Effective boson number calculations near the $Z=64$ subshell

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The effect of the partial subshell closure near $Z=64$ is studied by introducing the effective boson number in the framework of the interacting boson model. The energy spectra and the $B(E2)$ values of the Sm, Gd, and Dy isotope series are calculated. It is found that the agreements between the theoretical results and the observed data are very satisfactory when the partial closure effect is taken into account by a smooth variation of the effective proton-boson numbers.

I. INTRODUCTION

In recent years there are considerable interests in studying the subshell closure effects at $Z=64$.[1,2] It has been shown[3,4] that the $Z=64$ subshell had significant effect for $N\leq 88$ transition nuclei, but disappears as N approaches 90 due to the increasing importance of the n-p interactions. This subshell closure also introduces significant effects on the calculations using the interacting boson model (IBA). In IBA, the number of active bosons is usually determined by counting particles above, or below the nearest spherical-shell closure. Therefore, the counting of the boson numbers becomes ambiguous when some subshell closure exists. Wolf *et al.*[4] have calculated the magnetic moment of the first 2^+ states of the transitional nuclei Ba, Nd, Sm, and Gd in the framework of IBA-2, and Gill *et al.*[3] have performed an IBA calculation near $Z=64$ subshell. They made the assumption of a drastic change in the proton-boson numbers at $N=88$, namely, the assumption of a $Z=64$ shell for $N\leq 88$ and a $Z=50$ shell for $N\geq 90$. It is interesting to note that for those nuclei with $N\leq 88$ the counting schemes from the $Z=64$ subshell yielded better agreement with the experimental data. Scholten[5] has proposed a method to calculate the number of "effective bosons" in a microscopic model. It is interesting to see that when the number of effective bosons is calculated, it does yield a minimum value at $Z=64$. However, the minimum value is about 2.4 instead of 0 as required by a full closure structure. This means that there is considerable washing out of the shell closure effect at $Z=64$. A similar result has also been obtained by Maino and Venture[6] using the Nilsson model on the basis of BCS approximation. Federman and Pittel,[7] Federman *et al.*,[8] and Chen *et al.*[9] studied the shape transition of Zr and Mo isotopes in the framework of a shell model. It was found that the isospin $T=0$ component of the n-p interaction is responsible for the onset of deformation. It was suggested that the promotion of the neutron into the $h_{9/2}$ orbit near $N\simeq 90$ leads to a concurrent polarization of protons into the $h_{11/2}$ orbit, via the strong n-p interaction between particles in spin-orbit partners orbits, and results in an eradication of the subshell gap. However, it is well known that the shell-model single particle level spacings are dependent on the model space. Therefore, the assumption of sudden disappearance of the $Z=64$ subshell at $N=88$ made by Wolf *et al.* and Gill *et al.* seems to be oversimplified in the sense that it does not take the effects of partial closure of $Z=64$ and the smooth neutron number dependence into account. In order to investigate these partial subshell closure effects, some preliminary studies[10,11] on Sm and Er isotopes with $N=86$–96 have been done. It is found that satisfactory results can be achieved only when smooth variation of the proton-boson numbers for $N\leq 88$ are considered.

In this paper, we present a systematic study on the $^{148-158}$Sm, $^{150-158}$Gd, and $^{152-160}$Dy isotopes to simulate the effects of partial closure and the smooth neutron dependence by considering the effective proton-boson numbers in IBA. A unified Hamiltonian and an $E2$ transition operator for each isotope series are used in our calculation.

II. MODEL

In the calculation of energy levels, the most general Hamiltonian with nine parameters of IBA-1 was used:[12]

$$
\begin{aligned}
H_{sd}=&\epsilon_s(s^\dagger\cdot\tilde{s})+\epsilon_d(d^\dagger\cdot\tilde{d})+\sum_{L=0,2,4}\tfrac{1}{2}\sqrt{(2L+1)}C_L\times\{[d^\dagger\times d^\dagger]^{(L)}\times[\tilde{d}\times\tilde{d}]^{(L)}\}^{(0)}\\
&+(\tfrac{1}{2})^{1/2}v_2\{[d^\dagger\times d^\dagger]^{(2)}\times[\tilde{d}\times\tilde{s}]^{(2)}+[d^\dagger\times s^\dagger]^{(2)}\times[\tilde{d}\times\tilde{d}]^{(2)}\}^{(0)}\\
&+(\tfrac{1}{2})^{1/2}v_0\{[d^\dagger\times d^\dagger]^{(0)}\times[\tilde{s}\times\tilde{s}]^{(0)}+[s^\dagger\times s^\dagger]^{(0)}\times[\tilde{d}\times\tilde{d}]^{(0)}\}^{(0)}\\
&+u_2\{[d^\dagger\times s^\dagger]^{(2)}\times[\tilde{d}\times\tilde{s}]^{(2)}\}^{(0)}+\tfrac{1}{2}u_0\{[s^\dagger\times s^\dagger]^{(0)}\times[\tilde{s}\times\tilde{s}]^{(0)}\}^{(0)}\,. \qquad (1)
\end{aligned}
$$

For comparison, two calculations were performed. We first consider the conventional IBA (hereafter denoted as *MI*). Then the effective proton-boson numbers are included (hereafter denoted as *MII*) to investigate the partial subshell closure effects of the $Z=64$ and $N=82$. In both models, the number of neutron bosons N_ν is counted as usual, $N_\nu=\frac{1}{2}(N-82)$ where N is the number of neutrons. For the proton bosons, N_π is counted from $Z=50$ closed shell in *MI*. However, in *MII*, we relax all the shell closure restriction for the proton boson but maintain the only requirement that they must be integers. Therefore, we try different sets of proton-boson numbers for the nuclei in each isotope series in our calculations. Once we chose a set of N_π for the isotope series, a least-squares search for the interaction parameters is then carried out in the framework of IBA to fit the experimental data of these nuclei. It was found that the best set for the effective proton-boson numbers for Sm isotopes is $N_\pi=2$ for ^{148}Sm, $N_\pi=4$ for ^{150}Sm, and $N_\pi=6$ for all other Sm isotopes. The best set of the effective proton-boson numbers of the Gd isotopes is $N_\pi=2$ for ^{150}Gd, $N_\pi=5$ for ^{152}Gd, and $N_\pi=7$ for all other Gd isotopes. For Dy isotopes, the best set is $N_\pi=5$ for ^{152}Dy, $N_\pi=7$ for ^{154}Dy, and $N_\pi=8$ for all other Dy isotopes. It is worth noting that for each isotope series, both *MI* and *MII* count the same N_π for $N\geq 90$. This is consistent with the disappearance of the $Z=64$ subshell for $N\geq 90$ in this mass region as pointed out in previous works.[3,4] However, the linear variation of N_π for $N\leq 88$ in *MII* for each isotope series manifests the effects of the partial closure and smooth neutron number dependence.

In the calculations, 71 reliable energy levels in Sm, 94 levels in Gd, and 100 levels in Dy isotopes were included in the least-squares fittings. It is well known that not all the parameters in the Hamiltonian are linearly independent. Since we are concerned with excitation energies only, the effect of ϵ_s can be absorbed into ϵ_d. Also the parameter u_0 is kept at zero because it can be absorbed into other parameters.[13] The resulting interaction parameters and the overall root-mean-square deviations for Sm, Gd, and Dy isotopes are listed in Table I. It is worth noting that unified interaction parameters can be found for each isotope series. It is also seen from the table that the parameters change smoothly from Sm isotopes to Dy isotopes.

TABLE I. The interaction strengths for Sm, Gd, and Dy isotopes and the overall energy level root-mean-square deviations in MeV.

		ϵ_s	ϵ_d	C_0	C_2	C_4	u_0	u_2	v_2	v_0	Overall rmsd (MeV)
MI	Sm	0.0	1.933	−0.544	−0.344	−0.344	0.0	−0.413	−0.126	−0.082	0.146
	Gd	0.0	2.081	−0.526	−0.272	−0.356	0.0	−0.431	−0.072	−0.081	0.170
	Dy	0.0	0.948	−0.258	−0.108	−0.07	0.0	−0.34	−0.062	−0.117	0.184
MII	Sm	0.0	0.817	−0.343	−0.113	−0.113	0.0	−0.115	0.141	−0.091	0.084
	Gd	0.0	0.802	−0.338	−0.107	−0.105	0.0	−0.106	0.099	−0.104	0.090
	Dy	0.0	0.750	−0.252	−0.106	−0.102	0.0	−0.067	0.054	−0.116	0.086

III. RESULTS

The ground-band levels of the Sm, Gd, and Dy isotopes are shown in Fig. 1. We can see that the energy values for each J states change almost linearly for $N\leq 88$ and become quite flat for $N\geq 90$. This is consistent with the onset of the $Z=64$ subshell effect. Furthermore, the linear falling of the energy values for each J state as N goes from 86 to 88 seems to justify the linear variation of the effective proton-boson number N_π found in *MII* in each isotope series.

The partial closure effect for $Z=64$ subshell can be investigated by plotting the effective proton-boson numbers versus the number of protons for each mass number A as

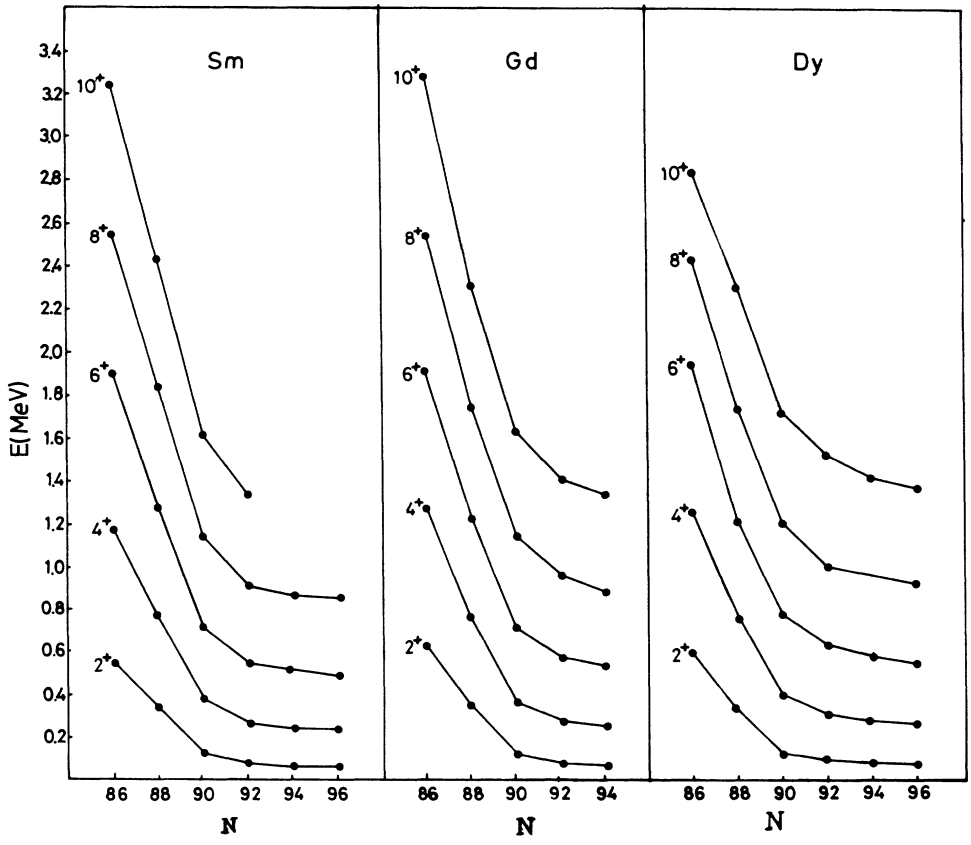

FIG. 1. The general trend of the experimental ground-band level energies of the Sm, Gd, and Dy isotopes.

shown in Fig. 2. For $A=150$, there is a clear minimum $N_\pi=2$ occurring at $Z=64$. The nonzero value of the minimum indicates that the closure of the $Z=64$ subshell is not complete but only partial. This result is consistent with that obtained by Scholten[5] as shown in the dashed line of the figure. However, in Scholten's result, there is no calculation to study the way the subshell closure will be decreasing as the neutron number N_ν approaches 90. We have also studied this tendency of washing out the subshell closure at $Z=64$ as $N\approx 90$. This can be clearly seen from the figure that the partial closure effect of $Z=64$ starts to decrease as A goes from 152 to 154 and is completely eliminated at $A=156$, where the curve becomes a straight line indicating the spherical closure of $Z=50$ for $N\geq 90$. The gradual decreasing of the partial closure effect in the mass region $A=150-156$ manifests the smooth neutron number dependence due to the increasing importance of the *n-p* interaction.

The calculated energy spectra of Sm, Gd, and Dy isotopes compared with the experimental values are shown in Figs. 3–8. In these figures different quasibands are separated in order to have a clear comparison. One can see that the traditional IBA (in the column *MI*) cannot reproduce well the energy spectra of these isotopes. The calculated ground state energies in *MI* in general have much lower values compared to the observed data especially for some higher spin states, except for ^{156}Dy and ^{158}Dy where the calculated values are higher than the ex-

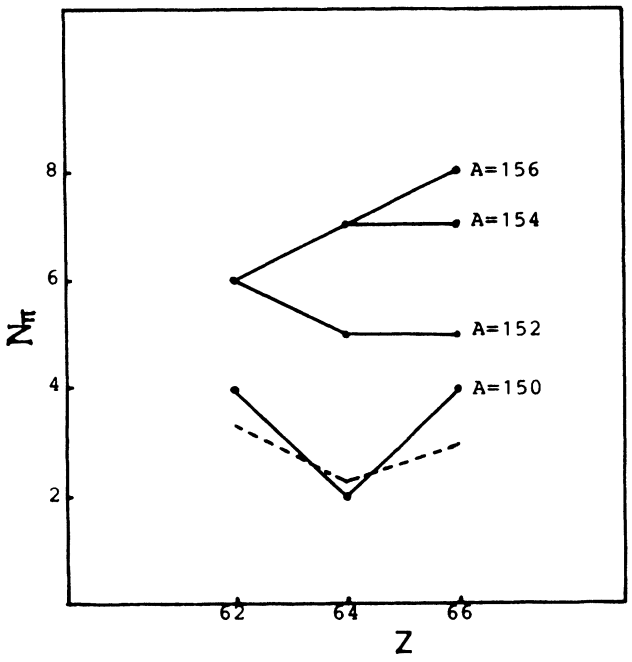

FIG. 2. The effective proton-boson numbers versus the number of protons for each mass number A. The dashed line is from the results of Scholten (Ref. 5).

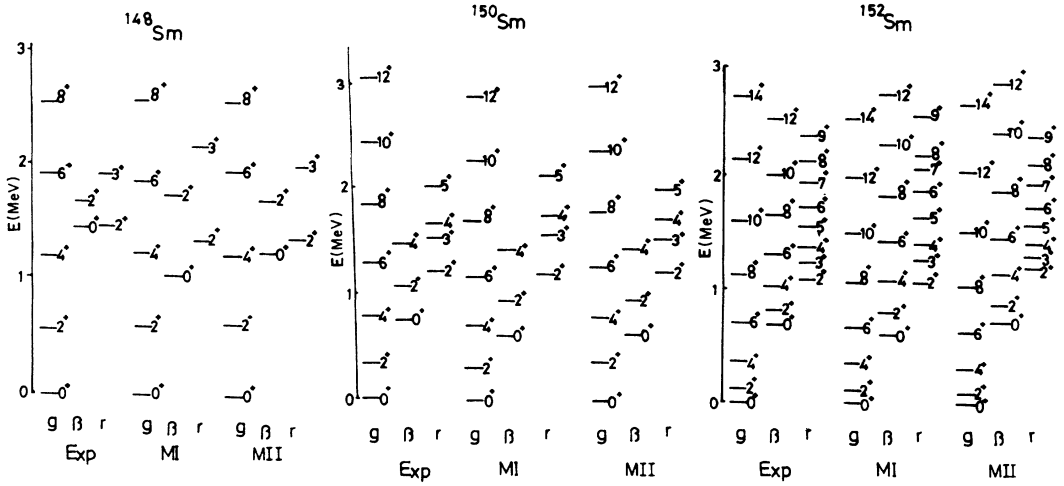

FIG. 3. Experimental and calculated energy spectra for ^{148}Sm, ^{150}Sm, and ^{152}Sm.

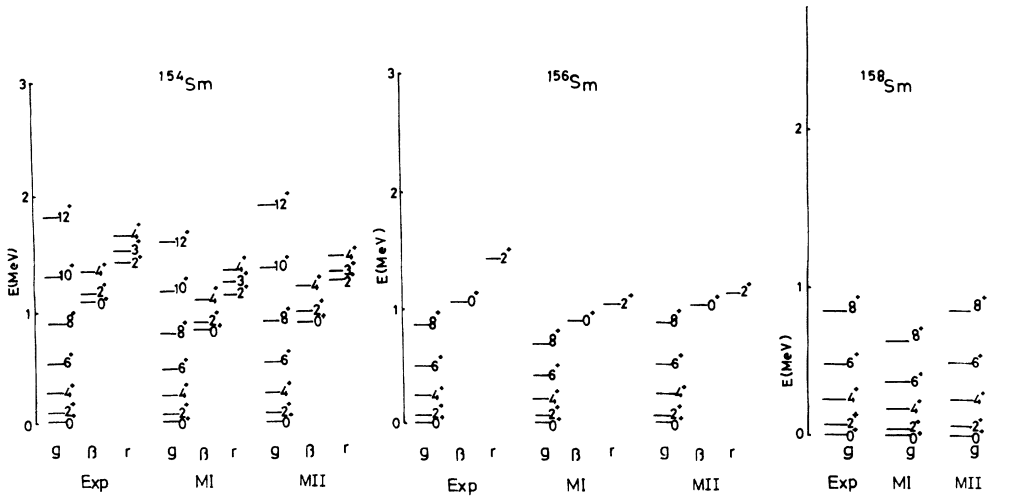

FIG. 4. Experimental and calculated energy spectra for ^{154}Sm, ^{156}Sm, and ^{158}Sm.

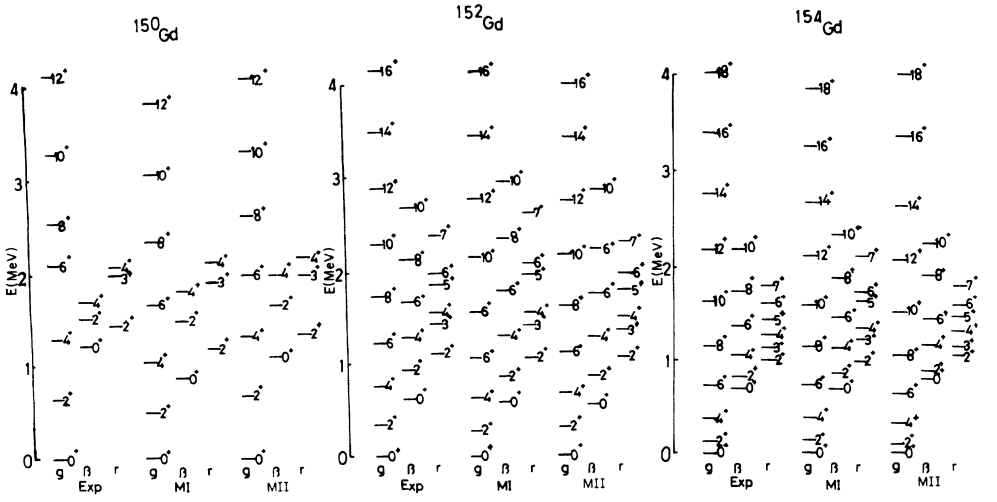

FIG. 5. Experimental and calculated energy spectra for ^{150}Gd, ^{152}Gd, and ^{154}Gd.

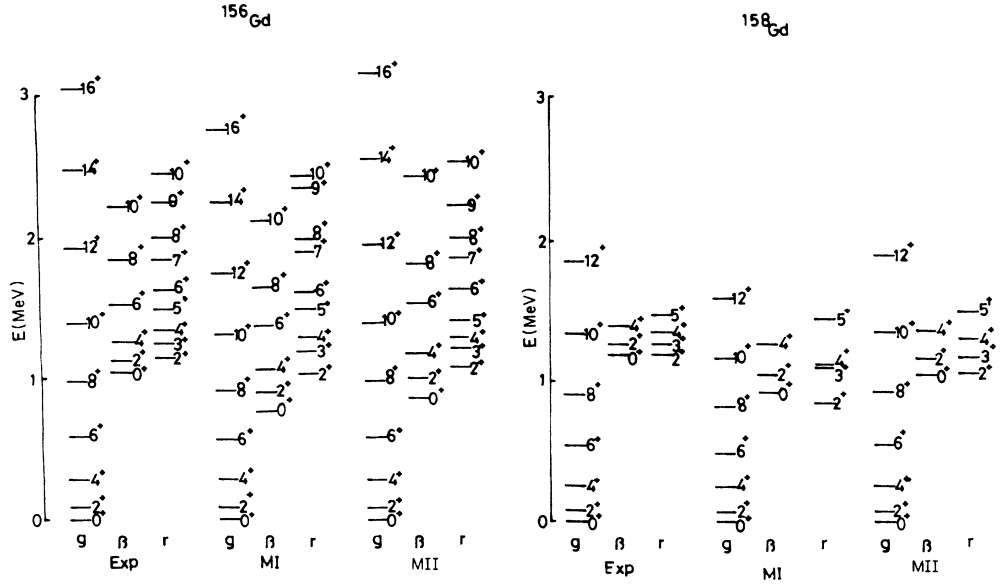

FIG. 6. Experimental and calculated energy spectra for ^{156}Gd and ^{158}Gd.

perimental data. The energy spacings calculated in *MI* are also in considerable disagreement with the observed data in the β band and the γ band. Some states are even in reversed order in the γ band of ^{156}Dy. Thus the results manifest that the pure IBA model is unable to simulate the energy level structure for the nuclei near $Z=64$. The reason for these discrepancies is that in the calculation of the nuclei near $Z=64$ with $86\leq N\leq 96$, the introduction of the neutron-proton interaction will become increasingly important when there are more neutrons outside the $N=82$ shell, and thus tend to eliminate the $Z=64$ subshell closure as mentioned above. Hence, it is reasonable to expect that the calculation which includes the proper number of proton bosons will yield better results for the energy level fittings. This is indeed true as shown in the figures (under the column *MII*) where the effective

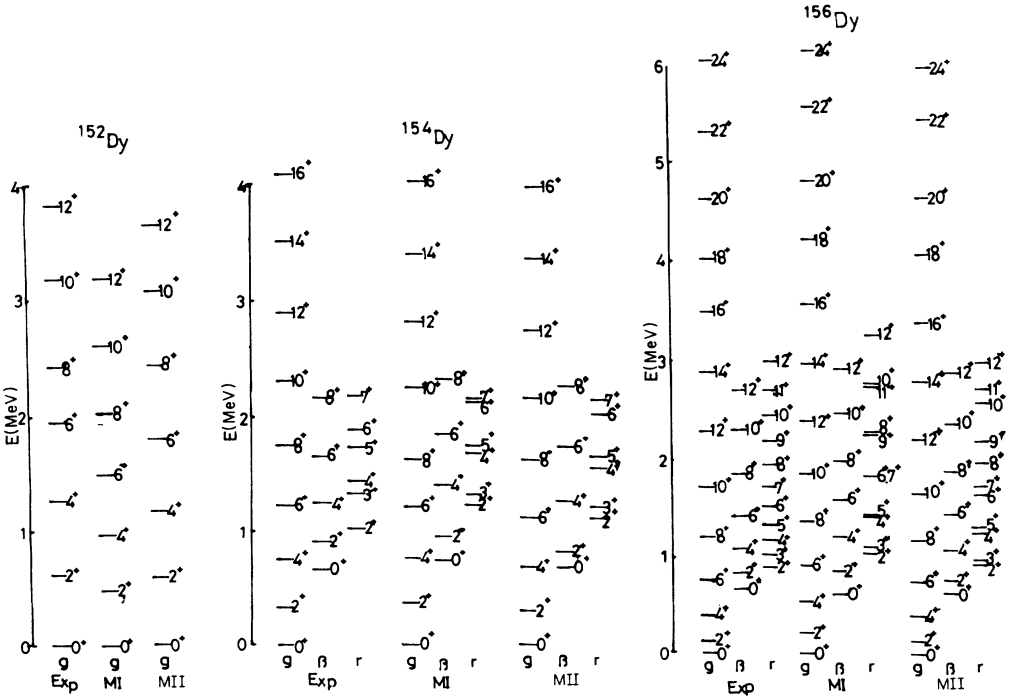

FIG. 7. Experimental and calculated energy spectra for ^{152}Dy, ^{154}Dy, and ^{156}Dy.

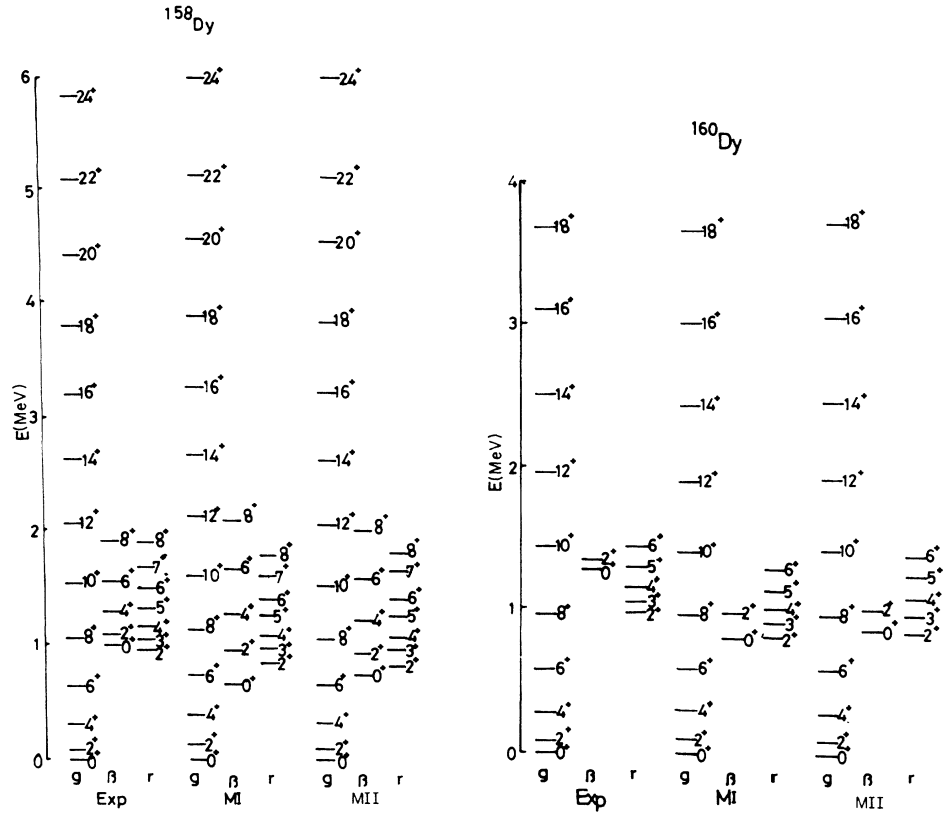

FIG. 8. Experimental and calculated energy spectra for ^{158}Dy and ^{160}Dy.

TABLE II. $B(E2)$ values (in e^2b^2) and branching ratios for Sm isotopes.

			Theory			
			This work		Previous works	
Nucleus	$J_i \rightarrow J_f$	Expt.	*MI*	*MII*		
^{148}Sm	$2_1 \rightarrow 0_1$	0.151	0.225	0.131		
	$4_1 \rightarrow 2_1$	0.25[a]	0.399	0.20		
^{150}Sm	$2_1 \rightarrow 0_1$	0.274[b]	0.355	0.318	0.275[b]	0.41[c]
	$4_1 \rightarrow 2_1$	0.53[b]	0.708	0.566	0.51[b]	0.73[c]
	$8_1 \rightarrow 6_1$	0.435[b]	0.934	0.609		
	$10_1 \rightarrow 8_1$	0.447[b]	0.70	0.512		
	$4_1 \rightarrow 2_2$	0.010 6[b]	0.08	0.05	0.139[b]	0.18[c]
	$2_2 \rightarrow 0_1$	0.003 6[b]	0.09	0.021	0.02[b]	0.008[c]
	$2_3 \rightarrow 0_1$	0.008 8[b]	0.000 1	0.003 5	0.02[b]	0.015[c]
	$2_2 \rightarrow 2_1$	0.27[b]	0.14	0.11	0.181[b]	0.12[c]
	$2_3 \rightarrow 2_1$	0.038 7[b]	0.005	0.079	0.024[b]	0.029[c]
	$2_3 \rightarrow 4_1$	0.017 4[b]	0.036	0.025		
	$3_1 \rightarrow 2_1/4_1$	0.296[b]	0.165	0.342	1.09[b]	0.54[d]
	$3_1 \rightarrow 2_2/2_1$	24±5[b]	8.54	20.62	4.34[b]	16.8[d]
^{152}Sm	$2_1 \rightarrow 0_1$	0.67[b]	0.41	0.861	0.673[b]	0.75[g]
	$4_1 \rightarrow 2_1$	1.017[b]	0.68	1.197	0.98[b]	1.0[g]
	$6_1 \rightarrow 4_1$	1.2[a]	0.88	1.256	1.193[f]	0.97[g]
	$8_1 \rightarrow 6_1$	1.39[a]	0.971	1.220	1.373[f]	0.83[g]
	$10_1 \rightarrow 8_1$	1.103[d]	0.907	1.131		
	$12_1 \rightarrow 10_1$	1.123[d]	0.826	1.001		
	$2_3 \rightarrow 0_1$	0.016 3[d]	0.007	0.010 2		
	$2_3 \rightarrow 2_1$	0.041 7[d]	0.011	0.026 7		
	$2_3 \rightarrow 4_1$	0.004 16[d]	0.002	0.001 9		
	$2_3 \rightarrow 2_1/0_1$	2.44[e]	1.57	2.618		

TABLE II. (*Continued*).

Nucleus	$J_i \rightarrow J_f$	Expt.	Theory This work *MI*	Theory This work *MII*	Previous works
	$2_3 \rightarrow 2_1/4_1$	11.9[e]	5.50	14.28	
	$3_1 \rightarrow 2_1/4_1$	0.95[e]	0.428	1.0	
	$4_3 \rightarrow 2_1/4_1$	0.088[e]	0.000 1	0.054	
	$2_2 \rightarrow 0_2/2_1$	<860[e]	50.8	273.7	
	$2_2 \rightarrow 0_1/2_1$	0.2[e]	0.081	0.256	
	$6_2 \rightarrow 4_1/6_1$	0.08[e]	0.308	0.658	
	$6_2 \rightarrow 4_2/6_1$	54[e]	128.5	330.8	
	$8_2 \rightarrow 6_1/8_1$	>0.06[e]	3.59	4.55	
	$8_2 \rightarrow 6_2/8_1$	>58[e]	173.2	175	
	$5_1 \rightarrow 4_1/6_1$	0.43[e]	0.102	0.391	
^{154}Sm	$2_1 \rightarrow 0_1$	0.922[d]	0.657	0.978	
	$4_1 \rightarrow 2_1$	1.186[d]	0.96	1.364	
	$8_1 \rightarrow 6_1$	1.497[d]	1.01	1.416	
	$10_1 \rightarrow 8_1$	1.538[d]	1.05	1.333	
	$12_1 \rightarrow 10_1$	1.565[d]	0.964	1.212	
	$2_3 \rightarrow 0_1$	0.013[d]	0.002	0.006 8	
	$2_3 \rightarrow 2_1$	0.02[d]	0.012	0.018	
	$2_3 \rightarrow 4_1$	0.000 8[d]	0.001 8	0.001 2	
	$2_3 \rightarrow 2_1/0_1$	1.56[d]	6.0	2.65	
	$3_1 \rightarrow 2_1/4_1$	2.5[d]	0.29	1.0	
	$4_3 \rightarrow 2_1/4_1$	0.055[d]	0.001 3	0.111	
^{156}Sm	$2_1 \rightarrow 0_1$	1.22[h]	0.784	1.089	
^{158}Sm	$2_1 \rightarrow 0_1$	1.28[h]	0.647	1.205	

[a]Reference 17.
[b]Reference 18.
[c]Reference 19.
[d]Reference 20.
[e]Reference 21.
[f]Reference 22.
[g]Reference 15.
[h]Reference 23.

TABLE III. $B(E2)$ values (in e^2b^2) for Gd isotopes.

Nucleus	$J_i \rightarrow J_f$	Expt.	Theory This work *MI*	Theory This work *MII*	Previous work	
^{152}Gd	$2_1 \rightarrow 0_1$	0.33[a]	0.506	0.463	0.33[b]	0.33[c]
	$4_1 \rightarrow 2_1$	0.64[a]	0.803	0.77	0.620[b]	0.69[c]
	$6_1 \rightarrow 4_1$	0.95[a]	0.68	0.84	0.76[b]	0.79[d]
	$4_1 \rightarrow 2_2$	0.096[a]	0.017	0.036	0.06[b]	0.011[d]
	$2_2 \rightarrow 2_1$	0.077[a]	0.201	0.116	0.164[b]	0.006[c]
	$2_2 \rightarrow 0_1$	0.001 4[a]	0.049	0.027	0.026[b]	0.009[c]
	$2_2 \rightarrow 0_2$	0.21[a]	0.085	0.263		
^{154}Gd	$2_1 \rightarrow 0_1$	0.773[e]	0.545	0.954	0.773[e]	0.87[c]
	$4_1 \rightarrow 2_1$	1.178[e]	1.07	1.132	1.098[e]	1.18[c]
	$6_1 \rightarrow 4_1$	1.39[e]	1.16	1.385	1.19[e]	1.17[c]
	$8_1 \rightarrow 6_1$	1.526[e]	1.17	1.345	1.20[e]	
	$10_1 \rightarrow 8_1$	1.73[e]	1.11	1.235	1.18[e]	
	$0_2 \rightarrow 2_1$	0.258[e]	0.03	0.02	0.136[e]	
	$2_2 \rightarrow 0_1$	0.004	0.027	0.017	0.016 8	
	$2_2 \rightarrow 2_1$	0.033[e]	0.018	0.017	0.025[e]	
	$2_2 \rightarrow 4_1$	0.09[e]	0.005	0.021	0.087[e]	
	$4_2 \rightarrow 2_1$	0.003[e]	0.009	0.027	0.016[e]	
	$4_2 \rightarrow 6_1$	0.071[e]	0.001 7	0.018	0.089[e]	
	$6_2 \rightarrow 4_1$	0.002 7[e]	0.001 3	0.047	0.013[e]	
	$6_2 \rightarrow 6_1$	0.033[e]	0.009	0.019	0.008 5[e]	

Table III. (*Continued*).

Nucleus	$J_i \rightarrow J_f$	Expt.	Theory This work *MI*	Theory This work *MII*	Theory Previous work
	$2_3 \rightarrow 0_1$	0.004 59	0.005	0.006 7	0.005 2
	$2_3 \rightarrow 2_1$	0.0	0.071	0.027	0.01[e]
	$2_3 \rightarrow 4_1$	0.001 39	0.004 3	0.001 2	0.000 38
	$3_1 \rightarrow 2_1$	0.010 3	0.011	0.008 2	0.017 6
	$3_1 \rightarrow 4_1$	0.01	0.055	0.017	0.01
	$4_3 \rightarrow 2_1$	0.001 4	0.001 3	0.004 8	0.001 9
	$4_3 \rightarrow 4_1$	0.01	0.061	0.028	0.01
	$4_3 \rightarrow 6_1$	0.004 3	0.003 9	0.003 6	0.000 81
	$5_1 \rightarrow 4_1$	0.007 4	0.001 1	0.003 4	0.01
^{156}Gd	$2_1 \rightarrow 0_1$	0.914[e]	0.86	1.04	0.914[e]
	$4_1 \rightarrow 2_1$	1.299[e]	1.12	1.45	1.286[e]
	$6_1 \rightarrow 4_1$	1.470[e]	1.29	1.53	1.38[e]
	$8_1 \rightarrow 6_1$	1.57[e]	1.28	1.50	1.38[e]
	$10_1 \rightarrow 8_1$	1.59[e]	1.23	1.42	1.34[e]
	$12_1 \rightarrow 10_1$	1.45[e]	1.22	1.26	1.26[e]
	$0_2 \rightarrow 2_1$	0.029[e]	0.197	0.124	0.025[e]
	$2_2 \rightarrow 0_1$	0.003 16	0.001	0.002 3	0.004
	$2_2 \rightarrow 2_1$	0.016[e]	0.002 8	0.015	0.006[e]
	$2_2 \rightarrow 4_1$	0.018[e]	0.007 9	0.043	0.015[e]
	$4_2 \rightarrow 2_1$	0.006 1[e]	0.011	0.045	0.004 7[e]
	$4_2 \rightarrow 4_1$	0.014[e]	0.003	0.035	0.004 8[e]
	$4_2 \rightarrow 6_1$	0.009 1[e]	0.012	0.048	0.014 8[e]
	$2_3 \rightarrow 2_1$	0.035 5[e]	0.026	0.023	0.033 3[e]
	$2_3 \rightarrow 4_1$	0.003 2[e]	0.001 4	0.003 8	0.002 3[e]
	$3_1 \rightarrow 2_1$	0.036 4[e]	0.002 8	0.008	0.037[e]
	$3_1 \rightarrow 4_1$	0.028[e]	0.005	0.015	0.019[e]
	$4_3 \rightarrow 2_1$	0.007 8[e]	0.002 5	0.002 6	0.011[e]
	$4_3 \rightarrow 4_1$	0.046[e]	0.002 1	0.005	0.039[e]
	$5_1 \rightarrow 4_1$	0.029 5[e]	0.000 1	0.000 6	0.028 9[e]
	$5_1 \rightarrow 6_1$	0.041[e]	0.003	0.014	0.025[e]
^{158}Gd	$2_1 \rightarrow 0_1$	1.008[e]	0.75	1.13	1.008[e]
	$2_3 \rightarrow 0_1$	0.001 6[e]	0.000 5	0.013	0.001 9[e]
	$2_3 \rightarrow 2_1$	0.001 2[e]	0.001 7	0.000 2	0.000 4[e]
	$2_3 \rightarrow 4_1$	0.007 1[e]	0.06	0.018	0.003 4[e]
	$4_3 \rightarrow 2_1$	0.004[e]	0.008	0.007	0.004 7[e]
	$4_3 \rightarrow 4_1$	0.002 2[e]	0.008 2	0.029	0.001 5[e]
	$4_3 \rightarrow 6_1$	0.009 4[e]	0.000 4	0.002 4	0.005 6[e]
	$2_2 \rightarrow 0_1$	0.016 9[e]	0.031	0.030	0.023 6[e]
	$2_2 \rightarrow 2_1$	0.029[e]	0.03	0.057	0.039[e]
	$2_2 \rightarrow 4_1$	0.001 3[e]	0.067	0.077	0.001 9[e]
	$3_1 \rightarrow 2_1$	0.029 7[e]	0.001 7	0.001 2	0.043[e]
	$3_1 \rightarrow 4_1$	0.017 7[e]	0.005 8	0.006	0.021[e]
	$4_2 \rightarrow 2_1$	0.006[e]	0.014	0.037	0.009 2[e]
	$4_2 \rightarrow 4_1$	0.04[e]	0.013	0.018	0.04[e]
	$4_2 \rightarrow 6_1$	< 0.01[e]	0.034	0.07	0.002 8[e]
	$5_1 \rightarrow 4_1$	0.023 5[e]	0.002	0.000 8	0.035[e]
	$5_1 \rightarrow 6_1$	0.019 4[e]	0.016	0.014	0.028[e]
	$4_3 \rightarrow 2_3$	1.37[e]	0.25	1.3	1.13[e]
	$4_2 \rightarrow 2_2$	0.6[e]	0.57	0.48	0.42[e]
	$5_1 \rightarrow 3_1$	1.01[e]	0.49	0.66	0.70[e]
	$4_3 \rightarrow 2_2$	0.037[e]	0.504	0.35	0.020 7[e]
	$4_3 \rightarrow 3_1$	0.14[e]	0.637	0.67	0.16[e]

[a]Reference 24.
[b]Reference 25.
[c]Reference 26
[d]Reference 15
[e]Reference 27

TABLE IV. $B(E)$ values (in e^2b^2) and branching ratios for Dy isotopes.

			Theory		
			This work		Previous work
Nucleus	$J_i \rightarrow J_f$	Expt.	*MI*	*MII*	
^{154}Dy	$2_2 \rightarrow 0_1/2_1$	0.015[a]	0.095	0.051	0.011[b]
	$4_2 \rightarrow 2_2/4_1$	5.1±0.5[a]	2.13	2.92	1.11[b]
	$6_2 \rightarrow 4_2/6_1$	14±2[a]	1.15	6.82	2.16[b]
^{156}Dy	$2_1 \rightarrow 0_1$	3.79[c]	1.04	1.17	1.01[b]
	$4_2 \rightarrow 2_2$	1.29[d]	0.75	0.792	0.75[b]
	$6_2 \rightarrow 4_2$	1.42[d]	1.15	1.25	1.07[b]
	$8_2 \rightarrow 6_2$	1.49[d]	1.19	1.36	1.01[b]
	$10_2 \rightarrow 8_2$	1.53[d]	1.13	1.32	0.81[b]
	$12_2 \rightarrow 10_2$	1.56[d]	0.86	0.97	0.56[b]
	$14_2 \rightarrow 12_2$	1.58[d]	0.68	0.80	0.27[b]
	$4_2 \rightarrow 4_1$	0.054[d]	0.098	0.09	
	$4_2 \rightarrow 2_1$	0.002 4[d]	0.018	0.079	
	$6_2 \rightarrow 6_1$	0.078[d]	0.53	0.76	
	$6_2 \rightarrow 4_1$	0.001 4[d]	0.011	0.004 5	
	$8_2 \rightarrow 8_1$	0.054[d]	0.031	0.057	
	$8_2 \rightarrow 6_1$	0.000 6[d]	0.002 9	0.001 7	
	$10_2 \rightarrow 10_1$	0.077[d]	0.014	0.038	
	$10_2 \rightarrow 8_1$	0.001[d]	0.004 6	0.002	
	$12_2 \rightarrow 12_1$	<0.32[d]	0.012	0.036	
	$12_2 \rightarrow 10_1$	0.006[d]	0.001 7	0.005 3	
	$14_2 \rightarrow 12_1$	$<0.001\,5$[d]	0.003 3	0.002 3	
^{158}Dy	$2_1 \rightarrow 0_1$	0.93[e]	1.34	1.40	

[a]Reference 28.
[b]Reference 15.
[c]Reference 25.
[d]Reference 29.
[e]Reference 30.

proton-boson number is taken into account. The results in *MII* show that the calculated ground-band energy levels are in good agreement with the observed data. The energy spacings of the levels in β and γ bands improved much better. The level ordering in the γ band of ^{156}Dy is also well reproduced.

To test the wave function, we also calculate the electromagnetic transitions. The general form of the $E2$ operator was used,

$$T^{(2)}=\alpha[(d^\dagger\times\tilde{s}+s^\dagger\times\tilde{d})^{(2)}+\beta(d^\dagger\times\tilde{d})^{(2)}] . \quad (2)$$

The parameters α and β were determined directly from least-squares fitting to the observed $B(E2)$ values. There are abundant observed $B(E2)$ data for the Sm, Gd, and Dy isotopes. We have calculated the $B(E2)$ both in *MI* and in *MII*. In the calculation of *MII*, it is interesting to note that a unified parameter $\alpha=0.165$ and $\beta=-0.105$ can be found to reproduce the $B(E2)$ values quite well for all the three isotope series. The resulting values of α and β obtained in our calculation agree approximately with those obtained in previous works.[14–16] A similar calculation is also performed in *MI* with unified parameters $\alpha=0.135$ and $\beta=0.115$. In general, the results in *MII* are much better than those in *MI*. Tables II, III, and IV show the $B(E2)$ values and some branching ratios. Some results of the previous works are also included for comparison. It can be seen from the tables that our calculated results in *MII* are in better agreement with the observed values than those in *MI*, especially for the transitions within the ground band. For the cross band transitions the agreement is also more satisfactory for the *MII* calculation.

IV. CONCLUSIONS

This paper presents a scheme to study the effect of subshell closure at $Z=64$ for the Sm, Gd, and Dy isotopes in the framework of IBA. We found that large discrepancies occur when $Z=50$ and $N=82$ is treated as a closed shell in this mass region. But this discrepancy can be improved very much when the partial subshell closure effect at $Z=64$ is taken into account. Therefore, in presenting a scheme for calculating nuclei near $Z=64$, it is necessary to recognize not only that there is a subshell closure at $Z=64$, but also that the neutron-proton interaction among nucleons will gradually wash out the subshell gap when there are more neutrons outside the $N=82$ shell. The existence of partial subshell closure causes ambiguities in counting boson numbers in the IBA-type calculations. Lacking a precise microscopic counting scheme, we may try to simulate these effects by introducing

effective boson numbers in the phenomenological calculations. It is found that the energy spectra and the $B(E2)$ values of the Sm, Gd, and Dy isotopes can be well explained when the partial subshell closure effect is taken into account by gradually increasing the effective proton bosons as N approaches 90. The results of our phenomenological calculations indicate that the effective boson approach in IBA is a rather encouraging approach. It may be helpful to study the foundation of this approach in a more microscopic model, such as the shell model which can manifest the effects of the n-p interaction explicitly.

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