sufficiently low, over a few tens of kilometres of fibre, to allow AM/VSB lightwave systems to operate from 50 to 550 MHz in the 1550 nm fibre window.

Conclusions: The feasibility of operating an analogue lightwave system over dispersive fibre using a directly modulated DFB laser has been demonstrated for the first time. The effect of fibre dispersion was to produce carrier suppression and second order sum products. By limiting the modulation depth to the linear region of the laser, a single octave, eight channel, broadcast quality, video transmission system was operated over 100 km of dispersive fibre.

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THROUGHPUT PERFORMANCE OF MULTISTAGE INTERCONNECTION NETWORKS IN PRESENCE OF MULTIPLE MEMORY HOT SPOTS

Indexing terms: Memories, Networks

A recursive formula to evaluate the throughput performance of multistage interconnection networks having two memory hot spots is derived. Classes of nonuniform traffic pattern are defined according to the relative locations of the hot spots. Numerical results show that the degradations are roughly the same under different classes of nonuniform traffic pattern.

Introduction: Most results regarding the performance of multistage interconnection networks (MINs) have been derived based on uniform traffic models. A uniform traffic model means that each processor independently generates memory access requests with identical rates and each memory module (MM) is equally likely to be the destination of any request. In a real world system, it is likely that one or more MMs store common variables of all processors and hence will be accessed more frequently than the others. This type of locality results in the so called 'hot spot' nonuniform traffic pattern. The bandwidth degradation of unbuffered MINs caused by the existence of a single hot MM had been studied in Reference 3. In this letter systems with multiple hot spots are considered.

System model: The study is focussed on systems with two hot MMs. The MIN studied in this letter consists of n stages, each stage has α^{n-1} crossbar networks of size $\alpha \times \alpha$. For mathematical tractability, we make the following assumptions:

(*a*) The number of independent processors is equal to the number of MMs and is denoted by $N = \alpha^n$.

(*b*) The network is operated synchronously and requests are not combined.

(*c*) The requests generated by a processor are independent of the requests generated by the other processors. The requests generated in a network cycle are also independent of the requests generated in the previous cycles.

(*d*) Each processor generates at most one request in a network cycle and the probability that a processor generates a request in a network cycle is equal to $r (0 \le r \le 1)$.

(*e*) One of the hot MMs attracts requests with probability $h_1 + (1 - h_1 - h_2)/N$, the other with probability $h_2 + (1 - h_1 - h_2)/N$. A cool (nonhot) MM attracts requests with probability $(1 - h_1 - h_2)/N$.

(*f*) Blocked requests are lost.

Performance analysis: To analyse the throughput performance, the relative locations of the two hot MMs must be known. For an n-stage MIN, the outputs can be numbered from top to bottom by α-ary sequences of length n. Without loss of generality, we assume the upmost output to be a hot MM which attracts requests with probability $h_1 + (1 - h_1 - h_2)/N$. Define the traffic as the class i nonuniform pattern if the ith digit, counted from right to left, of the α-ary representation of the other hot MM is the leftmost digit which is not a 0. Fig. 1 illustrates an example of class 2 nonuniform traffic pattern of a three-stage baseline network. For convenience, we call the traffic pattern of the degenerated case (i.e. only one hot MM which attracts requests with probability $h_1 + h_2 + (1 - h_1 - h_2)/N$) the class 0 nonuniform traffic pattern.

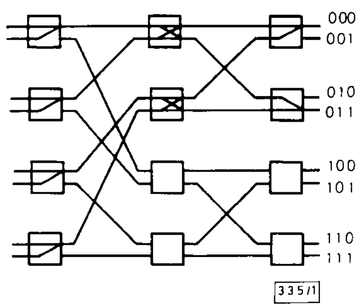

Fig. 1 *Nonuniform traffic pattern*
Eight post baseline network
Class 2

Let us consider the uniform traffic model ($h_1 = h_2 = 0$) first. Let $S_U(n; r)$ denote the throughput of an n-stage MIN with input rate r. Then $S_U(n; r)$ can be computed by

$$S_U(n; r) = \alpha S_U(n - 1; r') \tag{1}$$

where

$$r' = 1 - (1 - r/\alpha)^\alpha \tag{2}$$

The boundary condition of eqn. 1 is given by $S_U(0, r) = r$.

Next, let us consider systems having a single hot spot. Let $S_1(n; r, h)$ denote the throughput of an n-stage MIN under the single hot spot nonuniform traffic. Then we have

$$S_1(n; r, h) = S_1(n - 1; r', h') + (\alpha - 1)S_U(n - 1; r'') \tag{3}$$

where

$$r' = 1 - \left[1 - rh - \frac{r(1 - h)}{\alpha}\right]^\alpha \tag{4}$$

$$h' = \frac{\alpha h}{1 + (\alpha - 1)h} \tag{5}$$

and

$$r'' = 1 - \left[1 - \frac{r(1-h)}{\alpha}\right]^{\alpha} \tag{6}$$

The boundary condition of eqn. 3 is given by $S_1(0; r, h) = r$ for $0 \le h \le 1$. Eqn. 3 results in different throughput performance from that given in Reference 3. The reason is that the model in Reference 3 allows each processor to independently generate two requests in a network cycle. It is not difficult to derive eqns. 4 and 6. Let us consider the expression for h' given in eqn. 5. Suppose a hot port (i.e. a link which can reach the hot MM) which is an output of a switching element at stage 1 receives a request. Let t_h and t_{nh} denote the probabilities that the request received by the hot port is destined to the hot MM and a regular MM reachable from the hot port, respectively. It is clear that t_h is equal to the probability that the request is destined to the hot MM provided it is destined to one of the MMs reachable from the hot port. After some calculations, we conclude that $t_h = [h + (1-h)/N]/[h + (1-h)/\alpha]$. It is also possible to show that $t_{nh} = [(1-h)/N]/[h + (1-h)/\alpha]$. The excess probability for the hot MM of a request received by the hot port is equal to $h' = \alpha h/[1 + (\alpha - 1)h]$.

Finally, let us consider systems having two hot MMs. Let $S(n, i; r, h_1, h_2)$ denote the throughput of an n-stage MIN under the class i nonuniform traffic pattern. Then we have

$$S(n, i; r, h_1, h_2) = \begin{cases} S(n-1, i; r', h_1', h_2') + (\alpha - 1)S_U(n-1; r'') \\ \quad 0 \le i \le n-1 \\ S_1(n-1; r^*, h^*) + S_1(n-1; r^{**}, h^{**}) + (\alpha - 2)S_U(n-1; r^{***}) \\ \quad i = n \end{cases} \tag{7}$$

where

$$r' = 1 - \left[1 - r(h_1 + h_2) - \frac{r(1 - h_1 - h_2)}{\alpha}\right]^{\alpha} \tag{8}$$

$$h_1' = \frac{\alpha h_1}{1 + (\alpha - 1)(h_1 + h_2)} \tag{9}$$

$$h_2' = \frac{\alpha h_2}{1 + (\alpha - 1)(h_1 + h_2)} \tag{10}$$

$$r'' = 1 - \left[1 - \frac{r(1 - h_1 - h_2)}{\alpha}\right]^{\alpha} \tag{11}$$

$$r^* = 1 - \left[1 - rh_1 - \frac{r(1 - h_1 - h_2)}{\alpha}\right]^{\alpha} \tag{12}$$

$$h^* = \frac{\alpha h_1}{1 + (\alpha - 1)h_1 - h_2} \tag{13}$$

$$r^{**} = 1 - \left[1 - rh_2 - \frac{r(1 - h_1 - h_2)}{\alpha}\right]^{\alpha} \tag{14}$$

$$h^{**} = \frac{\alpha h_2}{1 + (\alpha - 1)h_2 - h_1} \tag{15}$$

and

$$r^{***} = 1 - \left[1 - \frac{r(1 - h_1 - h_2)}{\alpha}\right]^{\alpha} \tag{16}$$

The boundary condition of eqn. 7 is given by $S(0, 0; r, h_1, h_2) = r$. The above expressions can be verified by the same arguments we gave for the case of single hot MM.

Results and discussion: Fig. 2 shows the bandwidth (i.e., the throughput when $r = 1$) of a 10-stage MIN with $\alpha = 2$ for various values of h_1 and h_2 under different classes of nonuniform traffic pattern. In this Figure, we choose $h_1 = h_2 = h$. One can see that a higher degree of nonuniformity (i.e. a larger value of h) results in a more severe bandwidth degradation. According to our numerical results, the bandwidth degradations under different classes of nonuniform traffic pattern are roughly the same. For example, when $h_1 = h_2 = 0{\cdot}1$, the degradation is within 11·63% (class 3) and 13·01% (class 10).

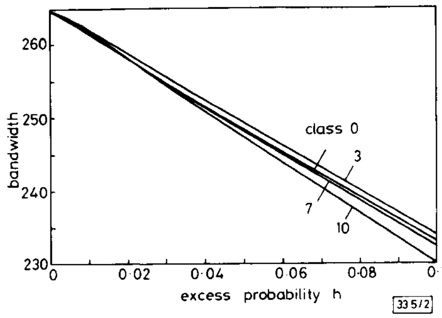

Fig. 2 *Bandwidth against request probability*
1024 × 1024 MIN
$\alpha = 2$

Although we focus our study on systems having two hot MMs, the analysis provided in this letter can be generalised to evaluate the throughput performance of systems having more than two hot MMs.

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FAST ALGORITHM FOR REAL JOINT TIME-FREQUENCY TRANSFORMATIONS OF TIME-VARYING SIGNALS

Indexing terms: Signal processing, Mathematical techniques, Fourier transforms

Instead of using the FFT, all the real joint time-frequency transformations (JTFT) can be evaluated by the same efficient transformation algorithm, using different time-indexed autocorrelation functions. The approach presented dramatically reduces the computation load. Our research suggests that a software package or a custom VLSI chip may implement real JTFTs.

Introduction: Most joint time-frequency transformations (JTFT) known can be written as the Fourier transformation of the time-indexed auto-correlation function

$$JTFT(t, w) = \int_{-\infty}^{\infty} R(t, \tau)e^{-j\omega\tau}\, d\tau \tag{1}$$

where the time-indexed auto-correlation function $R(t, \tau)$ determines the properties of JTFT. The most prominent subclass