Least-Squares Regenerative Hybrid Array for Adaptive Beamforming

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Abstract—**The regenerative hybrid adaptive antenna array has recently been proposed for data communications. It utilizes both the steering vector and reference signal, which is acquired from the array output through a detection-generation procedure, to preserve the desired signal while suppressing the interference. The regenerative hybrid adaptive array is much less sensitive to errors in the steering vector than the Applebaum-type array. The application of the least-squares method to the regenerative hybrid array to achieve fast convergence is investigated. Recursive algorithms such as the *QR* decomposition can be used to update the weights for computational efficiency. It is shown that the least-squares regenerative hybrid array converges to the steady state of the original regenerative hybrid array which is based on the gradient search algorithm. The array transient behavior and steady state performance are simulated by computer.**

I. Introduction

In adaptive beamforming, one can avoid the suppression of the desired signal by using a steering vector or a reference signal [1], [2]. Ideally, both these methods can achieve the maximum output signal-to-interference-plus-noise ratio (SINR). The Applebaum-type array, however, is very sensitive to errors in the steering vector, especially in the presence of a desired signal [3], [4]. As to the reference signal, it can be acquired only for limited types of signals, for example, the spread spectrum signal [5]. For point-to-point spread spectrum communications, the idea of a hybrid adaptive array has been described which utilizes both the steering vector and reference signal [6], [7]. It has been shown that the hybrid adaptive array is much less sensitive to errors of the steering vector than the Applebaum-type array and does not exhibit the weight cycling phenomenon reported in [8] for an imperfect reference signal.

Recently, a regenerative hybrid array has been proposed for data communications without using the spread spectrum technique [9], [10]. It consists of an Applebaum-type array and a regenerative reference loop in which the reference signal is generated through a detection–generation process. In adaption, the regenerative reference loop is first open and the array behaves like an Applebaum-type array. After weights converge, the loop is closed. The reference signal generated by the loop is highly correlated with the desired signal because of the detection-generation process. Therefore, it can be used to eliminate the desired signal component of the feedback. It has been shown that the weight vector of the regenerative hybrid array converges to that of the Applebaum-type array adapted without the desired signal present. Therefore, sensitivity to errors of the steering vector is reduced significantly.

The regenerative hybrid array described in [9], [10] is based on the gradient search algorithm, which converges slowly. In this paper, we investigate the utilization of the least-squares method [11]–[13] to adapt the regenerative hybrid array to achieve more rapid convergence. Recursive algorithms such as the *QR* decomposition can be used to update the weights. We analyze the least-squares solution of the weight vector and show that it converges to the steady state weight vector of the original regenerative hybrid array. Computer simulations are performed to simulate the array transient behavior and to compare its steady state performance with those of the Applebaum-type array and high-order derivative constraint arrays [14], [15].

II. Least-Squares Regenerative Hybrid Array

Let us consider the N element regenerative hybrid array shown in Fig. 1. It consists of a general sidelobe canceler (GSC) structure Applebaum-type array [16], [17] and a regenerative reference loop. The desired signal is assumed to be data modulated and its phase vector at the array is denoted by $\mathbf{s}_d$. The imperfect steering vector used for beamforming is $\mathbf{s}^*$. Initially the regenerative reference loop is open and the array behaves like an Applebaum-type array. Although the steering vector is imperfect, the interference in general will be suppressed by the Applebaum-type array as it reaches the steady state. The steering vector is assumed to be accurate enough so that the Applebaum-type array can generate a steady state output SINR of a few decibels. The reference loop is closed after the Applebaum array converges. This loop acquires the reference signal from the array output through a detection–generation procedure with the assumption of a perfect synchronization. The reference signal is intended to eliminate the desired signal component of the feedback so that the weight vector of the sidelobe canceler is adapted as if there were no desired signal present. Therefore, sensitivity to errors of the steering vector is significantly reduced. In the follow-

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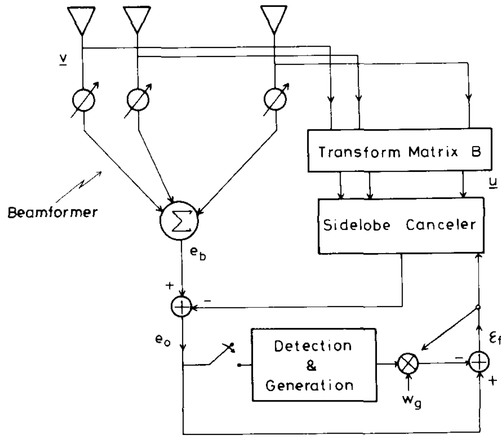

Fig. 1. Regenerative hybrid array.

ing, we investigate the application of the least-squares method to adapt the weights of the regenerative hybrid array in order to achieve fast convergence.

We first describe briefly the least-squares solution of the Applebaum-type array, i.e., the regenerative reference loop is open. From Fig. 1, the output of the beamformer at time k is

$$e_b(k) = \mathbf{v}^T(k)\mathbf{s}^* \tag{1}$$

where T and the asterisk denote transpose and conjugate, respectively, and $\mathbf{v}(k)$ is the array input vector. The input vector of the sidelobe canceler is

$$\mathbf{u}(k) = B\mathbf{v}(k) \tag{2}$$

where B is an $(N-1) \times N$ transform matrix with the null space spanned by $\mathbf{s}$. To find the least-squares solution of the weight vector of the sidelobe canceler $\mathbf{w}_u(k)$, we can choose the array output to be the error signal. That is, at each time k, we minimize the error function

$$\xi(k) = \sum_{i=1}^{k} |e_0(i)|^2 \tag{3}$$

where

$$e_0(i) = e_b(i) - \mathbf{u}^T(i)\mathbf{w}_u(k). \tag{4}$$

It is known that the solution of (3) is [11]

$$A_u^*(k)A_u^T(k)\mathbf{w}_u(k) = A_u^*(k)\mathbf{e}_b(k) \tag{5}$$

where $\mathbf{e}_b(k)$ is the desired response vector

$$\mathbf{e}_b(k) = [e_b(1), e_b(2), \cdots, e_b(k)]^T \tag{6}$$

and $A_u(k)$ is the data matrix

$$A_u(k) = [\mathbf{u}(1), \mathbf{u}(2), \cdots, \mathbf{u}(k)]. \tag{7}$$

Using (1) and (2), (5) can be written as

$$B^*A_v^*(k)A_v^T(k)[\mathbf{s}^* - B^T\mathbf{w}_u(k)] = 0 \tag{8}$$

where $A_v(k)$ is the matrix consisting of the input vectors,

$$A_v(k) = [\mathbf{v}(1), \mathbf{v}(2), \cdots, \mathbf{v}(k)]. \tag{9}$$

In (8) the term in brackets is the equivalent weight vector of the array, which is denoted by $\mathbf{w}_v(k)$,

$$\mathbf{w}_v(k) = \mathbf{s}^* - B^T\mathbf{w}_u(k). \tag{10}$$

From (8) and the fact that $\mathbf{s}$ spans the null space of B, the equivalent weight vector can be written in the form

$$\mathbf{w}_v(k) = \mu_1(k)[A_v^*(k)A_v^T(k)]^{-1}\mathbf{s}^*. \tag{11}$$

In (11), the constant $\mu_1(k)$ can be determined from the constraint

$$\mathbf{s}^T\mathbf{w}_v(k) = N, \tag{12}$$

which results in

$$\mu_1(k) = \frac{N}{\mathbf{s}^T[A_v^*(k)A_v^T(k)]^{-1}\mathbf{s}^*}. \tag{13}$$

With the assumption that the array input processes are ergodic and k is large, we have

$$A_v^*(k)A_v^T(k) = \sum_{i=1}^{k} \mathbf{v}^*(i)\mathbf{v}^T(i) \simeq kR \tag{14}$$

where R is the covariance matrix of $\mathbf{v}(i)$. Therefore, (11) converges to the steady state of the gradient search based Applebaum-type array.

After the Applebaum-type array reaches the steady state, the regenerative reference loop is closed. This loop regenerates a signal $e_g(k)$ from the array output through the detection-generation procedure. $e_g(k)$ is multiplied by a weight $w_g(k)$ to cancel the desired signal component of the feedback. $w_g(k)$ as well as $\mathbf{w}_u(k)$ are adapted by the least-squares method. In adaption, the feedback can be chosen to be the error signal, and the error function to be minimized can be defined as

$$\xi(k) = \beta \sum_{i=1}^{k_1} |e_0(i)|^2 + \sum_{i=k_1+1}^{k} |\epsilon_f(i)|^2. \tag{15}$$

In (15), $\epsilon_f(i)$ is the feedback

$$\epsilon_f(i) = e_b(i) - [\mathbf{u}^T(i) \;\; e_g(i)] \begin{bmatrix} \mathbf{w}_u(k) \\ w_g(k) \end{bmatrix}. \tag{16}$$

k_1 is the instant just before the regenerative reference loop is closed. β is a weighting factor between zero and unity,

$$0 < \beta \le 1 \tag{17}$$

which makes data received before k_1 less important. That choosing β properly could speed up the convergence rate is discussed in Section III.

From the fact that the regenerated signal $e_g(i)$ is zero for

$i \leq k_1$, (15) can be written as

$$\xi(k) = \beta \sum_{i=1}^{k_1} |\epsilon_f(i)|^2 + \sum_{i=k_1+1}^{k} |\epsilon_f(i)|^2. \quad (18)$$

The least-squares solution of (18) is

$$\begin{bmatrix} \hat{A}_u(k) \\ \mathbf{e}_g^T(k) \end{bmatrix}^* \begin{bmatrix} \hat{A}_u(k) \\ \mathbf{e}_g^T(k) \end{bmatrix}^T \begin{bmatrix} \mathbf{w}_u(k) \\ w_g(k) \end{bmatrix} = \begin{bmatrix} \hat{A}_u(k) \\ \mathbf{e}_g^T(k) \end{bmatrix}^* \hat{\mathbf{e}}_b(k). \quad (19)$$

In (19), $\hat{A}_u(k)$, $\mathbf{e}_g(k)$, and $\hat{\mathbf{e}}_b(k)$ are defined by

$$\hat{A}_u(k) = [\sqrt{\beta}\mathbf{u}(1), \cdots, \sqrt{\beta}\mathbf{u}(k_1), \mathbf{u}(k_1+1), \cdots, \mathbf{u}(k)] \quad (20)$$

$$\mathbf{e}_g(k) = [0, \cdots, 0, e_g(k_1+1), \cdots, e_g(k)]^T \quad (21)$$

and

$$\hat{\mathbf{e}}_b(k) = [\sqrt{\beta}e_b(1), \cdots, \sqrt{\beta}e_b(k_1), e_b(k_1+1), \cdots, e_b(k)]^T. \quad (22)$$

Equation (19) can be written as the two linear equations shown below:

$$\hat{A}_u^*(k)\hat{A}_u^T(k)\mathbf{w}_u(k) + \hat{A}_u^*\mathbf{e}_g(k)w_g(k) = \hat{A}_u^*(k)\hat{\mathbf{e}}_b(k) \quad (23)$$

and

$$\mathbf{e}_g^{T^*}(k)\hat{A}_u^T(k)\mathbf{w}_u(k) + \|e_g(k)\|^2 w_g(k) = \mathbf{e}_g^{T^*}(k)\hat{\mathbf{e}}_b(k). \quad (24)$$

In (24), $\|\cdot\|$ denotes the norm of a vector. From (24), $w_g(k)$ can be expressed as

$$w_g(k) = [\mathbf{e}_g^{T^*}(k)\hat{\mathbf{e}}_b(k) - \mathbf{e}_g^{T^*}(k)\hat{A}_u^T(k)\mathbf{w}_u(k)]/\|e_g(k)\|^2. \quad (25)$$

Using (4), (20), and (22), $w_g(k)$ can be written in the form

$$w_g(k) = \frac{\mathbf{e}_g^{T^*}(k)\hat{\mathbf{e}}_0(k)}{\|e_g(k)\|^2} \quad (26)$$

where

$$\hat{\mathbf{e}}_0(k) = [\sqrt{\beta}e_0(1), \cdots, \sqrt{\beta}e_0(k_1), e_0(k_1+1), \cdots, e_0(k)]. \quad (27)$$

To obtain $\mathbf{w}_u(k)$, we substitute (25) into (24), which results in

$$[\hat{A}_u^*(k)\hat{A}_u^T(k) - \hat{A}_u^*(k)\mathbf{e}_g(k)\mathbf{e}_g^{T^*}(k)\hat{A}_u^T(k)/\|\mathbf{e}_g(k)\|^2]\mathbf{w}_u(k) = [\hat{A}_u^*(k)\hat{\mathbf{e}}_b(k) - \hat{A}_u^*(k)\mathbf{e}_g(k)\mathbf{e}_g^{T^*}(k)\hat{\mathbf{e}}_b(k)/\|\mathbf{e}_g(k)\|^2]. \quad (28)$$

Or,

$$\mathbf{w}_u(k) = [\hat{A}_u^*(k)\hat{A}_u^T(k) - \hat{A}_u^*(k)\mathbf{e}_g(k)\mathbf{e}_g^{T^*}(k) \cdot \hat{A}_u^T(k)/\|\mathbf{e}_g(k)\|^2]^{-1} \cdot [\hat{A}_u^*(k)\hat{\mathbf{e}}_b(k) - \hat{A}_u^*(k) \cdot \mathbf{e}_g(k)\mathbf{e}_g^{T^*}(k)\hat{\mathbf{e}}_b(k)/\|\mathbf{e}_g(k)\|^2]. \quad (29)$$

It can be seen from (29) that if the regenerated signal $\mathbf{e}_g(k)$ is scaled by a complex number, $\mathbf{w}_u(k)$ remains the same. It can also be seen from Fig. 1 that if $\mathbf{e}_g(k)$ and $\mathbf{w}_g(k)$ are scaled by a complex number and the inverse of the number, respectively, $\mathbf{w}_u(k)$ remains unchanged.

To obtain the equivalent weight vector, $\mathbf{w}_v(k)$, we rewrite (28) by using (1) and (2)

$$B^*[\hat{A}_v^*(k)\hat{A}_v^T(k) - \hat{A}_v^*(k)\mathbf{e}_g(k)\mathbf{e}_g^{T^*}(k)\hat{A}_v^T(k)/\|\mathbf{e}_g(k)\|^2] \cdot [\mathbf{s}^* - B^T\mathbf{w}_u(k)] = 0 \quad (30)$$

where

$$\hat{A}_v(k) = [\sqrt{\beta}\mathbf{v}(1), \cdots, \sqrt{\beta}\mathbf{v}(k_1), \mathbf{v}(k_1+1), \cdots, \mathbf{v}(k)]. \quad (31)$$

Using (10) and the property that $\mathbf{s}$ spans the null space of B, we have from (30) that

$$\mathbf{w}_v(k) = \mu_2(k)[\hat{A}_v^*(k)\hat{A}_v^T(k) - \hat{A}_v^*(k)\mathbf{e}_g(k)\mathbf{e}_g^{T^*}(k) \cdot \hat{A}_v^T(k)/\|e_g(k)\|^2]^{-1}\mathbf{s}^*. \quad (32)$$

In (32), $\mu_2(k)$ is a constant which can be determined from (12),

$$\mu_2(k) = \frac{N}{\mathbf{s}^T[\hat{A}_v^*(k)\hat{A}_v^T(k) - \hat{A}_v^*(k)\mathbf{e}_g(k)\mathbf{e}_g^{T^*}(k)\hat{A}_v^T(k)/\|e_g(k)\|]\mathbf{s}^*}. \quad (33)$$

III. Convergence Properties

In this section, the convergence properties of the least-squares regenerative hybrid array are investigated. Since the interference has been suppressed as the regenerative reference loop is closed, $e_g(i)$ can be assumed to be uncorrelated with the interference. Furthermore, $e_g(i)$ is assumed to be uncorrelated with noises of the array elements. The term $\hat{A}_v^*(k)\mathbf{e}_g(k)\mathbf{e}_g^{T^*}(k)\hat{A}_v^T(k)$ in (32) in first examined below. Let $\alpha(i)$ denote the complex waveform of the desired signal. $\hat{A}_v(k)$ can be written

$$\hat{A}_v(k) = \mathbf{s}_d\hat{\boldsymbol{\alpha}}^T(k) + \hat{A}_n(k), \quad (34)$$

where $\mathbf{s}_d$ is the phase vector of the desired signal,

$$\hat{\boldsymbol{\alpha}}(k) = [\sqrt{\beta}\alpha(1), \cdots, \sqrt{\beta}\alpha(k_1), \alpha(k_1+1), \cdots, \alpha(k)]^T \quad (35)$$

and $\hat{A}_n(k)$ is the interference-plus-noise component of $\hat{A}_v(k)$. With the above assumptions, $\hat{A}_v^*(k)\mathbf{e}_g(k)\mathbf{e}_g^{T^*}(k)\hat{A}_v^T(k)$ can be approximated by

$$\hat{A}_v^*(k)\mathbf{e}_g(k)\mathbf{e}_g^{T^*}(k)\hat{A}_v^T(k) \simeq |\hat{\boldsymbol{\alpha}}^{T^*}(k)\mathbf{e}_g(k)|^2\mathbf{s}_d^*\mathbf{s}_d^T. \quad (36)$$

Since the first k_1 entries of $\mathbf{e}_g(k)$ are zero, (36) can be written

$$\hat{A}_v^*(k)\mathbf{e}_g(k)\mathbf{e}_g^{T^*}(k)\hat{A}_v^T(k) = |\tilde{\boldsymbol{\alpha}}^{T^*}(k)\tilde{\mathbf{e}}_g(k)|^2\mathbf{s}_d^*\mathbf{s}_d^T \quad (37)$$

where

$$\tilde{\alpha}^{T^*}(k) = [\alpha(k_1+1), \alpha(k_1+2), \cdots, \alpha(k)]^T \quad (38)$$

and

$$\tilde{\mathbf{e}}_g(k) = [e_g(k_1+1), e_g(k_1+2), \cdots, e_g(k)]^T. \quad (39)$$

The correlation coefficient between the regenerated signal and the desired signal can be defined as

$$\rho(k) = \frac{\tilde{\alpha}^{T^*}(k)\tilde{\mathbf{e}}_g(k)}{\|\tilde{\alpha}(k)\| \, \|\tilde{\mathbf{e}}_g(k)\|}. \quad (40)$$

Using (40), (37) becomes

$$\hat{A}_v^*(k)\mathbf{e}_g(k)\mathbf{e}_g^{T^*}(k)\hat{A}_v^T(k) = |\rho(k)|^2\|\tilde{\alpha}(k)\|^2\|\tilde{\mathbf{e}}_g(k)\|^2\mathbf{s}_d^*\mathbf{s}_d^T. \quad (41)$$

With the assumption that the desired signal is uncorrelated with the interference and noises, $\hat{A}_v^*(k)\hat{A}_v^T(k)$ can be approximated

$$\hat{A}_v^*(k)\hat{A}_v^T(k) \simeq \|\hat{\alpha}(k)\|^2\mathbf{s}_d^*\mathbf{s}_d^T + \hat{A}_n^*(k)\hat{A}_n^T(k). \quad (42)$$

Substituting (41) and (42) into (32) and using $\|\mathbf{e}_g(k)\| = \|\tilde{\mathbf{e}}_g(k)\|$, we have

$$\mathbf{w}_v(k) = \mu_2(k)\{[\|\hat{\alpha}(k)\|^2 - |\rho(k)|^2\|\tilde{\alpha}(k)\|^2] \cdot \mathbf{s}_d^*\mathbf{s}_d^T + \hat{A}_n^*(k)\hat{A}_n^T(k)\}^{-1}\mathbf{s}^*. \quad (43)$$

For ergodic array input processes, the following approximations can be made when k_1 and $k - k_1$ are large:

$$\|\hat{\alpha}(k)\|^2\mathbf{s}_d^*\mathbf{s}_d^T \simeq [\beta k_1 + (k-k_1)]R_d \quad (44)$$

$$\|\tilde{\alpha}(k)\|^2\mathbf{s}_d^*\mathbf{s}_d^T \simeq (k-k_1)R_d \quad (45)$$

and

$$\hat{A}_n^*(k)\hat{A}_n^T(k) \simeq [\beta k_1 + (k-k_1)]R_n. \quad (46)$$

In the above, R_d and R_n are the desired signal component and the interference-plus-noise component of the covariance matrix R, respectively. Substituting (44)–(46) into (43), we have

$$\mathbf{w}_v(k) = \mu_2(k)\{[\beta k_1 + (k-k_1)(1-|\rho(k)|^2)]R_d + [\beta k_1 + (k-k_1)]R_n\}^{-1}\mathbf{s}^*. \quad (47)$$

In (47), $\rho(k)$ is the correlation coefficient between the regenerated signal and the desired signal defined by (40). Statistically, $|\rho(k)|$ is close to unity even for an error probability as large as 10^{-2}–10^{-1}. That $|\rho(k)|$ converges to a value almost equal to one is discussed in detail in [9], [10]. Therefore, as $k \gg k_1$, we have

$$[\beta k_1 + (k-k_1)(1-|\rho(k)|^2)]R_d \ll [\beta k_1 + (k-k_1)]R_n, \quad (48)$$

and (47) becomes

$$\mathbf{w}_v(k) \simeq \mu_2(k)\{[\beta k_1 + (k-k_1)]R_n\}^{-1}\mathbf{s}^*. \quad (49)$$

We can conclude from (49) that the steady state equivalent weight vector of the least-squares regenerative hybrid array is

$$\mathbf{w}_v(\infty) = \frac{N}{\mathbf{s}^T R_n^{-1}\mathbf{s}^*}R_n^{-1}\mathbf{s}^*, \quad (50)$$

which is the same as that of the original regenerative hybrid array. Expression (50) is also the steady state weight vector of the Applebaum-type array adapted without the desired signal present. Therefore, sensitivity to errors of the steering vector is reduced significantly.

The array transient behavior depends on the value of β. It can be seen from (48) that in order to achieve fast convergence, β should be a small number. In that case the error function given by (18) is dominated by the error signal with the regenerative reference loop closed. However, when β is too small, the adaption of weights in the first several iterations after the loop is closed might be strongly affected by "snapshots" with large sampled noises. That could result in a temporary drop of the output SINR as shown by computer simulations presented in Section IV.

IV. Computer Simulations

Computer simulations were carried out to examine the transient behavior of the least-squares regenerative hybrid array. Comparison between the steady state performance of the regenerative hybrid array and high-order derivative constraint arrays was made. In simulating the transient behavior, the QR decomposition algorithm [11]–[13] is applied to update weights recursively. In each iteration, the algorithm computes a unitary matrix $Q(k)$, which triangularizes the input data matrix $A(k)$,

$$Q(k)A^T(k) = \begin{bmatrix} R(k) \\ 0 \end{bmatrix} \quad (51)$$

where $R(k)$ is a triangular matrix, and

$$A(k) = \begin{cases} A_u(k), & k \le k_1 \\ \begin{bmatrix} \hat{A}_u(k) \\ \mathbf{e}_g^T(k) \end{bmatrix}, & k > k_1. \end{cases}$$

The multiplication of $Q(k)$ and the desired response vector $\mathbf{d}(k)$ is denoted by

$$Q(k)\mathbf{d}(k) = \begin{bmatrix} \mathbf{p}(k) \\ v(k) \end{bmatrix} \quad (52)$$

where $\mathbf{d}(k)$ equals $\mathbf{e}_b(k)$ and $\hat{\mathbf{e}}_b(k)$ for $k \le k_1$ and $k > k_1$, respectively. The optimal weight vector is obtained by solving

$$R(k)\mathbf{w}(k) = \mathbf{p}(k) \quad (53)$$

by a process of back substitution. In our simulations, when $k > k_1$, the error function to be minimized is defined by (18) in which the error signal power before k_1 is reduced by a factor of β. Therefore, during the (k_1+1)th iteration, $R(k_1)$ and $\mathbf{p}(k_1)$ are multiplied by a factor of $\sqrt{\beta}$, and $R(k_1+1)$ and $\mathbf{p}(k_1+1)$ are computed by using $\sqrt{\beta}R(k_1)$ and $\sqrt{\beta}\mathbf{p}(k_1)$, respectively.

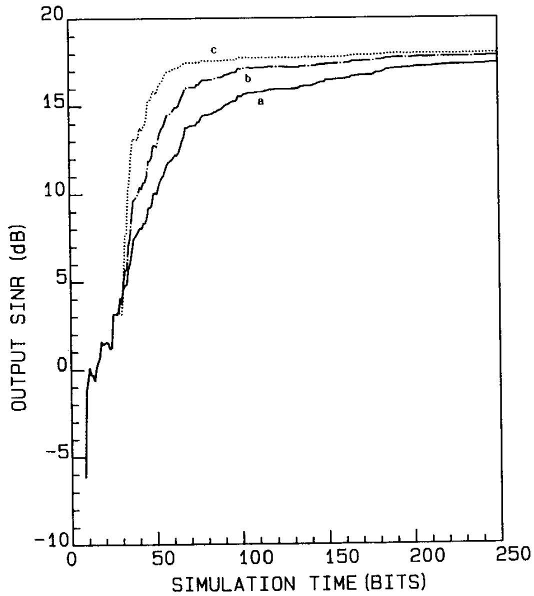

Fig. 2. Transient behavior. Pointing error: 1° k_1 = 25. a: β = 0.2; b: β = 0.1; c: β = 0.04.

In the simulations, a seven-element linear array with element locations 0, 1 λ, 2.5 λ, 3.5 λ, 6.5 λ, 8.0 λ and 8.5 λ, where λ is the wavelength, are utilized. The desired signal is a binary phase shift keying (BPSK) signal with an angle of arrival of 2° and power of 10 dB relative to the noise. Two Gaussian interferors are assumed present. Their angles of arrival are −2° and 7° with powers of 20 and 10 dB relative to noise, respectively. We first simulate the case where pointing error exists. With a preset steering direction of 1°, the output SINR of the array before adaption and the Applebaum-type array are −5.13 and 2.12 dB, respectively. Fig. 2 is the simulation with k_1 equal to 25, i.e., the regenerative reference loop is closed at the end of the twenty-fifth iteration. Three curves are plotted with β equal to 0.2, 0.1, and 0.04, respectively. The plot begins at the eighth iteration because the *QR* decomposition algorithm is under initialization for the first seven iterations. At the end of the twenty-fifth iteration, the Applebaum-type array results in an output SINR of about 3 dB. Once the regenerative reference loop is closed, the array output SINR increases rapidly. For example, at the thirty-fifth iteration, the SINR is 6.0, 7.5, and 10.9 dB for β equal to 0.2, 0.1, and 0.04, respectively. All three curves converge toward the steady state of the Applebaum-type array adapted without the desired signal, for which the output SINR is 18.1 dB. The convergence speed increases with decreasing β as predicted in Section III.

In above simulations, the reference signal is perfectly acquired in the detection-generation process. In order to see the effect of detection error on the transient behavior, we show in Fig. 3 simulations which contain detection error. With the same scenario except that k_1 equals 50, three curves are plotted in Fig. 3 with β equal to 0.2, 0.1, and 0.02, respectively. The detection error occurs at the fifty-third bit for all three curves. Simulation shows that the detection error causes a temporary drop of the output SINR for a β of 0.02. That is because for such a small β, array weights are strongly effected by the "noisy" snapshot. Moreover, the detection error slows the convergence rate, which one can see by comparing Fig. 3 with Fig. 2. However, the convergence rate in this case is still much faster than that of the gradient search based regenerative hybrid array, which takes more than one thousand iterations to generate an output SINR greater than 15 dB [9].

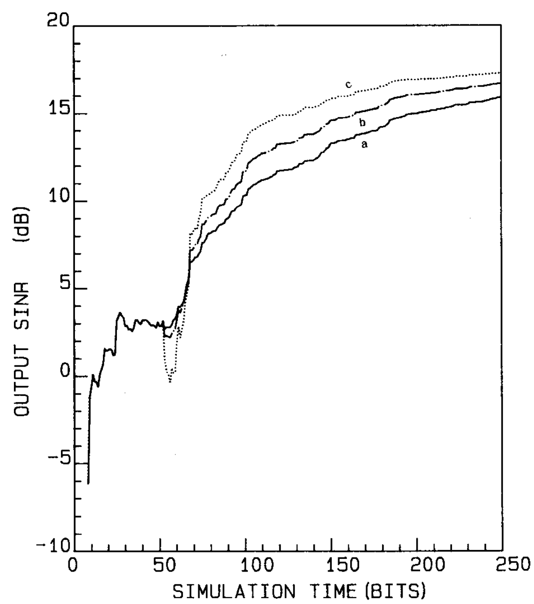

Fig. 3. Transient behavior. Pointing error: 1° k_1 = 50. a: β = 0.2; b: β = 0.1; c: β = 0.02.

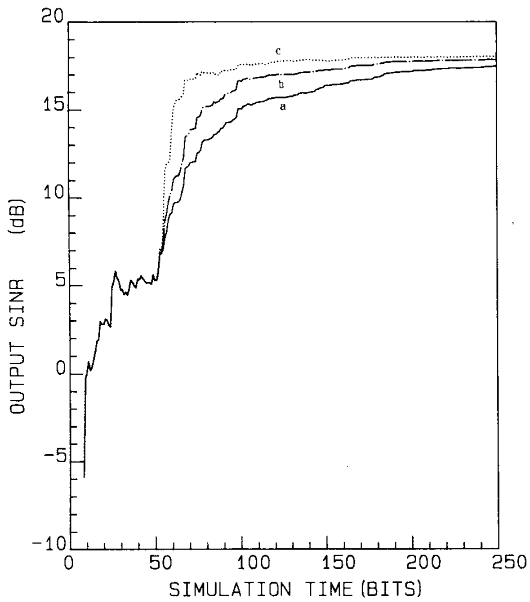

Fig. 4. Transient behavior. Pointing error: 0.5° $k_1 = 50$. a: $\beta = 0.2$; b: $\beta = 0.1$; c: $\beta = 0.02$.

TABLE I
ARRAY STEADY STATE PERFORMANCE

	Output SINR (dB)	
	Pointing error 1°	Position error deviation 0.038 λ
Regenerative hybrid array	18.12	18.08
Applebaum-type array	2.12	-0.37
1st order derivative constraint array	9.53	0.19
1st + 2nd order derivative constraint array	-2.49	-8.45

In simulating Fig. 3, the pointing error is 1° for which the Applebaum-type array has a steady state output SINR of 2.12 dB. Therefore, the noisy snap shot of the fifty-third iteration may have a strong impact on the adaption of weights. If the pointing error is reduced so that higher output SINR is obtained initially, that phenomenon can be eliminated. In Fig. 4 we show the simulation results with a presteering direction of 1.5°, i.e., the pointing error is 0.5°. With the pointing error, the Applebaum-type array has a steady state output SINR of 7.87 dB. Before we close the regenerative reference loop at the end of the fiftieth iteration, the output SINR is about 5.29 dB. Therefore, no detection error occurs in the fifty-third iteration, and the convergence rate is similar to that of Fig. 2.

The steady state performance of the regenerative hybrid array is simulated to compare with those of the Applebaum-type array and high-order derivative constraint arrays. For a presteering direction of 1°, the steady state output SINRs are 18.12, 2.12, 9.53, and −2.49 dB for the regenerative hybrid array, the Applebaum type array, the 1st order derivative constraint array, and the first-plus-second-order derivative constraint array, respectively, as shown in Table I. The steady state array power patterns are plotted in Fig. 5 with the weight vector length normalized to one. Results show that the regenerative hybrid array, the Applebaum-type array, and the first-order derivative constraint array have a good suppression of the interference. However, first-plus-second-order derivative constraint array fails, due to its broad beamwidth, to suppress the interferor with an angle of arrival of −2°. Among the three arrays which suppress the interference, only the regen-

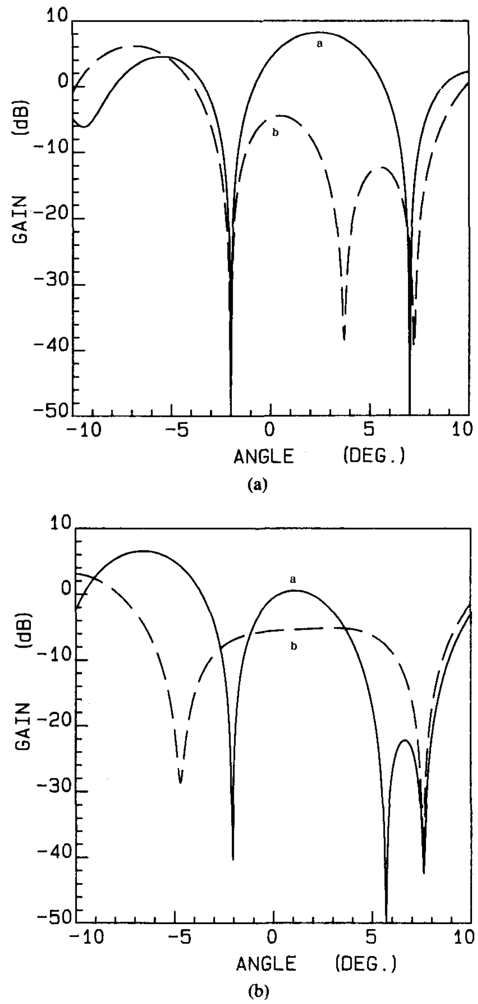

Fig. 5. (a) Array power pattern. a: Regenerative hybrid array. b: Applebaum-type array. (b) a: First-order derivative constraint array; b: first-plus-second-order derivative constraint array.

TABLE II
SAMPLED ARRAY ELEMENT POSITION

Array element	1	2	3	4	5	6	7
X axis (λ)	−0.04	1.02	2.47	3.52	6.50	7.94	8.52
Y axis (λ)	0.02	0.02	0.01	0.07	−0.02	−0.01	−0.01

erative hybrid array can generate a high gain in direction of the desired signal.

We have also simulated the transient behavior of the regenerative hybrid array with two-dimensional random element position errors. The probability density function of random errors in each dimension is assumed Gaussian with zero mean and standard deviation 0.038 λ. The sampled array element locations are shown in Table II. The desired signal and two interferors remain the same, and the array is accurately pointed. The array transient responses with k_1 equal to 25 and 50 are plotted in Figs. 6 and 7, respectively. Similar to simulations of the pointing error case, no detection error occurs for $k_1 = 25$, and one in the fifty-third iteration for $k_1 = 50$. Steady state performances of the regenerative hybrid array and other arrays are shown in Table I. The regenerative hybrid array has an output SINR of 18.08 dB. However, the SINRs of the

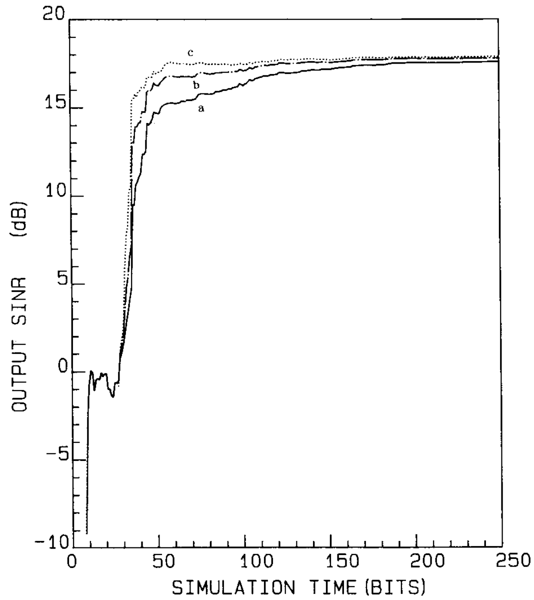

Fig. 6. Transient behavior. Position error deviation: 0.038 λ $k_1 = 25$. a: $\beta = 0.2$. b: $\beta = 0.1$. c: $\beta = 0.04$.

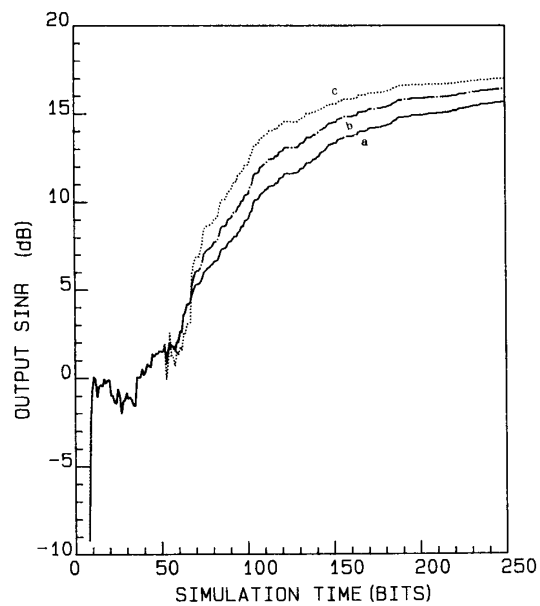

Fig. 7. Transient behavior. Position error deviation: 0.038 λ. a: $\beta = 0.2$; b: $\beta = 0.1$; c: $\beta = 0.02$.

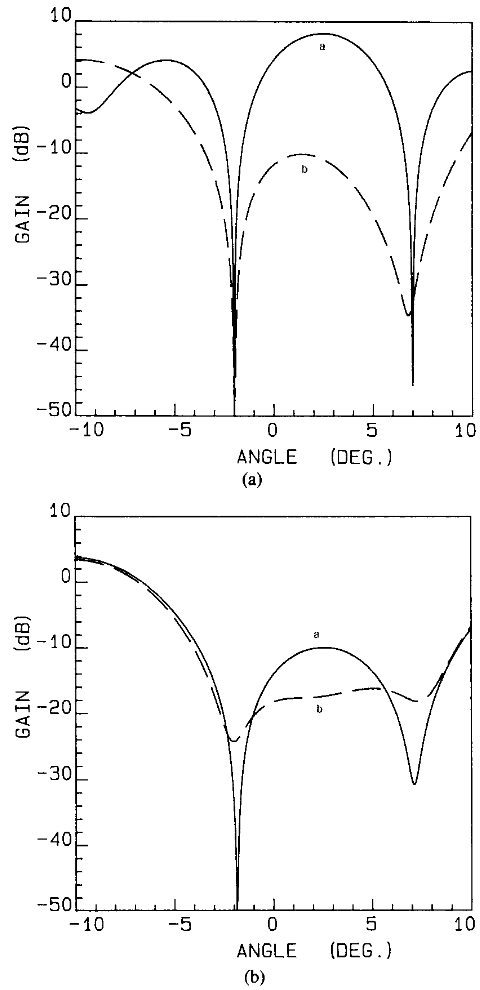

Fig. 8. (a) Array power pattern. a: Regenerative hybrid array; b: Applebaum-type array. (b) a: First-order derivative constraint array; b: First-plus-second-order derivative constraint array.

other three arrays are all below 0 dB. Array power patterns are shown in Fig. 8. One can clearly see that the regenerative hybrid array generates a high gain in direction of the desired signal, but the other arrays suppress the desired signal.

V. Conclusion

We have applied the least-squares method to adapt the regenerative hybrid array which utilizes both the steering vector and reference signal acquired from the array output to preserve the desired signal. We show that similar to the gradient search based regenerative hybrid array, the proposed array converges to the steady state of the Applebaum-type array adapted without the desired signal present. The array transient behavior is simulated with weights updated by the *QR* decomposition algorithm. Results show that the least-squares regenerative hybrid array converges much faster than the original regenerative hybrid array. Simulations of the steady state performance show that the regenerative hybrid array performs better than high-order derivative constraint arrays.

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