Chapter 1

Introduction

Traditional inventory management provides many analytical models which are more relevant and applicable in practice. The adequacy of these models has some important effects on the inventory control. For this reason given above, inventory management is a practical subject and plays an important role in scientific disciplines. In Section 1.1, we present the background of the inventory management, especially, focus on the newsboy problem which is one of the important subjects in the inventory management. Section 1.2 describes the motivation of this thesis. Section 1.3 shows the description of the problems. In Section 1.4, we provide the exact approaches to tackle these problems. At the end of this chapter, the organization of the thesis is presented in Section 1.5.

1.1 Background

Newsboy problem, also known as the single period inventory problem is very significant in terms of both theoretical and practical consideration. It focuses on the products which have the limited selling period (or short shelf-life) such as daily newspapers, monthly/weekly magazines, milks, seasonal products, fresh food and many others. The classical newsboy model assumes that if the surplus products are subject to storage for a short period of time, one ought to pay additional costs to dispose these items. If the unsatisfied demand is lost, the opportunity cost may be occurred. Generally, the demand presented in the classical newsboy problem is unknown and assumed to be a random variable with a known probability distribution. Consequently, the determination of the ordering quantity is critical for achieving designated objective function in the newsboy problem. Nowadays, several literatures on newsboy problem have provided very useful framework for making decision on advanced booking of orders. Hadley and Whitin [10] were the first researchers to introduce the newsboy problem. Some researchers have extended the classical newsboy problem based on various considerations. For example, Shore [39] considered a newsboy problem with random lot-size. Keisuke [14] extended the multi-period newsboy problem. Li *et al*. [25] and Lau and Lau [19] investigated the multiple products, Lau and Lau [22] further studied capacitated multiple products. Shao and Ji [37] studied the multi-product constrained fuzzy newsboy problem and solved by using credibility measure. Panda *et al*. [32] considered a multiple products manufacturing system under chance and imprecise constraints. For the excess inventory, some researchers have assumed that the surplus products can be sold by using the discount. It was the pioneer work of Hadley and Whitin [10], in which they considered the single product and single discount. A common case that arises in practice is one in which multiple discounts are progressively used to sell excess inventory. Multiple discounts are especially common in the apparel industry where discounts get steeper as the season draws to an end. Some retailers advertise a system in which products remaining in inventory are progressively discounted to attract more customers to their stores. Khouja [16, 17] studied the multiple discounts in the newsboy problem. Khouja and Mehrez [18] further combined the multiple products and multiple discounts.

There is an excellent survey of the literature on the various objective functions such as minimizing the expected cost (Nahmias [29]), maximizing the expected profit (Khouja [16]), maximizing the expected utility (Ismail and Louderback [12] and Lau [21]), and maximizing the probability of achieving a target profit (Ismail and Louderback [12], Shih [38], Lau [20], and Sankarasubramanian and Kumaraswamy [35]). However, so far, existing researches never care about the value of the maximum expected profit and the probability of achieving a target profit. These values can be expressed the product's profitability.

Whenever the demand is uncertain, several literatures always assumed that the demand is a random variable and follows a common distribution with known parameter(s). For example, the normal is preferred when the demand per cycle is relatively large, while the Poisson is better for low-demand items because it is discrete. Lau [21] has pointed that some seasonal or fashion products which have very high demand uncertainties may be more suitably modeled by the exponential distribution.

In practical work, the parameter(s) of demand distribution is/are unknown and depend(s) on the estimation technique. Berk *et al*. [3] used the frequentist and the Bayesian approaches for demand estimation. Also, most of the researches focused on the distribution-free newsboy problem, where the form of the demand distribution is unknown but only the mean and variance are specified. It was the pioneer work of Scarf [36], in which the minimax approach applied to minimize the maximum cost resulting from the worst possible demand distribution. This approach can derive a simple closed-form expression for the ordering quantity that maximizes expected profit. Moon and Choi [27] studied a distribution-free newsboy problem with balking, in which customers are allowed to balk when inventory level is low. Ouyang and Wu [31] presented an inventory model with mixture of backorders and lost sales, which relaxes the assumption about the normal distribution of lead-time demand. Ouyang and Chang [30] modified the continuous review inventory models involving variable lead time with a mixture of backorders and lost sales. They utilized the minimax distribution-free procedure for finding the optimal inventory strategy in the fuzzy sense where information about the lead time demand distribution is partial. Alfares and Elmorra [2] extended the analysis of the distribution-free newsboy problem to the case when shortage cost is taken into consideration. Mostard *et al*. [28] derived a simple closed-form formula to determine the order quantity for the distribution-free newsboy inventory problem with returns. It was shown in Mostard *et al*. [28] that the distribution-free order rule performs well when the coefficient of variation (*cv*) is at most 0.5, but is far from optimal when the *cv* is large. Liao *et al*. [24] considered a linear penalty cost for lost sales in the model under customer balking, which occurs when the available inventory reaches a threshold level. Lee and Hsu [23] developed for the decision-maker in a distribution-free newsboy problem to determine the expenditure on advertising and the order quantity. Recently, Kevork [15] developed appropriate estimators for the optimal ordering quantity and the maximum expected profit when demand is normally distributed. From the work of Kevork [15] who investigated the statistical properties for both small and large samples analytically and through Monte Carlo simulation.

All the above research works have been done under a random environment with stochastic demand based on probability theory. In reality, most of the evaluations are imprecise, fuzzy and cannot be quantified. The fuzzy set theory, introduced by Zadeh [43] is the best form that adapts all the uncertainty set to the model. When subjective evaluations are considered, the possibility theory takes the place of the probability theory (Zadeh [44]). The fuzzy set theory can represent linguistic data which cannot be easily modeled by other methods (Dubois and Prade [6]). In present, several researchers extended newsboy problem into fuzzy environment by using fuzzy set theory. Petrovi`c *et al*. [34] presented two models, (1) the newsboy problem with discrete fuzzy demand, and (2) the newsboy problem with imprecise costs. Li *et al*. [26] also proposed two models, but in the first model the demand was probabilistic and costs were fuzzy and in the other the costs were deterministic and the demand was fuzzy. They used fuzzy ordering of fuzzy numbers to obtain the optimal order quantity. Ishill and Konno [11] assumed that the demand is stochastic and fuzziness is restricted to shortage cost which is given by an

L-shape fuzzy number. Kao and Hsu [13] supposed that the demand was also described by a fuzzy number. Shao and Ji [37] proposed three types of models (EVM model, DCP model and CCP model) for multi-product newsboy problem under budget constraint in an imprecise environment, and solved by using credibility measure. Zhen and Xiaoyu [45] considered the multi-product newsboy problem with fuzzy demands under budget constraint. Panda *et al*. [32] extended the single period inventory problem in a multi-product manufacturing system under chance and imprecise constraints. Wee *et al*. [42] constructed a multi-objective joint replenishment deteriorating items inventory model, where the demand and shortage cost were assumed to be fuzzy variables. Dutta *et al*. [8] dealt with a single-period inventory model with a reordering strategy and fuzzy demand. The optimal policy is obtained via profit maximization. In present, Dutta and Chakraborty [7] presented a fuzzy single-period inventory model for two-item with one-way substitution policy where the opportunity for the product substitution was taken into consideration.

1.2 Motivation

The objective of traditional inventory management is to maintain optimum levels of inventory consistent with customer demands and plant capacity. Stated simply, traditional inventory management encompasses the principles, concepts and techniques for deciding: (1) what to order; (2) how much to order; (3) when to order; (4) where to store it; (5) when it is needed (Fogarty *et al*. [9]). Nowadays, the inventory management causes more problems due to science and technology changing with each new day. For example, the old product whether is unworthy of being ordered due to the spatial constraint in the warehouse as the new product is introduced. Furthermore, if more than two old products are considered to compare each other, which one should be substituted by new product? For the other case, if the capital investment in profitability improvement is implemented, one should focus on the product which has lower profitability due to the budgetary constraint. Therefore, in order to hold the competitive advantage, the inventory management ought to consider as more parameters as above five parameters.

1.3 Problem Statement

In this dissertation, we investigate two practical problems frequently occurred in the inventory systems, product evaluation problem and product selection problem.

Product evaluation problem

In the present market, the new products are unceasingly introduced. Although the new products are not immediately accepted by the customers, it means that a new trend is going to be popularly adopted in the market. If the managers are unable to accept the new product timely, the business may lose the competition. Therefore, the inventory manager should observe the requirement of customers, and order the new product at the right moment. In fact, when the new product is ordered, the old product may be curtailed due to the spatial constraint in the warehouse. Even, the old product may be substituted completely by a new product if the capacity of the old product is not good enough. In order to judge whether the old product is unworthy of being ordered in a competitive market, we conducted a product evaluation which examined whether the profitability meets a designated requirement. Consequently, the inventory management should add to determine the sixth parameter: (6) whether to order.

Product selection problem

If more than two old products are considered to compare each other, which one should be substituted by new product when the new product is introduced? To reflect this phenomenon, it is necessary to consider the product selection problem which deals with comparing all old products and selecting the one that has a significantly lower capacity. Consequently, the inventory management should add to determine the seventh parameter: (7) which to eliminate (or substitute). Note that the demands of products are usually irrelevant each other due to the preference and identity. Therefore, the demands are independent random variables. However, few of products have certain relation. For example, two different brands of apples can be substituted each other, the milk essence is an accessory to coffee. In these cases, the random demands are dependent, and the joint probability density function of demands must be used. In order to initially construct the product selection problem, we preliminary make the simplifying assumption that the product demands are independent, then the dependent case will be further studied in the future research based on the assumptions and formulations of this study.

Before exploring above two problems, one should select an appropriate criterion for measuring a product's capacity. To the best of our knowledge, criteria such as profitability, quality, reputation, fashion, and performance can express a product's capacity, especially, the profitability is a common criterion. For example, Trubint *et al*.

[41] adopted profitability, quality of service and urban construction as the criteria for finding optimal retail outlet locations. Steers [40] measured organizational effectiveness with profitability and market share. With regards to profitability evaluation, Pekka and Jukka [33] investigated profitability evaluation for intelligent transport system (ITS) investments, and Chen and Zhu [5] applied a projection pursuit model to evaluate the profitability of enterprise.

In this thesis, we consider the newsboy-type product and focus on the normal demand distribution, $N(\mu,\sigma)$. Note that the normal is a procedure well-established in mathematical statistics. In addition, we set the profitability is to be a criterion for measuring product capacity, and define the profitability as the probability of achieving the target profit under optimal ordering condition. In order to make the problems more relevant and applicable in practice, we assumed that the demand mean μ and demand standard deviation σ are unknown. Under the above assumptions, we ought to collect past demand data and implement statistical estimation for investigating product evaluation problem and product selection problem.

1.4 Research approach

Since the form of the profitability ought to be complex, it is hard to effectively find the statistical estimation of profitability when μ and σ are not given. This motivated us to develop a simple index combined with product's profitability. A new index is called "Achievable Capacity Index" and denoted as I_A. To the best of our knowledge, the index depends on μ and σ if the selling price and the related costs are given. We collect past demand data, and develop an unbiased and effective estimator of I_A to estimate actual I_A. For the demand data, the demand is the sum of the sales volume and the unsatisfied demand. It seems as if the unsatisfied demand is unable to observed or record. Practically, in order to understand the actual demand for controlling inventory and diminishing the lost sale opportunity cost, the retailers not only care about the sales volume but also try to record unsatisfied demand. Some products would appear to fit these conditions such as high-profit products and new products. Another kind of possibility is that the product is purchased by using the order. At this time, the order can be referred to demand. If the unsatisfied demand is unable to be observed or recorded, the historical sales data does not truly represent the demand and are termed censored demand data. Agrawal and Smith [1] mentioned that the negative binomial is an appropriate demand distribution for retail inventory management applications, and developed a parameter estimation methodology that compensates for the effects of unobservable lost sales.

By using the proposed index I_A, we adopt the statistical hypothesis testing methodology to examine these two problems. For the product evaluation problem, we implement the following hypothesis testing,

$$H_0 : I_A \le C \quad \text{versus} \quad H_1 : I_A > C,$$

where C is a designated requirement of I_A. Given a level of Type I error α (i.e., the chance of incorrectly judging $I_A \le C$ as $I_A > C$), the decision rule is to reject H_0 if the testing statistic is large than the critical value. Note that the p-value can be also adopted for making decisions in this testing, which presents the actual risk of misjudging $I_A \le C$ as $I_A > C$. If p-value$<\alpha$, the null hypothesis is rejected.

For the product selection problem, we note the indices I_{A1} and I_{A2} to present the profitability of Products I and II, respectively. The following hypothesis testing for comparing two I_A values is

$$H_0 : I_{A2} - I_{A1} \le h \quad \text{versus} \quad H_1 : I_{A2} - I_{A1} > h$$

where $h \ge 0$ is a designated outperformance. Note that if $h = 0$, the test is only to determine whether the Product II has a significantly better profitability than the Product I . Given a level of Type I error α (i.e., the chance of incorrectly judging $I_{A2} - I_{A1} \le h$ as $I_{A2} - I_{A1} > h$), the decision rule is to reject H_0 if the testing statistic is large than the critical value. Note that the p-value can be also adopted for making decisions in this testing, which presents the actual risk of misjudging $I_{A2} - I_{A1} \le h$ as $I_{A2} - I_{A1} > h$. If p-value $< \alpha$, the null hypothesis is rejected.

If the selling price and related cost are different between two products, the above hypothesis testing should be modified, i.e.,

$$H_0 : I^c_{A2} - I_{A1} \le \delta \quad \text{versus} \quad H_1 : I^c_{A2} - I_{A1} > \delta,$$

where $\delta \ge 0$ is a designated outperformance and I^c_{A2} is the correction of I_{A2}. Note that if $\delta = 0$, the test is only to determine whether the Product II has a significantly better profitability than the Product I. Given a level of Type I error α (i.e., the chance of incorrectly judging $I^c_{A2} - I_{A1} \le \delta$ as $I^c_{A2} - I_{A1} > \delta$), the decision rule is to reject H_0 if the testing statistic is large than the critical value. Note that the p-value can be also adopted for making decisions in this testing, which presents the actual risk of misjudging $I^c_{A2} - I_{A1} \le \delta$ as $I^c_{A2} - I_{A1} > \delta$. If p-value $< \alpha$, the null hypothesis is rejected.

1.5 Thesis organization

The main purpose of this dissertation is to develop a new index I_A, which can accurately and simply measure the profitability of newsboy-type product with normally distributed demand. By using this index, we then study two common problems in the inventory management, product evaluation problem and product selection problem. This dissertation is organized by five chapters as follows:

Chapter 1 is an introduction, which shows the background of the newsboy problem and the assumption of the uncertain demand, research motivation and problem statement.

In Chapter 2, we investigate the product evaluation problem. Statistical hypothesis testing methodology is utilized to tackle this product evaluation problem. The critical value of the test is calculated to determine the evaluation results. The sample size required for the designated power and confidence level is also investigated. An application example for a fresh food product is provided to illustrate the utilization of the proposed approach.

The majority of the results related to the distributional properties of the estimators were obtained based on the assumption of having a single sample. However, from a practical perspective, several stores have observed a weekly-based (or daily-based) demand records for monitoring profitable status such as fast food restaurants, dairy industries, chemical industries, and so on. Therefore, in these particular environments, the demand data is collected from multiple samples rather than single sample. In Chapter 3, we implement the statistical hypothesis testing methodology based on multiple samples. Critical values of the test based on multiple samples are calculated to determine the evaluation results. Furthermore, for practitioners' convenience, we provide a simple procedure to use in making decision on whether the profitability meets designated requirement. A real case on the sales of donuts is presented to illustrate the applicability of our approach

In Chapter 4, we study the product selection problem. Statistical hypothesis testing methodology is performed to tackle this selection problem. Critical value of the test is calculated to determine the selection decision. Sample size required for a designated power and confidence level is also investigated. An application example on comparing English-teaching magazines is presented to illustrate the practicality of our approach.

Chapter 5 presents some conclusions based on results of the investigation, and recommendations for the future investigations.

Chapter 2

Profitability Evaluation for Newsboy-Type Product with Normally Distributed Demand

In this chapter, we consider the newsboy-type product with normally distributed demand, $N(\mu, \sigma)$, and define the product's profitability as the probability of achieving the target profit under optimal ordering condition. In order to determine whether the product is unworthy of being ordered in a competitive market, we conduct a product evaluation which examine whether the profitability meets a designated requirement. Since the parameters μ and σ are always unknown, we introduce a new index "Achievable Capacity Index", I_A which has a simple form expression of the product's profitability. An unbiased and effective estimator of I_A is also derived. By using this index, we utilize a statistical hypothesis testing methodology to tackle the product evaluation problem. The critical value of the test is calculated to determine the evaluation results. The sample size required for the designated power and confidence level is also investigated. An application example for a fresh food product is provided to illustrate the utilization of the proposed approach.

This chapter is organized as follows: In the Section 2.1, the notations and assumptions related to this thesis are presented. Section 2.2 examines the profitability measurement. In this section, by using the relationship between demand properties (μ and σ) and target demand, we attempt to develop the achievable capacity index, I_A. Then, we explore the relationship between the profitability and the value of I_A. In the Section 2.3, we find an unbiased and effective estimator of I_A to estimate the actual I_A, and then implement the product evaluation. The critical value and sample size required are calculated. In the last section, an application example is presented.

2.1 Notations and assumptions

We consider the newsboy-type product with normally distributed demand. The surplus stock and unsatisfied demand must pay the disposal cost and opportunity cost, respectively. In addition, we define the profitability as the probability of achieving the target profit under the optimal ordering condition, in which the target profit is predetermined according to the product property and the sales experience. For convenience, the notations used in this thesis are as below:

p	selling price per unit.
c	purchasing/manufacturing cost per unit.
c_p	net profit per unit (i.e., $c_p = p - c$).
c_d	disposal cost for a surplus product.
c_e	excess cost per unit (i.e., $c_e = c_d + c$).
c_s	shortage cost per unit (i.e., the lost sale opportunity cost).
k	target profit.
T	target demand.
Q	ordering quantity.
D	demand during a period, which is a random variable.
$f(\cdot)$	probability density function of D.
Z	profit during a period.
AC	profitability of the newsboy-type product.

The following assumptions are used throughout this thesis:

A 1 Consider the newsboy-type product with normally distributed demand, $N(\mu, \sigma)$.

A 2 The selling price and related costs are given and constant.

A 3 To make the problem more relevant, the parameters μ and σ are unknown, but satisfied that $cv = \sigma / \mu < 0.3$ for neglecting the negative tail, i.e., $f(D<0) = \Phi(-\mu/\sigma) = \Phi(-1/cv) < \Phi(-1/0.3) \approx 0$.

A 4 The target demand is the minimal demand required for satisfying the target profit, i.e., $T = k/(p-c) = k/c_p$.

A 5 In order to possibly achieve the target profit, the ordering quantity must be greater than or equal to target demand, i.e., $Q \geq T$.

2.2 Profitability measurement

2.2.1 Achievable capacity index I_A

If the related parameters (p, c, c_d, c_s, and T) are given, the optimal ordering quantity and the level of profitability depend on the demand mean $E(D)$ and the demand standard deviation $\sqrt{Var(D)}$. Therefore, we develop a new index, which is a function of $E(D)$ and $\sqrt{Var(D)}$ to express the product's profitability, and so-called "Achievable Capacity Index (ACI)". Under the assumption that the demand is normally distributed, the achievable capacity index which we denote as I_A is defined:

$$I_A = \frac{E(D)-T}{\sqrt{Var(D)}} = \frac{\mu - T}{\sigma}.$$

The numerator of I_A provides the difference between demand mean and target demand. The denominator gives demand standard deviation. Obviously, it is desirable to have an I_A as large as possible.

2.2.2 Interrelationship between profitability and I_A

Based on the literature Sankarasubramanian and Kumaraswamy [35], the profit Z depends on the demand D and the ordering quantity Q, which are formulated as follows:

$$Z = \begin{cases} pD - c_d(Q-D) - cQ = (c_p + c_e)D - c_e Q, & 0 \le D \le Q \\ pQ - c_s(D-Q) - cQ = -c_s D + (c_p + c_s)Q, & D > Q \end{cases}$$

Note that if the surplus products can be salvaged, the value of c_d is negative and redefine into salvage price. For any $Q \ge T$, Z is strictly increasing in $D \in [0, Q]$ and strictly decreasing in $D \in [Q, \infty)$, and has a maximum at point $D = Q$. The maximum value of Z is equal and higher than k, i.e., $Z = pD - cQ = c_p D = c_p Q \ge c_p T = k$. The target profit will be realized when D is equal to either $LAL(Q)$ or $UAL(Q)$, so the target profit will be achieved in $D \in [LAL(Q), UAL(Q)]$, where

$$LAL(Q) = \frac{c_e Q + k}{c_p + c_e} \text{ and } UAL(Q) = \frac{(c_p + c_s)Q - k}{c_s}$$

are the lower and upper achievable limits, respectively, and both are the functions of Q. Under the assumption that the demand is normally distributed, the probability of achieving the target profit is:

$$\Pr(Z \ge k) = \Phi\left(\frac{UAL(Q) - \mu}{\sigma}\right) - \Phi\left(\frac{LAL(Q) - \mu}{\sigma}\right), \qquad (2.1)$$

where $\Phi(\cdot)$ is the cumulative distribution function of the standard normal distribution.

Before calculating the profitability, we first find the optimal ordering quantity that maximizes $\Pr(Z \ge k)$. We take the first-order of $\Pr(Z \ge k)$ with respect to Q, i.e.,

$$\frac{d\Pr(Z \ge k)}{dQ} = \frac{1}{\sqrt{2\pi}}\left[\frac{c_p + c_s}{c_s} e^{-\frac{1}{2}\left(\frac{UAL(Q)-\mu}{\sigma}\right)^2} - \frac{c_e}{c_p + c_e} e^{-\frac{1}{2}\left(\frac{LAL(Q)-\mu}{\sigma}\right)^2}\right].$$

It is well known that the necessary condition for Q to be optimal must satisfy the equation $\mathrm{d}\Pr(Z \geq k)/\mathrm{d}Q = 0$, which implies

$$\mu = \frac{UAL(Q)+LAL(Q)}{2} - \frac{\omega\sigma^2}{UAL(Q)-LAL(Q)}, \tag{2.2}$$

where $\omega = \ln[1 + c_p A / c_s c_e]$ and $A = c_p + c_e + c_s$. For $Q \geq T$, we solve Eq. (2), then obtain the unique optimal ordering quantity

$$Q^* = T + \frac{c_s(c_p+c_e)(c_p\mu - k)}{c_p(c_pA+2c_ec_s)} + \sqrt{\left[\frac{c_s(c_p+c_e)(c_p\mu-k)}{c_p(c_pA+2c_ec_s)}\right]^2 + \frac{2c_s{}^2(c_p+c_e)^2\omega\sigma^2}{c_pA(c_pA+2c_ec_s)}} > T. \tag{2.3}$$

In addition, the sufficient condition is also calculated as follows:

$$\left.\frac{\mathrm{d}^2\Pr(Z \geq k)}{\mathrm{d}Q^2}\right|_{Q=Q^*} = -\frac{(c_p+c_s)}{\sqrt{2\pi}\sigma^3 c_s{}^2(c_p+c_e)} \exp\left[-\frac{1}{2}\left(\frac{UAL(Q^*)-\mu}{\sigma}\right)\right]$$

$$\times\left\{\frac{\left[UAL(Q^*)-LAL(Q^*)\right](c_pA+2c_ec_s)}{2} + \frac{c_pA\omega\sigma^2}{UAL(Q^*)-LAL(Q^*)}\right\} < 0.$$

We can conclude that the stationary point Q^* is a global maximum. By using Eq. (2.2) and substituting Eq. (2.3) into Eq. (2.1), the profitability, AC, is obtained as follows:

$$AC = \Phi\left(G + \frac{\omega}{2G}\right) - \Phi\left(-G + \frac{\omega}{2G}\right), \tag{2.4}$$

where

$$G = \frac{UAL(Q^*)-LAL(Q^*)}{2\sigma} = M\left(\frac{\mu-T}{\sigma}\right) + \sqrt{M^2\left(\frac{\mu-T}{\sigma}\right)^2 + M\omega} = MI_A + \sqrt{M^2 I_A^2 + M\omega} > 0,$$

and

$$M = \frac{c_pA}{2(c_pA+2c_ec_s)} > 0.$$

It is easy to see that AC is a function of I_A. Taking the first-order derivative of $AC(I_A)$ with respect to I_A, we obtain

$$\frac{\mathrm{d}AC(I_A)}{\mathrm{d}I_A} = \frac{MG}{\sqrt{2\pi(MI_A^2+M\omega)}}\left[e^{\omega} + 1 + \frac{\omega}{2G^2}\left(e^{\omega}-1\right)\right]e^{-\frac{1}{2}\left(G+\frac{\omega}{2G}\right)^2} > 0.$$

As a result, $AC(I_A)$ is a strictly increasing function of I_A. Therefore, we can express the product's profitability according to the value of I_A, and the value of I_A is as large as possible. Based on the parameters $p=20$, $c=10$, $c_d=-5$, and $c_s=3$, Figure 2.1 plots the profitability versus various values of I_A for the effects of changes in the parameters p, c, c_d, and c_s. From Figure 2.1, the following observations can be made:

(1) With increase in the value of I_A, the product's profitability increases. Obviously, it is desirable to have a I_A as large as possible.

(2) When the value of parameter p increases, the product's profitability increases. It implies that if the customers can satisfy the price changes, the product's profitability is going to be increased when the selling price increases.

(3) The product's profitability decreases as c, c_d, and c_s increase. If the purchasing (or manufacturing) cost per unit, disposal cost for a surplus product and shortage cost per unit could be reduced effectively, the product's profitability could be improved.

2.3 Estimation of I_A based on single sample

The historical data of the demand ought to be collected based on single simple in order to estimate the actual I_A due to unknown μ and σ. First, the natural estimator $\hat{I}_A$ is considered. If a sample of size n is given as $\{x_1, x_2, \ldots x_n\}$, the natural estimator $\hat{I}_A$ is obtained by replacing the μ and σ by their estimators $\bar{x}=\sum_{i=1}^{n} x_i/n$ and $s=[\sum_{i=1}^{n}(x_i-\bar{x})^2/(n-1)]^{1/2}$, i.e.,

$$\hat{I}_A = \frac{\bar{x}-T}{s}.$$

Furthermore, the natural estimator $\hat{I}_A$ can be written as

$$\hat{I}_A = \frac{\bar{x}-T}{s} = \frac{1}{\sqrt{n}} \times \frac{\dfrac{\bar{x}-\mu}{\sigma/\sqrt{n}} + \left(\dfrac{\mu-T}{\sigma/\sqrt{n}}\right)}{\sqrt{\dfrac{(n-1)s^2}{\sigma^2}/(n-1)}} = \frac{1}{\sqrt{n}} \times \frac{Z+\sqrt{n}I_A}{\sqrt{\dfrac{\chi^2(n-1)}{n-1}}} = \frac{1}{\sqrt{n}} \times t_{n-1}(\theta). \qquad (2.5)$$

Therefore, the estimator $\hat{I}_A$ is distributed as $n^{-1/2} t_{n-1}(\theta)$, where $t_{n-1}(\theta)$ is a non-central t random variable with $n-1$ degree of freedom and the non-centrality parameter $\theta=\sqrt{n}I_A$. Because of

$$E(\hat{I}_A) = \frac{[\frac{n-1}{2}]^{\frac{1}{2}}\Gamma[\frac{n-2}{2}]}{\Gamma[\frac{n-1}{2}]} \times I_A \neq I_A ,$$

the estimator $\hat{I}_A$ is biased. To tackle this problem, we add the correction factor $b = [2/(n-1)]^{1/2}\Gamma[(n-1)/2]/\Gamma[(n-2)/2]$ to $\hat{I}_A$. Then we obtain unbiased estimator $b\hat{I}_A$ which we denote as $\tilde{I}_A$. Since $b < 1$ ($n > 2$), $Var(\tilde{I}_A) < Var(\hat{I}_A)$. The estimator $\tilde{I}_A$ is based only on the complete and sufficient statistics $(\bar{x}, s^2)$, consequently $\tilde{I}_A$ is the uniformly minimum variance unbiased estimator (UMVUE) of I_A.

We first define $R = \tilde{I}_A = b(\bar{x} - T)/s = Y/V$, where $Y = b(\bar{x} - T)/\sigma$ and $V = \sqrt{s^2/\sigma^2}$. Since $D \sim N(\mu, \sigma^2)$, we have $Y \sim N(b(\mu - T)/\sigma, b^2/n)$. In addition, it is well known that the random variable $(n-1)s^2/\sigma^2$ follows the chi-squared distribution with $n-1$ degree of freedom, we then have $V^2 = s^2/\sigma^2 \sim Gamma((n-1), 2/(n-1))$. By using the technique of change-of-variable, the probability density of V is derived as follows:

$$f_V(v) = \frac{2v^{n-2}}{\Gamma\left(\frac{n-1}{2}\right)\left(\frac{2}{n-1}\right)^{\frac{n-1}{2}}} \exp\left\{-\frac{n-1}{2}v^2\right\}, \qquad v > 0.$$

Because Y and V are independent continuous random variables, the probability density function of R can be obtained by the *Jacobian approach*, i.e.,

$$f_R(r) = \int_0^\infty f_Y(vr) f_V(v) |v| dv$$

$$= \frac{\sqrt{2n}\left(\frac{n-1}{2}\right)^{\frac{n-1}{2}}}{b\sqrt{\pi}\Gamma\left[(n-1)/2\right]} \int_0^\infty v^{n-1} \exp\left\{-\frac{1}{2}\left[\frac{n\left(vr - bI_A\right)^2}{b^2} + (n-1)v^2\right]\right\} dv, \qquad -\infty < r < \infty.$$

Figure 2.2 plots the probability density function of R, I_A = 1.0, 1.5, 2.0, 2.5, and n = 30, 50, 100, 150, 200 (from bottom to top in plots). From Figure 2.2, we can see significantly that

(1) The larger the value of I_A, the larger the variance of $R = \tilde{I}_A$.

(2) The distribution of R is unimodal and is rather symmetric to I_A even for small sample sizes.

(3) The larger the sample sizes *n*, the smaller the variance of *R*.

2.4 Hypothesis testing with I_A and evaluation results

To judge whether the profitability meets the designated requirement, we ought to consider the hypothesis testing:

$$H_0 : AC \le \mathbb{C} \text{ versus } H_1 : AC > \mathbb{C} ,$$

where $\mathbb{C}$ is the designated requirement. However, the statistical property of the estimator of AC is difficult to describe. Even, it is impossible to define the unbiased estimator of AC. From the last subsection, we have proven that I_A can express the profitability. Therefore, we consider the following hypothesis testing:

$$H_0 : I_A \le C \text{ versus } H_1 : I_A > C ,$$

where C is the designated requirement of I_A. Based on the probability density function of R and a given level of Type I error α (i.e., the chance of incorrectly judging $I_A \le C$ as $I_A > C$); the decision rule is to reject H_0 if the testing statistic $R > c_0$, where c_0 is the critical value that satisfies

$$\Pr\{R \ge c_0 \,|\, I_A = C, n\} = \alpha .$$

Table 1 shows the critical values for I_A = 1.0(0.2)3.0, n = 30(10)200 and α = 0.05.

2.5 Required sample size

In the previous subsection, the procedure is to test whether the profitability meets the designated requirement for given α risk (Type I error). But, the β risk (Type II error: the probability of incorrectly judging H_1 as H_0) is not taken into account. Once the sample size and the α risk are defined, the power of test, $1-\beta$, can be calculated. The power of the test for C = 1.0, 2.0 versus various values of I_A, n = 30, 50, 100, 150, 200, and α = 0.05 is showed in Figure. 2.3. It is seen that the larger the sample size, the larger the power of test, and consequently, the smaller the β risk. The required sample size for designated α and β risks can be calculated by recursive search method with the following two probability equations:

$$\Pr\{R \ge c_0 \,|\, H_0 : I_A \le C, n\} \le \alpha \quad \text{and} \quad \Pr\{R \ge c_0 \,|\, H_1 : I_A > C, n\} \ge 1-\beta .$$

In Table 2.2, we tabulate the sample sizes required for α = 0.05, designated power = 0.90, 0.95, 0.975, 0.99, designated requirement C = 1.0, 1.2, 1.4, 1.6, and difference of expected I_A and designated requirement $I_A - C = 0.3(0.1)1.0$.

2.6 Profitability evaluation for a fresh food

We consider a fresh food industry in Hsinchu, Taiwan, in which provides more than twenty different kinds of lunch boxes, breads, sandwiches for shopping malls and convenience stores. These fresh food products are prepared each day and have relatively short shelf-life (about one or two days). The overdue products can not be sold and need additional cost to dispose them. If the manufacturing quantity can not satisfy the order from the malls and stores, then the supplier must pay the lost sale opportunity cost. Therefore, these fresh food products exactly belong to the newsboy-type products.

Now, a new lunch box is recommended, the manufacturing quantity of the existing lunch box which has the lowest profitability should be curtailed due to the capacity constraints (manpower or machines). Note that in order to maintain fresh, the lunch boxes are prepared in the morning and the life cycle is only 12 hours. However, the supplier would like to know whether the profitability of the existing lunch box is higher than some level. If the existing lunch box is incapable, it must be replaced with the new one. The selling price of the existing lunch box is \$20 per unit, the manufacturing cost is \$10 per unit, and the target profit is \$200, 000. In addition, the lost sale opportunity cost is \$3 per unit. The surplus (overdue) lunch boxes can be manufactured into fertilizers, then the salvage price is \$5 per unit. Table 2.3 displays the demand units in thousand for the existing lunch box with sample size n = 100. Due to the company's propertied restriction, the prices, costs, and sample data were modified. We first use the Kolmogorov-Smirnov test for the sample data from Table 2.3 to confirm if the data is normally distributed. A test result in *p-value* > 0.05, which means that data is normally distributed. Histogram of the data is shown in Figure 2.4. If the designated requirement of the I_A value is C = 1.2, we implement the hypothesis testing: $H_0 : I_A \le 1.2$ versus $H_1 : I_A > 1.2$. For the data displayed in Table 2.3, we calculate the sample mean, sample standard deviation, and sample estimator, and obtain that $\bar{x}$ = 23.593, s = 1.882 and R = 1.894. Based on the Table 2.1, the critical value is 1.427 as C = 1.2, n = 100 and α = 0.05. Since R = 1.894 > 1.427 = c_0, we conclude that I_A is more than 1.2 with 95% confidence level. Therefore, the supplier only curtails output of the the existing lunch box. Furthermore, we calculate the critical value for C = 1.4, 1.5, 1.6, 1.61, 1.62, 1.63 with n = 100. The decision of the hypotheses are shown in Table 2.4. Based on the testing results, we can conclude that the profitability of the existing lunch box is higher than 1.62 with 95% confidence level. Assume that the expected I_A is 1.6. We use a hypothesis testing with a designated power of 0.95, the sample size required to sample is 135 as in Table 2.2. In this example, the sample size is less than 135, the power for testing $H_0: I_A \le 1.2$ versus $H_1: I_A > 1.2$ would be less than 0.95. In fact, the power of test for the expected I_A = 1.6 is 0.8766, that is the β risk is up to 0.0734. In order to reduce the β risk, we would suggest the supplier to sample for a designated power with as large sample size as in Table 2.2.

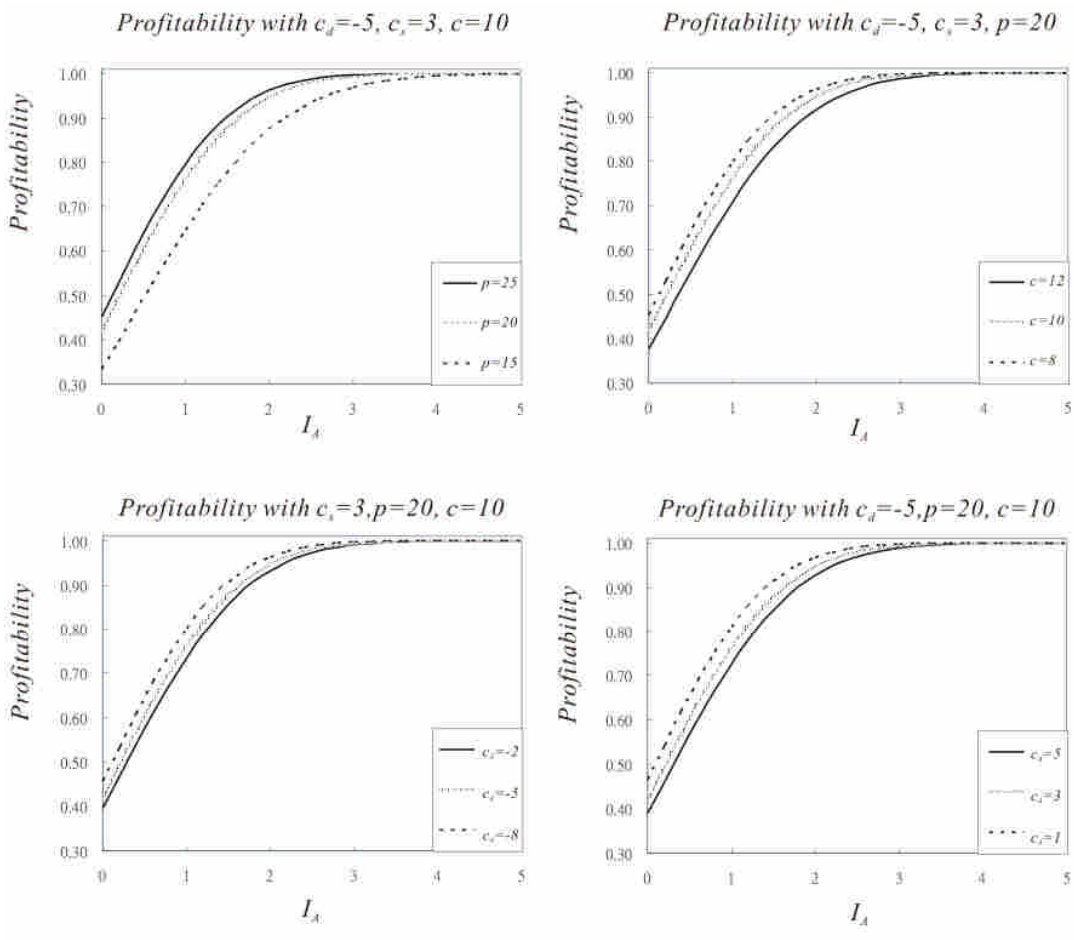

Figure 2.1. Profitability versus various values of I_A for the effects of changes p, c, c_d, c_s.

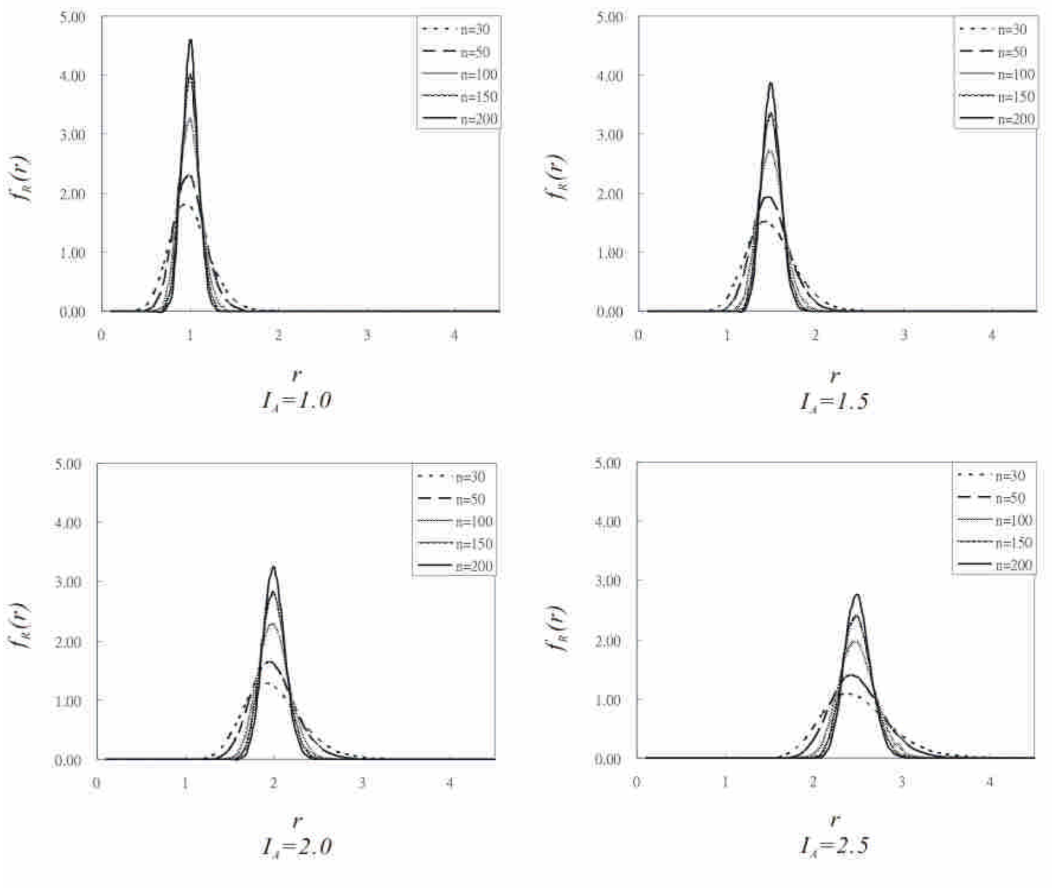

Figure 2.2. PDF plots of R for sample sizes n = 30, 50, 100, 150, 200.
(from bottom to top in plots)

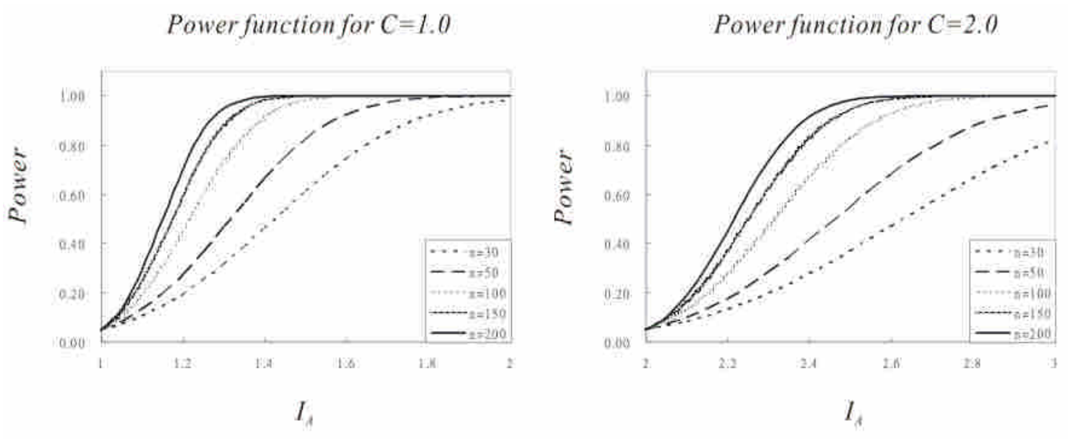

Figure 2.3. Power curves for C = 1.0, 2.0, with sample sizes n = 30, 50, 100, 150, 200.

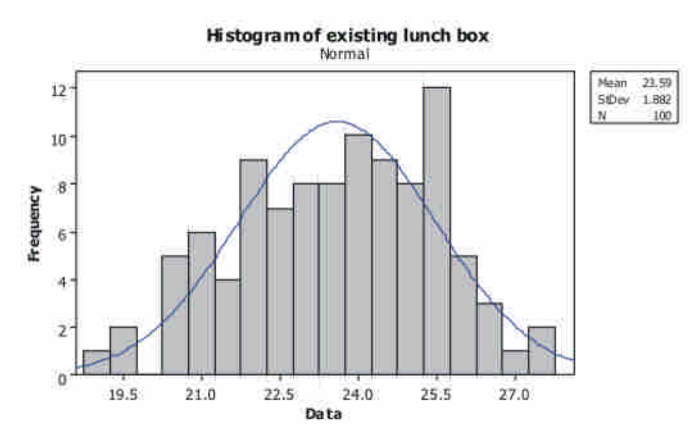

Figure 2.4. Histogram of demand data.

Table 2.1 Critical values for rejecting $I_A \le C$ with n = 30(10)200 and α = 0.05.

n	C										
	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4	2.6	2.8	3.0
30	1.402	1.635	1.871	2.108	2.348	2.589	2.830	3.073	3.317	3.562	3.806
40	1.343	1.571	1.801	2.032	2.265	2.500	2.735	2.971	3.208	3.445	3.683
50	1.304	1.528	1.754	1.982	2.211	2.441	2.672	2.904	3.136	3.369	3.602
60	1.276	1.497	1.721	1.946	2.172	2.399	2.627	2.855	3.085	3.314	3.544
70	1.254	1.474	1.695	1.918	2.142	2.367	2.592	2.819	3.045	3.272	3.500
80	1.237	1.455	1.675	1.896	2.118	2.341	2.565	2.789	3.014	3.239	3.465
90	1.223	1.440	1.658	1.878	2.099	2.320	2.542	2.765	2.989	3.212	3.436
100	1.211	1.427	1.644	1.863	2.082	2.303	2.524	2.745	2.967	3.189	3.412
110	1.200	1.416	1.632	1.850	2.068	2.288	2.508	2.728	2.949	3.170	3.392
120	1.191	1.406	1.622	1.839	2.056	2.275	2.494	2.713	2.933	3.153	3.374
130	1.184	1.397	1.613	1.829	2.046	2.263	2.481	2.700	2.919	3.139	3.358
140	1.177	1.390	1.604	1.820	2.036	2.253	2.471	2.689	2.907	3.125	3.344
150	1.170	1.383	1.597	1.812	2.028	2.244	2.461	2.678	2.896	3.114	3.332
160	1.165	1.377	1.591	1.805	2.020	2.236	2.452	2.669	2.886	3.103	3.321
170	1.160	1.372	1.585	1.799	2.013	2.229	2.444	2.660	2.877	3.094	3.311
180	1.155	1.367	1.579	1.793	2.007	2.222	2.437	2.653	2.869	3.085	3.301
190	1.151	1.362	1.574	1.787	2.001	2.216	2.430	2.646	2.861	3.077	3.293
200	1.147	1.358	1.570	1.782	1.996	2.210	2.424	2.639	2.854	3.070	3.285

Table 2.2 Sample size required for testing $H_0: I_A \leq C$ versus $H_1: I_A > C$.

C	I_A	Power				C	I_A	Power			
		0.90	0.95	0.975	0.99			0.90	0.95	0.975	0.99
1.0	1.3	161	201	239	288	1.2	1.5	185	231	275	331
	1.4	94	117	139	167		1.6	109	135	160	192
	1.5	63	78	91	110		1.7	73	90	106	127
	1.6	46	56	67	79		1.8	53	65	77	91
	1.7	35	43	51	60		1.9	41	50	58	69
	1.8	28	35	40	48		2	33	40	46	55
	1.9	24	28	33	39		2.1	27	33	38	45
	2	20	24	28	33		2.2	23	28	32	38
1.4	1.7	213	266	316	381	1.6	1.9	245	306	364	438
	1.8	125	155	184	221		2	143	178	212	254
	1.9	83	103	122	146		2.1	96	119	140	168
	2	61	75	88	105		2.2	69	86	101	121
	2.1	46	57	67	80		2.3	53	65	77	92
	2.2	37	45	53	63		2.4	42	52	61	72
	2.3	31	37	44	52		2.5	35	43	50	59
	2.4	26	32	37	43		2.6	30	36	42	49

Table 2.3 Sample data with 100 observations.

demand units in thousand /day									
26.56	25.51	22.00	22.60	23.20	23.37	25.44	24.64	23.16	22.70
22.37	20.87	22.20	24.14	25.34	24.26	23.24	21.90	22.67	22.83
23.02	25.50	25.46	26.60	22.66	21.24	21.42	21.95	21.62	27.57
24.11	26.89	24.64	24.10	22.03	24.59	25.36	19.40	20.70	25.93
23.72	23.33	25.22	23.31	23.19	24.86	24.96	23.89	24.49	19.60
20.81	24.78	21.12	21.14	23.96	24.29	26.07	22.57	24.85	23.65
22.60	24.94	25.72	24.27	25.40	20.84	23.05	20.45	23.24	20.56
24.24	25.36	22.09	23.43	26.36	27.38	20.56	23.52	24.95	21.51
22.20	25.31	23.83	24.23	24.31	25.97	22.03	26.13	18.99	21.51
22.17	20.44	25.18	25.50	23.82	23.50	24.54	25.45	25.91	24.20

Table 2.4 Critical values and decisions of testing the existing lunch box.

C	1.40	1.50	1.60	1.61	1.62	1.63
c_0	1.644	1.753	1.863	1.874	1.885	1.896 > R
Decision	Reject H_0	Reject H_0	Reject H_0	Reject H_0	Reject H_0	Accept H_0

Chapter 3

Assessing Profitability of a Newsboy-type Product with Normally Distributed Demand Based on Multiple Samples

Practically, the market information regarding demand is obtained from multiple samples rather than single sample. In this chapter, we estimate and test I_A based on multiple samples. A hypothesis testing based on multiple samples for product evaluation is presented. Critical values of the test are calculated to determine the evaluation results. Finally, a real case on the sales of donuts is presented to illustrate the applicability of our approach.

The rest of this chapter is organized as follows. In the Section 3.1, we derive an unbiased estimator $\tilde{I}_A$ to estimate actual I_A based on multiple samples. The distribution of $\tilde{I}_A$ is also given. In the Section 3.2, the critical value of the test is calculated to determine the evaluation results. Section 3.3 presents an example for donuts to illustrate the practicality of the approach to data collected from a donut store for profitability evaluation.

3.1 Estimation of I_A based on multiple samples

The historical data of the demand ought to be collected in order to estimate the actual I_A due to unknown μ and σ. For multiple samples of m groups each of size n is given as $\{x_{i1}, x_{i2}, \ldots, x_{in}\}$, where $i = 1, 2, \ldots, m$, let $\bar{x}_i = \sum_{j=1}^{n} x_{ij} / n$ and $s_i^2 = \sum_{j=1}^{n} (x_{ij} - \bar{x}_i)^2 / (n-1)$ be the ith sample mean and sample standard deviation, respectively. We first consider the natural estimator $\hat{I}_A$ which is obtained by replacing the μ and σ by their unbiased estimators $\bar{\bar{x}} = \sum_{i=1}^{m} \bar{x}_i / m$ and $s_p = [\sum_{i=1}^{m} s_i^2 / m]^{1/2}$ i.e.,

$$\hat{I}_A = \frac{\bar{\bar{x}} - T}{s_p}.$$

Furthermore, the natural estimator $\hat{I}_A$ can be written as

$$\hat{I}_A = \frac{\bar{\bar{x}} - T}{s_p} = \frac{1}{\sqrt{mn}} \times \frac{\dfrac{\bar{\bar{x}} - \mu}{\sigma / \sqrt{mn}} + \dfrac{\mu - T}{\sigma / \sqrt{mn}}}{\sqrt{\dfrac{m(n-1) s_p^2 / \sigma^2}{m(n-1)}}} = \frac{1}{\sqrt{mn}} \times \frac{Z + \sqrt{mn} I_A}{\sqrt{\dfrac{W}{m(n-1)}}} = \frac{1}{\sqrt{mn}} \times \frac{Z_A}{\sqrt{\dfrac{W}{m(n-1)}}},$$

where $Z_A = Z + \sqrt{mn} I_A \sim N(\sqrt{mn} I_A, 1)$, $Z \sim N(0,1)$, $W = m(n-1) s_p{}^2 / \sigma^2 \sim \chi^2{}_{m(n-1)}$. Since Z_A and W are independent, the estimator $\hat{I}_A$ is distributed as $(mn)^{-1/2} t_{m(n-1)}(\theta)$, where $t_{m(n-1)}(\theta)$ is a non-central t distribution with $m(n-1)$ degree of freedom and the non-centrality parameter $\theta = (mn)^{1/2} I_A$. Since

$$E(\hat{I}_A) = \frac{[m(n-1)/2]^{1/2} \Gamma[(m(n-1)-1)/2]}{\Gamma[m(n-1)/2]} \times I_A \neq I_A,$$

the natural estimator $\hat{I}_A$ is biased. To tackle this problem, we add a correction factor as follows

$$b = \frac{[2/m(n-1)]^{1/2} \Gamma[m(n-1)/2]}{\Gamma[(m(n-1)-1)/2]}.$$

Then we can obtain unbiased estimator $b\hat{I}_A$, which is denoted by $\tilde{I}_A$. Since $\tilde{I}_A$ is based solely on the complete and sufficient statistics $(\bar{\bar{x}}, s_p{}^2)$, it leads to the conclusion that the estimator $\tilde{I}_A$ is the uniformly minimum variance unbiased estimator (UMVUE) of I_A based on multiple samples. We first define $R = b(\bar{\bar{x}} - T)/s_p = Y/V$, where $Y = b(\bar{\bar{x}} - T)/\sigma$ and $V = s_p/\sigma$. It is easy to see that if the demand is normally distributed, we have $Y \sim N(b(\mu - T)/\sigma, b^2/mn)$. Since $m(n-1)s_p{}^2/\sigma^2$ follows the chi-squared distribution with $m(n-1)$ degree of freedom, we then have $V^2 \sim Gamma[m(n-1)/2, 2/m(n-1)]$. By using the technique of change-of-variable, the probability density function of V is derived as follows:

$$f_V(v) = \frac{2v^{m(n-1)-1}}{\Gamma\left(\frac{m(n-1)}{2}\right)\left(\frac{2}{m(n-1)}\right)^{\frac{m(n-1)}{2}}} \exp\left\{-\frac{m(n-1)}{2} v^2\right\}.$$

Because Y and V are independent continuous random variables, the probability density function of R can be obtained by the *Jacobian approach*, i.e.,

$$f_R(r) = \frac{\sqrt{2mn}\left(\frac{m(n-1)}{2}\right)^{\frac{m(n-1)}{2}}}{b\sqrt{\pi}\Gamma\left(\frac{m(n-1)}{2}\right)} \int_0^\infty v^{m(n-1)} \exp\left\{-\frac{1}{2}\left[\frac{(vr - bI_A)^2}{b^2/mn} + m(n-1)v^2\right]\right\} dv, -\infty < r < \infty.$$

Figure 3.1 plots the probability density function of R, $I_A = 1.0, 1.5, 2.0$, $n = 3, 4, 5$, and $m = 10, 25, 40$ (from bottom to top in plots). From Figure. 3.1, we can see that (1) for fixed sample sizes m and n, the variance of $\tilde{I}_A = R$ increases as I_A increases; (2) for a fixed n and I_A, the variance of $\tilde{I}_A = R$ decreases as m increases; and (3) for a fixed m and I_A, the variance of $\tilde{I}_A = R$ decreases as n increases.

Discussion

For the case with unequal sample sizes, the natural estimator of I_A can straightforwardly be expressed as:

$$\tilde{I}'_A = \frac{\bar{\bar{x}}' - T}{s'_p}$$

where $\bar{\bar{x}}' = \sum_{i=1}^{m} n_i \bar{x}_i / N$ is the grand mean of the overall sample, $N = \sum_{i=1}^{m} n_i$ is the number of observation in the total sample, and ${s'_p}^2 = \sum_{i=1}^{m} (n_i - 1) s_i^2 / (N - m)$ is the pooled sample variance. The estimator $\tilde{I}'_A$ can be rewritten as

$$\tilde{I}'_A = \frac{\bar{\bar{x}}' - T}{s'_p} = \frac{1}{\sqrt{N}} \times \frac{\dfrac{\bar{\bar{x}}' - \mu}{\sigma / \sqrt{N}} + \dfrac{\mu - T}{\sigma / \sqrt{N}}}{\sqrt{\dfrac{(N-m){s'_p}^2 / \sigma^2}{N-m}}} = \frac{1}{\sqrt{N}} \times \frac{Z + \sqrt{N} I_A}{\sqrt{\dfrac{W'}{N-m}}} = \frac{1}{\sqrt{N}} \times \frac{Z'_A}{\sqrt{\dfrac{W'}{N-m}}},$$

where $Z'_A = Z + \sqrt{N} I_A \sim N(\sqrt{N} I_A, 1)$, $W' = (N-m){s'_p}^2 / \sigma^2 \sim {\chi^2}_{N-m}$. Since Z'_A and W' are independent, the estimator $\hat{I}'_A$ is distributed as $(N)^{-1/2} t_{N-m}(\theta')$, where $t_{N-m}(\theta')$ is a non-central t distribution with $N-m$ degree of freedom and the non-centrality parameter $\theta' = (N)^{1/2} I_A$. Similarly, we also obtain the unbiased estimator $\tilde{I}'_A = b' \hat{I}'_A$, where $b' = [2/N-m]^{1/2} \Gamma[N-m/2] / \Gamma[(N-m-1)/2]$ is the correction factor of $\hat{I}'_A$.

3.2 Testing I_A based on multiple samples

In order to judge whether the product's profitability meets the designated requirement, the achievable capacity index I_A is adopted to be a criterion. We consider the following hypothesis testing:

$$H_0 : I_A \le C \quad \text{versus} \quad H_1 : I_A > C,$$

where C is the designated requirement of I_A. The critical value is used for making decision in profitability performance testing with designated Type I error α (i.e., the chance of incorrectly judging $I_A \le C$ as $I_A > C$). Since $\tilde{I}_A$ is distributed as $b(mn)^{-1/2} t_{m(n-1)}(\theta)$, the critical value, c_0, is determined by:

$$\alpha = \Pr\left\{\tilde{I}_A \ge c_0 \middle| I_A = C\right\} = \Pr\left\{\frac{b t_{m(n-1)}(\theta)}{\sqrt{mn}} \ge c_0 \middle| I_A = C\right\} = \Pr\left\{t_{m(n-1)}(\theta) \ge \frac{\sqrt{mn} c_0}{b} \middle| I_A = C\right\}.$$

Thus, we have

$$c_0 = \frac{b t_{m(n-1),\,\alpha}(\theta)}{\sqrt{mn}},$$

where $c_0 = t_{m(n-1),\,\alpha}(\theta)$ is the upper α quantile of a non-central t distribution with $m(n-1)$ degrees of freedom satisfying $\Pr\{t_{m(n-1)}(\theta) \ge t_{m(n-1),\,\alpha}(\theta)\} = \alpha$. If the observed value of the statistic $\tilde{I}_A = w$ is higher than the critical value, the null hypothesis is rejected. We then conclude that the profitability is better than designated requirement with $(1-\alpha)\times 100\%$ confidence level. Note that the p-value can be also adopted for making decisions in this testing, which presents the actual risk of misjudging $I_A \le C$ as $I_A > C$, i.e.,

$$p - \text{value} = \Pr\left\{\tilde{I}_A \ge w \,\middle|\, I_A = C\right\}$$

$$= \Pr\left\{\frac{b t_{m(n-1)}(\theta)}{\sqrt{mn}} \ge w \,\middle|\, I_A = C\right\} = \Pr\left\{t_{m(n-1)}(\theta) \ge \frac{w\sqrt{mn}}{b} \,\middle|\, I_A = C\right\}$$

If p-value$<\alpha$, the null hypothesis is rejected. We conclude that the profitability is better than designated requirement with the actual type I error p-value (rather than α). Table 3.1 displays the critical values for $\alpha = 0.05, 0.025, 0.01$ based on multiple samples $n = 3(1)5$, $m = 10(2)40$, and $I_A = 1.0(0.2)2.0$. Next, we also calculate the β risk. Once the sample size and the α risk are defined, the power function, $Power(I_A)$, may be expressed by:

$$Power(I_A) = \Pr\left\{\tilde{I}_A \ge c_0 \,\middle|\, I_A > C\right\}$$

$$= \Pr\left\{\frac{b t_{m(n-1)}(\theta)}{\sqrt{mn}} \ge c_0 \,\middle|\, I_A > C\right\} = \Pr\left\{t_{m(n-1)}(\theta) \ge \frac{c_0\sqrt{mn}}{b} \,\middle|\, I_A > C\right\}.$$

The power of the test for $C = 1.0, 1.4, 1.8$ versus various values of I_A, $n = 3, 4, 5$, $m = 10(10)40$, and $\alpha = 0.05$ is showed in Figure. 3.2. It is seen that the larger the sample size, the larger the power of test, and consequently, the smaller the β risk.

Profitability evaluation procedure

In the following, we develop a simple step-by-step procedure for the practitioners to use for judging whether the profitability meets the designated requirement.

Step 1 Determine the value of the designated requirement C, α-risk, and sample size (m, n).

Step 2 Calculate the value of the estimator, $\tilde{I}_A$, form the given sample.

Step 3 Find the corresponding critical value, $\tilde{I}_A$, based on α, C, m and n form the Table 3.1. Also we calculate the *p*-value based on C, m and n.

Step 4 Conclude that the profitability meets the designated requirement if $\tilde{I}_A > c_0$ (or *p*-value< α). Otherwise, the profitability does not meet the designated requirement.

3.3 Application example

We consider a dessert store, which provides delicious donuts made fresh daily in Taipei, Taiwan. This store is a Japanese-owned incarnation of a donut franchise formerly out of America. Fifty varieties of donuts are offered, one half of them are American style and another half of them are Japanese style. All of the donuts range from NT$20-35. Besides, each donut comes with a label indicating its level of sweetness. However, these donuts only have approximate 12 hours shelf-life due to texture deterioration. In order to provide the best texture, this store prepares the donut each day and disposes the overdue donuts after closing store. If the manufacturing quantity can not satisfy the demand, then the manager must pay the lost sale opportunity cost. Therefore, the donut exactly belongs to the newsboy-type product. Now, the manager would like to know whether the profitability of the designated donut is higher than some level. If it is incapable, the manager is going to plan a sale promotion. The selling price of the donut is NT$25 per unit, the manufacturing cost is NT$10 per unit, and the target profit is NT$2500. In addition, the lost sale opportunity cost is NT$3 per unit. The disposal cost for overdue donut is NT$1 per unit. Table 3.2 displays the profitability for $(p, c, c_d, c_s, k) = (25, 10, 1, 3, 2500)$ and $I_A = 0.00(0.01)3.09$. For the demand data, because of Saturday and Sunday are always have high demand. In order to avoid these extreme values, we only consider the demand on Monday-Friday. Note that the unsatisfied demand is record. Twenty samples of size five (i.e., twenty weeks demand) are displayed in Table 3.3. Due to the store's propertied restriction, the prices, costs, and sample data were modified. If the designated requirement of the I_A value is C = 1.8, we implement the hypothesis testing: $H_0 : I_A \leq 1.8$ versus $H_1 : I_A > 1.8$. We first use the Kolmogorov-Smirnov test for the sample data from Table 3.3 to confirm if the data is normally distributed. A test result in p-value> 0.05, which means that data is normally distributed. For the data displayed in Table 3.3, we calculate the overall sample mean, pooled sample variance, and sample estimator, and obtain that $\bar{\bar{x}} = 200.48$, ${s_p}^2 = 237.10$, and $\tilde{I}_A = 2.1753$. If the type I error α-risk set to 0.05, the critical value with $n = 20$, $m = 5$ and $C = 1.8$ is 2.1050 form Table 3.1. Since $\tilde{I}_A = 2.1753 > 2.1050 = c_0$, we conclude that the profitability meets the designated requirement, than it is unnecessary to plan a sale promotion. For calculating the *p*-value, we obtain *p*-value = 0.0244<0.05. Therefore, it suggests the same evaluation result.

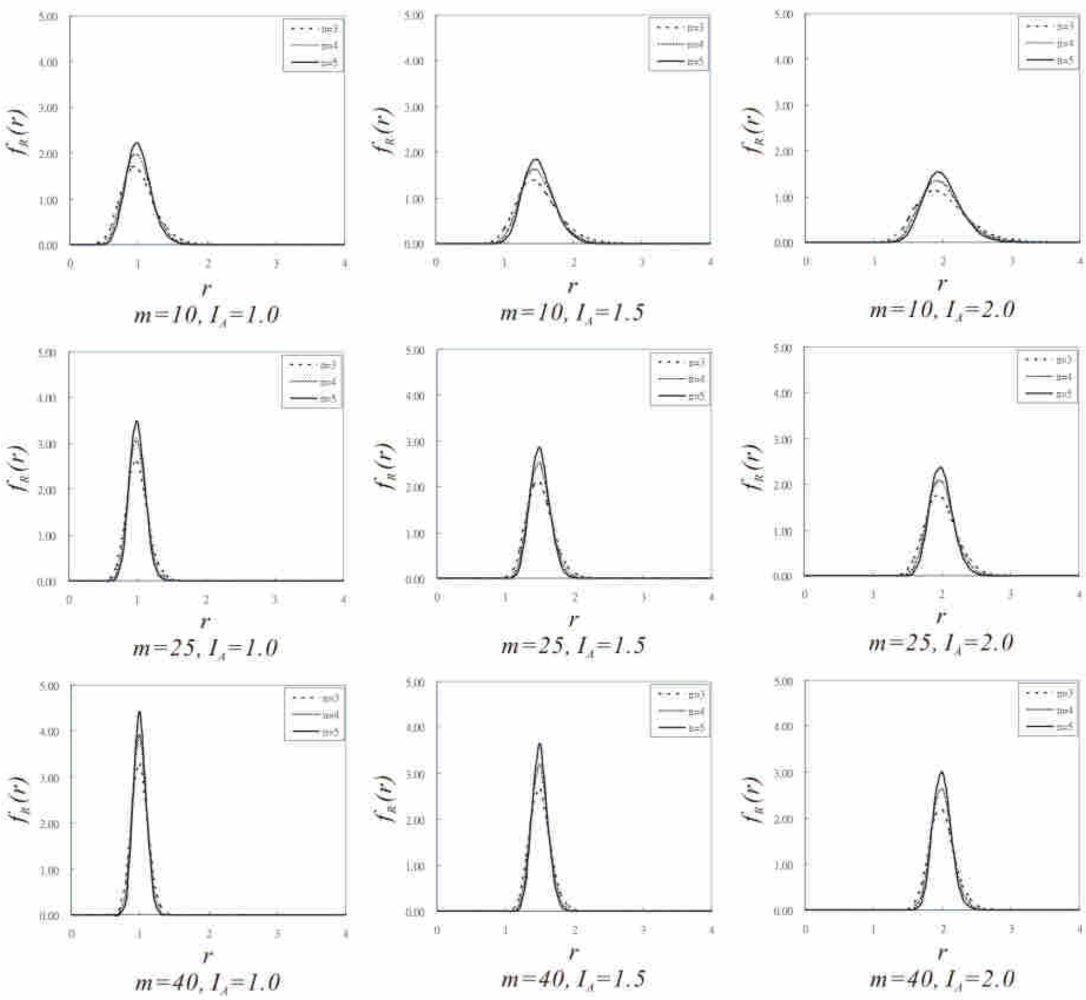

Figure 3.1 PDF plots of r for $n = 3, 4, 5,$ and $m = 10, 25, 40$.
(from bottom to top in plots)

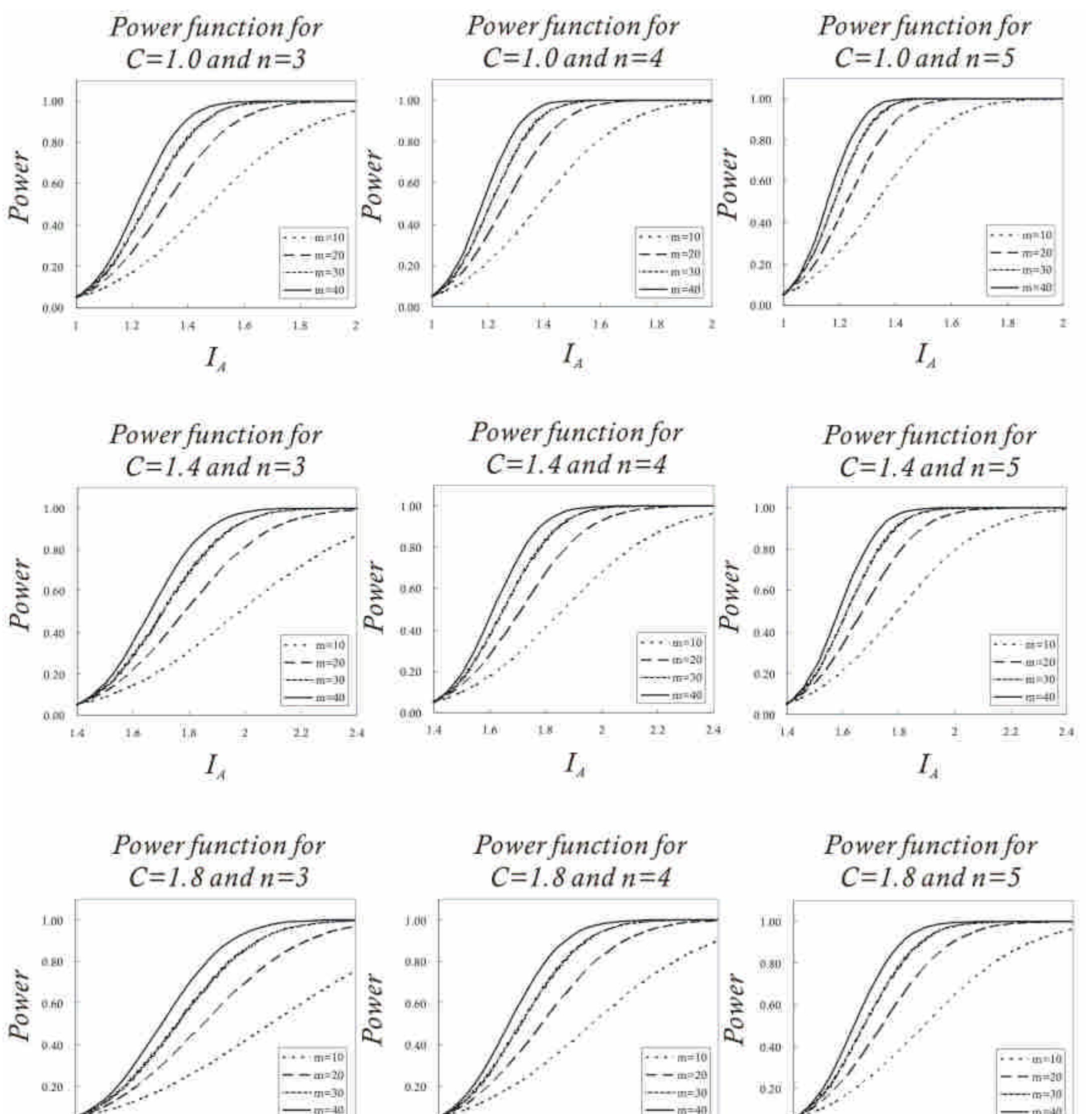

Figure 3.2 Power curves for C = 1.0, 1.4, 1.8, with sample s izes n =3, 4, 5 and m=10, 20, 30, 40.

Table 3.1. Critical values c_0 for $\alpha = 0.05, 0.025, 0.01$ based on multiple samples with $n = 3(1)5$, $m = 10(2)40$, and $C = 1.0(0.2)2.0$.

$\alpha = 0.05$	$C = 1.0$			$C = 1.2$			$C = 1.4$		
	n			n			n		
m	3	4	5	3	4	5	3	4	5
10	1.445	1.367	1.319	1.690	1.601	1.548	1.938	1.838	1.778
11	1.422	1.348	1.303	1.664	1.580	1.530	1.910	1.815	1.759
12	1.402	1.332	1.289	1.642	1.563	1.515	1.885	1.796	1.742
13	1.385	1.318	1.277	1.623	1.547	1.502	1.864	1.778	1.728
14	1.369	1.305	1.267	1.606	1.533	1.490	1.845	1.763	1.715
15	1.355	1.294	1.257	1.590	1.521	1.479	1.828	1.750	1.704
16	1.343	1.284	1.248	1.577	1.510	1.470	1.813	1.738	1.693
17	1.332	1.275	1.240	1.564	1.500	1.461	1.799	1.727	1.684
18	1.321	1.267	1.233	1.553	1.491	1.453	1.787	1.717	1.675
19	1.312	1.259	1.227	1.543	1.483	1.446	1.775	1.708	1.668
20	1.304	1.252	1.221	1.533	1.475	1.440	1.765	1.700	1.660
21	1.296	1.246	1.215	1.524	1.468	1.434	1.755	1.692	1.654
22	1.288	1.240	1.210	1.516	1.461	1.428	1.746	1.685	1.647
23	1.281	1.234	1.205	1.509	1.455	1.423	1.738	1.678	1.642
24	1.275	1.229	1.200	1.502	1.449	1.418	1.730	1.672	1.636
25	1.269	1.224	1.196	1.495	1.444	1.413	1.723	1.666	1.631
26	1.264	1.219	1.192	1.489	1.439	1.409	1.716	1.660	1.627
27	1.258	1.215	1.188	1.483	1.434	1.405	1.710	1.655	1.622
28	1.253	1.211	1.185	1.478	1.430	1.401	1.704	1.650	1.618
29	1.249	1.207	1.182	1.472	1.426	1.397	1.698	1.646	1.614
30	1.244	1.204	1.178	1.468	1.422	1.394	1.693	1.641	1.610
31	1.240	1.200	1.175	1.463	1.418	1.390	1.688	1.637	1.607
32	1.236	1.197	1.172	1.458	1.414	1.387	1.683	1.633	1.603
33	1.232	1.194	1.170	1.454	1.411	1.384	1.678	1.629	1.600
34	1.228	1.191	1.167	1.450	1.407	1.381	1.674	1.626	1.597
35	1.225	1.188	1.165	1.446	1.404	1.379	1.669	1.622	1.594
36	1.222	1.185	1.162	1.443	1.401	1.376	1.665	1.619	1.591
37	1.218	1.182	1.160	1.439	1.398	1.374	1.662	1.616	1.588
38	1.215	1.180	1.158	1.436	1.396	1.371	1.658	1.613	1.586
39	1.212	1.177	1.156	1.433	1.393	1.369	1.654	1.610	1.583
40	1.210	1.175	1.154	1.429	1.391	1.367	1.651	1.607	1.581

Table 3.1. (Continued).

$\alpha = 0.05$	$C = 1.6$			$C = 1.8$			$C = 2.0$		
	n			n			n		
m	3	4	5	3	4	5	3	4	5
10	2.188	2.076	2.011	2.441	2.316	2.244	2.694	2.558	2.479
11	2.157	2.052	1.990	2.406	2.290	2.222	2.657	2.529	2.454
12	2.130	2.030	1.972	2.377	2.266	2.202	2.625	2.504	2.433
13	2.106	2.012	1.956	2.351	2.246	2.185	2.597	2.482	2.415
14	2.086	1.995	1.942	2.328	2.228	2.170	2.572	2.462	2.398
15	2.067	1.981	1.929	2.308	2.212	2.156	2.550	2.445	2.384
16	2.051	1.967	1.918	2.290	2.198	2.144	2.530	2.430	2.371
17	2.036	1.955	1.908	2.274	2.185	2.133	2.513	2.416	2.359
18	2.022	1.945	1.899	2.259	2.173	2.123	2.497	2.403	2.348
19	2.010	1.935	1.890	2.245	2.162	2.114	2.482	2.391	2.338
20	1.998	1.925	1.882	2.233	2.153	2.105	2.468	2.380	2.329
21	1.988	1.917	1.875	2.221	2.143	2.097	2.456	2.371	2.320
22	1.978	1.909	1.868	2.211	2.135	2.090	2.444	2.361	2.312
23	1.969	1.902	1.862	2.201	2.127	2.083	2.434	2.353	2.305
24	1.960	1.895	1.856	2.191	2.120	2.077	2.424	2.345	2.298
25	1.952	1.889	1.851	2.183	2.113	2.071	2.414	2.337	2.292
26	1.945	1.883	1.845	2.175	2.106	2.065	2.405	2.330	2.286
27	1.938	1.877	1.841	2.167	2.100	2.060	2.397	2.324	2.280
28	1.931	1.872	1.836	2.160	2.094	2.055	2.389	2.317	2.275
29	1.925	1.867	1.832	2.153	2.089	2.050	2.382	2.312	2.270
30	1.919	1.862	1.828	2.147	2.084	2.046	2.375	2.306	2.265
31	1.914	1.857	1.824	2.141	2.079	2.042	2.368	2.301	2.260
32	1.908	1.853	1.820	2.135	2.074	2.038	2.362	2.296	2.256
33	1.903	1.849	1.817	2.129	2.070	2.034	2.356	2.291	2.252
34	1.898	1.845	1.813	2.124	2.065	2.030	2.350	2.286	2.248
35	1.894	1.841	1.810	2.119	2.061	2.027	2.345	2.282	2.244
36	1.889	1.838	1.807	2.114	2.057	2.023	2.340	2.278	2.241
37	1.885	1.834	1.804	2.110	2.054	2.020	2.335	2.274	2.237
38	1.881	1.831	1.801	2.105	2.050	2.017	2.330	2.270	2.234
39	1.877	1.828	1.798	2.101	2.047	2.014	2.326	2.266	2.231
40	1.873	1.825	1.796	2.097	2.044	2.011	2.321	2.263	2.228

Table 3.1. (Continued).

$\alpha = 0.025$	$C = 1.0$			$C = 1.2$			$C = 1.4$		
	n			n			n		
m	3	4	5	3	4	5	3	4	5
10	1.559	1.455	1.393	1.817	1.698	1.629	2.079	1.945	1.868
11	1.528	1.431	1.373	1.783	1.672	1.607	2.041	1.915	1.843
12	1.502	1.410	1.355	1.753	1.649	1.587	2.008	1.890	1.822
13	1.479	1.392	1.340	1.728	1.629	1.570	1.980	1.868	1.803
14	1.459	1.376	1.326	1.705	1.611	1.555	1.955	1.849	1.787
15	1.441	1.362	1.314	1.685	1.595	1.542	1.933	1.832	1.772
16	1.425	1.349	1.303	1.668	1.581	1.530	1.913	1.816	1.759
17	1.410	1.337	1.293	1.652	1.569	1.519	1.896	1.802	1.748
18	1.397	1.327	1.284	1.637	1.557	1.510	1.879	1.790	1.737
19	1.385	1.317	1.276	1.624	1.547	1.501	1.865	1.778	1.727
20	1.374	1.308	1.269	1.612	1.537	1.492	1.851	1.768	1.718
21	1.364	1.300	1.262	1.600	1.528	1.485	1.839	1.758	1.710
22	1.355	1.293	1.255	1.590	1.520	1.478	1.828	1.749	1.702
23	1.346	1.286	1.249	1.580	1.512	1.471	1.817	1.740	1.695
24	1.338	1.279	1.244	1.571	1.505	1.465	1.807	1.732	1.688
25	1.330	1.273	1.238	1.563	1.498	1.459	1.798	1.725	1.682
26	1.323	1.267	1.233	1.555	1.492	1.454	1.789	1.718	1.676
27	1.317	1.262	1.229	1.548	1.486	1.449	1.781	1.712	1.670
28	1.310	1.257	1.224	1.541	1.480	1.444	1.773	1.705	1.665
29	1.304	1.252	1.220	1.534	1.475	1.439	1.766	1.700	1.660
30	1.299	1.248	1.216	1.528	1.470	1.435	1.759	1.694	1.655
31	1.293	1.243	1.213	1.522	1.465	1.431	1.753	1.689	1.651
32	1.288	1.239	1.209	1.516	1.461	1.427	1.747	1.684	1.647
33	1.284	1.235	1.206	1.511	1.457	1.423	1.741	1.679	1.643
34	1.279	1.231	1.202	1.506	1.452	1.420	1.735	1.675	1.639
35	1.275	1.228	1.199	1.501	1.449	1.417	1.730	1.671	1.635
36	1.270	1.225	1.196	1.497	1.445	1.413	1.725	1.667	1.632
37	1.266	1.221	1.194	1.492	1.441	1.410	1.720	1.663	1.628
38	1.263	1.218	1.191	1.488	1.438	1.407	1.715	1.659	1.625
39	1.259	1.215	1.188	1.484	1.435	1.404	1.711	1.655	1.622
40	1.255	1.212	1.186	1.480	1.431	1.402	1.707	1.652	1.619

Table 3.1. (Continued).

$\alpha = 0.025$	$C = 1.6$			$C = 1.8$			$C = 2.0$		
	n			n			n		
m	3	4	5	3	4	5	3	4	5
10	2.344	2.194	2.108	2.610	2.445	2.350	2.879	2.697	2.594
11	2.301	2.162	2.081	2.564	2.409	2.321	2.829	2.659	2.562
12	2.266	2.134	2.058	2.525	2.379	2.296	2.786	2.626	2.535
13	2.234	2.110	2.038	2.491	2.353	2.274	2.749	2.598	2.511
14	2.207	2.089	2.020	2.461	2.330	2.255	2.716	2.573	2.490
15	2.183	2.070	2.004	2.434	2.310	2.238	2.687	2.551	2.472
16	2.161	2.053	1.990	2.411	2.291	2.222	2.662	2.531	2.455
17	2.142	2.038	1.977	2.389	2.275	2.208	2.639	2.513	2.440
18	2.124	2.024	1.965	2.370	2.260	2.195	2.618	2.496	2.426
19	2.108	2.011	1.955	2.353	2.246	2.184	2.598	2.482	2.414
20	2.093	2.000	1.945	2.336	2.233	2.173	2.581	2.468	2.402
21	2.080	1.989	1.936	2.322	2.222	2.163	2.565	2.455	2.392
22	2.067	1.979	1.927	2.308	2.211	2.154	2.550	2.444	2.382
23	2.055	1.970	1.920	2.295	2.201	2.146	2.536	2.433	2.373
24	2.044	1.961	1.912	2.283	2.192	2.138	2.523	2.423	2.364
25	2.034	1.953	1.905	2.272	2.183	2.130	2.511	2.413	2.356
26	2.025	1.946	1.899	2.262	2.175	2.123	2.500	2.404	2.349
27	2.016	1.939	1.893	2.252	2.167	2.117	2.490	2.396	2.341
28	2.008	1.932	1.887	2.243	2.160	2.111	2.480	2.388	2.335
29	2.000	1.926	1.882	2.234	2.153	2.105	2.470	2.381	2.328
30	1.992	1.920	1.877	2.226	2.146	2.099	2.461	2.374	2.323
31	1.985	1.914	1.872	2.219	2.140	2.094	2.453	2.367	2.317
32	1.978	1.909	1.867	2.211	2.134	2.089	2.445	2.361	2.312
33	1.972	1.904	1.863	2.204	2.129	2.084	2.437	2.355	2.306
34	1.966	1.899	1.859	2.198	2.124	2.080	2.430	2.349	2.302
35	1.960	1.894	1.855	2.191	2.119	2.076	2.423	2.344	2.297
36	1.954	1.890	1.851	2.185	2.114	2.071	2.417	2.339	2.292
37	1.949	1.885	1.847	2.179	2.109	2.067	2.411	2.334	2.288
38	1.944	1.881	1.844	2.174	2.105	2.064	2.405	2.329	2.284
39	1.939	1.877	1.841	2.169	2.100	2.060	2.399	2.324	2.280
40	1.934	1.874	1.837	2.163	2.096	2.057	2.393	2.320	2.276

Table 3.1. (Continued).

$\alpha = 0.01$	$C = 1.0$			$C = 1.2$			$C = 1.4$		
	n			n			n		
m	3	4	5	3	4	5	3	4	5
10	1.705	1.564	1.484	1.980	1.820	1.730	2.260	2.079	1.978
11	1.663	1.533	1.458	1.934	1.785	1.701	2.208	2.041	1.947
12	1.628	1.506	1.436	1.894	1.756	1.676	2.165	2.008	1.919
13	1.598	1.483	1.416	1.860	1.730	1.655	2.127	1.980	1.896
14	1.571	1.463	1.399	1.831	1.707	1.636	2.094	1.955	1.875
15	1.548	1.444	1.384	1.805	1.687	1.619	2.065	1.933	1.857
16	1.527	1.428	1.370	1.781	1.669	1.604	2.039	1.913	1.840
17	1.508	1.414	1.358	1.760	1.653	1.590	2.016	1.895	1.825
18	1.491	1.400	1.347	1.742	1.638	1.578	1.995	1.879	1.812
19	1.476	1.388	1.336	1.724	1.625	1.567	1.976	1.864	1.799
20	1.462	1.377	1.327	1.709	1.613	1.556	1.959	1.851	1.788
21	1.449	1.367	1.318	1.694	1.601	1.547	1.943	1.838	1.777
22	1.437	1.357	1.310	1.681	1.591	1.538	1.928	1.827	1.768
23	1.425	1.348	1.303	1.668	1.581	1.530	1.914	1.816	1.759
24	1.415	1.340	1.296	1.657	1.572	1.522	1.902	1.806	1.750
25	1.405	1.333	1.289	1.646	1.564	1.515	1.890	1.797	1.742
26	1.396	1.325	1.283	1.636	1.556	1.508	1.879	1.788	1.735
27	1.388	1.319	1.277	1.627	1.548	1.502	1.868	1.780	1.728
28	1.380	1.312	1.272	1.618	1.541	1.496	1.859	1.772	1.722
29	1.372	1.306	1.267	1.610	1.535	1.490	1.849	1.765	1.716
30	1.365	1.301	1.262	1.602	1.529	1.485	1.841	1.758	1.710
31	1.359	1.295	1.257	1.594	1.523	1.480	1.833	1.752	1.704
32	1.352	1.290	1.253	1.587	1.517	1.475	1.825	1.746	1.699
33	1.346	1.285	1.249	1.581	1.512	1.470	1.817	1.740	1.694
34	1.340	1.281	1.245	1.574	1.507	1.466	1.810	1.734	1.689
35	1.335	1.276	1.241	1.568	1.502	1.462	1.804	1.729	1.685
36	1.330	1.272	1.237	1.562	1.497	1.458	1.797	1.724	1.680
37	1.325	1.268	1.234	1.557	1.493	1.454	1.791	1.719	1.676
38	1.320	1.264	1.230	1.551	1.488	1.451	1.785	1.714	1.672
39	1.315	1.260	1.227	1.546	1.484	1.447	1.780	1.710	1.669
40	1.311	1.257	1.224	1.541	1.480	1.444	1.774	1.706	1.665

Table 3.1. (Continued).

$\alpha = 0.01$	$C = 1.6$			$C = 1.8$			$C = 2.0$		
	n			n			n		
m	3	4	5	3	4	5	3	4	5
10	2.543	2.341	2.229	2.829	2.606	2.482	3.117	2.872	2.737
11	2.486	2.299	2.195	2.767	2.560	2.444	3.049	2.822	2.696
12	2.438	2.263	2.165	2.714	2.520	2.412	2.991	2.779	2.661
13	2.396	2.232	2.139	2.668	2.486	2.384	2.942	2.742	2.630
14	2.360	2.205	2.116	2.629	2.456	2.359	2.899	2.710	2.604
15	2.328	2.180	2.096	2.594	2.430	2.337	2.860	2.681	2.580
16	2.300	2.159	2.078	2.562	2.406	2.318	2.827	2.656	2.559
17	2.274	2.139	2.062	2.534	2.385	2.300	2.796	2.633	2.540
18	2.251	2.122	2.047	2.509	2.366	2.284	2.769	2.612	2.522
19	2.230	2.105	2.034	2.486	2.348	2.269	2.744	2.593	2.506
20	2.211	2.091	2.021	2.465	2.332	2.256	2.721	2.575	2.492
21	2.194	2.077	2.010	2.446	2.318	2.244	2.700	2.559	2.478
22	2.177	2.065	1.999	2.429	2.304	2.232	2.681	2.544	2.466
23	2.162	2.053	1.989	2.412	2.291	2.221	2.663	2.531	2.455
24	2.148	2.042	1.980	2.397	2.279	2.212	2.647	2.518	2.444
25	2.135	2.032	1.972	2.383	2.268	2.202	2.631	2.506	2.434
26	2.123	2.023	1.964	2.369	2.258	2.194	2.617	2.495	2.424
27	2.112	2.014	1.956	2.357	2.248	2.185	2.603	2.484	2.416
28	2.101	2.005	1.949	2.345	2.239	2.178	2.591	2.474	2.407
29	2.091	1.997	1.942	2.334	2.231	2.170	2.579	2.465	2.399
30	2.082	1.990	1.936	2.324	2.223	2.164	2.567	2.456	2.392
31	2.073	1.983	1.930	2.314	2.215	2.157	2.557	2.448	2.385
32	2.064	1.976	1.924	2.305	2.208	2.151	2.547	2.440	2.378
33	2.056	1.970	1.919	2.296	2.201	2.145	2.537	2.433	2.372
34	2.048	1.964	1.914	2.287	2.194	2.139	2.528	2.425	2.366
35	2.041	1.958	1.909	2.279	2.188	2.134	2.519	2.419	2.360
36	2.034	1.952	1.904	2.272	2.182	2.129	2.511	2.412	2.355
37	2.027	1.947	1.900	2.264	2.176	2.124	2.503	2.406	2.349
38	2.021	1.942	1.895	2.257	2.170	2.119	2.495	2.400	2.344
39	2.015	1.937	1.891	2.251	2.165	2.115	2.488	2.394	2.339
40	2.009	1.932	1.887	2.244	2.160	2.111	2.481	2.389	2.335

Table 3.2. The profitability for $(p, c, c_d, c_s, k) = (25, 10, 1, 3, 2500)$ and $I_A = 0.00(0.01)3.09$

I_A	**0.00**	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**
0.0	0.4249	0.4285	0.4322	0.4359	0.4395	0.4432	0.4469	0.4506	0.4543	0.4580
0.1	0.4617	0.4654	0.4691	0.4728	0.4765	0.4802	0.4839	0.4876	0.4913	0.4950
0.2	0.4987	0.5024	0.5061	0.5099	0.5136	0.5173	0.5210	0.5246	0.5283	0.5320
0.3	0.5357	0.5394	0.5431	0.5467	0.5504	0.5541	0.5577	0.5614	0.5650	0.5687
0.4	0.5723	0.5759	0.5795	0.5831	0.5868	0.5903	0.5939	0.5975	0.6011	0.6046
0.5	0.6082	0.6117	0.6152	0.6188	0.6223	0.6258	0.6293	0.6327	0.6362	0.6396
0.6	0.6431	0.6465	0.6499	0.6533	0.6567	0.6601	0.6634	0.6668	0.6701	0.6734
0.7	0.6768	0.6800	0.6833	0.6866	0.6898	0.6931	0.6963	0.6995	0.7026	0.7058
0.8	0.7090	0.7121	0.7152	0.7183	0.7214	0.7245	0.7275	0.7305	0.7335	0.7365
0.9	0.7395	0.7425	0.7454	0.7483	0.7512	0.7541	0.7570	0.7598	0.7627	0.7655
1.0	0.7683	0.7710	0.7738	0.7765	0.7792	0.7819	0.7846	0.7873	0.7899	0.7925
1.1	0.7951	0.7977	0.8002	0.8028	0.8053	0.8078	0.8103	0.8127	0.8151	0.8176
1.2	0.8200	0.8223	0.8247	0.8270	0.8293	0.8316	0.8339	0.8361	0.8384	0.8406
1.3	0.8428	0.8449	0.8471	0.8492	0.8513	0.8534	0.8555	0.8575	0.8596	0.8616
1.4	0.8636	0.8655	0.8675	0.8694	0.8713	0.8732	0.8751	0.8769	0.8788	0.8806
1.5	0.8824	0.8841	0.8859	0.8876	0.8893	0.8910	0.8927	0.8944	0.8960	0.8976
1.6	0.8992	0.9008	0.9024	0.9039	0.9054	0.9070	0.9084	0.9099	0.9114	0.9128
1.7	0.9142	0.9156	0.9170	0.9184	0.9197	0.9211	0.9224	0.9237	0.9250	0.9262
1.8	0.9275	0.9287	0.9299	0.9311	0.9323	0.9335	0.9346	0.9358	0.9369	0.9380
1.9	0.9391	0.9401	0.9412	0.9423	0.9433	0.9443	0.9453	0.9463	0.9473	0.9482
2.0	0.9492	0.9501	0.9510	0.9519	0.9528	0.9537	0.9545	0.9554	0.9562	0.9570
2.1	0.9579	0.9587	0.9594	0.9602	0.9610	0.9617	0.9625	0.9632	0.9639	0.9646
2.2	0.9653	0.9660	0.9666	0.9673	0.9680	0.9686	0.9692	0.9698	0.9704	0.9710
2.3	0.9716	0.9722	0.9728	0.9733	0.9739	0.9744	0.9749	0.9754	0.9759	0.9764
2.4	0.9769	0.9774	0.9779	0.9784	0.9788	0.9793	0.9797	0.9801	0.9806	0.9810
2.5	0.9814	0.9818	0.9822	0.9826	0.9830	0.9833	0.9837	0.9841	0.9844	0.9848
2.6	0.9851	0.9854	0.9857	0.9861	0.9864	0.9867	0.9870	0.9873	0.9876	0.9879
2.7	0.9881	0.9884	0.9887	0.9889	0.9892	0.9894	0.9897	0.9899	0.9902	0.9904
2.8	0.9906	0.9908	0.9911	0.9913	0.9915	0.9917	0.9919	0.9921	0.9923	0.9925
2.9	0.9926	0.9928	0.9930	0.9932	0.9933	0.9935	0.9937	0.9938	0.9940	0.9941
3.0	0.9943	0.9944	0.9945	0.9947	0.9948	0.9949	0.9951	0.9952	0.9953	0.9954

Table 3.3. The 5 Sample data each of 20 observations.

Demand units /day					
Group	Observations in sample of size five				
(Week)	MON	TUE	WED	THU	FRI
1	185	169	189	201	192
2	221	220	191	180	203
3	208	213	217	212	196
4	224	195	208	214	224
5	202	218	208	197	189
6	189	198	212	204	225
7	219	196	190	229	198
8	188	215	188	191	185
9	189	206	194	191	186
10	215	225	198	191	212
11	178	173	186	224	212
12	183	214	244	212	217
13	221	194	187	194	174
14	172	217	205	216	214
15	191	199	183	196	179
16	187	223	183	219	198
17	176	205	211	216	198
18	199	184	235	186	184
19	187	183	206	212	203
20	192	178	210	180	195

Chapter 4

Product Selection for Newsboy-type Products with Normal Demands and Unequal Costs

In this chapter, we consider two newsboy-type products with unequal prices and costs. Both demands are independent and follow normal distributions with unknown parameters μ and σ. We study the product selection problem which deals with comparing two products and selecting the one that has a significantly higher profitability. The statistical hypothesis testing methodology is performed to tackle this selection problem. Critical value of the test is calculated to determine the selection decision. Sample size required for a designated power and confidence level is also investigated. An application example on comparing English-teaching magazines is presented to illustrate the practicality of our approach.

The rest of this chapter is organized as follows. An application example on comparing English-teaching magazines with different level is introduced in the Section 41. In the Section 4.2, the statistical hypothesis testing methodology is performed to tackle the product selection problem. The critical value and the sample size required for a designated power and confidence level are provided. In the Section 4.3, the English-teaching magazine selection is implemented to illustrate the practicality of our approach.

4.1 English-teaching magazine selection

The English-teaching magazine is one of the monthly magazines. It provides practical, interesting articles to improve English conversation skills. Radio and television programs also accompany each article and air Monday through Saturday. The publisher only provides the magazines in the beginning of each month. If the demand can not be satisfied, the publisher must pay the lost sale opportunity cost. The surplus magazines can not be sold in the next mouth, and need additional cost to dispose it. Therefore, this monthly magazine exactly belongs to newsboy-type product.

Next, we introduce a magazine publisher in Taipei, Taiwan, in which provides three level of English-teaching magazines, basic, intermediate, and high. The basic and intermediate magazines are the best teaching materials to the junior and senior students, respectively. The high magazine covers a wide range of topics. Most are reprinted from international magazines providing readers with a “Window on the World”. Therefore, it is most suitable for university students and business professionals. Note that these magazines can not be substituted each other. In this paper, we consider following two examples on comparing English-teaching magazines.

Example 1

The magazine publisher would like to know whether the profitability of intermediate magazine (Magazine II) is better than basic magazine (Magazine I). If not, the magazine publisher is going to plan a sale promotion for senior students. The price (p), purchasing cost (c), disposal cost (c_d), and shortage cost (c_s) for two magazines are presented as follows:

Magazine I

$p_1 = 12$ dollars/unit, $c_1 = 2$ dollars/unit, $c_{d1} = 3$ dollars/unit, and $c_{s1} = 3$ dollars/unit.

Magazine II:

$p_2 = 15$ dollars/unit, $c_2 = 3$ dollars/unit, $c_{d2} = 4$ dollars/unit, and $c_{s2} = 5$ dollars/unit.

Example 2

The magazine publisher would like to know whether the intermediate magazine (Magazine II) is the highest profitability of three magazines. The price and costs of the high magazine (Magazine III) are presented as follows:

Magazine III:

$p_3 = 20$ dollars/unit, $c_3 = 5$ dollars/unit, $c_{d3} = 5$ dollars/unit, and $c_{s3} = 10$ dollars/unit.

In order to match these examples, the following formulation is developed based on the above parameters. Table 4.1 displays the demand units in thousand for the three magazines with sample size $n_1 = n_2 = n_3 = 100$. Due to the publisher’s propertied restriction, the data, prices, and costs were modified.

4.2 Development of the exact method

To compare the two newsboy-type products with unequal prices and costs (Product I: $c_{p1} = p_1 - c_1$, $c_{e1} = c_{d1} + c_1$, c_{s1}; Product II: $c_{p2} = p_2 - c_2$, $c_{e2} = c_{d2} + c_2$, c_{s2}), we consider the hypothesis testing for comparing the two AC values,

$$H_0 : AC_2 - AC_1 \le h \quad \text{versus} \quad H_1 : AC_2 - AC_1 > h,$$

where $0 \le h < 1$ is a designated outperformance. $h = 0$, the test is only to determine whether the Product II has a significantly better profitability than the Product I. However, the statistical properties of the estimator of AC are difficult to describe. Even, it is impossible to define the unbiased estimator of AC. From the chapter 2, we have proven that the achievable capacity index I_A can express the product's profitability. Therefore, we adopt the indices I_{A1} and I_{A2} to present the profitability of Product I and Product II, respectively. First, we assume that two products' profitability are equal, i.e., $AC_1(I_{A1}) = AC_2(I_{A2})$. Because $AC_1(I_{A1})$ and $C_2(I_{A2})$ are monotonically increasing functions of $I_{A1} \in (-\infty, \infty)$ and $I_{A2} \in (-\infty, \infty)$, respectively, and their ranges are $(0, 1)$. For any $I_{A2} \in (-\infty, \infty)$, there exists an unique $I_{A1} \in (-\infty, \infty)$ such that $AC_1(I_{A1}) = AC_2(I_{A2})$ holds, and vice versa. Then we can show that $I_{A1} = AC_1^{-1}(AC_2(I_{A2}))$ and $I_{A2} = AC_2^{-1}(AC_1(I_{A1}))$, where $AC_1^{-1}(\bullet)$ and $AC_2^{-1}(\bullet)$ are the inverse functions of AC_1 and AC_2, respectively. Therefore, if the value of I_{A2} is ϕ, the corresponding value of I_{A1} is $I_{A1} = AC_1^{-1}(AC_2(\phi))$. From the above results, we can adopt the following hypothesis testing for comparing two I_A values:

$$H_0 : I_{A2}^c - I_{A1} \le \delta \quad \text{versus} \quad H_1 : I_{A2}^c - I_{A1} > \delta,$$

where $I_{A2}^c = AC_1^{-1}(AC_2(I_{A2}))$ and $\delta \ge 0$ is a designated outperformance. Note that if $\delta = 0$, the test is only to determine whether the Product II has a significantly better profitability than the Product I.

4.2.1 Sampling distribution of $I_{A2}^c - I_{A1}$

Before implementing this test, we should first derive the cumulative distribution function (CDF) and probability density function (PDF) of the test statistic $W = \tilde{I}_{A2}^c - \tilde{I}_{A1}$. If the sample sizes of Product I and Product II are n_1 and n_2, the PDF of the estimators $\tilde{I}_{A1} = R_1$ and $\tilde{I}_{A2} = R_2$ are

$$f_{R_1}(r_1)=\frac{\sqrt{2n_1}\left(\frac{n_1-1}{2}\right)^{\frac{n_1-1}{2}}}{b_1\sqrt{\pi}\Gamma[(n_1-1)/2]}\int_0^{\infty} v_1^{n_1-1}\exp\left\{-\frac{1}{2}\left[\frac{n_1(v_1r_1-b_1I_{A1})^2}{b_1^2}+(n_1-1)v_1^2\right]\right\}dv_1,$$

and

$$f_{R_2}(r_2)=\frac{\sqrt{2n_2}\left(\frac{n_2-1}{2}\right)^{\frac{n_2-1}{2}}}{b_2\sqrt{\pi}\Gamma[(n_2-1)/2]}\int_0^{\infty} v_2^{n_2-1}\exp\left\{-\frac{1}{2}\left[\frac{n_2(v_2r_2-b_2I_{A2})^2}{b_2^2}+(n_2-1)v_2^2\right]\right\}dv_2,$$

where $b_1=[2/(n_1-1)]^{1/2}\Gamma[(n_1-1)/2]/\Gamma[(n_1-2)/2]$, $b_2=[2/(n_2-1)]^{1/2}\Gamma[(n_2-1)/2]/\Gamma[(n_2-2)/2]$, $-\infty<r_1<\infty$, and $-\infty<r_2<\infty$. Since $I_{A2}^c=AC_1^{-1}(AC_2(I_{A2}))$, we set $R_2^c = \tilde{I}_{A2}^c = AC_1^{-1}(AC_2(\tilde{I}_{A2}))$, and derive CDF of R_2^c as follows:

$$F_{R_2^c}(r_2^c)=\Pr(R_2^c\le r_2^c)=\Pr(AC_1^{-1}(AC_2(R_2))\le r_2^c)=\Pr(R_2\le AC_2^{-1}(AC_1(r_2^c)))$$

$$=F_{R_2}(AC_2^{-1}(AC_1(r_2^c)))=\int_{-\infty}^{AC_2^{-1}(AC_1(r_2^c))} f_{R_2(r_2)}dr_2\ ,$$

where $-\infty<r_2^c<\infty$. Under the assumptions that two products are independent, we can easily obtain the CDF of $W=R_2^c-R_1$, i.e.,

$$F_W(w)=\Pr(W\le w)=\Pr(R_2^c-R_1\le w)=\int_{-\infty}^{\infty}\Pr(R_2^c-R_1\le w\,|\,R_1=r_1)f_{R_1}(r_1)dr_1$$

$$=\int_{-\infty}^{\infty}\Pr(R_2^c\le w+r_1)f_{R_1}(r_1)dr_1=\int_{-\infty}^{\infty}F_{R_2^c}(w+r_1)f_{R_1}(r_1)dr_1$$

$$=\int_{-\infty}^{\infty}\int_{-\infty}^{AC_2^{-1}(AC_1(w+r_1))}f_{R_1}(r_1)f_{R_2}(r_2)dr_2dr_1\ ,$$

where $-\infty<w<\infty$. Taking the first-order derivative of $F_W(w)$ with respect to w, the PDF of W can be obtained as follows:

$$f_W(w)=\int_{-\infty}^{\infty}f_{R_1}(r_1)f_{R_2}(AC_2^{-1}(AC_1(w+r_1)))\times\frac{dAC_2^{-1}(AC_1(w+r_1))}{dw}dr_1\ .$$

Figure 4.1 plots the CDF and PDF of W for I_{A1} = 2.0, 2.5, I_{A2}^c = 2.0, 2.5 and n_1 =

$n_2 = n = 30, 50, 100, 150, 200$. From Figure 4.1, we can see that

(1) The larger the value of $I_{A2}^{c} - I_{A1}$, the larger the variance of W.

(2) The PDF of W is unimodal and is rather symmetric to $I_{A2}^{c} - I_{A1}$ even for small sample sizes.

(3) The larger the sample sizes n, the smaller the variance of W.

4.2.2 Selection determine

Assume that the minimum requirement of I_{A1} and I_{A2}^{c} values are E, we consider the hypothesis testing: $H_0 : I_{A2}^{c} - I_{A1} \le \delta$ versus $H_1 : I_{A2}^{c} - I_{A1} > \delta$. Given a level of Type I error α (i.e., the chance of incorrectly judging $I_{A2}^{c} - I_{A1} \le \delta$ as $I_{A2}^{c} - I_{A1} > \delta$), the decision rule is to reject H_0 if the testing statistic $W \ge c_0$, where c_0 is the critical value that satisfies

$$\Pr\{W \ge c_0 \,\big|\, H_0 : I_{A2}^{c} - I_{A1} \le \delta, n_1, n_2, I_{A1} \ge E \text{ and } I_{A2} \ge E\} \le \alpha .$$

For all combinations of $(I_{A2}, I_{A2}^{c}, I_{A2})$ under H_0, the maximal critical value occurs at $I_{A1} = E$ and $I_{A2} = E + \delta$, and the larger the α, the smaller the critical value. Thus, we calculate the critical value c_0 with the probability

$$\Pr\{W \ge c_0 \,\big|\, I_{A1} = E, I_{A2}^{c} = E + \delta \; I_{A2} = AC_2^{-1}(AC_1(E + \delta)), n_1, n_2\} = \alpha .$$

If the test rejects the null hypothesis H_0, then there is sufficient information to conclude that Product II is significantly better than Product I by a magnitude of δ. Table 4.2 shows some critical values for some minimum level requirement E = 2.0, 2.2, 2.4, 2.6, the magnitude δ = 0.0(0.1)0.5 of the difference between the two products, $n_1 = n_2 = n$ = 30(10)200, and α = 0.05.

Discussion

If more than two products are considered, the multiple comparison test can be adopted to tackle product selection problem. Assume that k products are compared, we implement $m = C_2^k$ tests to decide the one which has the highest profitability, i.e.,

$$H_{0i}: I_{Aa}^{c} - I_{Ab} \le 0 \quad \text{versus} \quad H_{1i}: I_{Aa}^{c} - I_{Ab} > 0$$

where i = 1, 2, ...,m; a, b = 1, 2, ..., k and $a \ne b$. By imitating the method of the Bonferroni test [4], the level of significance α is adjusted by the number of comparisons m to correct for type I error inflation. If the *p-value* of the test is less than or equal α / m, the test rejects the null hypothesis, then there is sufficient information to conclude that Product a is significantly better than Product b. After integrating conclusion of these tests, we can find the profitability order, and then the highest profitability is decided.

4.2.3 Required sample size

In last subsection, the product selection procedure is developed for given α risk, the probability of incorrectly judging H_0 as H_1, which does not take into account the β risk (Type II error: the probability of incorrectly judging H_1 as H_0). When the sample sizes and the α risk are defined, the power of test, $1-\beta$, can be calculated. Figure 4.2 plots the power of the test for I_{A1} = 2.0(0.2)2.6 versus various values of I_{A2}^{c}, $n_1 = n_2 = n$ = 30, 50, 100, 150, 200, and α = 0.05. It can be seen that the larger the sample size, the larger the power of test, and consequently, the smaller the β risk.

To reduce the β risk and at the same time maintain the α risk at the required level, one could increase the sample sizes. By calculating the power for a specific value of I_{A2}^{c}, we may obtain the minimal sample size required for designated power and α risk. The required sample size can be calculated by recursive search method with the following two probability equations:

$$\Pr\{W \ge c_0 \,|\, H_0 : I_{A2}^{c} \le I_{A1}, n_1, n_2, I_{A1} \ge E \text{ and } I_{A2} \ge E\} \le \alpha \text{, and}$$

$$\Pr\{W \ge c_0 \,|\, H_1 : I_{A2}^{c} > I_{A1}, n_1, n_2, I_{A1} \ge E \text{ and } I_{A2} \ge E\} \ge 1-\beta \text{ .}$$

Table 4.3 shows the sample sizes required for various designated selection power $1-\beta$ = 0.90, 0.95, 0.975, 0.99, the minimal level requirement E = 2.0(0.2)2.6, and the magnitude of difference $I_{A2}^{c} - I_{A1}$ = 0.5(0.1)1.0.

4.3 Magazine selection implementations

The English-teaching magazines in the publisher have a minimal requirement of profitability. The minimal requirement of the I_A values for three magazines is I_{A1} = I_{A2}^c = I_{A3}^c = 2.0, and the target profit for three magazines is T = 200, 000 dollars/month. We first use the Kolmogorov-Smirnov test for the demand data from Table 4.1 to confirm if the data is normally distributed. A test result in *p-value* > 0.05, which means that data is normally distributed. Histograms of the data are shown in Figure 4.3. Now, we consider two examples presented in the Section 4.1 as follows:

Example 1

To determine if the Magazine II's profitability is higher than Magazine I, we perform the hypothesis testing: $H_0: I_{A2}^c - I_{A1} \leq \delta$ versus $H_1: I_{A2}^c - I_{A1} > \delta$. For the demand data of the two magazines displayed in Table 4.1, we calculate the sample means, sample standard deviations and the sample estimators for both magazines, and obtain that $\bar{x}_1$ = 25.180, $\bar{x}_2$ = 27.010, s_1 = 2.124, s_2 = 2.751, $\tilde{I}_{A1}$ = 2.420, $\tilde{I}_{A2}$ = 3.731, $\tilde{I}_{A2}^c$ = 3.480 , and thus W = 1.059. If α = 0.05, from Table 4.2, the critical value for n_1 = n_2 = n = 100, I_{A1} = 2.0 (the minimum requirement of I_A), δ = 0 is 0.399. Since the test statistic W = 1.059 > 0.399, we therefore conclude that the Magazine II's profitability is higher than Magazine I with 95% confidence level. We also calculate the critical value for δ = 0.56, 0.57, 0.58, 0.59, 0.60, 0.61 with n_1 = n_2 = n = 100, I_{A1} = 2.0. The decision of the hypotheses is shown in Table 4.4. Based on the testing results, we can conclude that the Magazine II's profitability is higher than Magazine II by a magnitude of 0.60, i.e. $I_{A2}^c > I_{A1} + 0.60$.

If the expected $I_{A2}^c = 0.60$ and selection power is 0.95, the sample size required is 195 as in Table 4.3. Since the sample sizes of two magazines are smaller than 195, the selection power for testing $H_0: I_{A2}^c - I_{A1} \leq 0$ versus $H_1: I_{A2}^c - I_{A1} > 0$ would be less than 0.95. In fact, the power of test for $I_{A2}^c = 2.60$ is 0.7723, that is the β risk of incorrectly accepting $I_{A2}^c \leq I_{A1}$ while actually $I_{A2}^c > I_{A1}$ is true is up to 0.1777. In order to reduce the β risk, we would suggest the manager to collect more demand data for satisfying a designated power.

Example 2

To determine if the Magazine II is the highest profitability of three magazines, we perform the following $m = C_2^3 = 3$ tests:

$$H_{01} : I_{A2}^c - I_{A1} \le 0 \quad \text{versus} \quad H_{11} : I_{A2}^c - I_{A1} > 0,$$

$$H_{02} : I_{A3}^c - I_{A1} \le 0 \quad \text{versus} \quad H_{12} : I_{A3}^c - I_{A1} > 0,$$

$$H_{03} : I_{A3}^c - I_{A2} \le 0 \quad \text{versus} \quad H_{13} : I_{A3}^c - I_{A2} > 0.$$

If $\alpha = 0.05$, we calculate the *p-value* for three tests, and obtain that $P_1 = 0.00002 < \alpha / m = 0.01667$, $P_2 = 0.00017 < \alpha / m = 0.01667$, and $P_3 = 0.78698 > \alpha / m = 0.01667$. We can conclude that Magazine II is significantly better than Magazine I (reject H_{01}), Magazine III is significantly better than Magazine I (reject H_{02}), and Magazine II is significantly better than Magazine III (accept H_{03}). Then, the Magazine II's profitability is the highest with 95% confidence level (i.e., Magazine II > Magazine III > Magazine I).

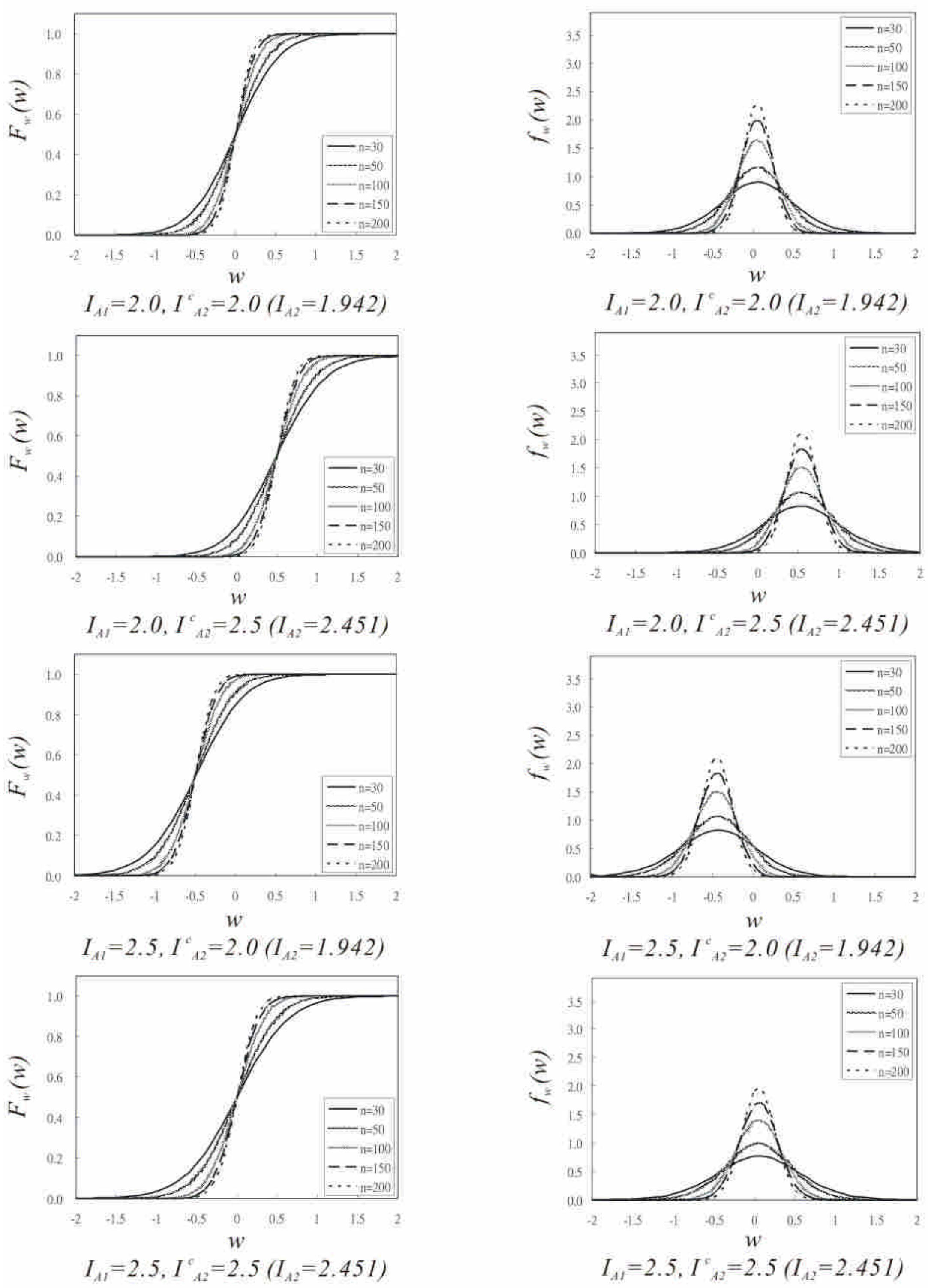

Figure 4.1 CDF and PDF plots of *W* for sample sizes n = 30, 50, 100, 150, 200.
(from bottom to top in plots)

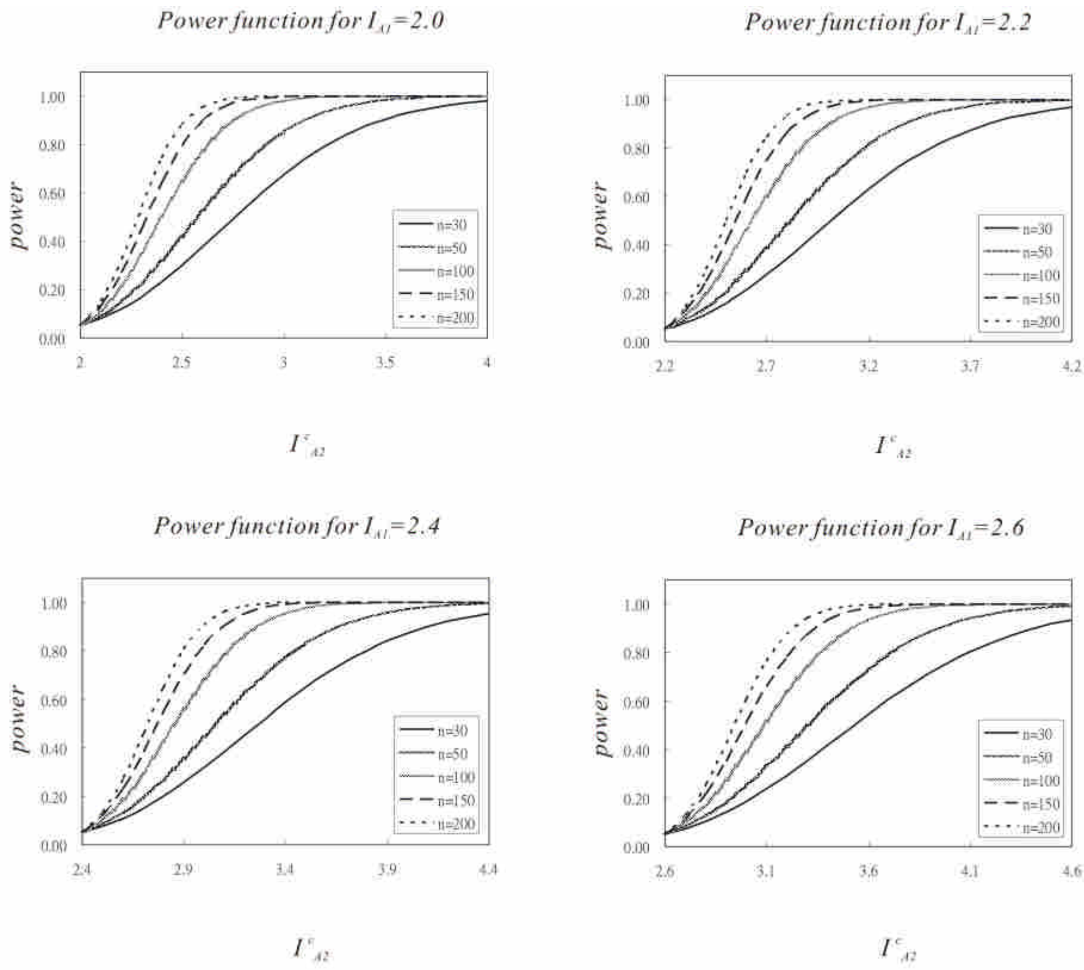

Figure 4.2 Power curves for $I_{A1} = 2.0, 2.2, 2.4, 2.6$, with sample sizes n = 30, 50, 100, 150, 200.

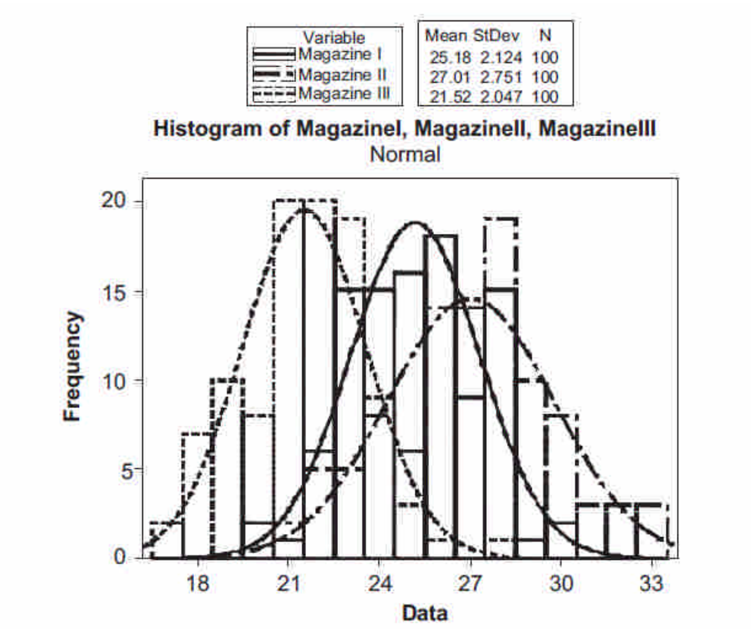

Figure 4.3 Histograms of the demand data for the Magazine I, Magazine II, and Magazine III.

Table 4.1 The demand units (in thousand)/month for the three magazines.

Basic magazine (Magazine I)																			
23	21	26	24	24	28	24	25	26	23	24	24	26	25	26	28	27	27	28	24
22	28	25	26	24	23	28	20	28	24	26	26	28	28	26	25	28	24	25	23
27	27	24	26	30	26	26	27	30	23	23	24	28	22	23	23	25	29	25	26
25	22	22	26	24	27	28	26	25	25	26	23	24	28	22	27	26	27	23	27
28	26	25	24	26	23	23	24	25	28	23	25	22	20	25	23	25	25	23	28
Intermediate magazine (Magazine II)																			
30	29	26	27	30	24	27	30	28	24	32	24	27	26	28	31	26	24	27	22
28	23	21	29	27	26	29	23	25	22	30	28	31	26	28	25	27	26	28	25
25	28	27	28	30	22	26	27	29	27	26	28	25	28	22	26	29	29	28	32
23	27	24	27	31	30	28	27	22	24	23	29	33	26	24	28	28	28	26	28
28	28	26	29	29	24	25	27	32	33	26	30	30	33	29	27	21	26	28	23
High magazine (Magazine III)																			
21	22	23	21	25	22	25	19	23	19	18	18	28	20	23	19	21	21	18	19
21	24	24	20	22	21	20	21	21	17	24	26	19	22	18	20	19	23	22	23
23	23	23	18	25	21	21	22	22	22	24	23	24	22	19	21	23	23	18	20
22	21	23	19	23	22	22	21	21	24	22	22	23	24	17	23	24	22	21	22
23	22	24	21	23	22	21	22	23	22	19	20	18	20	21	21	20	23	21	19

Table 4.2 Critical values for rejecting $I_{A2}^{c} - I_{A1} \le \delta$ with n = 30(10)200 and $\alpha = 0.05$.

I_{A1}	n	(I_{A2}^{c}, I_{A2})					
		(2.0, 1.908) $\delta = 0.0$	(2.1, 2.005) $\delta = 0.1$	(2.2, 2.102) $\delta = 0.2$	(2.3, 2.199) $\delta = 0.3$	(2.4, 2.296) $\delta = 0.4$	(2.5, 2.450) $\delta = 0.5$
2.0	30	0.747	0.863	0.980	1.095	1.215	1.333
	40	0.641	0.754	0.868	0.983	1.097	1.212
	50	0.570	0.682	0.794	0.907	1.019	1.133
	60	0.519	0.629	0.740	0.851	0.963	1.074
	70	0.479	0.589	0.699	0.809	0.919	1.030
	80	0.448	0.556	0.665	0.775	0.884	0.994
	90	0.422	0.530	0.638	0.747	0.856	0.965
	100	0.399	0.507	0.615	0.723	0.832	0.940
	110	0.380	0.488	0.595	0.703	0.811	0.919
	120	0.364	0.471	0.578	0.686	0.793	0.901
	130	0.349	0.456	0.563	0.670	0.777	0.885
	140	0.336	0.443	0.550	0.656	0.763	0.871
	150	0.325	0.431	0.537	0.644	0.751	0.858
	160	0.314	0.420	0.527	0.633	0.739	0.846
	170	0.305	0.411	0.517	0.623	0.729	0.836
	180	0.296	0.402	0.508	0.614	0.720	0.826
	190	0.288	0.394	0.499	0.605	0.711	0.817
	200	0.281	0.386	0.492	0.597	0.703	0.809
I_{A1}	n	(I_{A2}^{c}, I_{A2})					
		(2.2, 2.102) $\delta = 0.0$	(2.3, 2.199) $\delta = 0.1$	(2.4, 2.296) $\delta = 0.2$	(2.5, 2.393) $\delta = 0.3$	(2.6, 2.490) $\delta = 0.4$	(2.7, 2.587) $\delta = 0.5$
2.2	30	0.801	0.918	1.035	1.153	1.271	1.388
	40	0.686	0.800	0.915	1.029	1.144	1.260
	50	0.611	0.723	0.835	0.948	1.061	1.174
	60	0.555	0.666	0.777	0.889	1.001	1.112
	70	0.513	0.623	0.733	0.843	0.954	1.065
	80	0.479	0.588	0.697	0.807	0.917	1.027
	90	0.451	0.559	0.669	0.777	0.886	0.996
	100	0.427	0.535	0.643	0.752	0.861	0.969
	110	0.407	0.514	0.622	0.730	0.839	0.947
	120	0.389	0.496	0.604	0.712	0.819	0.927
	130	0.374	0.481	0.588	0.695	0.803	0.910
	140	0.360	0.467	0.573	0.680	0.788	0.895
	150	0.348	0.454	0.561	0.667	0.774	0.881
	160	0.336	0.443	0.549	0.655	0.762	0.869
	170	0.326	0.432	0.538	0.645	0.751	0.858
	180	0.317	0.423	0.529	0.635	0.741	0.847
	190	0.308	0.414	0.520	0.626	0.732	0.838
	200	0.300	0.406	0.512	0.617	0.723	0.829

Table 4.2 (Continued).

I_{A1}	n	(I_{A2}^c, I_{A2})					
		(2.4, 2.296) $\delta=0.0$	(2.5, 2.393) $\delta=0.1$	(2.6, 2.490) $\delta=0.2$	(2.7, 2.587) $\delta=0.3$	(2.8, 2.683) $\delta=0.4$	(2.9, 2.780) $\delta=0.5$
2.4	30	0.855	0.972	1.090	1.208	1.326	1.445
	40	0.733	0.848	0.962	1.077	1.192	1.308
	50	0.652	0.764	0.877	0.990	1.103	1.217
	60	0.593	0.704	0.816	0.927	1.039	1.151
	70	0.548	0.655	0.766	0.879	0.990	1.101
	80	0.511	0.621	0.730	0.840	0.950	1.060
	90	0.481	0.590	0.699	0.808	0.917	1.027
	100	0.456	0.564	0.673	0.781	0.890	0.999
	110	0.434	0.542	0.650	0.758	0.867	0.975
	120	0.415	0.523	0.630	0.738	0.846	0.954
	130	0.399	0.506	0.613	0.721	0.828	0.936
	140	0.384	0.491	0.598	0.705	0.812	0.920
	150	0.371	0.477	0.582	0.691	0.798	0.905
	160	0.359	0.465	0.572	0.678	0.785	0.892
	170	0.348	0.454	0.560	0.667	0.773	0.880
	180	0.338	0.444	0.550	0.656	0.763	0.869
	190	0.329	0.435	0.541	0.647	0.753	0.859
	200	0.321	0.426	0.532	0.638	0.744	0.850

I_{A1}	n	(I_{A2}^c, I_{A2})					
		(2.6, 2.490) $\delta=0.0$	(2.7, 2.587) $\delta=0.1$	(2.8, 2.683) $\delta=0.2$	(2.9, 2.780) $\delta=0.3$	(3.0, 2.877) $\delta=0.4$	(3.1, 3.058) $\delta=0.5$
2.6	30	0.911	1.029	1.147	1.265	1.383	1.502
	40	0.781	0.896	1.010	1.126	1.241	1.357
	50	0.694	0.807	0.920	1.033	1.147	1.260
	60	0.631	0.743	0.854	0.966	1.078	1.190
	70	0.583	0.693	0.804	0.915	1.026	1.137
	80	0.544	0.654	0.764	0.873	0.984	1.094
	90	0.512	0.621	0.730	0.840	0.949	1.059
	100	0.485	0.594	0.702	0.811	0.920	1.029
	110	0.462	0.570	0.678	0.787	0.895	1.004
	120	0.442	0.550	0.658	0.765	0.873	0.982
	130	0.425	0.532	0.639	0.747	0.854	0.962
	140	0.409	0.516	0.623	0.730	0.838	0.945
	150	0.395	0.501	0.608	0.715	0.822	0.930
	160	0.382	0.487	0.595	0.702	0.809	0.916
	170	0.370	0.477	0.583	0.690	0.796	0.903
	180	0.360	0.466	0.572	0.678	0.785	0.891
	190	0.350	0.456	0.562	0.668	0.774	0.881
	200	0.341	0.447	0.553	0.659	0.765	0.871

Table 4.3 Sample size required for testing $H_0: I_{A2}^c - I_{A1} \le \delta$ versus $H_1: I_{A2}^c - I_{A1} > \delta$.

I_{A1}	I_{A2}^c	***Power***				I_{A1}	I_{A2}^c	***Power***			
		0.90	0.95	0.975	0.99			0.90	0.95	0.975	0.99
2.0	2.5	216	275	332	405	2.2	2.7	246	313	379	461
	2.6	153	195	235	288		2.8	174	222	268	327
	2.7	115	146	177	216		2.9	130	166	201	246
	2.8	90	114	138	169		3.0	101	130	157	192
	2.9	72	92	112	136		3.1	82	105	126	155
	3.0	60	76	92	113		3.2	68	86	105	128
I_{A1}	I_{A2}^c	***Power***				I_{A1}	I_{A2}^c	***Power***			
		0.90	0.95	0.975	0.99			0.90	0.95	0.975	0.99
2.4	2.9	279	356	429	523	2.6	3.1	314	400	483	590
	3.0	197	251	304	371		3.2	221	283	341	417
	3.1	147	188	227	278		3.3	166	212	255	312
	3.2	115	147	177	217		3.4	129	165	196	244
	3.3	93	118	143	175		3.5	104	133	160	197
	3.4	76	97	118	144		3.6	86	109	132	162

Table 4.4 Critical values and decisions of testing the two magazines.

I_{A1}	2.00	2.00	2.00	2.00	2.00	2.00
I_{A2}^c	2.56	2.57	2.58	2.59	2.60	2.61
δ	0.56	0.57	0.58	0.59	0.60	0.61
c_0	1.006	1.017	1.028	1.038	1.049	**1.060>1.059**
Decision	Reject H_0	Reject H_0	Reject H_0	Reject H_0	Reject H_0	Accept H_0

Chapter 5

Conclusions and Future Research

In this thesis, we considered the newsboy-type products with normally distributed demand, and investigated the product evaluation problem and product selection problem. In addition, we developed a new index which has a simple form expression of profitability. Note that the profitability presented in this thesis defines as the probability of achieving the target profit under the optimal ordering condition. The proposed index, which we refer to as the Achievable Capacity Index (*ACI*, I_A), can reduce the difficulty of effective estimation when the demand mean μ and the demand standard deviation σ are unknown. For example, the unbiased and effective estimator $\tilde{I}_A$ is found effortlessly, and the distribution of estimator $\tilde{I}_A$ can be derived. By utilizing the proposed index, we adopted the statistical hypothesis testing methodology to tackle these two problems. In this chapter, we make conclusions and provide possible extensions of the present work for the further research.

6.1 Conclusions

In Chapter 2, we investigated the product evaluation problem which examined whether the profitability meets a designated requirement. We presented the hypothesis test to solve the evaluation problem, i.e., $H_0: I_A \le C$ as against $H_1: I_A > C$, where C is the designated requirement of I_A. Some tables (Table 2.1-2.4) are shown to practitioners or managers for deciding whether the old product is unworthy of being ordered as the new product is introduced under the accepted risks (Type I and Type II errors). Finally, a real-world application of a fresh food product is presented to illustrate the practicality of the exact approach.

Chapter 3 investigated the product evaluation problem as well as considered the demand data is collected from multiple samples rather than single sample. An unbiased and effective estimator of I_A based on multiple samples is also derived. The critical value of the test is calculated to determine evaluation result under the preset risk (Type I error). The implementation of the existing statistical theory for the profitability of Newsboy-type product makes it possible to apply the complicated theoretical results to the actual productions. For convenience, we also provided a simple step-by-step procedure for the practitioners to use in making decisions. Finally, a real-world example on the sales of donuts is presented to illustrate the practicality of the exact approach.

In Chapter 4, we investigated the product selection problem which deals with comparing products and selecting the one that has a significantly higher profitability. We provided the hypothesis testing to solve this selection problem, i.e., $H_0: I_{A2}^c - I_{A1} \leq \delta$ versus $H_1: I_{A2}^c - I_{A1} > \delta$, where $\delta \geq 0$. Some tables are shown the selection decisions and sample size required under the designated risks (Type I and Type II errors). Note that we also used the multiple comparison test for comparing more than two products. Our product selection procedure can be applied to cases with unequal sample sizes. The special case related the two products with equal price and costs can be suited this selection procedure. Finally, a real-world application comparing English-teaching magazines is presented to illustrate the practicality of the exact approach.

6.2 Future Research

The results of our study suggest five dimensions which could be addressed by future research.

1. We can further consider the imprecise demand and combine the fuzzy set concepts.

2. The demand follows the truncated normal distribution for relaxing the assumption of $cv < 0.3$.

3. In the product selection problem, we consider that the random demands of two products are dependent each other. Then, the joint probability density function of demands ought to be derived.

4. If the demand observations are costly or the data are sparse over time, we can adopt the sequential tests which can significantly reduce the sampling costs.

5. If the non-normal demand is considered, the bootstrap resampling method can be used, which handles more general distributions. The bootstrap resampling method does not rely on any distributional assumptions about the underlying population, which has been proved useful in many existing research for those cases. By applying this method, we can use the lower confidence bound of I_A to implement conservative profitability evaluation.

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