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Theory of the square to rhomb structural phase transitions in the vortex lattice of type-II superconductors

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Abstract

The theory of structural transformation of the vortex lattice in a fourfold symmetric in *a–b* plane type-II superconductors is constructed using both the London and the lowest Landau level Ginzburg–Landau model. Thermal fluctuations and quenched disorder influence the location of the square to rhomb structural transition line. The self consistent harmonic approximation for lattice anharmonicity within the fourfold symmetric generalization of the London model is used. Without disorder in the *T–B* plane the slope of the line is generally negative: thermal fluctuations favour a more symmetric square lattice. The theoretical line's location and slope are in good agreement with the experimental ‘‘second magnetization peak'' line measured in both LaSCCO and YBCO in a wide range of doping. We find that, while the thermal fluctuations are negligible for the transitions in low T_c materials, disorder plays an important role and creates a positive slope in the *T–B* plane since disorder favours a symmetric square lattice.
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1. Introduction

Structural phase transitions (SPT) between crystalline systems possessing different lattice symmetry is an old and still not a sufficiently well developed branch of the phase transitions physics. The simplest such transition is the square–rhomb transformation in the vortex lattice. In this case, single crystals of a tetragonal material like those of the borocarbide family are placed into an external magnetic field oriented along ‘‘*c*'' crystallographic axis to preserve fourfold symmetry in the basal plane.

The location in *T–H* plane of the critical line of the square–rhomb SPT in the vortex crystal at low temperatures is mapped precisely by various techniques and explained well by the nonlocal London (NLL) theory proposed by Kogan and collaborators [1]. The agreement with theoretical predictions is readily understood conceptually because the SPT in question occurs at low magnetic fields (much lower than H_{c2}), that is under the conditions when the conventional London theory of the vortex lattice is known to be very reliable. It is assumed that vortices are well separated although the vortex core structure is also accounted for phenomenologically by a cut-off. The NLL theory then includes additionally the four derivative terms which bring in the anisotropy effects essential to trigger the SPT between the vortex lattice phases. The more symmetric square vortex crystal, stable at a stronger magnetic field (higher density of vortices), transforms into a less symmetric rhombic vortex crystal as the magnetic field weakens (density of vortices decreases). At higher temperatures and closer to the $H_{c2}(T)$ curve the detailed data became available only recently. A neutron scattering experiment of Eskildsen et al. [2] on $LuNi_2B_2C$ suggests that the SPT line sharply bends upwards and even become re-entrant in this region. This is very surprising if one assumes that fluctuations of some kind are responsible for such behaviour. Gurevich and Kogan [3] were first to consider the thermal fluctuations. They worked

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in the NLL framework and succeeded in producing transition curves similar to the experimental ones. At a first glance, the structural phase transition in such a system, even at finite temperature (below the melting temperature of course), is driven by interactions on scales smaller that $C(L)a_0$, where $C(L) = 0.1$ is the Lindemann constant, and consequently have nothing to do with anisotropy of the vortex core. However, it was claimed in a recent theory of thermal fluctuations that the core anisotropy is crucial. Thus the problem should be considered from a more fundamental approach.

2. Theory

In this paper we show that the spatial disorder rather than thermal fluctuations is responsible for SPT line upwards and re-entrant behavior. We are primarily interested in the part of the phase diagram $B \ll H_{c2}(T)$, where the vortex core size is much smaller than the distance a between vortices. A standard approach to the crystal structure of point-like (or line-like) objects at finite temperature requires a sufficiently comprehensive account of the lattice anharmonicity. The simplest version of such a theory takes into account the interacting phonon excitations self consistently (the self consistent harmonic approximation – SCHA) using the microscopic derivation of the vortex–vortex interaction for a d-wave superconductor by Yang. In our previous paper [4] we obtain a structural phase transition line with a negative slope in the B–T plane. In our theory, unlike the preceding ones, no cutoff is required. Here we expand our approach taking into account perturbatively the influence of quenched disorder on the structural phase transition line. A microscopic manifestation of the structural phase transition is the softening of the elastic squash modulus $C_{sq} = 2(C_{11} + C_{12}) - C_{66}$ at the transition line. Disorder introduces the random pinning potential $U(r_a)$ with $\overline{U(r)U(r')} = K(r - r')$. The energy expansion in the main order in the displacements of vortices $\mathbf{u}_\alpha$ from their equilibrium square lattice position $\mathbf{R}_\alpha$ and pinning potential reads

$$E[u_q] = E_0 + \sum_{BZ} U_q + \sum_{BZ} iq_\alpha U_q u_q^\alpha + \frac{1}{2}\sum_{BZ} \Lambda_{\alpha\beta}(q) u_q^\alpha u_{-q}^\beta;$$
$$E_0 = \frac{1}{2}\sum_{m,n} w(\vec{G}_{mn});$$
$$u_a^\alpha = \sum_{BZ} u_q^\alpha \exp(iqR_a);$$
$$U(r) = \sum_{BZ} U_q \exp(iqR_a). \quad (1)$$

$w(G_{nm})$ is the Fourier transform of the fourfold vortex–vortex interaction, G_{nm} are the reciprocal vectors. Elastic modules $C_{\alpha\beta\gamma\delta}$ of the clean sample are given by the expansion of $\Lambda_{\alpha\beta}$ to the second order of q: $C_{\alpha\beta\gamma\delta} q_\gamma q_\delta$, $v(q)$ is the Fourier transform of the isotropic pair vortex–vortex interaction energy. For rhombic vortex lattice with the opening angle 2Θ where $\vec{G} = n\vec{q}_1 + m\vec{q}_2$, with

$$\vec{q}_1 = (2\tan\Theta)^{-1/2}(1, \tan\Theta);$$
$$\vec{q}_2 = (2\tan\Theta)^{-1/2}(1, -\tan\Theta). \quad (2)$$

For a given vortex structure one generally obtain the vortex displacement (strain) due to disorder $u_q^\alpha = \Lambda_{\alpha\beta}^{-1} iq_\beta U_q$, which when substituted into Eq. (1) gives after averaging over disorder:

$$\overline{E}_{2D} = E_0(\Theta) - \frac{1}{2}\sum_{BZ} \Lambda_{\alpha\beta}^{-1}(q) q_\alpha q_\beta K(q). \quad (3)$$

Here $K(q) = \sum_{BZ} K(q)\exp(i\vec{q}\vec{r})$. The energy $\overline{E}_{2D}$ should be minimized to obtain the lattice with lowest energy. Result essentially depends on disorder amplitude, in-plane anisotropy, magnetic field and the temperature as it presented in Fig. 1.

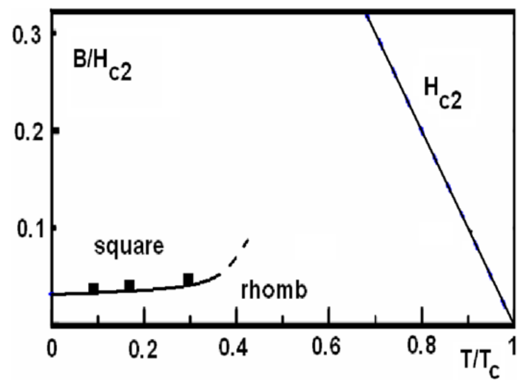

Fig. 1. Square and rhombic vortex lattice structures under disorder (black squares are experimental data of a $LuNi_2{\cdot}B_2C$ superconductor).

3. Summary

In low T_c superconductors the slope of the rhomb-square coexistence line can be positive.

Quenched disorder rather than temperature fluctuations are responsible for such behaviour. These results can be easily generalized to the case of three dimensional fourfold symmetric superconductor.

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