited for most non-IT industry company, system developer have to set up their priority, control budget and maximize users' satisfaction at the same time. We now apply Gower plot to this issue and try to provide the total solution of priority and budget for system developer and end users.

First of all, we find six factors: System stability, System efficiency, User Interface comfort, Data/ Report reliability, System compatibility and System scalability, that were dig out from the issue log of i2, and then design the survey based on six factors.

To make the allocation of budget and resource is proper, we make some change in the question to add the ratio concept into factor to factor, for instance, the question to end users will describe as "What is proper ratio for factor A : factor B? 1:1, 1:2,…."

Table 31 is the result of tournament for ordinal Gower plot. Also, the calculation method is the same as case study I. Table 32 is come out by Table 31.

1. Table 31- Response data sheet of Ordinal Gower Plot

Reponse Category	Strongly disagree:	Disagree:	Neither agree nor disagree:	Agree:	Strongly Agree:	Ordinal Tournament	Sample Size
System stability is important than System efficiency	0	1	2	4	4	1	11
System efficiency is important than User Interface comfort	0	7	1	1	2	-1	11
User Interface comfort is important than Data/ Report reliability	2	3	2	4	0	-1	11
Data/ Report reliability is important than System compatibility	0	0	1	7	3	1	11
System compatibility is important than System scalibility	1	1	2	6	1	1	11
System stability is important than User Interface comfort	0	0	2	5	4	1	11
System efficiency is important than Data/ Report reliability	1	4	1	5	0	0	11
User Interface comfort is important than System compatibility	0	2	2	5	2	1	11
Data/ Report reliability is important than System scalibility	0	0	1	6	4	1	11
System stability is important than Data/ Report reliability	1	1	3	5	1	1	11
System efficiency is important than System compatibility	0	3	2	5	1	1	11
User Interface comfort is important than System scalibility	0	2	2	5	2	1	11
System stability is important than System compatibility	0	1	3	6	1	1	11
System efficiency is important than System scalibility	0	2	2	5	2	1	11
System stability is important than System scalibility	0	1	2	4	4	1	11

2. Table 32- Response Matrix for Ordinal Gower Plot

	System stability	System efficiency	User Interface comfort	Data/ Report reliability	System compatibility	System scalability
System stability	0	1	1	1	1	1
System efficiency	-1	0	-1	0	1	1
User Interface comfort	-1	1	0	-1	1	1
Data/ Report reliability	-1	0	1	0	1	1
System compatibility	-1	-1	-1	-1	0	1
System scalability	-1	-1	-1	-1	-1	0

We can find the preference ranking in Figure 22: System stability > Data/ Report reliability > User Interface comfort > System efficiency > System compatibility > System scalability. Form the point of view of the result; we have found that the concern of end users is different from the result what we have got in the previous paragraph (User Interface comfort > System efficiency > System compatibility). It is strength of Gower plot to detect the conflict between factor and factor. With the conflict detection, Gower plot finally can derive out the real concern or real preference ranking without any noise.

3. Figure 22- Ordinal Gower Plot

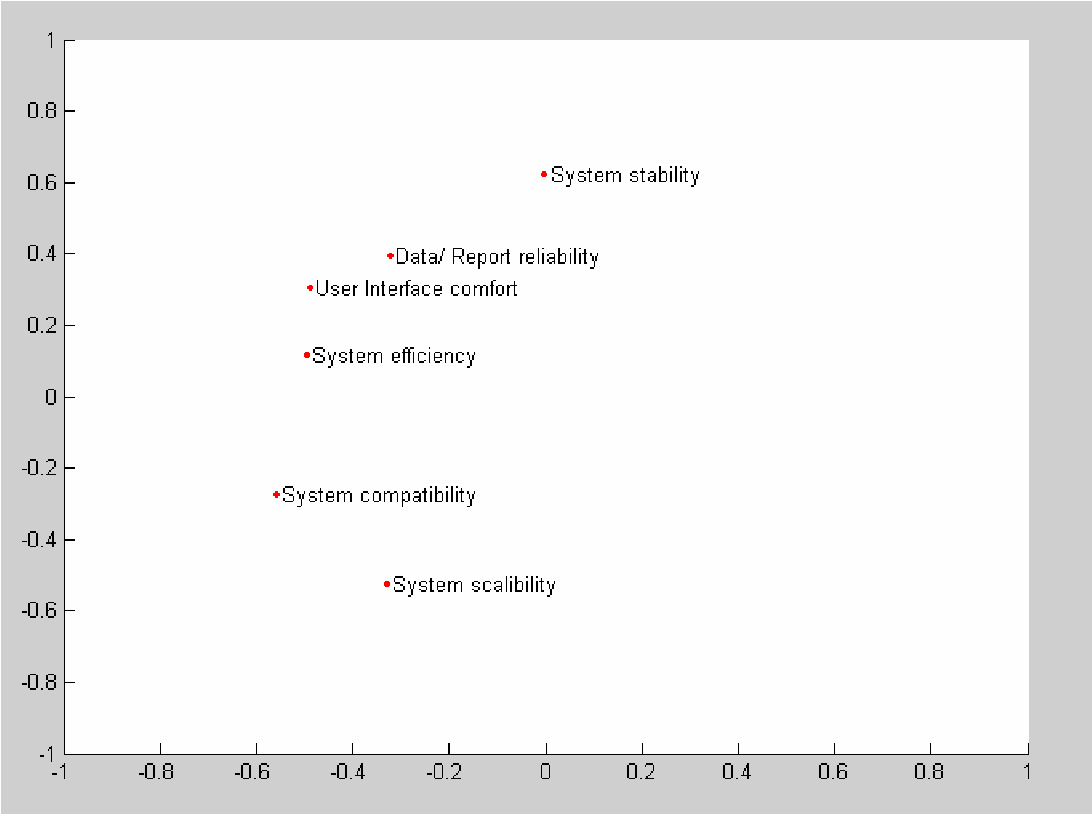


Table 33 is the result of response data sheet for cardinal Gower plot. Also, the calculation method is the same as case study I. Table 34 is come out by Table 33.

4. Table 33- Response data sheet of Cardinal Gower Plot

Reponse Category	1:1	1:2	1:3	1:4	1:5	1:6	1:7	1:8	1:9	1:10	10:1	9:1	8:1	7:1	6:1	5:1	4:1	3:1	2:1	Cardinal	Sample
Point	1.00	0.50	0.33	0.25	0.20	0.17	0.14	0.13	0.11	0.10	10.00	9.00	8.00	7.00	6.00	5.00	4.00	3.00	2.00	Geomean	Size
System stability vs.System efficiency	4	0	0	0	1	0	0	0	0	0	2	0	1	2	0	0	0	0	1	2.407	11
System efficiency vs. User Interface comfort	4	4	0	0	1	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0.671	11
User Interface comfort vs. Data/ Report reliability	3	1	2	0	2	0	0	0	0	0	0	0	0	2	0	0	0	1	0	0.903	11
Data/ Report reliability vs. System compatibility	3	0	0	0	0	0	0	0	0	0	0	0	4	4	0	0	0	0	0	4.322	11
System compatibility vs. System scalibility	5	0	0	0	0	0	0	0	0	0	1	0	1	3	0	0	0	0	1	2.697	11
System stability vs. User Interface comfort	4	0	0	0	0	0	0	0	1	0	2	0	2	1	0	0	0	0	1	2.309	11
System efficiency vs. Data/ Report reliability	3	2	2	0	0	0	0	0	0	1	0	0	0	2	1	0	0	0	0	0.982	11
User Interface comfort vs. System compatibility	5	1	1	0	0	0	0	0	0	0	0	1	0	2	0	0	0	1	0	1.633	11
Data/ Report reliability vs. System scalibility	3	0	0	0	0	0	0	0	0	0	0	3	1	2	0	0	0	1	1	3.688	11
System stability vs. Data/ Report reliability	5	0	1	0	0	0	0	0	0	1	0	0	0	3	0	0	1	0	0	1.416	11
System efficiency vs. System compatibility	5	2	0	0	0	0	0	0	0	0	0	1	1	2	0	0	0	0	0	1.853	11
User Interface comfort vs. System scalibility	4	0	1	0	0	0	0	0	0	0	0	1	0	4	0	1	0	0	0	2.596	11
System stability vs. System compatibility	3	1	0	1	0	0	0	0	0	0	2	0	0	3	0	1	0	0	0	2.476	11
System efficiency vs. System scalibility	5	1	0	0	0	0	0	0	0	0	0	1	1	3	0	0	0	0	0	2.355	11
System stability vs. System scalibility	3	0	0	0	0	0	0	0	0	0	2	1	1	3	0	1	0	0	0	4.413	11

5. Table 34- Response Matrix for Cardinal Gower Plot

	System stability	System efficiency	User Interface comfort	Data/ Report reliability	System compatibility	System scalibility
System stability	1	2.41	2.31	1.42	2.48	4.41
System efficiency	0.42	1	0.67	0.98	1.85	1.42
User Interface comfort	0.43	1.49	1	0.90	1.63	2.60
Data/ Report reliability	0.71	1.02	1.11	1	4.32	3.69
System compatibility	0.40	0.54	0.61	0.23	1	2.70
System scalibility	0.23	0.71	0.39	0.27	0.37	1

Below, we will introduce the method to allocate the budget.

(A) Find the regression line of the data points in cardinal Gower plot

(B) Adjust the data points into regression line without change the Y coordinates

(C) The ratio of measure of length of those data points will be the budget ratio of those factors.

(D) According to the prove as below, we can easily get the budget ratio as the ratio of

measure of length $= \sqrt{[-0.6453-(-0.5329)]^2+[0.3985-0.1767]^2}$:

$\sqrt{[-0.5329-(-0.4287)]^2+[0.1767-(-0.0289)]^2}$:

$\sqrt{[-0.4287-(-0.3888)]^2+[-0.0289-(-0.1077)]^2}$:

$\sqrt{[-0.3888-(-0.1926)]^2+[-0.1077-(-0.4949)]^2}$:

$\sqrt{[-0.1926-(-0.0667)]^2+[-0.4949-(-0.7433)]^2}$

=31:27:4:94:39

6. Figure 23-Cardinal Gower Plot

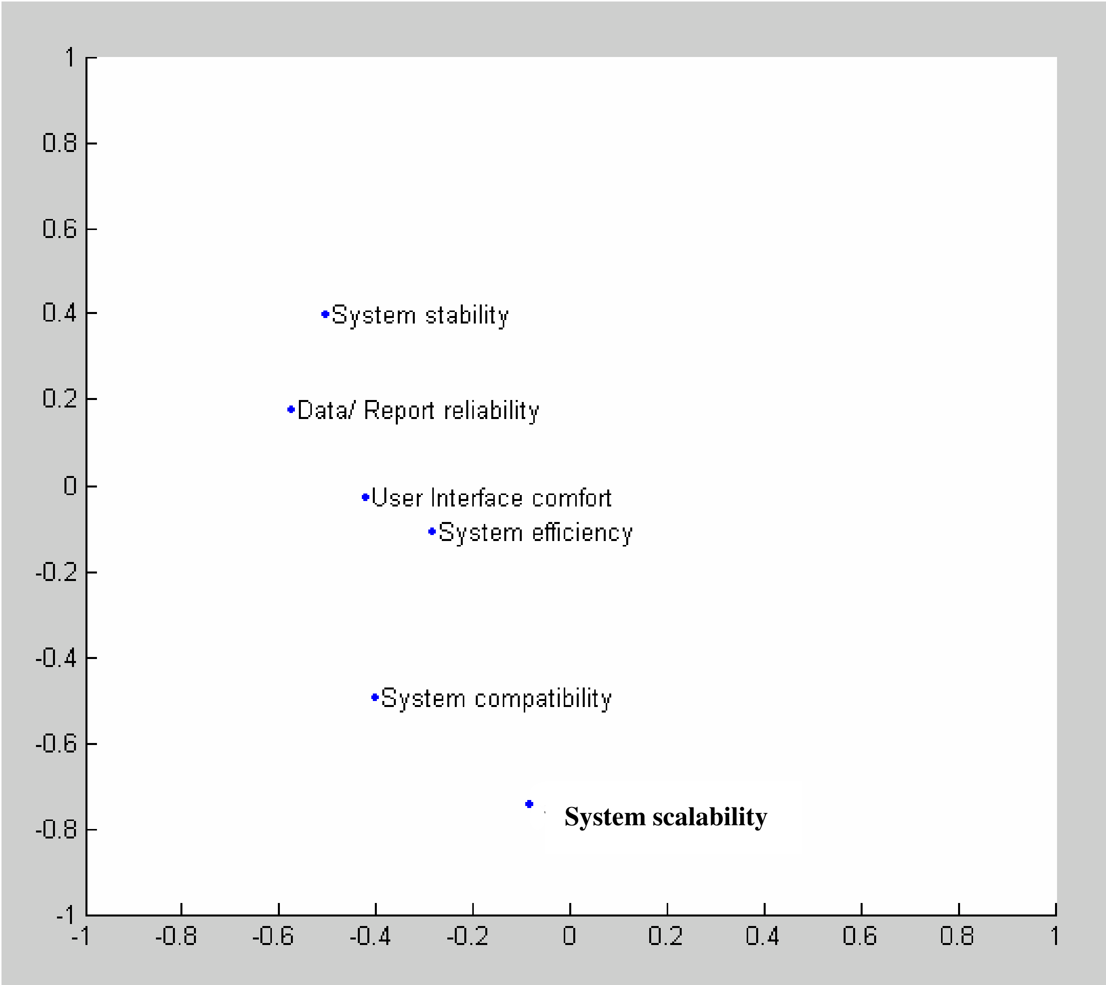


Figure 24- Figure 23 with coordinate data

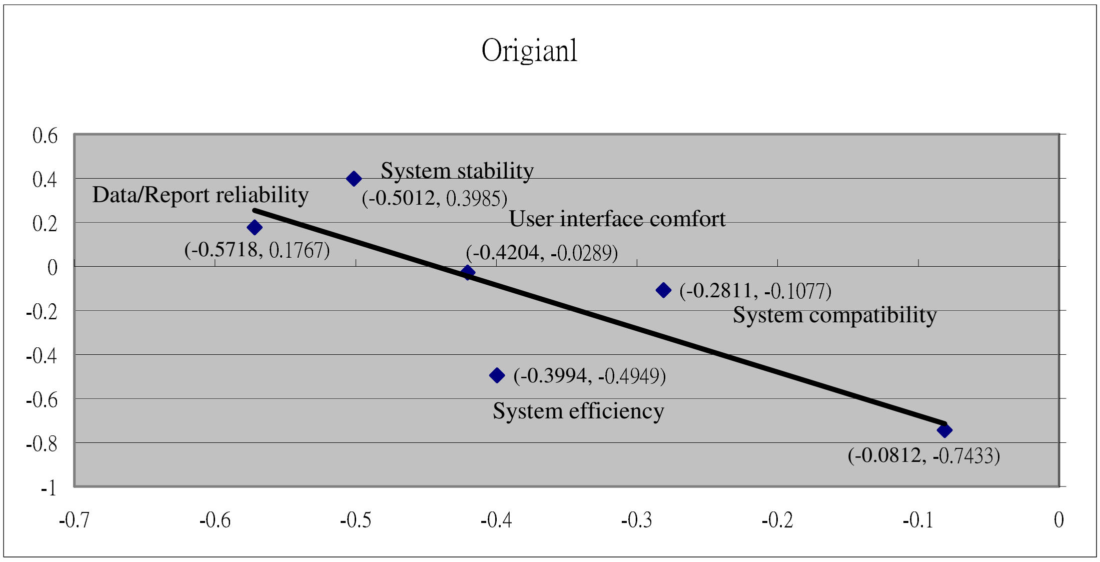
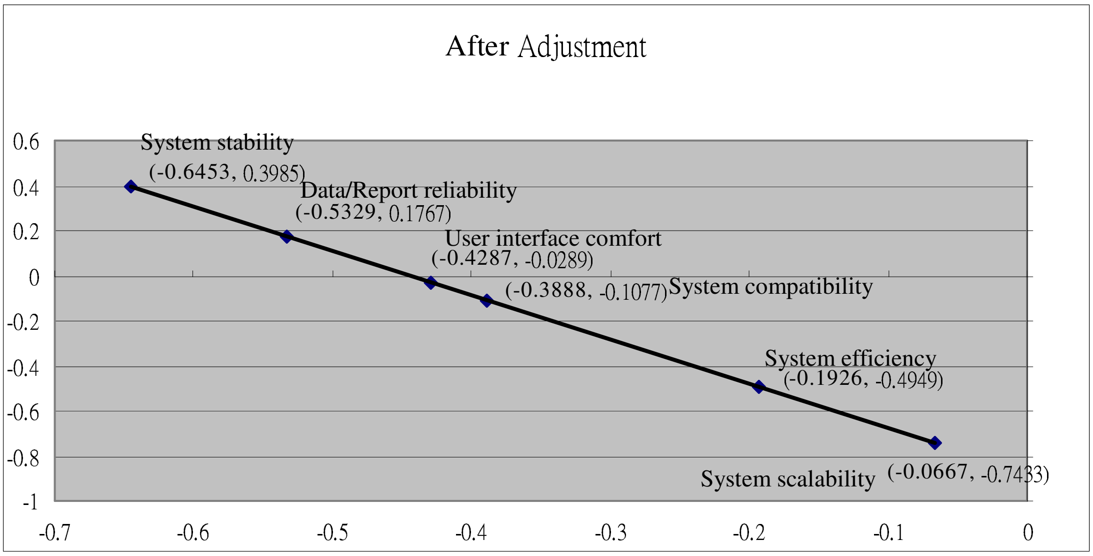
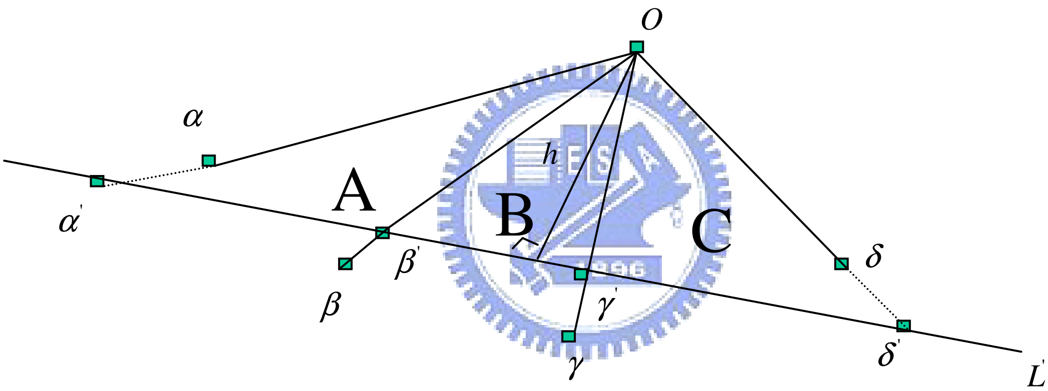


Figure 25- Data points relocate after taking regressing line





Let O denote any point in the graph, but not in the regression line $L^{'}$. The ratio of measure of area of $A:B:C$ will represent the ratio of calculation: they will approximate to the ratio of measure of length of $\overline{\alpha^{'}\beta^{'}}:\overline{\beta^{'}\gamma^{'}}:\overline{\gamma^{'}\delta^{'}}$. We can get the ratio of length to represent the ratio of calculation.

Prove:

Area A: Area B: Area C

$=\overline{\alpha^{'}\beta^{'}}\times h:\overline{\beta^{'}\gamma^{'}}\times h:\overline{\gamma^{'}\delta^{'}}\times h$

$=\overline{\alpha^{'}\beta^{'}}:\overline{\beta^{'}\gamma^{'}}:\overline{\gamma^{'}\delta^{'}}$

6. Reference

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7. Appendix

Singular Value Decomposition

Consider a matrix A that is of dimension $m \times n$ where $m \geq n$. This assumption is made for convenience only; all the results will also hold if $m < n$ As it turns out, the vectors in the the expansion of A are the eigenvectors of the square matrices AA^T and $A^T A$. The former is a outer product and results in a matrix that is spanned by the row space of A. The latter is a inner product and results in a matrix that is spanned by the column space (i.e., the range) of A.

The singular values are the nonzero square roots of the eigenvalues from AA^T and $A^T A$. The eigenvectors of AA^T are called the "left" singular vectors (U) while the eigenvectors of $A^T A$ are the "right" singular vectors (V). By retaining the nonzero eigenvalues $k = \min(m, n)$, a singular value decomposition (SVD) can be constructed. That is

$$A = U \Lambda V^T \tag{1}$$

where U is an $m \times m$ orthogonal matrix ($U^T U = I$), V is an $n \times n$ orthogonal matrix ($V^T V = I$), and Λ is an $m \times n$ matrix whose off-diagonal entries are all 0's and whose diagonal elements satisfy

$$\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_n \geq 0 \tag{2}$$

Example-

The covariance matrix Σ is an example of a square-symmetric matrix. Consider the following

$$\Sigma = \begin{bmatrix} 2.2 & 0.4 \\ 0.4 & 2.8 \end{bmatrix}$$

The matrix is not singular since the determinant $|\Sigma| = 6$ therefore Σ^{-1} exists. The eigenvalues and eigenvectors are obtained directly from Σ since it is already square. Furthermore, the left and right singular vectors (U; V) will be the same due to symmetry. We solve for the eigenvalues to obtain $\lambda_1 = 3$ and $\lambda_2 = 2$ which are also the singular values in this case.We then compute the corresponding eigenvectors to obtain $e_1^T = \left[1/\sqrt{5}, 2/\sqrt{5}\right]$ and $e_2^T = \left[2/\sqrt{5}, -1/\sqrt{5}\right]$. Finally we factor Σ into a singular value decomposition.

$$\Sigma = U\Lambda V^T = \begin{bmatrix} \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} & \frac{-1}{\sqrt{5}} \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} & \frac{-1}{\sqrt{5}} \end{bmatrix}^T = \begin{bmatrix} 2.2 & 0.4 \\ 0.4 & 2.8 \end{bmatrix}$$

It is now trivial to compute Σ^{-1} and $\Sigma^{-1/2}$.

$$\Sigma^{-1} = U\Lambda^{-1}V^T = \begin{bmatrix} \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} & \frac{-1}{\sqrt{5}} \end{bmatrix} \begin{bmatrix} \frac{1}{3} & 0 \\ 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} & \frac{-1}{\sqrt{5}} \end{bmatrix}^T = \begin{bmatrix} 0.47 & -0.07 \\ -0.07 & 0.37 \end{bmatrix}$$

$$\Sigma^{-1/2} = U\Lambda^{-1/2}V^T = \begin{bmatrix} \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} & \frac{-1}{\sqrt{5}} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{3}} & 0 \\ 0 & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} & \frac{-1}{\sqrt{5}} \end{bmatrix}^T = \begin{bmatrix} 0.68 & -0.05 \\ -0.05 & 0.60 \end{bmatrix}$$

Program - Ordinal Gower Plot

```
label={'Price','Color','Shape','Function','Disposition','Brand','Porpuse',}

r=[0 1   1   -1  -1  -1  -1;
-1   0   -1  -1  -1  -1  -1;
-1   1   0   -1  -1  -1  -1;
1    1   1   0   1   1   1;
1    1   1   -1  0   1   -1;
1    1   1   -1  -1  0   -1;
1    1   1   -1  1   1   0;]

a=r
[u,s,v]=svd(a)
for i=1:7
        sigma(i)=s(i,i);
   end

sigma

U=-u(:, 1)
V=-v(:, 1)
a_start=sigma(1)*(U*V'-V*U')
faithfulness=sigma(1)^2/(sigma(1)^2+sigma(3)^2+sigma(5)^2)
```

```
for i=1:7
end

for i=1:7
    for j=1:7
        if a(i,j) > 0
            t(i,j)=1;
        elseif a(i,j) < 0
                t(i,j)=-1;
        else
                t(i,j)=0;
        end
    end
end

t

[ut,st,vt]=svd(t)

for i=1:7
    sigma_t(i)=st(i,i);
end

sigma_t

Ut=-ut(:,1)
Vt=-vt(:,1)

t_star=sigma_t(1)*(Ut*Vt'-Vt*Ut')
faithfulness_t=sigma_t(1)^2/(sigma_t(1)^2+sigma_t(3)^2+sigma_t(5)^2)

for i=1:7
    %Ordernal
    line(Ut(i),Vt(i),'marker','.','lineWidth',3,'markersize',10,'color','r');
    text(Ut(i)+0.01,Vt(i),label(i));
end
xlim([-1 1]);
ylim([-1 1]);
```

Program - Cardinal Gower Plot

```
label={'Price','Color','Shape','Function','Disposition','Brand','Porpuse',}

r=[1        2.34    1.15    0.71    1.07    0.93    0.60;
0.43    1       0.73    0.45    0.63    0.47    0.50;
0.87    1.37    1       0.68    0.90    0.73    0.52;
1.41    2.22    1.46    1       2.22    2.18    1.49;
0.93    1.60    1.11        0.45    1       1.13    0.78;
1.07    2.11        1.37    0.46    0.89    1       0.73;
1.65    2.00    1.93    0.67    1.27    1.37    1;]

a=log(r)
[u,s,v]=svd(a)
for i=1:7
        sigma(i)=s(i,i);
  end

sigma

U=-u(:, 1)
V=-v(:, 1)
a_start=sigma(1)*(U*V'-V*U')
faithfulness=sigma(1)^2/(sigma(1)^2+sigma(3)^2+sigma(5)^2)
for i=1:7
     %Cardnal
     line(U(i),V(i),'marker','.','markersize',10);
     text(U(i)+0.01,V(i),label(i));
end

for i=1:7
     for j=1:7
          if a(i,j) > 0
               t(i,j)=1;
          elseif a(i,j) < 0
                    t(i,j)=-1;
```

```
            else
                        t(i,j)=0;
            end
      end
end

t

[ut,st,vt]=svd(t)

for i=1:7
      sigma_t(i)=st(i,i);
end

sigma_t

Ut=-ut(:,1)
Vt=-vt(:,1)

t_star=sigma_t(1)*(Ut*Vt'-Vt*Ut')
faithfulness_t=sigma_t(1)^2/(sigma_t(1)^2+sigma_t(3)^2+sigma_t(5)^2)

for i=1:7

end

xlim([-1 1]);
ylim([-1 1]);
```

Program Fix

```
label={'Price','Color','Shape','Function','Disposition','Brand','Porpuse',}

r=[1        2.34    1.15    0.71    1.07    0.93    0.60;
0.43    1       0.73    0.45    0.63    0.47    0.50;
0.87    1.37    1       0.68    0.90    0.73    0.52;
1.41    2.22    1.46    1       2.22    2.18    1.49;
0.93    1.60    1.11        0.45    1       1.13    0.78;
1.07    2.11        1.37    0.46    0.89    1       0.73;
1.65    2.00    1.93    0.67    1.27    1.37    1;]

a=r
a=log(r)
[u,s,v]=svd(a)
for i=1:7
        sigma(i)=s(i,i);
  end

sigma

U=-u(:, 1)
V=-v(:, 1)
a_start=sigma(1)*(U*V'-V*U')
faithfulness=sigma(1)^2/(sigma(1)^2+sigma(3)^2+sigma(5)^2)
for i=1:7
     %Cardnal
     line(-U(i),-V(i),'marker','.','markersize',10);
     text(-U(i)+0.01,-V(i),label(i));
end

for i=1:7
     for j=1:7
          if a(i,j) > 0
               t(i,j)=1;
          elseif a(i,j) < 0
```

```
                    t(i,j)=-1;
                else
                    t(i,j)=0;
                end
        end
end

t

[ut,st,vt]=svd(t)

for i=1:7
        sigma_t(i)=st(i,i);
end

sigma_t

Ut=-ut(:,1)
Vt=-vt(:,1)

t_star=sigma_t(1)*(Ut*Vt'-Vt*Ut')
faithfulness_t=sigma_t(1)^2/(sigma_t(1)^2+sigma_t(3)^2+sigma_t(5)^2)

for i=1:7
        %Ordernal
        %line(-Ut(i),-Vt(i),'marker','.','lineWidth',3,'markersize',10,'color','r');
        %text(-Ut(i)+0.01,-Vt(i),label(i));
end

xlim([-1 1]);
ylim([-1 1]);
```

Ordinal result of Case study I

```
label =
```

```
  Columns 1 through 6

    'Price'    'Color'    'Shape'    'Function'    'Disposition'    'Brand'

    'Porpuse'


r =

     0     1     1    -1    -1    -1    -1
    -1     0    -1    -1    -1    -1    -1
    -1     1     0    -1    -1    -1    -1
     1     1     1     0     1     1     1
     1     1     1    -1     0     1    -1
     1     1     1    -1    -1     0    -1
     1     1     1    -1     1     1     0


a =

     0     1     1    -1    -1    -1    -1
    -1     0    -1    -1    -1    -1    -1
    -1     1     0    -1    -1    -1    -1
     1     1     1     0     1     1     1
     1     1     1    -1     0     1    -1
     1     1     1    -1    -1     0    -1
     1     1     1    -1     1     1     0


u =

   -0.5345   -0.0000    0.5268    0.0907   -0.5344   -0.0091    0.3780
   -0.3333    0.4179   -0.5140    0.1468    0.1278   -0.5190    0.3780
   -0.4816    0.2319    0.0288    0.5337    0.3261    0.4235   -0.3780
    0.1189   -0.5211    0.2575    0.4684    0.4855   -0.2237    0.3780
   -0.3333   -0.4179   -0.4352   -0.3103    0.1100    0.5231    0.3780
   -0.4816   -0.2319    0.2057   -0.4934    0.3404   -0.4121   -0.3780
   -0.1189   -0.5211   -0.3994    0.3553   -0.4776   -0.2401   -0.3780
```

s =

4.3813	0	0	0	0	0	0
0	4.3813	0	0	0	0	0
0	0	1.2540	0	0	0	0
0	0	0	1.2540	0	0	0
0	0	0	0	0.4816	0	0
0	0	0	0	0	0.4816	0
0	0	0	0	0	0	0.0000

v =

0.0000	-0.5345	0.0907	-0.5268	0.0091	-0.5344	-0.3780
-0.4179	-0.3333	0.1468	0.5140	0.5190	0.1278	-0.3780
-0.2319	-0.4816	0.5337	-0.0288	-0.4235	0.3261	0.3780
0.5211	0.1189	0.4684	-0.2575	0.2237	0.4855	-0.3780
0.4179	-0.3333	-0.3103	0.4352	-0.5231	0.1100	-0.3780
0.2319	-0.4816	-0.4934	-0.2057	0.4121	0.3404	0.3780
0.5211	-0.1189	0.3553	0.3994	0.2401	-0.4776	0.3780

sigma =

4.3813	4.3813	1.2540	1.2540	0.4816	0.4816	0.0000

U =

0.5345
0.3333
0.4816
-0.1189
0.3333
0.4816
0.1189

V =

-0.0000
0.4179
0.2319
-0.5211
-0.4179
-0.2319
-0.5211

a_start =

0	0.9787	0.5431	-1.2204	-0.9787	-0.5431	-1.2204
-0.9787	0	-0.5431	-0.5431	-1.2204	-1.2204	-0.9787
-0.5431	0.5431	0	-0.9787	-1.2204	-0.9787	-1.2204
1.2204	0.5431	0.9787	0	0.9787	1.2204	0.5431
0.9787	1.2204	1.2204	-0.9787	0	0.5431	-0.5431
0.5431	1.2204	0.9787	-1.2204	-0.5431	0	-0.9787
1.2204	0.9787	1.2204	-0.5431	0.5431	0.9787	0

faithfulness =

0.9141

t =

0	1	1	-1	-1	-1	-1
-1	0	-1	-1	-1	-1	-1
-1	1	0	-1	-1	-1	-1
1	1	1	0	1	1	1
1	1	1	-1	0	1	-1
1	1	1	-1	-1	0	-1
1	1	1	-1	1	1	0

ut =

-0.5345	-0.0000	0.5268	0.0907	-0.5344	-0.0091	0.3780
-0.3333	0.4179	-0.5140	0.1468	0.1278	-0.5190	0.3780
-0.4816	0.2319	0.0288	0.5337	0.3261	0.4235	-0.3780
0.1189	-0.5211	0.2575	0.4684	0.4855	-0.2237	0.3780
-0.3333	-0.4179	-0.4352	-0.3103	0.1100	0.5231	0.3780
-0.4816	-0.2319	0.2057	-0.4934	0.3404	-0.4121	-0.3780
-0.1189	-0.5211	-0.3994	0.3553	-0.4776	-0.2401	-0.3780

st =

4.3813	0	0	0	0	0	0
0	4.3813	0	0	0	0	0
0	0	1.2540	0	0	0	0
0	0	0	1.2540	0	0	0
0	0	0	0	0.4816	0	0
0	0	0	0	0	0.4816	0
0	0	0	0	0	0	0.0000

vt =

0.0000	-0.5345	0.0907	-0.5268	0.0091	-0.5344	-0.3780
-0.4179	-0.3333	0.1468	0.5140	0.5190	0.1278	-0.3780
-0.2319	-0.4816	0.5337	-0.0288	-0.4235	0.3261	0.3780
0.5211	0.1189	0.4684	-0.2575	0.2237	0.4855	-0.3780
0.4179	-0.3333	-0.3103	0.4352	-0.5231	0.1100	-0.3780
0.2319	-0.4816	-0.4934	-0.2057	0.4121	0.3404	0.3780
0.5211	-0.1189	0.3553	0.3994	0.2401	-0.4776	0.3780

sigma_t =

4.3813	4.3813	1.2540	1.2540	0.4816	0.4816	0.0000

Ut =

0.5345
0.3333
0.4816
-0.1189
0.3333
0.4816
0.1189

Vt =

-0.0000
0.4179
0.2319
-0.5211
-0.4179
-0.2319
-0.5211

t_star =

0	0.9787	0.5431	-1.2204	-0.9787	-0.5431	-1.2204
-0.9787	0	-0.5431	-0.5431	-1.2204	-1.2204	-0.9787
-0.5431	0.5431	0	-0.9787	-1.2204	-0.9787	-1.2204
1.2204	0.5431	0.9787	0	0.9787	1.2204	0.5431
0.9787	1.2204	1.2204	-0.9787	0	0.5431	-0.5431
0.5431	1.2204	0.9787	-1.2204	-0.5431	0	-0.9787
1.2204	0.9787	1.2204	-0.5431	0.5431	0.9787	0

faithfulness_t =

0.9141

Cardinal result of Case study I

label =

Columns 1 through 6

'Price' 'Color' 'Shape' 'Function' 'Disposition' 'Brand' 'Porpuse'

r =

1.0000	2.3400	1.1500	0.7100	1.0700	0.9300	0.6000
0.4300	1.0000	0.7300	0.4500	0.6300	0.4700	0.5000
0.8700	1.3700	1.0000	0.6800	0.9000	0.7300	0.5200
1.4100	2.2200	1.4600	1.0000	2.2200	2.1800	1.4900
0.9300	1.6000	1.1100	0.4500	1.0000	1.1300	0.7800
1.0700	2.1100	1.3700	0.4600	0.8900	1.0000	0.7300
1.6500	2.0000	1.9300	0.6700	1.2700	1.3700	1.0000

a =

0	0.8502	0.1398	-0.3425	0.0677	-0.0726	-0.5108

-0.8440	0	-0.3147	-0.7985	-0.4620	-0.7550	-0.6931
-0.1393	0.3148	0	-0.3857	-0.1054	-0.3147	-0.6539
0.3436	0.7975	0.3784	0	0.7975	0.7793	0.3988
-0.0726	0.4700	0.1044	-0.7985	0	0.1222	-0.2485
0.0677	0.7467	0.3148	-0.7765	-0.1165	0	-0.3147
0.5008	0.6931	0.6575	-0.4005	0.2390	0.3148	0

u =

-0.4074	0.1717	0.2951	-0.5970	0.0653	-0.4006	0.4431
-0.5223	-0.5040	0.2912	0.2956	-0.5347	0.0918	0.0811
-0.3656	-0.0968	0.0520	-0.4566	0.2696	0.6320	-0.4167
0.0280	0.6546	0.6498	0.2069	-0.1856	0.1433	-0.2250
-0.3899	0.1253	-0.0078	0.5341	0.6324	0.1445	0.3552
-0.4751	0.1966	-0.3107	0.1330	0.0042	-0.5171	-0.5949
-0.2213	0.4736	-0.5536	-0.0449	-0.4502	0.3505	0.3044

s =

2.2444	0	0	0	0	0	0
0	2.2380	0	0	0	0	0
0	0	0.5630	0	0	0	0
0	0	0	0.5628	0	0	0
0	0	0	0	0.1984	0	0
0	0	0	0	0	0.1932	0
0	0	0	0	0	0	0.0028

v =

0.1723	0.4044	-0.5817	-0.2968	0.3977	0.0715	-0.4659
-0.5037	0.5234	0.2950	-0.2967	-0.0975	-0.5310	-0.0758
-0.0970	0.3649	-0.4745	-0.0534	-0.6127	0.2697	0.4257
0.6534	-0.0274	0.2053	-0.6527	-0.1376	-0.1761	0.2329
0.1235	0.3874	0.5365	0.0176	-0.1665	0.6327	-0.3442
0.1976	0.4795	0.1306	0.3131	0.5295	0.0029	0.5792
0.4753	0.2203	-0.0493	0.5447	-0.3603	-0.4570	-0.2962

sigma =

2.2444	2.2380	0.5630	0.5628	0.1984	0.1932	0.0028

U =

0.4074
0.5223
0.3656
-0.0280
0.3899
0.4751
0.2213

V =

-0.1723
0.5037
0.0970
-0.6534
-0.1235
-0.1976
-0.4753

a_start =

0	0.6626	0.2301	-0.6083	0.0379	0.0031	-0.3491
-0.6626	0	-0.2996	-0.7344	-0.5855	-0.7688	-0.8074
-0.2301	0.2996	0	-0.5301	-0.1862	-0.2656	-0.4382
0.6083	0.7344	0.5301	0	0.5795	0.7092	0.3543
-0.0379	0.5855	0.1862	-0.5795	0	-0.0413	-0.3546
-0.0031	0.7688	0.2656	-0.7092	0.0413	0	-0.4087
0.3491	0.8074	0.4382	-0.3543	0.3546	0.4087	0

faithfulness =

0.9339

t =

0	1	1	-1	1	-1	-1
-1	0	-1	-1	-1	-1	-1
-1	1	0	-1	-1	-1	-1
1	1	1	0	1	1	1
-1	1	1	-1	0	1	-1
1	1	1	-1	-1	0	-1
1	1	1	-1	1	1	0

ut =

-0.0000	-0.4780	0.0523	-0.8148	0.1216	-0.2538	-0.1601
0.5573	-0.1342	0.0000	0.0000	0.1758	0.6401	-0.4804
0.4352	-0.3732	0.0000	0.0000	-0.6584	0.0842	0.4804
-0.5573	-0.1342	0.0000	-0.0000	-0.6090	0.2639	-0.4804
0.0000	-0.4780	0.6795	0.4527	0.1216	-0.2538	-0.1601
-0.0000	-0.4780	-0.7318	0.3621	0.1216	-0.2538	-0.1601
-0.4352	-0.3732	-0.0000	0.0000	0.3468	0.5660	0.4804

st =

4.1529	0	0	0	0	0	0
0	4.1529	0	0	0	0	0
0	0	1.7321	0	0	0	0
0	0	0	1.7321	0	0	0
0	0	0	0	0.8682	0	0
0	0	0	0	0	0.8682	0
0	0	0	0	0	0	0.0000

vt =

-0.4780	0.0000	-0.8148	-0.0523	0.2538	0.1216	-0.1601
-0.1342	-0.5573	0.0000	-0.0000	-0.6401	0.1758	-0.4804
-0.3732	-0.4352	-0.0000	0.0000	-0.0842	-0.6584	0.4804
-0.1342	0.5573	0.0000	0.0000	-0.2639	-0.6090	-0.4804
-0.4780	0.0000	0.4527	-0.6795	0.2538	0.1216	-0.1601
-0.4780	0.0000	0.3621	0.7318	0.2538	0.1216	-0.1601
-0.3732	0.4352	0.0000	-0.0000	-0.5660	0.3468	0.4804

sigma_t =

4.1529	4.1529	1.7321	1.7321	0.8682	0.8682	0.0000

Ut =

0.0000
-0.5573
-0.4352

0.5573
-0.0000
0.0000
0.4352

Vt =

0.4780
0.1342
0.3732
0.1342
0.4780
0.4780
0.3732

t_star =

0	1.1063	0.8638	-1.1063	0.0000	0.0000	-0.8638
-1.1063	0	-0.6213	-0.6213	-1.1063	-1.1063	-1.1063
-0.8638	0.6213	0	-1.1063	-0.8638	-0.8638	-1.3489
1.1063	0.6213	1.1063	0	1.1063	1.1063	0.6213
-0.0000	1.1063	0.8638	-1.1063	0	-0.0000	-0.8638
-0.0000	1.1063	0.8638	-1.1063	0.0000	0	-0.8638
0.8638	1.1063	1.3489	-0.6213	0.8638	0.8638	0

faithfulness_t =

0.8212

Cardinal result of Case study II

Columns 1 through 3

'System stability'   'System efficiency'   [1x22 char]

Columns 4 through 6

[1x24 char] [1x20 char] 'System scalability'

r =

1.0000	2.4100	2.3100	1.4200	2.4800	4.4100
0.4200	1.0000	0.6700	0.9800	1.8500	1.4200
0.4300	1.4900	1.0000	0.9000	1.6300	2.6000
0.7100	1.0200	1.1100	1.0000	4.3200	3.6900
0.4000	0.5400	0.6100	0.2300	1.0000	2.7000
0.2300	0.7100	0.3900	0.2700	0.3700	1.0000

a =

0	0.8796	0.8372	0.3507	0.9083	1.4839
-0.8675	0	-0.4005	-0.0202	0.6152	0.3507
-0.8440	0.3988	0	-0.1054	0.4886	0.9555
-0.3425	0.0198	0.1044	0	1.4633	1.3056
-0.9163	-0.6162	-0.4943	-1.4697	0	0.9933
-1.4697	-0.3425	-0.9416	-1.3093	-0.9943	0

u =

0.5012	0.3999	0.5927	0.2214	0.3416	-0.2680
0.2811	-0.1053	-0.2483	-0.6708	0.0859	-0.6252
0.4204	-0.0300	0.3341	-0.3142	-0.7229	0.2992
0.5718	0.1772	-0.5349	-0.0292	0.3029	0.5128
0.3994	-0.4994	-0.2300	0.6051	-0.2525	-0.3290
0.0812	-0.7398	0.3694	-0.1876	0.4448	0.2769

s =

3.2115	0	0	0	0	0

0	3.2083	0	0	0	0
0	0	0.9854	0	0	0
0	0	0	0.9723	0	0
0	0	0	0	0.0340	0
0	0	0	0	0	0.0260

v =

-0.3985	0.4990	-0.2187	0.5949	0.2766	0.3376
0.1077	0.2819	0.6690	-0.2465	0.6299	0.0622
0.0289	0.4173	0.3103	0.3379	-0.3183	-0.7164
-0.1767	0.5761	0.0325	-0.5341	-0.5021	0.3137
0.4949	0.3985	-0.6100	-0.2276	0.3217	-0.2624
0.7433	0.0820	0.1876	0.3661	-0.2641	0.4491

sigma =

3.2115	3.2083	0.9854	0.9723	0.0340	0.0260

U =

-0.5012
-0.2811
-0.4204
-0.5718
-0.3994
-0.0812

V =

0.3985
-0.1077
-0.0289
0.1767

-0.4949

-0.7433

a_start =

0	0.5332	0.5845	0.4474	1.3077	1.3004
-0.5332	0	-0.1193	-0.3573	0.3087	0.6430
-0.5845	0.1193	0	-0.2916	0.6311	0.9960
-0.4474	0.3573	0.2916	0	1.1356	1.4112
-1.3077	-0.3087	-0.6311	-1.1356	0	0.8242
-1.3004	-0.6430	-0.9960	-1.4112	-0.8242	0

faithfulness =

0.9139

t =

0	1	1	1	1	1
-1	0	-1	-1	1	1
-1	1	0	-1	1	1
-1	1	1	0	1	1
-1	-1	-1	-1	0	1
-1	-1	-1	-1	-1	0

ut =

0.0000	0.5774	-0.1509	0.5573	-0.0465	0.5755
0.5774	-0.0000	-0.5573	-0.1509	-0.5755	-0.0465
0.5000	0.2887	0.1509	-0.5573	0.4751	0.3280
0.2887	0.5000	0.5573	0.1509	-0.2475	-0.5216
0.5000	-0.2887	-0.1509	0.5573	0.5216	-0.2475
0.2887	-0.5000	0.5573	0.1509	-0.3280	0.4751

```
st =

    3.7321         0         0         0         0         0
         0    3.7321         0         0         0         0
         0         0    1.0000         0         0         0
         0         0         0    1.0000         0         0
         0         0         0         0    0.2679         0
         0         0         0         0         0    0.2679


vt =

   -0.5774    0.0000   -0.5573   -0.1509    0.5755    0.0465
    0.0000    0.5774    0.1509   -0.5573   -0.0465    0.5755
   -0.2887    0.5000    0.5573    0.1509    0.3280   -0.4751
   -0.5000    0.2887   -0.1509    0.5573   -0.5216    0.2475
    0.2887    0.5000   -0.5573   -0.1509   -0.2475   -0.5216
    0.5000    0.2887   -0.1509    0.5573    0.4751    0.3280


sigma_t =

    3.7321    3.7321    1.0000    1.0000    0.2679    0.2679


Ut =

   -0.0000
   -0.5774
   -0.5000
   -0.2887
   -0.5000
   -0.2887


Vt =
```

0.5774
-0.0000
0.2887
0.5000
-0.2887
-0.5000

t_star =

0	1.2440	1.0774	0.6220	1.0774	0.6220
-1.2440	0	-0.6220	-1.0774	0.6220	1.0774
-1.0774	0.6220	0	-0.6220	1.0774	1.2440
-0.6220	1.0774	0.6220	0	1.2440	1.0774
-1.0774	-0.6220	-1.0774	-1.2440	0	0.6220
-0.6220	-1.0774	-1.2440	-1.0774	-0.6220	0

faithfulness_t =

0.9285

12.