國立交通大學

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碩 士 論 文

在多孔介質中兩相不可壓縮不相容的流體的

局部質量守恆計算法

A locally conservative scheme for two-phase incompressible immiscible flows in porous media

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摘　　要

應用於本論文的水流問題的數學模型可分為兩部分。一部分就是壓力方程式，另一部分就是 saturation 方程式。其中 saturation 方程式又分為 transport 和 diffusion 兩部分。在此論文中我們主要著重於解 transport 的部分。在此文中，我們模擬一個長兩百五十六公尺、寬兩百五十六公尺的一個儲油槽。Locally conservative Eulerian-Lagrangian methods (LCELM)是一個有效率的數值方法並且發展來改善在計算 transport 方程式中水流質量守恆的部分。從數值模擬的結果，我們可以了解時間變化與流體狀況的關係。

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ABSTRACT

The mathematical model of the waterflood problem which is applied in this paper can be divided into two sections. One is the pressure equation and the other is the saturation equation. And the saturation equation also can be partitioned into the transport stage saturation and the diffusive stage saturation. However, we will pay more attention to solve the transport stage saturation in this research. Here we construct a 256×256 meters reservoir system for simulation. An efficient numerical method, locally conservative Eulerian-Lagrangian methods (LCELM), is developed to compute the transport equation to improve the conservation of waterflood. From the results of the numerical simulations, we can realize the relation between temporal variation and the flow condition.

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此外，要感謝我的碩士班的同學，讓我能順利完成碩士的課業，同時也讓我學習到一些除了課業以外的東西。

Contents

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1 Introduction

In this research, we develop an efficient method for the two-phase incompressible immiscible flows in porous media. We will consider the waterflooding problem in this paper. The methods in this paper should also be of value in the numerical simulation of recently developed double porosity models for swelling clay soils [1]. The numerical method combines hybridized mixed finite elements, and a new version of the modified method of characteristics, a sophisticated operator-splitting procedure for separating the transport part from the diffusion part of the saturation equation.

Our approach is to write the governing equations in terms of a global pressure [2, 3, 4]; this leads to coupled pair of equations, an elliptic equation for the global pressure and a parabolic equation for the water saturation. The primary operator splitting separates the computation of the global pressure from the saturation. This permits the use of different time steps for pressure and saturation. The second operator splitting separates the effect of transport from diffusion in the saturation calculation; this allows the use of smaller steps for the transport than for the diffusion.

In section two, we present the mathematical model of the waterflood problem. Then the temporal discretization and the spatial discretization are shown in section three and section four, respectively. And we present an overview process of numerical simulation in section three. Furthermore, the section five shows how to calculate the pressure and the saturation. And the numerical results are displayed in section six.

2 The waterflood problem

We discuss the equations for two-phase, incompressible, immiscible flow in porous media. By mass conservation, we can get the equations form [4]

$$\Phi(x)\frac{\partial S(x,t)}{\partial t} - div(K\Lambda\Lambda_w(S)\nabla_x\Psi_w(x,t)) = q_{ext,w}(x,S), \tag{1}$$

$$-\Phi(x)\frac{\partial S(x,t)}{\partial t} - div(K\Lambda\Lambda_o(S)\nabla_x\Psi_o(x,t)) = q_{ext,o}(x,S), \tag{2}$$

$$P_c(S) = \Psi_c(x,t) + (\rho_o - \rho_w)gz, \Psi_c(x,t) = \Psi_o(x,t) - \Psi_w(x,t), \tag{3}$$

for $x \in \Omega$, where Φ is the porosity, S is the water saturation, and K is the absolute permeability tensor of the media. The total mobility is defined as

$$\Lambda = \frac{K_{rw}(S)}{\mu_w} + \frac{K_{ro}(S)}{\mu_o},$$

where K_{rw} and K_{ro} are the relative permeabilities of the water and oil phases, and μ_w and μ_o are the viscosity of the water and oil phases. The phase mobilities are

$$\Lambda\Lambda_w(S) = \frac{K_{rw}(S)}{\mu_w}, \qquad \Lambda\Lambda_o(S) = \frac{K_{ro}(S)}{\mu_o},$$

Ψ_α is the α-phase potential; $P_c = P_c(S) = P_o - P_w$ is the capillary pressure (note that $P_c' < 0$); ρ_α is the (constant) density of the α-phase; g is the gravitational constant; and z is the depth. Pressures are related to potentials by the relations

$$\Psi_\alpha = P_\alpha - \rho_\alpha gz, \qquad \alpha = w, o$$

The relative permeability functions $K_{r\alpha}$ and the capillary pressure P_c are assumed in this paper to be independent of x . If the global pressure [2, 3, 4] given by

$$P \equiv \frac{1}{2}(P_o + P_w + \int_0^{P_c}(\Lambda_o(P_c^{-1}(\zeta)) - \Lambda_w(P_c^{-1}(\zeta)))d\zeta) \tag{4}$$

is introduced, equations(1)-(3) can be written as a uniformly elliptic equation for global pressure and a convection-dominated parabolic equation for water saturation as follows :

$$U = -K(x)\Lambda(S)(\nabla P - (\Lambda_w \rho_w + \Lambda_o \rho_o) g \nabla z), \tag{5}$$

$$divU = q, \tag{6}$$

$$\Phi \frac{\partial S}{\partial t} + div(U\Lambda_w) + div((K\Lambda\Lambda_w\Lambda_o)(\nabla P_c + (\rho_w - \rho_o)g\nabla z)) = q^+ - \Lambda_w q^-, \tag{7}$$

where $q = q_{ext,w} + q_{ext,o}$, $q^+ = max(q, 0)$ and $q^- = max(-q, 0)$. The right-hand side of equation(7) results from assuming that only water is injected and, at a production point, the flow splits according to mobility between water and oil.

We shall assume" no flow "boundary conditions on $\partial\Omega$:

$$U \cdot \vec{n}|_{\partial\Omega} = 0, \tag{8}$$

$$K\Lambda\Lambda_w\Lambda_o(\nabla P_c - (\rho_w + \rho_o)g\nabla z) \cdot \vec{n}|_{\partial\Omega} = 0, \tag{9}$$

Where $\vec{n}$ is the unit outward normal vector to $\partial\Omega$. Compatibility to the incompressibility of the fluids requires that

$$\int_\Omega q dx = 0. \tag{10}$$

The initial condition of the system is determined by the single relation.

$$S(x, 0) = S_{init}(x), \quad for\ x \in \Omega \tag{11}$$

It will be convenient in the discussion of MMOC procedures to the saturation equation (7) in nondivergence form; a short calculation show that

$$\Phi \frac{\partial S}{\partial t} + \Lambda_w'(S) U \cdot \nabla S - div(KD(x, S)(\nabla S + \frac{(\rho_w - \rho_o)g\nabla z}{P_c'})) = (1 - \Lambda_w)q^+ \tag{12}$$

where

$$D(x, S) = -(\Lambda\Lambda_w\Lambda_o P_c')(S)$$

3 Discretization in temporal domain

We shall employ a time-discretization procedure based on operator splitting concepts. Assume that

$$\Delta t_p = i_1 \Delta t_s, \qquad \Delta t_s = i_2 \Delta t_{st}$$

where i_1 and i_2 are positive integers. Let $t^m = m\Delta t_p$ and give a function evaluated at time t^m by f^m . Similarly, let $t_n = n\Delta t_s$ and $t_{n,k} = t_n + k\Delta t_{st}$ and let $f_n = f(t_n)$ and $f_{n,k} = f(t_{n,k})$, respectively.

The pressure will be calculated at t^m , $m = 0, 1, 2 \cdots$. Δt_s is the time step for the diffusive stage saturation calculation. Δt_{st} is the time step for the microstep for the transport stage saturation calculation.

The time steps for the pressure and saturation variables will be allowed to be different. The numerical solution is obtained sequentially. Saturation solutions are computed for increasing values of the discretized time followed by the solution of the global pressure system after several saturation steps.

(1) At the beginning, we can use the initial condition $S^0 = S_{init}$ to determine $\{P^0, U^0\}$ by solving the pressure equation (in mixed form)

$$U^0 = -K(x)\Lambda(S^0)(\nabla P^0 - (\Lambda_w(S^0)\rho_w + \Lambda_o(S^0)\rho_o)g\nabla z)$$

$$divU^0 = q^0$$

(2) Let $E_1 U$ denote the extrapolation of U given by

$$(E_1 U)(t) = \begin{cases} U^0, & 0 < t \le t^1, \\ \frac{t-t^{m-1}}{\Delta t_p} U^m - \frac{t-t^m}{\Delta t_p} U^{m-1}, & t^m < t \le t^{m+1}. \end{cases} \tag{13}$$

For $t^m < t_{n+1} \leq t^{m+1}$, $m \geq 0$, solve the saturation equation (now expressed in nondivergence, mixed form)

$$V = -KD(x,S)(\nabla S + \frac{(\rho_w - \rho_o)g\nabla z}{P_c'}), \tag{14}$$

$$\Phi\frac{\partial S}{\partial t} + \Lambda_w'(S)(E_1U)\cdot\nabla S + divV = (1-\Lambda_w)q^+, \tag{15}$$

$$S(x,t^m) = S^m(x), \tag{16}$$

In (3.4), $S^m(x)$ denotes the final values from the $[t^{m-1}, t^m]$. $S^0(x)$ is the initial saturation.

(3) For $t = t^{m+1}$, solve for U^{m+1} and P^{m+1}

$$U^{m+1} = -K(x)\Lambda(S^{m+1})(\nabla P^{m+1} - (\Lambda_w(S^{m+1})\rho_w + \Lambda_o(S^{m+1})\rho_o)g\nabla z)$$

$$divU^{m+1} = q^{m+1}$$

(4) If $t^{m+1} \leq T$, go to (2); otherwise, stop.

In (2) , the algorithm for this is as follows:

(i) Let t_{n_1} and assume known $\{P, U, S\}$ for $t \leq t_{n_1}$.

(ii) For $n = n_1, \cdots\cdots, n_2 = n_1 + i_1 - 1$

(a) For $k = 0, \cdots, i_2 - 1$, compute the transport over $[t_{n,k}, t_{n,k+1}]$ by solving the system

$$\Phi\frac{\partial \zeta_{n,k}}{\partial t} + \Lambda_w'(\zeta_{n,k})(E_1U)\cdot\nabla\zeta_{n,k} = (1-\Lambda_w)q^+, \qquad x \in \Omega \tag{17}$$

$$(E_1U)\cdot\overrightarrow{n} = 0, \qquad x \in \partial\Omega \tag{18}$$

$$\zeta_{n,k}(x,t_{n,k}) = \begin{cases} S_n(x), & k = 0, \\ \zeta_{n,k-1}(x,t_{n,k}), & k = 1, \ldots i_2 - 1. \end{cases} \tag{19}$$

(b) Set $\overline{S}_n(x) = \zeta_{n,i_2-1}(x, t_{n,i_2}) = \zeta_{n,i_2-1}(x, t_{n+1})$.

(c) Compute the diffusive effects over $[t_n, t_{n+1}]$ by solving

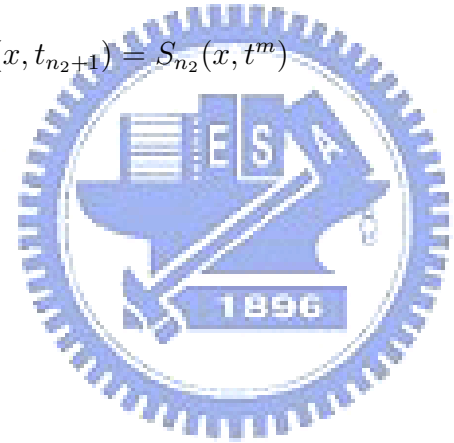
$$V = -KD(x,S)(\nabla S + \frac{(\rho_w - \rho_o)g\nabla z}{P_c'}), \qquad x \in \Omega \tag{20}$$

$$\Phi\frac{\partial S}{\partial t} + divV = 0, \qquad x \in \Omega \tag{21}$$

$$V \cdot \overrightarrow{n} = 0, \qquad x \in \partial\Omega \tag{22}$$

$$S(x, t_n) = \overline{S}_n(x), \qquad x \in \Omega \tag{23}$$

(iii) Set $S^{m+1}(x) = \overline{S}_{n_2}(x, t_{n_2+1}) = S_{n_2}(x, t^m)$



4 Discretization in spatial domain

We get the discretization from [9].

Let us return to the differential system (5)-(7) and restate it completely in mixed form by introducing a saturation flux variable in addition to the volumetric flow rate variable U. Then the equations take form

$$U = -K(x)\Lambda(S)(\nabla P - (\Lambda_w(S)\rho_w + \Lambda_o(S)\rho_o)g\nabla z), \tag{24}$$

$$divU = q, \tag{25}$$

$$V = -KD(x,S)(\nabla S + \frac{(\rho_w - \rho_o)g\nabla z}{P_c'}), \tag{26}$$

$$\Phi\frac{\partial S}{\partial t} + div(\Lambda_w U) + divV = q^+ - \Lambda_w q^-. \tag{27}$$

It should be noted that gravity terms enter into both flux variables. The no-flow boundary condition are expressed by

$$U \cdot \overrightarrow{n} = V \cdot \overrightarrow{n} = 0, \qquad x \in \partial\Omega. \tag{28}$$

We wish to approximate each pair, $\{U, P\}$ and $\{V, S\}$, by mixed finite elements. It is completely feasible and can be computationally advantageous [5, 6] to define the finite element methods for the pressure equation and saturation equation over different partitions of the domain, but in order to simplify our presentation we shall restrict our considerations to the use of the same partition for the two sets of variables. Let

$$\Omega = [0, LX] \times [0, LY]$$

and set $H = \{HX, HY\}$, where $HX = LX/NX$ and $HY = LY/NY$. Then let $X_i = iHX$ and $Y_j = jHY$, and define the elements of the partition $\tau = \tau(H) =$

$\{B_{ij} : \ i = 1, 2, \cdots, NX, \quad j = 1, 2, \cdots, NY\}$ by $B_{ij} = [X_{i-1}, X_i] \times [Y_{j-1}, Y_j]$; will serve for both the pressure and the saturation equations.

Let

$$\eta = \eta(H) = \{ \ \overrightarrow{\nu} \in H(div, \Omega) : \ \overrightarrow{\nu}|_{B_{ij}} \in P_{1,0} \times P_{0,1} \ \ and \ \ \overrightarrow{\nu} \cdot \overrightarrow{n} = 0 \ on \ \partial\Omega\}$$

$$W = W(H) = \{ \ \omega : \ \omega|_{B_{ij}} \in P_0\} \subset L^2(\Omega)$$

Where P_k denotes the set of polynomials of total degree k and $P_{k,l}$ denotes the tensor product of polynomials of degree k in x by those of degree l in y . Then set

$$M = M(H) = \eta \times W;$$

i.e., the lowest index Raviart-Thomas mixed finite element space over the partition τ .

We shall seek an approximate solution to the system (24)-(27) such that

(1) $\{U^m, P^m\} \in M, \qquad m = 0, 1, 2, \cdots$.

(2) $\{V_n, S_n\} \in M, \qquad n = 0, 1, 2, \cdots$.

In our computation, we let $LX = 256$, $LY = 256$, $NX = 256$ and $NY = 256$. Then we can get

$$\Omega = [0, 256] \times [0, 256]$$

and set $H = \{HX, HY\}$, where $HX = LX/NX = 1$ and $HY = LY/NY = 1$. We let $X_i = i$ and $Y_j = j$, and define the elements of the partition $\tau = \tau(H) = \{B_{ij} : \ i = 1, 2, \cdots, 256, \quad j = 1, 2, \cdots, 256\}$ by $B_{ij} = [X_{i-1}, X_i] \times [Y_{j-1}, Y_j]$;

5 Algorithm

5.1 The pressure equation

By [7], we can get the method of solving the pressure equation.

In each block B_{ij} , U_β, $\beta = L, R, B, T$ is the out normal component of the total flux across the edges of the block. H is the side length of block. The equations to be solved for P^m and U^m are :

$$U_L^m + U_R^m + U_B^m + U_T^m = q^m H,$$

$$(1 + \chi_{g\beta}\xi_{g\beta}^m)U_\beta^m + \xi_{g\beta}^m P^m = -\chi_{g\beta}\xi_{g\beta}^m \widetilde{U_{\beta'}^m} - \xi_{g\beta}^m \widetilde{\ell_{\beta'}^m}$$

$$+ K\Lambda_\beta^m(\Lambda_w^m\rho_w + \Lambda_o^m\rho_o)_\beta g\nabla z \cdot n_\beta,$$

where $\beta = L, R, B, T$, $\xi_{g\beta}^m = 2K\Lambda(\ell_{s\beta}^m)/H$, $\Lambda_\alpha^m = \Lambda_\alpha(\ell_{s\beta}^m)$, $\alpha = w, o$, and $\ell_{s\beta} = \frac{KS+\widetilde{K}\widetilde{S}}{K+\widetilde{K}}$, and $\widetilde{\ell_{g\beta'}} = \frac{KP+\widetilde{K}_\beta\widetilde{P}_\beta}{K+\widetilde{K}_\beta} + \frac{H}{2}g\frac{K-\widetilde{K}_\beta}{K+\widetilde{K}_\beta}(\Lambda_w\rho_w + \Lambda_o\rho_o)(\ell_{s\beta})\nabla z \cdot n_\beta$.

In the above, $\widetilde{P}_\beta$ and $\widetilde{K}_\beta$ indicate the value of P and K in the element across the $\beta-$interface, respectively.

5.2 The transport equation

The LCELM is the variational type of the MMOC. We shall explain the MMOC briefly.

5.2.1 MMOC procedure

The MMOC procedure for the waterflood problem is based on introducing a characteristic derivative for the transport part of the saturation equation written in non-divergence form.

$$\Theta(x, S, U) = \sqrt{\Phi(x)^2 + |\Lambda_w'(S)U|^2}, \tag{29}$$

$$\Theta\frac{\partial}{\partial\varphi} = \Phi\frac{\partial}{\partial t} + \Lambda_w'(S)U \cdot \nabla \tag{30}$$

(Note that the characteristic direction φ depends on x , the saturation, and the fluid velocity.) Thus, equation(15) can be written as

$$\Theta\frac{\partial S}{\partial\varphi} + divV = (1 - \Lambda_w)q^+$$

and equation(17) can be written as

$$\Theta\frac{\partial\zeta_{n,k}}{\partial\varphi} = (1 - \Lambda_w(\zeta_{n,k}))q^+, \qquad x \in \Omega, \quad t_{n,k} < t \le t_{n,k+1} \tag{31}$$

We compute the transport microstep by solving equation(31) with the initial values given by equation(19). As seen in Figure 1, the fundamental concept in the MMOC is the discretization of the characteristic derivative by backwards differencing along the tangent to the characteristic through the point $(x, t_{n,k+1})$ back to the time level $t_{n,k}$ for whatever $x-$point arise in the quadratures used in the finite element scheme. If we ignore for the moment sources and sink and the boundary, the transported values over a micro-step would be defined by

$$\overline{x_{n,k}}(x) = x - \frac{\Lambda_w'(\zeta_{n,k})(E_1U)(x, t_{n,k+1})\Delta t_{st}}{\Phi} \tag{32}$$

$$\zeta_{n,k+1}(x) = \zeta_{n,k}(\overline{x_{n,k}}(x)) \tag{33}$$

The only reasonable criterion [8] for conservation of mass globally is that the map (32) have Jacobian identically one.

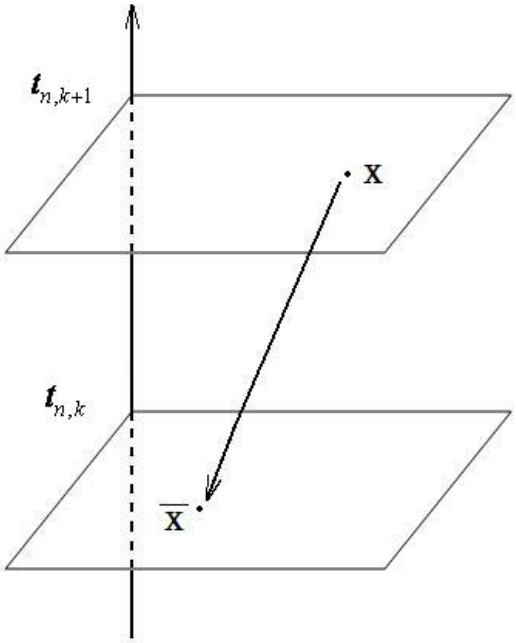


Figure 1: The point $(x, t_{n,k+1})$ back to the time level $t_{n,k}$.

5.2.2 LCELM procedure

Recall that the saturation equation can be written in divergence form as

$$\nabla_{t,x} \cdot \begin{pmatrix} \Phi S \\ \\ \Lambda_w U \end{pmatrix} + divV = q^+ - \Lambda_w q^- \tag{34}$$

Then the transport equation can be written as

$$\nabla_{t,x} \cdot \begin{pmatrix} \Phi S \\ \\ \Lambda_w U \end{pmatrix} = q^+ - \Lambda_w q^- \tag{35}$$

followed by the diffusive part given by

$$\Phi \frac{\partial S}{\partial t} + divV = 0 \tag{36}$$

Let $\Gamma = \Omega \times [t_{n,k}, t_{n,k+1}]$. Let G be a reasonable shaped, simply-connected subset of Ω , and define a subset $C = C_{n,k}(G)$ of Γ as follows. For each $x \in \partial G$, construct the solution

$y(x;t)$ of the final value problem

$$\frac{dy}{dt} = \frac{\Lambda_w U}{\Phi S}, \qquad t_{n,k+1} > t \geq t_{n,k} \tag{37}$$

$$y(x; t_{n,k+1}) = x \tag{38}$$

and set

$$\overline{x_{n,k}}(x) = y(x; t_{n,k}) \tag{39}$$

Then, let $\overline{G} = \overline{G_{n,k}}$ be the interior of the set $\{\overline{x_{n,k}}(x) \ : \ x \in \partial G\}$, and let C be the tube determined by G ,$\overline{G}$ and the integer curves (37)-(38); see Figure 2 and Figure 3 for an example C in a single space variable setting. (We let Δt_{st} be sufficiently small. Then the map $x \ \rightarrow \overline{x_{n,k}}$ is one-to-one, so that this construction can be carried out.) Now, denote the outward normal to ∂C by $\sigma(x,t)$ and note that it is orthogonal to the vector $\begin{pmatrix} \Phi S \\ \Lambda_w U \end{pmatrix}$ on the lateral surface of C . Then, integrate (35) over C :

$$\int_C \nabla_{t,x} \cdot \begin{pmatrix} \Phi S \\ \Lambda_w U \end{pmatrix} dxdt = \int_{\partial C} \begin{pmatrix} \Phi S \\ \Lambda_w U \end{pmatrix} \cdot \sigma dA$$

$$= \int_G \Phi S(t_{n,k+1}, x)\ dx \ - \ \int_{\overline{G}} \Phi S(t_{n,k}, x)\ dx \tag{40}$$

$$= \int_C (q^+ - \Lambda_w q^-)\ dxdt$$

Thus, mass is conserved locally in the transport step, as defined in equation(35) or equations(17)-(19) above, if

$$\int_G \Phi S(t_{n,k+1}, x)\ dx \ = \ \int_{\overline{G}} \Phi S(t_{n,k}, x)\ dx \ + \ \int_C (q^+ - \Lambda_w q^-)\ dxdt$$

The no-flow boundary condition is handled in a natural way in equation(40), since the integral curves (32)-(34) do not exit Ω in this case. In fact, if $x \in \partial\Omega$, then

the integral curve remain in $\partial\Omega$ and C has a portion of its lateral surface contained in $\Gamma = \Omega \times [t_{n,k}, t_{n,k+1}]$. Hence, no special cares arise for subsets G close to the boundary for these boundary conditions.

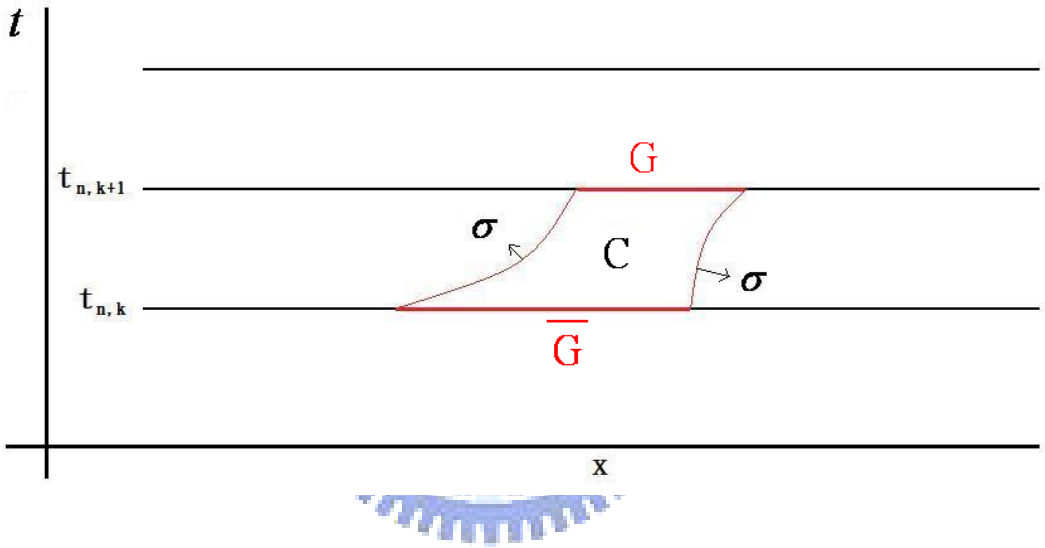


Figure 2: The space-time domain C.

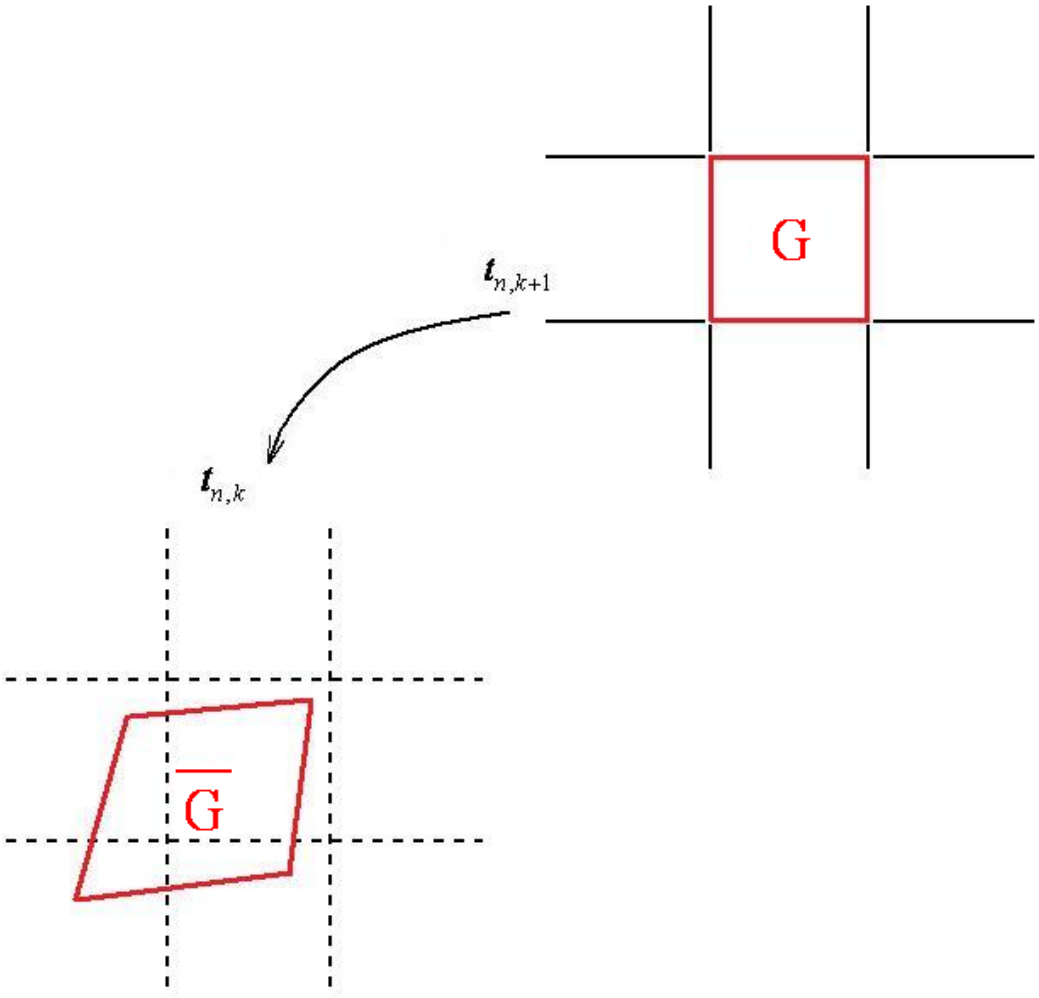


Figure 3: The area $(G, t_{n,k+1})$ back to the time level $t_{n,k}$.

5.3 Diffusive fractional step

We shall apply backward differencing in time over $[\ t_n, t_{n+1}\ ]$

$$V_{n+1} = -KD(\overline{S_n})(\nabla S_{n+1} + \frac{(\rho_w - \rho_o)g\nabla z}{P_c'(\overline{S_n})}),$$

$$\Phi\frac{S_{n+1} - \overline{S_n}}{\Delta t_s} + divV_{n+1} = 0,$$

with the no-flow boundary condition

$$V_{n+1} \cdot \overrightarrow{n} = 0.$$

Thus, the mixed finite element equations take the form

$$(\frac{1}{KD(\overline{S_n})}V_{n+1}\ ,\ \overrightarrow{n_1}) - (S_{n+1}\ ,\ div\overrightarrow{n_1})$$

$$= -(\frac{1}{P_c'(\overline{S_n})}(\rho_w - \rho_o)g\nabla z\ ,\ \overrightarrow{n_1}) \qquad ,\overrightarrow{n_1} \in \eta$$

$$(\Phi\frac{S_{n+1} - \overline{S_n}}{\Delta t_s}\ ,\ n_2) + (divV_{n+1}\ ,\ n_2) = 0 \qquad ,n_2 \in W.$$

6 Numerical results

The following data and functions are held fixed for the computational results:

Viscosity	$\mu_w = 0.5 \ \ cP$	$\mu_o = 10 \ \ cP$
Density	$\rho_w = 1 \ \ g/cm^3$	$\rho_o = 0.7 \ \ g/cm^3$
Porosity	$\Phi = 0.2$	
Residual saturations	$S_{rw} = 0.2$	$S_{ro} = 0.15$
Absolute permeability	$K = 6 \ \ mdarcy$	
Relative permeability	$K_{rw}(S) = \frac{(S-S_{rw})^2}{(1-S_{rw})^2}$	
	$K_{ro}(S) = \left(1 - \frac{S}{1-S_{ro}}\right)^2$	
Capillary pressure	$P_c(S) = \eta\left(\frac{1}{(S-S_{rw})^2} - \frac{\zeta}{(1-S^2}\right)$	
	$\zeta = S_{ro}^2(1 - S_{ro} - S_{rw})^{-2}$	
	$\eta = 3000 \ \ dynes/cm^2$	

In this study, the geometric domain is 256×256 meters and the grid size of the model is 1×1 meters. In order to solve the transport equation, we use two kinds of partitions as shown in Figure 4 and Figure 5 to compute the injection and other sections, respectively. The red line in Figure 4 and Figure 5 represents the computational domain. In the following figures, water is injected at the lower right corner at a uniform rate and a mixture of water and oil produced at the top left corner.

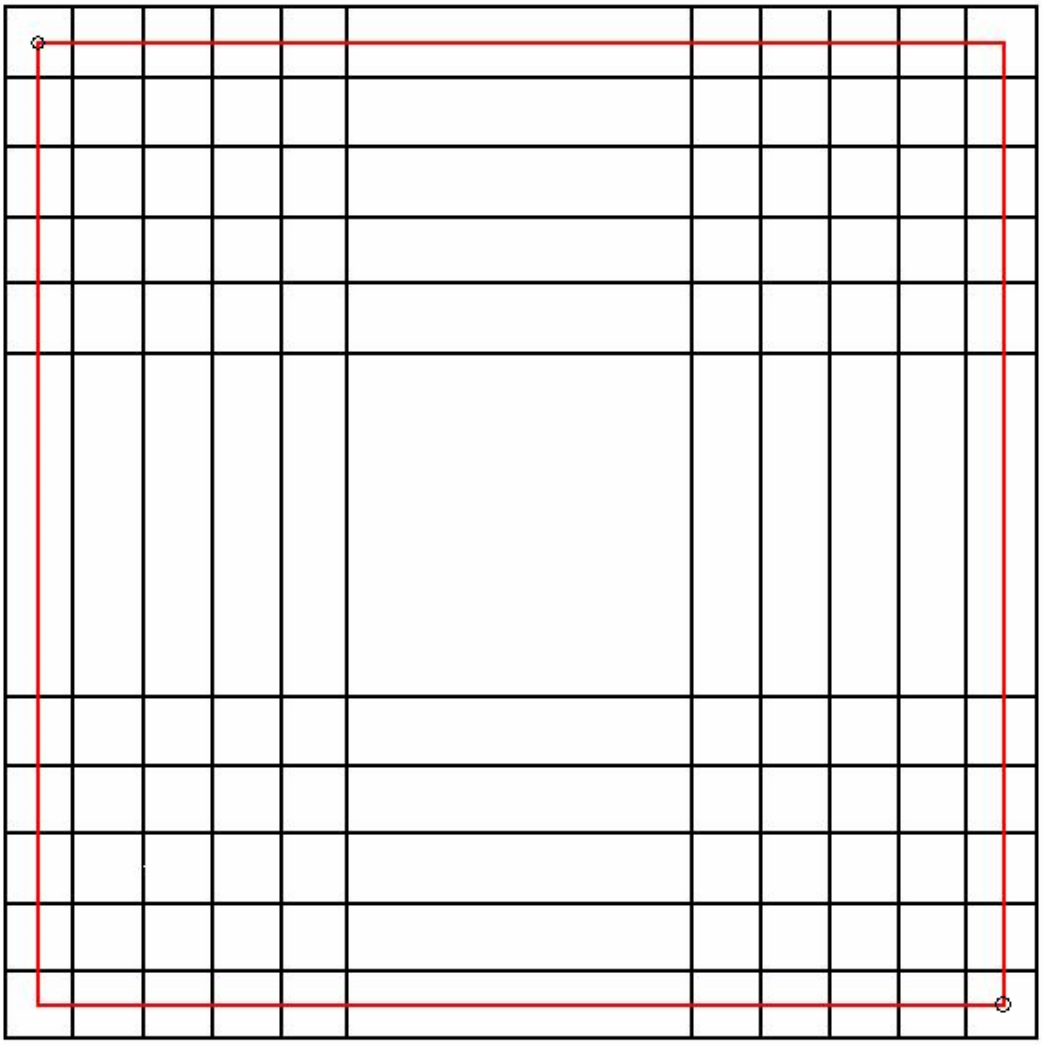


Figure 4: The kind of partition (black line) is used to calculate the whole domain except the injection.

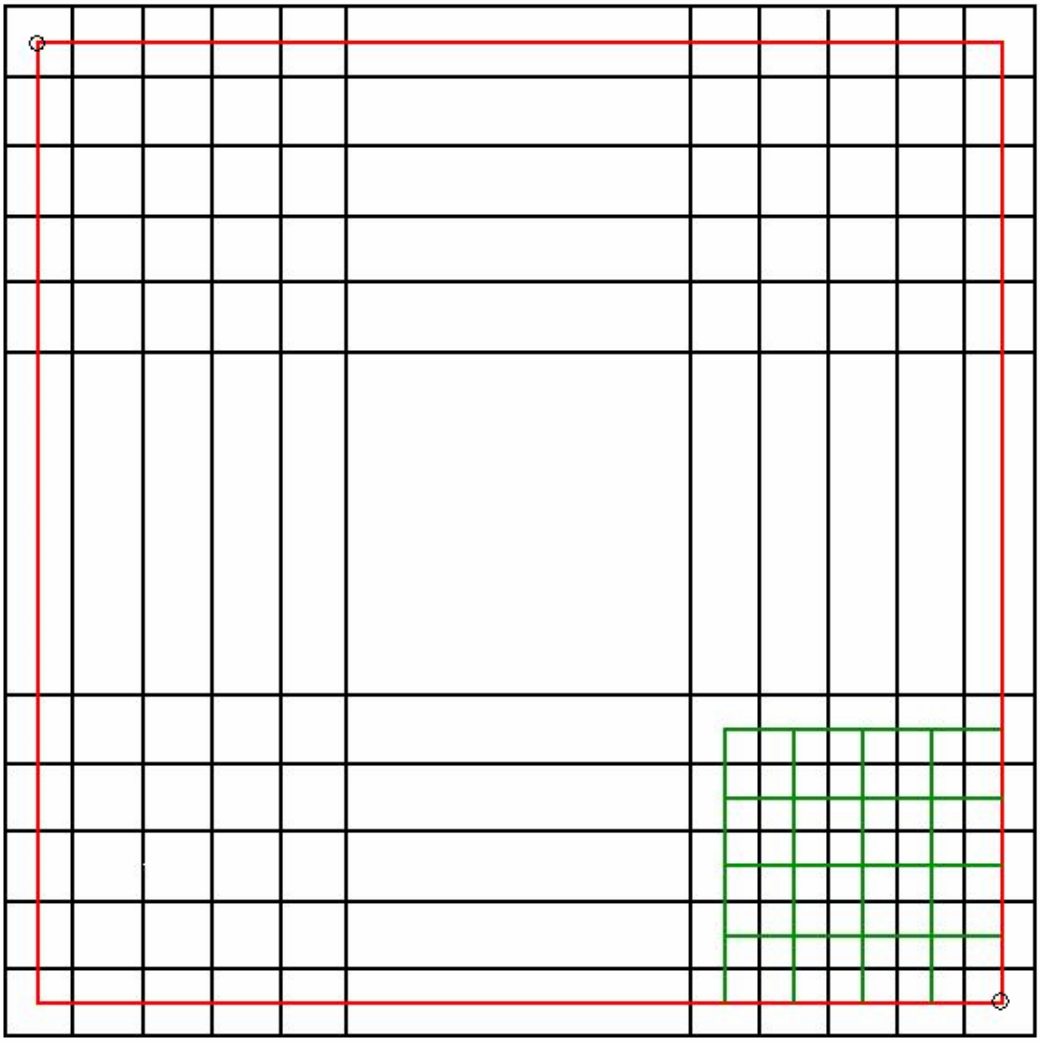


Figure 5: This kind of partition (green line) is used to calculate the injection.

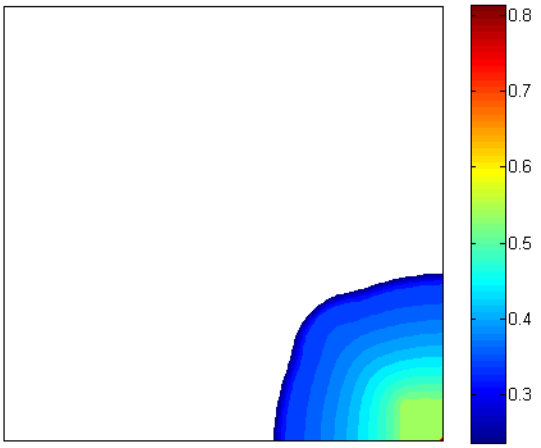


Figure 6: The water saturation at the 40th day.

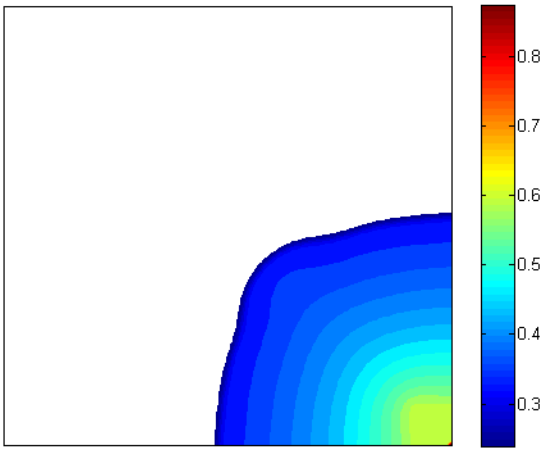


Figure 7: The water saturation at the 80th day.

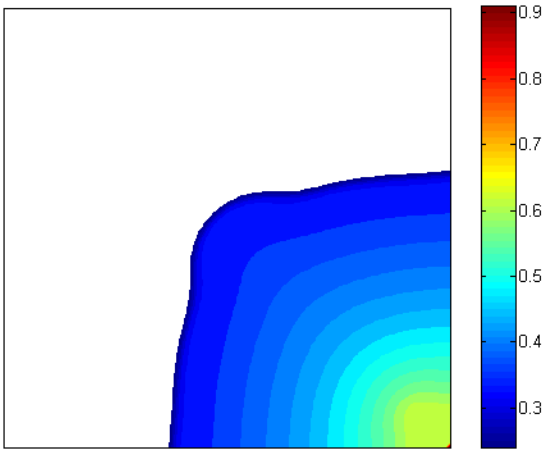


Figure 8: The water saturation at the 120th day.

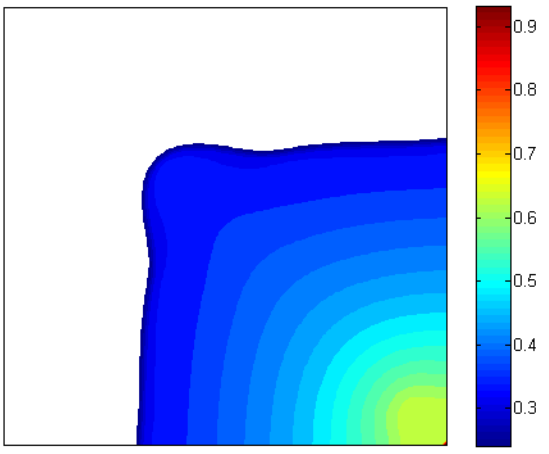


Figure 9: The water saturation at the 160th day.

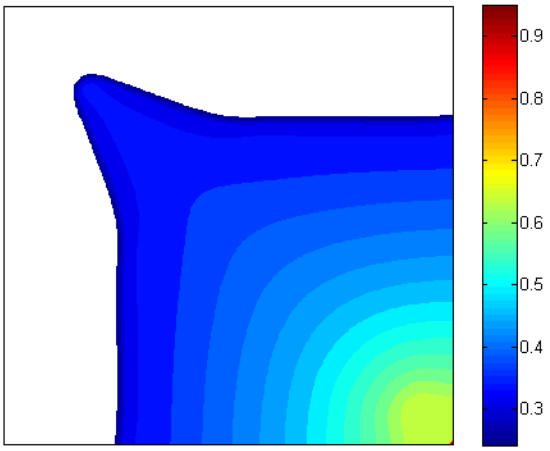


Figure 10: The water saturation at the 200th day.

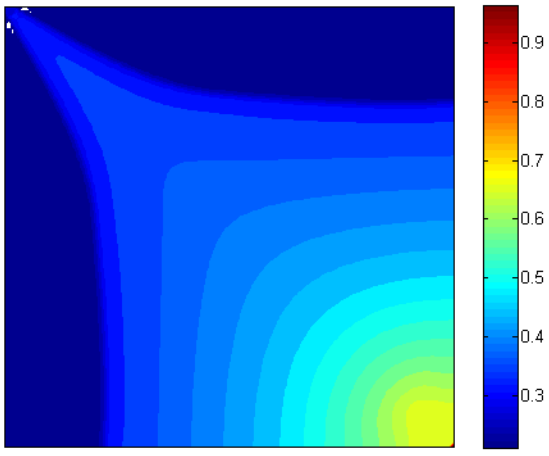


Figure 11: The water saturation at the 240th day.

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