國立交通大學

資訊科學與工程研究所

博　士　論　文

比較模組下多處理器系統診斷能力之局部量測

Determining Diagnosability of Multiprocessor Systems with a Local Approach under the Comparison Model

研 究 生 ： 江玠峰

指導教授 ： 譚建民　博士

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研 究 生：江玠峰　　Student：Chieh-Feng Chiang

指導教授：譚建民 博士　　Advisor：Dr. Jimmy J.M. Tan

國 立 交 通 大 學 資 訊 學 院

資 訊 工 程 系

博 士 論 文

A Dissertation submitted to Department of Computer Science

College of Computer Science

National Chiao Tung University

in Partial Fulfillment of the Requirements

for the Degree of

Doctor of Philosophy

in

Computer Science

June 2011

Hsinchu, Taiwan, R.O.C.

中 華 民 國 一 百 年 六 月

誌謝

本篇論文能夠順利完成，要感謝許多人的幫忙與協助。首先，最感謝我的指導老師譚建民教授，在博士班期間的悉心指導，使我學到許多專業領域的知識以及做研究的嚴謹態度；謝謝實驗室夥伴們長期的陪伴，國晃(晃哥)、倫閔(嚕嚕米)、宗翰(韓吉)、元翔(皮卡丘)、自良(龔學長)、政寬(寬哥)、圓鋼(阿鋼)，在辛苦的研究生活中注入一股活力。

除此之外，非常感謝論文計畫指導委員曾文貴教授、陳榮傑教授，校內口試委員蔡錫鈞教授，以及校外口試委員張貿翔教授、趙坤茂教授、王炳豐教授、與徐力行教授，因為各位教授的不吝賜教，讓這本論文能夠更趨完善。

最後，要把所有的感謝獻給我的家人。感謝爸媽從小到大的栽培，在我遇到人生抉擇與困頓時給予我勇氣、勇敢面對任何逆境；感謝老婆瑋琳長期的支持，懷我在求學過程中無後顧之憂；謝謝哥姊的關心，在我讀書過程中能感受到家人的溫暖。

對於週遭所有曾經幫助過我的師長朋友們，在此我都致上最深的謝意。

比較模組下多處理器系統診斷能力之局部量測

研究生：江玠峰　　　　　　指導教授：譚建民 博士

國立交通大學資訊工程系

摘要

系統診斷對於多處理器系統的可靠度來說是個相當重要的議題，而計算出一系統的診斷力也在近年來許多的研究上被廣為討論。在這篇論文當中，我們提出了一個新穎的診斷觀念—點診斷力，即為一局部的策略、從一點的觀點來計算出整個系統的全域診斷力，而這個局部點診斷力和整個系統的全域診斷力可謂息息相關。利用此局部的概念，我們將重心放在任意挑選的一點之上，以比較模組作為探討的基礎，我們提出了一個充分的條件，以計算出某點的局部點診斷力；我們也提出了一個稱之為延伸星狀圖的有用架構，以利我們判斷出某點的點診斷力之大小。除此之外，基於此局部診斷的觀點，我們也討論了一重要的性質—強點診斷性質；此性質說明了當某種條件成立時，我們可確保在一系統中，某點的局部點診斷力會等同於他自身的維度。最後，利用延伸星狀圖的特點，我們提出了一個能夠偵測出系統上所有點好壞的演算法；若一多處理系統上的總點數為 N，我們提出的演算法將能達到 $O(N \log N)$的表現，運用到大多數遞迴建構起的多處理器系統上都會有著不錯的效能。

關鍵字：偵錯診斷，比較診斷模組，點診斷力，延伸星狀結構，強點診斷性質，診斷演算法。

Determining Diagnosability of Multiprocessor Systems with a Local Approach under the Comparison Model

Student: Chieh-Feng Chiang Advisor: Dr. Jimmy J. M. Tan

Department of Computer Science
College of Computer Science
National Chiao Tung University

ABSTRACT

Diagnosis is an essential subject for the reliability of a multiprocessor system. Determining a system's diagnosability is a widely discussed issue on recent research. In this thesis, we present a novel idea on system diagnosis called node diagnosability. The node diagnosability can be viewed as a local strategy toward system diagnosability. There is a strong relationship between the node diagnosability and the traditional diagnosability. For this local sense, we focus more on a single processor and require only identifying the status of this particular processor correctly. Under the comparison diagnosis model, we propose a sufficient condition to determine the node diagnosability of a given processor. We also propose a useful local structure called an extended star to guarantee the node diagnosability. Based on the local sense of diagnosis, the strongly node-diagnosable property is discussed; this property describes the equivalence of the local diagnosability of a node and its degree. Furthermore, we provide an efficient algorithm to determine the faulty or fault-free status of each processor based on this structure. For a multiprocessor system with total number of processors N, the time complexity of our algorithm to diagnose a given processor is $O(\log N)$ and that to diagnose all the faulty processors is $O(N \log N)$ under the comparison model, provided that there is an extended star structure at each processor and that the time for looking up the testing result of a comparator in the syndrome table is constant. This newly proposed diagnosis algorithm has a well performance on most recursively constructed multiprocessor systems.

Keywords: fault diagnosis, comparison diagnosis model, node diagnosability, extended star structure, strongly node-diagnosable property, diagnosis algorithm.

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Chapter 1

Introductions and Motivations

Recently, high-speed multiprocessor systems have become more and more popular in computer technology. The reliability of the processors in a system is significant since even few faulty processors may cause the system failure. Whenever processors are found faulty, we should replace the faulty ones with fault-free ones to maintain the reliability of the system. Identifying all the faulty processors of a system is called diagnosis of the system. The maximum number of faulty processors that can be ensured to be identified is called the diagnosability of the system. A system G is t-diagnosable if all the faulty processors can be precisely pointed out given that the number of faulty processors is at most t. The maximum number t for which G is t-diagnosable is called the diagnosability of G.

Multiprocessor systems consist of processors and communication links between the processors. Practically, most multiprocessor systems are based on an underlying bus structure or fabric, and perfectly feasible for a central test controller (an independent processor acting as a controller) to check each processor in the system. In such scheme, the central controller itself can be tested externally. Several relevant papers are selected in the following, concerning the network on chip (NoC) issue: Pande et al. [28] developed an evaluation methodology to compare the performance and characteristics of a variety of NoC topologies; Bartic et al. [2] presented an NoC design, which is suitable for building networks with irregular topologies.

Throughout this dissertation, each processor in a system is presented as a node, and a single edge between two arbitrary nodes represents the communication bus or fabric. A diagnosis testing signal is supposed to be delivered from one node to another node through the communication bus at one time. A system performs system-level diagnosis by making each processor as a tester to test each of the directly connected ones, and such scheme contains no central test controller instead. All assumptions are given in order to be consistent with the classic comparison diagnosis model proposed by Maeng and Malek [26].

Several well-known approaches on diagnosis have been developed. One major approach, called the PMC diagnosis model, was first proposed by Preparata et al [29]. It performs diagnosis by sending a test-signal from a processor to another linked one and getting a returning response in the reverse direction. According to the collective testing results, the faulty or fault-free status of all processors in a system can be identified. Following the PMC model, Dahbura et al. [12] proposed a diagnosis algorithm with time complexity $O(N^{2.5})$ to identify all the faulty processors in a system with N processors. Another major diagnosis approach is called the comparison model which was proposed by Maeng and Malek [26][27]. In this model, the diagnosis is performed by simultaneously sending two identical signals from a processor to two other linked ones and comparing the responses. Under the comparison model, Sengupta and Dahbura [30] discussed some characterizations of a t-diagnosable system, and gave a polynomial time algorithm with time complexity $O(N^5)$ to diagnose a system of N processors.

Following the diagnosis models above, most previous studies focused on the diagnosis ability of a system in a global sense, but ignored some local systematic details. A system is t-diagnosable if all the faulty processors can be identified whenever the number of faulty processors is at most t. However, it is possible to correctly point out all the faulty processors in a t-diagnosable system when the number of faulty processors is greater than t. For example, consider two hypercube systems Q_m and Q_n which are m-diagnosable and n-diagnosable [31], respectively, where m and n are integers and $m \gg n$. A new system can be generated by integrating these two systems with few communication links in some way, and such way may cause the diagnosability of the new system to become n. However, the strong diagnosis ability of some part of the entire system, the part of the original m-diagnosable subsystem Q_m, is ignored. Thus, if only considering the global status, we lose some local details of the system. See Figure 1.1 for an illustration.

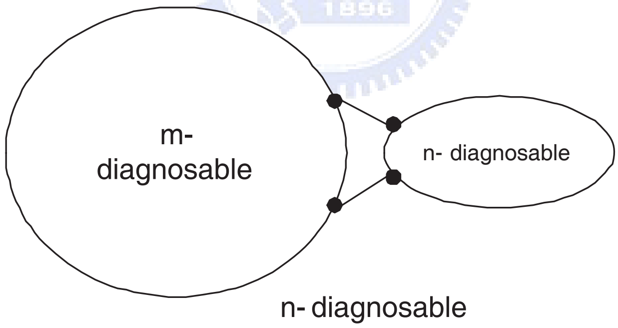

Figure 1.1: an n-diagnosable system generated by integrating an n-diagnosable subsystem and an m-diagnosable subsystem.

In some circumstances, however, we are only concerned about some substructure of a multiprocessor system which is implementable in very large scale integration (VLSI). For example, such substructure can be a ring structure or a path structure. If all processors in such a substructure can be guaranteed to be fault-free, the procedure is still workable even though a few processors in some other part of the system are faulty. Thus, the local substructure of a system is more critical than the global status of the entire system.

In this dissertation, we present a novel idea on system diagnosis which we call the node diagnosability. The node diagnosability can be viewed as a local strategy toward system diagnosability. There is a strong relationship between the node diagnosability and the traditional one. For this local sense, we focus more on a single processor, and require only identifying the status of this particular processor correctly. More specifically, every processor in a system has its own node diagnosability. Under the comparison diagnosis model, we propose a sufficient condition to determine the node diagnosability of a given processor x. On the basis of this sufficient condition, we propose an useful local structure called an extended star at processor x to guarantee its node diagnosability. Following the concept of node diagnosability, we discuss a property so called the strongly node-diagnosable property, which states the equivalence of the node diagnosability of a node and its degree. At last, we have an efficient algorithm to determine the faulty or fault-free status of each processor based on the extended star structure. For most practical multiprocessor systems, the number of links connecting to each processor is in the order of $\log N$, where N is the total number of processors. The time complexity of our algorithm to diagnose a given processor is bounded by $O(\log N)$ and to diagnose all the faulty processors in a system with N processors is bounded by $O(N \log N)$ under the comparison model, provided that there is an extended star structure at each processor, and that the time for looking up the testing result of a comparator in the syndrome table is constant. In general, the time complexity of our algorithm can be represented as $O(N\Delta)$, where Δ is the maximum degree of a processor in the N-processors system.

1.1 Basic Terms and Notations

The topology of a multiprocessor system can be modeled as an undirected graph $G = (V, E)$, where V represents the set of all processors and E represents the set of all connecting links between the processors. Throughout this dissertation, all systems represented as graphs are assumed to be undirected graphs without loops. That is, each pair of adjacent nodes are connected by exactly one edge between them.

The degree of a vertex v in a graph G, written $d_G(v)$ or $deg(v)$, is the number of edges incident with v. The maximum degree of G is denoted by $\Delta(G)$, whereas the minimum degree is denoted as $\delta(G)$. A graph G is called k-regular if the maximum degree and the minimum degree both equal to k. The neighborhood set of a node v, denoted by $N_G(v)$ or $N(v)$, is defined as the set of all nodes adjacent to v.

A node cover of G is a subset of nodes $Q \subseteq V$ such that every edge of E has at least one end node in Q. A node cover with the minimum cardinality is called a minimum node cover.

Let A and B be two sets of nodes, the symmetric difference of sets A and B is defined as the set $A \Delta B = (A \cup B) - (A \cap B)$. For any set of nodes $U \subseteq V(G)$, $G[U]$ denotes the subgraph of G induced by the node subset U. Let H be a subgraph of G and v be a node in H, $deg_H(v)$ denotes the degree of v in subgraph H. For a given set of nodes $S \subseteq V(G)$, we use $G - S$ to denote the induced subgraph $G[V(G) - S]$. Let S be a set of nodes and x be a node *not* in S, we use $C_{x,S}$ to denote the connected component which x belongs to in $G - S$.

1.2 Dissertation Organization

The rest of this doctorial thesis is organized as follows. Chapter 2 provides preliminaries and necessary background for the comparison diagnosis model. Chapter 3 introduces the concepts of node diagnosability and provides a sufficient condition to check whether a system is t-diagnosable at a given processor. The extended star local structure for guaranteeing a processor's node diagnosability is also introduced in this chapter. In Chapter 4, a property called strongly node-diagnosable property is discussed here. In Chapter 5, we propose an efficient algorithm to determine the faulty or fault-free status of a given processor. Finally, some conclusions and discussions are given in Chapter 6.

Chapter 2

The Comparison Diagnosis Model

2.1 Diagnosis Procedure of the Comparison Model

Under the comparison model [26][27], also called the MM model, the system diagnosis is performed by a specific testing procedure. For each processor w which has two distinct links to two other processors u and v, the diagnosis can be performed by simultaneously sending two identical signals from w to u and from w to v, and then comparing their returning responses in the reverse direction. Furthermore, in the MM* model [30], it is assumed that a comparison is performed by each processor for each pair of distinct connected neighbors. Throughout this dissertation, all discussions are based on the MM* model, the complete version of MM model.

This diagnosis-by-comparison strategy can be modeled as a labeled multigraph $M = (V, C)$, called a comparison graph, where V represents the set of all processors same as that in G and C represents the set of labeled edges. For each labeled edge $(u, v)_w \in C$, w is a label on the edge, which means that processors u and v are being compared by a comparator, the processor w.

In order to be consistent with the MM model, several assumptions are needed [30]:

1. all faults are permanent;
2. a faulty processor produces incorrect output for each of its given testing tasks;
3. the output of a comparison performed by a faulty processor is unreliable; and,
4. two faulty processors with the same input do not produce the same output.

The output on a labeled edge $(u, v)_w \in C$ is denoted by $r((u, v)_w)$, which represents the comparison result of w for the two responses from u and v. An agreement is denoted by $r((u, v)_w) = 0$, whereas a disagreement is denoted by $r((u, v)_w) = 1$. Since the comparator processor itself might be faulty or not, the testing result might be unreliable. For this reason, some conclusions are made: if $r((u, v)_w) = 1$, at least one member of $\{u, v, w\}$ is faulty; or, if $r((u, v)_w) = 0$ and w is known to be fault-free, both u and v are fault-free.

After the completion of testing on each comparator in a system, the collection of all testing results can be defined as a function $\sigma : C \rightarrow \{0, 1\}$ and referred to be a syndrome of the diagnosis. For a given syndrome σ, a subset of nodes $F \subset V(G)$ is consistent with σ if the syndrome σ can be produced from the situation that all nodes in F are faulty and all nodes in $V - F$ are fault-free. Let σ_F denote the set of syndromes which are consistent with F, that is, the collection of all possible syndromes which can be produced if F is the faulty set. Notice that for a syndrome σ, there might be more than one faulty subset of V which are consistent with σ.

2.2 Preliminaries for the Comparison Diagnosis Model

A system is defined to be diagnosable if, for every syndrome σ, an unique set of nodes $F \subseteq V$ is consistent with it. In addition, we call a system t-diagnosable if the system is diagnosable as long as the number of faulty processors is at most t. The maximum number t for a system to be t-diagnosable is called the diagnosability of the system. Two distinct subsets of nodes $F_1, F_2 \subset V$ are distinguishable if and only if $\sigma_{F_1} \cap \sigma_{F_1} = \phi$; otherwise, F_1 and F_2 are indistinguishable.

The following is an useful characterization, proposed by Sengupta and Dahbura [30], for the distinguishability of two sets of nodes under the comparison model.

Lemma 1 *[30] For every two distinct subsets of nodes F_1 and F_2, that is, $F_1 \neq F_2$ and $F_1, F_2 \subset V$, (F_1, F_2) is a distinguishable pair if and only if at least one of the following conditions is satisfied (as illustrated in Figure 2.1):*

1) $\exists\, u, w \in V - F_1 - F_2$ and $\exists\, v \in F_1 \Delta F_2$ such that $(u, v)_w \in C$,

2) $\exists\, u, v \in F_1 - F_2$ and $\exists\, w \in V - F_1 - F_2$ such that $(u, v)_w \in C$, or

3) $\exists\, u, v \in F_2 - F_1$ and $\exists\, w \in V - F_1 - F_2$ such that $(u, v)_w \in C$.

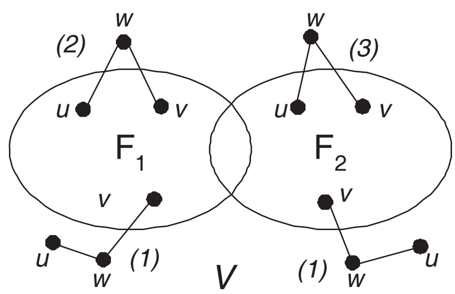

Figure 2.1: illustration of Lemma 1 — the distinguishability of two distinct subsets of nodes.

The detailed proof of this lemma was demonstrated by Sengupta and Dahbura [30]. For the completeness of this thesis, we sketch the proof briefly. If one of the three conditions holds, the distinguishability is absolutely determined:

i) Suppose condition 1) is satisfied. If $v \in F_1 - F_2$ then $r((u,v)_w) = 0$ for each syndrome in σ_{F_2}, and $r((u,v)_w) = 1$ for each syndrome in σ_{F_1}. Similarly, if $v \in F_2 - F_1$ then $r((u,v)_w) = 0$ for each syndrome in σ_{F_1}, and $r((u,v)_w) = 1$ for each syndrome in σ_{F_2}. Either case implies $\sigma_{F_1} \cap \sigma_{F_2} = \phi$.

ii) Suppose condition 2) is satisfied. Then $r((u,v)_w) = 0$ for each syndrome in σ_{F_2} and $r((u,v)_w) = 1$ for each syndrome in σ_{F_1}, which lead to $\sigma_{F_1} \cap \sigma_{F_2} = \phi$.

iii) Suppose condition 3) is satisfied, a similar argument is used as condition 2).

On the contrary, if none of the three conditions holds. We consider a syndrome such that for each $(u,v)_w \in C$, the comparison result can be classified to the following nine situations [30]:

i) If $u, v, w \in V - F_1 - F_2$ then $r((u,v)_w) = 0$.

ii) If $w \in V - F_1 - F_2$ and $u, v \in F_1$ then $r((u,v)_w) = 1$.

iii) If $w \in V - F_1 - F_2$ and $u, v \in F_2$ then $r((u,v)_w) = 1$.

iv) If $w \in V - F_1 - F_2$ and $u \in F_1$ and $v \in F_2$ then $r((u,v)_w) = 1$.

v) If $w \in F_1 - F_2$ and $v \in V - F_2$ and $u \in V - F_1 - F_2$ then $r((u,v)_w) = 0$.

vi) If $w \in F_2 - F_1$ and $v \in V - F_1$ and $u \in V - F_1 - F_2$ then $r((u,v)_w) = 0$.

vii) If $w \in F_1 - F_2$ and $u \in F_2$ then for all v, $r((u,v)_w) = 1$.

viii) If $w \in F_2 - F_1$ and $u \in F_1$ then for all v, $r((u, v)_w) = 1$.

ix) Other arbitrary comparison results.

Then the syndrome above belongs to $\sigma_{F_1} \cap \sigma_{F_2}$, and therefore F_1 and F_2 are indistinguishable. For example, if $w \in V - F_1 - F_2$, $u \in F_1 \cap F_2$, and $v \in F_1 - F_2$, then $r((u, v)_w) = 1$ whenever the faulty set of nodes is either F_1 or F_2. In such circumstance, pair (F_1, F_2) can not be distinguished only with such few information.

Let $G = (V, E)$ be a graph and let $M = (V, C)$ be the comparison graph of G. Define the order graph [30] of a node $u \in V$ to be a digraph $G_u = (X_u, Y_u)$, where $X_u = \{v \mid \text{either } (u, v) \in E \text{ or } (u, v)_w \in C \text{ for some } w\}$ and $Y_u = \{(v, w) \mid v, w \in X_u \text{ and } (u, v)_w \in C\}$.

For a given node $u \in V$, the order of u, $order(u)$, is defined as the cardinality of a minimum node cover of G_u. For a subset of nodes $U \subset V$, define $T(G, U)$ to be the set $\{v \mid (u, v)_w \in C \text{ and } u, w \in U \text{ and } v \in V - U\}$.

Next is a characterization proposed by Sengupta and Dahbura which gives a sufficient condition for a system being t-diagnosable.

Lemma 2 *[30] A system $G(V, E)$ with N nodes is t-diagnosable if*

1) $N \geq 2t + 1$,

2) each node has order at least t, and

3) for each $U \subset V$ such that $\mid U \mid = N - 2t + p$ and $0 \leq p \leq t - 1$, $\mid T(G, U) \mid > p$.

Chapter 3

The Local Approach to Determining System Diagnosability

There were some studies on system diagnosability of some well-known networks under the comparison model. For example, Wang [31][32] presented that the diagnosability of an n-dimensional hypercube Q_n is n for $n \geq 5$ and the diagnosability of an n-dimensional enhanced hypercube is $n+1$ for $n \geq 6$. Fan [16][17] showed that the diagnosability of an n-dimensional Crossed cube is n, and the diagnosability of an n-dimensional Möbius cube is n, for $n \geq 4$. Lai et al. [24] proposed that the diagnosability of the matching composition network is n for $n \geq 4$.

As we observe, the traditional system diagnosability describes the global status of a system. The purpose of this dissertation for considering the node diagnosability is to keep the local connective detail of a system that we might neglect. For example, for any two integers m and n with $m >> n \geq 4$, the diagnosability of two hypercube systems Q_m and Q_n is m and n [31][24], respectively. Combining these two systems with few communication links in some way may cause the diagnosability of the new topology to become n. In this situation, the strong diagnosis ability of some part of the entire system, the substructure Q_m, is ignored. Therefore, the need of keeping local information of each node is concerned.

In the previous studies on diagnosis, most results focused on the diagnosis ability of a system in a global sense: a system is t-diagnosable if all the faulty nodes can be identified given that there are at most t faulty nodes. In contrast to the global sense, we emphasize more on a single node x in a local sense: we require only identifying the status of one particular node correctly. More specifically, if x belongs to a set of faulty nodes, we must correctly identify x to be faulty; or, x is identified to be fault-free if x is indeed fault-free. In a word, we are only concerned about the status of the node x.

3.1 Node Diagnosability

We now introduce the concept of a system being t-diagnosable at a given node.

Definition 1 *A system $G(V, E)$ is t-diagnosable at node $x \in V(G)$ if, given a test syndrome $\sigma \in \sigma_F$ produced by the system under the presence of a set of faulty nodes F containing node x with $|F| \leq t$, every set of faulty nodes F' consistent with σ and $|F'| \leq t$, must also contain node x.*

An equivalent way of stating the above definition is given below.

Proposition 1 *A system $G(V, E)$ is t-diagnosable at node $x \in V(G)$ if, for each pair of distinct sets $F_1, F_2 \subset V(G)$ such that $F_1 \neq F_2$, $|F_1|, |F_2| \leq t$, and $x \in F_1 \Delta F_2$, (F_1, F_2) is a distinguishable pair.*

Then, we define the node diagnosability of a given node as follows.

Definition 2 *The node diagnosability $t_l(x)$ of a node $x \in V(G)$ in a system $G(V, E)$ is defined to be the maximum number of t for G being t-diagnosable at x, that is, $t_l(x) = max\{t \mid G\ is\ t - diagnosable\ at\ x\}$.*

The concept of a system being t-diagnosable at a node is consistent with the traditional concept of a system being t-diagnosable in the global sense. The relationship between these two is as follows.

Proposition 2 *A system $G(V, E)$ is t-diagnosable if and only if G is t-diagnosable at every node.*

Proof. We prove the necessary condition first. Suppose that there exists a node $y \in V(G)$ such that G is not t-diagnosable at y. By Proposition 1, there exists an indistinguishable pair (F_1, F_2) with $|F_i| \leq t, i = 1, 2$, and $y \in F_1 \Delta F_2$. This contradicts that G is t-diagnosable. Next, we prove the sufficiency. Suppose G is not t-diagnosable. Then there exists an indistinguishable pair (F_1, F_2) with $|F_i| \leq t, i = 1, 2$. Pick any node y in $F_1 \Delta F_2$, the system is not t-diagnosable at y by Proposition 1, which is a contradiction. □

Proposition 3 *The diagnosability $t(G)$ of a system $G(V, E)$ is equal to the minimum value among the node diagnosability of every node in G, that is, $t(G) = min\{t_l(x) \mid x \in V(G)\}$.*

Proof. The result follows trivially from Definition 2 and Proposition 2. □

From Proposition 2 and 3, the relationship between the traditional diagnosability and the node diagnosability was pointed out. Through this concept, the system diagnosability can be determined by testing the node diagnosability of each node. Especially in some well-known regular networks, the diagnosability can be easily identified because of the system symmetry. For example, in some graphs like hypercubes, cube-connected cycles, or complete graphs, the system diagnosability and the node diagnosability of each node in the system are the same, and such result can be applied in other applications.

In the following, we propose a sufficient condition for verifying whether a system G is t-diagnosable at a given node x.

Theorem 1 *A system $G(V, E)$ is t-diagnosable at a given node $x \in V(G)$ if, for every set of nodes $S \subset V(G)$, $|S| = p$, $0 \leq p \leq t-1$, and $x \notin S$, the cardinality of every node cover including x of the component $C_{x,S}$ is at least* $2(t-p)+1$.

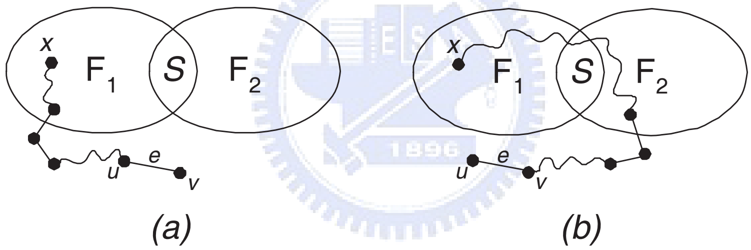

Figure 3.1: illustration of the proof of Theorem 1 — at least one edge lies in $C_{x,S} - F_1 \Delta F_2$.

Proof. We prove it using contradiction. Suppose system G is *not* t-diagnosable at node x. According to Proposition 1, there exists an indistinguishable pair of distinct node set (F_1, F_2) with $|F_1| \leq t$, $|F_2| \leq t$, and $x \in F_1 \Delta F_2$. Let S be the intersection of node sets F_1 and F_2, then the cardinality of S is less than or equal to $t-1$. (Otherwise, if $|S| = t$, then $F_1 = F_2$.) According to the condition that $x \notin S$ and $0 \leq |S| \leq t-1$, the cardinality of every node cover including x of the component $C_{x,S}$ is at least $2(t-p)+1$. Comparing this number with $|F_1 \Delta F_2| \leq 2(t-p)$, and $x \in F_1 \Delta F_2$, we get the fact that $F_1 \Delta F_2$ can not be a node cover of $C_{x,S}$. In other words, at least one member (a node) of the node cover of $C_{x,S}$ is outside $F_1 \Delta F_2$ (and also outside S according to the definition of component $C_{x,S}$). Consequently, by the property of node cover, there exists an edge $e = (u, v)$ in $C_{x,S}$ but outside $F_1 \Delta F_2$. Since edge e, nodes u, v, and node x belong to the same connected component $C_{x,S}$, there is a path leading from edge e to node x through set F_1 (as shown in Figure 3.1(a)) or F_2 (as shown in Figure 3.1(b)). Then by condition 1 of Lemma 1, (F_1, F_2) is a distinguishable pair. This is a contradiction, and the result follows. □

3.2 The Extended Star Structure

Let us introduce a structure first.

Definition 3 *Let x be a node in a graph $G(V, E)$ with $deg_G(x) \geq n$. Define $H(x; n)$ to be a subgraph of G of order n at node x, where the set of nodes is $\{x\} \cup \{v_{i1}, v_{i2} \mid 1 \leq i \leq n\}$ and the set of edges is $\{(x, v_{i1}), (v_{i1}, v_{i2}) \mid 1 \leq i \leq n\}$. (Figure 3.2 depicts the structure.)*

We notice that the term "order" is used in two different places, one is the order of a node x, $order(x)$, and the other is the order of the substructure defined here.

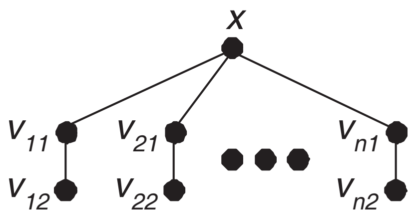

Figure 3.2: subgraph $H(x; n)$ of G of order n at node x.

Proposition 4 *Let $G(V, E)$ be a graph and x be a node in G. The order of x is at least n if G contains a subgraph $H(x; n)$ of order n at node x.*

Proposition 5 *Let $G(V, E)$ be a graph without cycles of length three, and x be a node in G. G contains a subgraph $H(x; n)$ of order n at node x if the order of x is at least n.*

Proof. Let S_1 and S_2 be two sets of nodes with distance 1 and 2 to the node x, respectively. Since G contains no cycles of length 3, there is no edge with both ends in S_1. Therefore, the order graph of x forms a bipartite graph with the partition (S_1, S_2). Because the node x has order at least n, which means the cardinality of a minimum node cover of the order graph of x is at least n. By a classical theorem of König [13] and Egerváry [14], the cardinality of a minimum node cover of a bipartite graph equals the maximum size of a matching in the bipartite graph. Then, there is a matching between S_1 and S_2 with the maximum size n. Consequently, G contains a subgraph $H(x; n)$ of order n at node x. □

The above two propositions state that the order of node x is at least n if and only if the system contains a subgraph $H(x; n)$ of order n at x. It implies that if the node diagnosability of node x is n, then G contains a subgraph $H(x; n)$ at x, provided that G has no cycles of length 3. However, having the substructure $H(x; n)$ at x, it does not necessarily guarantee that the node diagnosability of node x is at least n.

We now propose a substructure at node x, called an extended star, which can guarantee the node diagnosability of node x.

Definition 4 *Let x be a node in a graph $G(V, E)$. For $n \leq deg_G(x)$, an extended star $ES(x; n)$ of order n at node x is defined as $ES(x; n) = (V(x; n), E(x; n))$, where the set of nodes $V(x; n) = \{x\} \cup \{v_{ij} \mid 1 \leq i \leq n, 1 \leq j \leq 4\}$ and the set of edges $E(x; n) = \{(x, v_{k1}), (v_{k1}, v_{k2}), (v_{k2}, v_{k3}), (v_{k3}, v_{k4}) \mid 1 \leq k \leq n\}$. (See Figure 3.3 for an illustration.)*

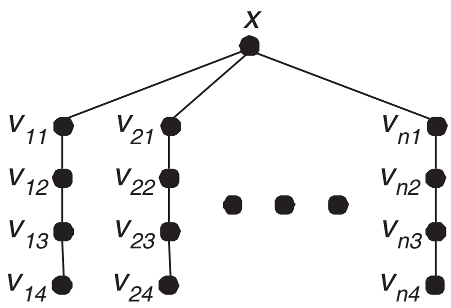

Figure 3.3: extended star structure $ES(x;n)$.

We say that there is an extended star structure $ES(x;n) \subseteq G$ at node x if G contains an extended star $ES(x;n)$ of order n at node x as a subgraph. Note that in the definition of the extended star, each node and each edge can occur only once in this structure. In other words, the problem of setting up the extended star structure turns into the problem of finding n node-disjoint paths of length 4 (3 hops) with dedicated starting nodes. In addition, such problem can be done off-line by the systematic structure of most well-known multiprocessor systems.

Theorem 2 *Let x be a node in a system $G(V,E)$. The node diagnosability of x is at least n if there exists an extended star $ES(x;n) \subseteq G$ at x.*

Proof. We use Theorem 1 to prove this result. First, we define $l_k = (v_{k1}, v_{k2}, v_{k3}, v_{k4})$ to be a quadruple of four consecutive nodes for any k, $1 \le k \le n$, with respect to $ES(x;n)$. We note that l_k is a path of length 3. Accordingly, the cardinality of a node cover of each l_k is at least 2. Let $S \subset V(G)$ be a set of nodes in G with $|S| = p$, $0 \le p \le n-1$, and $x \notin S$. After deleting S from $V(G)$, there are at least $(n-p)$ complete l_k's still remaining in $ES(x;n)$, where the word "complete" means that all v_{k1}, v_{k2}, v_{k3}, and v_{k4} of an l_k have not been deleted in $G - S$. Thus, the cardinality of a node cover including

x of the connected component $C_{x,S}$ is at least $1+2(n-p)$. Therefore, the system G with an extended star $ES(x;n)$ is n-diagnosable at x by Theorem 1. By Definition 2, the node diagnosability of x is at least n, that is, $t_l(x) \geq n$. □

Proposition 6 *Let x be a node in a system $G(V,E)$ with $deg_G(x) = n$. The node diagnosability of x is at most n.*

By Theorem 2 and Proposition 6, we have the following result.

Theorem 3 *Let x be a node in a system $G(V,E)$ with $deg_G(x) = n$. The node diagnosability of x is n if there exists an extended star $ES(x;n) \subseteq G$ at x.*

We observe that for an extended star structure, if the set of nodes is of the form $V(x;n) = \{x\} \cup \{v_{ij} \mid 1 \leq i \leq n, 1 \leq j \leq 3\}$ and the set of edges is of the form $E(x;n) = \{(x, v_{k1}), (v_{k1}, v_{k2}), (v_{k2}, v_{k3}) \mid 1 \leq k \leq n\}$, the node diagnosability n of node x cannot be guaranteed simply by this kind of substructure. For example, let F_1 be the set of nodes $\{x, v_{11}, v_{12}, v_{13}\}$ with $|F_1| = 4$, and F_2 be the set of nodes $\{v_{k2} \mid 1 \leq k \leq n\}$ with $|F_2| = n$ (as shown in Figure 3.4), (F_1, F_2) is not a distinguishable pair according to Lemma 1 unless there are other edges or nodes. Thus, the node diagnosability of x cannot be guaranteed to be n.

In most multiprocessor systems or interconnection networks, an extended star substructure at a given processor does exist. For example, the well-known multiprocessor systems such as the Hypercube, the Crossed cube [15], the Twisted cube [19], the Möbius cube [11], the Star [1], the mesh, and other hypercube-like graphs, in which an extended

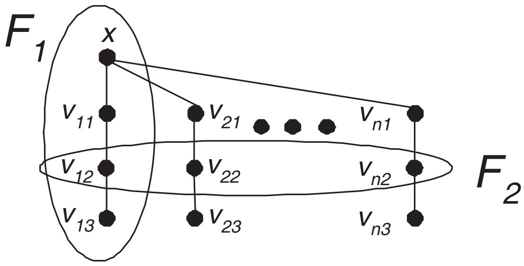

Figure 3.4: an example of an indistinguishable pair in an incomplete extended star structure with only set of nodes $\{x\} \cup \{v_{ij} \mid 1 \leq i \leq n, 1 \leq j \leq 3\}$ and set of edges $\{(x, v_{k1}), (v_{k1}, v_{k2}), (v_{k2}, v_{k3}) \mid 1 \leq k \leq n\}$.

star at a given processor can be carefully found because of the regular recursive construction, as long as the dimension n is suitably large.

3.3 Diagnosability of the Hypercube

Among all well-known interconnection networks, the Hypercube is one of the most popular ones. Following the structure of the Hypercube, lots of similar networks had been proposed, such as the Crossed cube [15], the Twisted cube [19], and the Möbius cube [11]. We call the category of these systems a cube family. For each cube in the cube family, an n-dimensional cube can be constructed in recurrence from two identical $(n-1)$-dimensional subcubes by adding a perfect matching between the two subcubes. A different perfect matching leads to a different cube. Because of the recursive construction, an n-dimensional cube has 2^n nodes in it. Each node in the cube is usually represented by an n-bit binary string. A binary string x of length n can be written as $x = x_n x_{n-1} \ldots x_2 x_1$, where x_i is 0 or 1, $1 \leq i \leq n$.

For each node x in an n-dimensional Hypercube, there are n distinct nodes adjacent to it and with 1-bit complement to it. It is easy to find an extended star structure $ES(x; n)$

at x in an n-dimensional Hypercube with $n \geq 5$ as following:

For each node $x = x_n x_{n-1} \ldots x_2 x_1$, there are n nodes adjacent to it, namely, $\overline{x_n} x_{n-1} \ldots x_2 x_1$, $x_n \overline{x_{n-1}} \ldots x_2 x_1$, ..., and $x_n x_{n-1} \ldots x_2 \overline{x_1}$, where the overline denotes the complement bit. Let $v_{n,1}$, $v_{n-1,1}$, ..., and $v_{1,1}$ be these nodes respectively. For each $v_{k,1}$, $v_{k,1} = x_n x_{n-1} \ldots \overline{x_k} \ldots x_2 x_1$, there are n nodes adjacent to it also. We can find one of these nodes with the $(k+1)(mod\ n)$-th bit complement to $v_{k,1}$, for all $1 \leq k \leq n$, and name it $v_{k,2}$. Then, $v_{k,2} = x_n x_{n-1} \ldots \overline{x_{k+1} x_k} \ldots x_2 x_1$. Moreover, we can find $v_{k,3} = x_n x_{n-1} \ldots \overline{x_{k+2} x_{k+1} x_k} \ldots x_2 x_1$ and $v_{k,4} = x_n x_{n-1} \ldots \overline{x_{k+3} x_{k+2} x_{k+1} x_k} \ldots x_2 x_1$ in the same way, where the indices are modulo n. (Figure 3.5)

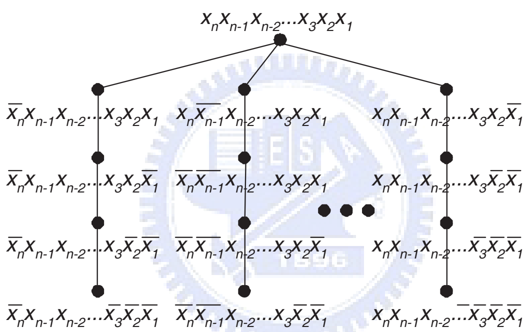

Figure 3.5: an extended star structure in an n-dimensional Hypercube with $n \geq 5$.

All these nodes do not have the same address (string bits) since the bit length is at least five. Thus, the procedure described above provides an extended star $ES(x;n)$ for every node x in $V(Q_n)$, for $n \geq 5$. Consequently, the node diagnosability of each node $x \in V(Q_n)$ is n and the diagnosability of Q_n is n, for $n \geq 5$, which is the same conclusion as that proposed by Wang [31]. Note that there are more than one way for searching an extended star in a Hypercube.

3.4 Diagnosability of the Star Graph

As another example, we show that the Star graph [1] with dimension 4 or more contains an extended star structure as a subgraph at each node . Let n be a positive integer. The Star graph of dimension n, denoted by S_n, is a graph whose set of nodes consists of all permutations of $\{1, 2, \ldots, n\}$. Each node is uniquely assigned a label $x_1x_2 \ldots x_n$, and is adjacent to the nodes $x_ix_2 \ldots x_{i-1}x_1x_{i+1} \ldots x_n$, for $2 \leq i \leq n$, that is, nodes obtained by a transposition of the first symbol with the ith symbol of the node. Consequently, there are $n!$ nodes in an n-dimensional Star graph, and each node has degree $n-1$. We can find an extended star structure $ES(x; n-1)$ at a given node x in S_n with $n \geq 5$ as follows.

For each node $x = x_1x_2 \ldots x_n$, there are $n-1$ nodes adjacent to it, namely, $x_2x_1x_3x_4 \ldots x_n$, $x_3x_2x_1x_4 \ldots x_n$, $\ldots$, $x_ix_2x_3x_4 \ldots x_{i-1}x_1x_{i+1} \ldots x_n$, $\ldots$, and $x_nx_2x_3x_4 \ldots x_{n-1}x_1$. Let $v_{2,1}$, $v_{3,1}$, $\ldots$, $v_{i,1}$, $\ldots$, and $v_{n,1}$ be these nodes respectively. For convenience of description, we say that two nodes are adjacent to each other with a $(1\ i)$ edge if one node can be obtained by a transposition of the first symbol with the ith symbol of the other node. Accordingly, x is adjacent to $v_{k,1}$ with a $(1\ k)$ edge, for all $2 \leq k \leq n$. For each $v_{k,1}$, there are $(n-2)$ more nodes adjacent to it except for x. We can choose one of these adjacent nodes of $v_{k,1}$ with a $(1\ \ k+1)$ edge if $2 \leq k \leq n-1$, and with a $(1\ \ \ ((k+2) mod\ n))$ edge if $k = n$. Let $v_{k,2}$ be these nodes, for all $2 \leq k \leq n$, respectively. We then find $v_{k,3}$ as one of the adjacent nodes of $v_{k,2}$ with a $(1\ \ k+2)$ edge if $2 \leq k \leq n-2$, and with a $(1\ \ \ ((k+3) mod\ n))$ edge if $n-1 \leq k \leq n$. Finally, we find $v_{k,4}$ as one of the adjacent nodes of $v_{k,3}$ with a $(1\ \ k+3)$ edge if $2 \leq k \leq n-3$, and with a $(1\ \ \ ((k+4) mod\ n))$ edge if $n-2 \leq k \leq n$. (Figure 3.6)

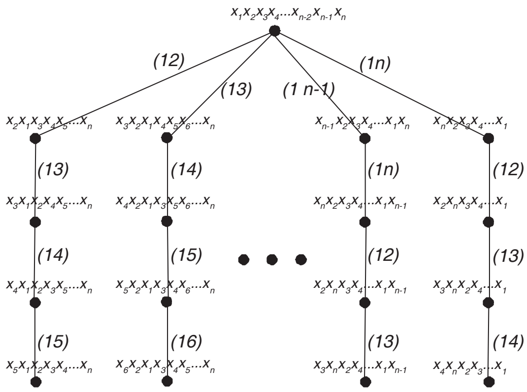

Figure 3.6: an extended star structure in an n-dimensional Star graph with $n \geq 5$.

Therefore, an extended star $ES(x; n-1)$ at every node $x \in V(S_n)$ can be retrieved for $n \geq 4$. We note however, for $n = 4$, the construction strategy described above has to be modified a little bit, since the construction strategy in the last paragraph will cause all $v'_{k,4}s$ to be the same node, for all $2 \leq k \leq n$. We can choose $v_{k,4}$ as one of the adjacent nodes of $v_{k,3}$ with a (1 3) edge for $k = 2$, a (1 4) edge for $k = 3$, and a (1 2) edge for $k = 4$, as a modified strategy. Therefore, for $n \geq 4$, the node diagnosability of each node $x \in V(S_n)$ is $n-1$ and the diagnosability of S_n is $n-1$, which is the same conclusion as that proposed by Zheng et al [35].

For most multiprocessor systems or interconnection networks, an extended star at a given node can be carefully found, as long as the dimension n is suitably large. This explains the fact that the node diagnosibility of a given node matches its degree in many cases.

3.5 An Example of Combining Two Hypercubes of Different Dimensions

One more example, consider an m-dimensional Hypercube system Q_m and an n-dimensional Hypercube system Q_n, for $m \geq n \geq 5$. The node diagnosability of each node in Q_m (Q_n, respectively) is m (n, respectively). Let u be a node in Q_m and v be a node in Q_n. A new system can be formed by adding an edge (u, v) between Q_m and Q_n. Applying the extended star structure, the node diagnosability of each node in Q_m (Q_n, respectively) remains m (n, respectively) except u (v, respectively), while the node diagnosability of node u (v, respectively) increases to $m+1$ ($n+1$, respectively). Overall, the diagnosability of this new system is n.

Chapter 4

Strongly Node-Diagnosable Property

In this chapter, we discuss the strongly node-diagnosable property, which states the relationship between a processor's node diagnosability and its degree. A processor is defined to be strongly node-diagnosable if the node diagnosability of it equals to its degree, where degree refers to as the number of links incident with it. A system is defined to be strongly node-diagnosable if all the processors in this system are strongly node-diagnosable. We shall prove that both an n-dimensional hypercube and an n-dimensional star graph have this property.

In some circumstances, some links in a multiprocessor system may be missing. A missing edge stands for a link which is broken or failure between two processors for some reasons. The existence of missing edges in a system may reduce the diagnosability of the whole system and change the node diagnosability of each node. We shall prove that both an n-dimensional hypercube and an n-dimensional star graph keep the strongly node-diagnosable property even if there is a bounded amount of missing edges.

We first introduce the definition of a node (respectively, a graph) being strongly node-diagnosable as follows.

Definition 5 *Let x be a node in a graph $G(V, E)$. Node x is strongly node-diagnosable if the node diagnosability of x equals to its degree in G. That is, $t_l(x) = deg_G(x)$.*

Definition 6 *Let $G(V, E)$ be a graph. Graph G is strongly node-diagnosable if the node diagnosability of every node equals to its degree in G. That is, $t_l(x) = deg_G(x)$, for all $x \in V(G)$.*

4.1 Strongly Node-Diagnosable Property of the Hypercube

Following the definitions of the so called strongly node-diagnosable property of a node or of a graph, we shall declare that an n-dimensional hypercube with $n \geq 5$ has the strongly node-diagnosable property in this section. First of all, a lemma is needed to show that there exists an extended star $ES(x; n) \subseteq Q_n$ of order n at every node x in Q_n, for $n \geq 5$.

Lemma 3 *For each node x in an n-dimensional hypercube Q_n with $n \geq 5$, there exists an extended star $ES(x; n) \subseteq Q_n$ of order n at x.*

Proof. Since Q_n is node symmetric, we arbitrarily choose $\mathbf{x} = x_n x_{n-1} \ldots x_1$ to be the root of an $ES(\mathbf{x}; n)$, and try to find an extended star $ES(\mathbf{x}; n)$ as a subgraph of the n-dimensional hypercube Q_n at the node $\mathbf{x}$. In this proof, we follow the notations in Definition 4.

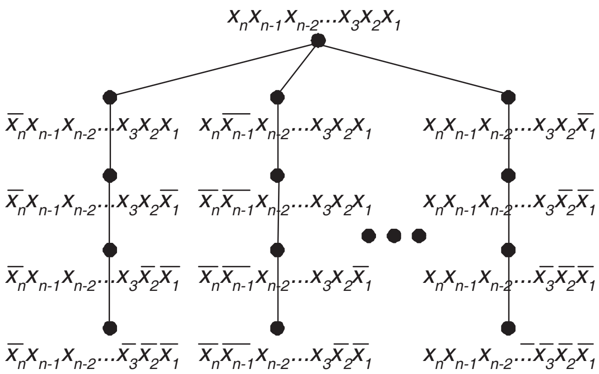

Figure 4.1: illustration of the proof of Lemma 3, an $ES(\mathbf{x};n)$ of order n at $\mathbf{x} = x_nx_{n-1}\ldots x_1$.

For all $n \geq 5$, we can find an extended star $ES(\mathbf{x};n)$ at node $\mathbf{x} = x_nx_{n-1}\ldots x_1$ (as shown in Figure 4.1), where the set of nodes is

$$\{\mathbf{x}\} \cup \{v_{k1} = x_nx_{n-1}\ldots\overline{x_k}\ldots x_1 \mid 1 \leq k \leq n\}$$

$$\cup \{v_{k2} = x_nx_{n-1}\ldots\overline{x_{(k+1)(mod\ n)}}\,\overline{x_k}\ldots x_1 \mid 1 \leq k \leq n\}$$

$$\cup \{v_{k3} = x_nx_{n-1}\ldots\overline{x_{(k+2)(mod\ n)}}\,\overline{x_{(k+1)(mod\ n)}}\,\overline{x_k}\ldots x_1 \mid 1 \leq k \leq n\}$$

$$\cup \{v_{k4} = x_nx_{n-1}\ldots\overline{x_{(k+3)(mod\ n)}}\,\overline{x_{(k+2)(mod\ n)}}\,\overline{x_{(k+1)(mod\ n)}}\,\overline{x_k}\ldots x_1 \mid 1 \leq k \leq n\}$$

and the set of edges is $\{(\mathbf{x}, v_{k1}), (v_{k1}, v_{k2}), (v_{k2}, v_{k3}), (v_{k3}, v_{k4}) \mid 1 \leq k \leq n\}$.

As a result, there exists an extended star $ES(\mathbf{x};n) \subseteq Q_n$ of order n at each node $\mathbf{x} \in V(Q_n)$ for $n \geq 5$. □

Theorem 4 *Let Q_n be an n-dimensional hypercube and $n \geq 5$. Each node x in Q_n is strongly node-diagnosable; and graph Q_n is strongly node-diagnosable.*

Proof. By Theorem 3 and Lemma 3, the node diagnosability of each node $x \in V(Q_n)$ is n, since the degree of x in Q_n is n and there exists an extended star $ES(x;n)$ of order

n at x, for $n \geq 5$. Thus, every node in an n-dimensional star Q_n with $n \geq 5$ is strongly node-diagnosable. So graph Q_n is strongly node-diagnosable. □

Up to now, we facilitate the procedure of proving that the diagnosability of Q_n is n, for $n \geq 5$. As mentioned before, a multiprocessor system may have some links broken or failure. Consequently, it may affect the reliability of the whole system. Now, we are proving that even with a total amount $n-2$ missing edges, an n-dimensional hypercube Q_n still keeps the strongly node-diagnosable property, for $n \geq 5$.

Note that for a given set of edges $L \subseteq E(G)$ in a system G, we use $G - L$ to denote the subgraph with node set $V(G)$ and edge set $E(G) - L$.

Lemma 4 *Let Q_n be an n-dimensional hypercube with $n \geq 5$, and let F be an arbitrary set of missing edges with $|F| \leq n-2$. For each node x in Q_n, there exists an extended star $ES(x; deg_{Q_n - F}(x)) \subseteq Q_n$ at x, where $deg_{Q_n - F}(x)$ denotes the remaining degree of node x in $Q_n - F$.*

Proof. We prove this lemma by induction on n.

For the base case $n = 5$, each node in Q_n is labeled as $x = x_5x_4x_3x_2x_1$, for each $x_i = 0$ or 1, $1 \leq i \leq 5$. Since the hypercube is node symmetric, we arbitrarily choose a node $\mathbf{x} = x_5x_4x_3x_2x_1$ for description. Since $n - 2 = 3$ for $n = 5$, there are at most 3 missing edges in this incomplete hypercube. It is straightforward but tedious to see that there indeed exists an extended star $ES(x; deg_{Q_n - F}(x)) \subseteq Q_n$ at each node x, for $n = 5$ and $|F| = 0$, 1, 2, or 3.

For induction hypothesis, suppose that for $n \geq 6$ and $|F| \leq n-3$, there exists an $ES(x; deg_{Q_{n-1}-F}(x)) \subseteq Q_{n-1}$ at each node $x \in V(Q_{n-1})$, where F is the set of missing edges.

Now we claim that for a set of missing edges F with $|F| \leq n-2$, there exists an $ES(x; deg_{Q_n-F}(x)) \subseteq Q_n$ at each node $x \in V(Q_n)$, for $n \geq 6$. Assume that the number of missing edges is at most $n-2$ in an n-dimensional hypercube Q_n, for $n \geq 6$. Let $f = (u, v)$ be an arbitrarily missing edge. The n-dimensional hypercube Q_n can be seen as the composition of two subgraphs Q^0_{n-1} and Q^1_{n-1}, where $u \in V(Q^0_{n-1})$ and $v \in V(Q^1_{n-1})$. Note that each Q^i_{n-1} is isomorphic to an $(n-1)$-dimensional hypercube Q_{n-1}, $i = 0$ or 1. Then, the number of all missing edges except f in both Q^0_{n-1} and Q^1_{n-1} is at most $n-3$. Therefore, there is an extended star $ES(x; n-1)$ of order $n-1$ at every node x in this faulty hypercube. Consider an arbitrary node $\mathbf{x}$ in Q_n, $\mathbf{x}$ is in one of the two induced subgraphs Q^i_{n-1}, $i = 0$ or 1. Without loss of generality, we let $\mathbf{x} \in Q^0_{n-1}$. If the incident edge of $\mathbf{x}$ that has the other end in Q^1_{n-1} is missing (as shown in Figure 4.2(a)), we are done. That is, the order of the extended star at $\mathbf{x}$ equals to $\mathbf{x}$'s remaining degree in this $Q_n - F$, and both are $n-1$. Otherwise, the incident edge of $\mathbf{x}$ that has the other end in Q^1_{n-1} is fault-free (as shown in Figure 4.2(b)). Then we can find an $ES(\mathbf{x}; n)$ of order n at node $\mathbf{x}$. As a result, there is an $ES(\mathbf{x}; deg_{Q_n-F}(\mathbf{x})) \subseteq Q_n$ at $\mathbf{x}$, for $n \geq 6$ and $|F| \leq n-2$. □

Theorem 5 *Let Q_n be an n-dimensional hypercube and $n \geq 5$, and let F be an arbitrary set of missing edges with $|F| \leq n-2$. For each node x in Q_n with missing edges F, node x is strongly node-diagnosable; and graph $Q_n - F$ is strongly node-diagnosable.*

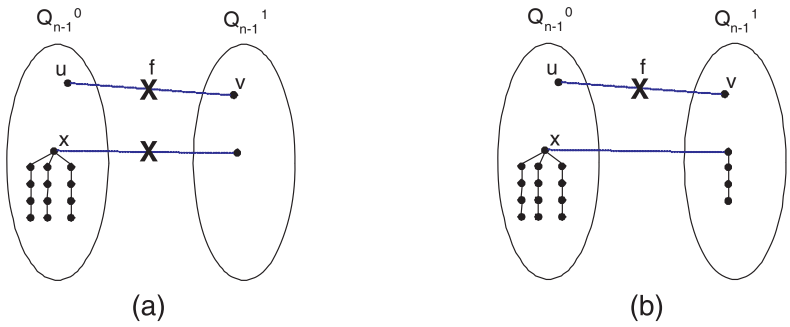

Figure 4.2: illustration for the inductive step in the proof of Lemma 4.

Proof. By Theorem 3 and Lemma 4, the node diagnosability of each node x in an incomplete n-dimensional hypercube $Q_n - F$ is equal to its remaining degree, for $n \geq 5$ and $|F| \leq n-2$. Thus, every node in $Q_n - F$ is strongly node-diagnosable. Consequently, graph $Q_n - F$ is strongly node-diagnosable. □

4.2 Strongly Node-Diagnosable Property of the Star Graph

In this section, we show that an n-dimensional star with $n \geq 4$ has the strongly node-diagnosable property. Same as that for the hypercube in the last section, we need to explicitly state that there exists an extended star $ES(x; n-1) \subseteq S_n$ of order $n-1$ at every node x in S_n, for $n \geq 4$.

Lemma 5 *For each node x in an n-dimensional star S_n with $n \geq 4$, there exists an extended star $ES(x; n-1) \subseteq S_n$ of order $n-1$ at x.*

Proof. We use the notations in Definition 4 to find an extended star $ES(x; n-1)$ as a subgraph of an n-dimensional star S_n at a given node x. Since S_n is node symmetric, we arbitrarily choose $\mathbf{x} = x_1 x_2 \ldots x_n$ to be the root of an $ES(\mathbf{x}; n-1)$.

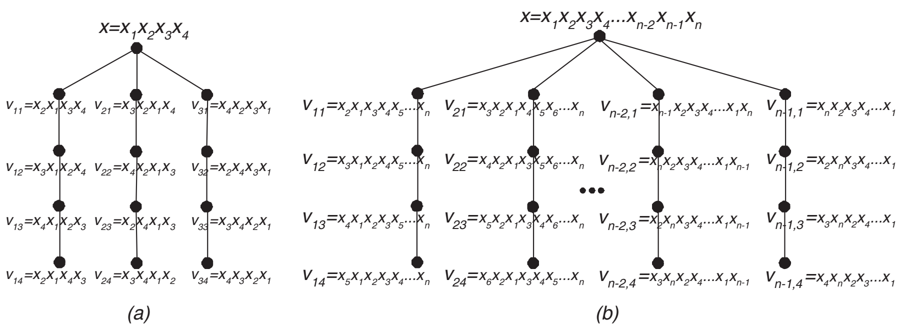

Figure 4.3: illustration of the proof of Lemma 5, a) an $ES(\mathbf{x};3)$ of order 3 at $\mathbf{x}=x_1x_2x_3x_4$; b) an $ES(\mathbf{x};n-1)$ of order $n-1$ at $\mathbf{x}=x_1x_2\ldots x_n$.

For $n=4$, we can find an extended star $ES(\mathbf{x};3)$ of order 3 at node $\mathbf{x}=x_1x_2x_3x_4$ (as shown in Figure 4.3(a)), where the set of nodes contains $\mathbf{x}$, $v_{11}=x_2x_1x_3x_4$, $v_{12}=x_3x_1x_2x_4$, $v_{13}=x_4x_1x_2x_3$, $v_{14}=x_2x_1x_4x_3$, $v_{21}=x_3x_2x_1x_4$, $v_{22}=x_4x_2x_1x_3$, $v_{23}=x_2x_4x_1x_3$, $v_{24}=x_3x_4x_1x_2$, $v_{31}=x_4x_2x_3x_1$, $v_{32}=x_2x_4x_3x_1$, $v_{33}=x_3x_4x_2x_1$, and $v_{34}=x_4x_3x_2x_1$, and the set of edges is $\{(\mathbf{x},v_{k1}),(v_{k1},v_{k2}),(v_{k2},v_{k3}),(v_{k3},v_{k4})\mid 1\le k\le 3\}$.

For $n\ge 5$, we can find an extended star $ES(\mathbf{x};n-1)$ at node $\mathbf{x}=x_1x_2\ldots x_n$ (as shown in Figure 4.3(b)), where the set of nodes is

$$
\begin{aligned}
&\{\mathbf{x}\}\ \cup\ \{v_{k1}=x^{k+1}\mid 1\le k<n\}\\
&\quad\cup\ \{v_{k2}=v_{k1}^{k+2}\mid 1\le k<n-1\}\ \cup\ \{v_{k2}=v_{k1}^{(k+3)\ mod\ n}\mid n-1\le k<n\}\\
&\quad\cup\ \{v_{k3}=v_{k2}^{k+3}\mid 1\le k<n-2\}\ \cup\ \{v_{k3}=v_{k2}^{(k+4)\ mod\ n}\mid n-2\le k<n\}\\
&\quad\cup\ \{v_{k4}=v_{k3}^{k+4}\mid 1\le k<n-3\}\ \cup\ \{v_{k4}=v_{k3}^{(k+5)\ mod\ n}\mid n-3\le k<n\}
\end{aligned}
$$

and the set of edges is $\{(\mathbf{x},v_{k1}),(v_{k1},v_{k2}),(v_{k2},v_{k3}),(v_{k3},v_{k4})\mid 1\le k\le n-1\}$.

As a result, there exists an extended star $ES(\mathbf{x};n-1)\subseteq S_n$ of order $n-1$ at each node $\mathbf{x}\in V(S_n)$ for $n\ge 4$. $\square$

Theorem 6 *Let S_n be an n-dimensional star and $n \geq 4$. Each node x in S_n is strongly node-diagnosable; and graph S_n is strongly node-diagnosable.*

Proof. By Theorem 3 and Lemma 5, the node diagnosability of each node $x \in V(S_n)$ is $n-1$, since the degree of x in S_n is $n-1$ and there exists an extended star $ES(x; n-1)$ of order $n-1$ at x, for $n \geq 4$. Thus, every node in an n-dimensional star S_n with $n \geq 4$ is strongly node-diagnosable. So graph S_n is strongly node-diagnosable. □

By the theorem above, we conclude that the diagnosability of S_n is $n-1$, for $n \geq 4$, which is the same result as that proposed by Zheng et al [35]. In the following, we show that an n-dimensional star S_n keeps the strongly node-diagnosable property even with up to $n-3$ missing edges, for $n \geq 4$.

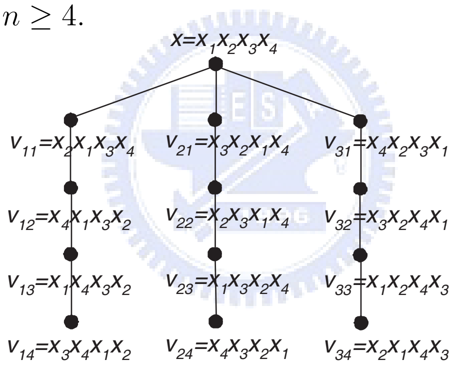

Figure 4.4: an alternative extended star $ES(\mathbf{x}; 3)$ at $\mathbf{x} = x_1x_2x_3x_4$ described in the proof of Lemma 6.

Lemma 6 *Let S_n be an n-dimensional star with $n \geq 4$, and let F be an arbitrary set of missing edges with $|F| \leq n-3$. For each node x in S_n, there exists an extended star $ES(x; deg_{S_n - F}(x)) \subseteq S_n$ at x, where $deg_{S_n - F}(x)$ denotes the remaining degree of node x in $S_n - F$.*

Proof. We prove this lemma by induction on n.

For $n = 4$, $n - 3 = 1$, each node in S_n is labeled as a permutation on $\langle 4 \rangle$. We now consider the situation when the number of missing edges is 0 or 1. Since the star graph is node symmetric, we choose $\mathbf{x} = x_1x_2x_3x_4$ for description. There is an extended star structure $ES(\mathbf{x}; 3)$ as described in the proof of Lemma 5. If there is no missing edges in S_4, then we are done. (See Figure 4.3(a).) If there is one missing edge in S_4, one of three cases happens: 1) if the missing edge is not in the set of edges of the $ES(\mathbf{x}; 3)$ found previously, an $ES(\mathbf{x}; 3)$ at $\mathbf{x}$ certainly exists; 2) if the missing edge is the 2nd, 3rd or 4th edge of $\mathbf{x}$, the degree of $\mathbf{x}$ is 2 and there is an $ES(\mathbf{x}; 2)$ at $\mathbf{x}$; 3) if the missing edge is one of the edges in the original $ES(\mathbf{x}; 3)$ above except the 2nd, 3rd or 4th edge of $\mathbf{x}$, we can alternatively find a new $ES(\mathbf{x}; 3)$ to avoid the missing edge. This alternative extended star of order 3 at $\mathbf{x}$ (as shown in Figure 4.4) contains the node set { $\mathbf{x}$, $v_{11} = x_2x_1x_3x_4$, $v_{12} = x_4x_1x_3x_2$, $v_{13} = x_1x_4x_3x_2$, $v_{14} = x_3x_4x_1x_2$, $v_{21} = x_3x_2x_1x_4$, $v_{22} = x_2x_3x_1x_4$, $v_{23} = x_1x_3x_2x_4$, $v_{24} = x_4x_3x_2x_1$, $v_{31} = x_4x_2x_3x_1$, $v_{32} = x_3x_2x_4x_1$, $v_{33} = x_1x_2x_4x_3$, $v_{34} = x_2x_1x_4x_3$ } and the edge set $\{(\mathbf{x}, v_{k1}), (v_{k1}, v_{k2}), (v_{k2}, v_{k3}), (v_{k3}, v_{k4}) \mid 1 \leq k \leq 3\}$. Thus, there exists an extended star $ES(x; deg_{S_n - F}(x)) \subseteq S_n$ at each node x, for $n = 4$ and $|F| = 0$ or 1.

For induction hypothesis, suppose that for $n \geq 5$ and $|F| \leq n - 4$, there exists an $ES(x; deg_{S_{n-1} - F}(x)) \subseteq S_{n-1}$ at each node $x \in V(S_{n-1})$, where F is the set of missing edges.

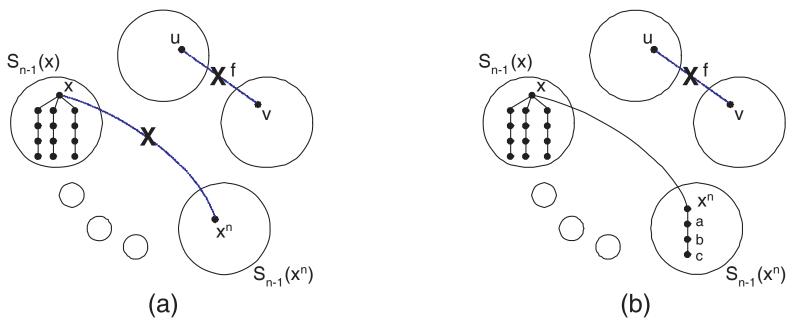

Figure 4.5: illustration for the inductive step in the proof of Lemma 6.

Now we claim that for a set of missing edges F with $|F| \leq n-3$, there exists an $ES(x; deg_{S_n-F}(x)) \subseteq S_n$ at each node $x \in V(S_n)$, for $n \geq 5$. Assume that the number of missing edges is at most $n-3$ in an n-dimensional star S_n, for $n \geq 5$. Let $f = (u, v)$ be an arbitrarily missing edge. Since the star graph is edge symmetric, without lost of generality, we let $v = u^n$. The n-dimensional star S_n can be seen as the composition of n subgraphs S_n^k, for $1 \leq k \leq n$, where S_n^k is a subgraph of S_n induced by the nodes $\mathbf{z}'s$ with $(\mathbf{z})_n = k$. Thus, the number of all missing edges except f in S_n is at most $n-4$. Consider a node $\mathbf{x}$ in S_n, $\mathbf{x}$ is in one of the n induced subgraphs S_n^k, $1 \leq k \leq n$, and each S_n^k is isomorphic to an $(n-1)$-dimensional star S_{n-1}. Let $S_{n-1}(\mathbf{x})$ be the substar which $\mathbf{x}$ belongs to. By the induction hypothesis, there is an extended star $ES(\mathbf{x}; deg_{S_{n-1}(\mathbf{x})-F'}(\mathbf{x})) \subseteq S_{n-1}(\mathbf{x})$ at $\mathbf{x}$, where F' is the set of all missing edges in $S_{n-1}(\mathbf{x})$ and $|F'| \leq n-4$. If the nth edge of $\mathbf{x}$ is missing (Figure 4.5(a)), the degree of $\mathbf{x}$ in $S_{n-1}(\mathbf{x}) - F'$ is equal to the degree of $\mathbf{x}$ in this incomplete star $S_n - F$ with at most $n-3$ missing edges. If the nth edge of $\mathbf{x}$ is not missing (Figure 4.5(b)), $\mathbf{x}$ is adjacent to its nth neighbor, denoted by $\mathbf{x}^n$, through the nth edge. Let $S_{n-1}(\mathbf{x}^n)$ be the subgraph which $\mathbf{x}^n$ belongs to. Since $|F| \leq n-3$, the remaining degree of each node in $S_n - F$ is at least 2. Then $\mathbf{x}^n$ is adjacent to another node $\mathbf{a}$ in $S_{n-1}(\mathbf{x}^n)$, $\mathbf{a}$ is adjacent to another node $\mathbf{b}$ in $S_{n-1}(\mathbf{x}^n)$, and $\mathbf{b}$ is adjacent to another node $\mathbf{c}$ in $S_{n-1}(\mathbf{x}^n)$. As a result, there is an $ES(\mathbf{x}; deg_{S_n-F}(\mathbf{x})) \subseteq S_n$ at $\mathbf{x}$, for $n \geq 5$ and $|F| \leq n-3$. The proof is complete. □

Theorem 7 *Let S_n be an n-dimensional star and $n \geq 4$, and let F be an arbitrary set of missing edges with $|F| \leq n-3$. For each node x in S_n with missing edges F, node x is strongly node-diagnosable; and graph $S_n - F$ is strongly node-diagnosable.*

Proof. By Theorem 3 and Lemma 6, the node diagnosability of each node x in an incomplete n-dimensional star $S_n - F$ is equal to its remaining degree, for $n \geq 4$ and $|F| \leq n - 3$. Thus, every node in $S_n - F$ is strongly node-diagnosable. Consequently, graph $S_n - F$ is strongly node-diagnosable. □

4.3 Some Conclusions for the Strongly Node-Diagnosable Property

At the last part of this chapter, we give some conclusions for the strongly node-diagnosable property. As the previous two sections showed, we observe that both the n-dimensional hypercube and the n-dimensional star are strongly node-diagnosable if there are at most $deg(x) - 2$ missing edges, for any node x in the regular hypercube Q_n or star S_n. The number $deg(x) - 2$ is tight in the sense that the strongly node-diagnosable property can not be guaranteed if there are $deg(x) - 1$ missing edges. We have an example to show that an H_n or an S_n may not keep the strongly node-diagnosable property if there are $deg(x) - 1$ missing edges. Let $\mathbf{x}$ be an arbitrary node in H_n (respectively, S_n). Suppose there are $deg(x) - 1$ missing edges in H_n (respectively, S_n), which are all incident with node $\mathbf{x}$ (as shown in Figure 4.6). Then, the remaining degree of $\mathbf{x}$ in this incomplete hypercube (respectively, star) with missing edges is 1. Let $\mathbf{y}$ be the only node adjacent to $\mathbf{x}$. Let F_1 be the set of nodes $\{\mathbf{y}\} \cup N(\mathbf{y}) - \{\mathbf{x}\}$ with $|F_1| = deg(\mathbf{y})$, and F_2 be the set of nodes $N(\mathbf{y})$ with $|F_2| = deg(\mathbf{y})$. By Lemma 1, (F_1, F_2) is not a distinguishable pair under the comparison diagnosis model, and this incomplete hypercube (respectively, star) with missing edges is not $deg(\mathbf{y})$-diagnosable at $\mathbf{y}$. Since the node diagnosability of $\mathbf{y}$ (which is less than $deg(\mathbf{y})$) does not equal to its degree (which is $deg(\mathbf{y})$) in this incomplete network, node $\mathbf{y}$ is not strongly node-diagnosable anymore. So an incomplete hypercube H_n (respectively, star S_n) with $deg(x)-1$ missing edges can not be guaranteed to be strongly node-diagnosable.

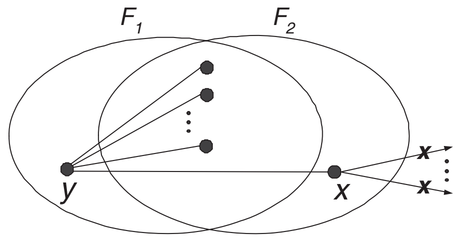

Figure 4.6: an example showing that a $deg(x)$-regular network is not strongly node-diagnosable with $deg(x)-1$ missing edges, for some node x in this regular network.

Chapter 5

A Diagnosis Algorithm for the Comparison Diagnosis Model

Given an extended star structure at a node, we shall present a diagnosis algorithm to determine whether this node is faulty or not for a given syndrome under the comparison model. As stated in Theorem 3, the node diagnosability of a node can be determined by the neighboring nodes (processors) around it. Intuitively, a node's faulty/fault-free status should also be determined by the comparison outputs of the nodes surrounding it, and Theorem 8 provides an algorithm for performing such procedure.

Let $ES(x;n)$ be an extended star at a given node x in $V(G)$, the diagnosing signals are sent back and forth inside $ES(x;n)$. Since there are communication links between x and v_{k1}, v_{k1} and v_{k2}, v_{k2} and v_{k3}, and v_{k3} and v_{k4}, for all $1 \leq k \leq n$, v_{k1}, v_{k2} and v_{k3} can be the comparators of the comparison model. After the comparison test, each comparator has a testing result denoted by 0 (1, respectively) representing the agreement (disagreement, respectively). Given an extended star $ES(x;n)$ at a node x, we define $r_k = (r^1, r^2, r^3)$ to be the testing result of an ordered triple (v_{k1}, v_{k2}, v_{k3}) with respect to $ES(x;n)$, where r^1 is the comparison result of v_{k1} for the two responses from x and v_{k2}, r^2 is the comparison result of v_{k2} for the two responses from v_{k1} and v_{k3}, and r^3 is the comparison result of v_{k3} for the two responses from v_{k2} and v_{k4}. Then, r_k can be in one of the eight different states which are $r(0) = (0,0,0)$, $r(1) = (0,0,1)$, $r(2) = (0,1,0)$, $r(3) = (0,1,1)$, $r(4) = (1,0,0)$, $r(5) = (1,0,1)$, $r(6) = (1,1,0)$ and $r(7) = (1,1,1)$. Let $R(i)$ be the set of the collection of all $r(i)$, for all $0 \leq i \leq 7$. Obviously, the summation of the cardinality of $R(0)$ to $R(7)$ is n, that is, $\sum_{i=0}^{7} |R(i)| = n$.

Let x be a node in a system. Suppose that the degree of x is n and suppose that there is an extended star $ES(x;n)$ at x. Then the node diagnosability of x is n, which means we may not be able to identify all the faulty nodes, if the number of faulty nodes in $ES(x;n)$ is $n+1$ or more. Therefore, we assume that the number of faulty nodes is at most n. Under this assumption, we have an efficient algorithm to determine whether node x is faulty or not.

5.1 The Diagnosis Algorithm

Theorem 8 *Let x be a node with degree n in a system $G(V,E)$. Suppose that there is an extended star $ES(x;n) \subseteq G$ at x. Define $r_k = (r^1, r^2, r^3)$ to be the testing result of (v_{k1}, v_{k2}, v_{k3}) with respect to $ES(x;n)$. Then, r_k can be in one of the eight states (as illustrated in Figure 5.1):*

$r(0) = (0,0,0)$, $r(1) = (0,0,1)$, $r(2) = (0,1,0)$, $r(3) = (0,1,1)$, $r(4) = (1,0,0)$, $r(5) = (1,0,1)$, $r(6) = (1,1,0)$, and $r(7) = (1,1,1)$.

Let $R(i)$ be the set of the collection of all $r(i)$, and $|R(i)|$ be the cardinality of $R(i)$. Then, under the assumption that the number of faulty nodes is at most n, *i) x is fault-free, if $|R(0)| \geq |R(4)|$; or,*

ii) x is faulty, if $|R(0)| < |R(4)|$.

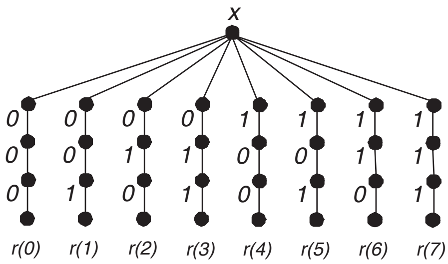

Figure 5.1: eight different output states for Theorem 8.

Proof. Let $l_k = (v_{k1}, v_{k2}, v_{k3}, v_{k4})$ be an ordered quadruple, $1 \leq k \leq n$, with respect to $ES(x; n)$. We prove the first part of this theorem by contradiction. Suppose that the number of faulty nodes in $ES(x; n)$ is at most n and suppose that x is faulty, the counting of all the other faulty nodes is as follows:

For those l_k with result $r(0)$, there are at least 3 faulty nodes which are v_{k1}, v_{k2} and v_{k3}.

For those l_k with result $r(1)$, there are at least 2 faulty nodes which are v_{k1} and v_{k2}.

For those l_k with result $r(2)$, there is at least 1 faulty node which is v_{k1}.

For those l_k with result $r(3)$, there are at least 2 faulty nodes which are v_{k1} and one of v_{k2}, v_{k3} and v_{k4} since the output of v_{k3} is disagreement.

For those l_k with result $r(4)$, the number of faulty nodes is uncertain.

For those l_k with result $r(5)$, there is at least 1 faulty node which is one of v_{k2}, v_{k3} and v_{k4} since the output of v_{k3} is disagreement.

For those l_k with result $r(6)$, there is at least 1 faulty node which is one of v_{k1}, v_{k2} and v_{k3} since the output of v_{k2} is disagreement.

For those l_k with result $r(7)$, there is at least 1 faulty node which is one of v_{k2}, v_{k3} and v_{k4} since the output of v_{k3} is disagreement.

Thus, the number of faulty nodes is at least

$1 + 3|R(0)| + 2|R(1)| + |R(2)| + 2|R(3)| + |R(5)| + |R(6)| + |R(7)| = \Sigma_{i=0}^{7}|R(i)| + (1 + 2|R(0)| + |R(1)| + |R(3)| - |R(4)|)$.

By the assumption that $|R(0)| \geq |R(4)|$, the number of faulty nodes is strictly more than $\Sigma_{i=0}^{7}|R(i)|$ which is equal to n. This contradicts to the assumption that the number of faulty nodes in $ES(x;n)$ is at most n. Therefore, x is fault-free.

Now, we prove the second part of this theorem. Suppose that the number of faulty nodes in $ES(x;n)$ is at most n and suppose that x is fault-free, the counting of all the other faulty nodes is as follows:

For those l_k with result $r(0)$, the number of faulty nodes is uncertain.

For those l_k with result $r(1)$, there is at least 1 faulty node which is one of v_{k2}, v_{k3} and v_{k4} since the output of v_{k3} is disagreement.

For those l_k with result $r(2)$, there is at least 1 faulty node which is one of v_{k1}, v_{k2} and v_{k3} since the output of v_{k2} is disagreement.

For those l_k with result $r(3)$, there is at least 1 faulty node which is one of v_{k1}, v_{k2} and v_{k3} since the output of v_{k2} is disagreement.

For those l_k with result $r(4)$, there are at least 2 faulty nodes for the reasons that: *i*) if v_{k1} is faulty, v_{k2} must be faulty since the comparison result of v_{k2} is wrong; or, *ii*) if v_{k1} is fault-free, v_{k2} must be faulty and v_{k3} must be faulty too.

For those l_k with result $r(5)$, there is at least 1 faulty node which is one of v_{k1}, v_{k2} since the output of v_{k1} is disagreement.

For those l_k with result $r(6)$, there is at least 1 faulty node which is one of v_{k1}, v_{k2} since the output of v_{k1} is disagreement.

For those l_k with result $r(7)$, there is at least 1 faulty node which is one of v_{k1}, v_{k2} since the output of v_{k1} is disagreement.

Thus, the number of faulty nodes is at least

$$|R(1)|+|R(2)|+|R(3)|+2|R(4)|+|R(5)|+|R(6)|+|R(7)| = \Sigma_{i=0}^{7}|R(i)|+(|R(4)|-|R(0)|).$$

By the assumption that $|R(0)| < |R(4)|$, the number of faulty nodes is larger than $\Sigma_{n=i}^{7}|R(i)|$ which is equal to n. This contradicts to the assumption that the number of faulty nodes in $ES(x;n)$ is at most n. Therefore, x is faulty. □

Roughly speaking, the collections of testing results $R(0)$ and $R(4)$, with respect to the extended star $ES(x;n)$ found at node x, dominate the faulty/fault-free status of x. We can determine the faulty of fault-free status of a node by just comparing the number of the testing results $r(0)$'s and $r(4)$'s on an arbitrary extended star we found.

5.2 Analysis of the Time Complexity for the Diagnosis Algorithm

We now measure the time complexity to diagnose all the faulty nodes in a system. For most of the practical systems with N nodes, the degree of each node is in the order of $\log N$. For example, the n-dimensional Hypercube Q_n has $N = 2^n$ nodes and the degree of each node is n, $n = \log N$; the n-dimensional Star S_n has $N = n!$ nodes and the degree of each node is $n - 1 = O(n) = O(\frac{\log N}{\log n}) = O(\frac{\log N}{\log \log N})$. We assume that a testing result of each comparator for each pair of distinct neighbors with which it can communicate directly is stored in a syndrome table. Given an extended star structure $ES(x; n)$ at a node x, assume the time for looking up the testing result of a comparator in the syndrome table is constant c. Then, the time needed for determining the faulty or fault-free status of node x is $3c \log N = O(\log N)$. Consequently, the total time for diagnosing all the faulty nodes is $O(N \log N)$.

As a result, for most practical multiprocessor systems, especially some well-known symmetric and regular topologies like hypercube systems, the time for self diagnosis is $O(N \log N)$, where N is the total number of processors in it. On the other hand, the presented diagnosis algorithm is not restricted to symmetric systems only. We can apply such method to diagnose a system node by node, and consequently to diagnose the whole system. In general, the time complexity is $O(N\Delta)$, where Δ is the maximum degree of a node in this system.

The time complexity $O(N \log N)$ obtained here is based on the symmetry of most recently practical multiprocessor systems. Applying the traditional approach by Sengupta and Dahbura [30] results in an initiate result of time complexity $O(N^5)$. However, under some constraints like symmetry or regularity of the systems, using the classical approach may result in a better computational complexity than $O(N^5)$, especially on some special cases of hypercubes or other well-known topologies. A recent paper can be referred on this issue; Yang and Tang [33] address the fault identification of diagnosable multiprocessor systems under the MM* comparison model, and present an $O(N\Delta^3\delta)$ time diagnosis algorithm for an N-node system, where Δ and δ are the maximum and minimum degrees of a node, respectively.

Chapter 6

Conclusions and Discussions

The issue of identifying all the faulty processors is important in the design of interconnection networks or multiprocessor systems, which is implementable in very large scale integration (VLSI). The process of identifying all the faulty processors is called diagnosis of a system. Under the asymmetric comparison diagnosis model, each processor acts as a comparator to test each pair of adjacent two processors. Further, Sengupta and Dahbura [30] proposed a polynomial time algorithm with time complexity $O(N^5)$ to diagnose a system with total number N of processors. In some circumstances, it is not necessary to judge the status of all processors but several ones in some substructure of the system such as a ring structure or a path structure.

In this dissertation, we proposed a novel idea on system diagnosis called node diagnosability. Opposite to that of the traditional diagnosability, the concept of node diagnosability put more focus on a single processor, and require only identifying the status of this particular processor correctly. Estimating the node diagnosability of each processor in a system also provides a new viewpoint for checking the diagnosability of the whole system. Under the comparison diagnosis model, we proposed a sufficient condition to determine a given processor's node diagnosability, and an efficient algorithm to determine whether a processor is faulty based on the local syndrome of a given extended star structure. All these concepts can be applied to many well-known interconnection networks. For most practical multiprocessor systems, the number of links connecting to each processor is in the order of $\log N$, where N is the total number of processors. The time complexity of our algorithm to diagnose a given processor is $O(\log N)$ and to diagnose all the faulty processors in a system is $O(N \log N)$.

Finally, we propose a research topic worth studying at the end of this paper, which is the issue of the underlying assumptions consistent with the comparison diagnosis model. As referred to those assumptions, all faults are permanent, and the comparison output performed by a faulty processor is unreliable. However, in future technologies it is likely that many faults will be transient or non-permanent, making fixed diagnosis strategies more complex, and violating the comparison diagnosis strategy we are based on. Furthermore, a faulty processor may be able to perform self-diagnosis and identify itself as faulty. So violating each assumption of the comparison model may lead to a different situation, and each of the modifications will be an interesting problem for further research.

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