國立交通大學

工業工程與管理學系

博士論文

具有多項異常原因之 $\bar{X}$ 管制圖經濟設計

Economic Design of $\bar{X}$ Control Chart for Multiple Assignable Causes

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摘要

本論文主要研究製程異常原因發生為多項異常原因之 $\bar{X}$ 管制圖經濟設計。研究中製程生產方式分為斷續工件生產及連續流動生產兩種；製程失效機構則考慮指數分配與韋氏分配兩種。根據生產方式及製程失效機構，在論文中共提出二項研究主題：(1)連續流動生產且製程失效機構屬指數分配之 $\bar{X}$ 管制圖經濟設計，(2)斷續生產且製程失效機構屬韋氏分配之 $\bar{X}$ 管制圖經濟設計。

對於這兩種經濟設計模式，我們利用抽樣方法和成本結構來建構損失成本函數，並在損失成本最小化下來找尋最佳的 n (樣本大小)，h (抽樣間隔時間)，k (管制界限係數)。由於經濟設計模式進行敏感度研究，可以提供管理者或工程師了解輸入參數對模式的影響。因此，我們也將對最佳的 n，h，k 值進行敏感度分析，藉此分析來了解時間參數或成本參數的變動後，對於最佳 n，h，k 值之影響。最後，我們有提供數值結果並討論之。

除此之外，我們將對多項異常原因下僅一個異常發生之模式和多項異常原因下有二個異常發生之模式進行比較分析，藉由數值結果可以得知考慮多項異常原因下有二個異常發生之模式對於降低品質成本和增加其在斷續生產之競爭力是有一個有用的方法。

關鍵字：經濟設計，$\bar{X}$ 管制圖，多項異常原因，韋氏衝擊模氏，指數衝擊模式，斷續生產，連續流動生產。

Economic Design of $\overline{X}$ Control Chart for Multiple Assignable Causes

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Abstract

In this dissertation, we analyze the economic design of $\overline{x}$-control charts and extend the model for the case of multiple assignable causes to allow for the second occurrence of an assignable cause following the first occurrence. In addition, two process failure mechanisms are investigated in different manufacturing environments. One is the Exponential failure mechanism in a continuous flow process and another is the Weibull failure mechanism in a discrete part process.

For those two models, the expected loss-cost functions are established by the sampling scheme and cost structure. Optimal values of the economic design parameters including the sampling size(n), the sampling intervals (h) and control limit coefficient (k) are determined by minimizing loss-cost functions. Because of sensitivity investigation on the model with critical input parameters may provide some answers for the model analyst. A sensitivity analysis is provided to discuss how the model can be affected by the time parameters or cost parameters in the investigated model. For illustration purpose, numerical results are also presented.

Subsequently, we perform comparative analysis between the model that once an assignable cause occurs, no further assignable causes will occur and the modified model that allow for the second occurrence of an assignable cause following the first occurrence.

Our numerical investigations showed that a modified model should be helpful in reducing the quality cost and increasing competitiveness in a discrete part process.

Key words: Continuous Flow Process, Discrete Part Process, Economic design, Exponential shock model, Multiple assignable causes, Weibull shock model, $\bar{x}$-control chart.

Preface

The objective of this dissertation is to provide optimal economically-based control charts for use in detecting out-of-control conditions when monitoring continuous flow process or discrete part process.

I wish to express sincere appreciation to my major adviser, Dr. W. L. Pearn, for his guidance, assistance, and encouragement throughout this research and during my doctoral studies. Thanks also to my another adviser, Dr. Y. M. Yang, for his interest and assistance.

I also wish to thank my committee members, Dr. Kai-Bin Huang, Dr. H. W. Su, Dr. Dong-Yuh Yang, for their assistance. Thanks are extended to my sisters for their moral support to give me the opportunity to fulfill this dissertation.

Finally, I wish to dedicated this dissertation to my parents and my husband, Hung-Chia Kuo, for their sacrifice, understanding, encouragement, and love.

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Chapter 1

Introduction

Control charts are the most theoretical technology in magnificent seven. Control charts are the primary quality improvement tools in statistical process control (SPC), which used to establish and maintain statistical control of manufacturing process. The effective use of control charts are largely dependent upon their design. For this reason given above, control charts are practical subjects and play important roles in SPC. In Section 1.1, we describe the background of the control charts. Section1.2 is devoted to introduce inspection intervals principle. In Section 1.3, we relate our problem to earlier works in the literature. Section 1.4 shows the description of the models in this thesis. At the end of this chapter, the scope of the thesis is presented in Section 1.5.

1.1 Background

The control chart was originated in 1924 by Shewhart [37] as a means to differentiate between the normal, expected random causes and the special or assignable causes of the process variability. In the development of new processes or products, or in the restudy of existing ones, the Shewhart control charts are still the basic tools for establishing a state of statistical control (Gibra [17]). The $\bar{X}$ control chart is an online control tool used to detect the mean shifts of a process. When a $\bar{X}$ control chart is used to monitor the process mean, three questions are concerned by the quality control engineers. These are (1) How large a sample should be employed? (2) At what interval should the samples be taken? (3) What multiple of sigma should be used in determining the control limits? (Duncan [14]). Selection of these decision variables based on some subjective and/or objective criteria. Shewhart [37] developed the use of 3-sigma control limits as action limits. The justification of 3-sigma limits is based on empirical-economic considerations rather than on a formal statistical basis. Shewhart settled on subgroup sizes of 4 or 5 for $\bar{X}$- and R-charts and left the quality control engineer or other personnel. The design can affect the cost, statistical properties, and ultimately user confidence. Cost considerations are important for obvious reasons.

The concept of an economic design was first introduced by Girshick and Rubin [18]. Although the optimal control rules in their model are too complex to have practical value, their work provided the basis for most cost-based models in control chart designs. The pioneer investigation of the economic design of an $\bar{X}$-chart was made by Duncan [14]. He formulated an excellent model for the determination of the optimal parameters of an $\bar{X}$-chart.

However, these reviews are centered on piece part manufacturing. In continuous processes, there is not a well defined production unit. Almost any chemical, petroleum, bulk liquid, or otherwise semi-homogenized product is a case of this kind. The problem is compounded by the fact that the sample may have been taken from a vat or pipeline in which there is a homogeneous mixture resulting from flow or agitation. To pull n samples instantaneously from a continuous flow process would usually result in ranges of zero, or in the range being an almost pure measure of test variation. Due to the number of such process, there is a need to develop appropriate quality control techniques for continuous flow processes.

Koo and Case [23] first proposed an economic design of $\bar{X}$ control charts for using in monitoring continuous flow process, where the amount of time the process remains in control can be formulated as exponential distribution. A sampling scheme in a continuous flow process is to take one sample from the process at each sampling time and then combine n analytical results into a subgroup. That is considerably different from pulling n samples at one time as in a discrete piece-part process.

In practice, many production processes are affected by several assignable causes. Therefore, processes subject to single assignable cause are not common. In such situation, an investigation of the case where there is a multiplicity of assignable causes is therefore desirable. Duncan [15] has generalized his single assignable model to a situation in which there are s assignable causes, however, where different special causes will shift the process mean by different amounts. Duncan's multiple causes model is divided into two types: Model I presents "a single occurrence" model. It is assumed that once assignable cause A_i occurs, the process remains in that other assignable causes occur no longer till assignable cause A_i is detected; Model II presents "double occurrence". It is assumed that the model allows for the second occurrence of an assignable cause following the first occurrence.

Many interesting research results are investigated on the process failure mechanism (see Duncan [14,15], Hu [21], Banerjee and Rahin [3]). Past work regarding process failure mechanism may be classified into three categories: (i) Exponential, (ii) Weibull, and (iii) Gamma. The time between occurrences of successive special causes are exponentially distributed with a specified mean value, and thus, a constant failure rate for the process is implied. Banerjee and Rahin [3] pointed out that the use of a constant sampling interval is counterintuitive in the case of a process with an increasing failure rate. A more realistic approach is to shorten the sampling interval because the process deteriorates further as time goes by.

In this thesis, we deal with two process failure mechanisms in different manufacturing environments with multiple assignable causes. One is the Exponential failure mechanism in a continuous flow process and another is the Weibull failure mechanism in a piece-part process. For those two models, the expected loss-cost functions are established by the sampling scheme and cost structure. Optimal values of the economic design parameters including the sampling size(n), the sampling intervals (h) and control limit coefficient (k) are determined by minimizing loss-cost functions. Analytical results for sensitivity analysis are also obtained.

1.2 Inspection interval principle

In this section, we introduce the inspection interval principle (Banerjee and Rahim [3]).We attempt to obtain a near-optimal solution by imposing some sort of restriction on the lengths of sampling intervals. In this connection, we note that the uniform sampling intervals for Markovian shock models provide a constant integrated hazard over each interval. Being motivate by this, we propose that the lengths of the sampling intervals should be chosen in such a way that the integrated hazard over each interval should be equal.

The process is monitored by drawing random samples of size n at times $h_1,\ h_1+h_2,\ h_1+h_2+h_3,\ \cdots$ and so on. Where h_j is the j th sampling interval. For convenience, let W_j be the time of the j th sample, $W_j=\sum_{i=1}^{j} h_i,\ j=1,\ 2,\ \cdots$ and $W_0=0$. For a non-uniform sampling, the length of the sampling interval h_j ($j=1,\ 2,\ \cdots$) is chosen such that the probability of a process shift in an interval, given no shift until the start of the interval, is constant for all intervals. This amounts to choosing the h_j such that the integrated hazard rate over each interval is constant. Specifically, the h_j is chosen such that

$$\int_{w_j}^{w_{j+1}} r(t)\ dt = \int_0^{w_1} r(t)\ dt\ ,\ \forall j=0,\ 1,\ 2,\ \cdots \tag{1.1}$$

and the Weibull hazard rate is defined by

$$r(t)=\lambda \upsilon t^{\upsilon-1}. \tag{1.2}$$

Utilizing equations (1.1) and (1.2) and induction, one obtains $w_j = j^{1/\upsilon} h_1$, and therefore,

$$h_j = \left[j^{1/\upsilon} - (j-1)^{1/\upsilon} \right] h_1 . \tag{1.3}$$

From the above Equation (1.3), the h_j is a non-increasing function of h_1. The W_j goes to infinity as j goes to infinity. That is $h_1 \geq h_2 \geq h_3 \geq \cdots$ and $\lim_{j \to \infty} W_j = \infty$.

1.3 Literature Review

The pioneer investigation of the economic design of $\bar{x}$-control charts was made by Duncan [14]. He formulated an excellent model for the determination of the optimal parameters of $\bar{x}$-control charts. These parameters (the sample size, the time interval between taking successive samples, and the control limits) were derived to maximize the approximate average net income of a process. He also made a number of specific assumptions. For example, he assumed an exponential time to failure of the process and that the process is subject to the occurrence of an assignable cause of variation which takes the form of a shift, of constant magnitude, in the process mean. The standard deviation is assumed to remain stable. Furthermore, he assumed that the process is not shut down while the search for the assignable cause is in progress. Since the work by Duncan, numerous authors have made a wide variety of changes to Duncan's modeling assumptions, the distribution of the time of assignable cause, approach, and *et al*. on their economic design of process control charts. Earlier work in this area was summarized by Montgomery [27] and Vance [40]. Both are excellent references. Several works focusing on the economic design of $\bar{X}$ control charts will be discussed.

1.3.1 Single special cause

The classical model for the economic design of $\bar{x}$-control charts subject to a single assignable cause was first introduced by Duncan [14]. An approximation to the optimal design was found. Goel, *et al*. [19] developed an algorithm to find the exact optimal solution of Duncan's model by computer. Gibra [16] developed a model for the determination of the optimal parameters of $\bar{x}$-control charts. He assumed that the sum of times required to take, inspect a sample, compute and plot a sample average and to discover and eliminate the assignable cause has an Erlangian distribution. In Duncan's model, the author assumed that the time to discover and eliminate the assignable cause is constant.

Chiu and Wetherill [11] proposed a simple semi-economic scheme for the design of a control plan using an $\bar{x}$-control chart. They simplified the loss-cost function proposed by Duncan by eliminating some insignificant terms. The essential characteristic of the semi-economic plan is to specify the probability of true alarms at a value of 0.9 or 0.95. They found that of the 25 semi-economic plans 17 were better than Duncan's approximate plans and the remaining 8 showed a loss-cost within 1.7% above Duncan's corresponding values.

Lorenzen and Vance [25] provided one unified approach to the economic design of process control charts. They considered a general process model that applied to all control charts. Collani [13] discussed two process control strategies: (1) As soon as one sample average falls outside the control limits, the process is shut down and action is taken to search the assignable cause of variation. (2) The process is shut down in every h hours. The expected cost produced was given as investigating the assignable cause. Tagaras [39] considered the probability of shift, the correlation of process variance and mean, and error of measurement on the optimal design parameters.

1.3.2 Warning limits

Gordon and Weindling [20] presented the economic design of $\bar{x}$-control charts with warning limits. They consider a single assignable cause model, and minimize the long run average cost per good part produced. Chiu and Cheung [10] presented a study which starts where Gordon and Weindling stop to investigate the economic design of $\bar{x}$

-control charts with both warning and action limits, based on a widely studied process model. They assumed that a search for the assignable cause is undertaken when either (1) any point exceeds the action limit, or (2) a run of N points fall between the action and warning limits. The loss-cost function is following Duncan's [14] arguments with straightforward modifications.

1.3.3 Joint economic designs

Two charts are usually employed together to monitor the process. One is for monitoring the shift in the process mean; the other for monitoring the change in the process variation. In Duncan's model, the standard deviation is assumed to remain stable. Saniga [34] was the first to study the joint economic design of $\bar{X}$ and R charts in which Duncan's [14] approach is not used. He assumed that two shifts can occur during the production of a specified number of units. Therefore, the parameters of R chart are considered. Saniga [35] also investigated the sensitivity of the economic design to the type of process model using a discrete-time cost model developed by Barker [4]. Furthermore, Saniga [36] was the first considered an application of economic statistical principle to the joint design of an $\bar{X}$ and R charts. The objective is to minimize the expected total cost per unit time subject to constraints on the Type I error probability, Type II error probability and average time to signal (ATS).

Still another useful paper is that of Rahim, *et al*. [33]. Their cost model for their economically-based $\bar{X}$ and variance charts followed the unified approach of Lorenzen and Vance [40]. The joint $\bar{X}$ and variance charts were compared to $\bar{X}$ and R charts. Results showed that the $\bar{X}$ and variance charts have lower cost and higher power.

1.3.4 Non-Exponential process failure

Most of the work of the economic design of quality control charts assume that the underlying distribution of the process failure mechanism is exponential. That is, the times between occurrences of successive special causes are exponentially distributed with a specified mean value, and thus, a constant failure rate for the process is implied. For some processes that deteriorate with time, the exponential assumption may not be appropriate.

Banerjee and Rahim [2] utilized a renewal theory approach to design and evaluate economically-based control charts. Examples were given for the situations where the distributions of the process failure mechanism was geometric and where it was Poisson. The case of the gamma shock model was also thoroughly discussed. They showed that certain non-Markovian models can be analyzed by adopting a renewal equation approach. However, the issue of nonuniform sampling scheme had not been addressed until Banerjee and Rahim [3] pointed out that the use of a constant sampling interval is counterintuitive in the case of a process with an increasing failure rate. Therefore, they proposed an economic model of the $\bar{X}$ chart under Weibull shock using a varying sampling interval. They compared three cases and found that increasing the frequency of sampling with the age of the system yields a lower operational cost per hour for an increasing failure rate Weibull distributed shock model.

Parkhideh and Case [30] extended and generalized the model of Banerjee and Rahim [3] to develop six design parameters of economically-based dynamic $\bar{x}$-control chart. They, in addition to adopting the rich Weibull failure mechanism, allowed the control chart design parameters to vary over time. Comparisons between the dynamic $\bar{x}$-control chart and the traditional $\bar{x}$-control chart under a wide range of situations were made. They reported that the dynamic $\bar{x}$-control chart is always superior to Duncan's [14] $\bar{x}$-control chart when the underlying distribution of the process failure mechanism is Weibull. Hiroshi and Rahim [24] simplified the dynamic model of Parkhideh and Case

[30] to develop three design parameters of economically-based dynamic $\bar{x}$-control chart. Zhang and Berardi [45] proposed an economic statistical design model subject to constraints on the Type I error probability, power and average time to signal (ATS), which basically followed the model of Banerjee and Rahim [3]. In addition, Rahim [32] presented a FORTRAN program for the optimal economic design of $\bar{x}$-control charts based on the economic model of Banerjee and Rahim [3].

Mcwillians [26] conducted a sensitivity analysis of the effects of misspecification of the underlying distribution of the process failure mechanism on the optimal control chart design parameters and the resulting operating loss. The Weibull distribution was selected to represent the underlying distribution of the process failure mechanism and it was implemented in Lorenzen and Vance's [25] model. He found that the economic control chart design is not sensitive to the shape of the Weibull distribution.

1.3.5 Multiple special causes

Duncan [15] extended his single cause model [14] to develop an economic model for the $\bar{x}$-control chart subject to a multiplicity of special causes. Each special cause produces a shift of know magnitude in the process mean. Two models designed Model I and Model II were considered. Model I assumes that once a special cause occurs, it continues to affect the process until it is detected and during this period it is undisturbed by the occurrence of other special causes. Model II allows for the second occurrence of a special cause following the first occurrence. This problem has also been addressed by several other researchers. All of them used only one set of control limits to maintain the process under control. There are situations, however, where different special causes will shift the process mean by different amounts; also, different cost and restoration procedures are required to repair the process for different shifts.

Tagaras and Lee [38] applied the Model I of Duncan [15] to propose an economic model of multiple control limits and correct procedures with multiple corresponding levels of process shifts. The criterion used for determining the design parameters was the expected loss per time unit. It was reported that the proposed model showed a significant improvement over the single-cause model. Jaraiedi and Zhuang [22] presented a computer program which followed the Model I of Duncan [15] to determine the optimal design parameters. Thus, Chung [12] simplified the Model I of Duncan [15] to develop the feasible solution of the design parameters. Chen and Yang [8] extended the time of occurrence of assignable causes in Duncan [15] multiplicity-cause model from exponential distribution to Weibull distribution.

Arnold [1] applied Collani's [13] alternative sampling policies to design a model of $\bar{X}$ control charts subject to a multiplicity of special causes and assumed that there are $(m+1)$ states in which a process can exist. That is, there is one state δ_0 that indicates the process is in a state of statistical control and m states that indicate the process mean has shifted.

1.3.6 Continuous flow process

Most of the applications of $\bar{x}$-control charts are in a piece-part manufacturing environment. Koo and Case [23] applied the $\bar{x}$-control chart procedure to a continuous flow process and developed an economic model. The underlying distribution of the process failure mechanism is the exponential distribution. In their procedure, a sampling scheme is to take one sample from the process at each sampling time and then combine n analytical results into subgroup that is considerably different from pulling n samples at one time. Chen and Yang [6] modified Koo and Case [23] to develop an economic design of $\bar{x}$-control charts with single assignable cause in a continuous flow process. They employed the Weibull distribution as the underlying distribution of the process failure mechanism. In addition, Chen and Yang [8] exposed an economic design of $\bar{x}$

-control chart with multiple assignable causes in a continuous flow process based on the Model I of Duncan [15]. They also assumed that the underlying distribution of the process failure mechanism is the Weibull distribution.

More recently, focus on other control chart including Chen and Yang [7], Yang and Rahim [42], Zhang, *et al*. [44] provided fine reviews of the economic design of process control charts. Chen and Yang [7] proposed a model of a moving average control chart (MA control chart) with a Weibull failure mechanism from an economic viewpoint. In Zhang, *et al*.'s [44] paper, it is proposed to monitor the cumulative number of samples inspected until a nonconforming sample is encountered. An economic model is developed for designing such a generalized CCC chart. Yang and Rahim [42] extended the research which conducted by Banerjee and Rahim [3]. Their general approach is now applied to a multivariate control chart instead of a univariate control chart. A cost model for the economic statistical design of a Hotelling T^2 control chart is derived to deal with situations involving a Weibull shock model with an increasing failure rate.

1.4 Problem Statement

In this dissertation, we investigate the economic design of $\bar{x}$-control charts for discrete part Weibull process and for continuous flow exponential process with multiple assignable causes.

For discrete part Weibull process, samples of size n are drawn in every h_j hours of production, and the sample means are plotted on the $\bar{x}$ control charts which has an control limit at $\mu_0 \pm k\,\sigma/\sqrt{n}$. The sampling method and plotting on $\bar{x}$-control charts in a discrete part process is shown in Figure 1.1. And the average cycle length for discrete part process is illustrated in Figure 1.2.

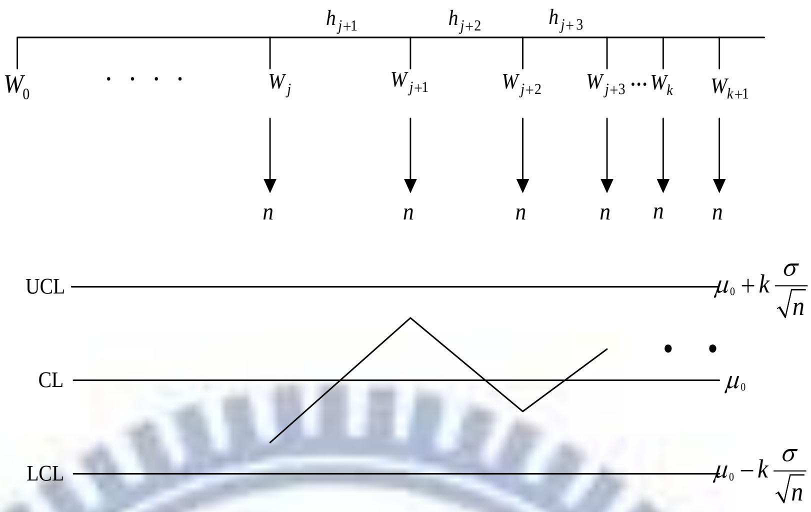

Figure 1.1 Sampling method and plotting on $\bar{x}$ -control charts in a discrete part process

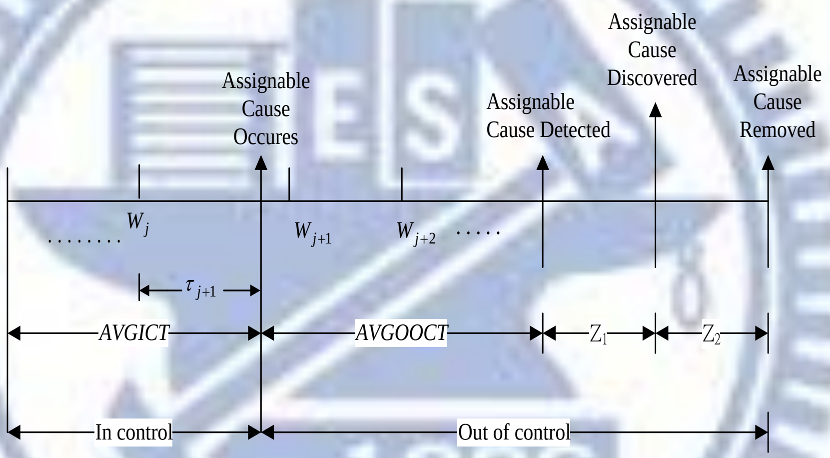

Figure 1.2 The average cycle length for discrete part process

For continuous flow exponential process, sampling scheme is to take one sample from the process at each sampling time and then combine n analytical results into a subgroup. The sample means are plotted on $\bar{x}$ -control charts which has an control limit at $\mu_0 \pm k\,\sigma/\sqrt{n}$. The sampling method and plotting on $\bar{x}$ -control charts in a continuous flow process is shown in Figure 1.3. And the average cycle length for continuous flow process is illustrated in Figure 1.4.

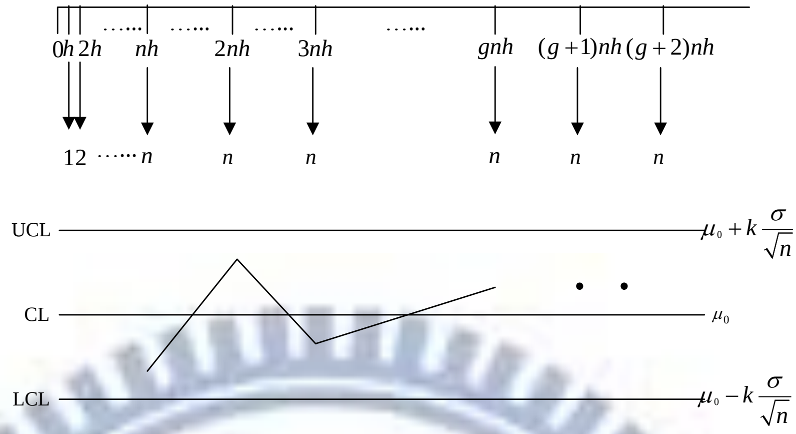

Figure 1.3 Sampling method and plotting on $\bar{x}$ -control charts in a continuous flow process

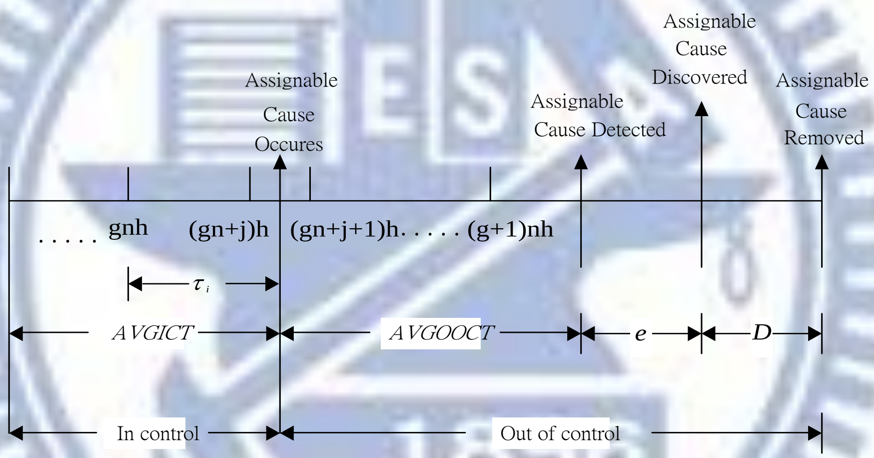

Figure 1.4 The average cycle length for continuous flow process

We assume that there are s possible assignable causes and the occurrence time of any one assignable cause follows Weibull or Exponential distribution. The occurrences of s assignable causes are independent to each other. Once assignable cause A_i occurs, it continues to affect the process until it is detected by control chart, and during this period it is allowed for the second occurrence of an assignable cause following the first occurrence. The process at any time is either in control or out of control which resulted in a $\delta_i\sigma$ shift amount in the process mean by the occurrence of the i th assignable cause A_i. Duncan [15] considered three different shape parameters (the λ_i 's are non-increasing functions of the δ_i's), they are J-shaped, rectangular and half-bell shaped. These distributions would cover the distribution most likely to meet in reality. Therefore, three prior distributions of δ_i are considered. There are negative-exponential ($(1/2)\exp(-\delta_i/2)$), uniform and half-normal ($(1/\sqrt{2\pi})\exp(-(0.5\delta_i)^2/2)$). Finally, to simplify the analysis, we assume that the joint effect of the two assignable causes is always to produce a shift of $\Delta\delta\sigma$ in the process mean regardless of what two assignable causes occur jointly. Consequently, there is no need to consider the prior distribution of the second assignable causes.

A production cycle begins when a new system is installed and ends when the process is brought back to an in-control state after a system failure is detected and repaired. The objective is to find optimal values for sample size, control limit coefficient, and sample interval such that the expected loss-cost per unit time is minimized.

1.5 Scope of Dissertation

The main purposes of this dissertation are to analyze: (i) the economic design of $\bar{x}$-control charts for discrete part Weibull process with multiple assignable causes; and (ii) the economic design of $\bar{x}$-control charts for continuous flow exponential process with multiple assignable causes. This dissertation is organized by four chapters as follows:

Chapter 1 is an introduction, which shows the background of the control charts, earlier studies on the economic design of $\bar{x}$-control charts. The inspection interval principle relevant to this study is also presented.

In Chapter 2, we study the economic design of $\bar{x}$-control charts for discrete part Weibull process with multiple assignable causes. The expected cycle length and the total expected cost per cycle are derived by using the inspection intervals principle. Next, the expected loss-cost function is constructed by the ratio of the expected cycle length and the total expected loss-cost per cycle. We determine the optimal design parameters (n, k, and h_1) of the model to minimize the expected loss-cost function. In addition, comparison is also investigated. Finally, we provide numerical results among the optimal design parameters, the minimal expected loss-cost function and the specific value of design parameters.

In Chapter 3, we consider another the economic design of $\bar{x}$-control charts for continuous flow exponential process with multiple assignable causes. The occurrence time of any one assignable cause follows Exponential distribution. For such type of process, we construct the expected loss-cost function based on the the expected cycle length and the total expected loss-cost per cycle. In addition, the optimal design parameters (n, k, and h) of the model can be analytically determined to minimize the expected loss-cost function. Finally, sensitivity analysis is investigated, and a numerical result is also provided.

Chapter 4 presents some conclusions based on results of the investigation, and recommendations for the future investigations.

Chapter 2

Economic Design of $\bar{x}$-Control Charts for Discrete Part Weibull Process with Multiple Assignable Causes

In this chapter, we study the economic design of $\bar{x}$-control charts for discrete part Weibull process with multiple assignable causes. A modified version of Chen and Yang's model [8] for the $\bar{x}$-control charts is proposed to deal with situations involving the multiple assignable causes. In Chen and Yang's model, it is assumed that once an assignable cause occurs, no further assignable causes will occur. To ascertain the effect of this assumption, a study is conducted in this chapter that allows for the second occurrence of an assignable cause following the first occurrence. For manufacturers, the economic objective of production is very important and has to be optimized. An economic approach is developed for the design of $\bar{x}$-control charts. Therefore, we adopt Duncan's multiple causes model [15], Banerjee and Rahim's sampling scheme [3], and Chen and Yang's cost structure [8] to develop a modified model. A modified model is the economic design of $\bar{x}$-control charts for discrete part process with Weibull in control times which subjects to a multiplicity of assignable causes.

This chapter is organized as follows: In Section 2.1, we give some basic definition and assumptions of the model under study and give some notations. Section 2.2 presents the formulation of the expected cycle length by using the inspection interval principle. In Section 2.3, we develop the total expected loss-cost per cycle. And the expected loss-cost function is constructed. In Section 2.4, we determine the optimal design parameters. Finally, in Section 2.5, a numerical example will be presented to compare the optimal results between the modified model and Chen and Yang's model.

2.1 Definitions, Assumptions and Notations

(1) The occurrence time of the ith assignable cause (denoted as A_i, $i=1, 2, \cdots, s$) that the process remains in the in-control state follows a Weibull distribution and the probability density function is given by

$$f_i(t) = \lambda_i \upsilon\, t^{\upsilon-1} e^{-\lambda_i t^{\upsilon}},\ t \ge 0,\ \upsilon \ge 1,\ \lambda_i > 0,\ i=1,\ 2,\ \cdots,\ s. \tag{2.1}$$

The hazard rate is $r_i(t) = \lambda_i \upsilon\, t^{\upsilon-1}$, where λ_i, $i=1, 2, \cdots, s$, is a scale parameter and υ is a shape parameter.

(2) The process is normally distributed and characterized by an in-control state μ_0, because of the occurrence of an assignable cause A_i which occurs at random, resulting in a shift in the mean from μ_0 to either $\mu_0 + \delta_i \sigma$ or $\mu_0 - \delta_i \sigma$. Where μ_0, σ, and δ_i are, respectively, the process mean, the process standard deviation, and shift parameter.

(3) The occurrence of the ith assignable cause A_i does not affect the process variability, that is, the process mean and the process variability are independent.

(4) The shift in the process mean is instantaneous.

(5) The time to sample and to draw control point is negligible and production ceases during the searches and repair.

(6) Define p_{ij} $(i=1, 2, \cdots, s, j=1, 2, \cdots)$ as a conditional probability that the ith assignable cause A_i will occur during the sampling interval h_{j+1}, given that the cause A_i is not occur at time W_j, that is

$$p_{ij} = \frac{\int_{W_j}^{W_{j+1}} f_i(t)dt}{\int_{W_j}^{\infty} f_i(t)dt} = \frac{e^{-\lambda_i (W_j)^{\upsilon}} - e^{-\lambda_i (W_{j+1})^{\upsilon}}}{e^{-\lambda_i (W_j)^{\upsilon}}} = 1 - \exp(\lambda_i (h_1)^{\upsilon}). \tag{2.2}$$

From the above Equation (2.2), the p_{ij} is a function of λ_i, h_1, and υ only. Let $p_{ij} = p_i$, for $i=1, 2, \cdots, s, j=1, 2, \cdots$.

(7) Define q_{ij} as a unconditional probability that the ith assignable cause A_i will

occur during the sampling interval h_{j+1}. The q_{ij} can be obtained from Equation (2.2).

$$q_{ij} = \int_{w_j}^{w_{j+1}} f_i(t)dt = (1-p_{ij})^j p_{ij}, \ \text{for } i=1,\ 2,\ \cdots,\ s,\ j=1,\ 2,\ \cdots. \tag{2.3}$$

(8) Let τ_{ij} be the expected duration of the in-control period within the sampling interval h_{j+1}, given that the i th assignable cause A_i occurred during this sampling interval, that is

$$\tau_{ij} = \frac{\int_{w_j}^{w_{j+1}} (t-W_j) f_i(t)dt}{q_{ij}}. \tag{2.4}$$

Thus, the expected in-control time τ_i during any one sampling interval in which the transition is from an in-control state to an out-of-control state is given by

$$\begin{aligned}\tau_i &= \sum_{j=0}^{\infty} \tau_{ij} \cdot q_{ij} = \sum_{j=0}^{\infty} \int_{w_j}^{w_{j+1}} (t-W_j) f_i(t)dt \\ &= \sum_{j=0}^{\infty} \int_{w_j}^{w_{j+1}} t \cdot f_i(t)dt - \sum_{j=0}^{\infty} (j)^{1/\upsilon} h_1 (1-p_i)^j p_i = (\frac{1}{\lambda_i})^{1/\upsilon} \Gamma(1+\frac{1}{\upsilon}) - h_1 p_i (1-p_i) A_{(1-pi)},\end{aligned} \tag{2.5}$$

where $A_{(x)} = \sum_{l=0}^{\infty} (l+1)^{1/\upsilon} \cdot x^l$, for $|x|<1$, and $\Gamma(y)$ is gamma function, $y \ge 1$.

In this chapter, the following notations shall be used in the formulation of the loss-cost function.

n — the sample size (decision variable)

h_j — the length of the j th sampling interval, where $j=1,\ 2,\ \cdots,\ h_0=0$ (decision variable)

k — the control limit coefficient (decision variable)

Z_0 — the expected search time associated with the false alarm

Z_1 — the expected time to discover assignable cause A_i once the cause A_i

has been detected

Z_{2i} — the expected time to repair process once the cause A_i has been discovered, where $i = 1,\ 2,\ \cdots,\ s$

Z_2' — the expected time to repair process once the joint assignable cause has been discovered

D_0 — the quality cost per unit time while producing in control

Y — the cost per false alarm while producing in control

D_{1i} — the quality cost per unit time while producing out of control owing to the occurrence of the i th assignable cause A_i, where $i = 1,\ 2,\ \cdots,\ s$

D_1' — the quality cost per unit time while producing out of control owing to the occurrence of the joint assignable cause

w_i — the cost to locate and repair the i th assignable cause A_i, where $i = 1,\ 2,\ \cdots,\ s$

w' — the cost to locate and repair the joint assignable cause

a — the fixed sample cost

b — the cost per unit sampled

2.2 Formulation of the expected cycle length

We assume that there are s possible assignable causes and the occurrence time of any one assignable cause follows Weibull distribution. The occurrences of s assignable causes are independent to each other. After being disturbed by an assignable cause A_i, the process will be affected by any other assignable causes. In the other hand, if the first assignable cause continues undetected, the second assignable cause (possibly a repetition of the first) is assumed to occur at random in a later intervals.

The occurrence time of the second assignable cause after the taking of the first sample being distributed Weibull with the mean $(1/\lambda')^{1/\upsilon}\Gamma(1+1/\upsilon)$ and the probability density function of occurrence will be $f'(t)=\lambda'\upsilon\, t^{\upsilon-1}e^{-\lambda' t^{\upsilon}},\ \upsilon\geq 1,\ \lambda'>0,\ t\geq 0$. The process is assumed to be in one of the three states. It is (1) in a state of in control or (2) it has been disturbed by the occurrence of the ith assignable cause A_i which produces a shift of $\delta_i\sigma$ in the process mean or (3) it has been disturbed by the occurrence of the second assignable cause following the first, the joint effect of which in every case is arbitrarily assumed to produce a shift of $\Delta\delta\sigma$ in the process mean. The expected cycle length consists of three states, which can be derived as follows:

(1) State 1:

The probability at time t in control is

$$P(T>t)=P(A_1>t,A_2>t,\cdots,A_s>t)=P(A_1>t)\cdot P(A_2>t)\cdot\cdots\cdot P(A_s>t)=e^{-\lambda_0\cdot t^{\upsilon}}, \quad (2.6)$$

where $\lambda_0=\sum_{i=1}^{s}\lambda_i$ and then the probability density function of occurrence of multiple assignable causes will be $f_0(t)=\lambda_0\upsilon\, t^{\upsilon-1}\exp(-\lambda_0 t^{\upsilon}),\ \upsilon\geq 1,\ \lambda_0>0,\ t\geq 0$, thus the average time in control is $(1/\lambda_0)^{1/\upsilon}\Gamma(1+1/\upsilon)$.

Therefore, the process is in an in-control state and the expected time that the assignable cause will occur is $(1/\lambda_0)^{1/\upsilon}\Gamma(1+1/\upsilon)$.

(2) State 2:

The process has been disturbed by the occurrence of the first assignable cause and produces a shift of $\delta_i\sigma$ in the process mean. When the process is in State 2, it is assumed that no further disturbance occurs until after the first sample is taken. The process can be classified into two situations.

Situation 1:

Consider Situation 1 for Figure 2.1. Situation 1 is the period that the second assignable cause will not to be occurring until assignable cause A_i is detected.

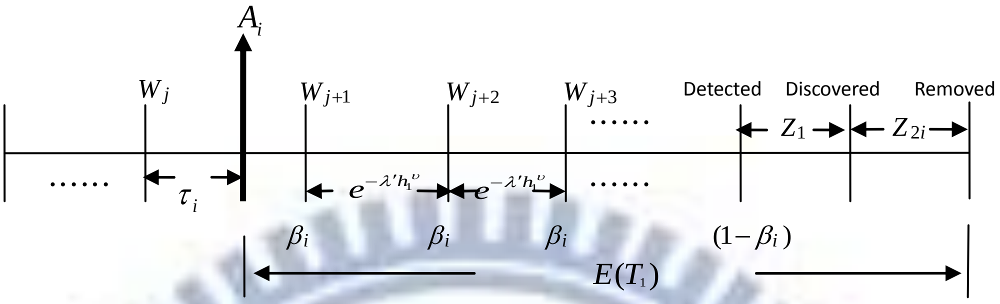

Figure 2.1. The process of Situation 1 for State 2 in a discrete part process.

Let β_i be the probability of type II error, that is, the probability that control point falls inside control limits after the occurrence of the first assignable cause. Thus, $\beta_i = \Phi(k-\delta_i\sqrt{n}) - \Phi(-k-\delta_i\sqrt{n})$, where $\Phi(\cdot)$ is the cumulative distribution of the standard normal.

Define $E(T_1)$ as the expected time that is from the occurrence of the first assignable cause A_i to the cause A_i has detected, discovered, and removed, which can be expressed as:

$$
\begin{aligned}
E(T_1) &= \left\{ \sum_{k=1}^{\infty} \left[\sum_{j=0}^{\infty} \int_{W_j}^{W_{j+1}} f_i(t)dt \cdot \left(W_{j+k} - W_j\right) \right] \cdot \left(\beta_i e^{-\lambda' h_1^{\upsilon}}\right)^{k-1} \left(1-\beta_i\right) \right\} - \tau_i + Z_1 + Z_{2i} \\
&= \left\{ \left(1-\beta_i\right) \cdot \sum_{k=1}^{\infty} \sum_{j=0}^{\infty} (1-p_i)^j p_i \cdot \left(W_{j+k} - W_j\right) \cdot (\beta_i{}')^{k-1} \right\} - \tau_i + Z_1 + Z_{2i} \\
&= \frac{(1-\beta_i)\cdot P_i(1-P_i)}{(1-P_i-\beta_i{}')(1-\beta_i{}')} P_i h_1 A_{(1-P_i)} - \frac{(1-\beta_i)\cdot \beta_i{}'}{1-P_i-\beta_i{}'} P_i h_1 A_{(\beta_i{}')} - \tau_i + Z_1 + Z_{2i},
\end{aligned}
\tag{2.7}
$$

where $\beta_i{}' = \beta_i e^{-\lambda' h_1^{\upsilon}}$, $p_i = 1 - e^{-\lambda_i h_1^{\upsilon}}$, and τ_i is given by Equation (2.5).

Situation 2:

Consider Situation 2 for Figure 2.2. Situation 2 is the period that from the occurrence of the first assignable cause to the occurrence of the second assignable cause and during this period that the assignable cause A_i is never to be detected.

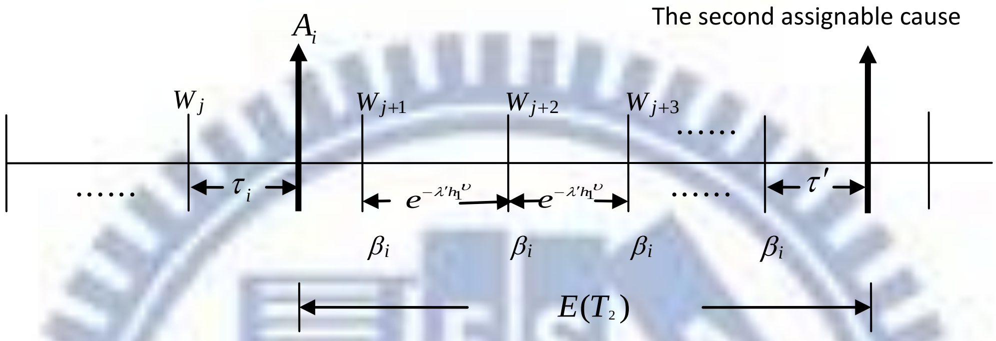

Figure 2.2. The process of Situation 2 for State 2 in a discrete part process.

Let p'_2 be the probability that the second assignable cause occurs between W_{j+i} and $W_{j+(i+1)}$. That is

$$p'_2 = \frac{\int_{W_{j+1}}^{W_{j+2}} f'(t)dt}{\int_{W_{j+1}}^{\infty} f'(t)dt} = \frac{e^{-\lambda W_{j+1}^{\upsilon}} - e^{-\lambda W_{j+2}^{\upsilon}}}{e^{-\lambda W_{j+1}^{\upsilon}}} = 1 - \exp(-\lambda' h_1^{\upsilon}). \tag{2.8}$$

From the above Equation (2.8), the p'_2 is a function of λ', h_1, and υ only. Let $p'_2 = p'$.

Define $E(T_2)$ as the expected time that is from the occurrence of the first assignable cause A_i to the occurrence of the second assignable cause, which can be expressed as:

$$
\begin{aligned}
&E(T_2)\\
&=\left(\sum_{k=1}^{\infty}\left[\sum_{j=0}^{\infty}\int_{W_j}^{W_{j+1}} f_i(t)dt\cdot(W_{j+k}-W_j)\right]\cdot(\beta_i e^{-\lambda' h_1^{\upsilon}})^{k-1}\beta_i(1-e^{-\lambda' h_1^{\upsilon}})\right)-\tau_i+\tau'\\
&=\left(\frac{\beta_i(1-e^{-\lambda' h_1^{\upsilon}})}{\beta_i'}\sum_{k=1}^{\infty}\sum_{j=0}^{\infty}(1-p_i)^j p_i\cdot(W_{j+1+k}-W_{j+1})\cdot(\beta_i')^{k-1}\right)-\tau_i+\tau'\\
&=\left(\frac{\beta_i(1-e^{-\lambda' h_1^{\upsilon}})}{\beta_i'}p_i h_1\sum_{k=1}^{\infty}\sum_{j=0}^{\infty}(1-p_i)^j\cdot\left((j+k)^{\frac{1}{\upsilon}}-j^{\frac{1}{\upsilon}}\right)\cdot(\beta_i')^k\right)-\tau_i+\tau'\\
&=\left(\frac{(\beta_i-\beta_i')(1-p_i)}{1-p_i-\beta_i'}p_i h_1 A_{(1-pi)}-\frac{(\beta_i-\beta_i')\beta_i'}{1-p_i-\beta_i'}p_i h_1 A_{(\beta_i')}-\frac{(\beta_i-\beta_i')}{1-\beta_i'}(1-p_i)p_i h_1 A_{(1-pi)}\right)-\tau_i+\tau',
\end{aligned}
\tag{2.9}
$$

where $\beta_i' = \beta_i e^{-\lambda' h_1^{\upsilon}}$, $p_i = 1-e^{-\lambda_i h_1^{\upsilon}}$, and $\tau' = (1/\lambda')^{1/\upsilon}\Gamma(1+1/\upsilon) - h_1 p'(1-p')A_{(1-p')}$.

Let $AVGOOCT_{2i}$ be the expected time to detect the first assignable cause, once the ith assignable cause A_i has occurred. To summarize the Situation 1 and 2, we obtain the $AVGOOCT_{2i}$ as following:

$$AVGOOCT_{2i} = E(T_1) - Z_1 - Z_{2i} + E(T_2). \tag{2.10}$$

Then the average length of runs in State 2 resulting from the initial assignable cause A_i , respectively, is.

$$E[State2] = E(T_1) + E(T_2). \tag{2.11}$$

(3) State 3:

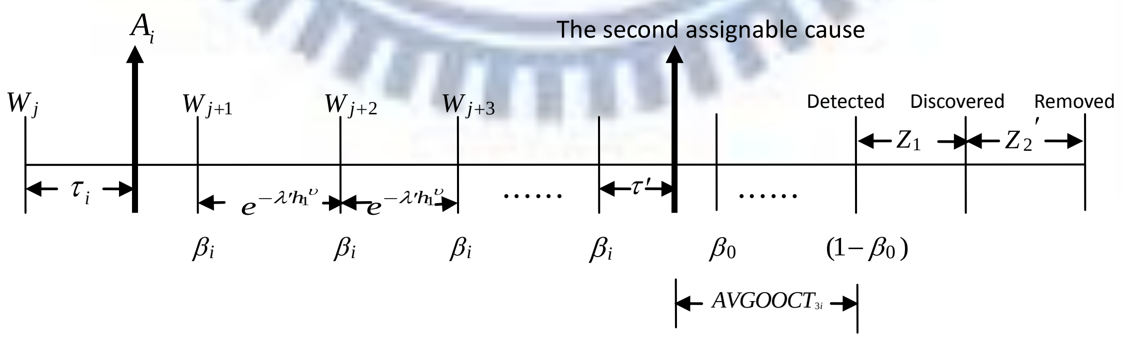

Figure 2.3. The process for State 3 in a discrete part process.

Consider State 3 for Figure 2.3. State 3 is the period that from the occurrence of the second assignable cause to the joint assignable cause detected. The joint effect is assumed to produce a shift of $\Delta\delta\sigma$ in the process mean. Let the probability of detecting a shift of $\Delta\delta\sigma$ be β_0, $\beta_0 = \Phi(k - \Delta\delta\sqrt{n}) - \Phi(-k - \Delta\delta\sqrt{n})$.

Define P_{2i} $(i = 1,\ 2,\ \cdots,\ s)$ as the probability that the second assignable cause will occur after the occurrence of the first assignable cause, which can be expressed as:

$$\begin{aligned} P_{2i} &= \sum_{k=1}^{\infty}\left(\sum_{j=0}^{\infty}\int_{W_j}^{W_{j+1}} f_i(t)dt\right)(\beta_i e^{-\lambda' h_1^{\upsilon}})^{k-1}\cdot\beta_i(1-e^{-\lambda' h_1^{\upsilon}}) \\ &= \beta_i(1-e^{-\lambda' h_1^{\upsilon}})\sum_{k=1}^{\infty}(\beta_i')^{k-1}\sum_{j=0}^{\infty}p_i(1-p_i)^j \\ &= \frac{\beta_i(1-e^{-\lambda' h_1^{\upsilon}})}{1-\beta_i'} = \frac{\beta_i - \beta_i'}{1-\beta_i'}, \end{aligned} \tag{2.12}$$

where $\beta_i' = \beta_i e^{-\lambda' h_1^{\upsilon}}$.

Let $AVGOOCT_{3i}$ be the expected time of detecting the joint assignable cause. The expression for the $AVGOOCT_{3i}$ is given by

$$\begin{aligned} &AVGOOCT_{3i} \\ &= \left\{\left(\sum_{k=1}^{\infty}\left[\sum_{j=0}^{\infty}\int_{W_j}^{W_{j+1}} f'(t)dt\cdot\left(W_{j+k}-W_j\right)\right]\left(\beta_0\right)^{k-1}\left(1-\beta_0\right)\right)-\tau'\right\}\cdot P_{2i} \\ &= \left\{\frac{p'(1-p')}{1-p'-\beta_0}p'h_1A_{(1-p')} - \frac{(1-\beta_0)\beta_0}{1-p'-\beta_0}p'h_1A_{(\beta 0)} - \tau'\right\}\cdot P_{2i}\ . \end{aligned} \tag{2.13}$$

Let $E(State3)$ as the average length of runs in State 3 resulting from the initial assignable cause A_i is

$$E(State3) = AVGOOCT_{3i} + \left(Z_1 + Z_2'\right)\cdot P_{2i}\ . \tag{2.14}$$

Let r_1 be the expected number of samples taken in the "in-control" period. Then the r_1 can be expressed as:

$$r_1 = \sum_{j=1}^{\infty} j\cdot\int_{W_j}^{W_{j+1}} f_0(t)dt = \sum_{j=1}^{\infty} j\cdot(1-p_0)^j p_0 = (\frac{1-p_0}{p_0}), \tag{2.15}$$

where $p_0 = 1 - e^{-\lambda_0 h_1^{\upsilon}}$.

The expected number of false alarms per cycle before the process goes out of control will be α times the expected number of samples taken in the "in-control" period, where $\alpha = 2[1-\Phi(k)]$. The expected number of false alarms per cycle will thus be $\alpha \cdot r_1$. Therefore, the expected time of finding false alarms will be $Z_0 \cdot (\alpha \cdot r_1)$.

The occurrence rate of the assignable cause A_i in total s assignable causes is assumed to be λ_i / λ_0. The average length of runs in some "out-of-control" state that result from the initial assignable cause A_i is the sum of the average run length in State 2 plus the average run length in State 3. The overall mean time for a cycle will thus be

$$E(T) = (\frac{1}{\lambda_0})^{1/\upsilon} \Gamma(1+\frac{1}{\upsilon}) + \sum_{i=1}^{s} \frac{\lambda_i}{\lambda} \left(E(State2) + E(State3) \right) + Z_0 \cdot \alpha \frac{1-p_0}{p_0}. \qquad (2.16)$$

2.3. Formulation of the expected loss-cost per cycle

Based upon the above derivation of the expected cycle length, the ingredients of expected loss-cost per cycle $E(C)$ are as follows:

(1) The expected quality cost per cycle of in-control state is

$$D_0 \cdot (\frac{1}{\lambda_0})^{1/\upsilon} \Gamma(1+\frac{1}{\upsilon}). \qquad (2.17)$$

(2) The expected cost per cycle of out-of-control state is

$$\sum_{i=1}^{s} \frac{\lambda_i}{\lambda_0} \left[D_{1i} \cdot AVGOOCT_{2i} + D_1' \cdot AVGOOCT_{3i} \right]. \qquad (2.18)$$

(3) The expected cost per cycle to locate and repair the assignable cause is

$$\sum_{i=1}^{s} \frac{\lambda_i}{\lambda_0} \left[(1-P_{2i}) w_i + P_{2i} \cdot w' \right]. \qquad (2.19)$$

(4) The expected sampling cost per cycle is

$$(a+bn) \cdot \sum_{i=1}^{s} \frac{\lambda_i}{\lambda_0} \cdot (r_1 + r_{2i} + r_{3i}). \qquad (2.20)$$

(5) The expected cost per cycle of finding false alarms is

$$Y\cdot(\alpha\cdot r_1). \tag{2.21}$$

(6) Let r_{2i} be the expected number of samples taken in State 2 when the assignable cause A_i has occurred. Let r_{3i} be the expected number of samples taken in State 3. The r_{2i} and r_{3i} are expressed as Equation (2.22) and (2.23).

$$\begin{aligned}
r_{2i} &= \left(\sum_{k=1}^{\infty} k\cdot\left[\sum_{j=0}^{\infty}\int_{w_j}^{w_{j+1}} f_i(t)dt\right]\cdot(\beta_i\exp(-\lambda' h_1^{\upsilon}))^{k-1}(1-\beta_i)\right)\\
&\quad+\left(\sum_{k=1}^{\infty} k\cdot\left[\sum_{j=0}^{\infty}\int_{w_j}^{w_{j+1}} f_i(t)dt\right]\cdot(\beta_i\exp(-\lambda' h_1^{\upsilon}))^{k-1}\beta_i(1-\exp(-\lambda' h_1^{\upsilon}))\right)\\
&=\frac{1-\beta_i}{(1-\beta_i')^2}+\frac{\beta_i(1-\exp(-\lambda' h_1^{\upsilon}))}{(1-\beta_i')^2}\\
&=\frac{1-\beta_i'}{(1-\beta_i')^2},
\end{aligned} \tag{2.22}$$

where $\beta_i' = \beta_i\exp(-\lambda' h_1^{\upsilon})$.

$$\begin{aligned}
r_{3i} &= P_{2i}\cdot\left(\sum_{k=1}^{\infty} k\cdot\left[\sum_{j=0}^{\infty}\int_{w_j}^{w_{j+1}} f_i(t)dt\right]\cdot(\beta_0)^{k-1}(1-\beta_0)\right)\\
&= P_{2i}\cdot\left((1-\beta_0)\sum_{k=1}^{\infty} k\cdot(\beta_0)^{k-1}\right)\\
&= P_{2i}\cdot\frac{1}{1-\beta_0}.
\end{aligned} \tag{2.23}$$

To summarize, we obtain the expected loss-cost per cycle as following:

$$\begin{aligned}
E(C) &= D_0\cdot\left(\frac{1}{\lambda_0}\right)^{1/\upsilon}\Gamma\left(1+\frac{1}{\upsilon}\right)\\
&\quad+\sum_{i=1}^{s}\frac{\lambda_i}{\lambda_0}\left[D_{1i}\cdot AVGOOCT_{2i}+D_1'\cdot AVGOOCT_{3i}+(1-P_{2i})w_i+P_{2i}\cdot w'\right]\\
&\quad+\left[(a+bn)\cdot\sum_{i=1}^{s}\frac{\lambda_i}{\lambda_0}(r_1+r_{2i}+r_{3i})\right]\\
&\quad+Y\cdot\alpha\cdot\left(\frac{1-p_0}{p_0}\right).
\end{aligned} \tag{2.24}$$

Finally, the expected loss-cost function $E(A)$ is constructed by the $E(T)$ and $E(C)$. Our objective is to find the optimal design parameters n, h_1 and k to minimize the $E(A)=E(T)/E(C)$ based on the given values of time, cost and shift parameters.

2.4. Determination of optimal design parameters

The parameters involving in the expected loss-cost function can be classified into cost parameters ($Y; D_0; D_{1i}; D_1'; w_i; \mathrm{w}'; a\ ; b$), time parameters ($\mathrm{Z}_0; Z_1; \mathrm{Z}_{2i}; Z_2'$), shift parameters ($\delta_i; \Delta\delta$), Weibull distribution parameters ($\lambda_i;\ \upsilon$) and design parameters ($n\ ; h_1\ ; L$). A numerical example is used to illustrate the performance of the model. We assume that $Y=\$2000$ per false alarm, $D_0=\$210$ per unit time, $D_1'=\$4000$ per unit time, $a=\$20$ per sampled, $b=\$20$ per unit sampled, $w'=\$1000$ per cycle, $Z_0=1.25$ hours, $Z_1=1.25$ hours, $Z_2'=2$ hours, $\lambda'=0.02$, $\Delta\delta=2$ and they remain the same for the different assignable causes. D_{1i}, w_i, Z_{2i}, and λ_i are taken to be a function of δ_i and the rules of selection are as follows:

(1) The λ_i is a non-increasing function of the δ_i for all *i*. When the cause A_i occurs, μ_0 will shift to $\mu_0 \pm \delta_i\sigma$. D_{1i} is proportional to the resulting increase in the percent of product outside specification ($\beta_{0i}=1-\Phi(3-\delta_i)+\Phi(-3-\delta_i)\approx 1-\Phi(3-\delta_i)$, for $i=1,\ 2,\ \cdots,\ s$). From the repeated production experiment, the more the occurrence of shift, the lower values of w_i and Z_{2i} are.

(2) Assume the process exists seven assignable causes $(A_i,\ i=1,\ 2,\ \cdots,\ 7)$, those causes will produce 1σ, 1.5σ, 1.8σ, 2σ, 2.2σ, 2.5σ, and 3σ shift amount, respectively, the occurrence of any one assignable cause are random and independent.

(3) Z_{2i}, D_{1i}, and w_i are functions of δ_i. Assume that $\delta_i=2$, D_{1i}, w_i, and Z_{2i} are set to be \$4000, \$1000, and 2 hours.

(4) Owing to obtain λ_i, we assume that $\sum_{i=1}^{s} \lambda_i \cdot D_{1i} = \lambda \cdot D_1 = 80$.

Let PD_i denoted the prior distribution of δ_i ($\delta_1 = 1,\ \delta_2 = 1.5,\ \delta_3 = 1.8, \cdots,\ \delta_7 = 3$). In this section, the negative-exponential, uniform, and half- normal distribution are considered for PD_i. Let the prior distribution of δ_4 =2 be PD_4. We set up the values of the time parameters and the cost parameters for δ_4 =2 as "base case", and in one set, λ_i are chosen as proportional to PD_i. According to the discussion of above (1), (2), (3), and (4) rules, we assume that $w_i = (PD_i / PD_4) \times 1000$, $Z_{2i} = (PD_i / PD_4) \times 2$, $D_{1i} = (\beta_{0i} / \beta_{04}) \times 4000$ since δ_4 =2 and $\lambda_i = (PD_i / PD_1) \times \lambda_1$. The values of w_i, Z_{2i}, and λ_i for different prior distributions and the values of D_{1i} for different δ_i are displayed in Table 2.1.

Table 2.1
The set of cost and probability parameters.[a]

A_i	δ_i	β_{0i}	PD_i			D_{1i}	Z_{2i}			w_i			$\lambda_i (\times 10^{-3})$		
			NE_i	Un_i	HN_i		NE_i	Un_i	HN_i	NE_i	Un_i	HN_i	NE_i	Un_i	HN_i
1	1	0.0228	0.303	0.143	0.352	575	3.29	2	2.90	1647	1000	1454	4.566	2.294	4.220
2	1.5	0.0668	0.236	0.143	0.301	1684	2.56	2	2.48	1283	1000	1244	3.557	2.294	3.608
3	1.8	0.1151	0.203	0.143	0.266	2901	1.10	2	2.19	1103	1000	1099	3.059	2.294	3.190
4	2	0.1587	0.184	0.143	0.242	4000	2	2	2	1000	1000	1000	2.772	2.294	2.901
5	2.2	0.2119	0.166	0.143	0.218	5341	1.80	2	1.80	902	1000	901	2.502	2.294	2.612
6	2.5	0.3085	0.143	0.143	0.183	7776	1.55	2	1.51	777	1000	756	2.155	2.294	2.194
7	3	0.5	0.112	0.143	0.130	12602	1.21	2	1.07	609	1000	537	1.689	2.294	1.557

[a] NE_i, Negative-exponential; Un_i, Uniform; HN_i, Half-normal.

2.5. Comparison with Chen and Yang's model

Chen and Yang [8] assumed once assignable cause A_i occurs, it continues to affect the process until it is detected by the control chart, and during this period it is undisturbed by the occurrence of other assignable causes. Owing to compare the model, the computer program developed in this work is available from us. We used search technique which is developed by Rahim [32] to find the optimal values of the decision variables n, h_1, and

k. Table 2.2 compares the effect that variation in Weibull parameter values has on the economic design of $\bar{x}$-control charts. With the same data, for several sets of υ, cost comparisons between the modified model and Chen and Yang's model for different sets of Weibull shape parameter.

Table 2.2 indicates the following general conclusions:

(1) When shape parameter υ increases, the sampling interval h_1 and the expected loss-cost increase significantly and then decrease.

(2) When shape parameter υ increases, the control limit coefficient L decrease, but there is no significant change on the sampling size n.

(3) Comparisons between the modified model and Chen and Yang's model, Chen and Yang's model has smaller L value but larger h_1 than the modified model. Roughly, the range of difference of loss-cost between the two models is from 10% to 38%. and

(4) When $\upsilon = 1$ (that is exponential distribution), the modified model will have larger loss-cost than Chen and Yang's model. But there is no significant differences on the control limit coefficient L between the two models.

(5) Among the economic design for three prior distributions, the half-normal prior distribution is the best while uniform prior distribution is the worse. The range of difference of loss-cost between half-normal and uniform prior distribution is from 3% to 12%.

(6) For the modified model, possibly we should use charts with 2 sample size.

Table 2.2
Optimal design parameters between the modified model and Chen and Yang's model at three different prior distributions as υ changes .[a]

υ	Prior Distribution	Model	Optimal design			$E(A)=E(C)/E(T)$
			n	h_1	L	
1	NE	modified model	3	1.01	2.66	390.691
		Chen and Yang's model	3	1.39	2.51	353.500
	Un	modified model	2	0.83	2.61	384.760
		Chen and Yang's model	3	1.39	2.59	346.356
	HN	modified model	3	1.01	2.66	389.378
		Chen and Yang's model	3	1.39	2.51	352.114
1.5	NE	modified model	2	1.60	2.59	472.544
		Chen and Yang's model	3	2.62	2.44	399.002
	Un	modified model	2	1.59	2.66	482.880
		Chen and Yang's model	3	2.60	2.56	398.134
	HN	modified model	2	1.60	2.58	468.948
		Chen and Yang's model	3	2.61	2.45	394.466
2	NE	modified model	2	1.73	2.66	500.725
		Chen and Yang's model	3	2.75	2.40	410.598
	Un	modified model	2	1.72	2.72	526.204
		Chen and Yang's model	3	2.73	2.53	411.885
	HN	modified model	2	1.73	2.65	495.239
		Chen and Yang's model	3	2.74	2.40	403.104
3	NE	modified model	2	1.63	2.83	504.199
		Chen and Yang's model	3	2.45	2.32	406.463
	Un	modified model	2	1.61	2.86	546.441
		Chen and Yang's model	3	2.42	2.47	405.930
	HN	modified model	2	1.62	2.81	497.603
		Chen and Yang's model	3	2.44	2.32	395.317
4	NE	modified model	2	1.51	2.98	493.117
		Chen and Yang's model	3	2.16	2.25	397.437
	Un	modified model	2	1.49	3.00	543.670
		Chen and Yang's model	3	2.13	2.41	393.925
	HN	modified model	2	1.50	2.95	486.836
		Chen and Yang's model	3	2.15	2.26	384.435

[a] NE , Negative-exponential; Un , Uniform; HN , Half-normal.

Chapter 3

Economic Design of $\bar{x}$-Control Charts for Continuous Flow Process with Multiple Assignable Causes

In this chapter, we study the economic design of $\bar{x}$-control charts for continuous flow process with multiple assignable causes. A modified version of Koo and Case's model [23] for the $\bar{x}$-control charts is proposed to deal with situations involving the multiple assignable causes. In Koo and Case's model [23], it is assumed that once an assignable cause occurs, no further assignable causes will occur. To ascertain the effect of this assumption, a study is made in this chapter that allows for the second occurrence of an assignable cause following the first occurrence. The probability of the assignable causes following exponential distribution and the process-failure mechanism having a fixed hazard rate. In addition, an economic approach is developed in this chapter for the design of $\bar{x}$-control charts. Therefore, we adopt Duncan's multiple causes model [15], Koo and Case's sampling scheme [23] and cost structure of Koo et al. [24] to develop a modified model. A modified model is the economic design of $\bar{x}$-control charts for continuous flow process which subjects to a multiplicity of special causes. Thus, the expected loss-cost function is presented and the optimal values of the design parameters (the sample size, the sampling interval and control limit coefficient) are determined by minimizing the expected loss-cost function.

This chapter is organized as follows: In Section 3.1, we give some basic definition and assumptions of the model under study and give some notations. In Section 3.2, we propose the formulation of the expected cycle length. Section 3.3 constructs the expected loss-cost function. In Section 3.4, we determine the optimal design parameters. Finally, sensitivity analysis are suggested in Section 3.5.

3.1 Definitions, Assumptions and Notations

The definitions and assumptions considered in our model are as follows:

(9) The occurrence time of the ith assignable cause (denoted as A_i, $i = 1, 2, \cdots, s$) that the process remains in the in-control state follows an exponential distribution and the probability density function is given by

$$f_i(t) = \lambda_i e^{-\lambda_i t^{\upsilon}}, \ t > 0, \ \ \lambda_i > 0, \ i = 1, \ 2, \ \cdots, \ s. \tag{3.1}$$

(2) The time at which the process goes out of control is distributed as the minimum of n independent exponentials with means $1/\lambda_1$, $1/\lambda_2$, $\cdots$, $1/\lambda_n$ and thus has an exponential distribution with mean $1/\lambda$, where

$$\lambda = \sum_{i=1}^{n} \lambda_i \ . \tag{3.2}$$

(3) The process is normally distributed and characterized by an in-control state μ_0, because of the occurrence of an assignable cause A_i which occurs at random, resulting in a shift in the mean from μ_0 to either $\mu_0 + \delta_i \sigma$ or $\mu_0 - \delta_i \sigma$, where μ_0, σ, and δ_i are, respectively, the process mean, the process standard deviation, and shift parameter.

(4) The occurrence of an assignable cause A_i does not affect the process variability, that is, the process mean and the process variability are independent.

(5) The process mean is not shifting slowly, but instantaneously.

(6) The time to sample and draw control point is negligible and production ceases during the searches and repair.

(7) Define p_{ij} $(i = 1, 2...., s)$ as the probability that the assignable cause A_i will occur during the sampling interval j^{th} and $(j+1)^{st}$, that is

$$P_{ij} = \frac{\int_{jh}^{(j+1)h} \lambda_i e^{-\lambda_i t} dt}{\int_0^{nh} \lambda_i e^{-\lambda_i t} dt} = \frac{e^{-\lambda_i (j+1)h} + e^{-\lambda_i jh}}{1 - e^{-\lambda_i nh}} = \frac{e^{-\lambda_i jh}(1 - e^{-\lambda_i h})}{1 - e^{-\lambda_i nh}}, \text{ for } i = 1,\ 2,\ \cdots,\ s. \quad (3.3)$$

When the process is out-of-control, the mean of the process will shift to $\mu_0 + \delta_i \sigma$. If the shift occurs during the sampling interval j^{th} and $(j+1)^{st}$, then the mean of the process in this subgroup will be $\mu = \mu_0 + ((n-j)/n) \cdot \delta_i \sigma$. Let the probabilities of detecting an assignable cause in a shift occurring subgroup and the next subsequent subgroups after the occurrence of assignable cause A_i be P_i' and P_i, respectively, P_i' and P_i are formulated as follows:

$$P_i' = \sum_{j}^{n-1} P_{ij} \cdot \left[1 - \Phi\left(k - \frac{n-j}{\sqrt{n}} \delta_i \right) + \Phi\left(-k - \frac{n-j}{\sqrt{n}} \delta_i \right) \right], \quad (3.4)$$

$$P_i = 1 - \Phi\left(k - \delta_i \sqrt{n}\right) + \Phi\left(-k - \delta_i \sqrt{n}\right), \quad (3.5)$$

where $\Phi(\cdot)$ is the cumulative density function of the standard normal distribution.

(8) Let τ_i be the expected time between the samples taken just prior to the occurrence of assignable cause A_i and the occurrence itself, that is

$$\tau_i = \frac{\int_{jnh}^{(j+1)nh} e^{-\lambda_i t} \lambda_i (t - jnh) dt}{\int_{jnh}^{(j+1)nh} e^{-\lambda_i t} \lambda_i dt} = \frac{e^{-\lambda_i jnh} \int_0^{nh} t e^{-\lambda_i t} \lambda_i dt}{e^{-\lambda_i jnh} \int_0^{nh} e^{-\lambda_i t} \lambda_i dt} = \frac{1 - (1 + \lambda_i nh) e^{-\lambda_i nh}}{\lambda_i (1 - e^{-\lambda_i nh})}. \quad (3.6)$$

(9) Define $\hat{p}_{ij}$ $(i = 1,\ 2,\ \cdots,\ s)$ as the probability that a second assignable cause will occur during the sampling interval j^{th} and $(j+1)^{st}$ after the occurrence of assignable cause A_i, and p_{ij} is given by Equation (3.3), $\hat{p}_{ij}$ is the same formula with λ' replacing λ_i.

Let the probabilities of detecting joint assignable causes in a shift occurring subgroup and the next subsequent subgroups be $\hat{P}_i'$ and $\hat{P}_i$, respectively, $\hat{P}_i'$ and $\hat{P}_i$ are formulated as follows:

$$\hat{P}_i{}' = \sum_{j=0}^{n-1} \hat{P}_{ij} \cdot (1-\beta_0)$$

$$= \sum_{j=0}^{n-1} \left[\frac{(1-e^{-\lambda' h})e^{-\lambda' jh}}{(1-e^{-\lambda' nh})} \right] \left[1-\Phi\left(k - \frac{n\Delta\delta + j(\delta_i - \Delta\delta)}{\sqrt{n}} \right) + \Phi\left(-k - \frac{n\Delta\delta + j(\delta_i - \Delta\delta)}{\sqrt{n}} \right) \right], \quad (3.7)$$

$$\hat{P}_i = 1-\Phi\left(k - \Delta\delta\sqrt{n}\right) + \Phi\left(-k - \Delta\delta\sqrt{n}\right). \quad (3.8)$$

In this chapter, the following notations shall be used in the formulation of the loss-cost function.

n — the sample size (decision variable)

h — the interval between samples measured in hours (decision variable)

k — the control limit coefficient (decision variable)

e — the average time of sampling, inspecting, evaluating and plotting

D_i — the average time taken to find assignable cause A_i after a point has been found to fall outside the control limits , where $i = 1, 2, \cdots, s$

D' — the average time taken to find the combined assignable causes after a point has fallen outside the control limits

M_i — the increased loss per hour of operation due to the presence of assignable cause A_i

M' — the additional loss per hour of operations when the process is in State 3

T — the average cost of looking for an assignable cause when a false alarm occurs

w_i — the average cost of finding assignable cause A_i when it occurs , where $i = 1, 2, \cdots, s$

w' — the average cost of finding the combined assignable causes, assumed to be independent of the assignable causes

c — the variable cost per item of sampling, inspecting, evaluating and plotting

b — the fixed cost per sampling of sampling, inspecting, evaluating and plotting

3.2 Formulation of the Expected Cycle Length

We assume that there are s possible assignable causes. The occurrence time of any one assignable cause is assumed to be independently exponentially distributed with mean time $1/\lambda_i$ for $i=1, 2, \cdots, s$. The occurrence time of the first assignable cause has a exponential distribution with mean time $1/\lambda$ where λ is the summation of λ_i and the occurrence time of a second assignable cause has a exponential distribution with mean time $1/\lambda^{'}$ where $\lambda^{'}$ is a function of λ. After being disturbed by an assignable cause A_i, the process will be affected by any other assignable causes. In the other hand, if the first assignable cause continues undetected, the second assignable cause (possibly a repetition of the first) is assumed to occur at random in a later intervals.

The process is assumed to be in one of the three states. It is (1) in a state of in-control or (2) it has been disturbed by the occurrence of an assignable cause A_i which produces a shift of $\delta_i\sigma$ in the process mean or (3) it has been disturbed by the occurrence of a second assignable cause following the first, the joint effect of which in every case is arbitrarily assumed to produce a shift of $\Delta\delta\sigma$ in the process mean. The expected cycle length consists of three states, which can be derived as follows:

(1) State 1:

The probability at time t in control is

$$P(T>t)=P(A_1>t, A_2>t, \cdots, A_s>t)=P(A_1>t)\cdot P(A_2>t)\cdot\cdots\cdot P(A_s>t)=e^{-\lambda t}, \quad (3.9)$$

where $\lambda=\sum_{i=1}^{s}\lambda_i$ and then the probability density function of occurrence of multiple assignable causes will be $f(t)=\lambda e^{-\lambda t},\ \lambda>0,\ t\geq 0$, thus the average time in control is

$1/\lambda$.

Therefore, the process is in an in-control state and the expected time that the assignable cause will occur is $1/\lambda$.

(2) State 2:

The process has been disturbed by the occurrence of the first assignable cause A_i and produces a shift of $\delta_i\sigma$ in the process mean. When the process is in State 2, it is assumed that no further disturbance occurs until after the first sample is taken. The process can be classified into two situations.

Situation 1:

Consider Situation 1 for Figure 3.1. The process is the period that the second assignable cause will not to be occurring until assignable cause A_i is detected. From the above Equations (3.4) and (3.5), the probability that a point falls outside the control limits at the first sampling interval or at the other sampling interval after the occurrence of the first assignable cause is P_i' or $(1-P_i')e^{-\lambda' nh}[(1-P_i)e^{-\lambda' nh}]^{r-2}\cdot P_i,\ r=2,\ 3,\ \cdots$.

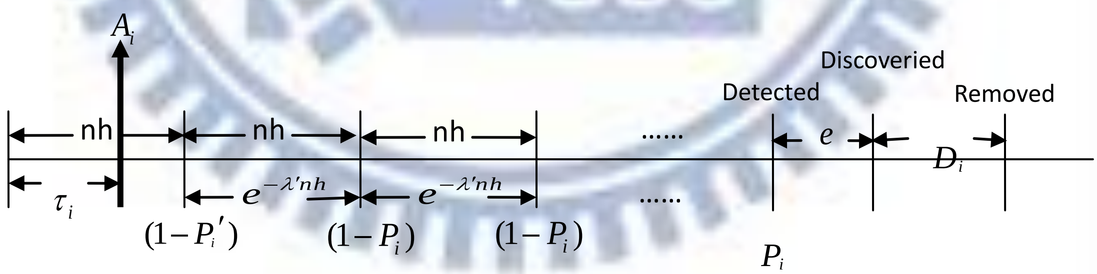

Figure 3.1. The process of Situation 1 for State 2 in a continuous flow process.

Then let the average time in Situation 1 be E_1. The E_1 is

$$E_1 = nh\left\{P_i' + \sum_{r=2}^{\infty} r \cdot \left[(1-P_i')e^{-\lambda' nh}\left((1-P_i)e^{-\lambda' nh}\right)^{r-2} \cdot P_i\right]\right\} - \tau_i + e + D_i$$

$$= nh\left[P_i' + r_i' P_i \frac{2-r_i}{(1-r_i)^2}\right] - \tau_i + e + D_i, \tag{3.10}$$

where $r_i' = (1-P_i')e^{-\lambda' nh}$ and $r_i = (1-P_i)e^{-\lambda' nh}$.

Situation 2:

Consider Situation 2 for Figure 3.2. The process is from the occurrence of the first assignable A_i to the occurrence of a second assignable cause, during the period that the cause A_i is never to be detected. The conditional probability that a second assignable cause will occur between the first and the second subgroup is $(1-P_i')(1-e^{-\lambda' nh})$, then will occur between the $(r+1)$ and the $(r+2)$ subgroup is $(1-P_i')(1-e^{-\lambda' nh})[(1-P_i)e^{-\lambda' nh}]^{r-1}$, $r = 1, 2, \cdots$.

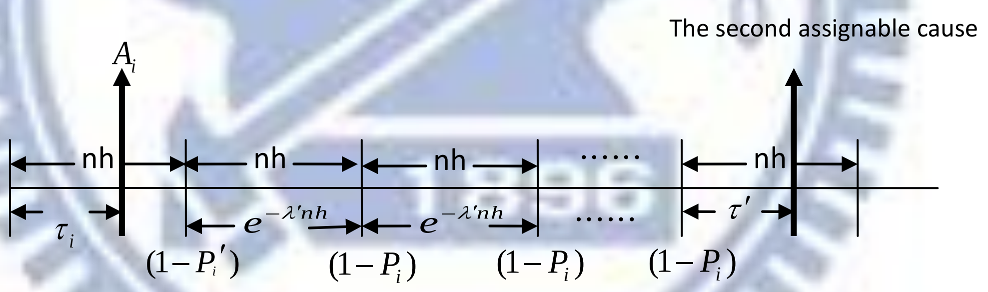

Figure 3.2. The process of Situation 2 for State 2 in a continuous flow process.

Let the average time in Situation 2 be E_2. The E_2 is

$$E_2 = nh\left\{\sum_{r=1}^{\infty} r(1-P_i')(1-e^{-\lambda' nh})\left[(1-P_i)e^{-\lambda' nh}\right]^{r-1}\right\} - \tau_i + \tau'$$

$$= nh\left[\frac{1-P_i'-r_i'}{(1-r_i)^2}\right] - \tau_i + \tau', \tag{3.11}$$

where τ_i is given by Equation (3.6), τ' is the same formula and definition with λ' replacing λ_i.

It follows from what has been derived that the average time of State 2, respectively, is $E[State2]$.

$$E[State2] = E_1 + E_2$$
$$= \left\{ nh\left[P_i' + r_i' P_i \frac{2-r_i}{(1-r_i)^2} \right] - \tau_i + e + D_i \right\} + \left\{ nh\left[\frac{1-P_i' - r_i'}{(1-r_i)^2} \right] - \tau_i + \tau' \right\}. \quad (3.12)$$

(3) State 3:

Consider State 3 for Figure 3.3. State 3 is the period that from the occurrence of the first assignable A_i to the joint assignable cause detected. The joint effect is to produce a shift of $\Delta\delta\sigma$ in the process mean. Define $\beta_0 = \Phi(k - \Delta\delta\sqrt{n}) - \Phi(-k - \Delta\delta\sqrt{n})$ is the probability that a point falls inside the control limits after the occurrence of a second assignable cause, then the probability that the joint assignable cause detected is $1-\beta_0$.

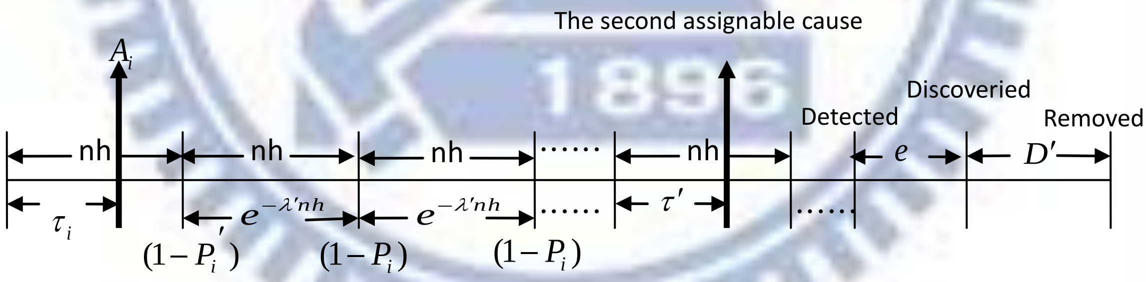

Figure 3.3. The process for State 3 in a continuous flow process.

Let the probability on the first sampling interval of a point falling outside the control limits after the occurrence of joint effect be $\hat{P}_i'$ and the probability on the other sampling interval of a point falling outside the control limits after the occurrence of joint effect be $(1-\hat{P}_i')(1-\hat{P}_i)^{r-2} \cdot \hat{P}_i$, for $r = 2,\ 3,\ \cdots$.

The D' is assumed to be independent of the assignable causes. The D' is not changed by the joint effect of cause A_i and a second assignable cause. Then the average time of State 3 is given by $E(State3)$, will be

$$E(State3)=\left\{nh\left[\hat{P}_i{}' + \sum_{r=2}^{\infty} r(1-\hat{P}_i{}')(1-\hat{P}_i)^{r-2}\hat{P}_i\right]-\tau'+e+D'\right\}\cdot\left(\frac{1-P_i{}'-r_i{}'}{(1-r_i)}\right)$$

$$=\left[nh\left(\frac{1+\hat{P}_i-\hat{P}_i{}'}{\hat{P}_i}\right)-\tau'+e+D'\right]\cdot\left(\frac{1-P_i{}'-r_i{}'}{(1-r_i)}\right). \tag{3.13}$$

Summing up the various average time of State 1, State 2 and State 3. The overall mean time for a cycle will thus be

$$E(T)=\frac{1}{\lambda}+\frac{\sum_{i=1}^{s}\lambda_i}{\lambda}\left(E(State2)+E(State3)\right). \tag{3.14}$$

3.3. Formulation of the Loss-Cost Function

Based upon the above derivation of the expected cycle length, the ingredients of the expected loss-cost function per unit time $E(C)$ are as follows:

1. The average hourly loss when out of control is

$$L_1=\frac{\frac{\sum_{i=1}^{s}\lambda_i}{\lambda}\left(E(State2)\cdot M_i\right)+\frac{\sum_{i=1}^{s}\lambda_i}{\lambda}\left(E(State3)\cdot M'\right)}{E(T)}. \tag{3.15}$$

2. The expected number of false alarms before the process goes out-of-control will be the probability of a false alarm (α) times the expected number of subgroups taken in an in-control period. Hence, the expected number (ENF) of false alarms per hour of operation will be $ENF=\alpha\cdot[\exp(-\lambda nh)/(1-\exp(-\lambda nh))$. Thus, the average hourly false-alarm cost is

$$L_2=\frac{ENF\cdot T}{E(T)}. \tag{3.16}$$

3. The average hourly cost of finding and repairing the assignable cause is

$$L_3 = \frac{\frac{\sum_{i=1}^{s}\lambda_i}{\lambda}\left[W_i \cdot \left(1-\frac{1-P_i{}'-r_i{}'}{1-r_i}\right)\right] + \frac{\sum_{i=1}^{s}\lambda_i}{\lambda}\left[W' \cdot \left(1-\frac{1-P_i{}'-r_i{}'}{1-r_i}\right)\right]}{E(T)} . \tag{3.17}$$

4. The average hourly cost of maintaining the control chart is

$$L_4 = \frac{b+cn}{nh} = \frac{b}{nh} + \frac{c}{h} . \tag{3.18}$$

To summarize, we obtain the expected loss-cost function as following:

$$E(C) = L_1 + L_2 + L_3 + L_4 . \tag{3.19}$$

Finally, the expected loss-cost function $E(C)$ is constructed. Our objective is to find the optimal design parameters n, h and k to minimize the $E(C)$ based on the given values of time and cost parameters.

3.4. Determination of Optimal Design Parameters

In this section, we will find the optimal design parameters n, h, and k. The parameters involving in the expected loss-cost function ($E(C)$) can be classified into cost parameters (T, b, c, M_i, M', W_i, W'), time parameters (e, D_i, D'), shift parameters (δ_i, $\Delta\delta$), exponential distribution parameters (λ_i, λ') and design parameters (n, h, k). A numerical example will be used to illustrate of Koo and Case Model [23], value for T=2000, b=20, c=20, M'=4000, W'=1000, $\Delta\delta$=2, λ'=0.02, D'=2, e=1.25 are not changed by assignable cause A_i. The M_i, W_i, D_i and λ_i are taken to be a function of, δ_i the rule of selection is as follows:

(1) The λ_i is a non-increasing function of δ_i. When the cause A_i occurs, μ_0 shift to $\mu_0+\delta_i\sigma$, M_i is proportional to the resulting increase in the percent of product outside specification $(1-\beta_i)$, where $\beta_i = \Phi(3-\delta_i)$. As δ_i varies above and

below 2, the percent beyond specifications increases and decreases to cause corresponding variations in M_i.

(2) Assume the process exists seven assignable causes $(A_i,\ i=1,\ 2,\ \cdots,\ 7)$. Those causes will produce 1σ, 1.5σ, 1.8σ, 2σ, 2.2σ, 2.5σ and 3σ shift and the occurrence of each assignable cause randomly and indecently produce single shift.

(3) Like that D_i, M_i and W_i are function of δ_i. For δ_i=2, M_i, W_i and D_i is equal to 4000, 1000 and 2.

(4) When the parameters, T, b, c and e, are kept fixed, the numerical examples used. For example, the parameter $\Delta\delta$ is varied from 1 to 1.5 to 2 to 2.5. The parameter λ' is varied from 0.005 to 0.01 to 0.02 to 0.04. Then D', M' and W' are the same as D_i, M_i and W_i of δ_i =2. Owing to obtain λ_i, assume $\sum_{i=1}^{s}\lambda_i M_i = \lambda \cdot M' = 80$.

(5) When the parameters, $\Delta\delta$, λ', D', M' and W', are kept fixed, the numerical examples used. For example, the parameter T is varied from 1000 to 2000 to 3000. The parameter b is varied from 10 to 20 to 30. The parameter c is varied from 10 to 20 to 30. The parameter e is varied from 0.625 to 1.25 to 1.875.

Let PD_i denoted prior distribution of $\delta_i\ (\delta_1=1,\ \delta_2=1.5,\ \delta_3=1.8,\ \cdots,\ \delta_7=3)$. In this section, the negative-exponential, uniform and half- normal are considered for PD_i. Prior distribution of δ_4=2 is PD_4, we set up the time and cost values for δ_4=2 as “base case”, and in one set, λ_i are chosen as proportional to PD_i. According to the discussion of above (1), (2), (3), (4) and (5) rules, we have $W_i=(PD_i/PD_4)\times 1000$, $D_i=(PD_i/PD_4)\times 2$, $M_i=(PD_i/PD_4)\times 4000$, and $\lambda_i=(PD_i/PD_1)\times\lambda_1$. The values of W_i, D_i and λ_i for different prior distribution and the values of M_i for different δ_i are listed in Table 3.1.

Table 3.1
The reference set of cost and probability parameters.[a]

A_i	δ_i	$1-\beta_i$	PD_i			M_i	D_i			W_i			$\lambda_i(\times 10^{-3})$		
			NE_i	Un_i	HN_i		NE_i	Un_i	HN_i	NE_i	Un_i	HN_i	NE_i	Un_i	HN_i
1	1	0.0228	0.303	0.143	0.352	575	3.293	2	2.909	1647	1000	1454	4.566	2.294	4.220
2	1.5	0.0668	0.236	0.143	0.301	1684	2.565	2	2.488	1283	1000	1244	3.557	2.294	3.608
3	1.8	0.1151	0.203	0.143	0.266	2901	1.103	2	2.198	1103	1000	1099	3.059	2.294	3.190
4	2	0.1587	0.184	0.143	0.242	4000	2	2	2	1000	1000	1000	2.772	2.294	2.901
5	2.2	0.2119	0.166	0.143	0.218	5341	1.804	2	1.802	902	1000	901	2.502	2.294	2.612
6	2.5	0.3085	0.143	0.143	0.183	7776	1.554	2	1.512	777	1000	756	2.155	2.294	2.194
7	3	0.5	0.112	0.143	0.130	12602	1.217	2	1.074	609	1000	537	1.689	2.294	1.557

[a] NE_i, Negative-exponential; Un_i, Uniform; HN_i, Half-normal.

3.5. Sensitivity Analysis

We used search technique which is developed by Rahim [32] to determine the optimal design parameters. The code was considered to minimize the expected loss-cost function $E(C)$, and provides economically optimal values of n, h and k. The effects of changes in the cost parameters on the minimum the expected loss-cost function $E(C)$ design are listed in Table 3.2 along with other data.

(1) For λ'=0, the value of $E(C)$ is equal to the value of the model of Koo et al. [24]. Therefore, one result stood out clearly. It was noted that if λ' is decreased, the loss-cost of the model approaches to the loss-cost of the model of Koo et al.

(2) With the same value of the parameters in both models, the loss-cost of the model is larger than the loss-cost of the model of Koo et al., but smaller than the loss-cost of Koo and Case's Model. If the conservative designing point of view is applying, the multiplicity-cause model can replace the single-cause model.

(3) T, b, c and e are kept fixed at the reference values listed in Table 3.2. Variation in $\Delta\delta$, D' and W' has little effect on the loss-cost, but variation in λ' and M' has their dominant effect on the loss-cost.

(4) Among the economic design for three prior distributions, the negative-exponential prior distribution is the best while uniform prior distribution is the worse. But the difference between the loss-cost for half-normal and negative-exponential prior distribution is less than 0.7%.

(5) For the negative-exponential prior distribution, variation in λ' has its primary effect upon the optimal value of k. The sample size and the frequency of sampling are affected moderately. Thus, for large λ', we should use charts with 2.5 sigma limits.

(6) For the negative-exponential prior distribution, variation in M' has its dominant effect on the optimal value of k. When M' is relatively large, k should be small; when M' is relatively small, k should be large. Variation in M' has little effect on the optimal values of n and h.

(7) $\Delta\delta$, λ', M', D', and W' are kept fixed at the reference values listed in Table 3.2. Variation in T and b has little effect on the loss-cost, but variation in c and e has more effect on the loss-cost.

(8) For the negative-exponential prior distribution, variation in c affects all three of the elements of design. For high values of c, the optimal design calls for taking small samples, possibly only samples of 2, at large intervals between samples and with control limits at low multiples of sigma.

(9) For the negative-exponential prior distribution, variation in e affects primarily the optimal value of k, possibly we should use charts with 2.5 sigma limits. It also has a moderate affect on the frequency of sampling.

Table 3.2

Optimum design parameters for model at three different prior distributions [a]

T	b	c	e	$\Delta\delta$	λ'	M'	D'	W'	Prior Distribution	Optimal design n	h	k	loss-cost
2000	20	20	1.25	1	0.02	4000	2	1000	*NE*	3	0.362	2.471	429.260
									Un	3	0.352	2.568	442.129
									HN	3	0.365	2.495	431.020
				1.5					*NE*	3	0.367	2.538	423.742
									Un	2	0.451	2.587	438.291
									HN	3	0.346	2.592	427.627
				2					*NE*	3	0.368	2.557	422.427
									Un	2	0.433	2.621	437.589
									HN	2	0.464	2.498	428.724
				2.5					*NE*	3	0.369	2.560	422.115
									Un	2	0.415	2.617	440.229
									HN	2	0.428	2.524	433.159
				2	0.005	4000	2	1000	*NE*	2	0.483	2.512	417.650
									Un	2	0.442	2.637	433.239
									HN	2	0.464	2.504	421.480
					0.01				*NE*	2	0.438	2.545	420.378
									Un	2	0.510	2.507	435.130
									HN	2	0.442	2.529	424.079
					0.02				*NE*	2	0.487	2.445	425.596
									Un	2	0.433	2.621	437.589
									HN	2	0.464	2.498	428.724
					0.04				*NE*	2	0.423	2.481	433.107
									Un	2	0.457	2.565	442.146
									HN	2	0.407	2.502	436.311
				2	0.02	575	2	1000	*NE*	2	0.444	2.585	410.458
									Un	2	0.505	2.556	427.322
									HN	2	0.444	2.585	414.505
						1684			*NE*	2	0.459	2.514	415.286
									Un	2	0.460	2.545	430.957
									HN	2	0.496	2.500	419.230
						4000			*NE*	2	0.487	2.445	425.596
									Un	2	0.433	2.621	437.589
									HN	2	0.464	2.498	428.724
						7776			*NE*	2	0.461	2.407	440.185
									Un	2	0.455	2.518	447.608
									HN	2	0.434	2.418	442.769

[a] *NE*, Negative-exponential; *Un*, Uniform; *HN*, Half-normal.

Table 3.2-(Continued) [a]

T	b	c	e	$\Delta\delta$	λ'	M'	D'	W'	Prior Distribution	Optimal design n	h	k	loss-cost
2000	20	20	1.25	2	0.02	4000	3.3	1000	NE	2	0.400	2.555	428.969
							0.5		Un	2	0.416	2.619	435.097
							2.9		HN	2	0.400	2.555	431.435
							2.6		NE	2	0.400	2.555	427.119
							1		Un	2	0.483	2.565	435.798
							2.5		HN	2	0.487	2.445	430.322
							2		NE	2	0.487	2.445	425.596
							2		Un	2	0.433	2.621	437.589
							2		HN	2	0.464	2.498	428.724
							1.6		NE	2	0.429	2.527	423.969
							3		Un	2	0.433	2.621	439.453
							1.5		HN	2	0.429	2.527	427.420
				2	0.02	4000	2	1647	NE	2	0.487	2.445	426.058
								250	Un	2	0.472	2.512	437.242
								1454	HN	2	0.487	2.445	429.413
								1283	NE	2	0.487	2.445	425.798
								500	Un	2	0.472	2.512	437.362
								1244	HN	2	0.464	2.498	428.898
								1000	NE	2	0.487	2.445	425.595
								1000	Un	2	0.433	2.621	437.589
								1000	HN	2	0.464	2.498	428.724
								777	NE	2	0.487	2.445	425.437
								1500	Un	2	0.483	2.565	436.061
								756	HN	2	0.464	2.498	428.551
1000	20	20	1.25	2	0.02	4000	2	1000	NE	2	0.436	2.250	410.201
									Un	2	0.493	2.389	424.369
									HN	2	0.436	2.250	413.827
2000									NE	2	0.487	2.445	425.596
									Un	2	0.433	2.621	437.589
									HN	2	0.464	2.498	428.724
3000									NE	3	0.315	2.756	433.948
									Un	2	0.432	2.697	446.118
									HN	3	0.324	2.761	437.141
2000	10	20	1.25						NE	2	0.411	2.541	413.229
									Un	2	0.390	2.645	425.844
									HN	2	0.411	2.541	416.752

[a] NE, Negative-exponential; Un, Uniform; HN, Half-normal.

Table 3.2-(Continued) [a]

T	b	c	e	$\Delta\delta$	λ'	M'	D'	W'	Prior Distribution	Optimal design n	h	k	loss-cost
	20								*NE*	2	0.487	2.445	425.596
									Un	2	0.433	2.621	437.589
									HN	2	0.464	2.498	428.724
	30								*NE*	2	0.465	2.486	435.951
									Un	2	0.538	2.482	448.340
									HN	2	0.445	2.506	439.800
2000	20	10	1.25						*NE*	3	0.271	2.751	393.807
									Un	3	0.288	2.722	408.808
									HN	3	0.257	2.743	397.111
		20							*NE*	2	0.487	2.445	425.596
									Un	2	0.433	2.621	437.589
									HN	2	0.464	2.498	428.724
		30							*NE*	2	0.482	2.454	446.222
									Un	2	0.569	2.486	458.027
									HN	2	0.544	2.389	449.364
2000	20	20	0.625						*NE*	2	0.432	2.522	382.936
									Un	2	0.467	2.563	393.987
									HN	2	0.432	2.498	428.724
			1.25						*NE*	2	0.487	2.522	386.522
									Un	2	0.433	2.621	437.589
									HN	2	0.464	2.498	449.364
			1.875						*NE*	2	0.464	2.498	466.341
									Un	2	0.472	2.512	480.262
									HN	2	0.464	2.498	469.766

[a] *NE*, Negative-exponential; *Un*, Uniform; *HN*, Half-normal.

Chapter 4

Conclusions and Future Research

In this thesis, we considered the economic design of $\bar{x}$-control charts for discrete part Weibull process and for continuous flow exponential process with multiple assignable causes. Using the time elements and cost elements, we constructed the loss-cost function to determine the optimal values of the design parameters (the sample size, the sampling interval and control limit coefficient). Then we obtained the optimal solutions for those two models at the minimal loss-cost. Sensitivity analysis was conducted to investigate the effect of changes in the time elements or the cost elements on the optimal values. In this chapter, we make conclusions and provide possible extensions of the present work for the further research.

4.1 Conclusions

In Chapter 2, we investigated the optimal economic design of $\bar{x}$-control charts for discrete part Weibull process with multiple assignable causes. We constructed the expected loss-cost function per unit time to determine the optimal values of the design parameters. A numerical example was provided to verify the effectiveness of the modified version as compared with the original model by Chen and Yang [8]. Through the comparison, we can indicated that the cost differences range for the original model and the modified version is from 10% to 38%. Such a model should be helpful in reducing the quality cost and increasing competitiveness in a discrete part process.

In Chapter 3, we depict the detailed development of an economic model for the optimal design of $\bar{x}$-control charts for continuous flow process. The process-failure mechanism is assumed with multiple assignable causes and each assignable cause follows an exponential distribution. Solutions of the optimal design parameters, n, h

and k, have been obtained according to the different values of the model parameters. Overall, this paper advances economically-based $\bar{x}$-control charts to the important area of multiple assignable causes process in continuous flow process.

4.2 Future Research

We have considered the multiple assignable causes in economic design. Possible future work with respect to economically-based control chart techniques when monitoring continuous flow process or discrete part process are as follows:

1. The same techniques developed in this thesis can be extended to other control chart methods such as: the *CUSUM* (cumulative sum) control chart, MA (moving average) control chart, etc.

2. In discrete part Weibull process, we have analyzed that the model allows for the second occurrence of an assignable cause following the first occurrence. We may study that the model allows for the multiple occurrence of assignable causes following the first occurrence.

3. In continuous flow process, we have investigated that the time of assignable cause follows a exponential distribution subjects to a multiple of assignable causes. We may study that the time of assignable cause follows Weibull distribution subjects to a multiple of assignable causes. Furthermore, the model allows for the multiple occurrence of assignable causes following the first occurrence.

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