terrain. All these considerations are being addressed by the ongoing effort.

References

[1] E. G. Gilbert and D. W. Johnson, "Distance functions and their application to robot path planning in the presence of obstacles," *IEEE J. Robotics Automat.*, vol. RA-1, no. 1, pp. 21-30, Mar. 1985.

[2] J. L. Crowley, "Navigation for an intelligent mobile robot," *IEEE J. Robotics Automat.*, vol. RA-1, no. 1, pp. 31-41, Mar. 1985.

[3] R. L. Jolly, "Path planning for obstacle avoidance using distance and penalty functions," *ROBEXS '86* (Houston, TX), June 1986.

[4] T. S. Chang, K. Qiu, and J. J. Nitao, "An obstacle avoidance algorithm for an automatic land vehicle," *Int. J. Robotics Automat.*, vol. 2, no. 1, pp. 21-25, 1987.

[5] V. H. L. Cheng and B. Sridhar, "Considerations for automated nap-of-the-earth rotorcraft flight," in *Proc. 1988 Amer. Contr. Conf.* (Atlanta, GA), June 15-17, 1988.

[6] E. K. Wong and K. S. Fu, "A hierarchical orthogonal space approach to three-dimensional path planning," *IEEE J. Robotics Automat.*, vol. RA-2, no. 1, pp. 42-53, Mar. 1986.

[7] O. Khatib, "Real-time obstacle avoidance for manipulators and mobile robots," in *Proc. 1985 IEEE Int. Conf. Robotics Automat.* (St. Louis, MO), Mar. 25-28, 1985, pp. 500-505.

[8] V. L. Lumelsky, "On the connection between maze-searching and robot motion planning algorithms," in *Proc. 27th Conf. Decision Contr.*, (Austin, TX), Dec. 1988.

[9] B. Sridhar and A. V. Phatak, "Simulation and analysis of image-based navigation system for rotorcraft low-altitude flight," presented at the *AHS Mtg. Automat. Appl. Rotorcraft* (Atlanta, GA), Apr. 4-6, 1988.

[10] L. Matthies, R. Szeliski, and T. Kanade, "Kalman filter-based algorithms for estimating depth from image sequences," in Proc. Image Understanding Workshop, Apr. 1988, vol. 1, pp. 199-213, DARPA Inform. Sci. Technol. Office.

[11] V. H. L. Cheng, "Obstacle-avoidance automatic guidance: A concept-development study," in *Proc. AIAA Guidance, Navigat. Contr. Conf.* (Minneapolis, MN), Aug. 15-17, 1988.

Comments on "Computation of Input Generalized Forces for Robots with Closed Kinematic Chain Mechanism"

SHIR-KUAN LIN

The above paper[1] by Luh and Zheng extended Wittenburg's reduced system [1] to derive a formula for the computation of inverse torques of robots with closed kinematic chains. Their result is a milestone and of great worth since the earlier works dealing with dynamics of closed kinematic chains all concentrated on the formulation of the forward dynamic equations. However, some equations in Luh and Zheng's paper[1] are confusing.

In their paper, if we substitute the Lagrange multipliers (24) into (22) and note (25) we get

$$J^0(q^0)\ddot{q}^0+f^0(q^0,\dot{q}^0)+g^0(q^0)=[\tau_1^c \cdots \tau_{n-m}^c\ 0 \cdots 0]^T. \quad \text{(C1)}$$

It is mathematically not equivalent to (17), which is repeated as follows:

$$J^0(q^0)\ddot{q}^0+f^0(q^0,\dot{q}^0)+g^0(q^0)=[\tau_1^0 \cdots \tau_n^0]^T. \quad \text{(C2)}$$

It should be remarked that the motion and configuration of joints in the virtual open-chain robot are assumed, although not explicitly in Luh and Zheng's paper,[1] identical with those in the orginal closed-chain robot. Thus $q^0 = q^c$, $J^0 = J^c$, $f^0 = f^c$, $g^0 = g^c$, where the superscripts "0" and "c" denote the virtual open chain and the orginal closed chain, respectively. But the actuator torques in the virtual open chain are not equal to those in the original closed chain, i.e., $\tau^0 \neq \tau^c$, otherwise there were no physical difference between a closed chain and its virtual open chain. The fact that the solutions of the above two sets of equations $\ddot{q}^0$ are not identical gives rise to doubt on the correctness of the result (25) in Luh and Zheng's paper.[1]

Using the Lagrange's equations with Lagrange multipliers [2], [3] to study the dynamics of a closed chain, the same result (25) has also been obtained. I think that the result of Luh and Zheng's Paper[1] is definitely correct, but the derivation is still confusing.

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[1] J. Y. S. Luh and Y. F. Zhang, *IEEE J. Robotics Automat.*, vol. RA-1, no. 2, pp. 95-103, June 1985.

Reply[2] by J. Y. S. Luh[3]

The comments by Shir-Kuan Lin assert that in our paper[1] we extend Wittenburg's reduced system. He then discredits our work based on the following exercise: Using our equations (22), (24), and (25), he derives his (C1) and claims that (C1) contradicts our (17).

Our response is as follows: In our paper, we refer to Wittenburg's method when we introduce our (20). There is no base to assert that any of our equations prior to (20) are derived from his work. This includes (17). Equation (17) represents the equation of motion for the virtual *open-chain* robot in which the term virtual open chain means that the original closed-chain robot is virtually (i.e., make-believe) cut open at one of its joints. We then treat it as if an open-chain robot. Thus (17) does not include the constraints. The constraints, however, are imposed when the virtual open chain is recoved to become the original closed chain whose equations of virtual work and motion are (20) and (22), respectively. Consequently, the derived equations (21)-(25) are for the closed-chain robots which include the constraints. Lin uses (22)-(25) to derive his equation (C1) which naturally is for closed-chain robots with constraints. Hence (C1) agrees with (20) and, of course, not with (17).

Perhaps we should have emphasized in our original paper[1] that (17) does not include the constraints which are introduced by the closed-chain mechanism. However, we believe that the statement "to form an n-jointed open-chain mechanism with a tree structure" in our paper includes that implication.

References

[1] J. Wittenburg, *Dynamics of Systems of Rigid Bodies.* Stuttgart, FRG: B. G. Teubner, 1977, sec. 5.3.

[2] H. Goldstein, *Classical Mechanics.* Reading, MA: Addison-Wesley, 1959, sec. 2.4.

[3] W. Greiner, *Theoretische Physik, Band 2: Mechanik II,* 4th ed. Thun, FRG: Harri Deutsch, 1986, pp. 236 and 261.

[2] Manuscript received February 3, 1989; revised August 2, 1989.
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