國 立 交 通 大 學

科技管理研究所

博 士 論 文

多個低階決策單位二階層規劃應用之研究
－以預算分配為例

Application of a Bilevel Multi-Follower Decision-Making Model for Budget Allocation Problems

研究生：楊有恆

指導教授：虞孝成教授

中 華 民 國 一 ○ ○ 年 十 月

多個低階決策單位二階層規劃應用之研究
－以預算分配為例

Application of a Bilevel Multi-Follower Decision-Making Model for Budget Allocation Problems

研究生：楊有恆　　Student: Yeou-Herng Yang

指導教授：虞孝成教授　　Advisor: Dr. Hsiao-Cheng Yu

國 立 交 通 大 學

科技管理研究所

博 士 論 文

Dissertation

Submitted to Institute of Management of Technology

College of Management

National Chiao Tung University

in partial Fulfillment of the Requirements

for the Degree of Ph. D

in

Management of Technology

October 2011

Hsinchu, Taiwan, Republic of China

中 華 民 國 一 〇 〇 年 十 月

多個低階決策單位二階層規劃應用之研究
—以預算分配為例

研究生：楊有恆　　　　　　　　　指導教授：虞孝成教授

國立交通大學科技管理研究所博士班

摘　要

過去許多學者們為解決當時社會上所發生的一些經濟與管理問題，促使了數學規劃相關理論的發展與突破，而層級結構的規劃模式可以清楚描繪出組織體系之間的決策模式與運作過程，促使多層級規劃的概念逐漸展開。而二階層規劃係為解決二個層級結構間分權式決策的尋優問題，可視為一個多層級規劃的特殊型式，在上階層之決策者稱為領導者，而在下階層之決策者稱為追隨者。通常在真實的環境中，在下階層結構中會出現超過一個以上的追隨者，這種型式的層級結構則被稱為多個低階決策單位之二階層規劃。此種規劃所產生的問題，係上階層領導者的決策不僅會受到這些下階層追隨者的決策影響，同時也會被這些下階層決策單位之間的相互關係所影響。

因此，在本篇論文中，試圖建構一種屬於一個高階決策與多個低階決策系統的二階層預算分配模型，並針對此預算分配模型，分別探討多個低階決策系統之間合作、非合作與部分合作變數所衍生的關連性問題。而本文新設計的二階層預算分配運作機制，係上階層領導者從所有下階層決策單位提出的全部計畫中，挑選出能為組織創造最大價值的計畫，並給予合理的預算支援，惟上階層領導者在尋優的決策過程中，應同時考量並兼顧下階層各單位的良性競爭與均衡發展，以避免資源的錯置或不當浪費。

本論文最後將採取兩個階段來解決上述所建構多個低階決策單位之二階層預算分配的尋優問題。第一階段係上階層領導者運用改良式資料包絡法，從全部的計畫中初步篩選出具有效率的計畫，此為預算分配前之重要決策程序；第二階段則運用灰關聯方法處理多個低階決策單位之間的關係，並發展出一種啟發式演算法，試圖求解有關合作、非合作與部分合作之決策變數所衍生的二階層預算分配問題，以獲得上、下兩個階層所有的決策者均可接受的可行有效解，俾提供上階層領導者作出最適當的決策，而這個新發展的演算法比過去的典型演算方式更為簡單容易。

關鍵字： 二階層規劃、多個低階決策單位、預算分配、改良式資料包絡法、灰關聯分析、啟發式演算法

Application of a Bilevel Multi-Follower Decision-Making Model for Budget Allocation Problems

Student: Yeou-Herng Yang Advisor: Dr. Hsiao-Cheng Yu

Institute of Management of Technology
National Chiao-Tung University

Abstract

The bilevel programming (BLP) problem can be viewed as an uncooperative, two-person game in unbalanced economic markets. The BLP problem is a special case of multilevel programming (MLP) problems with a two-level structure. A decision maker at the upper level is known as the leader, and, at the lower level, is known as the follower. Usually, in a real world situation, there is more than one follower in the lower level; this type of the hierarchical structure is called a bilevel multi-follower (BLMF) decision-making model. Therefore, the leader's decision will be affected not only by the reactions of the followers, but also by the relationships among the followers.

In this thesis, the budget allocation model is a bilevel decision-making system with one single upper level decision maker and multiple lower level decision-making units. There are two types of BLMF models for the budget allocation that has been developed; one is a classical module that uses the uncooperative variable, and the other is a new module with partial cooperative variables. In the new bilevel budget allocation models, the upper level chooses the better projects from multiple proposals to maximize the value of the lower level projects and to minimize the ratio of the funding differences among the divisions.

The budget distribution problems are solved using two-stage methods. In stage one, a new generalized data envelopment analysis (GDEA), an improvement of data envelopment analysis (DEA), is developed. It is an important procedure of distribution to guarantee the quality of the proposals from the upper level decision maker. In the next stage, the grey relational analysis and a new heuristic algorithm take advantage of solving this problem and present a feasible solution of this particular model. The algorithm is efficient, and solutions are acceptable for real world situations. It is simpler than the classical solution methods are.

Keywords: bilevel programming, multi-follower, budget allocation, grey relational analysis, generalized data envelopment analysis (GDEA), heuristic algorithm

誌　謝

擔任一位職業軍人是個人從小以來的志向，成為一位稱職的專業軍官亦是不斷努力追求的目標，而攻讀博士學位卻不曾出現在過去人生奮鬥的選項中，但改變我人生的奇蹟竟真實的發生了，這一切將歸功於李建中上將、虞孝成教授及李宗耀博士，是您們開啟了我人生的另一扇窗，給了我不一樣的天空，引領我進入了一流名校交通大學的學術殿堂，盡情吸收各領域的專業知識，並提升了洞察事物及解決問題的能力，對個人言，實獲益斐淺，交大、科管所，這是一個值得我一輩子引以為傲的地方。

「學海無涯勤是岸，青雲有路志為梯」一直為求學期間自我惕勵之座右銘，回顧過往，歷經入學考試、修習課業、資格考試及國內外期刊發表等漫漫長路，終於在口試委員包曉天老師、黎正中老師、朱詣尹老師、洪志洋老師及黃仕斌老師的指正下，通過博士學位考試，除了無比的高興之外，內心盡是充滿感謝之情，在學期間，感謝洪志洋老師、袁建中老師、徐作聖老師、林亭汝老師及曾國雄老師無私無我的悉心教導，不但讓我獲得有關財務策略、技術前瞻、產業分析、國際行銷等跨領域知識與研究方法，並擴大了我的宏觀視野，此外，最要感謝的是引領我入門的指導教授虞孝成老師，除了傳道、授業、解惑外，其豐富的學養及溫仁恭儉讓的處世態度是我永遠學習的典範，而另一位指導教授劉宜欣老師不畏病魔纏身所帶來的苦痛，全心全意毫不藏私的將其所學傳授予我，這份恩情沒齒難忘，在學習的過程中，因為有駕人學長、文漢學長、昕翰、芃婷、家立、立翰、雅迪、嘉祈等人的輔導，得以順利修習各類課程，一併獻上感謝之意。

感謝國防部後備事務司前司長謝雲龍中將准予在職進修博士學位之機會，及軍動處長葉宜生將軍、副座梁存孝上校、胡靖民老師、楊柏川、柏傳琦、史志明、朱希承、吳炬宏上校及黃小玲小姐的鼓勵與支持，亦要感謝後備司令部副司令李銘藤中將的提攜及指揮官李海同將軍、陳嘉尚將軍、動管處長邱國樑將軍及旅長吳繼桐上校對我在職求學的寬容；另要感謝 903 旅主任潘岱勳上校等同仁、南區副指揮官陳之望、易乃文上校、主任楊安、宋卓立上校、軍監組長李國良上校及林鴻彬、陳國榮、權代平中校等戰訓、後通組伙伴在工作上的協助；在博士論文最後的定稿階段，感謝林振和、洪仕貴、莊沛芳、王詠令、林振裕上校及召整組的世輝、柏青、國棟、信匡、家惲、重堯、文良、家倫、敏凱、元正、岳謙的幫助，使我能在不影響工作的前題下，順利完成博士學業。我將一秉初衷，貢獻所學，認真執行每一項任務，期不辜負各級長官的教誨與栽培。

在這不算短的求學過程中，時而孤獨、時而徬徨、時而寂寞、時而無助，真心感謝結縭二十年的妻子邱堡櫻女士，默默的陪伴著我，適時給予支持與鼓勵，常常不辭辛勞新竹來回奔波，並對家庭及子女無微不致的照顧，讓我無後顧之憂，終能順利完成學業，謹以此博士論文獻給我的父母、岳父母及我最摯愛的家人，我要說，有你們真好。

楊有恆　謹誌

中華民國一〇〇年十月

于國立交通大學科技管理研究所

Table of Contents

List of Tables

List of Figures

Chapter 1 Introduction

1.1 Background and Motivations

In the past six, seven decades, many researchers have developed mathematical programming methods in order to solve many decision-making problems. These methods included linear programming (LP), integer linear programming (ILP), multi-objective programming (MOP), and so on. Nevertheless, these methods are not well suited to solving decision-making or management problems in multiple hierarchical structures.

At first, in the late 1970s, the hierarchical structure method was developed from the concept of game theory by decision-making researchers, whom started to pay more attention to multilevel programming (MLP) problems. Then, the MLP method became an even more important tool in dealing with management/hierarchical decision problems. However, development of solution methods in MLP was limited, and the application of MLP was premature at the time. In 1977, Karwan Mark H. and Bialas Wayne F. formed a Decision System Group at SUNY Buffallo to study hierarchical decision problems. In the 1980s, many fundamental results were published in major journals [2-8] by the Decision System Group.

Meanwhile, Shuh-Tzy Hsu [34, 36] and his associates developed the concept that decision makers of each level should optimize one's own variables/objectives and follow the hierarchy to reach optimum results. Furthermore, by considering a MLP to be a composition of a bilevel programming (BLP), one could reach an optimal solution of MLP. When MLP involves only two levels, it is called a bilevel programming (BLP). In 1994, Yi-Hsin Liu et al, [39, 40] pointed out the geometry of optimal solutions of bilevel linear programming (BLLP) problems. In the late 1990s, the researchers [9, 17, 22] further developed the BLLP model to better fit reality. More recently, Chenggen Shi et al, [11-15, 23] have proposed a series of extended new definition for BLLP theories to deal in terms of deficiency. In fact, the researchers were uninterested in the multiple followers and the applications for the BLP.

The improved model involved one leader and several followers. Bilevel programming with multiple followers is a complicated problem that will be studied in this dissertation. Chapter 2 will discuss the histories and development of the hierarchical optimization and review the mathematical definitions of the continuous and the discrete variables of BLP. Chapter 3 will show the bilevel programming within a multi-follower (BLMF) involving uncooperative condition relationship. Chapter 4 will explain the problems of the partial cooperative relationship under the followers of a BLMF. Finally, a new solution algorithm will be developed to optimize the budget allocation.

1.2 Fields of Study

The research fields of this dissertation include the elements shown below:

(a) A bilevel programming model that includes the upper level and the lower level.

(b) A bilevel, multi-follower decision-making system i.e. one single upper level decision maker (DM) with multiple lower level decision making units (DMUs).

(c) The models of the uncooperative and the partial cooperative relationship of a bilevel involving a multi-follower decision maker.

(d) The limited budget (or resource) allocation models that have been built.

(e) The 0/1 knapsack model for an integer programming problem.

(f) The discrete lower level decision variables.

1.3 Objectives of Research

At the present time, none of the existing solution algorithm for the BLP can be treated as the simplex method in the LP. Therefore, an efficient heuristic algorithm is considered in this dissertation. The objectives of this research are listed below:

(a) Develop a bilevel programming model with multiple followers to solve budget allocation problems.

(b) Develop a bilevel budget allocation model to optimize each individual level of objectives.

(c) Discuss the bilevel multiple follower 0/1 programming problems involving uncooperative and partial cooperative variables.

(d) Develop a new GDEA, an improvement of DEA. It is an important procedure of distribution to guarantee the quality of the proposals from the upper level decision maker.

(e) Apply the theory of the grey relationship to obtain the grey relationship grade among all the lower level DMUs in the partial cooperational relationship.

(f) Develop a new heuristic algorithm for the budget allocation solution. It should take advantage of the nature of the bilevel programming problem and offer a feasible solution for this particular model.

1.4 Flow Chart of this Work

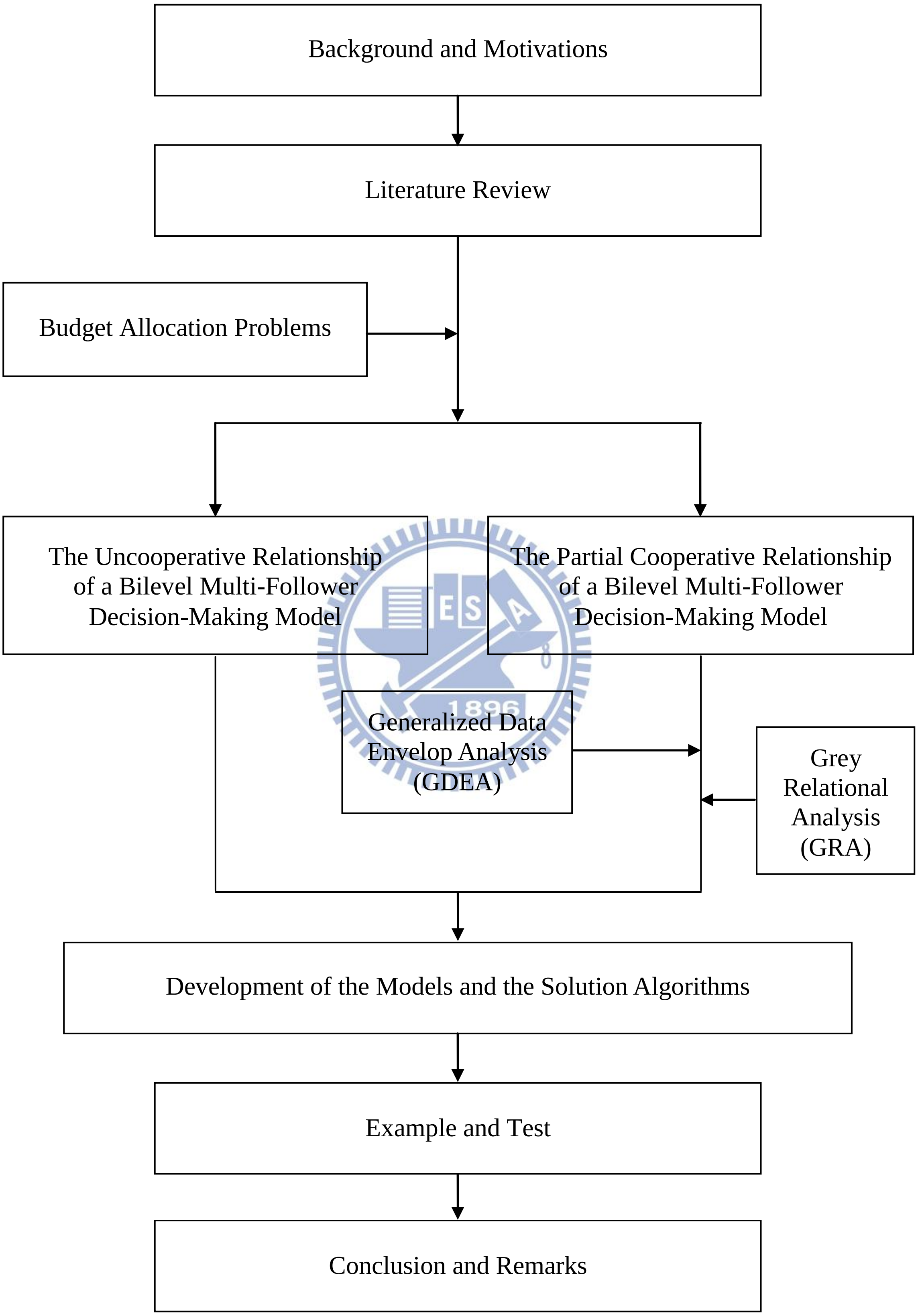


Figure 1.1 Research Flow Chart

Chapter 2 Literatures Review

2.1 History of the Hierarchical Optimization

In 1952, the bilevel programming (BLP) problem was viewed as an uncooperative, two-person game, as introduced by Von Stackelberg. In the basic model, the decision variables are partitioned among the players who seek to optimize their individual pay off functions. Perfect knowledge of information is assumed so that both players know the objective and feasible choices available to the other [26]. (Jonathan F. Bard, 1998)

Two-level planning had been proposed first by Kornai J. and Liptak T. H. in 1965 [27], and they presented a planning task formulated as a single linear programming problem of a maximizing type. This overall central information (OCI) problem is transformed into a two-level problem, in which the "central problem" is the need to create an allocation pattern where the sum of the maximal yields of the "sector problems" will be greatest. The general mode is written as equation (2.1):

$$\begin{aligned}
&\max\ c'x \quad Ax \le b,\ x \ge 0 \\
&\min\ y'b \quad y'A \ge c',\ y \ge o \\
\text{(OCI)}\quad & X = \{x : Ax \le b,\ x \ge 0\},\quad Y = \{y : y'A \ge c', y \ge 0\}, \\
& X^* = \{x^* : x^* \in X,\ c'x^* = \max_{x\in X} c'x\}, \\
& Y^* = \{y^* : y^* \in Y,\ y'^* b = \min_{y\in Y} y'b\}
\end{aligned} \tag{2.1}$$

be the forms of the primal and dual versions in the OCI problem[1]. The solution of the two-level program is achieved by a game-theoretical model.

In 1977, the Decision Systems Group began work on a class of *n*-person decentralized optimization problems that would be known as multilevel programming. Karwan Mark H. and Bialas Wayne F. (1978) [4] released their first report on multilevel programming for the optimization of hierarchical systems. The multilevel optimization techniques parcel out

[1] The primal variable x is called the OCI program; the dual variable y is the OCI shadow price system. Let X denotes the set of feasible OCI programs and X^* the set of optimal OCI programs, let Y be the set of feasible OCI shadow price systems and Y^* the set of optimal OCI shadow price systems.

control over the decision variables of an optimization problem among the decision makers. An important feature is that a planner at one level of the decision hierarchy may have his objective function determined, in part, by variables controlled at other levels.

Many decision making problems require compromises among the objectives of several interacting individuals or agencies. Often, these decision makers are arranged within an administrative or hierarchical structure with independent and perhaps conflicting objectives at each level. For example, the policies of the federal government affect the strategies of state officials. Those decisions, in turn, affect the activities of local governments. A multiple hierarchical structure of administrative is illustrated in Fig. 2.1 [2-5] (Bialas and Karwan, 1978-2002).

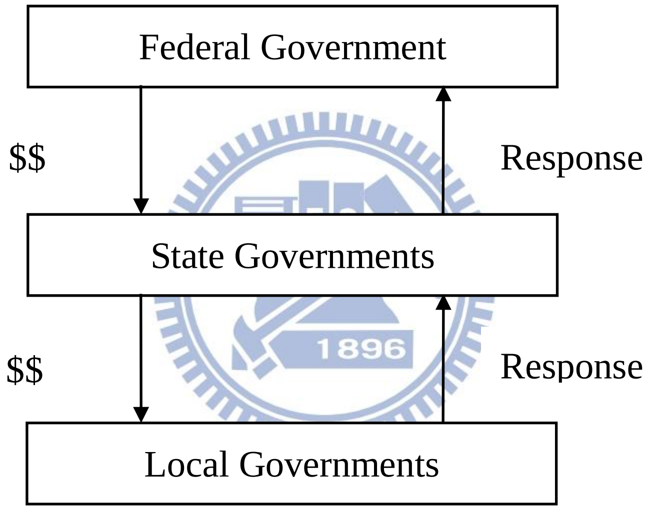


Figure 2.1 A Multiple Hierarchical Structure of Administration

Source: Bialas Wayne F., “Multilevel Optimization”, State University of New York at Buffalo, Industrial Engineering, 2002.

Bialas and Karwan (1984) have noted the following common characteristics of multilevel organizations [7]:

(a) The system has interacting decision-making units within a predominantly hierarchical structure.

(b) Each subordinate/lower level executes its policies after, and in view of, the decisions of the supreme/upper level.

(c) Each unit maximizes net benefits independently of other units, but is affected by the actions of other units externally.

(d) The external effect on a decision maker's problem is reflected in both his objective function and his set of feasible decisions.

When the hierarchical structure is restricted to only two levels, it is categorized as bilevel programming (BLP). The equation of a linear BLP problem is written below:

$$
\text{(BLP)} \qquad
\begin{aligned}
&\max_{x_2} \; c_{21}x_1 + c_{22}x_2 \quad \text{where } x_1 \text{ solves} \\
&\quad \max_{x_1|x_2} \; c_{11}x_1 + c_{12}x_2 \\
&\quad \text{s.t.} \quad A_1x_1 + A_2x_2 \le b \\
&\qquad\qquad x_1, x_2 \ge 0
\end{aligned}
\qquad (2.2)
$$

Jonathan F. Bard (1983) [24, 25] developed the bilevel multidivisional programming (BLMP) problem as a model for a decentralized organization. In particular, the model has a hierarchical structure comprised of one leader unit and M follower units. To formulate the problem mathematically, we assume the upper level decision maker wishes to maximize his objective function F and each of the M follower wishes to maximize his own objective function f^i. The corporate unit has first choice and selects a strategy $x^0 \in S^0$, followed by the M subordinate units that select their strategies $x^i \in S^i$ simultaneously. The strategy sets will be given explicit representation:

$$S^0 = \{x^0 : g^0(x^0) \le 0\}, \quad S^i = \{x^i : g^i(x^i) \le 0\}, \quad i = 1,2,\ldots,M\,.$$

With the BLMP problem defined as equation (2.3):

$$
\text{(BLMP)} \qquad
\begin{aligned}
&\max_{x^0} \; (F(x^0, x^{\bar{0}}) : g^0(x^0) \le 0 \\
&\text{where} \quad x^{\bar{0}} \equiv (x^1, x^2 \ldots, x^M) \text{ solves} \\
&\quad \max_{x^i} \; f^i(x^i, x^{\bar{i}}) \\
&\qquad \text{s.t.} \quad g^i(x^i, x^{\bar{i}}) \le 0 \\
&\qquad\qquad i = 1,2,\ldots,M.
\end{aligned}
\qquad (2.3)
$$

The main feature of the model[2] provides pairwise perfect information between leader and follower level payoffs, while permitting each unit to pursue its own goal.

Ben-Ayed O., Boyce D. E., and Blair C. E. (1988) [1] have applied bilevel formulations to the network design problem arising in transportation systems. In the accompanying formulation, a central planner controls investment costs at the system level, while operational costs depend on traffic flows, which are determined by the individual user's route selection. Assuming users make decisions to maximize their specific utility functions; their choices do not necessarily coincide with the choices that are optimal for the system.

More recently, Chenggen Shi et al, [11-15, 23] have proposed that the leader-level constraint functions in a linear bilevel programming problem are arbitrary in form. Shi, Lu, and Zhang provide not only a new definition for linear BLP theories to deal in terms of deficiency, but they also develop a series of extended approaches to solve the problem.

2.2 Theory of the Bilevel Programming

The bilevel programming (BLP) problem is a special case of multilevel programming (MLP) problems with a structure of two levels. The linear bilevel programming techniques were mainly developed for solving decentralized management problems with decision makers in a hierarchical structure. A decision maker at an upper level is known as the leader, and, at the lower level, is known as the follower [7] (Bialas and Karwan, 1984). Each decision maker (leader or follower) tries to optimize his own objective function with or without considering the objective of the other levels, but the decision of each level affects the objective optimization of the other level. Therefore, the leader may influence the behavior of the follower without completely controlling the follower's action. At the same time, the leader may be affected by the follower's behavior [26]. (Bard, 1998)

---

2 Notice that g^i is a function not only of x^i but each of the other decision variables, call them $x^{\bar{i}}$. This suggests the useful notation $x \equiv \{x^i, x^{\bar{i}}\}$.

For the special case of n = 2 levels, a bilevel programming problem is a problem with two decision makers. The decision maker 2 is the leader who controls x_2 and maximizes $c_2x = c_{21}x_1 + c_{22}x_2$, and the decision maker 1 is the follower who controls x_1 and maximizes $c_1x = c_{11}x_1 + c_{12}x_2$. The leader must announce his choice of x_2 before the follower selects x_1. Fig. 2.2 represents an example of a rational reaction set for a bilevel problem [2] (Bialas Wayne F., 2002).

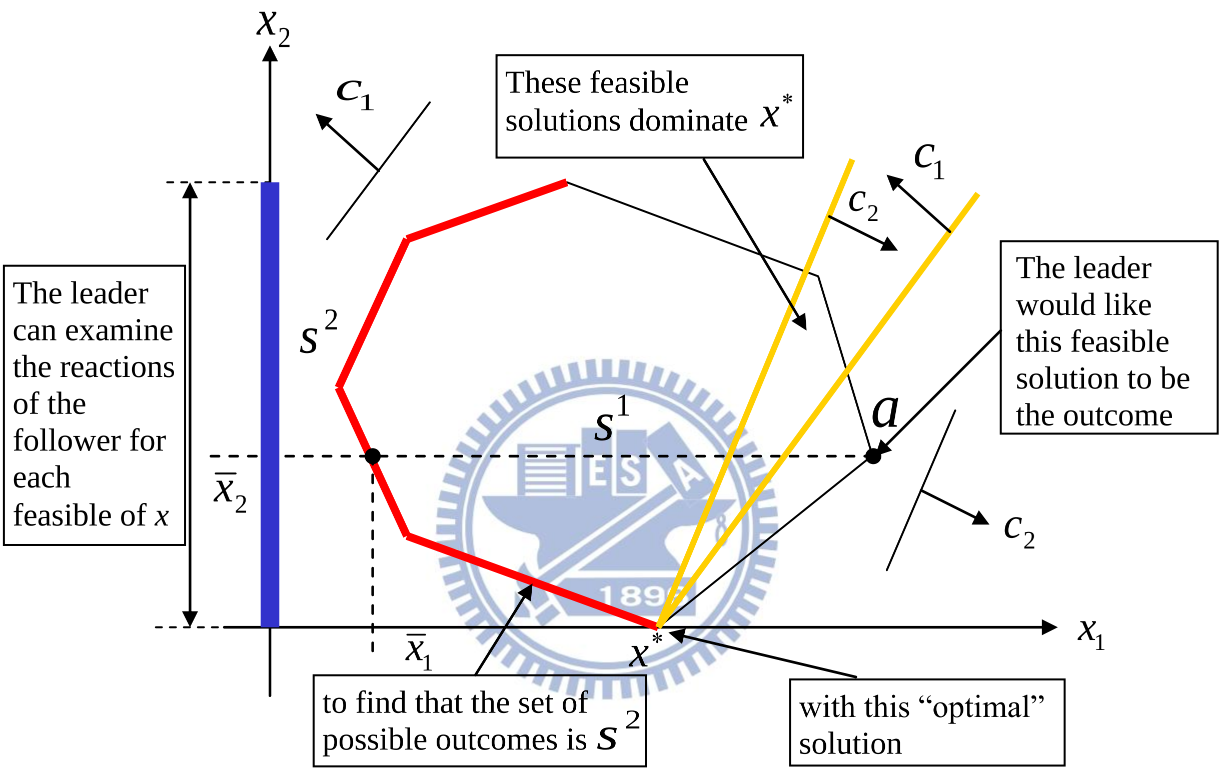


Figure 2.2 Example of a Rational Reaction Set for a Bilevel Problem

Source: Bialas Wayne F., "Multilevel Mathematical Programming: An Introduction", State University of New York at Buffalo, 2002.

The leader would like this feasible solution a to be the outcome, but if he chooses this value $\bar{x}_2$ at axis x_2, then the follower will maximize $c_1x = c_{11}x_1 + c_{12}x_2$ and respond with this axis x_1. $\bar{x}_1 = \psi(\bar{x}_2)$ as the follower's rational response, and it may not be a single value (see Fig. 2.3 A geometric relationship of interactions of the leader and the follower).

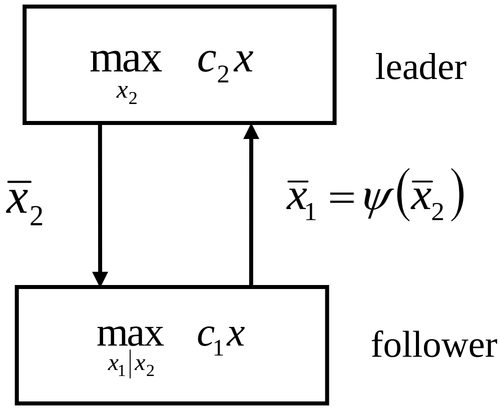


Figure 2.3 A Geometric Relationship of Interactions of the Leader and the Follower

For a given x_2 from the leader, the follower solves in (2.4)

$$
\text{(Follower)} \quad \begin{aligned} &\max_{x_1|x_2} c_{11}x_1 + c_{12}x_2 \\ &\text{s.t.} \quad A_1x_1 \le b - A_2x_2 \\ &\qquad x_1 \ge 0 \end{aligned} \tag{2.4}
$$

the leader can examine the reactions of the follower for each feasible choice of x_2 and find that the set of possible outcomes is S^2. The leader's problem is actually to find in (2.5)

$$
\text{(Leader)} \quad \begin{aligned} &\max_{x_2} c_{21}x_1 + c_{22}x_2 \\ &\text{s.t.} \quad (x_1, x_2) \in S^2, \quad \text{where } x_1 \text{ solves} \end{aligned} \tag{2.5}
$$

$x^* = (x_1^*, x_2^*)$ with this optimal solution in the bilevel programming problem. Even in the linear case, the bilevel programming problem is a non-convex optimization problem. The solution represents the stable outcome of the bilevel decision process that need not be Pareto optimal[3].

There is an important concept regarding a model with two (or more) objectives. Usually a model with two objectives is either a bilevel (hierarchical environment) or bi-objective (optimize two objectives simultaneously), and these two models are different.

[3] Given a set of alternative allocations of goods or income for a set of individuals, a change from one allocation to another that can make at least one individual better off without making any other individual worse off is called a Pareto improvement. An allocation is Pareto efficient or Pareto optimal when no further Pareto improvements can be made. The Pareto optimum is a state of allocating the resources where it is no longer possible to make anyone better off without making someone worse off.

2.3 The Variables of the Bilevel Programming

2.3.1 Continuous Variables

The hierarchical structure of the BLP problem imposes a strict order on the selection of the decision variables each planner controls. That is, the follower decision level executes its policies after, and in view of, the decision of the leader level, and the leader level optimizes its objective independently over the reactions from the follower level [39, 40]. (Yi-Hsin Liu, Stephen M. Hart and Thomas H. Spencer, 1994-1995)

Let the vectors $a,c,x \in R^{n_1}$, $b,d,y \in R^{n_2}$, and $u \in R^m$. Further, let A and B be two matrices with size $m{\times}n_1$ and $m{\times}n_2$, respectively. Given this, the BLP problem is the equation (2.6):

$$\text{(BLP I)} \qquad \begin{aligned} &\max_x \ F(x,y) = ax + by \quad \text{where } \ y \ \text{ solves} \\ &\max_y \ f(x,y) = cx + dy \\ &\text{s.t.} \quad (x,y) \in S \end{aligned} \tag{2.6}$$

The constraint set $S = \{(x,y): Ax + By \le u, (x, y \ge 0)\}$ is assumed to be a bounded, nonempty subset of $R^{n_1+n_2}$.

Since S is assumed bounded and nonempty, for each $\bar{x}$, the follower planner's problem,

$$\begin{aligned} &\max_y \ f(x,y) = dy \\ &\text{s.t.} \quad By \le u - A\bar{x}, \\ &\qquad\ y \ge 0 \end{aligned}$$

has an optimal solution. The set of all optimal solutions with respect to this $\bar{x}$ is called the feasible reaction set for the follower planner and is denoted $Y(\bar{x})$. The leader planner's feasible region, also referred to as the set of all rational reactions of f over S, is defined as

$$\psi(S) = \{(x,y): (x,y) \in S, \ \ y \in Y(x)\}.$$

a point $(x', y') \in \psi(S)$ such that $ax' + by' \geq ax'' + by''$ for all $(x'', y'') \in \psi(S)$ is called an optimal solution of the BLP.

The geometric properties of BLP problem have been well explored, and the pertinent ones are presented below:

(a) $\psi(S)$ is a connected subset of S.

(b) If there is an optimal solution to the BLP problem, then there is an extreme point of $\psi(S)$ that is an optimal solution of the BLP problem, and hence there is an extreme point of S that is an optimal solution of the BLP problem.

The following theorem provides a geometric characterization of an optimal solution of a BLP problem.

Theorem 1. If an optimal solution of the leader objective function over S is in $\psi(S)$, then it is an optimal solution to the BLP problem.

Theorem 2. If there exists an optimal solution of the leader objective function over S not in $\psi(S)$, there exists a boundary feasible extreme point that optimizes the BLP problem.

Example 1:

$$\max_x \; x + 3y \quad \text{where } y \text{ solves}$$

$$\max_y \; -y$$

$$\text{s.t.} \quad -x + y \leq 3$$

$$x + 2y \leq 12$$

$$4x - y \leq 12$$

$$x \geq 0, \; y \geq 0$$

The illustration of the method is presented in Fig. 2.4. It is easy to see that $\psi(S)$ is the set of points on line segments connecting (0, 0), (3, 0), and (4, 4), while the optimal solution of this BLP problem is the point (4, 4), which is a boundary feasible extreme point.

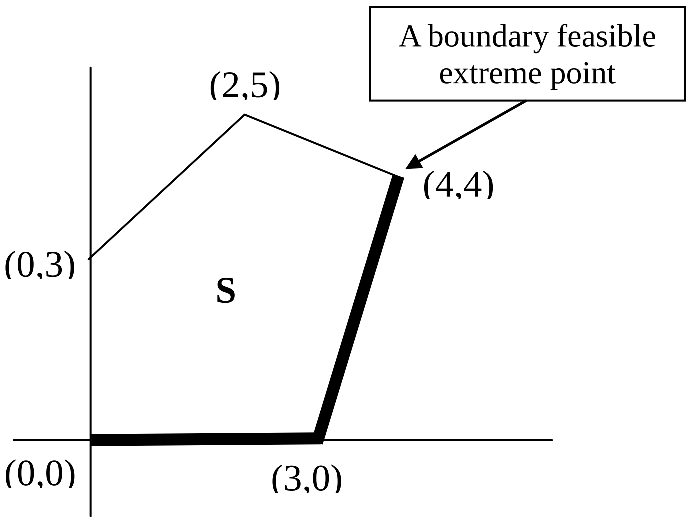


Figure 2.4 A Optimal Solution to the Linear BLP

Corresponding to the equation (2.6) and example 1, Bard (1998) [26] gave the following basic definition for a linear BLP solution:

(a) Constraint region of the BLP problem:

$$S = \{(x, y): x \in X,\ y \in Y,\ Ax + By \le u,\ (x, y \ge 0)\}.$$

(b) Feasible set for the follower for each fixed $x \in X$:

$$S(x) = \{y \in Y : By \le u - Ax\}$$

(c) Projection of S onto the leader's decision space:

$$S(X) = \{x \in X : \exists\, y \in Y,\ By \le u - Ax\}$$

(d) Follower's rational reaction set for $x \in S(X)$:

$$P(x) = \{y \in Y : y \in \arg\max[f(x, \hat{y}) : \hat{y} \in S(x)]\}$$

where $\arg\max[f(x, \hat{y}) : \hat{y} \in S(x)] = \{y \in S(x) : f(x, y) \ge f(x, \hat{y}),\ \hat{y} \in S(x)\}$

(e) Inducible region:

$$IR = \{(x, y) : (x, y) \in S,\ y \in P(x)\}$$

The rational reaction set $P(x)$ defines the response while the inducible region IR represents the set over which the leader may optimize his objective. Thus, in teams of the above notations, the linear BLP problem can be written as:

$$\max\{F(x, y) : (x, y) \in IR\}$$

To ensure that Fig. 2.5 has an optimal solution, Bard (1998) [26] gave the following assumption:

(a) S is nonempty and compact.

(b) For all decisions taken by the leader, the follower has some room to respond, i.e., $P(x) \neq \phi$.

(c) $P(x)$ is a point-to-point map.

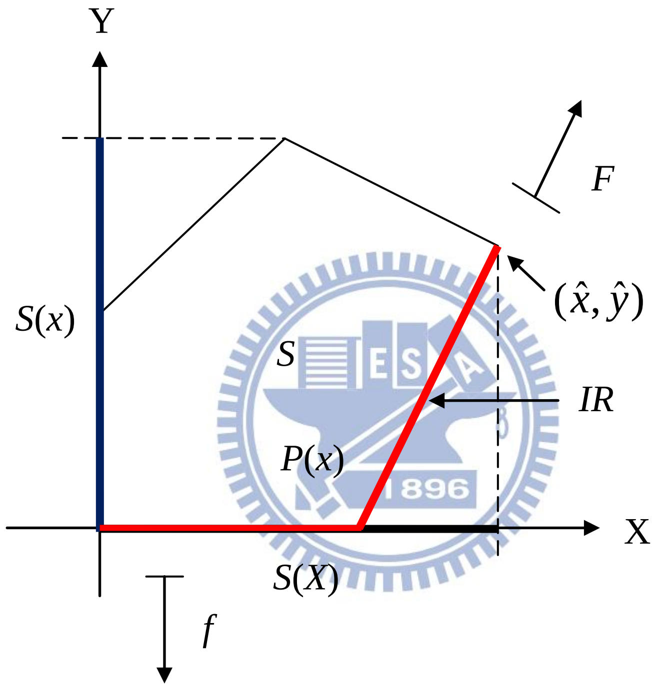


Figure 2.5 Illustration of BLP Solution by using Definition

2.3.2 Discrete Variables

In many optimization problems, a subset of the variables is restricted to only take on discrete value. This can complicate the problem. To specify the model, let x_1 be an n_1-dimensional vector of continuous variables and x_2 be an n_2-dimensional vector of discrete variable, where $x=(x_1,x_2)$ and $n=n_1+n_2$. Similarly, define y_1 as an m_1-dimensional vector of continuous variables and y_2 as an m_2-dimensional vector of discrete variable, where $y=(y_1,y_2)$ and $m=m_1+m_2$. This leads to

$$
\text{(BLP II)} \quad
\begin{aligned}
&\min_{x} \ F(x,y) = c_{11}x_1 + c_{12}x_2 + d_{11}y_1 + d_{12}y_2 \\
&\text{s.t.} \quad A_{11}x_1 + A_{12}x_2 + B_{11}y_1 + B_{12}y_2 \le b_1 \\
&\qquad x_1 \ge 0, \ \ y_1 \ge 0 \quad \text{integer} \\
&\quad \min_{y} \ f(y) = d_{21}y_1 + d_{22}y_2 \\
&\quad \text{s.t.} \quad A_{21}x_1 + A_{22}x_2 + B_{21}y_1 + B_{22}y_2 \le b_2 \\
&\qquad\quad x_2 \ge 0, \ \ y_2 \ge 0 \quad \text{integer}
\end{aligned}
\tag{2.7}
$$

where all vectors and matrices are of the conformal dimension, and the linear terms in *x* have been omitted from the follower's objective in function (2.7) [26] (Bard, 1998).

Note that it may be desirable to explicit include additional restrictions, such as upper and lower bounds, on the variables. In this case, let $x \in X = \{x : l_j^1 \le x_j \le u_j^1, \ j = 1,2,\ldots,n\}$ and $y \in Y = \{y : l_j^2 \le y_j \le u_j^2, \ j = 1,2,\ldots,m\}$.

Bard has investigated the properties of the zero-one linear BLP problem when some or all variables are restricted to binary values. Based on the specific instances of (2.7), it will be convenient to consider the problem in the form of (2.6) without reference to which variables are continuous and which are discrete; i.e.,

$$
\begin{aligned}
&\min_{x \in K} \ F(x,y) = c_1 x + d_1 y \\
&\text{subject to} \quad A_1 x + B_1 y \le b_1 \quad \text{where } \ y \ \text{ solves} \\
&\quad \min_{y \in Y} \ f(x,y) = d_2 y \\
&\quad \text{subject to} \quad A_2 x + B_2 y \le b_2 \\
&\qquad\qquad x \ge 0, \ \ y \ge 0 \quad \text{integer}
\end{aligned}
\tag{2.8}
$$

where $c_1 \in R^n$, $d_1, d_2 \in R^m$, $b_1 \in R^p$, $b_2 \in R^q$, $A_1 \in R^{p \times n}$, $B_1 \in R^{p \times m}$, $A_2 \in R^{q \times n}$, $B_2 \in R^{q \times m}$, $X \subseteq R^n$ and $Y \subseteq R^m$.

In addition to the definition in section 2.3.1, let $S_L(y) = \{x \in X : A_2 x \le b_2 - B_{2y}\}$, for all values of $y \in Y$, and $S_U(y) = \{(x,y) : A_1 x + B_{1y} \le b_1\}$.

For each $x \in X$, it will be assumed that the optimal solution of the lower level problem is unique. Along with the linear bilevel programming problem (L-BLPP) where $X=R^n$ and $Y=R^m$, there are three models as shown below:

(a) Discrete linear bilevel programming (DL-BLP) problem, where[4] $X=B^n$ and $Y=B^m$;

(b) Discrete-continuous linear bilevel (DCL-BLP) problem, where $X=B^n$ and $Y=R^m$; and

(c) Continuous-discrete linear bilevel (CDL-BLP) problem, where $X=R^n$ and $Y=B^m$.

Figure 2.6 depicts the inducible regions associated with the four problems.

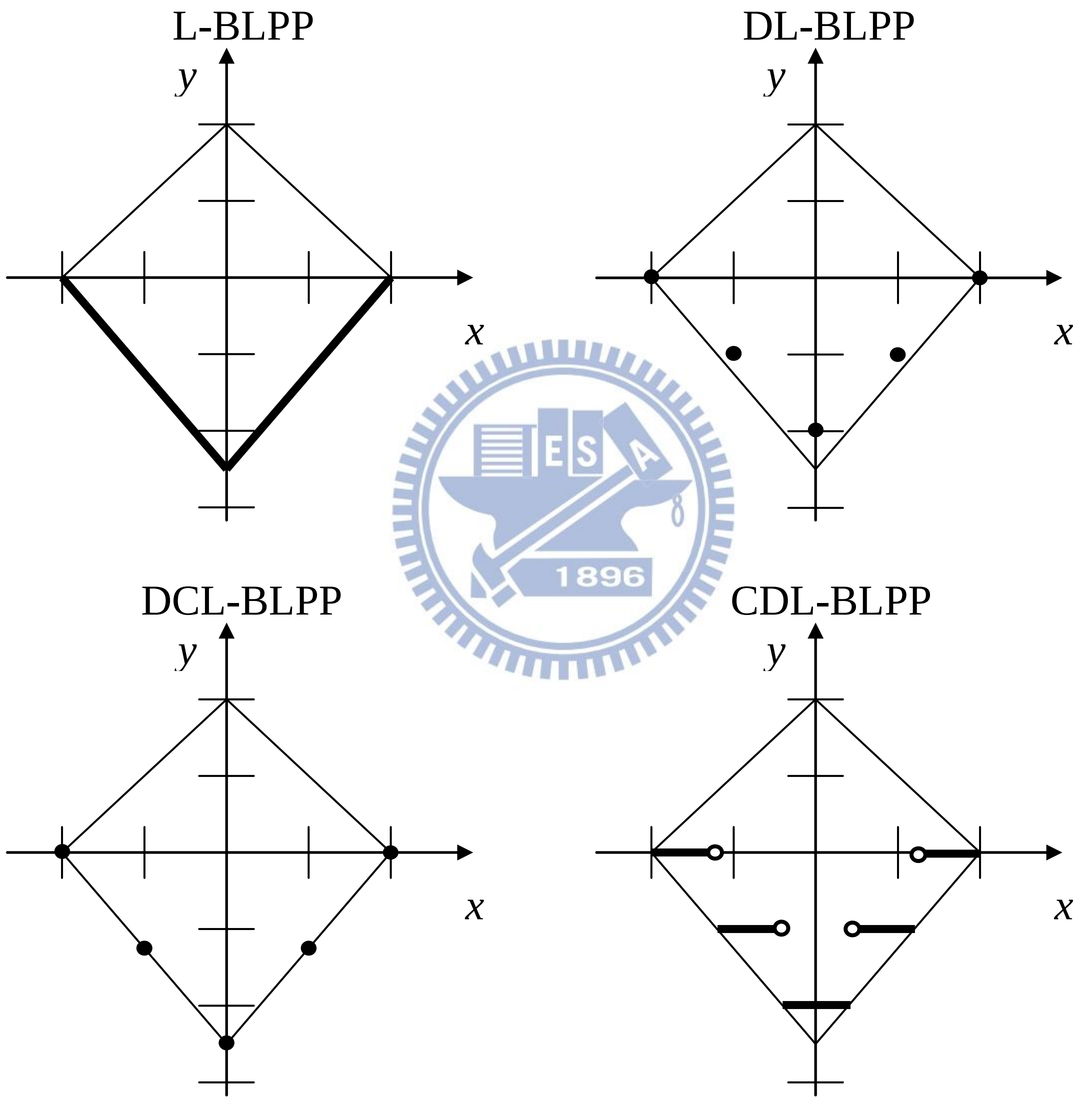


Figure 2.6 Inducible Regions for Versions of the Linear BLPP

Source: Bard Jonathan F., "Practical Bilevel Optimization: Algorithms and Applications", Kluwer Academic Publishers, Netherlands, pp.235, 1998.

[4] Where B^n is set of all binary n-tuples.

Corresponding to Fig. 2.6, Bard (1998) [26] presented some properties and theorems as shown below:

Property 1: If $S_U = R^{n+m}$, then IR is nonempty if $S \neq \phi$. If $S_U \neq R^{n+m}$, then IR is nonempty if there exists a $\bar{x} \in X$ such that $(\bar{x}, \bar{y}) \in S_U$.

Property 2: The inducible regions of DCL-BLPP and DL-BLPP are included in the inducible regions of L-BLPP and CDL-BLPP, respectively.

Property 3: For the L-BLPP, let S be a bounded set, i.e., a polytope. If $S_U = R^{n+m}$, then L-BLPP, DL-BLPP, and DCL-BLPP have an optimal solution if $S \neq \phi$. If $S_U \neq R^{n+m}$, then L-BLPP, DL-BLPP, and DCL-BLPP have an optimal solution if exists a $\bar{x} \in X$ such that $(\bar{x}, \bar{y}) \in S_U$.

Theorem: Let $S_U = R^{n+m}$, $S \neq \phi$ and suppose there exists an optimal solution (x^*, y^*) to CDL-BLLP. Then (x^*, y^*) is a boundary (bd) point of S.

Algorithms designed to solve integer programs generally rely on some separation, relaxation, and understanding to construct ever-tighter bounds on the solution.

2.4 Summaries

The bilevel programming problem can be viewed as an uncooperative, two-person game; in this model, the players seek to optimize their individual pay off functions. Since 1977, Karwan and Bialas formed a Decision System Group to study hierarchical decision problems. Many fundamental results were published in major journals by this group. An important feature is that a planner at one level of the decision hierarchy may have his objective function determined, in part, by variables controlled at other levels.

The BLP problem is a special case of the multilevel programming problem with a two-level structure. The BLP problem imposes a strict order on the selection of the decision variables each planner controls. That is, the follower decision level executes its policies after, and in view of, the decision of the leader level, and the leader level optimizes its objective independently over the reactions from the follower level.

The BLP problems that include continuous and discrete variables are examined in this chapter. The principal concepts and a formal definition of the continuous variables are presented by Yi-Hsin Liu. There are two geometric properties of the problem as shown:

(a) $\psi(S)$ is a connected subset of S.

(b) If there is an optimal solution to the BLP problem, then there is an extreme point of $\psi(S)$ that is an optimal solution of the BLP problem, and hence there is an extreme point of *S* that is an optimal solution of the BLP problem.

In many optimization problems, a subset of the variables is restricted to only take on discrete value. Bard has investigated the properties of the zero-one linear BLP problem when some or all variables are restricted to binary values. There are three models shown below:

(a) Discrete linear bilevel programming (DL-BLP) problem.

(b) Discrete-continuous linear bilevel (DCL-BLP) problem.

(c) Continuous-discrete linear bilevel (CDL-BLP) problem.

The common methods used to solve a linear bilevel programming problem and are related to the continuous variables are the *k*th-Best algorithm and Kuhn-Tucker approach. In addition, another method related to the discrete variable is Branch and Bound notation. All of the detail algorithms can be found in Bard's work (1998) [26] "Practical Bilevel Optimization: Algorithms and Applications".

Chapter 3 The Uncooperative Relationship of a Bilevel Multi-Follower Decision-Making Model

3.1 Introduction

A bilevel programming (BLP) problem is a special case of the multilevel programming (MLP) problem with a structure of two levels. The bilevel programming techniques are mainly developed for solving decentralized management problems with decision makers in a hierarchical organization (see Chapter 2). A decision maker at an upper level is known as the leader, and at the lower level is the follower [7] (Bialas Wayne F. and Karwan Mark H., 1984). Each decision maker (DM) (leader or follower) optimizes his/her own objective function with or without considering the objective of the other level, but the decision of each level affects the optimization of the other level. Therefore, the leader may influence the behavior of the follower without completely controlling the follower's action. At the same time, the leader may be affected by the follower's behavior [26] (Jonathan F. Bard, 1998).

Usually, in a real world situation, there is more than one follower in the lower level; this type of the hierarchical structure is called a bilevel multi-follower (BLMF) decision-making model. However, the different relationships among these followers might force the leader to use multiple different processes in deriving an optimal solution for leader decision making. Therefore, the leader's decision will be affected not only by the reactions of these followers, but also by the relationships among these followers. In general, there are three kinds of relationships among the followers; these relationships are determined by how decision variables [23] (Jie Lu, et al.) are shared among the followers. These scenarios are as follows:

(a) The cooperative situation where the followers totally share the decision variables in their objectives and constraints.

(b) The uncooperative situation where there is no sharing of decision variables among the followers.

(c) The partial cooperative situation where the followers partially share decision variables in their objectives and/or constraints.

If the cooperative situation among various followers in case (a) takes place, then the problem is equivalent to a situation where all units of the lower level act as a single unit with only one objective function. In such cases, the linear BLMF programming problem is reduced to a linear BLP one [34] (Shuh-Tzy Hsu, An-Der Huang and Ue-Pyng Wen, 1993).

The case (b) above will be discussed carefully in this chapter. The most problematic situation for a hierarchical structure of linear bilevel multi-follower decision-making is the uncooperative situation where there is no sharing of decision variables among the followers [34, 23] (e.g., Hsu S. T., Hung A. D., and Wen U. P., 1993; Lu J., Shi C., and Zhang G., 2006). This situation can be formulated as follows:

For $x \in X \subset R^n$, $y_i \in Y_i \subset R^{m_i}$, $Y = (Y_1, Y_2, ..., Y_K)^T$, $F : X \times Y_1 \times ... \times Y_K \to R^1$, $f_i : X \times Y_i \to R^1$ and $i = 1,2,...,K$, a linear BLMF decision problem in which $K\,(\geq 2)$ followers are involved and there is no shared decision variable, objective functions, and constraint function among them is defined as follows:

$$\min_{x \in X} \quad F(x, y_1, ..., y_k) = cx + \sum_{s=1}^{K} d_s y_s$$
$$\text{s.t.} \quad Ax + \sum_{s=1}^{K} B_s y_s \leq b \tag{3.1}$$

where $y_i\ (i = 1,2,...,K)$, for each value of *x*, is the solution of the lower level problem:

$$\max \quad f_i(x, y_i) = c_i x + e_i y_i$$
$$\text{s.t.} \quad A_i x + C_i y_i \leq b_i \tag{3.2}$$

where $c \in R^n$, $c_i \in R^n$, $d_i \in R^{m_i}$, $e_i \in R^{m_i}$, $b \in R^p$, $b_i \in R^{q_i}$, $A \in R^{p \times n}$, $B \in R^{p \times m}$, $B_i \in R^{p \times m_i}$, $A_i \in R^{q_i \times n}$, $C_i \in R^{q_i \times m_i}$, $i = 1,2,...K$.

According to the definitions above, there are four characteristics of the uncooperative relationship of a linear BLMF (BLMF-UC) decision-making problem as follows:

(a) A bilevel decision-making system with one single upper level DMU and multiple lower level DMUs.

(b) The upper level DM controls a set of variables, while each lower level DMU controls one's own decision variables.

(c) Each lower level DM's decision variables are independent, i.e. uncooperative.

(d) Each lower level DM optimizes its own objective, hence, the lower level solves multiple objective programming problems.

Based on the definitions and characteristics stated above, a budget allocation model is constructed to discuss the nature of BLMF-UC. Since the structure of the budget allocation problem is hierarchal with multi-followers, this problem not only involves two decision-making levels, but it also is an uncooperative situation where there is no sharing of decision variables among the followers.

Generally speaking, the structure of the budget allocation problem studied in this work is a bilevel programming problem with 0/1 variables, which is difficult to solve. Therefore, the purposes of this chapter are to formulate a new model and to propose an efficient heuristic algorithm for this problem.

3.2 Budget allocation problems

An organization's management requires information about the resources available to achieve the organization's purpose. Resources are acquired, allocated, and manipulated under the manager's control. The organization's purpose is sometimes stated as its vision or goal. The vision or goal is attained through the achievement of multiple, numerous, and often competing objectives [31] (Richard O. Mason and Swanson E. Burton, 1979).

There are a variety of ways to achieve a systematic and rational allocation of resources that will provide a competitive advantage to an organization. The methodology discussed below is quite flexible and can be adapted to a wide variety of situations and constraints. The methodology consists of the following steps [19] (Ernest H. Forman and Mary Ann Selly, 2001):

Step 1: Identify/design alternatives

Expertise in the art and science of identifying and/or designing alternatives lies in the domain of the decision makers, who have many years of study and experience with which to act on this task. The goal here is to help them attain better measurements and syntheses in order to better capitalize on their knowledge and experience.

Step 2: Identify and structure the organization's goals and objectives

The main message is that decisions must be made on the basis of achievement of objectives with resource allocation decisions. And so the entire enterprise's goals and objectives must be addressed. The executives understand these goals and objectives and can best make judgments about the relative importance of the main organizational objectives and, possibly, the sub-objectives.

Step 3: Prioritize the objectives and sub-objectives

The relative importance of the objectives and sub-objectives must be established in order to make a rational allocation of resources. The prioritization of the organization's objectives during the resource allocation process will lead to another important advantage—in the top management's quest for excellence, one will be able to respond to shifts in direction brought about by changes in the environment and competitive forces.

Step 4: Measure alternative's contribution

Having prioritized the organization's objectives and sub-objectives, the next step is to evaluate how much each proposed activity (or each possible level of funding for each activity) would contribute to each level's objectives.

Step 5: Find the best combination of alternatives

After prioritizing the organization's objectives and sub-objectives and rating the contribution of the competing activities, the lowest level objectives, etc., we have ratio scale measures of the relative contribution of each alternative combination to the organization's overall objectives.

Otherwise, in applied mathematics, the resource allocation (RA) problem is an optimization problem with a single constraint. Given a fixed amount of the resource *B* (this is the constraint), DM is asked to determine its allocation to *n* activities in such a way that the objective function under consideration is optimized. The simple structure of the resource allocation problem discussed is generally formulated as (3.3) [35] (Toshihide Ibaraki and Naoki Katoh, 1988):

$$
\text{(RA)}\quad
\begin{aligned}
&\text{maximize} && f(x_1, x_2, \ldots, x_n) \\
&\text{subject to} && \sum_{j=1}^{n} x_j \le B \\
& && x_j \ge 0, \quad j = 1, 2, \ldots, n.
\end{aligned}
\tag{3.3}
$$

That is, given one type of resource whose total amount is equal to *B*, DM wants to allocate it to *n* activities so that the objective value $f(x_1, x_2, \ldots, x_n)$ becomes as large as possible. The objective value may be interpreted as the profit or reward, and it is natural to maximize *f*. DM will sometimes considers minimization problems such as the cost, time, or loss.

In general, limited resources must be allocated among several activities, and linear programming often solves resource allocation problems. To use linear programming to allocate resources, Wayne L. Winston in 1991 [37] made three vital assumptions:

(a) The amount of a resource assigned to an activity may be any non-negative number.

(b) The benefit obtained from each activity is proportional to the amount of the resource assigned to the activity.

(c) The benefit obtained from more than one activity is the sum of the benefits obtained from the individual activities.

Wayne L. Winston (1991) [37] had considered a generalized resource allocation (GRA) problem. Suppose that the organization has *B* units of resource available and *n* activities to which the resource can be allocated. If activity *j* is implemented at a level x_j (assume x_j must be a nonnegative integer), then $g_j(x_j)$ units of the resource are used by activity *j*, and a benefit $v_j(x_j)$ is obtained. The problem of determining the allocation of resources that maximizes total benefit that is subject to the limited resource available may be written as the following equation (3.4):

$$(\text{GRA})\quad \begin{aligned} \max\ & \sum_{j=1}^{n} v_j(x_j) \\ \text{s.t.}\ & \sum_{j=1}^{n} g_j(x_j) \le B \\ & x_j \ge 0; \quad j = 1,2,\ldots,n \\ & x_j : \text{integer} \end{aligned} \tag{3.4}$$

The other important and very common algorism uses 0-1 variable to represent binary choice. Consider an event that may or may not occur and suppose that it is part of the problem in deciding between these two possibilities. To model such a binary, variable *x* is used and let

$$x = \begin{cases} 1 & \text{if the event occurs} \\ 0 & \text{if the event do not occur} \end{cases}$$

Suppose there are *n* projects. The *j*th project ($j = 1,2,\ldots,n$) has a cost c_j and a value of v_j. Each project is either done or not done; that is, it is not possible to do a fraction of any of the projects. Also, there is a budget of *B* available to fund the projects. The problem of choosing a subset of the projects to maximize the sum of the values while not exceeding the budget constraint is the 0-1-knapsack (KP) problem. It is written as the following equation (3.4) (George L., Nemhauser, Laurence A. and Wolsey, 1988) [21]:

$$(\text{KP-RA})\quad \begin{aligned} \max\ & \sum_{j=1}^{n} v_j x_j \\ \text{s.t.}\ & \sum_{j=1}^{n} c_j x_j \le B \\ & x \in \{0,1\} \end{aligned} \tag{3.5}$$

This problem is called the knapsack problem because of the analogy to the hiker's problem of deciding what should be put in a knapsack given a weight limitation on how much can be carried.

According to the methodology of resource allocation mentioned above, the organization makes resource decisions in a rational way in order to achieve its vision or goals. The organization must do the following:

(a) Identify/design alternatives.

(b) Identify and structure the organization's objectives.

(c) Prioritize the objectives and sub-objectives.

(d) Measure alternatives' contribution.

(e) Find the best combination of alternatives.

Suppose the headquarters (HQ) of an organization has a budget $\$B$ available and the budget will be distributed to its follower units (U_i). The problem of determining the allocation of resources is how one should maximize total contribution (or value) that is subject to the limited resource available. That is, a HQ should refer to the five principles above before a decision is made to allocate resources. The operating procedure includes:

(a) The HQ (upper level) draws out concrete resource allocation rules and measures from the organization's visions or goals.

(b) Based on these rules and measures, the lower level evaluates its sub-objectives and submits its resource requirement proposal (p_{ij}). Each proposal must contain cost (c_{ij}) and anticipated value (v_{ij}).

(c) The HQ examines proposals before finally issuing the optimal allocation of its limited resources.

Based on the procedure above, a two-stage reviewing process is used. Stage 1, the proposals are reviewed by a committee to ensure the significance of the proposal for the organization's visions or goals. In this stage, some proposals are disqualified. Stage 2, the committee decides whether the qualified proposals are to be funded or not and how much each should receive in funds. The funded proposals must maximize contribution within a limited budget. Fig. 3.1 is a diagram showing the hierarchical structure for resource allocation.

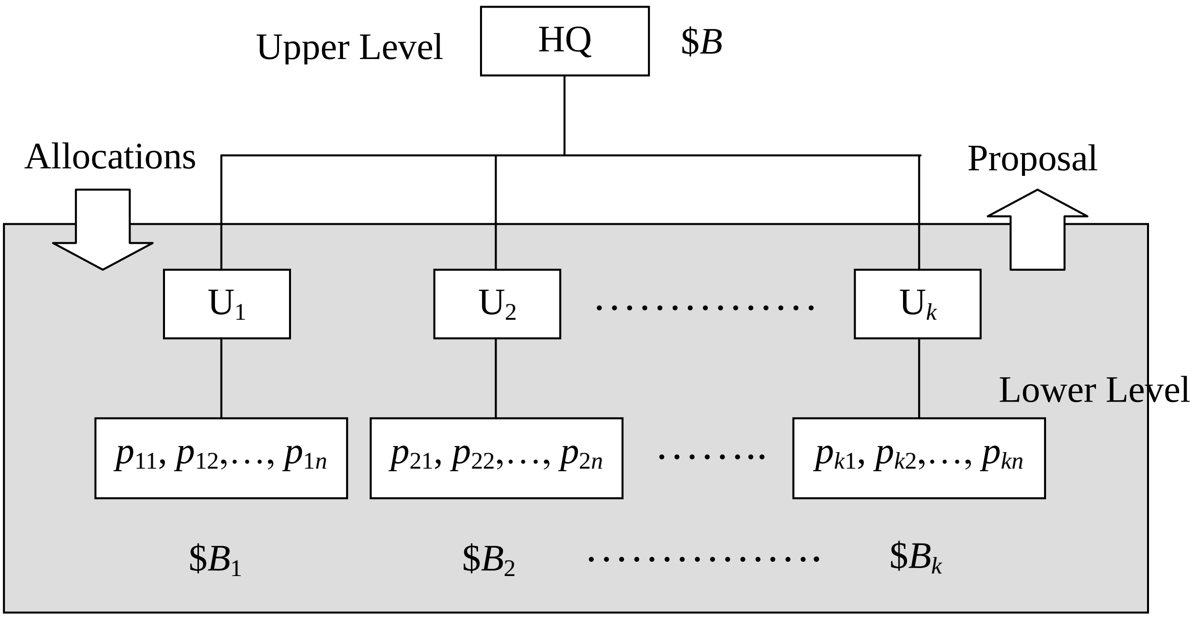


Figure 3.1 Diagram of Hierarchical Structure for Resource Allocation

Source: Study

The problem is the 0-1-knapsack problem. Basically, it may be written as an equation (3.6) in order to maximize the total value that is subjected to the limited resource available and to achieve an organization's objective.

$$
\begin{aligned}
&\max \sum_{j=1}^{n} v_{ij} x_{ij}, \quad i = 1,2,\ldots,k \\
&\text{s.t.} \quad \sum_{j=1}^{n} c_{ij} x_{ij} \le B_i, \quad i = 1,2,\ldots,k \\
&\qquad \sum_{i=1}^{k} B_i \le B \\
&\qquad x_{ij} \in \{0,1\}
\end{aligned}
\tag{3.6}
$$

v_{ij}: The anticipated value from the *j*th project of the *i*th unit.

c_{ij}: Cost required in the *j*th project of the *i*th unit.

x_{ij}: Decision variable of the *j*th project of the *i*th unit.

3.3 Model Development

These budget allocation problems can be modeled as a 0/1 integer program; however, computation efficiency is a concern. The particular problems studied in this research belong to the problem of NP hard[5], which suggests an efficient heuristic algorithm is necessary. This problem is formulated as a bilevel multi-follower programming involving the uncooperative decision variables (BLMF-UC), where the lower level decision maker's problem is a mathematical programming problem with independent, multiple objectives; then, the upper level DM, the leader, must solve the optimization problem over the lower level decision maker's rational reaction set.

The HQ of this particular company has funds of $*B* for the distribution to each of the *k* divisions under its supervision. The distribution process follows the rules below:

(a) Each division (follower or unit) summits proposal(s) to apply for funding.

(b) Each proposal (P) clearly and correctly states the work project, the cost (C) requirements to complete the project, and the value (V) of the project accomplishments.

(c) At least one proposal from each division must be funded.

(d) The overall efficiency/value is maximized (i.e. the subobjective of each lower level decision maker).

(e) The differences in the levels of satisfaction (as defined below) among the divisions are minimized (i.e. the objective of the upper level decision maker).

Note: Statement 3 above is necessary, otherwise the funding distribution process will be impossible to achieve.

---

[5] In computational complexity theory, NP-hard (Non-deterministic Polynomial-time hard) refers to the class of decision problems that contains all problems H such that for all decision problems L in NP there is a polynomial-time many-one reduction to H. Informally this class can be described as containing the decision problems that are at least as hard as any problem in NP. This intuition is supported by the fact that if we can find an algorithm A that solves one of these problems H in polynomial time then we can construct a polynomial time algorithm for every problem in NP by first executing the reduction from this problem to H and then executing the algorithm A.

In order to distribute the funds according to the rules above, a model is developed and is shown below. First, some basic definitions are given, and notations are introduced.

In a bilevel budget allocation programming problem with the uncooperative relationship, the goal of the upper level DM is to seek the most rational way of distributing the budget to each follower division simultaneously. The leader wants to balance the growth among all individual divisions and promote the strength of the entire organization within the industry as well. In order to do so, the objectives are to minimize the level of satisfaction among the individual divisions upon their funding approvals. Therefore, the upper level DM's problem is [P1]:

[P1] to minimize the level of satisfaction among the funded individual divisions, so that a balanced resource is allocated to lower units.

Under the resource allocation policy of the upper level DM, the lower level DM will pursue the largest value of each individual division in order to maximize the total value of the organization. Hence, this distribution is reasonable for lower level DMs.

In other words, one should understand the strengths and weaknesses among the divisions to make the rational decision. The rational decision of the lower level DM is to maximize the total value of his/her division. Therefore, the lower level DM's problem is [P2]:

[P2] to maximize the total value of each division while the costs stay within the funding limits.

In this decision-making problem, [P1] and [P2] are two closely related models; they are not separable. There is a natural hierarchical relationship tie between [P1] and [P2]. More precisely, the upper level DM selects the policy of resource allocation, and the lower level DM gives systematic reactions under this policy. The upper level DM then makes a rational decision after reviewing the systematic reactions of lower level DM. The relationship of [P1] and [P2] can be considered as the hierarchical programming problem [P]:

[P] to choose certain proposals to maximize the value of each division and to minimize the differences of the level of satisfaction among the divisions funded, simultaneously, the costs stay within the funding limits.

More precisely, the above is restated as:

[P] Minimize the difference of level of satisfaction among the division funded

where the chosen factors (proposals) will minimize the level of satisfaction and solve the problem

Maximize the value of each division

Subject to the costs stay within the funding limits

The hierarchical programming problem [P] above is a case in which the lower level decision variables are uncooperative for this type of bilevel multi-follower/multi-objective programming; it is the uncooperative relationship (BLMF-UC) problem. The mathematical model of a BLMF-UC problem is formulated as follows:

Let $U(i): i = 1,2,\cdots,k$ be k divisions/units of funds to be distributed, and let $P(i) = \{p(i,j):\ \ j = 1,2,\cdots,n\}$ be the collection of n proposals submitted by division i to request funding. Without loss of generality, we can assume that every division submits n proposals, and each proposal has a cost, $c(i,j)$, which denotes the cost required to accomplish $p(i,j)$. And $v(i,j)$ denotes the value obtained when $p(i,j)$ is accomplished.

The efficiency of project j in the division i is the ratio of the value to the requested funding.

i.e. $$e(i,j) = \frac{v(i,j)}{c(i,j)} \tag{3.7}$$

Otherwise, the level of satisfaction of division i is the ratio of the total funded amount to the total requested amount.

i.e. $$L(i) = \frac{\sum_{j=1}^{n} c(i,j)y(i,j)}{\sum_{j=1}^{n} c(i,j)} \tag{3.8}$$

Thus, the model of BLMF-UC for this problem aims to minimize any difference in the satisfaction levels among the divisions, while the total efficiency obtained from the funded proposal is maximized. In addition, the conditions below must be satisfied:

x is the variable of the leader, DM, while $y(i)$ is a vector valued variable of each lower level DM. The decision variable $y(i, j)$ is defined as below.

$$y(i,j) = \begin{cases} 1 & \text{if } p(i,j) \text{ is funded} \\ 0 & \text{otherwise} \end{cases}$$

In each division, the total cost of a funded proposal cannot exceed the dollar amount allotted to this division. Moreover, the total of the allotted amount for all divisions cannot exceed \$B, which is the available funding for distribution.

The mathematical model of BLMF-UC is then formulated in (3.9) and is shown as follows:

$$\text{(BLMF-UC)}\quad \begin{aligned} & \min\ x \\ & \text{s.t. } x \ge L(i) - L(m), 1 \le i, m \le k \\ & \text{where } y(i,j) \text{ solves} \\ & \quad \max \sum_{j=1}^{n} v(i,j) y(i,j), \forall i = 1,2,\ldots,k \\ & \quad \text{s.t.} \\ & \qquad \sum_{j=1}^{n} c(i,j) y(i,j) \le B(i), \quad \forall i = 1,2,\ldots k \\ & \qquad \sum_{j=1}^{n} y(i,j) \ge 1, \quad \forall i = 1,2,\ldots,k \\ & \qquad \sum_{i=1}^{k} B(i) \le B \\ & \qquad y(i,j) \in \{0,1\} \\ & \qquad B(i) \ge 0, \quad \forall i = 1,2,\ldots,k \\ & \qquad\qquad\qquad \forall j = 1,2,\ldots n \end{aligned} \tag{3.9}$$

Note: $x \ge L(i) - L(m),\ 1 \le i, m \le k$ implies x is greater than the difference between the maximum $L(i)$ and minimum $L(i)$ among all $i = 1,2,\cdots,k$.

This is a bilevel programming model where the lower level has multiple objectives. To solve this problem, the classic mixed integer program algorithm can be used; however, in cases that involve large numbers of variables in the problem, using the classical algorithm to find a solution becomes inefficient. Therefore, a heuristic solution algorithm is developed.

3.4 A Heuristic Algorithm

Based on the above budget distribution model of BLMF-UC, the heuristic algorithm process is as follows:

Step 1: Computing the requested dollar amount.

$$r = B / \sum_{i=1}^{k} \sum_{j=1}^{n} c(i,j)$$

$$r(i) = r \sum c(i,j)$$

Step 2: Computing efficiency of each project.

$$e(i,j) = \frac{v(i,j)}{c(i,j)}, \quad \forall i = 1,2,\cdots,k,\ j = 1,2,\ldots,n.$$

Let $(e(i,(j))$ be a decreasing rearrangement of $(e(i,j))$

i.e. $(e(i,(1)) \geq e(i,(2)) \geq \cdots \geq e(i,(n))$ for all $\forall i = 1,2,\cdots,k$.

Step 3: Determine $y(i,j)$.

$y(i,(j)) = 0, \forall i, j$

$\mathrm{T}(1) = 0$

For $i = 1$ to k

For $j = 1$ to n

$\mathrm{T}(i) = \mathrm{T}(i) + c(i,(j))$

If $\mathrm{T}(i) \leq r(i)$ then $y(i,(j)) = 1$

Next j

Next i

Step 4: Let $D = B - \sum c(i,(j))y(i,(j))$ then $D \geq 0$.

Step 5: If $D = 0$ or $c(i,(j)) > D > 0, \forall i, j$, then $(y(i,(j)))$ as obtained from 3 is optimal.

Step 6: Set x, let $r - 0.5x \leq L(m)$ and $r + 0.5x \geq L(i)$,

If $D \geq c(i',(j'))$ for some $1 \leq i' \leq k$, $1 \leq j' \leq n$ then among all i', j', Such that $(y(i',(j'))) = 0$, choose $\hat{i}, \hat{j}$ with $L(m) \leq L(\hat{i}) \leq L(i)$.

Step 7: Set $y(\hat{i},(\hat{j})) = 1$.

Step 8: Repeat 6 and 7 until 5 is true, then the problem is solved, where the final solution is $y(1,(1)) = \ldots y(u,(1)) = \ldots = y(u,(s-1)) = \ldots = y(u,(s)) = 1$ and all other y_i's are set to equal 0. Where $1 \leq u \leq k,\ 1 \leq s \leq n$.

Step 9: End.

It is easy to see that the final solution obtained from the algorithm is not necessarily optimal. However, it is a good approximation, and it is particularly important that final solutions are feasible and easily obtained.

A quick user guide and a source code of the heuristic algorithm for the BLMF-UC are written in the Appendix 1, 2.

Example 2:

Let the dollar amount to be distributed be NT $3,000 million; the data of the cost required and value obtained from each project of the units is shown in Table 3.1:

Table 3.1 The Data for Each Project in Example 2

		p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9	p_{10}	Σc
U(1)	*v*	46	519	368	328	91	129	785	831	462	—	
	c	14	134	139	91	44	34	212	214	130	—	1012
U(2)	*v*	145	134	855	237	250	190	454	—	—	—	
	c	27	46	183	75	91	45	112	—	—	—	579
U(3)	*v*	665	356	218	36	190	512	74	110	—	—	
	c	148	145	59	14	65	110	23	46	—	—	610
U(4)	*v*	232	145	263	210	106	438	142	728	312	536	
	c	149	76	108	92	59	118	83	135	145	188	1153
U(5)	*v*	792	665	181	308	419	420	—	—	—	—	
	c	335	269	71	96	109	130	—	—	—	—	1010

The budget allocation model of BLMF-UC is formulated as follows:

$$\min\ x$$

$$\text{s.t. } x \ge L(i) - L(m), 1 \le i, m \le k$$

where $y(i,j)$ solves

$$\max \begin{cases} 46y_{11} + 519y_{12} + 368y_{13} + 328y_{14} + 91y_{15} + 129y_{16} + 785y_{17} + 831y_{18} + 462y_{19} \\ 145y_{21} + 134y_{22} + 855y_{23} + 237y_{24} + 250y_{25} + 190y_{26} + 454y_{27} \\ 665y_{31} + 356y_{32} + 218y_{33} + 36y_{34} + 190y_{35} + 512y_{36} + 74y_{37} + 110y_{38} \\ 232y_{41} + 145y_{42} + 263y_{43} + 210y_{44} + 106y_{45} + 438y_{46} + 142y_{47} + 728y_{48} + 312y_{49} + 536y_{410} \\ 792y_{51} + 665y_{52} + 181y_{53} + 308y_{54} + 419y_{55} + 420y_{56} \end{cases}$$

s.t.

$$\sum_{j=1}^{10} c(i,j)y(i,j) \le B(i), \quad \forall i = 1,2,\ldots 5$$

$$\sum_{j=1}^{10} y(i,j) \ge 1, \quad \forall i = 1,2,\ldots,5$$

$$\sum_{i=1}^{5} B(i) \le 3000$$

$$y(i,j) \in \{0,1\}$$

$$B(i) \ge 0, \quad \forall i = 1,2,\ldots,5$$

$$\forall j = 1,2,\ldots 10$$

Follow the Heuristic Algorithm above.

Step 1: Compute the requested dollar amount.

$$r = B / \sum_{i=1}^{5}\sum_{j=1}^{10} c(i,j) = 3000/4368 = 68.7\%$$

$$r(1) = r\sum_{j=1}^{10} c(i,j) = 68\%(1013) = 696$$

$$r(2) = 68\%(579) = 398$$

$$r(3) = 68\%(612) = 419$$

$$r(4) = 68\%(1153) = 793$$

$$r(5) = 68\%(1010) = 694$$

Step 2: Compute efficiency of each project.

Let $e(i,j) = v(i,j)/c(i,j)$, $\forall i = 1,2,3,4$ and $j = 1,2,\ldots,5$ (see Table 3.2. Rows 4, 8, 12, 16, and 20).

Table 3.2 Values, Costs, Efficiencies and Rank for Each Project

		p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9	p_{10}
U(1)	v	46	519	368	328	91	129	785	831	462	—
	c	14	134	139	91	44	34	212	214	130	—
	e	3.3	3.9	2.6	3.6	2.1	3.8	3.7	3.9	3.6	
	(r)	(7)	(2)	(8)	(5)	(9)	(3)	(4)	(1)	(6)	
U(2)	v	145	134	855	237	250	190	454	—	—	—
	c	27	46	183	75	91	45	112	—	—	—
	e	5.4	2.9	4.7	3.2	2.7	4.2	4.1	—	—	—
	(r)	(1)	(6)	(2)	(5)	(7)	(3)	(4)	—	—	—
U(3)	v	665	356	218	36	190	512	74	110	—	—
	c	148	145	59	14	65	110	23	46	—	—
	e	4.5	2.5	3.7	2.6	2.9	4.7	3.2	2.4	—	—
	(r)	(2)	(7)	(3)	(6)	(5)	(1)	(4)	(8)	—	—
U(4)	v	232	145	263	210	106	438	142	728	312	536
	c	149	76	108	92	59	118	83	135	145	188
	e	1.6	1.9	2.4	2.3	1.8	3.7	1.7	5.4	2.2	2.9
	(r)	(10)	(7)	(4)	(5)	(8)	(2)	(9)	(1)	(6)	(3)
U(5)	v	792	665	181	308	419	420	—	—	—	—
	c	335	269	71	96	109	130	—	—	—	—
	e	2.4	2.5	2.5	3.2	3.8	3.2	—	—	—	—
	(r)	(6)	(5)	(4)	(3)	(1)	(2)	—	—	—	—

Then, rearrange $\{e(i,j): j = 1,2,\ldots,10\}$ in order to obtain a decreasing rearrangement $\{e(i,(j)): j = 1,2,\ldots,10\}$ (see Table 3.3)

Table 3.3 The Rearrangement of Each Unit's Project

U(i)		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
U(1)	v_1	831	519	129	785	328	462	46	368	91	—
	c_1	214	134	34	212	91	130	14	139	44	—
	e_1	3.9	3.9	3.8	3.7	3.6	3.6	3.3	2.6	2.1	—
	$L(1)$	21	34	38	59	68	80	82	96	100	—
U(2)	v_2	145	855	190	454	237	134	250	—	—	—
	c_2	27	183	45	112	75	46	91	—	—	—
	e_2	5.4	4.7	4.2	4.1	3.2	2.9	2.7	—	—	—
	$L(2)$	5	36	44	63	76	84	100	—	—	—
U(3)	v_3	512	665	218	74	190	36	356	110	—	—
	c_3	110	148	59	23	65	14	145	46	—	—
	e_3	4.7	4.5	3.7	3.2	2.9	2.6	2.5	2.4	—	—
	$L(3)$	18	42	52	56	66	68	92	100	—	—
U(4)	v_4	728	438	536	263	210	312	145	106	142	232
	c_4	135	118	188	108	92	145	76	59	83	149
	e_4	5.4	3.7	2.9	2.4	2.3	2.2	1.9	1.8	1.7	1.6
	$L(4)$	12	22	38	48	56	68	75	80	87	100
U(5)	v_5	419	420	308	181	665	792	—	—	—	—
	c_5	109	130	96	71	269	335	—	—	—	—
	e_5	3.8	3.2	3.2	2.5	2.5	2.4	—	—	—	—
	$L(5)$	11	24	33	40	67	100	—	—	—	—

Step 3: Let $L(i) \le r \le 68.7\%$, and Determine $y(i,j)$.

$$y(1,(1)) = y(1,(2)) = y(1,(3)) = y(1,(4)) = y(1,(5)) = 1$$

$$y(2,(1)) = y(2,(2)) = y(2,(3)) = y(2,(4)) = 1$$

$$y(3,(1)) = y(3,(2)) = y(3,(3)) = y(3,(4)) = y(3,(5)) = y(3,6) = 1$$

$$y(4,(1)) = y(4,(2)) = y(4,(3)) = y(4,(4)) = y(4,(5)) = y(4,(6)) = 1$$

$$y(5,(1)) = y(5,(2)) = y(5,(3)) = y(5,(4)) = y(5,(5)) = 1$$

Step 4: $D = B - \sum c(i,(j))y(i,(j))$

= 3000－(214+134+34+212+91)－(27+183+45+112)－(110+148+59+23+65+14)

－(135+118+188+108+92+145)－(109+130+96+71+269)

= 68

Step 5: Since D = 68, move to Step 6.

Step 6: Set $x = 10$, let $63 \le L(m)$ and $73 \ge L(i)$,

$L(1)$ = 68%; $L(2)$ = 63%; $L(3)$ = 68; $L(4)$ = 68%, $L(5)$ = 67%

choose $L(1)$.

Step 7: Let $y(1,(7)) = 1$

$L(1)$ = 70%.

Step 8: D = 68－14 = 54

$y(2,(6)) = 1$, $L(2)$ = 71%

D = 54－46 = 8 (By the above algorithm, a solution is obtained)

Step 9: End.

Summarize the above information. The variables of the lower level are given as follows:

$$
\text{Solution} \begin{cases}
y(1,(1)) = \ldots = y(1,(5)) = y(1,(7)) = 1, & y(1,(6)) = y(1,(8)) = y(1,(9)) = 0 \\
y(2,(1)) = \ldots = y(2,(4)) = y(2,(6)) = 1, & y(2,(5)) = y(2,(7)) = 0 \\
y(3,(1)) = \ldots = y(3,(6)) = 1, & y(3,(7)) = y(3,(8)) = 0 \\
y(4,(1)) = \ldots = y(4,(6)) = 1, & y(4,(7)) = \ldots = y(4,(10)) = 0 \\
y(5,(1)) = \ldots = y(5,(5)) = 1, & y(5,(6)) = 0
\end{cases}
$$

Table 3.4 The Decision Variables of the Lower Level for Each Unit's Project

	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9	p_{10}
U(1)	1	1	0	1	0	1	1	1	0	—
U(2)	1	1	1	0	0	1	1	—	—	—
U(3)	1	0	1	1	1	1	1	0	—	—
U(4)	0	0	1	1	0	1	0	1	1	1
U(5)	0	1	1	1	1	1	—	—	—	—

The maximized efficiency of each subdivision is below:

$$e(1) = \frac{v(1)}{c(1)} = \frac{831+519+129+785+328+46}{214+134+34+212+91+14} = \frac{2638}{699} = 3.77$$

$$e(2) = \frac{1778}{413} = 4.31$$

$$e(3) = \frac{1695}{419} = 4.05$$

$$e(4) = \frac{2487}{786} = 3.16$$

$$e(5) = \frac{1993}{675} = 2.95$$

$$E = \frac{V}{C} = \frac{2638+1778+1695+2487+1993}{699+413+419+786+675} = \frac{10591}{2992} = 3.54$$

These solutions give the following levels of satisfaction:

$L(1) = 70\%$

$L(2) = 71\%$

$L(3) = 69\%$

$L(4) = 68\%$

$L(5) = 67\%$

The minimum difference of the maximum and minimum level of satisfaction is 71- 67= 4. □

The total value obtained from this solution is 10,591, and the total efficiency is 3.54. □

A diagram of the heuristic algorithm, with a linear BLMF-UC problem, is proposed in the following Figure 3.2:

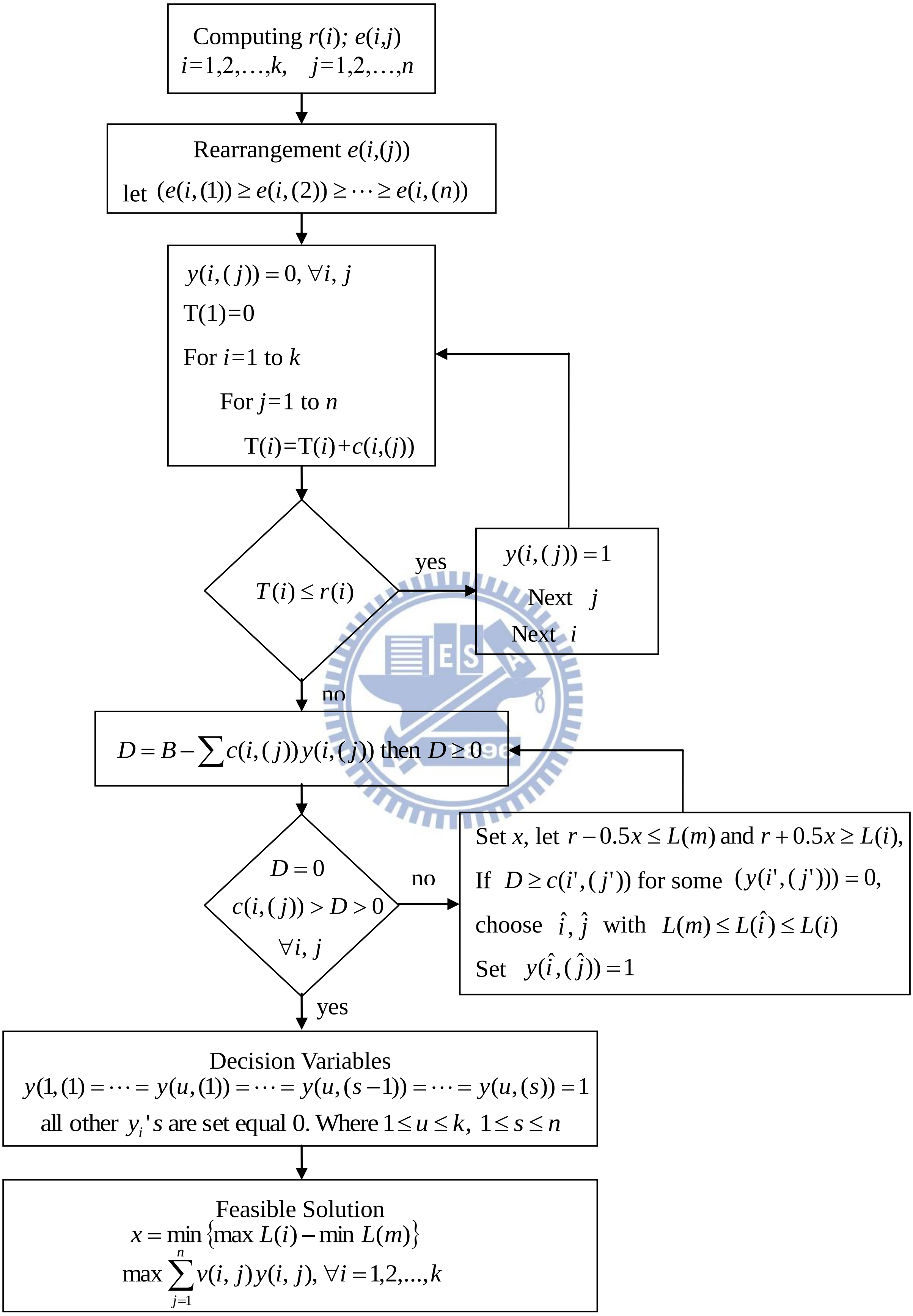


Figure 3.2 Diagram of the Heuristic Algorithm for BLMF-UC

Source: Study

3.5 Summaries

In a bilevel programming with multiple DMUs, the decision variables of followers can be cooperative, uncooperative, and partially cooperative. In the case of multiple lower level DMUs with cooperative decision variables, the problem can be treated as the case of a single DMU and single objective programming.

This chapter mainly investigates the case when multiple lower level DMUs work with uncooperative variables in a bilevel program. In this chapter, we discuss the definition of BLMF-UC (its characteristics) and structure a model of budget distribution, that is a more complex problem. The goal of the upper level DM is to minimize the level of satisfaction among the individual divisions upon their funding approvals. Under the policy of the upper level DM, the lower level DM will pursue the largest value of each individual division in order to maximize the total value of the organization.

Then, we develop a heuristic algorithm to overcome the difficulties due to the uncooperative decision variables. The model of the feature is as follows:

(a) An application of bilevel programming involved multiple followers.

(b) Lower level DM with multiple objectives.

(c) All DMUs are independent and have uncooperative decision variables.

(d) Lower level decision variables are 0/1 integer variable.

By the way, a quick user guide and a source code of the heuristic algorithm for the BLMF-UC are written in this dissertation. They are very useful to solve these problems.

Chapter 4 extends the BLMF to the case where decision variables are partially cooperative. This extension can model real world hierarchical structures more precisely.

Chapter 4 The Partial Cooperative Relationship of a Bilevel Multi-Follower Decision-making Model

4.1 Introduction

The multilevel decision-making system is a key type of decision-making model with a hierarchical organization, and a bilevel structure is the simplest type of all multilevel decision-making systems. Usually, the lower level involves multiple decision makers of the bilevel structure. Different reactions can be generated at the lower level towards each possible action conducted by the upper level; multiple followers are involved in a bilevel decision-making system.

In Chapter 3, a BLMF-UC decision-making process and its characteristic will be discussed. In this model, the lower level DMs optimize their objectives under control of the high level DMs' policy. The lower level is a multiple-objective programming problem in which the variables are independent. And hence, the lower level decision variables are uncooperative. This chapter extends the case of uncooperative variables to the partial cooperative relationship of a BLMF decision programming problem. This dissertation is based on a budget allocation problem and will construct a bilevel decision-making model as an example.

Please recall that the three kinds of relationship of decision variables among the followers, as categorized by Jie Lu, et al. (2006) [23], are as follows:

(a) The cooperative situation where the followers share all decision variables in their objectives and constraints.

(b) The uncooperative situation where there is no sharing of decision variables among the followers (see Chapter 3).

(c) The partial cooperative situation where the followers partially share decision variables in their objectives and/or constraints.

In fact, the cases (a) and (b) above can be treated as special cases of (c). The case (c) above will be discussed carefully in this chapter. The partial cooperative relationship of BLMF (BLMF-PC) decision-making problems are formulated below [12, 23]: (Jie Lu, et al.)

For $x \in X \subset R^n$, $y_i \in Y_i \subset R^{m_i}$, $Y = (Y_1, Y_2, ..., Y_K)^T$, $z \in Z \subset R^m$, $F: X \times Y_1 \times ... \times Y_K \times Z \to R^1$, $f: X \times Y_1 \times \cdots Y_K \times Z \to R^1$, and $i = 1,2,...,K$, a linear BLMF decision problem in which $K\ (\geq 2)$ followers are involved and there are partial shared decision variables, but separate objective functions and constraint functions among the followers are defined as follows:

The upper level decision maker's problem consists of solving

$$\begin{aligned} \min_{x \in X} \quad & F(x, y_1, ..., y_K, z) = cx + \sum_{s=1}^{K} d_s y_s + dz \\ \text{s.t.} \quad & Ax + \sum_{s=1}^{K} B_s y_s + B_z \leq b \end{aligned} \tag{4.1}$$

where y_i (i=1,2,…,K) and z, for each value of x, are the solution of the lower level problem:

$$\begin{aligned} \min_{y_i \in Y_i, z \in Z} \quad & f_i(x, y_1, ..., y_k, z) = c_i x + \sum_{s=1}^{K} e_{is} y_s + e_i z \\ \text{s.t.} \quad & A_i x + \sum_{s=1}^{K} C_{is} y_s + C_i z \leq b_i \end{aligned} \tag{4.2}$$

where $c \in R^n$, $c_i \in R^n$, $d \in R^m$, $d_i \in R^{m_i}$, $e_{is} \in R^{m_j}$, $b \in R^p$, $b_i \in R^{q_i}$, $A \in R^{p \times n}$, $A_i \in R^{q_i \times n}$, $B \in R^{p \times m}$, $B_i \in R^{p \times m_i}$, $C_i \in R^{q_i \times m}$, $C_{is} \in R^{q_i \times m_i}$, $i, s = 1,2,...,K$.

Note: in the model above,

(a) x, the decision variables of the leader.

(b) y_i, the decision variables of the ith followers, are not shared.

(c) z, the decision variables, shared by all of the followers.

The characteristics of the partial cooperative relationship of a BLMF-PC decision-making problem are as follows:

(a) A bilevel decision-making system with single leader and multiple followers.

(b) The follower is a multiple objective decision-making problem.

(c) The leader controls a set of decision variables while each follower controls two different sets of decision variables; one set includes independent decision variables and the other set includes cooperative decision variables.

In section one of this chapter, the mathematics model for BLP of multiple followers with partially cooperative variables has been discussed. In the second section, some formal definitions of partial cooperative variables are stated and budget distribution problems are solved effectively. Therefore, the leader and the followers can make good decisions.

4.2 Definition of the Problems

In a bilevel budget allocation problem of BLMF-PC, the goal of the upper level DM is to seek out the most efficient way of distributing the budget to each division as to balance the growth among all individual divisions and promote the strength of entire organization within the industry. In order to do so, first, the objectives must minimize the level of satisfaction among the individual divisions upon their funding approvals. Secondly, the constraints are to ensure the output efficiency is no less than the input efficiency.

Usually, useful information such as human, material, and financial are considered as input resources. The output values come in two forms: visible and invisible, namely, with explicit value and implicit value, respectively. Where the explicit value directly is created by the organization, and the implicit value is impacted by the divisions. Therefore, the upper level DM's problem is [P1]:

[P1] to minimize the level of satisfaction among the division funded under the condition that the output efficiency is no less than the input efficiency.

Under the budget allocation policy of the upper level DM, the lower level DM pursues the largest value of each individual division in order to maximize the total value of the organization. So, this distribution is reasonable for lower level DMs.

In other words, one must understand the strengths and weaknesses among the divisions so the optimal decision can be made. The optimal decision of the lower level DMs is to maximize the explicit and the implicit values of each proposal. Therefore, the lower level DMs' problem is [P2]:

[P2] to maximize the sum of the explicit and the implicit value of each proposal while the costs stay within the funding limits.

In this decision problem, [P1] and [P2] are two closely related models, they are not separable. There is a natural, hierarchical relationship tie between [P1] and [P2]. More precisely, the upper level DM makes decisions concerning the policy of resources allocation, and the lower level DM gives rational reactions under the policy. The upper level DM then makes an optimal decision after reviewing the rational reactions of lower level DM. The relationship of [P1] and [P2] can be considered as an extended hierarchical programming problem [P]:

[P] to choose certain proposals to maximize the value of the explicit and the implicit of each division within limited funfs, which in order to minimize the level of satisfaction among the divisions funded under the condition, so that the output efficiency is no less than input efficiency.

More precisely, the above is restated as

[P] Minimize the level of satisfaction among the division funded

Subject to the output efficiency is no less than input efficiency

where the chosed factors minimize the level of satisfaction and solve the problem

Maximize the sum of the explicit and the implicit value of each division

Subject to the costs stay within the funding limits

Note:

(a) The output values come in two forms, visible and invisible, namely, explicit value and implicit value, respectively. The explicit value is directly created by the organization, while the implicit value is impacted by the divisions.

(b) Human, material, and financial are considered as input resources.

(c) The explicit efficiency is defined as the explicit value times the received funded over the total input resource.

(d) The implicit efficiency is defined as the implicit value times the division's own funding over the total input resource.

(e) The explicit values are cooperative variables and implicit efficiencies are independent variables.

The extended hierarchical programming problem [P] above is a case where lower level decision variables are partially cooperative for a BLMF-PC programming problem. In order to construct a newly improved budget allocation model, in section three, a generalized data envelopment analysis (GDEA) is developed for checking the qualification of projects before evaluating the possibility of funding. Fig. 4.1 is a diagram showing the BLMF-PC programming problem for budget allocation.

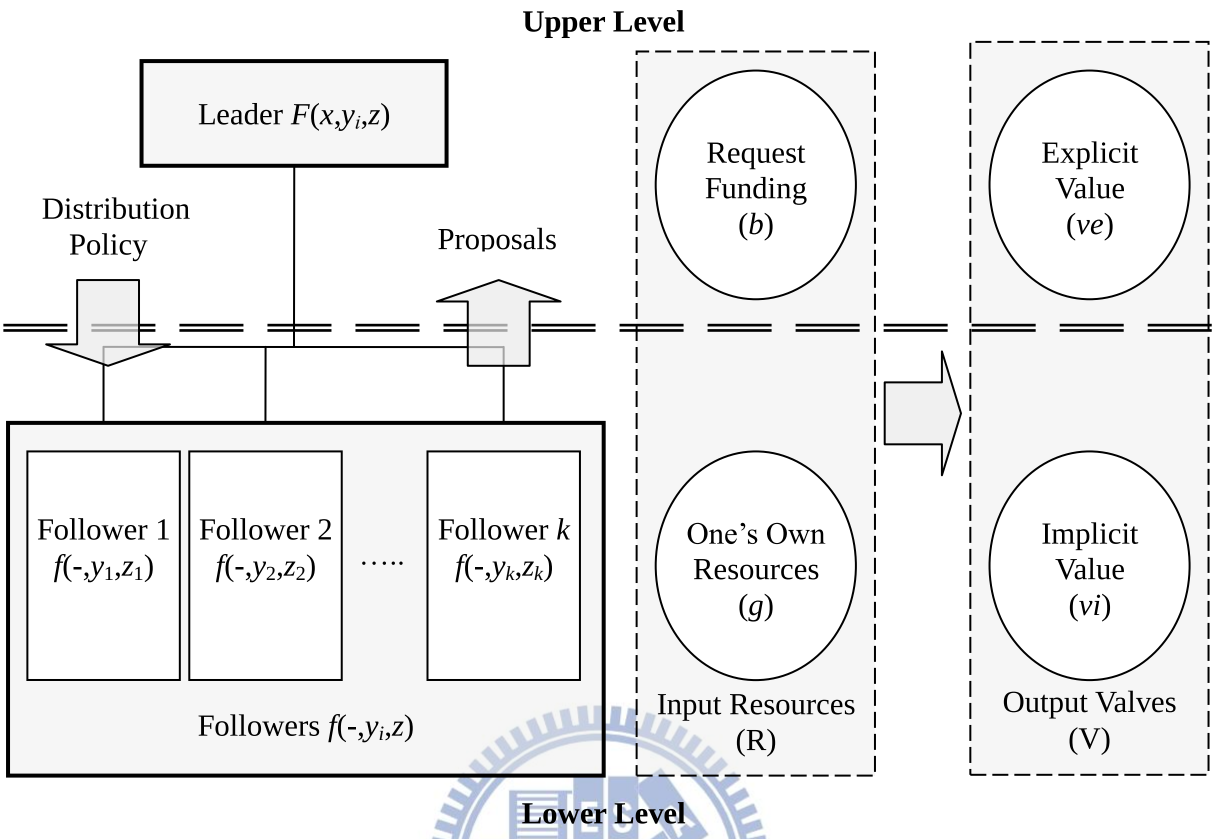


Figure 4.1 Diagram of the BLMF-PC Programming System for Budget Allocation

Source: Study

4.3 Generalized Data Envelopment Analysis

A decision-making system has k decision-making units; each unit's efficiency is determined by the ratio of the weighted sum of the output data and the weighted sum of the input data. A data envelop analysis (DEA) model determines the efficiency of a certain unit. However, to determine the efficiency of m units ($1 < m \leq k$) simultaneously, the DEA model can not be applied directly. This research develops a simple approach to handle this problem effectively.

The above mentioned problem has been studied in the last two decades. Many different approaches to the problem were published. For instance [16, 20, 33] were published; basically, each method constructs a weight by a different approach, to be used to determine the efficiencies, simultaneously. The mentioned weight is usually called common weight. So, this problem is frequently called the determination of common weight of a DEA problem

This research determines the common weight by solving a multiple objective fractional program (MOFP). Solving a MOFP is not trivial. Usually, it is difficult. This paper proposes a simple approach by taking advantage of the natural structure of this particular MOFP model and determining an efficient solution for this multiple objective problem. Consequently, an optimal common weight is determined.

4.3.1 Models and Solutions

Let S={ $u_i : i = 1,2,...k$ } be a decision-making system, and let each u_i be a decision-making unit (DMU). And let $u_i = (x_i, y_i)$ be a collection of input data $x_i \in R^{p^+}$ and output data $y_i \in R^{q^+}$ (Note: R^{p^+}, R^{q^+} denote the sets of positive *p*, *q* vectors, respectively). i.e. for each $x_i = (x_{i1}, x_{i2}, ..., x_{ip}) > 0$, $y_i = (y_{i1}, y_{i2}, ..., y_{iq}) > 0$. Also let $\alpha = (\alpha_1, \alpha_2, ..., \alpha_p)$, $\beta = (\beta_1, \beta_2, ..., \beta_q)$ be the relative weights of input data and output data, respectively. Then the efficiency of the *i*th DMU is $E_i = \dfrac{\beta y_i}{\alpha x_i}$ for certain real vectors α and β. Usually, these vectors are not known and need to be determined. A. Charnes and his associates [10] developed the following DEA model to determine α and β :

(DEA) $\max_{\alpha,\beta} E_{i0}$ subject to $E_i \le 1$, for all $i = 1,2,...,k$. (4.3)

This is a fractional program and can be solved easily by the following linear program (4.4):

$$\max_{\alpha,\beta} \beta y_{i0}$$

subject to

(DEALP) $\alpha x_{i0} = 1,$ (4.4)

$$\beta y_i - \alpha x_i \le 0, i = 1,2, . . k.,$$

$$\alpha, \beta \ge 0$$

This linear program can be solved easily. And, hence, the efficiency of the unit i_0 is determined. However, to determine the efficiency of the DMU i with $i \neq i_0$, this model is not suitable due to the fact that the objective function only considers the advantage of the unit i_0. Similarly, let S be a subset of the indexed set $\{1,2,...,k\}$; to consider the efficiency of all members u_i, $j \in S = \{s_i : i = 1,2,...,t\}$, simultaneously, the above linear program model is also not suitable. In order to handle the above mentioned problem, the following generalized DEA (GDEA) model is developed as equation (4.5):

$$\text{(GDEA)} \quad \max_{\alpha,\beta} E_j, \, j \in S \quad \text{subject to } E_i \leq 1, \text{ for all } i = 1,2,...,k. \tag{4.5}$$

This is a multiple objective fractional program, which is a special form of multiple objective programs. A multiple objective program (MOP) optimizes several objectives simultaneously. An optimal solution of the program is called an efficient solution. The precise definitions are given below.

Definition

Let F: $R^n \to R^k$ be a vector valued function defined on a subset X of R^n. The program

$$\text{(MOP)} \quad \max_{x \in X} F(x)$$

is called a multiple objective program. x^* is an efficient solution of MOP if for each x in X such that $F(x) \geq F(x^*)$ implies $F(x) = F(x^*)$.

In general, to find an optimal solution (efficient solution) for a multiple objective program can be difficult unless it can be solved as a single objective program. The following scarlarization theorem connects both multiple objectives and single objective programs.

4.3.2 Theorem (Scarlarization Theorem)

x^* is an efficient solution of MOP if and only if x^* solves the linear program ($P(\lambda)$) $\max_{x \in X} \lambda F(x)$ for some positive $\lambda \in R^k$.

Proof. Omitted. (see Dauer J. P., Liu Y. H., 1990 [18])

Solving this particular problem, GDEA, a solution method is developed using the nature of this model. This is a generalization of DEALP.

GDEA can be written explicit

$$\max_{\alpha,\beta} \frac{\beta y_j}{\alpha x_j} \quad j \in S$$

subject to

(GDEA) $$\frac{\beta y_i}{\alpha x_i} \le 1 \quad \text{for all } i = 1,2,\ldots,k,$$ (4.6)

$$\alpha, \beta \ge \varepsilon \succ 0 \quad \text{for some given } \varepsilon$$

Clearly, this is a generalization of DEA model, since in the case when *S* contains exactly one element, it is DEA model. To solve this GDEA program, observe DEALP, and the linear program GDEALP below is developed.

DEA has one objective function, which is nonlinear. However, with a proper transformation, $\alpha x_{i0} = 1$, in the constraints, the objective function becomes linear and the number of the linear constraints increases by one; the solution of DEA through DEALP is then straightforward. One can extend the transformation to develop GDEALP as follows:

Without loss of generality, let $S = \{1,2,\ldots t\}$ and $i_0 = 1$, then, $\alpha x_1 = 1$ in DEALP. Now, in GDEA one can similarly let $\alpha x_1 = 1$, which implies for each *j* in *S* there is a k_j such that $\alpha x_j = k_j$ (and $k_1 = 1$). Thus,

$$\max_{\alpha,\beta} \frac{\beta y_j}{k_j} \quad j \in S$$

subject to

(GDEA) $$\alpha x_j = k_j \quad j \in S$$ (4.7)

$$\frac{\beta y_i}{\alpha x_i} \le 1 \quad \text{for all } i = 1,2,\ldots,k.$$

$$\alpha, \beta \ge \varepsilon \succ 0.$$

It is easy to see that under the constraints the objective functions $\max_{\alpha,\beta} \frac{\beta y_j}{k_j} = \frac{\max_\beta \beta y_j}{\min_\alpha k_j}$ for all *j* in *S*. Since, here, one deals with positive numbers only. Furthermore, one can rewrite GDEA as the following multiple objective program of MOGDEA:

$$
\text{(MOGDEA)}\quad
\begin{aligned}
&\max_\beta \beta y_j \quad j \in S \\
&\min_\alpha k_j \quad\quad j \in S \\
&\text{subject to} \\
&\alpha x_j = k_j \quad j \in S \\
&\frac{\beta y_i}{\alpha x_i} \le 1 \text{ for all } i = 1,2,...,k. \\
&\alpha, \beta \ge \varepsilon \succ 0.
\end{aligned}
\qquad (4.8)
$$

It is nontrivial to rewrite GDEA as MOGDEA, the Lemma below shows the equivalence of two programs.

Lemma,

(α^*, β^*) is an efficient solution of GDEA if and only if (α^*, β^*) is an efficient solution of MOGDA.

Proof.

(α^*, β^*) is an efficient solution of GDEA iff for all *j*, $\frac{\beta y_j}{\alpha x_j} \ge \frac{\beta^* y_j}{\alpha^* x_j}$ implies $\frac{\beta y_j}{\alpha x_j} = \frac{\beta^* y_j}{\alpha^* x_j}$. For each feasible (α, β) in MOGDEA / GDEA, if $\beta y_j \ge \beta^* y_j$ and $\alpha x_j \le \alpha^* x_j$ for all *j*, then, since all terms are positive we have $\frac{\beta y_j}{\alpha x_j} \ge \frac{\beta^* y_j}{\alpha^* x_j}$, which implies $\frac{\beta y_j}{\alpha x_j} = \frac{\beta^* y_j}{\alpha^* x_j}$ again, all terms are positive implies $\beta y_j = \beta^* y_j$ and $\alpha x_j = \alpha^* x_j$. Thus, (α^*, β^*) is an efficient solution of MOGDEA. The converse can be shown by the similar argument. //

Applied with the scarlarization theorem and the Lemma above, the theorem is as follows:

Main Theorem

(α^*, β^*) solves

$$
\text{(GDEALP)}\quad
\begin{aligned}
&\max_{\alpha,\beta} \Sigma_{j\in S} (\beta y_j - \alpha x_j) \\
&\text{subject to} \\
&\alpha x_j = k_j \quad j \in S \quad (\text{Recall}: k_1 = 1) \\
&\beta y_i \le \alpha x_i \quad i = 1,2,\ldots,k. \\
&\alpha, \beta \ge 0.
\end{aligned}
\tag{4.9}
$$

if and only if (α^*, β^*) is an efficient solution of GDEA.

Proof. Omitted.

The theorem provides a solution method for the generalized DEA problem. Since after the determination of efficient weights (α^*, β^*), everything follows immediately. By plugging in the values found above to the ratios of the weighted average of the output and the input, one can obtain the results.

The proposed method is more efficient than the current existing methods are, since it requires running one linear program only. The following example exhibits the proposed method. (For an example related to GDEA, see Appendix 5.)

Next, section four, which is a continuation of section two, develops a multi-follower budget distribution model with partial cooperative variables. This problem is solved using the concepts of GDEA, which are discussed in section three, for preprocessing the data of the projects from each division to guarantee the quality of funded projects, so as to avoid unnecessary distributions.

4.4 Model Development

According to the definitions of problems of BLMF-PC in Chapter 4.2, the upper level DM's problem is to minimize the level of satisfaction among the division, which is funded under the condition that the output efficiency is no less than the input efficiency is. Hence, the level of satisfaction of division *i* is formulated as equation (4.10), and it is a ratio of the total funded amount and the total requested amount.

$$L(i) = \frac{\sum_{j=1}^{n} b(i,j) y(i,j)}{\sum_{j=1}^{n} b(i,j)}, \quad \forall i = 1,2,...k. \tag{4.10}$$

$L(i)$: The level of satisfaction of the *i*th division.

$b(i,j)$: The resources/budget requested by the *i*th division for the proposal *j*.

$y(i,j)$: The decision variable of the proposal *j* from the *i*th division.

Next, according to the theorems of GDEA in section three, the input-output efficiency $E(i,j)$ can be formulated as equation (4.11):

$$\begin{aligned} E(i,j) = \max\ & \alpha_1 \cdot ve(i,j) + \alpha_2 \cdot vi(i,j) \\ \text{s.t.}\quad & \beta_1 \cdot g(i,j) + \beta_2 \cdot b(i,j) = 1 \\ & (\alpha_1 \cdot ve(i,j) + \alpha_2 \cdot vi(i,j)) - (\beta_1 \cdot g(i,j) + \beta_2 \cdot b(i,j)) \le 0, \\ & \quad \forall i = 1,2,...,k \text{ and } j = 1,2,...,n. \\ & \alpha_1, \alpha_2, \beta_1, \beta_2 \ge \varepsilon > 0. \end{aligned} \tag{4.11}$$

$E(i,j)$: The input-output efficiency.

$ve(i,j)$: The explicit value of the proposal *j* by the *i*th division.

$vi(i,j)$: The implicit value of the proposal *j* by the *i*th division.

$g(i,j)$: The resources provided by the *i*th division, which are used in the proposal *j*.

α : The weight of the output value.

β : The weight of the input value.

Now, let μ be a given positive number and less than 1; the mathematical model of upper level is then formulated as follows:

$$
\begin{aligned}
&\min \quad x \\
&\text{s.t.} \quad x \geq L(i) - L(m), \quad 1 \leq i, m \leq k, \\
&\qquad E(i, j) \geq \mu, \quad \forall i = 1,2,\ldots k, \quad j = 1,2,\ldots n.
\end{aligned}
\tag{4.12}
$$

x : The decision variable of the upper level DM.

μ : The controlled efficiency factor.

According to a diagram of the BLMF-PC programming problem for budget allocation from Fig. 4.1, under the budget allocation policy of the upper level DM, the lower level DMs pursue the largest value of each individual division in order to maximize the total value of the organization. So, this distribution is reasonable for lower level DMs. Usually, the values are visible and invisible, namely, explicit value and implicit value, respectively. Where the explicit value is directly created by the organization, and whiles the implicit value is impacted by the divisions.

By the way, in each division, the total cost of a funded proposal cannot exceed the dollar amount allotted to this division. Moreover, the total of the allotted amount for all divisions cannot exceed \$*B*, which is the available funding for distribution. The mathematical model of lower level is then formulated as follows:

$$
\max \quad \sum_{j=1}^{n} vi(i,j)y(i,j) + ve(i)z(i) \quad \forall i = 1,2,\ldots,k.
$$

s.t.

$$
\begin{aligned}
&\sum_{j=1}^{n} b(i,j)y(i,j) \le B(i), \quad \forall i = 1,2,\ldots,k \text{ and } j = 1,2,\ldots,n,\\
&\sum_{i=1}^{k} B(i) \le B, \quad \forall i = 1,2,\ldots,k,\\
&B(i) \ge 0, \quad \forall i = 1,2,\ldots,k,\\
&\sum_{j=1}^{n} y(i,j) \ge 1, \quad \forall i = 1,2,\ldots,k \text{ and } j = 1,2,\ldots,n,\\
&y(i,j) \in \{0,1\} \quad \forall i = 1,2,\ldots,k \text{ and } j = 1,2,\ldots,n.
\end{aligned}
\tag{4.13}
$$

$ve(i)$: The total explicit value by the *i*th division.

$B(i)$: The limit of the budget to be obtained by the *i*th division for the proposal *j*.

B : The available resources from the organization.

$y(i,j)$: The decision variable of the proposal *j* from the *i*th division.

$z(i)$: The relational degree of explicit value from the *i*th division.

Note:

(a) $y(i,j)$ is a vector valued variable of uncooperation with each lower level DM.

(b) $z(i)$ is a cooperative variable in the lower level DMs.

(c) The following are some definitions of the terms used in the model above:

$$
\begin{aligned}
&e(i,j) = \frac{vi(i,j)}{I(i,j)}, \quad \forall i = 1,2,\ldots,k \text{ and } j = 1,2,\ldots,n,\\
&w(i,j) = \frac{ve(i,j)}{I(i,j)}, \quad \forall i = 1,2,\ldots,k \text{ and } j = 1,2,\ldots,n.
\end{aligned}
\tag{4.14}
$$

$e(i,j)$: The implicit efficiency of the proposal *j* by the *i*th division.

$w(i,j)$: The explicit efficiency of the proposal *j* by the *i*th division.

4.5 The Solution Algorithm

Let $\{U(i): i = 1,2,\cdots,k\}$ be a collection of k divisions/followers with a budget to be distributed, and let $P(i) = \{p(i, j): j = 1,2,\cdots,n\}$ be the collection of n proposals submitted by division i to request funding. Without loss of generality, we can assume that every division submits n proposals. Each proposal has input $I(i,j)$ and output data $O(i,j)$. The input data includes one's own resources $g(i,j)$ and the request for funding $b(i,j)$; the output data includes implicit value $vi(i,j)$ and explicit value $ve(i,j)$. The inputs and the outputs for the proposals of the divisions are listed in table 4.1.

Table 4.1 The Inputs and the Outputs for the Proposals of the Divisions

U	$p(i, j)$	$I(i, j)$		$O(i, j)$	
U(1)	$p(1,1)$	$g(1,1)$	$b(1,1)$	$vi(1,1)$	$ve(1,1)$
	$p(1,2)$	$g(1,2)$	$b(1,2)$	$vi(1,2)$	$ve(1,2)$
	⋮	⋮	⋮	⋮	⋮
	$p(1,n)$	$g(1,n)$	$b(1,n)$	$vi(1,n)$	$ve(1,n)$
U(2)	$p(2,1)$	$g(2,1)$	$b(2,1)$	$vi(2,1)$	$ve(2,1)$
	$p(2,2)$	$g(2,2)$	$b(2,2)$	$vi(2,2)$	$ve(2,2)$
	⋮	⋮	⋮	⋮	⋮
	$p(2,n)$	$g(2,n)$	$b(2,n)$	$vi(2,n)$	$ve(2,n)$
⋮	⋮	⋮	⋮	⋮	⋮
U(i)	$p(i,1)$	$g(i,1)$	$b(i,1)$	$vi(i,1)$	$ve(i,1)$
	$p(i,2)$	$g(i,2)$	$b(i,2)$	$vi(i,2)$	$ve(i,2)$
	⋮	⋮	⋮	⋮	⋮
	$p(i,n)$	$g(i,n)$	$b(i,n)$	$vi(i,n)$	$ve(i,n)$
⋮	⋮	⋮	⋮	⋮	⋮
U(k)	$p(k,1)$	$g(k,1)$	$b(k,1)$	$vi(k,1)$	$ve(k,1)$
	$p(k,2)$	$g(k,2)$	$b(k,2)$	$vi(k,2)$	$ve(k,2)$
	⋮	⋮	⋮	⋮	⋮
	$p(k,n)$	$g(k,n)$	$b(k,n)$	$vi(k,n)$	$ve(k,n)$

Preprocessing:

For all $i = 1,2,\ldots,k$ and $j = 1,2,\ldots,n.$

1. Let $SI = \phi.$

2. Compute the input-output efficiency $E(i, j)$ using the GDEA mathematical program as shown below:

$$
\begin{aligned}
E(i, j) &= \max\ \alpha_1 \cdot ve(i, j) + \alpha_2 \cdot vi(i, j) \\
\text{s.t.}\quad & \beta_1 \cdot g(i, j) + \beta_2 \cdot b(i, j) = 1 \\
& (\alpha_1 \cdot ve(i, j) + \alpha_2 \cdot vi(i, j)) - (\beta_1 \cdot g(i, j) + \beta_2 \cdot b(i, j)) \le 0, \\
& \qquad\qquad \forall i = 1,2,\ldots,k \text{ and } j = 1,2,\ldots,n. \\
& \alpha_1, \alpha_2, \beta_1, \beta_2 \ge \varepsilon > 0.
\end{aligned}
$$

3. Let $E(i, j) \ge \mu, \quad \forall i = 1,2,\ldots k, \quad j = 1,2,\ldots n,$ for given $1 > \mu > 0$.

4. If $E(i, j) < \mu$, then discard $p(i, j)$,

   otherwise $SI = SI \cup \{(i, j)\}$ (the set of indices of all selected projects)

Selection Process:

1. Apply the theory of the grey relationship to obtain the grey relationship grade z's among all the lower level DMUs.

   Let S be the space of grey relation factors

$$
\begin{aligned}
S = \{s_j \,|\, j \in J = \{0,1,2,\ldots,m\},\ m \ge 2 \\
s_j = (s_j(1), s_j(2),\ldots, s_j(n),\ n \ge 3\}
\end{aligned}
$$

   then $\nabla_{0j}(k) = \nabla_j(k) = \left| s_0(k) - s_j(k) \right|$

And grey relational coefficient

$$z(s_0(k), s_i(k)) = \frac{\min_j \min_k \nabla_j(k) + \zeta \max_j \max_k \nabla_j(k)}{\nabla_j(k) + \zeta \max_j \max_k \nabla_j(k)}, \zeta \in [0,1]$$

grey relationship grade $z(s_0, s_j) = \frac{1}{n}\sum_{k=1}^{n} z(s_0(k), s_j(k))$.

2. Compute $r = B / \sum_{i=1}^{k}\sum_{j=1}^{n} b(i,j)$ and $r(i) = r \cdot \sum_{j=1}^{n} b(i,j)$.

3. Compute $e(i,j) = \frac{vi(i,j)}{I(i,j)}, \quad \forall (i,j) \in SI.$

Let $\{e(i,(j))\}$ be a decreasing rearrangement of $e(i,j)$,

i.e. $e(i,(1)) \ge e(i,(2)) \ge \cdots \ge e(i,(n))$ for all $\forall (i,j) \in SI$.

Also compute $w(i,j) = \frac{ve(i,j)}{I(i,j)}, \quad \forall (i,j) \in SI.$

Let $\{w(i,(\bar{j}))\}$ be a decreasing rearrangement of $w(i,j)$,

i.e. $w(i,(\bar{1})) \ge w(i,(\bar{2})) \ge \cdots \ge w(i,(\bar{n}))$ for all $\forall (i,j) \in SI$.

4. Let $y(i,(j)) = 0, \forall (i,j) \in SI,$

$T(1) = 0,$

for $i = 1,2,\ldots,k,$

if $IJJ - SI \ne \phi$, then

for $1 \le j \le n, \quad 1 \le \bar{j} \le n,$

to compare the corresponding lower level objective value of $e(i,(j))$ and $w(i,(\bar{j}))$

i.e. to determine the value

$$LO(i^*, j^*) = \max \{LO(i, j) : LO(i, j) = \sum_{j \in JJ} vi(i, j) y(i, j) + ve(i) z(i), \quad \forall (i, j) \in SI.\}$$

$$0 < z(i) \le 1, \quad \forall (i, j) \in SI.$$

$T(i) = T(i) + b(i^*, (j^*)),$

if $T(i) \le r(i)$, then

$y(i^*, (j^*)) = 1, \quad IJJ = IJJ \cup \{(i^*, j^*)\}.$

otherwise, the end of comparison and the selection in the unit *i* is complete.

Next *j*

Next *i*.

5. Let $D = B - \sum_{(i,j) \in IJJ} b(i,(j)) y(i,(j))$ then $D \ge 0$.

6. If $D = 0$ or $D \ne 0$ and $b(i,(j)) > D, \forall i, j \in IJJ$, then $(y(i,(j)))$ obtained from 6 is feasible.

7. Set x, let $r - 0.5x \le L(m)$ and $r + 0.5x \ge L(i)$,

   If $D \ge b(i^*,(j^*))$ for some $1 \le i^* \le k, \quad 1 \le j^* \le n$, then among all i^*, j^*,

   sush that $(y(i^*,(j^*))) = 0$, choose $\hat{i}, \hat{j}$ with $L(m) \le L(\hat{i}) \le L(i)$.

8. Set $y(\hat{i},(\hat{j})) = 1$.

9. Repeat 7 and 8 until 6 is true; the problem is then solved, and the final solution is $y(1,(1)) = \ldots = y(l,(1)) = \ldots = y(l,(s-1)) = \ldots = y(l,(s)) = 1$ and all other y_i's are set equal 0. Where $1 \le l \le k,\ 1 \le s \le n$.

10. End.

A quick user guide and a source code for the grey relational analysis and the heuristic algorithm of the BLMF-PC are written in the Appendix 3, 4.

Example 3:

The government will distribute $6,000 millions to the energy industry of four fields, which including energy saving, renewable energy, new energy and energy technology etc. The amount to be distributed to each department of the field is determined by the proposals submitted by each division. Each proposal includes the input data $I(i,j)$ and the output data $O(i,j)$. The input contains one's own recourses $g(i,j)$ and the request for funding $b(i,j)$; the output includes implicit value $vi(i,j)$ and explicit value $ve(i,j)$. The data of inputs and outputs for the proposals are listed in table 4.2.

Table 4.2 The Data of Inputs and Outputs for the Proposals in Example 3

Fields	$p(i, j)$	$I(i, j)$		$O(i, j)$	
		$g(i, j)$	$b(i, j)$	$vi(i, j)$	$ve(i, j)$
F(1) (Energy Saving)	$p(1,1)$	147	424	8160	863
	$p(1,2)$	21	65	941	74
	$p(1,3)$	83	313	1486	3065
	$p(1,4)$	9	28	140	10
	$p(1,5)$	58	142	4761	213
	$p(1,6)$	117	321	3449	1531
	$p(1,7)$	121	294	1809	2013
	$p(1,8)$	21	67	823	39
F(2) (Renewable Energy)	$p(2,1)$	192	767	2992	7154
	$p(2,2)$	165	808	2853	8325
	$p(2,3)$	49	146	1210	162
	$p(2,4)$	143	569	5186	9898
	$p(2,5)$	197	794	4900	7276
	$p(2,6)$	36	140	743	300
	$p(2,7)$	120	455	4150	7481
	$p(2,8)$	22	487	678	380

F(3) (New Energy)	$p(3,1)$	50	170	410	40
	$p(3,2)$	293	481	2255	547
	$p(3,3)$	101	262	1989	62
	$p(3,4)$	31	100	395	60
	$p(3,5)$	125	42	55	30
	$p(3,6)$	55	34	123	24
	$p(3,7)$	37	137	330	77
	$p(3,8)$	92	229	866	55
	$p(3,9)$	20	51	26	395
F(4) (Energy Technology)	$p(4,1)$	102	282	296	4373
	$p(4,2)$	228	614	102	2320
	$p(4,3)$	38	114	66	4491
	$p(4,4)$	51	236	72	2266
	$p(4,5)$	49	111	760	1797
	$p(4,6)$	23	112	39	1884
	$p(4,7)$	24	254	18	4123
	$p(4,8)$	27	168	44	2680

Preprocessing:

For all $i = 1,2,\ldots,4$ and $j = 1,2,\ldots,9$.

1. Let $SI = \phi$.

2. Compute the input-output based efficiency $E(i, j)$ using the GDEA mathematical program for the Unit 1:

$$\max \quad 21569\beta_1 + 7807\beta_2 - 576\alpha_1 - 1653\alpha_2$$

$$\text{s.t.} \quad 147\alpha_1 + 424\alpha_2 - k_1 = 0$$

$$21\alpha_1 + 65\alpha_2 - k_2 = 0$$

$$83\alpha_1 + 133\alpha_2 - k_3 = 0$$

$$147\alpha_1 + 424\alpha_2 \text{-} 8160\beta_1 \text{-} 863\beta_2 \geq 0$$

$$21\alpha_1 + 65\alpha_2 - 941\beta_1 \text{-} 74\beta_2 \geq 0$$

$$83\alpha_1 + 133\alpha_2 \text{-} 1486\beta_1 \text{-} 3065\beta_2 \geq 0$$

$$9\alpha_1 + 28\alpha_2 \text{-} 140\beta_1 - 10\beta_2 \geq 0$$

$$58\alpha_1 + 142\alpha_2 \text{-} 4761\beta_1 \text{-} 231\beta_2 \geq 0$$

$$117\alpha_1 + 321\alpha_2 \text{-} 3449\beta_1 \text{-} 1531\beta_2 \geq 0$$

$$121\alpha_1 + 294\alpha_2 \text{-} 1809\beta_1 \text{-} 2013\beta_2 \geq 0$$

$$21\alpha_1 + 67\alpha_2 \text{-} 823\beta_1 \text{-} 39\beta_2 \geq 0$$

$$k_1 = 1$$

The above linear program GDEA using LINDO gives a set of optimal weights:

α_1 =0.000001 α_2 = 0.002357 β_1 = 0.000061 β_2 = 0.000211

Plug the values of α, β into $E_j = \beta y_j / \alpha x_j$ to obtain the efficiency of the Unit 1 for desired rankings. Repeat Step 2 to compute $E(i, j)$ for Unit 2, Unit 3, and Unit 4. The results showing the input-based efficiency of each proposal are listed in table 4.3.

Table 4.3 The Results Showing the Input-Output Based Efficiency of Each Proposal

Fields	*p*1	*p*2	*p*3	*p*4	*p*5	*p*6	*p*7	*p*8	*p*9
F(1)	0.68	0.48	1.00	0.16	1.00	0.70	0. 77	0.37	—
F(2)	0.44	0.49	0.67	1.00	0.69	0.56	0.97	0.84	—
F(3)	0.35	0.75	1.00	0.59	0.25	0.55	0.39	0.53	1.00
F(4)	0.47	0.10	1.00	0.38	1.00	0.66	1.00	0.73	—

3. Let $E(i,j) \geq \mu, \quad \forall i = 1,2,\ldots k, \quad j = 1,2,\ldots n,$ for given $1 > \mu > 0$.

4. If $E(i,j) < \mu$, the proposal is not qualified, and discard $p(i,j)$; otherwise, $SI = SI \cup \{(i,j)\}$.

When $\mu = 0.4$, using data in table 4.3, the proposals $p(1,4)$, $p(1,8)$, $p(3,1)$, $p(3,5)$, $p(3,7)$, $p(4,2)$ and $p(4,4)$ are deleted. Consequently, the index set of the qualified projects $SI = \{p(1,1), p(1,2), \ldots, p(4.8)\}$ is used in the selection process below.

Selection Process:

1. Apply the theory of the grey relationship to obtain the grey relationship grade z's among all the lower level DMUs.

   First, consider in unit 1 the input/output data of six proposals obtained by first selection stage:

$$\omega_1 = (\omega_1(1), \omega_1(2), \omega_1(3), \omega_1(4)) = (147, 424, 8160, 863)$$

$$\omega_2 = (\omega_2(1), \omega_2(2), \omega_2(3), \omega_2(4)) = (21, 65, 941, 74)$$

$$\omega_3 = (\omega_3(1), \omega_3(2), \omega_3(3), \omega_3(4)) = (83, 313, 1486, 3065)$$

$$\omega_5 = (\omega_5(1), \omega_5(2), \omega_5(3), \omega_5(4)) = (58, 142, 4761, 213)$$

$$\omega_6 = (\omega_6(1), \omega_6(2), \omega_6(3), \omega_6(4)) = (117, 321, 3449, 1531)$$

$$\omega_7 = (\omega_7(1), \omega_7(2), \omega_7(3), \omega_7(4)) = (121, 294, 1809, 2013)$$

to initialization ω_j, $j = 1,2,3,5,6,7$

$$s_1(k) = \frac{\omega_1(k)}{\omega_1(1)} = \frac{\omega_1(k)}{147},\ s_1(1) = \frac{\omega_1(1)}{147} = \frac{147}{147} = 1$$

$$s_1 = (s_1(1), s_1(2), s_1(3), s_1(4)) = (1, 2.88, 55.51, 5.87)$$

$$s_2 = (1, 3.1, 44.81, 3.52),\quad s_3 = (1, 3.77, 17.9, 36.93),$$

$$s_5 = (1, 2.45, 82.09, 3.67),\quad s_6 = (1, 2.74, 29.48, 13.09),$$

$$s_7 = (1, 2.43, 14.95, 16.64)$$

Find the difference sequence ∇_j; from Table 4.3, obtain $E(1,5) = 1$; let s_5 be the reference sequence, p_j, $j = 1,2,3,6,7$ be comparative sequence,

then

$$\nabla_{5j}(k) = \left|s_5(k) - s_i(k)\right|$$
$$\nabla_{5j} = (\nabla_{5j}(1), \nabla_{5j}(2), \nabla_{5j}(3), \nabla_{5j}(4))$$

let $j = 1,\ k = 1, 2, 3, 4$, then

$$\nabla_{51}(1) = \left|s_5(1) - s_1(1)\right| = \left|1 - 1\right| = 0,$$

$$\nabla_{51}(2) = \left|s_5(2) - s_1(2)\right| = \left|2.45 - 2.88\right| = 0.44,$$

$$\nabla_{51}(3) = \left|s_5(3) - s_1(3)\right| = \left|82.09 - 55.51\right| = 26.58,$$

$$\nabla_{51}(4) = \left|s_5(4) - s_1(4)\right| = \left|3.67 - 5.87\right| = 2.20$$

$$\nabla_{51} = (\nabla_{51}(1), \nabla_{51}(2), \nabla_{51}(3), \nabla_{51}(4)) = (0, 0.44, 26.58, 2.20)$$

similarly,

$$\nabla_{52} = (0, 0.68, 26.91, 33.4),\quad \nabla_{53} = (0, 1.32, 64.18, 33.26),$$

$$\nabla_{56} = (0, 1.03, 11.58, 23.84)\quad \nabla_{57} = (0, 1.34, 2.95, 20.29)$$

For ∇_{5j}, $j = 1,2,3,6,7$

$$\max_j \max_k |x_5(k) - x_j(k)| = \max_j \max_k \nabla_{5j} = 67.14$$

$$\min_i \min_j |x_5(j) - x_i(j)| = \min_i \min_j \nabla_{3i} = 0$$

Let distinguishing coefficient $\zeta = 0.5$, and the grey relational coefficient:

$$z(s_5(k), s_j(k)) = \frac{\min_j \min_k \nabla_{5j}(k) + 0.5 \max_j \max_k \nabla_{5j}(k)}{\nabla_{5j}(k) + 0.5 \max_j \max_k \nabla_{5j}(k)} = \frac{0.5 \times 67.14}{\nabla_{5j}(k) + 0.5 \times 67.14}$$

$$= \frac{33.57}{\nabla_{5j}(k) + 33.57}$$

for $j = 6, k = 1,2,3,4$

$$z(s_5(1), s_1(1)) = \frac{33.57}{\nabla_{51}(1) + 33.57} = \frac{33.57}{0 + 33.57} = 1$$

$$z(s_5(2), s_1(2)) = \frac{33.57}{0.44 + 33.57} = 0.99, \; z(s_5(3), s_1(3)) = \frac{33.57}{26.58 + 33.57} = 0.56$$

$$z(s_5(4), s_1(4)) = \frac{33.57}{2.20 + 33.57} = 0.94$$

for ζ_{51} be the sequence constructed using $z(s_5(k), s_1(k)), k = 1,2,3,4$, then

$$\zeta_{51} = (1, 0.99, 0.56, 0.94)$$

Similarly, $z(s_5(k), s_2(k))$, $z(s_5(k), s_3(k))$, $z(s_5(k), s_6(k))$ and $z(s_5(k), s_7(k))$ for ζ_{52}, ζ_{53}, ζ_{56}, ζ_{57}:

$$\zeta_{52} = (1, 0.98, 0.47, 1), \; \zeta_{53} = (1, 0.96, 0.34, 0.5),$$

$$\zeta_{56} = (1, 0.99, 0.39, 0.78), \; \zeta_{57} = (1, 1, 0.33, 0.72)$$

By definition of the grey relational grade

$$z(s_5, s_j) = \frac{1}{4}\sum_{k=1}^{4} z(s_5(k), s_j(k)),\ k = 1,2,3,4$$

for $j = 1$, then

$$z(s_5, s_1) = \frac{1}{4}\sum_{k=1}^{4} z(s_5(k), s_1(k)) = \frac{1}{4}(1 + 0.99 + 0.56 + 0.94) = 0.87$$

$$z(s_5, s_2) = 0.86,\quad z(s_5, s_3) = 0.70,\quad z(s_5, s_6) = 0.79,\quad z(s_5, s_7) = 0.76$$

Repeat the selection process 1 and obtain the grey relational grade for Field 2, Field 3, and Field 4; see Table 4.4 below:

Table 4.4 The Results Showing the Grey Relational Grade of Each Proposal

Fields	*p*1	*p*2	*p*3	*p*4	*p*5	*p*6	*p*7	*p*8	*p*9
F(1)	0.87	0.86	0.70	u*	**1.00**	0.79	0.76	u	—
F(2)	0.78	0.81	0.76	**1.00**	0.81	0.76	0.94	0.72	—
F(3)	u	0.81	**1.00**	0.85	u	0.79	u	0.87	0.67
F(4)	0.85	u	**1.00**	u	0.80	0.89	0.84	0.92	—

u*: Unqualified projects.

2. Compute $r = B / \sum_{i=1}^{4}\sum_{j=1}^{5} b(i, j) = 6000 / 7923 = 75.7\%$

$$r(1) = r\sum_{j=1}^{5} b(i, j) = 75.7\%(1559) = 1180$$

$$r(2) = 75.7\%(4166) = 3154$$

$$r(3) = 75.7\%(1157) = 876$$

$$r(4) = 75.7\%(1041) = 788$$

3. Compute $e(i,j) = \dfrac{vi(i,j)}{I(i,j)}, \quad \forall (i,j) \in SI.$

Let $\{e(i,(j))\}$ be a decreasing rearrangement of $e(i,j)$,

i.e. $e(i,(1)) \ge e(i,(2)) \ge \cdots \ge e(i,(n))$ for all $\forall (i,j) \in SI.$

Table 4.5 Implicit Efficiencies for Each Project of Divisions with Proposal Ranking

Fields		p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9
F(1)	vi_1	8160	941	1486	u	4761	3449	1809	u	—
	I_1	571	86	396	u	200	438	415	u	—
	e_1	14.29	10.94	3.75	u	23.80	7.87	4.36	u	—
		(2)	(3)	(6)	u	(1)	(4)	(5)	u	—
F(2)	vi_2	2992	2853	1210	5186	4900	743	4150	678	—
	I_2	959	973	195	712	991	176	575	509	—
	e_2	3.12	2.93	6.21	7.28	4.94	4.22	7.22	1.33	—
		(6)	(7)	(3)	(1)	(4)	(5)	(2)	(8)	—
F(3)	vi_3	u	2255	1989	395	u	123	u	866	26
	I_3	u	774	363	131	u	89	u	321	71
	e_3	u	2.91	5.48	3.02	u	1.38	u	2.70	0.37
		u	(3)	(1)	(2)	u	(5)	u	(4)	(6)
F(4)	vi_4	296	u	66	u	760	39	18	44	—
	I_4	384	u	152	u	160	135	278	195	—
	e_4	0.77	u	0.43	u	4.75	0.29	0.06	0.23	—
		(2)	u	(3)	u	(1)	(4)	(6)	(5)	—

Also compute $w(i,j) = \dfrac{ve(i,j)}{I(i,j)}, \quad \forall (i,j) \in SI.$

Let $\{w(i,(\bar{j}))\}$ be a decreasing rearrangement of $w(i,j)$,

i.e. $w(i,(\bar{1})) \geq w(i,(\bar{2})) \geq \cdots \geq w(i,(\bar{n}))$ for all $\forall (i,j) \in SI$.

Table 4.6 Explicit Efficiencies for Each Project of Divisions with Proposal Ranking

Fields		p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9
F(1)	ve_1	863	74	3065	u	213	1531	2013	u	—
	I_1	571	86	396	u	200	438	415	u	—
	w_1	1.51	0.86	7.74	u	1.07	3.50	4.85	u	—
		(4)	(6)	(1)	u	(5)	(3)	(2)	u	—
F(2)	ve_2	7154	8325	162	9898	7276	300	7481	380	—
	I_2	959	973	195	712	991	176	575	509	—
	w_2	7.46	8.56	0.83	13.90	7.34	1.70	13.01	0.75	—
		(4)	(3)	(7)	(1)	(5)	(6)	(2)	(8)	—
F(3)	ve_3	u	547	62	60	u	24	u	55	395
	I_3	u	774	363	131	u	89	u	321	71
	w_3	u	0.71	0.17	0.46	u	0.27	u	0.17	5.56
		u	(2)	(6)	(3)	u	(4)	u	(5)	(1)
F(4)	ve_4	4373	u	4491	u	1797	1884	4123	2680	—
	I_4	384	u	152	u	160	135	278	195	—
	w_4	11.39	u	29.55	u	11.23	13.96	14.83	13.74	—
	vo_1	(5)	u	(1)	u	(6)	(3)	(2)	(4)	—

4. $y(i,(j)) = 0, \forall (i,j) \in SI$

$T(1) = 0$

for $i = 1,2,3,4,$

if $IJJ - SI \neq \phi$ then

for $1 \le j \le 9, 1 \le \bar{j} \le 9,$

(i) To compare the corresponding lower level objective value of $e(1,(1))$ and $w(1,(\bar{1}))$, to determine the value

$$LO(i^*, j^*) = \max \begin{cases} (213 \times 1) + (4761)(1) = 4974 \\ (3065 \times 0.7) + (1486)(1) = 3632 \end{cases}$$

Between $e(1,(1))$ and $w(1,(\bar{1}))$, $e(1,(1))$ is chosen after computation and comparison. Therefore, we let $y(1,5) = 1$, i.e., to select $p(1,5)$, and to obtain

$$T(1) = T(1) + b(1,(1)) = T(1) + b(1,5) = 142 \le r(1) \le 1180$$

then $y(1,5) = 1, \quad IJJ = IJJ \cup \{(i^*, j^*)\}.$

(ii) Next j, choose $e(1,(2))$ and $w(1,(\bar{1}))$, then compute

$$\max \begin{cases} (213 \times 1 + 863 \times 0.87) + (4761 + 8160)(1) = 13885 \\ (213 \times 1 + 3065 \times 0.7) + (4761 + 1486)(1) = 8606 \end{cases}$$

Between $e(1,(2))$ and $w(1,(\bar{1}))$, $e(1,(2))$ is chosen after computation and comparison. Therefore, we let $y(1,1) = 1$, i.e., to select $p(1,1)$, and to obtain

$$T(1) = T(1) + b(1,(2)) = 142 + 424 = 566 \le r(1) \le 1180$$

then $y(1,1) = 1, \quad IJJ = IJJ \cup \{(i^*, j^*)\}.$

(iii) Next j, choose $e(1,(3))$ and $w(1,(\bar{1}))$, then compute

$$\max \begin{cases} (213 + 751 + 74 \times 0.86) + (4761 + 8160 + 941)(1) = 14890 \\ (213 + 751 + 3065 \times 0.7) + (4761 + 8160 + 1486)(1) = 17517 \end{cases}$$

Between $e(1,(3))$ and $w(1,(\bar{1}))$, $w(1,(\bar{1}))$ is chosen after computation and comparison. Therefore, we let $y(1,3)=1$, i.e., to select $p(1,3)$, and to obtain

$$T(1)=T(1)+b(1,(6))=142+424+313=879\leq r(1)\leq 1180$$

$$\text{then}\quad y(1,3)=1,\quad IJJ=IJJ\cup\{(i^*,j^*)\}.$$

(iv) Next j, choose $e(1,(3))$ and $w(1,(\bar{2}))$, then compute

$$\max\begin{cases}(213+751+2146+64)+(4761+8160+1486+941)(1)=18522\\(213+751+2146+1598)+(4761+8160+1486+1809)(1)=20924\end{cases}$$

Between $e(1,(3))$ and $w(1,(\bar{2}))$, $w(1,(\bar{2}))$ is chosen after computation and comparison. Therefore, we let $y(1,7)=1$, i.e., to select $p(1,7)$, and to obtain

$$T(1)=T(1)+b(1,(6))=142+424+313+294=1173\leq r(1)\leq 1180$$

$$\text{then}\quad y(1,7)=1,\quad IJJ=IJJ\cup\{(i^*,j^*)\}.$$

(v) Next j, choose $e(1,(3))$ and $w(1,(\bar{3}))$, then compute

$$\max\begin{cases}(4708+64)+(16216+941)(1)=21929\\(4708+1209)+(16216+3449)(1)=25582\end{cases}$$

Between $e(1,(3))$ and $w(1,(\bar{3}))$, $w(1,(\bar{3}))$ is chosen after computation and comparison. Therefore, we select $p(1,6)$ and to obtain

$$T(1)=T(1)+b(1,(6))=142+424+313+294+321=1494>r(1)>1180$$

$$\text{then}\quad y(1,7)=0,\quad y(i,j)\in y(i',(j')).$$

Next i.

Repeat the process 4; similarly, the rearrangement of each division's projects in table 4.7 is obtained.

Table 4.7 The Rearrangement of Each Field's Projects

F(1)	p	p_5	p_1	p_3	p_7	p_6	p_2		
	vi	4761	8160	1486	1809	3449	941		
	$ve \cdot z$	213	751	2146	1530	1209	64		
	V	4974	8911	3632	3339	4658	1005		
	b	142	424	313	294	321	65		
	$L(1)$	9.11	36.31	56.38	75.24	95.83	100		
F(2)	p	p_4	p_7	p_2	p_1	p_5	p_3	p_6	p_8
	vi	5186	4150	2853	2992	4900	1210	743	678
	$ve \cdot z$	9898	7032	6743	5580	5894	123	228	274
	V	15084	11182	9596	8572	10794	1333	971	952
	b	569	455	808	767	794	146	140	487
	$L(2)$	13.66	24.58	43.98	62.39	81.45	84.95	88.31	100
F(3)	p	p_3	p_4	p_2	p_8	p_9	p_6		
	vi	1989	395	2255	866	26	123		
	$ve \cdot z$	62	51	454	48	265	19		
	V	2051	446	2709	914	291	142		
	b	262	100	481	229	51	34		
	$L(3)$	22.64	31.29	72.86	92.65	97.06	100		
F(4)	p	p_3	p_7	p_5	p_1	p_8	p_6		
	vi	66	18	760	296	44	39		
	$ve \cdot z$	4491	3463	1438	3717	2466	1677		
	V	4557	3481	2198	4013	2510	1716		
	b	114	254	111	282	168	112		
	$L(4)$	10.95	35.35	46.01	73.10	89.24	100		

Thus, the decision variables are set to equal 1, i.e., the proposals are selected.

(Decision Variable)

$$y(1,1) = y(1,3) = y(1,5) = y(1,7) = 1$$

$$y(2,1) = y(2,2) = y(2,4) = y(2,7) = 1$$

$$y(3,2) = y(3,3) = y(3,4) = 1$$

$$y(4,1) = y(4,3) = y(4,5) = y(4,7) = 1$$

5. Let $D = B - \sum b(i,(j))x(i,(j))$

= 6000 − (142+424+313+294) − (569+455+808+767) − (262+100+481) − (114+254+111+282)

= 624

6. Since D = 624, into step 7.

7. Set $x = 20$, let $65.7 \le L(m)$ and $L(i) \le 85.7$,

$L(1)$ = 75.24%; $L(2)$ = 62.39%; $L(3)$ = 72.86; $L(4)$ = 73.10%,

choose $L(2)$.

8. $y(2,3) = y(2,6) = 1$.

9. Repeat step 6, D = 624 − 146 − 140 = 338, into step 7.

$L(1)$ = 75.24%; $L(2)$ = 69.25%; $L(3)$ = 72.86; $L(4)$ = 73.10%,

choose $L(1)$, $L(3)$ and $L(4)$, let $y(1,2) = y(3,9) = y(4,6) = 1$

D = 338 − 65 − 51 − 112 = 110, into step 7.

$L(1)$ = 79.41%; $L(2)$ = 69.25%; $L(3)$ = 77.27; $L(4)$ = 83.86%,

choose $L(3)$, let $y(3,6) = 1$,

D = 110 − 34 = 76 (By the above algorithm, a solution is obtained)

10. End.

$$\text{Solution} \quad y = \begin{bmatrix} 1,1,1,0,1,0,1,0 \\ 1,1,1,1,0,1,1,0 \\ 0,1,1,1,0,1,0,0,1 \\ 1,0,1,0,1,1,1,0 \end{bmatrix}.$$

Table 4.8 The Decision Variables of the Lower Level for Each Unit's Project

Fields	$p1$	$p2$	$p3$	$p4$	$p5$	$p6$	$p7$	$p8$	$p9$
F(1)	1	1	1	0	1	0	1	0	—
F(2)	1	1	1	1	0	1	1	0	—
F(3)	0	1	1	1	0	1	0	0	1
F(4)	1	0	1	0	1	1	1	0	—

This final solution gives the level of satisfaction are

$L(1) = 79.41$

$L(2) = 69.25$

$L(3) = 80.21$

$L(4) = 83.86$

The minimum difference of maximum level of satisfaction and minimum level of satisfaction $x = \{\max L(i, x_i) - \min L(i, x_i)\} = 83.86 - 69.25 = 14.61$ □

The total value obtained from this solution is $\sum_{j=1}^{n} vi(i, j)y(i, j) + ve(i)z(i) = 90{,}217.$ □

The following diagram is a flow chart of the solution algorithm of the BLMF-PC programming in figure 4.2 and 4.3.

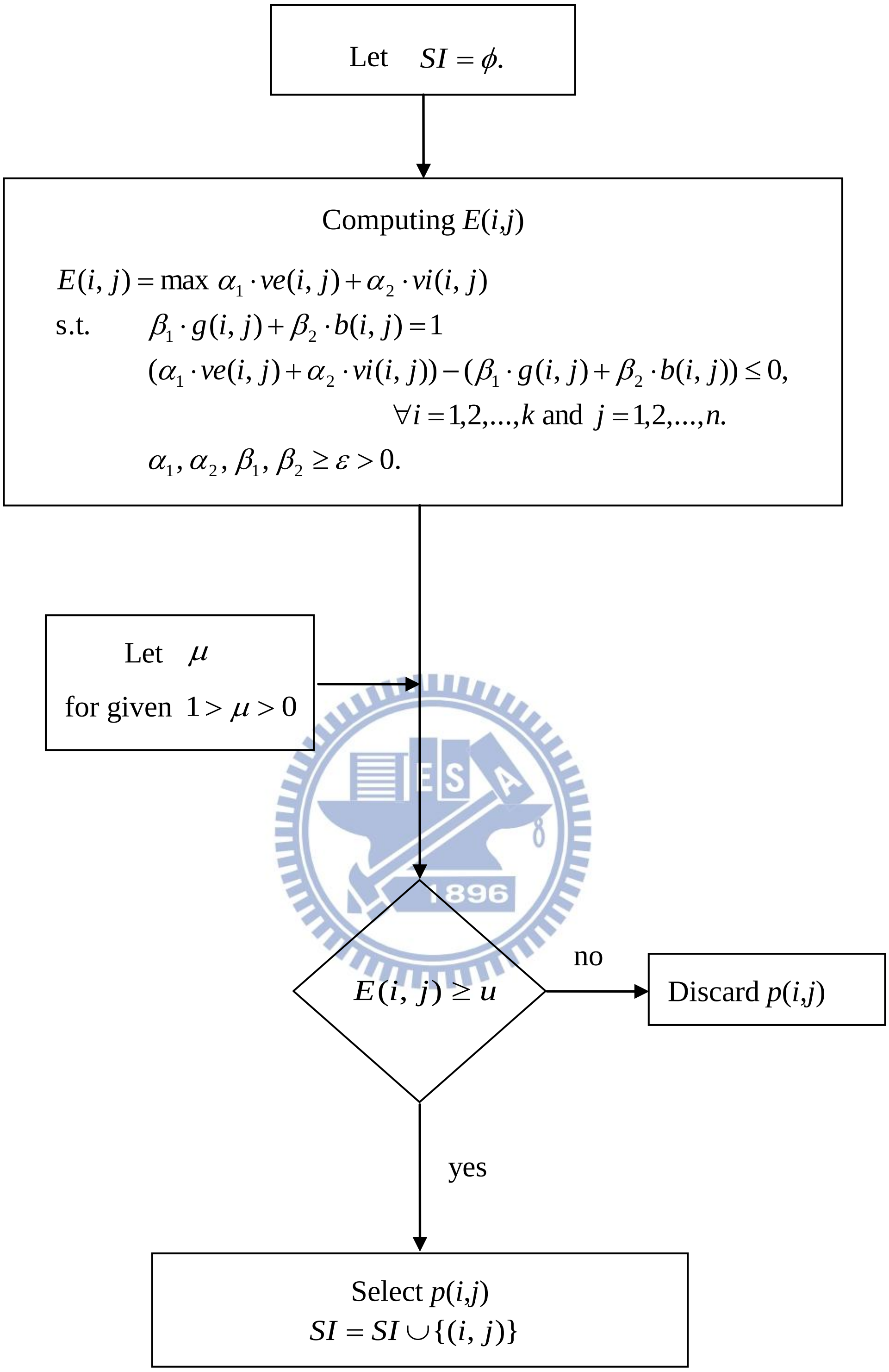


Figure 4.2 Diagram of the Preprocessing Stage for BLMF-PC

Source: Study

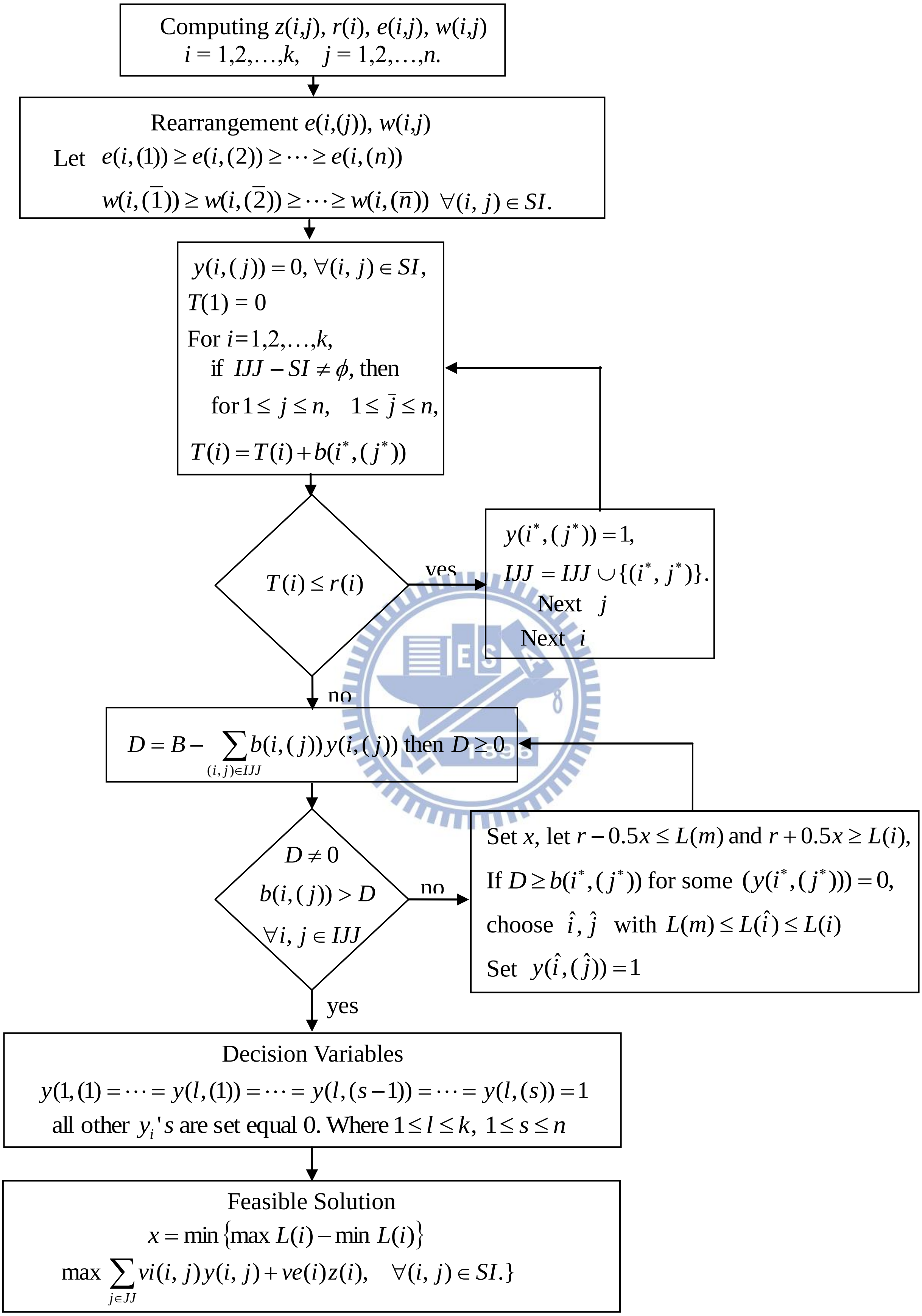


Figure 4.3 Diagram of the Heuristic Algorithm for BLMF-PC

Source: Study

4.6 Summaries

This chapter extends the BLMF problems to the case when decision variables are partially cooperative. The definitions and the characteristics of BLMF-PC are discussed, and a multi-follower budget distribution model with partial cooperative variables is constructed.

In this model, the goal of the upper level DM is to minimize the level of satisfaction among the individual divisions upon their funding approvals, and the constraints are to ensure the output efficiency is no less than the input efficiency. Under the budget allocation policy of the upper level DM, the lower level DM pursues the largest value of each individual division in order to maximize the total value of the organization. So, the optimal decision is to maximize the explicit and the implicit values of each proposal.

The BLMF-PC budget allocation problems are solved using the concepts of GDEA for preprocessing the data of the projects from each division to guarantee the quality of funded projects; this avoids unnecessary distributions. The grey relational analysis and the heuristic algorithm are then applied for a budget distribution.

This extension models the hierarchical structure of the real world more precisely. This chapter mainly investigates the case with multiple lower level DMUs with partially cooperative variables in a bilevel program. The model has the following properties:

(a) It is a bilevel programming model.

(b) Lower level DM has multiple objectives.

(c) All DMUs are partially dependent with partial cooperative decision variables.

(d) Lower level decision variables are discrete.

The two-stage solution algorithm is developed: stage 1: GDEA, and stage 2: the grey relational analysis and the heuristic algorithm. The final solution is feasible and can be optimal or near optimal. Most importantly, a quick user guide and a source code for the grey relational analysis and the heuristic algorithm of the BLMF-PC are written in this dissertation, they can easier to solve these difficult problems.

Chapter 5 Concluding Remarks

5.1 Conclusions

There are two types of the models for the budget allocation in this thesis. In Chapter 3, a classical bilevel programming model is developed. This simple model with BLMF-UC uses the uncooperative variable. In Chapter 4, a newly improved BLMF-PC model is devolved with the partial cooperative variables.

The budget distribution problems are solved using a two-stage method (stage 1: GDEA preprocessing, stage 2: the grey relational analysis and the heuristic algorithm) to obtain a final solution. However, the solution might be near optimal instead of optimal; using this method is much simpler than using a traditional algorithms.

This budget allocation model of the bilevel programming problems with the multiple followers has the following properties:

(a) It is a bilevel programming model; the upper level is called the leader and the lower level is called the follower.

(b) A bilevel decision-making system with one single upper level decision maker (DM) and multiple lower level decision-making units (DMUs).

(c) Each lower level DMU optimizes its own objective; hence, the lower level has multiple objective decision-making problems.

(d) The lower level decision variables are discrete.

(e) The leader controls a set of decision variables while each follower controls one's own decision variables.

(f) Each lower level DM's decision variables are uncooperative (called independent); the DM's cooperative decision variables are dependent.

The major contributions of this thesis are listed below:

(a) The budget allocation model of a bilevel multiple follower 0/1 programming problems involving uncooperative and partial cooperative variables are developed. In the new bilevel budget allocation models, the upper level chooses the better projects from given proposals to maximize the value of the lower level projects and to minimize the ratio of the funding differences among divisions.

(b) The output values are visible and invisible of proposed projects, namely, explicit value and implicit value, respectively. Where the explicit value is directly created by the organization, usually all variables are dependent. The implicit value is impacted by the divisions, and its variables are independent.

(c) A new GDEA, an improvement of DEA, is developed. It is an important procedure of distribution to make sure the quality of the proposals from the upper level decision maker above the controlled efficiency factor.

(d) Apply the theory of the grey relationship to obtain the grey relationship grade among all the lower level DMUs in order to solve the problem of lower levels with a partial cooperational relationship.

(e) A new heuristic algorithm is developed for the budget allocation solution. The solution algorithm takes advantage of the nature of this problem, which gives a feasible solution for this particular model. The algorithm is efficient, and solutions are acceptable for the real world situation. It is simpler than the classical solution methods are.

(f) The quick user guides and the source codes of the heuristic algorithm for the BLMF-UC and the BLMF-PC are written, which easier to solve these difficult problems of the BLP.

5.2 Further Research

The current model deals with a single leader and multiple followers; the multiple follower decision variables include uncooperative and partial cooperative variables. But this model is not suitable when more than one leader is involved in the upper level of the organization.

However, in reality, it is possible to have multiple leaders in a hierarchical decision-making structure/system. In this system, the leader group optimizes multiple objectives in the upper level, and, as before, the multiple followers in the lower level are considered. To model this problem, one would consider the cases where the variables are uncooperative and partially cooperative in addition to the multiple objectives of the leaders and the followers.

References

1. Ben-Ayed O., Boyce D. E., Blair, C. E., “A General Bilevel Linear Programming Formulation of the Network Design Problem,” Transportation Research, B22, pp. 311-318, 1988.

2. Bialas Wayne F., “Multilevel Mathematical Programming: An Introduction,” Department of Industrial Engineering, State University of New York at Buffalo, From: http://www.acsu.buffalo.edu/~bialas/IE664.html, 2002.

3. Bialas Wayne F., Karwan Mark H., “Mathematical Methods for Multilevel Planning,” Research Report No.79-2, the Operations Research Program, Department of Industrial Engineering, State University of New York at Buffalo, 1979.

4. Bialas Wayne F., Karwan Mark H., “Multilevel Linear Programming,” Technical Report 78-1, the Operations Research Program, Department of Industrial Engineering, State University of New York at Buffalo, 1978.

5. Bialas Wayne F., Karwan Mark H., “Multilevel Optimization: A Mathematical Programming Perspective,” the Operations Research Program, Department of Industrial Engineering, State University of New York at Buffalo, 1980.

6. Bialas Wayne F., Karwan Mark H., “On Two-Level Optimization,” IEEE Transactions on Automatic Control, AC-27 (1), pp. 211-214, 1982.

7. Bialas Wayne F., Karwan Mark H., “Two-Level Linear Programming,” Management Science, 30 (8), pp. 1004-1020, 1984.

8. Bialas Wayne F., Karwan Mark H., Shaw J. P., “A Parametric Complementary Pivot Approach for Two-level Linear Programming,” Research Report 80-2, the Operations Research Program, Department of Industrial Engineering, State University of New York at Buffalo, 1980.

9. Cao D., Leung L. C., “A Partial Cooperation Model for Non-Unique Linear Two-Level Decision Problems,” European Journal of Operational Research, 140, pp. 134-141, 2002.

10. Charnes A., Cooper W. W., Rhodes E. L., “Measuring the efficiency of decision making units,” European Journal of Operational Research, 2 (6), pp. 429-444, 1978.

11. Chenggen Shi, Guangquan Zhang, Jie Lu, “On the Definition of Linear Bilevel Programming Solution,” Applied Mathematics and Computation, 160, pp. 169-176, 2005.

12. Chenggen Shi, Hong Zhou, Jie Lu, Guangquan Zhang, Zhongwei Zhang, "The *K*th-best Approach for Linear Bilevel Multifollower Programming with Partial Shares Variables among Followers," Applied Mathematics and Computation, 188, pp. 1686-1698, 2007.

13. Chenggen Shi, Jie Lu, Guangquan Zhang, "An Extended *K*th-best Approach for Linear Bilevel Programming," Applied Mathematics and Computation, 164, pp. 843-855, 2005.

14. Chenggen Shi, Jie Lu, Guangquan Zhang, "An Extended Kuhn-Tucker Approach for Linear Bilevel Programming," Applied Mathematics and Computation, 162, pp. 51-63, 2005.

15. Chenggen Shi, Jie Lu, Guangquan Zhang, Hong Zhou, "An Extended Branch and Bound Algorithm for Linear Bilevel Programming," Applied Mathematics and Computation, 180, pp. 529-537, 2006.

16. Cook W. D., Kress M., "Data envelopment model for aggregating preference ranking," Management Science, 36 (11), pp. 1302-1310, 1990.

17. Dantzig G. B., Thapa M. N., Linear Programming 1: Introduction, New York Springer, 1997.

18. Dauer J. P., Yi-Hsin Liu, "Solving multiple objective linear programs in objective space," European Journal of Operational Research, 46, pp. 350-357, 1990.

19. Ernest H. Forman, Mary Ann Selly, Decision by Objectives, World Scientific Publishing Co., Singapore, 2001.

20. Franklin Liu, Hao H. P., "Ranking of units on the DEA frontier with common weights," Computers and Operations Research, 35 (5), pp. 1624-1637, 2008.

21. George L., Nemhauser, Laurence A., Wolsey, Integer and Combinatorial Optimization, John Wiley & Sons, Canada, 1988.

22. Gui-Ling Li, Mao-Xian Zhao, "The Further Research on the Optimal Solution of Bi-level Programming Problem," Journal of Shandong University of Science and Technology (Natural Science), 25 (3), pp. 100-102, 2006.

23. Jie Lu, Chenggen Shi, Guangquan Zhang, "On Bilevel Multi-Follower Decision Making: General Framework and Solutions", Information Science - An International Journal, 176, pp. 1607-1627, 2006.

24. Jonathan F. Bard, "Coordination of a Multidivisional Organization through Two levels of Management," OMEGA, 11 (5), pp. 457-468, 1983.

25. Jonathan F. Bard, James E. Falk, "An Explicit Solution to the Multi-Level Programming Problem," Computers and Operations Research, 9 (1), pp. 77-100, 1982.

26. Jonathan F. Bard, Practical Bilevel Optimization: Algorithms and Applications, Kluwer Academic Publishers, Netherlands, 1998.

27. Kornai J., Liptak T. H., "Two-Level Planning," Econometrica, 33 (1), pp. 141-168, 1965.

28. Land A. H., Doig A. G., "An automatic method of solving discrete programming problems." Econometrica, 28 (3), pp. 497-520, 1960.

29. Omar B. A., Blair E. C., "Computational Difficulties of Bilevel Linear Programming," Operations Research, 38 (4), pp. 556-560, 1990.

30. Pierre Hansen, Brigitte Jaumard, Gilles Savard, "New Branch-and-Bound Rules for Linear Bilevel Programming," SIAM Journal on Scientific and statistical Computing, 13 (5), pp. 1194-1217, 1992.

31. Richard O. Mason, Swanson E. Burton, "Measurement for Management Decision: A Perspective," the California Management Review, 21 (3), pp. 14, 1979.

32. Roy A. Bauer, Collar, Emilio, Tang, "Victor the Silverlake Project: Transformation at IBM," Oxford University Press, New York, 1992.

33. Saati S., "Determining a common set of weights in DEA by solving a linear program," Journal of Industrial Engineering International, 4 (6), pp. 51-56, 2008.

34. Shuh-Tzy Hsu, An-Der Huang, Ue-Pyng Wen, "Linear Multi-Division Bi-level Programming Problem," Journal of the Chinese of Industrial Engineers, 10 (4), pp. 257-262, 1993.

35. Toshihide Ibaraki, Naoki Katoh, Resource Allocation Problems: Algorithmic Approaches, The MIT Press, Cambridge and Massachusetts London, England, 1988.

36. Ue-Pyng Wen, Shuh-Tzy Hsu, "Linear Bilevel Programming Problems – A Review," Operational Research Society, 42 (2), pp. 125-133, 1991.

37. Wayne L. Winston, Introduction to Mathematical Programming: Applications and Algorithms, PWS-KENT Publishing Company, New York, 1991.

38. Yibing Lv, Tiesong Hu, Guangmin Wang and Zhongping Wan, "A Penalty Function Method Based on Kuhn-Tucker Condition for Solving Linear Bilevel Programming," Applied Mathematics and Computation, 188, pp. 808-813, 2007.

39. Yi-Hsin Liu and Stephen M. Hart, "Characterizing an Optimal Solution to the Linear Bi-level Programming Problem," European Journal of Operational Research, 73 (1), pp. 164-166, 1994.

40. Yi-Hsin Liu and Thomas H. Spencer, "Solving a Bilevel Linear Program When the Inner Decision Maker Controls Few Variables," European Journal of Operational Research, 81 (3), pp. 644-651, 1995.

Appendix 1: BLMF-UC Quick User Guide

System Requirement:

Any OS (Window/Mac/Linux…, etc.) with JavaScript supported explorer, such like Internet Explorer. Please note that you should open the security rule to make sure the explorer can read the local files in your hard disk.

1. Build a data sheet.

Each column means the input/output of different projects.

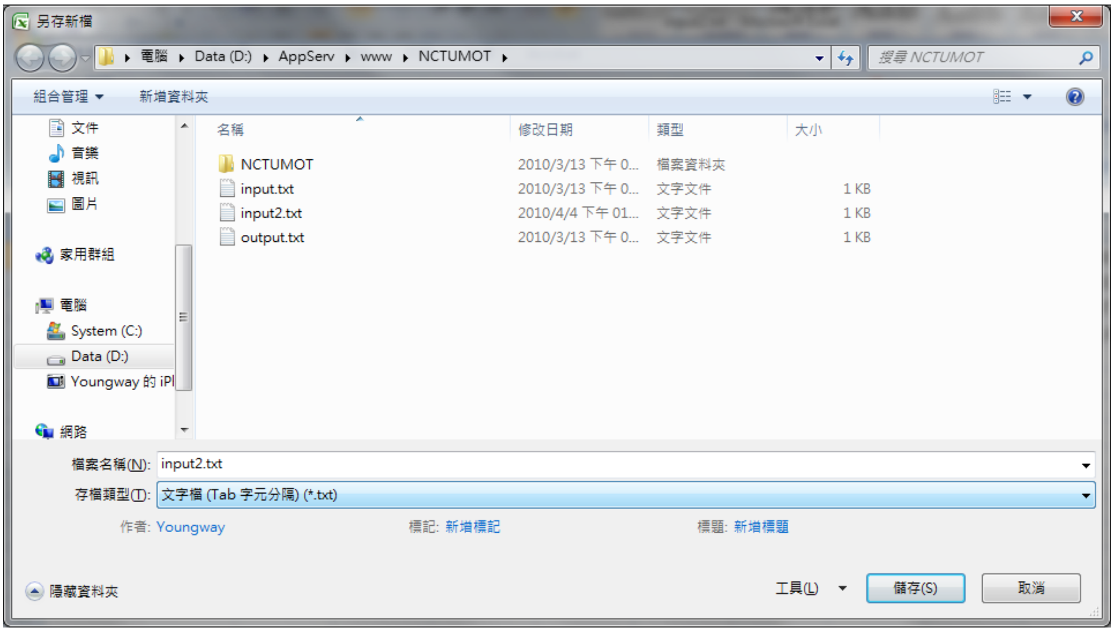
Each row means different fields/industries/business units, …, etc.

	A	B	C	D	E	F	G
1	Field	Project	Input	Output			
2	1	1	14	46			
3	1	2	134	519			
4	1	3	139	368			
5	1	4	91	328			
6	1	5	44	91			
7	1	6	34	129			
8	1	7	212	785			
9	1	8	214	831			
10	1	9	130	462			
11	2	1	27	145			
12	2	2	46	134			
13	2	3	183	855			
14	2	4	75	237			
15	2	5	91	250			
16	2	6	45	190			
17	2	7	112	454			
18	3	1	148	665			
19	3	2	145	356			
20	3	3	59	218			
21	3	4	14	36			
22	3	5	65	190			
23	3	6	110	512			
24	3	7	23	74			
25	3	8	46	110			
26	4	1	149	232			
27	4	2	76	145			
28	4	3	108	263			

2. Save the input/output data as text file.

Use “Save As…” function to save the data:

a. Choose type to “Text file (Separate by Tab)(*.txt)/文字檔(Tab 字元分隔)(*.txt)”



b. Save input/budget as “input.txt”, and save output as “output.txt”

3. Run “RunBLP.htm”

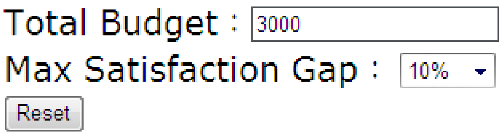
The program should integrate the input/output data if the data is valid. Shown below:

Input and Output Table

Field	Project	Cost	Value
F(0)	P(0)	14	46
	P(1)	134	519
	P(2)	139	368
	P(3)	91	328
	P(4)	44	91
	P(5)	34	129
	P(6)	212	785
	P(7)	214	831
	P(8)	130	462
F(1)	P(0)	27	145
	P(1)	46	134
	P(2)	183	855
	P(3)	75	237
	P(4)	91	250
	P(5)	45	190
	P(6)	112	454
F(2)	P(0)	148	665
	P(1)	145	356
	P(2)	59	218
	P(3)	14	36
	P(4)	65	190
	P(5)	110	512
	P(6)	23	74
	P(7)	46	110
F(3)	P(0)	149	232
	P(1)	76	145
	P(2)	108	263
	P(3)	92	210
	P(4)	59	106
	P(5)	118	438
	P(6)	83	142
	P(7)	135	728
	P(8)	145	312
	P(9)	188	536
F(4)	P(0)	335	792
	P(1)	269	665
	P(2)	71	181
	P(3)	96	308
	P(4)	109	419
	P(5)	130	420

If the program does not integrate the data, please check if the input/output file is in right format.

4. Set Total Budget and Max Satisfaction Gap.



5. Start calculation.

Press “Run” and the program will list the decision flow and final budget left/total output.

Caculation start...

Input Budget is valid.

The optimal satisfaction is 68.74% for each filed:

Field 1: $695;

Field 2: $398;

Field 3: $419;

Field 4: $792;

Field 5: $694;

***** 1st run -- Choose the project based on optimal satisfaction. Each Field cannot exceed 68.74%. *****

===Choose the project from Field 1. ===

choose Project 8. (Satisfaction: 21.15%)

choose Project 2. (Satisfaction: 34.39%)

choose Project 6. (Satisfaction: 37.75%)

choose Project 7. (Satisfaction: 58.7%)

choose Project 4. (Satisfaction: 67.69%)

(!)If choose project 1, the satisfaction (69.07%) will over 68.74%.

(!)If choose project 3, the satisfaction (81.42%) will over 68.74%.

(!)If choose project 5, the satisfaction (72.04%) will over 68.74%.

(!)If choose project 9, the satisfaction (80.53%) will over 68.74%.

===Choose the project from Field 2. ===
choose Project 1. (Satisfaction: 4.66%)
choose Project 3. (Satisfaction: 36.27%)
choose Project 6. (Satisfaction: 44.04%)
choose Project 7. (Satisfaction: 63.39%)
(!)If choose project 2, the satisfaction (71.33%) will over 68.74%.
(!)If choose project 4, the satisfaction (76.34%) will over 68.74%.
(!)If choose project 5, the satisfaction (79.1%) will over 68.74%.

===Choose the project from Field 3. ===
choose Project 6. (Satisfaction: 18.03%)
choose Project 1. (Satisfaction: 42.3%)
choose Project 3. (Satisfaction: 51.97%)
(!)If choose project 2, the satisfaction (75.74%) will over 68.74%.
choose Project 7. (Satisfaction: 55.74%)
(!)If choose project 2, the satisfaction (79.51%) will over 68.74%.
choose Project 5. (Satisfaction: 66.39%)
(!)If choose project 2, the satisfaction (90.16%) will over 68.74%.
choose Project 4. (Satisfaction: 68.69%)
(!)If choose project 2, the satisfaction (92.46%) will over 68.74%.
(!)If choose project 8, the satisfaction (76.23%) will over 68.74%.

===Choose the project from Field 4. ===
choose Project 8. (Satisfaction: 11.71%)
choose Project 6. (Satisfaction: 21.94%)
choose Project 10. (Satisfaction: 38.25%)
choose Project 3. (Satisfaction: 47.61%)
choose Project 4. (Satisfaction: 55.59%)
choose Project 9. (Satisfaction: 68.17%)
(!)If choose project 1, the satisfaction (81.09%) will over 68.74%.
(!)If choose project 2, the satisfaction (74.76%) will over 68.74%.
(!)If choose project 5, the satisfaction (73.29%) will over 68.74%.
(!)If choose project 7, the satisfaction (75.37%) will over 68.74%.

===Choose the project from Field 5. ===
choose Project 5. (Satisfaction: 10.79%)
choose Project 6. (Satisfaction: 23.66%)
choose Project 4. (Satisfaction: 33.17%)
choose Project 3. (Satisfaction: 40.2%)

(!)If choose project 1, the satisfaction (73.37%) will over 68.74%.
choose Project 2. (Satisfaction: 66.83%)

===1st run Caculation complete.===
Each field received (satisfaction):
Field 1: $685/$1012 (67.69%)
Field 2: $367/$579 (63.39%)
Field 3: $419/$610 (68.69%)
Field 4: $786/$1153 (68.17%)
Field 5: $675/$1010 (66.83%)

*****Second run -- Check the rest budget, if it still can be used on some projects, than consider those qualified projects.*****
Only choose the project which can meet satisfaction requirement (63.74% ~ 73.74%)

Field 1 Project 1 is OK for budget and satisfaction requirent. Choose this project.
Field 2 Project 2 is OK for budget and satisfaction requirent. Choose this project.

===2st run Caculation complete.===

*****Caculation is finished.*****
Each field received (satisfaction):
Field 1: $699/$1012 (69.07%)
Field 2: $413/$579 (71.33%)
Field 3: $419/$610 (68.69%)
Field 4: $786/$1153 (68.17%)
Field 5: $675/$1010 (66.83%)

Budget Used：$2992
Budget Left：$8
Total Output:10591
Total efficiency obtained：353.97%
Satification Level is 4%

Appendix 2: BLMF-UC Source Code

```
<!DOCTYPE html PUBLIC "-//W3C//DTD XHTML 1.0 Transitional//EN"
"http://www.w3.org/TR/xhtml1/DTD/xhtml1-transitional.dtd">
<html xmlns="http://www.w3.org/1999/xhtml">
<head>
<META http-equiv="Content-Type" content="text/html; charset=utf-8" />
<META NAME="robots" CONTENT="noindex,nofollow">
<META HTTP-EQUIV="CACHE-CONTROL" CONTENT="NO-CACHE">
<META HTTP-EQUIV="EXPIRES" CONTENT="0">
<META HTTP-EQUIV="PRAGMA" CONTENT="NO-CACHE">

<!-- jQuery & Plug-in -->
<script type="text/javascript" src="./jquery-1.4.2.min.js"></script>
<link rel="stylesheet" href="main.css" type="text/css">
<script type="text/javascript">

/*Basic Function Start-->*/
function explode (delimiter, string, limit) {
    /* Splits a string on string separator and return array of components. If limit is positive
only limit number of components is returned. If limit is negative all components except the
last abs(limit) are returned. */
    /* version: 909.322 */
    /* discuss at: http://phpjs.org/functions/explode        // +        original by: Kevin van
Zonneveld (http://kevin.vanzonneveld.net) */
    /* +        improved by: kenneth */
    /* +        improved by: Kevin van Zonneveld (http://kevin.vanzonneveld.net) */
    /* +        improved by: d3x */
    /* +        bugfixed by: Kevin van Zonneveld (http://kevin.vanzonneveld.net)        // *
example 1: explode(' ', 'Kevin van Zonneveld'); */
    /* *        returns 1: {0: 'Kevin', 1: 'van', 2: 'Zonneveld'} */
    /* *        example 2: explode('=', 'a=bc=d', 2); */
    /* *        returns 2: ['a', 'bc=d'] */
     var emptyArray = { 0: '' };

    /* third argument is not required */
    if ( arguments.length < 2 ||
        typeof arguments[0] == 'undefined' ||            typeof arguments[1] == 'undefined' )
    {
        return null;
    }
     if ( delimiter === '' ||
        delimiter === false ||
        delimiter === null )
    {
        return false;        }

    if ( typeof delimiter == 'function' ||
        typeof delimiter == 'object' ||
```

```
            typeof string == 'function' ||              typeof string == 'object' )
        {
            return emptyArray;
        }
          if ( delimiter === true ) {
            delimiter = '1';
        }

        if (!limit) {               return string.toString().split(delimiter.toString());
        } else {
            // support for limit argument
            var splitted = string.toString().split(delimiter.toString());
            var partA = splitted.splice(0, limit - 1);              var partB =
splitted.join(delimiter.toString());
            partA.push(partB);
            return partA;
        }
}
function statMsg(m){
  var q = $("label#stat")[0].innerHTML;
  q += m + "</br>";
  $("label#stat").html(q);
  return;
}

function reset(){
  for(var i in choose){
    for(var j in choose[j]){
      choose[i][j] = 0;
    }
  }
  for(var i in Lproj){
    Lproj[i] = 0;
  }
  totalOutput = 0;
  $("#LBudget").hide();
  return;
}
/*<--Basic Function End*/

/*Global variable Start-->*/
var maxproj=0;
var input = new Array();
var output = new Array();
var eff = new Array();
var choose = new Array();
var MAX_PROJ_NUM = 0;
var projnum = new Array();
var Lproj = new Array();
var totalOutput = 0;
```

```
var test = "";
var UnitInput = new Array();
/*<--Global variable End*/

/*Initial Data Start-->*/
function LoadData(){
  $.ajax({
    type: "POST",
    url: "data.txt",
    datatype: "text",
    success: function(data, status){
      var raw_data = explode("\n", data);
      for(i in raw_data){
        if (raw_data[i] != "") var sec_data = explode("\t", raw_data[i]);
        if (sec_data[0] == "Field") continue;
          /*fill into DATA*/
          var p = parseInt(sec_data[0])-1;
          var q = parseInt(sec_data[1])-1;
          if(input[p] == null) input[p] = new Array();
          if(input[p][q] == null) input[p][q] = new Array();
          if(output[p] == null) output[p] = new Array();
          if(output[p][q] == null) output[p][q] = new Array();
          if(eff[p] == null) eff[p] = new Array();
          if(eff[p][q] == null) eff[p][q] = new Array();
          if(choose[p] == null) choose[p] = new Array();
          if(choose[p][q] == null) choose[p][q] = new Array();

          input[p][q] = Number(sec_data[2]); /*own resource*/
          output[p][q] = Number(sec_data[3]); /*request funding*/
          if( input[p][q] > 0 ) eff[p][q]= Number(output[p][q]/input[p][q]);
          else eff[p][q] = 0;
          choose[p][q] = 0;

      }
      /*check max project number*/
      for(var i in input){
        projnum[i] = input[i].length;
        if(input[i].length > MAX_PROJ_NUM) MAX_PROJ_NUM = input[i].length-1;
      }
    },
    complete: function(){
      DATATab();
      EfficiencyTab();
    }
  });
}
function DATATab(){
  /*Header row*/
  var tar = document.getElementById("inputTab");
  var newRow = tar.insertRow(tar.rows.length);
```

```
  var cell = tar.rows[tar.rows.length-1].insertCell(0);
  cell.align = "center";
  cell.style.minWidth = "70px";
  cell.innerHTML = "Field";
  var cell = tar.rows[tar.rows.length-1].insertCell(1);
  cell.align = "center";
  cell.style.minWidth = "70px";
  cell.innerHTML = "Project";
  var cell = tar.rows[tar.rows.length-1].insertCell(2);
  cell.align = "center";
  cell.style.minWidth = "70px";
  cell.innerHTML = "Cost";
  var cell = tar.rows[tar.rows.length-1].insertCell(3);
  cell.align = "center";
  cell.style.minWidth = "70px";
  cell.innerHTML = "Value";
  /* contain rows*/
  for(var i in input){
    var FCol = 0;
    for(var j in input[i]){
      var newRow = tar.insertRow(tar.rows.length);
      if (FCol == 0){
        var cellfix = 0;
        var cell = tar.rows[tar.rows.length-1].insertCell(0);
        cell.rowSpan = input[i].length;
        cell.align = "center";
        cell.innerHTML = "F(" + i + ")";
      }else cellfix = -1;
      var cell = tar.rows[tar.rows.length-1].insertCell(cellfix+1);
      cell.align = "center";
      cell.innerHTML = "P(" + j + ")";
      var cell = tar.rows[tar.rows.length-1].insertCell(cellfix+2);
      cell.align = "center";
      cell.innerHTML = input[i][j];
      var cell = tar.rows[tar.rows.length-1].insertCell(cellfix+3);
      cell.align = "center";
      cell.innerHTML = output[i][j];
      FCol = null;
      cellfix = null;
    }
  }
}
function EfficiencyTab(){
  var tar = document.getElementById("effTab");
  var i=0;
  var newRow = tar.insertRow(tar.rows.length);
  var cell = tar.rows[tar.rows.length-1].insertCell(0);
  cell.align = "center";
  cell.style.minWidth = "70px";
  cell.innerHTML = "Unit\\Project";
```

```
  for(var j=0; j < MAX_PROJ_NUM; j++){
    var cell = tar.rows[tar.rows.length-1].insertCell(j+1);
    cell.align = "center";
    cell.style.minWidth = "70px";
    cell.innerHTML = "Project " + (j+1);
  }
  for(var i=0; i < input.length; i++){
    var newRow = tar.insertRow(tar.rows.length);
    var cell = tar.rows[tar.rows.length-1].insertCell(0);
    cell.align = "center";
    cell.innerHTML = "Unit" + (i+1);
    for(var j=0; j < MAX_PROJ_NUM; j++){
      var cell = tar.rows[tar.rows.length-1].insertCell(j+1);
      cell.align = "right";
      if(eff[i][j] != null) cell.innerHTML = parseInt(eff[i][j]*10000, 10)/10000;
      else cell.innerHTML = "---";
    }
  }
  ChoosenTab();
}
function ChoosenTab(){
  var tar = document.getElementById("choTab");
  var i=0;
  var newRow = tar.insertRow(tar.rows.length);
  var cell = tar.rows[tar.rows.length-1].insertCell(0);
  cell.align = "center";
  cell.style.minWidth = "70px";
  cell.innerHTML = "Unit\\Project";
  for(var j=0; j < MAX_PROJ_NUM; j++){
    var cell = tar.rows[tar.rows.length-1].insertCell(j+1);
    cell.align = "center";
    cell.style.minWidth = "70px";
    cell.innerHTML = "Project " + (j+1);
  }
  for(var i=0; i < input.length; i++){
    var newRow = tar.insertRow(tar.rows.length);
    var cell = tar.rows[tar.rows.length-1].insertCell(0);
    cell.align = "center";
    cell.innerHTML = "Unit" + (i+1);
    for(var j=0; j < MAX_PROJ_NUM; j++){
      var cell = tar.rows[tar.rows.length-1].insertCell(j+1);
      cell.align = "center";
      if(projnum[i] > j){
        if(choose[i][j] == 0) cell.innerHTML = "<font color='red'>o</font>";
        else cell.innerHTML = "<font color='green'>v</font>"
      } else {
        cell.innerHTML = "<font color='gray'>---</font>"
      }
    }
  }
```

```
  SatisfyTab();
}
function SatisfyTab(){
  var tar = document.getElementById("satTab");
  var itotal = new Array();
  for(var i in input){
    UnitInput[i] = 0;
    for (var j in input[i]){
      UnitInput[i] += parseInt(input[i][j]);
    }
  }
  for(var i in choose){
    itotal[i] = 0;
    for(var j in choose[i]){
      if(choose[i][j] == 1) itotal[i] += parseInt(input[i][j]);
    }
  }
  var updateSat = new Array();
  var MaxSat = 0, minSat = 100;
  for(var i=0; i < projnum.length; i++){
    updateSat[i] = parseInt(itotal[i]/UnitInput[i]*1000000, 10)/10000;
    var newRow = tar.insertRow(tar.rows.length);
    var cell = tar.rows[tar.rows.length-1].insertCell(0);
    cell.align = "center";
    cell.style.minWidth = "70px";
    cell.innerHTML = "Unit" + (i+1);
    var cell = tar.rows[tar.rows.length-1].insertCell(1);
    cell.align = "right";
    cell.style.minWidth = "70px";
    cell.innerHTML = updateSat[i] +"%";
  }
  for(y in updateSat){
    if(updateSat[y] > MaxSat) MaxSat = updateSat[y];
    if(updateSat[y] < minSat) minSat = updateSat[y];
  }
  var SDiff = parseInt($("#satset")[0].value)/100;
  if(SDiff*100 < parseInt((MaxSat - minSat))) {
    $("#CANTS").html("<font color=red size=+2>Satification Level cannot meet the
requirement ("+SDiff*100+"%).<br> Please double check the
configuration.<br><br></font>");
    $("#CANTS").show();
  }else $("#CANTS").hide();
  $("#SDiff").html("Satification Level is " + parseInt((MaxSat - minSat),10) + "%");
  if(parseInt(MaxSat - minSat)!=0) $("#SDiff").show();
}
/*<-- Initial Data End*/
/*Caculation Function Start-->*/
function Run(){
  $("#RUN").hide();
  reset();
```

```
  statMsg("Caculation start...");
  var p = $("input#MBudget")[0].value;
  if(isNaN(p)) {
    alert("Invalid Budget!");
    $("input#MBudget")[0].value = 0;
    statMsg("Invalid Budget! Please check Total Budget.");
    statMsg("Caculation terminated.");
    return;
  }else var b = parseInt(p);
  statMsg("Input Budget is valid.");
        /*Caculate RequestDollar */
  var s = new Array(); //total input for each unit
  var t = 0; //total input.
  for(var i=0; i < input.length; i++){
    s[i] = 0;
    for(var j=0; j < input[i].length; j++){
      s[i] += parseInt(input[i][j]);
    }
    t += parseInt(s[i]);
  }
  var RD = p/t;
  statMsg("The optimal satisfaction is "+parseInt(RD*10000)/100+"% for each filed:");
  var r = new Array();
  var lim = parseInt($("#satset")[0].value)/200;
  for(var i=0; i < input.length; i++){
    r[i] = s[i] * (RD-lim);
    statMsg("Field "+(i+1)+": $"+parseInt(r[i])+";");
  }
  $("#LBudget").show();
  $("#LBudget").html("Budget Left： " + b +", Total Output:" + totalOutput);
  statMsg("</br></br>***** 1st run -- Choose the project based on optimal satisfaction. Each
Field cannot exceed " + Math.round(RD*10000)/100 + "% - " + lim*100 + "% = " +
Math.round((RD-lim)*10000)/100 +"%. *****");
  for (i in eff){
    statMsg("<br>===Choose the project from Field "+(Number(i)+1)+". ===");
    var fin = 0;
    while( b > 0 && fin != 1){
      /*find the max most efficient project --- 1st run, will check Satisfaction */
      var Meff = 0, Mi = null, Mj = null;
        for (j in eff[i]){
          if(eff[i][j] > Meff){
            if( b > input[i][j] && choose[i][j] == 0){
              /*Check satisfaction*/
              if(chkSat( i, j, r[i], s[i])){
                /*if the satisfaction can fulfill the requirement, then choose the project.*/
                Meff = eff[i][j];
                Mi = i;
                Mj = j;
              }
            }
```

```
          }
        }
      if (Mi == null && Mj == null) fin = 1;
      else{
        b -= input[Mi][Mj];
        choose[Mi][Mj] = 1;
        Lproj[Mi] += 1;
        totalOutput += parseInt(output[Mi][Mj]);
        var itotal = 0;
        for(var j in choose[i]){
          if(choose[i][j] == 1) itotal += parseInt(input[i][j]);
        }
        var sof = Math.round(itotal/s[i]*10000)/100; /*the satisfaction of each field*/
        statMsg("choose Project "+ (parseInt(Mj)+1) + ". (Satisfaction: "+sof+"%)");


        $("#LBudget").html("Budget Used：$"+ (p-b) +"</br> Budget Left：$" + b +"</br> Total
Output:" + totalOutput);
        $("#choTab").html("");
        $("#satTab").html("");
        ChoosenTab();
      }
    }
  }
  statMsg("<br>===1st run Caculation complete.===");
  statMsg("Each field received (satisfaction):");
  for(var i=0; i < input.length; i++){
    var itotal = 0;
    for(var j in choose[i]){
      if(choose[i][j] == 1) itotal += parseInt(input[i][j]);
    }
    statMsg("Field "+(i+1)+": $"+itotal+"/$"+s[i]+"
("+Math.round((itotal/s[i])*10000)/100+"%)");
  }

  statMsg("<br><br>*****Second run -- Check the rest budget, if it still can be used on some
projects, than consider those qualified projects.*****");
  statMsg("Only choose the project which can meet satisfaction requirement
("+Math.round((RD-lim)*10000)/100+"% ~ "+Math.round((RD+lim)*10000)/100+"%)
<br><br>");
  fin = 0;
  while( b > 0 && fin != 1){
    /*find the max most efficient project --- 2nd run*/
    var Meff = 0, Mi = null, Mj = null;
    for(var i in eff){
      for (var j in eff[i]){
        if(eff[i][j] > Meff){
          if( b > input[i][j] && choose[i][j] == 0){
            if (ChkSatSec(i, j, RD, lim)){
              Meff = eff[i][j];
```

```
            Mi = i;
            Mj = j;
          }
        }
      }
    }
  }
  if (Mi == null && Mj == null) fin = 1;
  else{
    b -= input[Mi][Mj];
    choose[Mi][Mj] = 1;
    Lproj[Mi] += 1;
    totalOutput += parseInt(output[Mi][Mj]);
    statMsg("Field "+(Number(Mi)+1)+" Project "+(Number(Mj)+1)+" is OK for budget and
satisfaction requirent. Choose this project.");
    $("#LBudget").html("Budget Used：$"+ (p-b) +"</br> Budget Left：$" + b +"</br> Total
Output:" + totalOutput);
    $("#choTab").html("");
    $("#satTab").html("");
    ChoosenTab();
  }
 }

 statMsg("<br>===2st run Caculation complete.===");
 statMsg("<br>*****Caculation is finished.*****");
 statMsg("Each field received (satisfaction):");
 for(var i=0; i < input.length; i++){
  var itotal = 0;
  for(var j in choose[i]){
    if(choose[i][j] == 1) itotal += parseInt(input[i][j]);
  }
  statMsg("Field "+(i+1)+": $"+itotal+"/$"+s[i]+"
("+Math.round((itotal/s[i])*10000)/100+"%)");
 }
 $("#TETF").show();
 $("#TETF").html("Total efficiency obtained：" + parseInt(totalOutput/(p-b)*10000)/100 +
"%");
}
function chkSat( p, q, x, y){
 //Check if Satisfaction reach average
 var itotal = 0;
 for(var i in choose[p]){
  if(choose[p][i] == 1) itotal += parseInt(input[p][i]);
 }
 itotal += parseInt(input[p][q]);
 if(parseInt(itotal) > parseInt(x)){
  statMsg("(!)If choose project " + (Number(q)+1) + ", the satisfaction
("+Math.round((itotal/y)*10000)/100+"%) will over "+ Math.round((x/y)*10000)/100+"%.");
  return false;
 }else return true;
```

```
}

function ChkSatSec( p, q, x, y ){
  var r = parseInt($("#satset")[0].value);
  var itotal = new Array();
  for(var i in choose){
    itotal[i] = 0;
    for(var j in choose[i]){
      if(choose[i][j] == 1) itotal[i] += parseInt(input[i][j]);
    }
  }
  itotal[p] += parseInt(input[p][q]);
  var updateSat = new Array();
  for(var i in itotal){
    updateSat[i] = parseInt(itotal[i]/UnitInput[i]*1000000, 10)/1000000;
  }
  /*
  var MaxSat = 0, minSat = 1;
  var MaxUnit, minUnit;
  for(var i in updateSat){
    if(updateSat[i] > MaxSat) MaxSat = updateSat[i];
    if(updateSat[i] < minSat)  minSat = updateSat[i];
  }
  var SatDiff = MaxSat - minSat;
  */
  if(updateSat[p] > (x+y)){
    /*statMsg("Field "+(Number(p)+1)+" Project "+(Number(q)+1)+" is OK for budget, but if
choose this project, the satisfaction ("+Math.round(updateSat[p]*10000)/100+"%) will over
"+Math.round((x+y)*10000)/100+"%. Skip this project.");*/
    return false;
  }else if (updateSat[p] < (x-y)){
    /*statMsg("Field "+(Number(p)+1)+" Project "+(Number(q)+1)+" is OK for budget, but if
choose this project, the satisfaction ("+Math.round(updateSat[p]*10000)/100+"%) will less
than "+Math.round((x-y)*10000)/100+"%. Skip this project.");*/
    return false;
  }
  return true;

}
/*<-- Caculation Function End*/

$(document).ready(function(){
  LoadData();
});
</script>
</head>
<body>
<b>Input and Output Table</b>
<table id="inputTab" background="#000000">
</table>
```

```
</br>
<div id="caluse">
<b>Efficiency</b>
<table id="effTab">
</table>
</br>
</div>
<b>Choosen</b>
<table id="choTab">
</table>
</br>
<b>Satisfaction</b>
<table id="satTab">
</table>
</br>
Total Budget：<input id="MBudget" type="text" value="0"></br>
Max Satisfaction Gap：
<select id="satset"><script>
for(var i = 1; i <=100; i++){
document.write("<option value=" + i + ">" + i + "% </option>");}
</script></select></br>
<input id="RUN" type="button" value="RUN" onClick="Run();"><input type="button"
value="Reset" onClick="window.location.reload();"></br>
<label id="stat" class="msg"></label></br>
<label id="CANTS" style="display:none;"></label></br>
<label id="LBudget" style="display:none;"></label></br>
<label id="TETF" style="display:none;"></label></br>
<label id="SDiff" style="display:none;"></label></br>
</body>
</html>
```

Appendix 3: BLMF-PC Quick User Guide

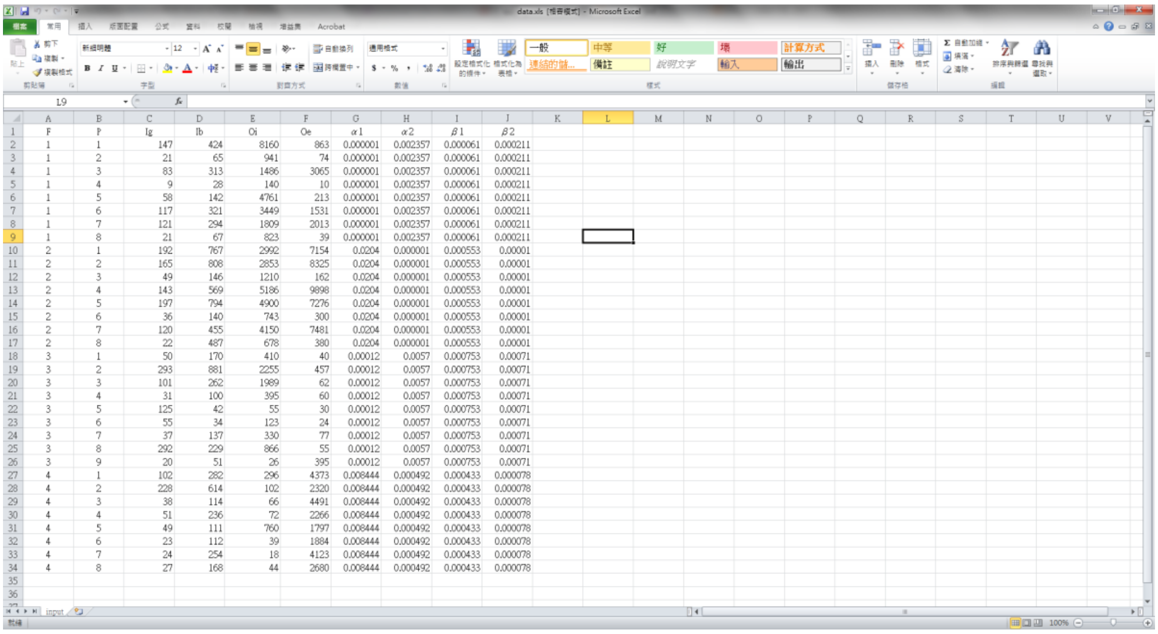
System Requirement:

Any OS (Window/Mac/Linux…, etc.) with JavaScript supported explorer, such like Internet Explorer. Please note that you should open the security rule to make sure the explorer can read the local files in your hard disk.

1. Build a data sheet.

Each column means the input/output of different projects.

Each row mean different fields/industries/business units, …, etc.

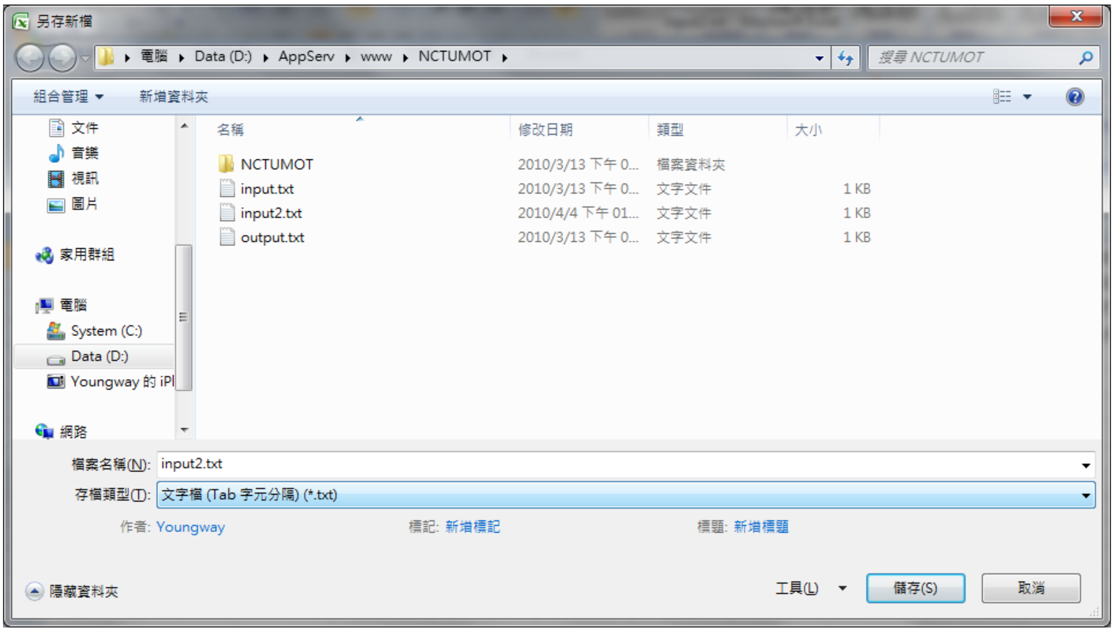


F	P	Ig	Ib	Oi	Oe	α1	α2	β1	β2
1	1	147	424	8160	863	0.000001	0.002357	0.000061	0.000211
1	2	21	65	941	74	0.000001	0.002357	0.000061	0.000211
1	3	83	313	1486	3065	0.000001	0.002357	0.000061	0.000211
1	4	9	28	140	10	0.000001	0.002357	0.000061	0.000211
1	5	58	142	4761	213	0.000001	0.002357	0.000061	0.000211
1	6	117	321	3449	1531	0.000001	0.002357	0.000061	0.000211
1	7	121	294	1809	2013	0.000001	0.002357	0.000061	0.000211
1	8	21	67	823	39	0.000001	0.002357	0.000061	0.000211
2	1	192	767	2992	7154	0.0204	0.000001	0.000553	0.00001
2	2	165	808	2853	8325	0.0204	0.000001	0.000553	0.00001
2	3	49	146	1210	162	0.0204	0.000001	0.000553	0.00001
2	4	143	569	5186	9898	0.0204	0.000001	0.000553	0.00001
2	5	197	794	4900	7276	0.0204	0.000001	0.000553	0.00001
2	6	36	140	743	300	0.0204	0.000001	0.000553	0.00001
2	7	120	455	4150	7481	0.0204	0.000001	0.000553	0.00001
2	8	22	487	678	380	0.0204	0.000001	0.000553	0.00001
3	1	50	170	410	40	0.00012	0.0057	0.000753	0.00071
3	2	293	881	2255	457	0.00012	0.0057	0.000753	0.00071
3	3	101	262	1989	62	0.00012	0.0057	0.000753	0.00071
3	4	31	100	395	60	0.00012	0.0057	0.000753	0.00071
3	5	125	42	55	30	0.00012	0.0057	0.000753	0.00071
3	6	55	34	123	24	0.00012	0.0057	0.000753	0.00071
3	7	37	137	330	77	0.00012	0.0057	0.000753	0.00071
3	8	292	229	866	55	0.00012	0.0057	0.000753	0.00071
3	9	20	51	26	395	0.00012	0.0057	0.000753	0.00071
4	1	102	282	296	4373	0.008444	0.000492	0.000433	0.000078
4	2	228	614	102	2320	0.008444	0.000492	0.000433	0.000078
4	3	38	114	66	4491	0.008444	0.000492	0.000433	0.000078
4	4	51	236	72	2266	0.008444	0.000492	0.000433	0.000078
4	5	49	111	760	1797	0.008444	0.000492	0.000433	0.000078
4	6	23	112	39	1884	0.008444	0.000492	0.000433	0.000078
4	7	24	254	18	4123	0.008444	0.000492	0.000433	0.000078
4	8	27	168	44	2680	0.008444	0.000492	0.000433	0.000078

2. Save the input/output data as text file.

Use “Save As…” function to save the data:

a. Choose type to “Text file (Separate by Tab)(*.txt)/文字檔(Tab 字元分隔)(*.txt)”



b. Save the file as “data.txt”

3. Run “RunBLP.htm”

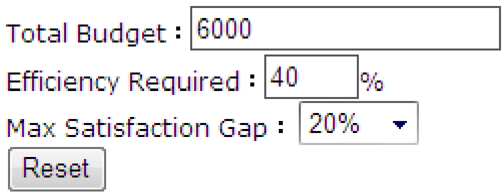
The program should integrate the input/output data if the data is valid. Shown below:

Input and Output Table

Field	Project	own resource	request funding	implicit output value	explicit output value	Selection results
F(1)	P(1)	147	424	8160	863	--
	P(2)	21	65	941	74	--
	P(3)	83	313	1486	3065	--
	P(4)	9	28	140	10	--
	P(5)	58	142	4761	213	--
	P(6)	117	321	3449	1531	--
	P(7)	121	294	1809	2013	--
	P(8)	21	67	823	39	--
F(2)	P(1)	192	767	2992	7154	--
	P(2)	165	808	2853	8325	--
	P(3)	49	146	1210	162	--
	P(4)	143	569	5186	9898	--
	P(5)	197	794	4900	7276	--
	P(6)	36	140	743	300	--
	P(7)	120	455	4150	7481	--
	P(8)	22	487	678	380	--
F(3)	P(1)	50	170	410	40	--
	P(2)	293	881	2255	547	--
	P(3)	101	262	1989	62	--
	P(4)	31	100	395	60	--
	P(5)	125	42	55	30	--
	P(6)	55	34	123	24	--
	P(7)	37	137	330	77	--
	P(8)	92	229	866	55	--
	P(9)	20	51	26	395	--
F(4)	P(1)	102	282	296	4373	--
	P(2)	228	614	102	2320	--
	P(3)	38	114	66	4491	--
	P(4)	51	236	72	2266	--
	P(5)	49	111	760	1797	--
	P(6)	23	112	39	1884	--
	P(7)	24	254	18	4123	--
	P(8)	27	168	44	2680	--

If the program does not integrate the data, please check if the input/output file is in right format.

4. Set Total Budget, required efficiency and Max Satisfaction Gap.



5. Start calculation.

Press "Run" and the program will list the decision flow and final budget left/total output.

Caculation start...

Field 1 Project 4 is disqualified   Field 1 Project 8 is disqualified
Field 3 Project 1 is disqualified   Field 3 Project 5 is disqualified
Field 3 Project 7 is disqualified   Field 4 Project 2 is disqualified
Field 4 Project 4 is disqualified

Field 1, choose 5 as reference sequence.
▽51=(0,0.44,26.58,2.2)   ▽52=(0,0.65,37.28,0.15)
▽53=(0,1.32,64.18,33.26)   ▽54=(0,0.66,66.53,2.56)
▽55=(0,0,0,0)   ▽56=(0,0.3,52.61,9.41)
▽57=(0,0.02,67.14,12.96)   ▽58=(0,0.74,42.9,1.82)
MAX▽58=67.14; min▽58=0

Field 2, choose 4 as reference sequence.
▽41=(0,0.02,20.68,31.96)   ▽42=(0,0.92,18.97,18.76)
▽43=(0,1,11.57,65.91)   ▽44=(0,0,0,0)
▽45=(0,0.05,11.39,32.28)   ▽46=(0,0.09,15.63,60.88)
▽47=(0,0.19,1.68,6.88)   ▽48=(0,18.16,5.45,51.94)
MAX▽48=65.91; min▽48=0

Field 3, choose 3 as reference sequence.
▽31=(0,0.81,11.49,0.19)   ▽32=(0,0.95,12,1.25)
▽33=(0,0,0,0)   ▽34=(0,0.63,6.95,1.32)
▽35=(0,2.26,19.25,0.37)   ▽36=(0,1.98,17.46,0.18)
▽37=(0,1.11,10.77,1.47)   ▽38=(0,0.1,10.28,0.02)
▽39=(0,0.04,18.39,19.14)
MAX▽39=19.25; min▽39=0

Field 4, choose 3 as reference sequence.
▽31=(0,0.24,1.17,75.31) ▽32=(0,0.31,1.29,108.01)
▽33=(0,0,0,0) ▽34=(0,1.63,0.33,73.75)
▽35=(0,0.73,13.77,81.51) ▽36=(0,1.87,0.04,36.27)
▽37=(0,7.58,0.99,53.61) ▽38=(0,3.22,0.11,18.92)
MAX▽38=108.01; min▽38=0
Let distinguishing coefficient ζ=0.5
Come out Grey Relational Degree of each Proposal. See the table above.
Come out Internal Efficiencies for each Project of Dvisions. See the table above.
Come out External Efficiencies for each Project of Dvisions. See the table above.
The qualified funding requirement for field #1 is: $1559
The qualified funding requirement for field #2 is: $4166
The qualified funding requirement for field #3 is: $1157
The qualified funding requirement for field #4 is: $1041
Total funding requirement for all qualified projects is: $7923
The best satisfied percentage of each field is 75.73%:

***** 1st run -- Choose the project based on optimal satisfaction. Each Field cannot exceed 75.73%. *****
Field 1 cannot exceed $1181
Field 2 cannot exceed $3155
Field 3 cannot exceed $876
Field 4 cannot exceed $788

===Choose the project from Field 1. ===

Step 1:
Option 1: project 5 - Output = 4761 + 213 = 4974
Option 2: project 3 - Output = 1486 + 2145.5 = 3631.5
Choose Project 5, which cost $142. Field 1 Budget Left: $1039. Satisfcation: 9.11%.

Step 2:
Option 1: project 1 - Output = 8160 + 752.9675000000001 = 8912.9675
Option 2: project 3 - Output = 1486 + 2145.5 = 3631.5
Choose Project 1, which cost $424. Field 1 Budget Left: $615. Satisfcation: 36.31%.

Step 3:
Option 1: project 2 - Output = 941 + 63.825 = 1004.825
Option 2: project 3 - Output = 1486 + 2145.5 = 3631.5
Choose Project 3, which cost $313. Field 1 Budget Left: $302. Satisfcation: 56.38%.

Step 4:
Option 1: project 2 - Output = 941 + 63.825 = 1004.825
Option 2: project 7 - Output = 1809 + 1534.9125 = 3343.9125
Choose Project 7, which cost $294. Field 1 Budget Left: $8. Satisfcation: 75.24%.

Step 5:
Option 1: project 2 - Output = 941 + 63.825 = 1004.825
Option 2: project 6 - Output = 3449 + 1209.49 = 4658.49
Budget quota is not sufficient to choose project 6!

===Choose the project from Field 2. ===

Step 1:
Option 1: project 4 - Output = 5186 + 9898 = 15084
Option 2: project 4 - Output = 5186 + 9898 = 15084
Choose Project 4, which cost $569. Field 2 Budget Left: $2586. Satisfcation: 13.66%.

Step 2:
Option 1: project 7 - Output = 4150 + 7050.8425 = 11200.8425
Option 2: project 7 - Output = 4150 + 7050.8425 = 11200.8425
Choose Project 7, which cost $455. Field 2 Budget Left: $2131. Satisfcation: 24.58%.

Step 3:
Option 1: project 3 - Output = 1210 + 123.12 = 1333.12
Option 2: project 2 - Output = 2853 + 6743.25 = 9596.25
Choose Project 2, which cost $808. Field 2 Budget Left: $1323. Satisfcation: 43.98%.

Step 4:
Option 1: project 3 - Output = 1210 + 123.12 = 1333.12
Option 2: project 1 - Output = 2992 + 5580.12 = 8572.12
Choose Project 1, which cost $767. Field 2 Budget Left: $556. Satisfcation: 62.39%.

Step 5:
Option 1: project 3 - Output = 1210 + 123.12 = 1333.12
Option 2: project 5 - Output = 4900 + 5911.75 = 10811.75
Budget quota is not sufficient to choose project 5!

===Choose the project from Field 3. ===

Step 1:
Option 1: project 3 - Output = 1989 + 62 = 2051
Option 2: project 9 - Output = 26 + 263.66249999999996 = 289.6625
Choose Project 3, which cost $262. Field 3 Budget Left: $614. Satisfcation: 22.64%.

Step 2:
Option 1: project 4 - Output = 395 + 51 = 446
Option 2: project 9 - Output = 26 + 263.66249999999996 = 289.6625
Choose Project 4, which cost $100. Field 3 Budget Left: $514. Satisfcation: 31.29%.

Step 3:
Option 1: project 2 - Output = 2255 + 444.4375 = 2699.4375
Option 2: project 9 - Output = 26 + 263.66249999999996 = 289.6625
Choose Project 2, which cost $481. Field 3 Budget Left: $33. Satisfcation: 72.86%.

Step 4:
Option 1: project 8 - Output = 866 + 47.712500000000006 = 913.7125
Option 2: project 9 - Output = 26 + 263.66249999999996 = 289.6625
Budget quota is not sufficient to choose project 8!

===Choose the project from Field 4. ===

Step 1:
Option 1: project 5 - Output = 760 + 1433.1075 = 2193.1075
Option 2: project 3 - Output = 66 + 4491 = 4557
Choose Project 3, which cost $114. Field 4 Budget Left: $674. Satisfcation: 10.95%.

Step 2:
Option 1: project 5 - Output = 760 + 1433.1075 = 2193.1075
Option 2: project 7 - Output = 18 + 3463.3199999999997 = 3481.32
Choose Project 7, which cost $254. Field 4 Budget Left: $420. Satisfcation: 35.35%.

Step 3:
Option 1: project 5 - Output = 760 + 1433.1075 = 2193.1075
Option 2: project 6 - Output = 39 + 1681.47 = 1720.47
Choose Project 5, which cost $111. Field 4 Budget Left: $309. Satisfcation: 46.01%.

Step 4:
Option 1: project 1 - Output = 296 + 3717.0499999999997 = 4013.05
Option 2: project 6 - Output = 39 + 1681.47 = 1720.47
Choose Project 1, which cost $282. Field 4 Budget Left: $27. Satisfcation: 73.1%.

Step 5:
Option 1: project 6 - Output = 39 + 1681.47 = 1720.47
Option 2: project 6 - Output = 39 + 1681.47 = 1720.47
Budget quota is not sufficient to choose project 6!

===1st run Caculation complete.===
Each field received (satisfaction):
Field 1: $1173/$1559 (75%)
Field 2: $2599/$4166 (62%)
Field 3: $843/$1157 (73%)
Field 4: $761/$1041 (73%)
Satisfaction level:13%
Budget left: $624

***** 2nd run Pick-Up -- Choose the candidate project from all field. The satisfaction of each field shall not exceed 75.73% + 10% = 85.73%. *****
Field 1 cannot exceed $1337.
Field 2 cannot exceed $3572.
Field 3 cannot exceed $992.
Field 4 cannot exceed $892.

Check is there any feasible project in Field 2...
Choose Field 2 Project 3, which cost $146. Then each field received (satisfaction):
Field 1: $1173/$1559 (75%)
Field 2: $2745/$4166 (66%)
Field 3: $843/$1157 (73%)
Field 4: $761/$1041 (73%)
Satisfaction level:9%

Check is there any feasible project in Field 2...
Choose Field 2 Project 6, which cost $140. Then each field received (satisfaction):
Field 1: $1173/$1559 (75%)
Field 2: $2885/$4166 (69%)
Field 3: $843/$1157 (73%)
Field 4: $761/$1041 (73%)
Satisfaction level:6%

Check is there any feasible project in Field 2...
Cannot find any more feasible projects in Field 2.

Check is there any feasible project in Field 4...
Choose Field 4 Project 6, which cost $112. Then each field received (satisfaction):
Field 1: $1173/$1559 (75%)
Field 2: $2885/$4166 (69%)
Field 3: $843/$1157 (73%)
Field 4: $873/$1041 (84%)
Satisfaction level:15%

Check is there any feasible project in Field 3...
Choose Field 3 Project 9, which cost $51. Then each field received (satisfaction):
Field 1: $1173/$1559 (75%)
Field 2: $2885/$4166 (69%)
Field 3: $894/$1157 (77%)
Field 4: $873/$1041 (84%)
Satisfaction level:15%

Check is there any feasible project in Field 1...
Choose Field 1 Project 2, which cost $65. Then each field received (satisfaction):
Field 1: $1238/$1559 (79%)
Field 2: $2885/$4166 (69%)
Field 3: $894/$1157 (77%)
Field 4: $873/$1041 (84%)
Satisfaction level:15%

Check is there any feasible project in Field 3...
Choose Field 3 Project 6, which cost $34. Then each field received (satisfaction):
Field 1: $1238/$1559 (79%)
Field 2: $2885/$4166 (69%)
Field 3: $928/$1157 (80%)
Field 4: $873/$1041 (84%)
Satisfaction level:15%

Check is there any feasible project in Field 1...
Cannot find any more feasible projects in Field 1.

Check is there any feasible project in Field 3...
Cannot find any more feasible projects in Field 3.

Check is there any feasible project in Field 4...
Cannot find any more feasible projects in Field 4.

*******Caculation complete.*******
Each field received (satisfaction):
Field 1: $1238/$1559 (79%)
Field 2: $2885/$4166 (69%)
Field 3: $928/$1157 (80%)
Field 4: $873/$1041 (84%)
Satisfaction level:15%
Budget left: $76
Total investment: $5924
Total Output: $90216.86
Total efficiency obtained: 1522.9%
****

Appendix 4: BLMF-PC Source Code

```
<!DOCTYPE html PUBLIC "-//W3C//DTD XHTML 1.0 Transitional//EN"
"http://www.w3.org/TR/xhtml1/DTD/xhtml1-transitional.dtd">
<html xmlns="http://www.w3.org/1999/xhtml">
<head>
<META http-equiv="Content-Type" content="text/html; charset=utf-8" />
<META NAME="robots" CONTENT="noindex,nofollow">
<META HTTP-EQUIV="CACHE-CONTROL" CONTENT="NO-CACHE">
<META HTTP-EQUIV="EXPIRES" CONTENT="0">
<META HTTP-EQUIV="PRAGMA" CONTENT="NO-CACHE">

<!-- jQuery & Plug-in -->
<script type="text/javascript" src="./jquery-1.4.2.min.js"></script>
<link rel="stylesheet" href="main.css" type="text/css">
<script type="text/javascript">

/*Basic Function Start-->*/
function explode (delimiter, string, limit) {
	/* Splits a string on string separator and return array of components. If limit is positive
only limit number of components is returned. If limit is negative all components except the
last abs(limit) are returned. */
	/* version: 909.322 */
	/* discuss at: http://phpjs.org/functions/explode	// +	original by: Kevin van
Zonneveld (http://kevin.vanzonneveld.net) */
	/* +	improved by: kenneth */
	/* +	improved by: Kevin van Zonneveld (http://kevin.vanzonneveld.net) */
	/* +	improved by: d3x */
	/* +	bugfixed by: Kevin van Zonneveld (http://kevin.vanzonneveld.net)	// *
example 1: explode(' ', 'Kevin van Zonneveld'); */
	/* *	returns 1: {0: 'Kevin', 1: 'van', 2: 'Zonneveld'} */
	/* *	example 2: explode('=', 'a=bc=d', 2); */
	/* *	returns 2: ['a', 'bc=d'] */
	  var emptyArray = { 0: '' };

	/* third argument is not required */
	if ( arguments.length < 2 ||
		typeof arguments[0] == 'undefined' ||		typeof arguments[1] == 'undefined' )
	{
		return null;
	}
	  if ( delimiter === '' ||
		delimiter === false ||
		delimiter === null )
	{
		return false;	}

	if ( typeof delimiter == 'function' ||
		typeof delimiter == 'object' ||
```

```
            typeof string == 'function' ||              typeof string == 'object' )
        {
            return emptyArray;
        }
          if ( delimiter === true ) {
            delimiter = '1';
        }

        if (!limit) {               return string.toString().split(delimiter.toString());
        } else {
            // support for limit argument
            var splitted = string.toString().split(delimiter.toString());
            var partA = splitted.splice(0, limit - 1);              var partB =
splitted.join(delimiter.toString());
            partA.push(partB);
            return partA;
        }
}

function sortNumber(a,b)
{
  return b - a;
}

function statMsg(m){
  var q = $("label#stat")[0].innerHTML;
  q += m + "</br>";
  $("label#stat").html(q);
  return;
}

function reset(){
  for(var i in SELECT){
    for(var j in SELECT[i]){
      SELECT[i][j] = 0;
    }
  }
  $("#LBudget").hide();
  return;
}
/*<--Basic Function End*/

/*Global variable Start-->*/
var maxproj=0;
/* the structre of DATA:
      DATA[x]: Fileds, x=0 no used.
      DATA[x][y]: Plans, y=0 no used.
      DATA[x][y][z]: 0: own resource
            1: request funding
            2: implicit output value
```

```
        3: explicit output value
        4:a1
        5:a2
        6:b1
        7:b2
        8:s1
        9:s2
        10:s3
        11:s4
        12:Nabla(1)
        13:Nabla(2)
        14:Nabla(3)
        15:Nabla(4)
        16:Zeta*/

var DATA = new Array();

/* the structure of EFF:
  EFF[x]: field, x=0 no used.
  EFF[x][y]: each projects, y=0 no used. */
var EFF = new Array();

/* the structure of SELECT:
  SELECT[x]: field, x=0 no used.
  SELECT[x][y]: each projects, y=0 no used. */

var SELECT = new Array();

var MAX_PROJ_NUM = 0;
var TOTAL_BUDGET = 0;
var BUDGET_LEFT = 0;
var EFF_REQ = 0;
var TOTAL_OUTPUT = 0;
var SAT = new Array();  /*Satisfaction of each field*/
var QFUN = new Array();  /*Qualified funding for each field*/
var AFUN = new Array();  /*Actual received funding for each field*/
/*<--Global variable End*/

/*Initial Data Start-->*/
function LoadData(){
  $.ajax({
    type: "POST",
    url: "data.txt",
    datatype: "text",
    success: function(data, status){
      var raw_data = explode("\n", data);
      for(i in raw_data){
        if (raw_data[i] != "") var sec_data = explode("\t", raw_data[i]);
        if (sec_data[0] == "Field") continue;
          /*fill into DATA*/
```

```
        var p = parseInt(sec_data[0]);
        var q = parseInt(sec_data[1]);
        if(DATA[p] == null) DATA[p] = new Array();
        if(DATA[p][q] == null) DATA[p][q] = new Array();


        DATA[p][q][0] = Number(sec_data[2]); /*own resource*/
        DATA[p][q][1] = Number(sec_data[3]); /*request funding*/
        DATA[p][q][2] = Number(sec_data[4]); /*implicit output value*/
        DATA[p][q][3] = Number(sec_data[5]); /*explicit output value*/
        DATA[p][q][4] = Number(sec_data[6]); /*a1*/
        DATA[p][q][5] = Number(sec_data[7]); /*a2*/
        DATA[p][q][6] = Number(sec_data[8]); /*b1*/
        DATA[p][q][7] = Number(sec_data[9]); /*b2*/

        /*create select information*/
        if(SELECT[p] == null) SELECT[p] = new Array();
        if(SELECT[p][q] == null) SELECT[p][q] = new Array();
        SELECT[p][q] = 0;

        /*create efficiency information*/
        if(EFF[p] == null) EFF[p] = new Array();
        if(EFF[p][q] == null) EFF[p][q] = new Array();
        var r = (Number(sec_data[4])*Number(sec_data[8]) +
Number(sec_data[5])*Number(sec_data[9]))/(Number(sec_data[2])*Number(sec_data[6]) +
Number(sec_data[3])*Number(sec_data[7]));
        EFF[p][q] = Math.round(r*1000)/1000;

      }
      /*check max project number*/
      for(var i in DATA){
        if(DATA[i].length > MAX_PROJ_NUM) MAX_PROJ_NUM = DATA[i].length-1;
      }
    },
    complete: function(){
      DATATab();
      EfficiencyTab();
    }
  });
}

function DATATab(){
  /*Header row*/
  var tar = document.getElementById("inputTab");
  var newRow = tar.insertRow(tar.rows.length);
  var cell = tar.rows[tar.rows.length-1].insertCell(0);
  cell.align = "center";
  cell.style.minWidth = "70px";
  cell.innerHTML = "Field";
  var cell = tar.rows[tar.rows.length-1].insertCell(1);
```

```
cell.align = "center";
cell.style.minWidth = "70px";
cell.innerHTML = "Project";
var cell = tar.rows[tar.rows.length-1].insertCell(2);
cell.align = "center";
cell.style.minWidth = "70px";
cell.innerHTML = "own resource";
var cell = tar.rows[tar.rows.length-1].insertCell(3);
cell.align = "center";
cell.style.minWidth = "70px";
cell.innerHTML = "request funding";
var cell = tar.rows[tar.rows.length-1].insertCell(4);
cell.align = "center";
cell.style.minWidth = "70px";
cell.innerHTML = "implicit output value";
var cell = tar.rows[tar.rows.length-1].insertCell(5);
cell.align = "center";
cell.style.minWidth = "70px";
cell.innerHTML = "explicit output value";
var cell = tar.rows[tar.rows.length-1].insertCell(6);
cell.align = "center";
cell.style.minWidth = "70px";
cell.innerHTML = "Selection results";
/* contain rows*/
for(var i in DATA){
  var FCol = 0;
  if(DATA[i] == "Field") continue;
  for(var j in DATA[i]){
    var newRow = tar.insertRow(tar.rows.length);
    if (FCol == 0){
      var cellfix = 0;
      var cell = tar.rows[tar.rows.length-1].insertCell(0);
      cell.rowSpan = DATA[i].length-1;
      cell.align = "center";
      cell.innerHTML = "F(" + i + ")";
    }else cellfix = -1;
    var cell = tar.rows[tar.rows.length-1].insertCell(cellfix+1);
    cell.align = "center";
    cell.innerHTML = "P(" + j + ")";
    var cell = tar.rows[tar.rows.length-1].insertCell(cellfix+2);
    cell.align = "center";
    cell.innerHTML = DATA[i][j][0];
    var cell = tar.rows[tar.rows.length-1].insertCell(cellfix+3);
    cell.align = "center";
    cell.innerHTML = DATA[i][j][1];
    var cell = tar.rows[tar.rows.length-1].insertCell(cellfix+4);
    cell.align = "center";
    cell.innerHTML = DATA[i][j][2];
    var cell = tar.rows[tar.rows.length-1].insertCell(cellfix+5);
    cell.align = "center";
```

```
      cell.innerHTML = DATA[i][j][3];
      var cell = tar.rows[tar.rows.length-1].insertCell(cellfix+6);
      cell.align = "center";
      if(SELECT[i][j] == 0) cell.innerHTML = "--";
      else if(SELECT[i][j] == "N"){
        cell.style.background = "black";
        cell.innerHTML = "<font color=white>Disqualified</font>";
      }else{
        cell.style.background = "green";
        cell.innerHTML = SELECT[i][j];
      }
      FCol = null;
      cellfix = null;
    }
  }
}
function EfficiencyTab(){
  var tar = document.getElementById("effTab");
  var newRow = tar.insertRow(tar.rows.length);
  var cell = tar.rows[tar.rows.length-1].insertCell(0);
  cell.align = "center";
  cell.style.minWidth = "70px";
  cell.innerHTML = "Fields";
  for(var i=0; i < MAX_PROJ_NUM; i++){
    var cell = tar.rows[tar.rows.length-1].insertCell(i+1);
    cell.align = "center";
    cell.style.minWidth = "70px";
    cell.innerHTML = "Project " + (i+1);
  }
  for(var i in EFF){
    var newRow = tar.insertRow(tar.rows.length);
    var cell = tar.rows[tar.rows.length-1].insertCell(0);
    cell.align = "center";
    cell.innerHTML = "F(" + i +")";
    for(var j=1; j <= MAX_PROJ_NUM; j++){
      var cell = tar.rows[tar.rows.length-1].insertCell(j);

      if(EFF[i][j]==null){
        cell.align = "center";
        cell.style.background = "black";
        cell.style.color = "white"
        cell.innerHTML = "---";
      }else{
        cell.align = "right";
        cell.innerHTML = parseInt(EFF[i][j]*10000, 10)/10000;
      }
    }
  }
}
/*<-- Initial Data End*/
```

```
/*Caculation Function Start-->*/
function Run(){
  $("#RUN").hide();
  reset();
  statMsg("Caculation start...");
  var p = $("input#MBudget")[0].value;
  if(isNaN(p)) {
    alert("Invalid Budget!");
    $("input#MBudget")[0].value = 0;
    statMsg("Invalid Budget! Please check Total Budget.");
    statMsg("Caculation terminated.");
    return;
  }else{
    TOTAL_BUDGET = Number(p);
    BUDGET_LEFT = Number(p);
  }
  p = $("input#EffReq")[0].value;
  if(isNaN(p)) {
    alert("Invalid Efficiency!");
    $("input#MBudget")[0].value = 0;
    statMsg("Invalid Efficiency! Please check Efficiency Required.");
    statMsg("Caculation terminated.");
    return;
  }else{
    EFF_REQ = Number(p)/100;
  }
  p = null;
        /*Filter unqualified projects*/
  for(var i in EFF){
    for(var j in EFF[i]){
      if( EFF[i][j] < EFF_REQ){
        SELECT[i][j] = "N";
        statMsg("Field " +i+ " Project " +j+" is disqualified");
        /*Update Efficiency Table*/
        var tar = document.getElementById("effTab").rows[i].cells[j];
        tar.style.background = "black";
        tar.style.color = "white"
      }
    }
  }
  /*Update DATA Table*/
  var rowcount = 1;
  for(var i in DATA){
    if(DATA[i] == "Field") continue;
    var tar = document.getElementById("inputTab");
    for(var j in DATA[i]){
      if (j==1) var cell = tar.rows[rowcount].cells[6];
      else var cell = tar.rows[rowcount].cells[5];
      cell.align = "center";
      if(SELECT[i][j] == 0) cell.innerHTML = "--";
```

```
    else if(SELECT[i][j] == "N"){
      cell.style.background = "black";
      cell.innerHTML = "<font color=white>Disqualified</font>";
    }else{
      cell.style.background = "green";
      cell.innerHTML = SELECT[i][j];
    }
    rowcount += 1;
  }
}
/*Calculate Grey Relational Degree of each proposal*/
/*Calculate Sj(1),Sj(2),Sj(3),Sj(4)*/
for(var i in DATA){
  for(var j in DATA[i]){
    DATA[i][j][8] = DATA[i][j][0]/DATA[i][j][0]; /*Sj(1)*/
    DATA[i][j][9] = DATA[i][j][1]/DATA[i][j][0]; /*Sj(2)*/
    DATA[i][j][10] = DATA[i][j][2]/DATA[i][j][0];/*Sj(3)*/
    DATA[i][j][11] = DATA[i][j][3]/DATA[i][j][0];/*Sj(4)*/
  }
}
for(var i in DATA){
  /*find the most efficient project of each field as reference sequence*/
  var Meff = 0, Mj = null;
  for (var j in EFF[i]){
    if(EFF[i][j] > Meff){
      Meff = EFF[i][j];
      Mj = j;
    }
  }
  statMsg("</br>Field "+i+", choose "+Mj+" as reference sequence.");
  /*Calculate nabla*/
  for(var j in DATA[i]){
    DATA[i][j][12] = Math.round((Math.abs(DATA[i][Mj][8]-DATA[i][j][8]))*100)/100;
/*nabla(1)*/
    DATA[i][j][13] = Math.round((Math.abs(DATA[i][Mj][9]-DATA[i][j][9]))*100)/100;
/*nabla(2)*/
    DATA[i][j][14] = Math.round((Math.abs(DATA[i][Mj][10]-DATA[i][j][10]))*100)/100;
/*nabla(3)*/
    DATA[i][j][15] = Math.round((Math.abs(DATA[i][Mj][11]-DATA[i][j][11]))*100)/100;
/*nabla(4)*/

statMsg("&nabla;"+Mj+j+"=("+DATA[i][j][12]+","+DATA[i][j][13]+","+DATA[i][j][14]+",
"+DATA[i][j][15]+")");
  }
  /*find the Max and min nabla of each field*/
  var Maxnab = 0, minbab=65535;
  for(var j in DATA[i]){
    for(var k = 12; k<=15; k++){
      if(DATA[i][j][k] > Maxnab){
        Maxnab = DATA[i][j][k];
```

```
        }
        if(DATA[i][j][k] < minbab){
          minbab = DATA[i][j][k];
        }
      }
    }
    statMsg("MAX&nabla;"+Mj+j+"="+Maxnab+"; min&nabla;"+Mj+j+"="+minbab);
    /*Let distinguishing coefficient ζ=0.5*/
    var zeta = 0.5;
    for(var j in DATA[i]){
      DATA[i][j][16] =
((Math.round(((zeta*(minbab+Maxnab))/(DATA[i][j][12]+(zeta*Maxnab)))*100))+
              (Math.round(((zeta*(minbab+Maxnab))/(DATA[i][j][13]+(zeta*Maxnab)))*100))+
              (Math.round(((zeta*(minbab+Maxnab))/(DATA[i][j][14]+(zeta*Maxnab)))*100))+

(Math.round(((zeta*(minbab+Maxnab))/(DATA[i][j][15]+(zeta*Maxnab)))*100)))/400;
  /*Zeta*/
      DATA[i][j][17] = Number(DATA[i][j][3] * DATA[i][j][16]); /* ve(i)*z(i) */
      DATA[i][j][18] = Math.round((DATA[i][j][2] + DATA[i][j][17])*10000)/10000; /* vi +
ve(i)*z(i) */
    }
  }
  /*create GRD Table*/
  statMsg("Let distinguishing coefficient ζ=0.5");
  statMsg("Come out Grey Relational Degree of each Proposal. See the table above.");
  $("#GRD").show();
  var tar = document.getElementById("GRDTab");
  var newRow = tar.insertRow(tar.rows.length);
  var cell = tar.rows[tar.rows.length-1].insertCell(0);
  cell.align = "center";
  cell.style.minWidth = "70px";
  cell.innerHTML = "Fields";
  for(var i=0; i < MAX_PROJ_NUM; i++){
    var cell = tar.rows[tar.rows.length-1].insertCell(i+1);
    cell.align = "center";
    cell.style.minWidth = "70px";
    cell.innerHTML = "Project " + (i+1);
  }
  for(var i in DATA){
    var newRow = tar.insertRow(tar.rows.length);
    var cell = tar.rows[tar.rows.length-1].insertCell(0);
    cell.align = "center";
    cell.innerHTML = "F(" + i +")";
    for(var j=1; j <= MAX_PROJ_NUM; j++){
      var cell = tar.rows[tar.rows.length-1].insertCell(j);
      cell.align = "center";
      if(DATA[i][j]==null){
        cell.style.background = "black";
        cell.style.color = "white"
        cell.innerHTML = "---";
```

```
    }else  cell.innerHTML = DATA[i][j][16];
    if(SELECT[i][j] == "N"){
      tar.rows[i].cells[j].style.background = "black";
      tar.rows[i].cells[j].style.color = "white"
      tar.rows[i].cells[j].innerHTML = "Disqualified"
    }
  }
}
/*Calculate external/internal efficiency and explicitly value*/
for(var i in DATA){
  for(var j in DATA[i]){
    DATA[i][j][19] = DATA[i][j][2]/(DATA[i][j][0]+DATA[i][j][1]); /*internal efficiency*/
    DATA[i][j][20] = DATA[i][j][3]/(DATA[i][j][0]+DATA[i][j][1]); /*external efficiency*/
  }
}
/*internal efficiency Table*/
statMsg("Come out Internal Efficiencies for each Project of Dvisions. See the table above.");
$("#IET").show();
var tar = document.getElementById("IETab");
var newRow = tar.insertRow(tar.rows.length);
var cell = tar.rows[tar.rows.length-1].insertCell(0);
cell.align = "center";
cell.style.minWidth = "70px";
cell.innerHTML = "Fields";
var cell = tar.rows[tar.rows.length-1].insertCell(1);
cell.align = "center";
cell.style.minWidth = "70px";
cell.innerHTML = "P";
for(var i=0; i < MAX_PROJ_NUM; i++){
  var cell = tar.rows[tar.rows.length-1].insertCell(i+2);
  cell.align = "center";
  cell.style.minWidth = "70px";
  cell.innerHTML = "Project " + (i+1);
}
for(var i in DATA){
  var newRow = tar.insertRow(tar.rows.length);
  var cell = tar.rows[tar.rows.length-1].insertCell(0);
  cell.rowSpan = 3;
  cell.align = "center";
  cell.innerHTML = "F(" + i +")";
  var cell = tar.rows[tar.rows.length-1].insertCell(1);
  cell.align = "center";
  cell.innerHTML = "vi<sub>"+i+"</sub>";
  for(var j=1; j <= MAX_PROJ_NUM; j++){
    var cell = tar.rows[tar.rows.length-1].insertCell(j+1);
    cell.align = "center";
    if(DATA[i][j]==null){
      cell.style.background = "black";
      cell.style.color = "white"
      cell.innerHTML = "---";
```

```
    }else  cell.innerHTML = DATA[i][j][2];
   }
   var newRow = tar.insertRow(tar.rows.length);
   var cell = tar.rows[tar.rows.length-1].insertCell(0);
   cell.align = "center";
   cell.innerHTML = "I<sub>"+i+"</sub>";
   for(var j=1; j <= MAX_PROJ_NUM; j++){
    var cell = tar.rows[tar.rows.length-1].insertCell(j);
    cell.align = "center";
    if(DATA[i][j]==null){
     cell.style.background = "black";
     cell.style.color = "white"
     cell.innerHTML = "---";
    }else  cell.innerHTML = (DATA[i][j][0]+DATA[i][j][1]);
   }
   var newRow = tar.insertRow(tar.rows.length);
   var cell = tar.rows[tar.rows.length-1].insertCell(0);
   cell.align = "center";
   cell.innerHTML = "e<sub>"+i+"</sub>";
   for(var j=1; j <= MAX_PROJ_NUM; j++){
    var cell = tar.rows[tar.rows.length-1].insertCell(j);
    cell.align = "center";
    if(DATA[i][j]==null){
     cell.style.background = "black";
     cell.style.color = "white"
     cell.innerHTML = "---";
    }else  cell.innerHTML = Math.round(DATA[i][j][19]*10000)/10000;
   }
  }
  /*external efficiency Table*/
  statMsg("Come out External Efficiencies for each Project of Dvisions. See the table
above.");
  $("#EET").show();
  var tar = document.getElementById("EETab");
  var newRow = tar.insertRow(tar.rows.length);
  var cell = tar.rows[tar.rows.length-1].insertCell(0);
  cell.align = "center";
  cell.style.minWidth = "70px";
  cell.innerHTML = "Fields";
  var cell = tar.rows[tar.rows.length-1].insertCell(1);
  cell.align = "center";
  cell.style.minWidth = "70px";
  cell.innerHTML = "P";
  for(var i=0; i < MAX_PROJ_NUM; i++){
   var cell = tar.rows[tar.rows.length-1].insertCell(i+2);
   cell.align = "center";
   cell.style.minWidth = "70px";
   cell.innerHTML = "Project " + (i+1);
  }
  for(var i in DATA){
```

```
  var newRow = tar.insertRow(tar.rows.length);
  var cell = tar.rows[tar.rows.length-1].insertCell(0);
  cell.rowSpan = 3;
  cell.align = "center";
  cell.innerHTML = "F(" + i +")";
  var cell = tar.rows[tar.rows.length-1].insertCell(1);
  cell.align = "center";
  cell.innerHTML = "ve<sub>"+i+"</sub>";
  for(var j=1; j <= MAX_PROJ_NUM; j++){
    var cell = tar.rows[tar.rows.length-1].insertCell(j+1);
    cell.align = "center";
    if(DATA[i][j]==null){
      cell.style.background = "black";
      cell.style.color = "white"
      cell.innerHTML = "---";
    }else  cell.innerHTML = DATA[i][j][3];
  }
  var newRow = tar.insertRow(tar.rows.length);
  var cell = tar.rows[tar.rows.length-1].insertCell(0);
  cell.align = "center";
  cell.innerHTML = "I<sub>"+i+"</sub>";
  for(var j=1; j <= MAX_PROJ_NUM; j++){
    var cell = tar.rows[tar.rows.length-1].insertCell(j);
    cell.align = "center";
    if(DATA[i][j]==null){
      cell.style.background = "black";
      cell.style.color = "white"
      cell.innerHTML = "---";
    }else  cell.innerHTML = (DATA[i][j][0]+DATA[i][j][1]);
  }
  var newRow = tar.insertRow(tar.rows.length);
  var cell = tar.rows[tar.rows.length-1].insertCell(0);
  cell.align = "center";
  cell.innerHTML = "w<sub>"+i+"</sub>";
  for(var j=1; j <= MAX_PROJ_NUM; j++){
    var cell = tar.rows[tar.rows.length-1].insertCell(j);
    cell.align = "center";
    if(DATA[i][j]==null){
      cell.style.background = "black";
      cell.style.color = "white"
      cell.innerHTML = "---";
    }else  cell.innerHTML = Math.round(DATA[i][j][20]*10000)/10000;
  }
}

var OE = new Array(); /*Efficiency*/
for(var i in DATA){
  for(var j in DATA[i]){
    if(OE[i] == null) OE[i] = new Array();
    OE[i][j] = Math.round((DATA[i][j][18] / DATA[i][j][1])*10000)/10000;
```

```
    }
  }

  /*Sum all qualified project funding requirement*/
  var totalFun = 0;
  var bFun = new Array();   /*Optimal received funding quota for each field*/
  var bFunLeft = new Array(); /*left received funding quota for each field*/
  for(var i in DATA){
    if(DATA[i] == "Field") continue;
    AFUN[i] = 0;
    bFun[i] = 0;
    QFUN[i] = 0;
    for(var j in DATA[i]){
      if(SELECT[i][j] != "N"){
        totalFun += DATA[i][j][1];
        QFUN[i] += DATA[i][j][1];
      }
    }
    statMsg("The qualified funding requirement for field #"+i+" is: $" + QFUN[i]);
  }
  statMsg("Total funding requirement for all qualified projects is: $" + totalFun);
  var RD = Math.round((TOTAL_BUDGET/totalFun)*10000)/100;
  var lim = parseInt($("#satset")[0].value)/2;
  var AdjustedRD = Math.round((RD-lim)*100)/100;
  if (AdjustedRD < 0) AdjustedRD =0;
  statMsg("The best satisfied percentage of each field is " + RD +"%:");

  $("#LBudget").show();
  $("#LBudget").html("Budget Left：" + BUDGET_LEFT +", Total Output:" +
TOTAL_OUTPUT);
  /*1st run, Choose the candidate project independently from each field */
  statMsg("</br></br>***** 1st run -- Choose the project based on optimal satisfaction. Each
Field cannot exceed " + RD + "% - " + lim + "% = " + AdjustedRD +"%. *****");
  for (var i in bFun){
    bFun[i] = Math.round((Number(QFUN[i]) * Number(AdjustedRD))/100);
    bFunLeft[i] = bFun[i];
    statMsg("Field " + i + " cannot exceed $" + bFun[i]);
  }

  for(var i in DATA){
    statMsg("<br>===Choose the project from Field "+i+". ===");
    var fin = 0;
    var step = 1;
    while( bFunLeft[i] > 0 && fin != 1){
      /* find the most internal/external-efficient one*/
      var Mieff = 0, Meeff = 0, Mii = null, Mei = null;
      for (j in DATA[i]){
        if(DATA[i][j][19] > Mieff && SELECT[i][j] == 0){
          Mieff = DATA[i][j][19];
          Mii = j;
```

```
        }
        if(DATA[i][j][20] > Meeff && SELECT[i][j] == 0){
          Meeff = DATA[i][j][20];
          Mei = j;
        }
      }
      if (Mii == null && Mei == null) fin = 1;
      else{
        /*Compare both options. Choose better one. If budget not sufficient, choose reachable
one.*/
        statMsg("<br>Step "+step+":");
        statMsg("Option 1: project "+Mii+" - Output = "+DATA[i][Mii][2]+" + "+
DATA[i][Mii][17]+" = "+DATA[i][Mii][18]);
        statMsg("Option 2: project "+Mei+" - Output = "+DATA[i][Mei][2]+" + "+
DATA[i][Mei][17]+" = "+DATA[i][Mei][18]);
        if(DATA[i][Mii][18] == DATA[i][Mei][18]){
          if (bFunLeft[i] > DATA[i][Mii][1])  Mj = Mii;
          else{
            statMsg("Budget quota is not sufficient to choose project "+Mii+"!");
            fin = 1;
            continue;
          }
        }else if(DATA[i][Mii][18] > DATA[i][Mei][18]){
          if (bFunLeft[i] > DATA[i][Mii][1])  Mj = Mii;
          else{
            statMsg("Budget quota is not sufficient to choose project "+Mii+"!");
            if(bFunLeft[i] > DATA[i][Mei][1]) Mj = Mei;
            else{
              statMsg("Budget quota is not sufficient to choose project "+Mei+"!");
              fin = 1;
              continue;
            }
          }
        }else if(DATA[i][Mii][18] < DATA[i][Mei][18]){
          if (bFunLeft[i] > DATA[i][Mei][1]) Mj = Mei;
          else{
            statMsg("Budget quota is not sufficient to choose project "+Mei+"!");
            if(bFunLeft[i] > DATA[i][Mii][1]) Mj = Mii;
            else{
              statMsg("Budget quota is not sufficient to choose project "+Mii+"!");
              fin = 1;
              continue;
            }
          }
        }
        /*Check OK, reveice the project*/
        BUDGET_LEFT -= DATA[i][Mj][1];
        bFunLeft[i] -= DATA[i][Mj][1];
        AFUN[i] += DATA[i][Mj][1];
        SAT[i] = Math.round(AFUN[i]/QFUN[i] * 100);
```

```
        SELECT[i][Mj] = "PASS";
        TOTAL_OUTPUT += DATA[i][Mj][18];
        statMsg("Choose Project "+ (parseInt(Mj)) + ", which cost $" +DATA[i][Mj][1]+ ". Field
"+i+" Budget Left: $"+bFunLeft[i]+". Satisfcation: "+
Math.round((AFUN[i]/QFUN[i])*10000)/100+"%.");
        $("#LBudget").html("Total investment: $"+ (TOTAL_BUDGET-BUDGET_LEFT) +",
Budget Left：" + BUDGET_LEFT +", Total Output:" +
Math.round(TOTAL_OUTPUT*100)/100);

        /*Update Efficiency Table*/
        var tar = document.getElementById("effTab").rows[i].cells[Mj];

        /*Update DATA Table*/
        var rowcount = 1;
        for(var k in DATA){
          if(DATA[k] == "Field") continue;
          var tar = document.getElementById("inputTab");
          for(var j in DATA[k]){
            if (j==1) var cell = tar.rows[rowcount].cells[6];
            else var cell = tar.rows[rowcount].cells[5];
            cell.align = "center";
            if(SELECT[k][j] == 0) cell.innerHTML = "--";
            else if(SELECT[k][j] == "N"){
              cell.style.background = "black";
              cell.innerHTML = "<font color=white>Disqualified</font>";
            }else{
              cell.style.background = "green";
              cell.innerHTML = SELECT[k][j];
            }
            rowcount += 1;
          }
        }
        step += 1;
      }
    }
  }
  statMsg("<br>===1st run Caculation complete.===");
  statMsg("Each field received (satisfaction):");
  for(var i in AFUN){  statMsg("Field "+i+": $"+AFUN[i]+"/$"+QFUN[i]+"
("+SAT[i]+"%)");}
  var SMax = 0, Smin = 100;
  for(var i in SAT){
    if(SAT[i] >= SMax) SMax = SAT[i];
    if(SAT[i] <= Smin) Smin = SAT[i];
  }
  statMsg("Satisfaction level:"+ (Number(SMax) - Number(Smin)) +"%");
  statMsg("Budget left: $"+ BUDGET_LEFT);
  fin = 0;
  /*2nd run, Choose the candidate project from all field. */
  var AdjustedRD = Math.round((RD+lim)*100)/100;
```

```
  if (AdjustedRD > 100) AdjustedRD = 100;
  statMsg("<br><br>***** 2nd run Pick-Up -- Choose the candidate project from all field.
The satisfaction of each field shall not exceed " + RD + "% + " + lim + "% = " + AdjustedRD
+"%. *****");
  for (var i in bFun){
    bFun[i] = Math.round((Number(QFUN[i]) * Number(AdjustedRD))/100);
    statMsg("Field " + i + " cannot exceed $" + bFun[i] +".");
  }
  statMsg("</br>");
  var fieldSkip = new Array();
  while( BUDGET_LEFT > 0 && fin != 1){
    var Meff = 0, Mi = null, Mj = null;
    /*Find the Field with min Satisfy*/
    var Smin = 100, MF;
    for(var i in SAT){
      if(fieldSkip[i] == null) fieldSkip[i] = 0;
      else if(fieldSkip[i] == 1) continue;
      if(SAT[i] <= Smin) {
        Smin = SAT[i];
        MF = i;
      }
    }
    statMsg("Check is there any feasible project in Field " + MF + "...");
    for (j in OE[MF]){
      if(OE[MF][j] > Meff){
        if( BUDGET_LEFT > DATA[MF][j][1] && SELECT[MF][j] == 0){
          if (ChkSatSec(MF, j, AdjustedRD)){
            Meff = OE[MF][j];
            Mi = MF;
            Mj = j;
          }
        }
      }
    }
    if (Mi == null && Mj == null){
      statMsg("Cannot find any more feasible projects in Field " + MF + ".</br>");
      fieldSkip[MF] = 1;
    }else{
      BUDGET_LEFT -= DATA[Mi][Mj][1];
      AFUN[Mi] += DATA[Mi][Mj][1];
      SAT[Mi] = Math.round(AFUN[Mi]/QFUN[Mi] * 100);
      SELECT[Mi][Mj] = "PASS";
      TOTAL_OUTPUT += DATA[Mi][Mj][18];
      statMsg("</br>Choose Field "+Mi+" Project "+ (parseInt(Mj)) + ", which cost $"
+DATA[Mi][Mj][1]+ ". Then each field received (satisfaction):");
      $("#LBudget").html("Total investment: $"+ (TOTAL_BUDGET-BUDGET_LEFT) +",
Budget Left： " + BUDGET_LEFT +", Total Output:" +
Math.round(TOTAL_OUTPUT*100)/100);
      var SMax = 0, Smin = 100;
      for(var i in SAT){
```

```
        if(SAT[i] >= SMax) SMax = SAT[i];
        if(SAT[i] <= Smin) Smin = SAT[i];
      }
      for(var i in AFUN){  statMsg("Field "+i+": $"+AFUN[i]+"/$"+QFUN[i]+"
("+SAT[i]+"%)");}
      statMsg("Satisfaction level:"+ (Number(SMax) - Number(Smin)) +"%</br>");
    }
    fin = 1;
    for(var i in fieldSkip){
      if (fieldSkip[i] == 0) fin = 0;
    }
  }
  var SMax = 0, Smin = 100;
  for(var i in SAT){
    if(SAT[i] >= SMax) SMax = SAT[i];
    if(SAT[i] <= Smin) Smin = SAT[i];
  }

  statMsg("<br><br>*****Caculation complete.*****");
  var SDIFF = parseInt($("#satset")[0].value);
  if ((Number(SMax) - Number(Smin)) > SDIFF){
    statMsg("(!) This result cannot meet the satisfaction requirement!! Please double check
your data and configuration!!");
    $("#CANTS").html("<font color=red size=+2>Satification Level cannot meet the
requirement ("+SDIFF+"%).<br> Please double check the configuration.<br><br></font>");
    $("#CANTS").show();
    $("#CANTS2").html("<font color=red size=+2>Satification Level cannot meet the
requirement ("+SDIFF+"%).<br> Please double check the configuration.<br><br></font>");
    $("#CANTS2").show();
  }
  statMsg("Each field received (satisfaction):");
  for(var i in AFUN){  statMsg("Field "+i+": $"+AFUN[i]+"/$"+QFUN[i]+"
("+SAT[i]+"%)");}
  statMsg("Satisfaction level:"+ (Number(SMax) - Number(Smin)) +"%");
  statMsg("Budget left: $"+ BUDGET_LEFT);
  statMsg("Total investment: $"+ (TOTAL_BUDGET-BUDGET_LEFT));
  statMsg("Total Output: $"+ Math.round(TOTAL_OUTPUT*100)/100);
  statMsg("Total efficiency obtained: "+
parseInt(TOTAL_OUTPUT/(TOTAL_BUDGET-BUDGET_LEFT)*10000)/100 + "%");
  statMsg("****************************");
  $("#TETF").show();
  $("#TETF").html("Total efficiency obtained: " +
parseInt(TOTAL_OUTPUT/(TOTAL_BUDGET-BUDGET_LEFT)*10000)/100 + "% </br>"
          +"Satisfaction level:"+ (Number(SMax) - Number(Smin)) +"%");
}

function ChkSatSec( p, q , x){
  var r = parseInt($("#satset")[0].value);
  var itotal = new Array();
  var updatedSat = new Array();
```

```
  for(var i in AFUN){
    itotal[i] = AFUN[i];
    if( i == p ) itotal[i] += DATA[p][q][1]
    updatedSat[i] = Math.round(itotal[i]*100/ QFUN[i]);
  }
  var MaxSat = 0, minSat = 100;
  var MaxUnit, minUnit;
  for(var i in updatedSat){
    if(updatedSat[i] >= MaxSat) MaxSat = updatedSat[i];
    if(updatedSat[i] <= minSat)  minSat = updatedSat[i];
  }
  var SatDiff = MaxSat - minSat;
  /*if(SatDiff > r) return false;*/
  if(updatedSat[p] > x) return false;
  else if(updatedSat[p] < (x-r)) return false;
  else return true;
}
/*<-- Caculation Function End*/

$(document).ready(function(){
  LoadData();
});
</script>
</head>
<body>
<b>Input and Output Table</b>
<table id="inputTab" background="#000000">
</table>
</br>
<div id="caluse">
<b>Efficiency</b>
<table id="effTab">
</table>
</br>
</div>
<div id="GRD" style="display:none;">
<b>Grey Relational Degree of each Proposal</b>
<table id="GRDTab">
</table>
<br>
</div>
<div id="IET" style="display:none;">
<b>Internal Efficiencies for each Project of Dvisions</b>
<table id="IETab">
</table>
<br>
</div>
<div id="EET" style="display:none;">
<b>External Efficiencies for each Project of Dvisions</b>
<table id="EETab">
```

```
</table>
<br>
</div>
<label id="SDiff"></label>
</br></br>
Total Budget：<input id="MBudget" type="text" value="0"></br>
Efficiency Required：<input id="EffReq" type="text" value="0" size=2>%</br>
Max Satisfaction Gap：
<select id="satset"><script>
for(var i = 1; i <=100; i++){
document.write("<option value=" + i + ">" + i + "% </option>");}
</script></select></br>
<input id="RUN" type="button" value="RUN" onClick="Run();"><input type="button"
value="Reset" onClick="window.location.reload();"></br>
<label id="LBudget" style="display:none;"></label></br>
<label id="TETF" style="display:none;"></label></br>
<label id="CANTS" style="display:none;"></label></br>
<label id="stat" class="msg"></label></br>
<label id="CANTS2" style="display:none;"></label></br>
</body>
</html>
```

Appendix 5: Example for GDEA

A decision system has ten decision making units; the input/output data is in the following table. Based on the efficiency of the units, rank the first three units according to the input/output data.

	Input			output	
Unit 1	0.7	250	12.4	88	76
Unit 2	0.5	300	11.5	85	70
Unit 3	0.8	320	13	83	81
Unit 4	0.7	440	14.2	83	78
Unit 5	0.8	400	12	85	79
Unit 6	0.7	330	11	80	76
Unit 7	0.4	370	10.5	77	75
Unit 8	0.6	400	12.5	80	78
Unit 9	0.7	390	14	86	78
Unit 10	0.5	340	13	85	75

Solving GDEA below:

$$\max \quad 256\beta_1 + 227\beta_2 - 2\alpha_1 - 870\alpha_2 - 36.9\alpha_3$$

s.t.

(GDEALP)

$$\begin{aligned}
&0.7\alpha_1 + 250\alpha_2 + 12.4\alpha_3 - k_1 = 0 \\
&0.5\alpha_1 + 300\alpha_2 + 11.5\alpha_3 - k_2 = 0 \\
&0.8\alpha_1 + 320\alpha_2 + 13\alpha_3 - k_3 = 0 \\
&0.7\alpha_1 + 250\alpha_2 + 12.4\alpha_3 - 88\beta_1 - 76\beta_2 \geq 0 \\
&0.5\alpha_1 + 300\alpha_2 + 11.5\alpha_3 - 85\beta_1 - 70\beta_2 \geq 0 \\
&0.8\alpha_1 + 320\alpha_2 + 13\alpha_3 - 83\beta_1 - 81\beta_2 \geq 0 \\
&0.7\alpha_1 + 440\alpha_2 + 14.2\alpha_3 - 83\beta_1 - 78\beta_2 \geq 0 \\
&0.8\alpha_1 + 400\alpha_2 + 12\alpha_3 - 85\beta_1 - 79\beta_2 \geq 0 \\
&0.7\alpha_1 + 330\alpha_2 + 11\alpha_3 - 80\beta_1 - 76\beta_2 \geq 0 \\
&0.4\alpha_1 + 370\alpha_2 + 10.5\alpha_3 - 77\beta_1 - 75\beta_2 \geq 0 \\
&0.6\alpha_1 + 400\alpha_2 + 12.5\alpha_3 - 80\beta_1 - 78\beta_2 \geq 0 \\
&0.7\alpha_1 + 390\alpha_2 + 14\alpha_3 - 86\beta_1 - 78\beta_2 \geq 0 \\
&0.5\alpha_1 + 340\alpha_2 + 13\alpha_3 - 85\beta_1 - 75\beta_2 \geq 0 \\
&k_1 = 1
\end{aligned}$$

The above linear program gives a set of optimal weights:

α_1=0.034 α_2=0.001 α_3=0.070 β_1=0.008 β_2=0.004

Plug the values of α, β into $E_j = \dfrac{\beta y_j}{\alpha x_j}$ to obtain the efficiency of the Unit 1, Unit 2, and Unit 3 for desired rankings.

	x_{i1}	x_{i2}	x_{i3}	y_{i1}	y_{i2}	E_i	Rank
Unit 1	0.7	250	12.4	88	76	0.999907	(1)
Unit 2	0.5	300	11.5	85	70	0.999883	(2)
Unit 3	0.8	320	13	83	81	0.911561	(3)
Unit 4	0.7	440	14.2	83	78	0.801268	-
Unit 5	0.8	400	12	85	79	0.949533	-
Unit 6	0.7	330	11	80	76	0.999873	-
Unit 7	0.4	370	10.5	77	75	0.999854	-
Unit 8	0.6	400	12.5	80	78	0.883908	-
Unit 9	0.7	390	14	86	78	0.845872	-
Unit 10	0.5	340	13	85	75	0.904834	-

After solving the linear programming GDEA, using LINDO based on the unit efficiency we obtain the following:

The first place is Unit 1

The second place is Unit 2

The third place is Unit 3

Note that if we set the weighted input value of the second (the third) unit is 1 the rankings of the units interested remain unchanged as expected. This result can be obtained easily by solving the corresponding GDEALP. □