國　立　交　通　大　學

電子工程學系　電子研究所碩士班

碩　士　論　文

以多層級貝氏賽局為基礎之感知無線網路頻譜買賣

Multistage Bayesian Game based Spectrum Trading for Cognitive Radio Networks

研 究 生: 李重佑

指導教授: 簡鳳村 博士

中 華 民 國 九 十 九 年 一 月

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以多層級貝氏賽局為基礎之感知無線網路頻譜買賣

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摘要

在本篇論文，我們以賽局理論的角度來研究感知無線電網路頻譜買賣。我們考慮一個由多主要服務者(primary service)和多的次要服務者(secondary service)所構成的感知無線電網路。主要服務者是此買賣賽局中的頻譜賣家，它們可以設定租借頻帶給次要服務者的單位頻帶價格；次要服務者是該賽局中的買家，它們要決定跟買家買多少頻帶。我們提出以多層級貝氏賽局為基礎的買賣模型來建立每個玩家可能未公開的私人資訊的情況，並在符合頻帶限制下依序地求得完美貝氏平衡點(perfect Bayesian equilibrium)。所謂的頻帶限制其實就是所有買家所要求的頻帶量加起來不能超過賣家所能負荷，而每個買家所要求的頻帶量也不能是負值。由倒推歸納原則，我們將買家的 Karush-Kuhn-Tucker (KKT) condition 轉化為賣家的最佳化問題之條件，並將所有賣家的 KKT condition 集合起來成為 joint KKT condition，符合該 joint KKT condition 的解即為此賽局的解。我們並提出以 active-set algorithm 來解該 joint KKT condition，並分析它的複雜度。此論文也探討了玩家的行動和對未知資訊的信念是否會收斂。在模擬中，我們比較了我們的作法和前人的作法，並且數值上探討

了該賽局之收斂行為。

Multistage Bayesian Game based Spectrum Trading for Cognitive Radio Networks

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Abstract

In this thesis, we study the problem of spectrum trading in cognitive radio (CR) networks from a game theoretical perspective. Particularly, we consider a CR network with multiple primary services (PSs) and multiple secondary services (SSs), where all PSs are sellers targeting at setting the prices for spectrum leasing and SSs are buyers deciding how much spectrum are demanded from each PS in the trading game. Aiming at dealing with the trading behaviors, we propose using a multistage Bayesian game based trading model to account for possible unknown private information in each player, and obtain the perfect Bayesian equilibrium (PBE) sequentially under a bandwidth constraint, which requires all SSs' demanded bandwidth not exceeding that the PS can possibly offer and each SS's demand should not be negative. Following the backward induction principle, we transfer the Karush-Kuhn-Tucker (KKT) condition of the SSs into each PS's optimization constraint, and collectively form joint KKT conditions that satisfy the bandwidth constraint. We present an active-set based algorithm to solve the joint KKT conditions, and analyze the corresponding complexity. Furthermore, the convergence behaviors of the action profiles and the beliefs of the unknown information are also investigated in the work. Finally, in the simulations, we compare the proposed approach with earlier work and numerically study the convergence behaviors of the proposed

multistage game.

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在新竹的這六年，還要謝謝大伯父李宗仁和如英姊的照顧。最後，我要感謝我的家人，謝謝你們一直以來的支持，你們永遠是我精神上最大的支柱。

在此，將此篇論文獻給所有愛我和我愛的人。

李重佑

民國九十九年一月 於新竹

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Chapter 1

Introduction

1.1 Significance of the Research

Nowadays we are facing a more congested spectrum than ever before. Most spectrum for long-distance radio transmissions has been allocated to licensed users, and there's no much room for emerging wireless applications. However, large temporal and geographical variations exist in licensed spectrum utilization, and almost only 2% are always in use according to the survey in [1]. That is, the efficiency of spectrum utilization is unacceptably low, so researchers start to think different spectrum allocation policies in order to tackle the problem. There're two main approaches to this issue. One is that unlicensed users can opportunistically utilize licensed spectrum if not interfering with licensed users, while the other is that spectrum trading (or active negotiation) between licensed and unlicensed users would be a promising solution [2, 3]. We attempt to address several issues in the spectrum trading problem in this thesis. The idea of spectrum trading comes from economic point of view because of the success of economical world. Both schemes are considered as possible solutions in dynamic spectrum management. Apparently both schemes could enhance the efficiency of spectrum utilization, but how the network behaves, how much efficiency can be increased, and how the fairness is guaranteed are open issues to be studied.

1.2 Motivation

The promise of providing anytime and anywhere multimedia services demands a large spectrum for broadband wireless communications. On one hand, this drives the advance of radio technology to faster, convenient and reliable communications. On the other hand, the enormous demand also unveils the problem of insufficiency and under-utilized inefficiency of current radio spectrum. Useful radio spectrum is a scarce resource in that the characteristic of spectrum on different frequency is different, e.g. the communication on 60 GHz is only suitable for short distance because of the absorption of radio signal by oxygen of the Atmosphere. Nowadays, the most useful spectrum band for median and long distance communication is below 5 GHz due to the characteristic of spectrum and current circuit technology. To tackle the problem, the idea of exploiting under-utilized licensed spectrum for more flexible and efficient transmissions is receiving significant attentions lately. In particular, the concept of cognitive radio (CR) [4] is considered as a promising technique to improve the efficiency of current radio spectrum.

A cognitive radio (CR) is a software-defined radio capable of intelligently sensing, adapting and responding to constantly varying environments, particularly the available spectrum temporarily not used by licensed users. However, there still exist many technical challenges before cognitive radios can be practically deployed. One critical challenge is how to invite the licensed service operators to accept coexistence with cognitive users so that they are willing to share their unused spectrum to unlicensed cognitive (secondary) services. Leasing available spectrum to unlicensed services is an attractive solution that provides an incentive for legitimate licensed operators to support deploying cognitive radios [3]. This gains monetary profits for licensed operators, while fulfilling unlicensed services' satisfaction requirements by renting.

1.2.1 Why Game Theory?

Conventional Media Access Control (MAC) theory is based on optimization, and the objective function it aims at optimizing is the network system utility or the network system utility in terms of fairness, *e.g.* proportional fairness. Although some problem formulations using optimization theory can be decomposed to problems to optimize network and user utility separately by dual-primal method [5], which makes distributed decision making possible, the solution for the optimization problem inherently couldn't always satisfy each user's individual utility.

In contrast to optimization-based approach, game theory is a mathematical tool to deal with interactions between multiple entities, each of which has its own utility function, and intrinsically looks for equilibrium solutions that maximizes each user's individual utility. Though the network system utility may not be optimized, the strategy obtained from the game theoretical perspective provides a solution that achieves efficiency and fairness under certain criteria.

1.2.2 Related Work and Our Approach

An overview of the general idea and recent developments about dynamic spectrum sharing games can be found in [6]. The auction mechanism for spectrum band in CR networks with multiple primary and multiple secondary users is considered in [7], where the authors discuss competitive equilibrium, cheating behaviors which may deteriorate the efficiency of of the spectrum sharing and propose using reserve prices and beliefs to prevent collusion. The work in [8] and [9] consider a game model which incorporates both monetary gain and quality-of-service (QoS) satisfaction of wireless services in utility functions. The authors in [9] explicitly model the price for available bandwidth as a function of demand, and obtain the Nash equilibrium (NE) for the spectrum sharing strategy in a network consisting of a single primary service (PS) and multiple secondary services (SS). The work in [8] considers the spectrum trading game in a CR network with multiple PS's and a single SS, and models the utilities of the PS and SS separately, wherein the demand of SS implicitly affects the price. However, under certain circumstances, the equilibrium bandwidth demand for the SS would be negative, and the corresponding NE turns out to be infeasible, though theoretically solvable. The work in [10] discusses the same problem as in [8], and compare different features such as market equilibrium as well as competitive and cooperative pricing strategies. In [11], the authors investigate the spectrum trading behaviors with a more general model in which multiple primary users (PUs) and multiple secondary users (SUs) are considered in the CR network. However, the utility model considered in [11] may not capture different QoS requirements of SUs and assumes that each PU sets the same price for all SUs. One key assumption underlying all the above work is that each player in the modeled game have complete knowledge about the other players' private information. This is in general not a realistic assumption. To account for the unknown private information within each player, one can resort to tools in Bayesian game or stochastic game to study the behaviors of spectrum trading in a sequential (dynamic) manner [12, 13]. In [12], we formulate the spectrum trading behaviors for a CR network with multiple PS's and a single SS as a Bayesian game, and study the corresponding solution concept, *i.e.* the perfect Bayesian equilibrium, sequentially. The work in [13] proposes to characterize the dynamics of spectrum access strategies under a stochastic game framework with the introduction of state transitions. The authors also propose to predict the future dynamics using approaches in learning theory in order to obtain better strategies for spectrum bidding.

In this work, extending the studies in [12], we address the problem of spectrum trading in a CR network consisting of multiple PS's and multiple SS's. We assume each player (PS or SS) in the game has its own private information, such as the number of active connections within each service or the channel conditions, that is unknown to other players. With the assumption, we formulate a multistage trading model based on the Bayesian game to statistically account for the unknown private information (incomplete information), and sequentially obtain the perfect Bayesian equilibrium (PBE) in the trading process. We further assume that each PS is allowed to set different prices to the SS's with different QoS, and SS's with different QoS can demand different bandwidth sizes to a particular PS in the considered model. Particularly, we consider a bandwidth constraint on the aggregate bandwidth demand from all SS's such that the total demand has to be within feasible supply regions provided by each PS.

1.3 Contribution

Aiming at dealing with the trading behaviors with that each play has its own private information, we propose using a multistage Bayesian game based trading model to account for possible unknown private information in each player, and obtain the perfect Bayesian equilibrium (PBE) sequentially under a bandwidth constraint, which requires all SS's demanded bandwidth not exceeding that the PS can possibly offer and each SS's demand should not be negative. We formulate the considered problem as a *multistage* game, since one-shot game can't capture the time-varying demands for resources due to the dynamic nature of wireless channels and wireless services. In multistage game, the allocation is performed repeatedly, and belief updates through observing others' actions can also be made possible. Our formulation captures different pricing and demand strategies for different seller and buyer pairs based on their QoS's. More specifically, on one hand we allow a primary service set different prices per unit bandwidth to different secondary services based on their operating conditions and QoS requirements. On the other hand, different secondary services can demand different bandwidth sizes from the same primary service. Following the backward induction principle, we transfer the Karush-Kuhn-Tucker (KKT) condition of the SS's into each PS's optimization constraint, and collectively form joint KKT conditions that satisfy the bandwidth constraint to guarantee our PBE is physically feasible. We present an active-set based algorithm to solve the joint KKT conditions, and analyze the corresponding complexity. Furthermore, we illustrate the spectrum trading game by an example with specific utility functions of PS's and SS's. The convergence behaviors of the action profiles and the beliefs of the unknown information are also investigated in the work. In the simulations, we compare the proposed approach with that in [8] and numerically study the convergence behaviors of the proposed multistage game.

As a final remark in the section, we would like to emphasize the general applicability of the joint KKT approaches to solve a game with constraints. Mathematically, we formulate the problem considered in the thesis as a game with constraints, which is often very difficult to solve. Relevant approaches are rarely seen in the field of pure game theory, not to mention in the literature related to wireless networks. In most studies that consider games with constraints, their problems usually have certain mathematical structure so that the solutions are always on the boundary set by the constraints. In this thesis, we attempt to solve a bandwidth-constrained game, where the constraints include budgets and feasible bandwidths, using the proposed joint KKT conditions. It is worthwhile to note that joint KKT condition is generally applicable to solve a constrained game. The solutions generally need not be on the boundary of the constraints.

Chapter 2

Cognitive Radio and Game Theory Preliminary

2.1 Cognitive Radio

Cognitive radio, which first appeared in Joseph Mitola's doctoral dissertation in 2000 [4], is defined as an intelligent wireless communication system that are capable of achieving highly reliable communication whenever and wherever needed by adjusting its own transmission parameters according to the radio environmental conditions it senses. CR is called "cognitive" in that it's equipped with structures supporting a cognition cycle consisting of Observe, Orient, Plan, Decide, and Act phases as Fig. 2.1[1] shows. As for realistic implementation, CR is built based on software defined radio and wide-band RF front end to achieve that. There're prototypes of CR already built, such as the first prototype CR1 by Mitola [4], CR and networking by Virginia Tech [14].

Although the initial aim of CR is not to efficiently utilize the radio spectrum, it serves as the natural candidate for the problem of spectrum under-utilization. CR can either opportunistically detect the spectrum hole and transmit or actively negotiate with primary users, *i.e* the existing licensed users, to access the spectrum. In recent years, there're tremendous amount of researches on CR-related topic. They can be classified into three

[1]This figure is adapted From Mitola, "Cognitive Radio: An Integrated Agent Architecture for Soft-ware Defined Radio", Doctor of Technology, Royal Inst. Technol. (KTH), 2000, pp 48

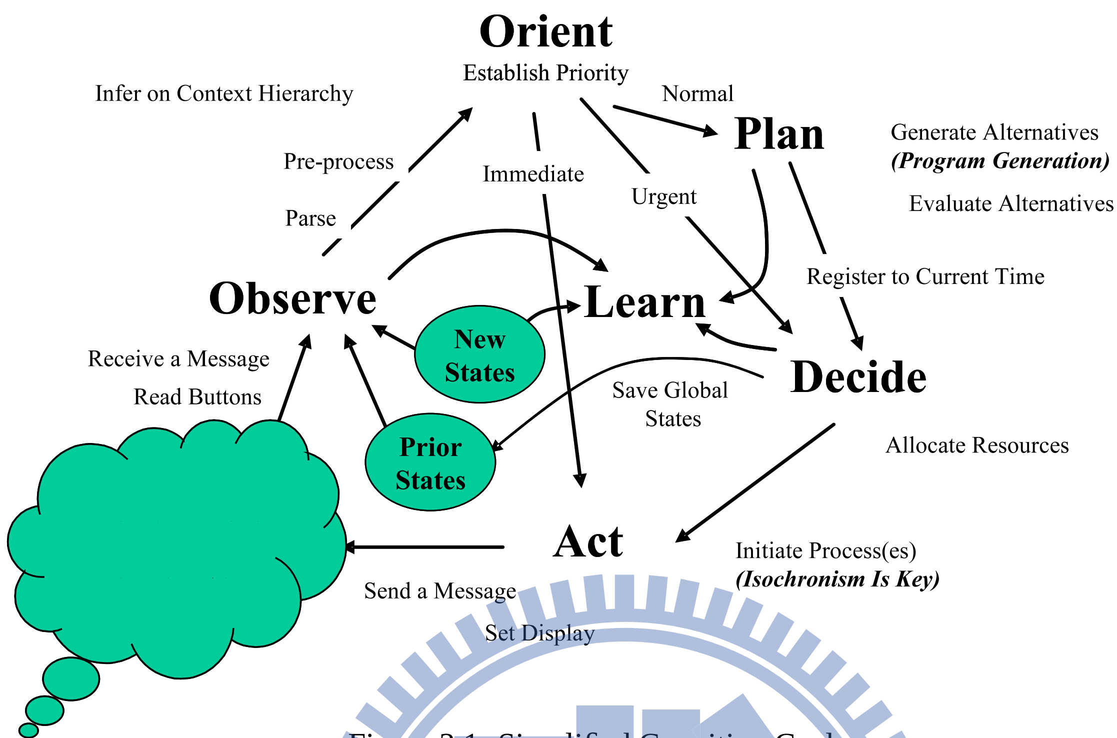


Figure 2.1: Simplified Cognition Cycle.

fundamental tasks [3],

1. Radio-scene analysis, which includes estimation of interference temperature of the radio environment and detection of spectrum holes.

2. Channel state estimation and predictive modeling, which encompasses estimation of channel-state information and prediction of channel capacity for use by the transmitter.

3. Transmit power control and dynamic spectrum management.

Our work is focus on dynamic spectrum management, which we adopt game theoretic approach to tackle with.

2.2 Game Theory

Game theory is a mathematical tool to predict the result of rational interactive decision makers. Predicting the result of such players has great merit in many fields such as chess, card game, gambling, business and economics, politics, international diplomatics, and also in wireless network in which we are interested since in those fields no player can achieve his goal or gain his own maximal profit without considering the competitors' behavior. Although sometimes the explicit model is difficult to be defined (e.g. politics) or too complex to predict the result and to derive the winning strategy [2] (e.g. in chess game, the problem can't easily be formulated as the math form with which we are familiar and strategy space is discrete, making it both not differentiable and too complex in number to examine the all strategy profile), game theory stands a important tool to provide either a solution to simplified problem or an insight. As for wireless network, applying game theory to predict and further to regulate the network is anticipated since the increasing complexity of wireless network results in significant interference and foreseeable dynamics of interactive users in cognitive network.

In this section, we introduce some basic knowledge of noncooperative game theory that are necessary for understanding our work, while interested reader can refer to [15] or [16] for deeper materials.

2.2.1 A Basic Game – Definitions and Theorems

A game in essence is that there're multiple players and each player possesses its own strategy (*e.g.* variable) which it can freely adjust and its own objective function (*e.g.* function) which depends on its and other players' strategy. In mathematics, a game is defined as

Definition 1 *A game* Γ *is*

$$\Gamma = \left\langle \mathcal{I},\ \{A_x\}_{x\in\mathcal{I}},\ \{u_x\}_{x\in\mathcal{I}} \right\rangle, \tag{2.1}$$

where $\mathcal{I} \equiv \{1, 2, \cdots, N\}$ *is the set of players,* A_x *is the set of actions available for player* x*, and we denote the set of all available actions for all players as* $A = A_1 \times A_2 \times \cdots \times A_N$*. A action taken by player* x *is* $a_x \in A_x$*, and the action profile of all players is* $\mathbf{a} = a_1 \times a_2 \times \cdots \times a_N \in A$*. For notational simplicity, we denote* $\mathbf{a}_{-x}$ *as the action profile*

[2] Actually, game theory predicts the equilibrium strategy instead of winning strategy, but one can pick out the equilibrium strategy most beneficial to him as winning strategy.

taken by all players except player x. u_x is player x's utility function which is a function of a_x and of $\mathbf{a}_{-x}$.

There're some assumptions in game theory. First, each player is rational and selfish so that each want to maximize its own utility. Readers should mind that "selfish" doesn't mean "malicious". A selfish player cares about its utility, while a malicious player aims at harming other players. It's also assumed that all players know the rules of the game, *i.e.* each knows all players' action set and utility, and each knows that other players know that and so on, and the action is perfectly observable by all. Indeed, the scenario is too ideal due to those assumptions, so other different kind of game models are developed by mathematicians to make the model more practical. For instance, Bayesian game, the game model we apply in this thesis, is a game that there're some private parameters in each player's utility function. The private parameters are not known to all in this kind of game, and it's also called game of incomplete information. The detail of Bayesian game will be introduced in the latter section. Lets go on the basic game.

What action or strategy would each player take? Apparently, each player choose the action that are best for it given the other players' action, and that action that player x would take is defined as follows,

Definition 2 *The best response $b_x(\mathbf{a}_{-x})$ of player x to the action profile $\mathbf{a}_{-x}$ is a action a_x such that:*

$$b_x(\mathbf{a}_{-x}) = arg \max_{a_x \in A_x} u_x(a_x, \mathbf{a}_{-x}) \tag{2.2}$$

Since best response is the best for player x, player x would stick to it.

Each knows that each player would take best response, so the result of game is the action profile that is best response for all, if it exists. This mutual best response point, which was found by the Nobel Laureate John Forbes Nash, is a equilibrium since every player would stick to it. The formal definition is as below,

Definition 3 *The pure strategy profile $\mathbf{a}^*$ constitutes a Nash equilibrium (NE) if, for each player x,*

$$u_x(a_x^*, \mathbf{a}_{-x}^*) \geq u_x(a_x, \mathbf{a}_{-x}^*), \forall a_x \in A_x \tag{2.3}$$

Note that this definition is for pure strategy, and there's corresponding NE for mixed strategy [3]. In the following, we address the condition for the existence of pure-strategy NE under different conditions,

Theorem 1 (Debrew 1952; Glicksberg 1952; Fan 1952 [16]) *Consider a strategic-form game whose strategy spaces A_x are nonempty compact convex subsets of an Euclidean space. If the payoff are continuous in $\mathbf{a}$ and quasi-concave in a_x, ther exists a pure-strategy Nash equilibrium.*

Theorem 2 (Dasgupta and Maskin [16]) *Let A_x be a nonempty, convex and compact subset of a finite-dimensional Euclidean space, for all x. If, for all x, u_x is quasi-concave in a_x, is upper semi-continuous in $\mathbf{a}$, and has a continuous maximum , there exists a pure-strategy Nash equilibrium.*

The definition of quasiconcave, upper semi-continuous and continuous maximum are illustrated as follows.

Definition 4 *If $f(\lambda x + (1-\lambda)y) \geq \min(f(x), f(y))$ for all x,$y \in$ **dom** f and $0 \leq \lambda \leq 1$, and **dom** f is convex [17], then f is quasiconcave.*

Definition 5 *A function $u_i(\cdot)$ on S is upper semi-continuous at $\mathbf{s}$, if, for any sequence $\mathbf{s}^n$ converging to $\mathbf{s}$, [16]*

$$\limsup_{n \to +\infty} u_i(\mathbf{s}^n) \leq u_i(\mathbf{s}) \tag{2.4}$$

Definition 6 *A function u_i has a continuous maximum if $u_i^*(\mathbf{s}_{-i}) \equiv \max_{s_i} u_i(s_i, \mathbf{s}_{-i})$ is continuous in $\mathbf{s}_{-i}$. [16]*

NE is thought to be the *solution concept*, *i.e.* the rule for predicting how the game will be played, of static game of complete and perfect information, and interested readers can find the corresponding theorem for mixed-strategy version in [16]. It's notable that there're different *solution concepts* for different kind of games, for example *perfect Bayesian equilibrium* for Bayesian game.

[3] Mixed strategy is randomization of pure strategy, which can be viewed as more general strategy than pure one. The condition for existence of mixed-strategy NE is also looser than for pure-strategy NE. However, we often like to find pure-strategy NE since it's more physically achievable.

2.2.2 Multistage Bayesian Game

Considering each player contains its own private information without knowing others' one, NE is not a suitable solution concept in such game due to the unknown information. This kind of problem happens often, for example, say two competing firms whose strategy is to determine the quantity of goods, what's the best strategy for each of them without knowing the other's operation cost details? Or more generally, how would the game proceed if there's some uncertainty about players' information? Bayesian game is a type of game aiming at this kind of situation. The Bayesian approach forms belief about the unknown information and allows each player to update its posterior beliefs, *i.e.* posterior probabilities, about the other players' private information by observing their actions in prior stages. Each player can act accordingly in the current stage based on the updated beliefs, and then the game proceeds in a way that each player maximizes its expected profit according to its belief.

Game Formulation

A multistage Bayesian game Γ can be formulated as follows,

$$\Gamma = \left\langle \mathcal{I},\ \{A_x(h^t)\},\ \{\theta_x \in \Theta_x\},\ \{u_x\},\ \{\mu_x(\boldsymbol{\theta}_{-x}|\theta_x, h^0)\} \middle| x \in \mathcal{I}, h^t \in \mathcal{H}^t, t = 0, 1, 2, \cdots, T \right\rangle, \tag{2.5}$$

where $\mathcal{I} \equiv \{1, 2, \cdots, N\}$ is the set of players, $A_x(h^t)$ is the set of actions available for player x given a history $h^t = (\mathbf{a}^0, \mathbf{a}^1, \cdots, \mathbf{a}^{t-1})$ at the beginning of stage t with the notation $\mathbf{a}^\tau = a_1^\tau \times a_2^\tau \times \cdots \times a_N^\tau$ the action profile at stage τ with $a_i^\tau \in A_i(h^\tau)$ being the action of the ith player at stage τ, $\mathcal{H}^t$ is the set of all history h^t with $h^0 = \emptyset$. T is the length of game. We denote the set of all available actions for all players at stage τ as $A(h^\tau) = A_1(h^\tau) \times A_2(h^\tau) \times \cdots \times A_N(h^\tau)$. θ_x is the private information, also known as type, of player x. Type, which is the incomplete information in Bayesian game, cannot be known but can be inferred by other players. The type profile $\boldsymbol{\theta} = \theta_1 \times \theta_2 \times \cdots \times \theta_N$, and $\boldsymbol{\theta}_{-x}$ denotes the type profile $\boldsymbol{\theta}$ excluding θ_x. The actual type value for player x is denoted by $\widehat{\theta}_x$, and the corresponding type profile is $\widehat{\boldsymbol{\theta}}$. The utility function u_x of player x is a mapping $u_x : \mathcal{H}^\tau \times \boldsymbol{\theta} \to \mathbb{R}$ from the space $\mathcal{H}^\tau \times \boldsymbol{\theta}$ to the set of real numbers $\mathbb{R}$. In other words, u_x is a function of all players' types, past and current actions. $\mu_x(\boldsymbol{\theta}_{-x}|\theta_x, h^0)$ is player x's belief about other players' type $\boldsymbol{\theta}_{-x}$ given its own type θ_x at history h^0. In contrast with the static game of incomplete information, the belief about others can be updated stage by stage. Bayesian game defines the rule of how players update their belief stage by stage, and the players' actions can change according to the newly updates of beliefs.

Solution Concept and Belief System

In the game theory literature, the *solution concept* in a multistage game of incomplete information is called the perfect Bayesian equilibrium (PBE), which is a parallel to the subgame perfect equilibrium (SPE) in a multistage game of complete information. As SPE serves as a refinement of the Nash equilibrium in a multistage game of complete information, PBE is a refinement of the Bayesian NE (BNE) in a multistage game of incomplete information. To obtain PBE, some restrictions and assumptions on the belief system must be satisfied, and players' behaviors must be *sequentially rational* [16]. For the purpose of self-contained exposition of this thesis, we list the definition for the pure-strategy PBE. The mixed-strategy version can also be found in [16].

Definition 7 *A perfect Bayesian equilibrium is a* (a^*, μ) *that satisfies (P) and B(i)-B(iv).*

B(i) Posterior beliefs are independent, and all types of player x have the same beliefs. For all $\boldsymbol{\theta}$, t, and h^t, we have

$$\mu_x(\boldsymbol{\theta}_{-x}|\theta_x, h^t) = \prod_{y \neq x} \mu_x(\theta_y|h^t). \tag{2.6}$$

B(ii) Bayes' rule is used to update beliefs from $\mu_x(\theta_y|h^t)$ to $\mu_x(\theta_y|h^{t+1})$ whenever possible. For all x, y, h^t, and $a_y^t \in A_y(h^t)$, if there exists $\breve{\theta}_y$ with $\mu_x(\breve{\theta}_y|h^t) > 0$ and $a_y^{t*}(\breve{\theta}_y) = a_y^{t*}(\widehat{\theta}_y)$, then, for all θ_y

$$\mu_x(\theta_y|h^{t+1}) = \frac{\mu_x(\theta_y|h^t)\delta\left(a_y^{t*}(\theta_y) - a_y^{t*}(\widehat{\theta}_y)\right)}{\sum_{\theta'_y : a_y^{t*}(\theta'_y) = a_y^{t*}(\widehat{\theta}_y)} \mu_x(\theta'_y|h^t)}, \tag{2.7}$$

where

$$\delta\left(a_y^{t*}(\theta_y) - a_y^{t*}(\widehat{\theta}_y)\right) = \begin{cases} 1, \text{ if } a_y^{t*}(\theta_y) = a_y^{t*}(\widehat{\theta}_y), \\ 0, \text{ otherwise.} \end{cases} \tag{2.8}$$

where $a_y^{t*}(\theta_y)$ denotes the best action for player y corresponds to type θ_y at stage t. Note that B(ii) doesn't restrict the way belief about player y are updated if player y's stage-t action had conditional probability 0, which is the very difference from SE.

B(iii) For all h^t, x, y, θ_y, $\mathbf{a}^t$, and $\tilde{\mathbf{a}}^t$,

$$\mu_x(\theta_y|(h^t, \mathbf{a}^t)) = \mu_x(\theta_y|(h^t, \tilde{\mathbf{a}}^t)) \text{ if } a_y^t = \tilde{a}_y^t \tag{2.9}$$

This condition means that even if player y does deviate at stage t, the updating process should not be influenced by the action of other players.

B(iv) For all h^t, θ_z, and $x \neq y \neq z$,

$$\mu_x(\theta_z|h^t) = \mu_y(\theta_z|(h^t)) = \mu(\theta_z|(h^t)) \tag{2.10}$$

the belief of player x, y about third player z are the same.
This condition implies that the posterior beliefs are consistent with a common joint distribution on Θ given h^t with

$$\mu(\boldsymbol{\theta}_{-x}|h^t)\mu(\theta_x|(h^t)) = \mu(\boldsymbol{\theta}|(h^t)) \tag{2.11}$$

(P) *Sequentially rational*: For each player x, type θ_x, and history h^t,

$$a_x^{t*}(\theta_x) = arg \max_{a_x^t \in A_x} \sum_{\boldsymbol{\theta}_{-x}} \mu_x(\boldsymbol{\theta}_{-x}|h^t) u_x(a_x^t, \mathbf{a}_{-x}^{t*}(\boldsymbol{\theta}_{-x})|\boldsymbol{\theta}, h^t), \tag{2.12}$$

Here we assume that Θ is discrete set. For continuous set, we replace the summation with integral for the condition (P), or we can do approximation by quantizing continuous set into discrete one.

Chapter 3

Multistage Bayesian Spectrum Trading Game

3.1 Problem Setup

We consider a cognitive radio network with N primary services (*e.g.* the existing cellular services) and M secondary services (*e.g.* an SS can be a small network with a CR base-station and multiple CR users), as shown in Fig. 3.1. The ith PS operates on its own exclusive spectrum W_i, from which the ith PS can lease available unused bandwidth b_{ji} to the jth SS who doesn't own the legal right to use the spectrum. To maximize each PS's profit, each PS offers different prices to different SS's. In the trading process, all PS's compete with each other in the prices offering to the SS's, and each SS decides from whom and how much of the available spectrum to rent. Specifically, we model the spectrum trading process as a multistage game in a manner that all PS's simultaneously set their own prices p_{ji}, for all i and j, in the first stage. And, in the subsequent stage the jth SS requests bandwidth b_{ji} from the ith PS, for $i = 1, 2, \cdots, N$ and $j = 1, 2, \cdots, M$. Practically, however, each player (PS or SS) may possess its own private information that is unknown to other players. Therefore, each player cannot predict the overall trading behaviors correctly, which makes the decisions of optimal strategies a challenging task. In this incomplete information game, we propose using the theory of multistage Bayesian game to deal with the problem. The dynamic Bayesian approach allows each player to

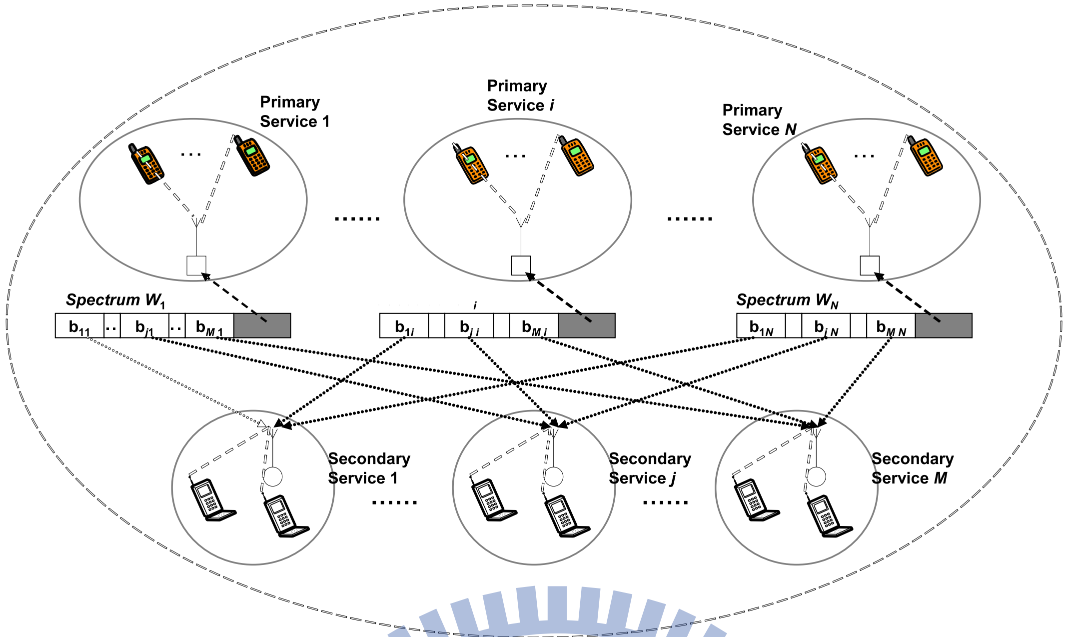


Figure 3.1: The cognitive radio network with multiple primary services and multiple secondary services.

update its posterior beliefs, *i.e.* posterior probabilities, about the other players' private information by observing their actions in prior stages. And, each player can act accordingly in the current stage based on the updated beliefs.

We assume that each player is selfish, but rational in the considered multistage sequential Bayesian game [16]. And the objective is to find the perfect Bayesian equilibrium for all players actions in a way that each player maximizes its expected profit as the game evolves sequentially.

As the private information may not be updated promptly and the channel conditions may change, we study a repeated version of the multistage Bayesian. The evolution of the repeated multistage game is illustrated in Fig. 3.2 with that one unit game composed of two stage is finished in one period.

Rather than learning in game to reach equilibrium, which spends time and energy on signaling and evolving, we believe that computing optimal strategy in one shot is more suitable for our scenario by the following reasons. First, the decision making of primary/secondary service is done by each primary/secondary base station, so the com-

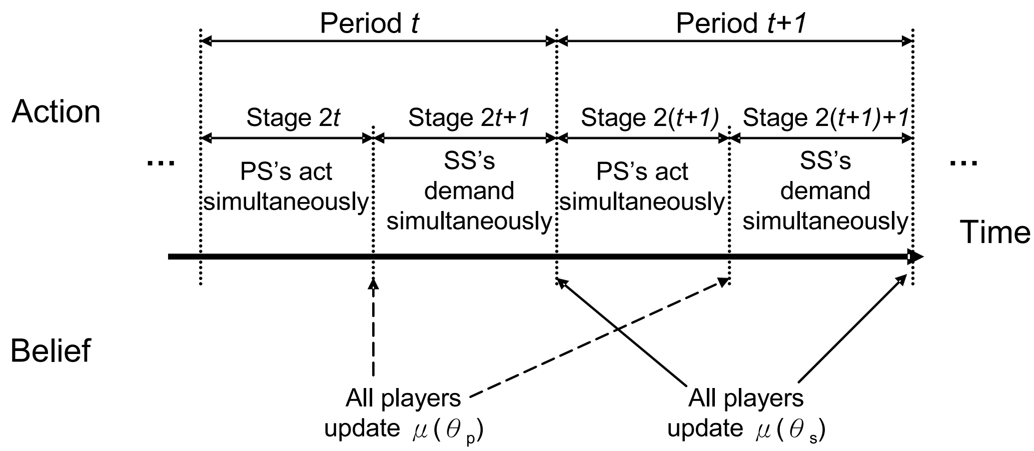


Figure 3.2: The evolution of the multistage sequential game.

plexity of computing optimization problem is affordable for them. Also, since the players are BS, the number N and M is not as much as the number of terminal wireless users in a normal cell.

3.2 Game Formulation

In this section, we describe the proposed multistage Bayesian game for spectrum trading with a general utility function. We will illustrate the idea by a specific example in Sec. 3.4.

We formulate the spectrum trading process as a multistage Bayesian game

$$\Gamma = \Big\langle \mathcal{I},\ \{A_x(h^t)\},\ \{\theta_x \in \Theta_x\},\ \{\mathcal{P}_x\},\ \{\mu_x(\boldsymbol{\theta}_{-x}|\theta_x, h^0)\}\Big| x \in \mathcal{I}, h^t \in \mathcal{H}^t, t = 0, 1, 2, ..., T\Big\rangle, \tag{3.1}$$

where $\mathcal{I} \triangleq \mathcal{I}_p \cup \mathcal{I}_s$ is the set of players with $\mathcal{I}_p = \{p_1, p_2, \ldots, p_N\}$ being the set of all PS's and $\mathcal{I}_s = \{s_1, s_2, \ldots, s_M\}$ the set of all SS's, $A_x(h^t)$ is the set of actions available for player x given a history $h^t = (\mathbf{a}^0, \mathbf{a}^1, \cdots, \mathbf{a}^{t-1})$ at the beginning of stage t with $\mathbf{a}^\tau = \mathbf{a}^\tau_{p_1} \times \mathbf{a}^\tau_{p_2} \times \cdots \times \mathbf{a}^\tau_{p_N} \times \mathbf{a}^\tau_{s_1} \times \mathbf{a}^\tau_{s_2} \times \cdots \times \mathbf{a}^\tau_{s_M}$ the action profile consisting of the actions from all players (including PS's and SS's) at stage τ with $\mathbf{a}^\tau_{p_i} \in A_{p_i}(h^\tau)$ and $\mathbf{a}^\tau_{s_j} \in A_{s_j}(h^\tau)$ being the action of the ith PS and the jth SS at stage τ, respectively. The set $\mathcal{H}^t$ contains all possible histories h^t's at time t with $\mathcal{H}^0$ the null set. We denote the set of all available actions for all players as $A(h^\tau) = A_{p_1}(h^\tau) \times A_{p_2}(h^\tau) \times \cdots \times A_{p_N}(h^\tau) \times A_{s_1}(h^\tau) \times A_{s_2}(h^\tau) \times \cdots \times A_{s_M}(h^\tau)$.

We denote $\mathbf{p}_i^p = (p_{1i}, p_{2i}, \cdots, p_{Mi})^T$, $\mathbf{p}_j^s = (p_{j1}, p_{j2}, \cdots, p_{jN})^T$, $\mathbf{b}_i^p = (b_{1i}, b_{2i}, \cdots, b_{Mi})^T$, $\mathbf{b}_j^s = (b_{j1}, b_{j2}, \cdots, b_{jN})^T$, $\mathbf{p}^\tau = \mathbf{p}_1^{p,\tau} \times \mathbf{p}_2^{p,\tau} \times \cdots \times \mathbf{p}_N^{p,\tau}$, and $\mathbf{b}^\tau = \mathbf{b}_1^{s,\tau} \times \mathbf{b}_2^{s,\tau} \times \cdots \times \mathbf{b}_M^{s,\tau}$. In each time period, PS i sets price $\mathbf{a}_{p_i} = \mathbf{p}_i^p$ at the even stage and stays silent (*i.e.* $\mathbf{a}_{p_i} = \boldsymbol{\phi}$) at the odd stage [16]. On the contrary, SS j performs "do nothing" (*i.e.* $\mathbf{a}_{s_j} = \boldsymbol{\phi}$) at the even stage and demands $\mathbf{a}_{s_j} = \mathbf{b}_j^s$ for bandwidth at the odd stage. Therefore, $\mathbf{a}^\tau = \mathbf{p}^\tau \times \boldsymbol{\phi}$ at even stage, and $\mathbf{a}^\tau = \boldsymbol{\phi} \times \mathbf{b}^\tau$ at odd stage where $\boldsymbol{\phi}$ is the action profile of "do nothing".

$\boldsymbol{\Theta}_x$ is the set of possible private information θ_x for player x. The type profile $\boldsymbol{\theta}_p = (\theta_{p_1}, \theta_{p_2}, \cdots, \theta_{p_N})$, and $\boldsymbol{\theta}_s = (\theta_{s_1}, \theta_{s_2}, \cdots, \theta_{s_M})$. $\boldsymbol{\theta}_{p_{-i}}$ denotes the type profile $\boldsymbol{\theta}_p$ excluding θ_{p_i}. Similarly, $\boldsymbol{\theta}_{s_{-j}}$ denotes the type profile $\boldsymbol{\theta}_s$ excluding θ_{s_j}. The overall type profile is $\boldsymbol{\theta} = (\boldsymbol{\theta}_s, \boldsymbol{\theta}_p)$. The actual type value for player x is denoted by $\widehat{\theta}_x$, and the actual type profile for PS's, SS's and overall players are $\widehat{\boldsymbol{\theta}}_p$, $\widehat{\boldsymbol{\theta}}_s$, $\widehat{\boldsymbol{\theta}}$, respectively. $\mathcal{P}_x$ standing for the profit function (*i.e.* the net utility) of player x is a mapping $\mathcal{P}_x : \mathcal{H}^\tau \times \boldsymbol{\theta} \rightarrow \mathbb{R}$ from the space $\mathcal{H}^\tau \times \boldsymbol{\theta}$ to the set of real numbers $\mathbb{R}$. $\mu_x(\boldsymbol{\theta}_{-x}|\boldsymbol{\theta}_x, h^t)$ is player x's beliefs about other players' types given its type $\boldsymbol{\theta}_x$ with history h^t. More details about the beliefs will be described in the next section.

In contrast with the static game of incomplete information, the belief about others can be updated stage by stage, and the players' actions can change according to the newly updates of beliefs.

3.3 General Formulation for Multiple Sellers and Multiple Buyers

3.3.1 Utility Model

As mentioned in the system model, PS i leases bandwidth b_{ji} to SS j The amount of b_{ji} affects the remaining available bandwidth of PS i, and thus affects the corresponding QoS satisfaction, which for PS i is denoted by the utility function $u_{p_i}(\mathbf{b}_i^p|\boldsymbol{\theta}, h^t)$. The monetary gain of trading is $\sum_j p_{ji} b_{ji} = (\mathbf{p}_i^p)^T \mathbf{b}_i^p$, where the superscript T means vector transpose.

And the total profit of PS i is given by

$$\mathcal{P}_{p_i}(\mathbf{p}_i^p, \mathbf{b}_i^p|\boldsymbol{\theta}) = u_{p_i}(\mathbf{b}_i^p|\boldsymbol{\theta}) + (\mathbf{p}_i^p)^T\mathbf{b}_i^p, \tag{3.2}$$

where $u_{p_i}(\mathbf{b}_i^p|\boldsymbol{\theta}, h^t)$ is denoted as $u_{p_i}(\mathbf{b}_i^p|\boldsymbol{\theta})$ for notational simplicity, also the condition on history h^t will be omitted in $\mathcal{P}_{p_i}$. We assume that $\mathcal{P}_{p_i}(\mathbf{p}_i^p, \mathbf{b}_i^p|\boldsymbol{\theta})$ is a concave function of $(\mathbf{p}_i^p, \mathbf{b}_i^p)$. Although such assumption is made, we will show in the explicit system that the assumption may not be the same as general formulation does to guarantee the joint KKT condition. Θ is assumed a discrete space in the formulation.

For secondary service, the utility of QoS $u_{s_j}(\mathbf{b}_j^s|\boldsymbol{\theta})$ is modeled as a concave function of $\mathbf{b}_j^s$. The cost of buying bandwidth is $\sum_i p_{ji}b_{ji} = (\mathbf{p}_j^s)^T\mathbf{b}_j^s$. The total profit of secondary service is

$$\mathcal{P}_{s_j}(\mathbf{p}_j^s, \mathbf{b}_j^s|\boldsymbol{\theta}) = u_{s_j}(\mathbf{b}_j^s|\boldsymbol{\theta}) - (\mathbf{p}_j^s)^T\mathbf{b}_j^s \tag{3.3}$$

which is still a concave function of $\mathbf{b}_j^s$.

3.3.2 Self-Optimization and KKT Translation

Since SS j is a follower of the game, it can observe the sellers' action $\mathbf{p}_j^s(\hat{\boldsymbol{\theta}}_p) = (p_{j1}(\hat{\theta}_{p_1}), p_{j2}(\hat{\theta}_{p2}), \cdots, p_{jN}(\hat{\theta}_{p_N}))^T$. Note that $\mathbf{p}_j^s(\hat{\boldsymbol{\theta}}_p)$ is the optimal price corresponds to type profile $\hat{\boldsymbol{\theta}}_p$, and SS just observes the prices without the knowledge of $\hat{\boldsymbol{\theta}}_p$. That is, SS may still don't know $\hat{\boldsymbol{\theta}}_p$ correctly ($\boldsymbol{\theta}_p$ is still random), but the prices corresponds to $\hat{\boldsymbol{\theta}}_p$ is deterministic. Based on that, SS j of type $\hat{\theta}_{s_j}$ would maximize its expected profit which can be formulated as

$$\mathbf{b}_j^{s*} = arg\max_{\mathbf{b}_j^s} E_{\boldsymbol{\theta}_{s_{-j}},\boldsymbol{\theta}_p}[\mathcal{P}_{s_j}(\mathbf{p}_j^s(\hat{\boldsymbol{\theta}}_p), \mathbf{b}_j^s|\hat{\theta}_{s_j}\boldsymbol{\theta}_{s_{-j}}\boldsymbol{\theta}_p)] \tag{3.4}$$

The KKT condition [17] for the profit maximization of SS j of type $\hat{\theta}_{s_j}$ is

$$\nabla_{\mathbf{b}_j^s} E_{\boldsymbol{\theta}_{s_{-j}},\boldsymbol{\theta}_p}[\mathcal{P}_{s_j}(\mathbf{p}_j^s(\hat{\boldsymbol{\theta}}_p), \mathbf{b}_j^s|\hat{\theta}_{s_j}\boldsymbol{\theta}_{s_{-j}}\boldsymbol{\theta}_p)]|_{\mathbf{b}_j^s=\mathbf{b}_j^{s*}} = \mathbf{0}, \tag{3.5}$$

which is equivalent to

$$\nabla_{\mathbf{b}_j^s} E_{\boldsymbol{\theta}_{s_{-j}},\boldsymbol{\theta}_p}[u_{s_j}(\mathbf{b}_j^s|\hat{\theta}_{s_j}, \boldsymbol{\theta}_{s_{-j}}, \boldsymbol{\theta}_p)]|_{\mathbf{b}_j^s=\mathbf{b}_j^{s*}} = \mathbf{p}_j^s(\hat{\boldsymbol{\theta}}_p), \tag{3.6}$$

here we observe that $\mathbf{b}_j^{s*}$ is a function of $\hat{\theta}_{s_j}$ and $\hat{\boldsymbol{\theta}}_p$, hence it can be can denote as $\mathbf{b}_j^{s*}(\hat{\theta}_{s_j}, \hat{\boldsymbol{\theta}}_p)$. Another observation is that SS's self-optimization are not coupled each other, namely, the profit maximization for SS j depends on all PS's and SS j itself but not other SS's. Hence, to be sequentially rational for SS j, it only needs to solve (3.5) or (3.6) without taking other SS's action into consideration.

But, how would PS's move with knowing that each SS is sequentially rational? It's widely known that the technique *backward induction* [16] is useful in solving finite dynamic game of perfect information. Here we apply the similar idea in solving trading game. All primary services know that SS j would ask the best demand $\mathbf{b}_j^{s*}(\hat{\theta}_{s_j}, \hat{\boldsymbol{\theta}}_p)$, or equivalently, they know the KKT condition for all SS's (3.6). However, since PS i doesn't know the exact type $\hat{\theta}_{s_j}$ and $\hat{\boldsymbol{\theta}}_{p_{-i}}$ exactly, PS i views $\mathbf{b}_j^{s*}(\theta_{s_j}, \hat{\theta}_{p_i}, \boldsymbol{\theta}_{p_{-i}})$ as a random variable with uncertain θ_{s_j} and $\boldsymbol{\theta}_{p_{-i}}$. Here, the objective of PS i is to maximize its expected profit based on the beliefs $\mu(\boldsymbol{\theta}_s, \boldsymbol{\theta}_{p_{-i}}|\boldsymbol{h}^t)$ about other players' private information, considering the KKT condition of SS's. The optimization for PS i of type $\widehat{\theta}_{p_i}$ is therefore given by

$$\mathbf{p}_i^{p*}(\hat{\theta}_{p_i}) = arg\max_{\mathbf{p}_i^p} E_{\boldsymbol{\theta}_s, \boldsymbol{\theta}_{p_{-i}}}[\mathcal{P}_{p_i}(\mathbf{p}_i^p, \mathbf{b}_i^{p*}(\boldsymbol{\theta}_s, \hat{\theta}_{p_i}, \boldsymbol{\theta}_{p_{-i}})|\boldsymbol{\theta}_s, \hat{\theta}_{p_i}, \boldsymbol{\theta}_{p_{-i}})], \tag{3.7}$$

$$\text{s.t. } 0 \leq b_{ji}^*(\theta_{s_j}, \hat{\theta}_{p_i}, \boldsymbol{\theta}_{p_{-i}}), \forall s_j, \forall \theta_{s_j} \in \Pi_{s_j}(h^t), \forall \boldsymbol{\theta}_{p_{-i}} \in \Pi_{p_{-i}}(h^t), \tag{3.8}$$

$$W_i \geq \sum_j b_{ji}^*(\theta_{s_j}, \hat{\theta}_{p_i}, \boldsymbol{\theta}_{p_{-i}}), \forall \boldsymbol{\theta}_s \in \Pi_s, \forall \boldsymbol{\theta}_{p_{-i}} \in \Pi_{p_{-i}}(h^t), \tag{3.9}$$

$$\mathbf{p}_j^s(\hat{\theta}_{p_i}, \boldsymbol{\theta}_{p_{-i}}) = \nabla_{\mathbf{b}_j^s} E_{\boldsymbol{\theta}_{s_{-j}}, \boldsymbol{\theta}_p}[u_{s_j}(\mathbf{b}_j^s|\boldsymbol{\theta})]|_{\mathbf{b}_j^s = \mathbf{b}_j^{s*}(\theta_{s_j}, \hat{\theta}_{p_i}, \boldsymbol{\theta}_{p_{-i}})}, \forall s_j, \forall \theta_{s_j} \in \Pi_{s_j}(h^t), \forall \boldsymbol{\theta}_{p_{-i}} \in \Pi_{p_{-i}}(h^t), \tag{3.10}$$

where $\Pi_{s_j}(h^t)$ is the set of all possible θ_{s_j}'s that satisfy $\mu(\theta_{s_j}|h^t) > 0$, Π_s is the set of all possible $\boldsymbol{\theta}_s$'s that satisfy $\mu(\boldsymbol{\theta}_s|h^t) > 0$ and $\Pi_{p_{-i}}(h^t)$ is the set of all possible $\boldsymbol{\theta}_{p_{-i}}$'s that satisfy $\mu(\boldsymbol{\theta}_{p_{-i}}|h^t) > 0$. The constraints in (3.8) and (3.9) limit the demand to be within the physically realizable spectrum region afforded by PS i under all possible type profiles of the other players. Note that there are numbers of inequalities in (3.8) and (3.9), but we can reduce them by finding the minimal set $\Theta_{m,i}(h^t)$ and $\Theta_{M,i}(h^t)$ to represent these two inequalities. The determination of $\Theta_{m,i}(h^t)$ and $\Theta_{M,i}(h^t)$ depends largely on the utility model. The constraint (3.10) is the KKT condition (3.6) for all SS's for any possible type profile. In this work, we call this approach the *KKT translation*. In this optimization problem, with the assumptions we've made, if the constraints (3.10) are affine functions, then the problem is a convex optimization problem, otherwise it's a optimization problem [17]. The difference lies in whether KKT condition is sufficient and necessary or purely necessary for the problem.

3.3.3 Perfect Bayesian Equilibrium and Joint KKT Condition

We are now ready to find the PBE at stage t of the multistage Bayesian game modeled in the considered cognitive radio network. The posterior belief is obtained by PBE updating rule B(i)-B(iv). With that, the condition (P) of PBE at any stage is

$$\mathbf{p}_i^{p*}(\theta_{p_i}) = arg\max_{\mathbf{p}_i^p} E_{\boldsymbol{\theta}_s,\boldsymbol{\theta}_{p_{-i}}}[\mathcal{P}_{p_i}(\mathbf{p}_i^p, \mathbf{b}_i^{p*}(\boldsymbol{\theta}_s, \theta_{p_i}, \boldsymbol{\theta}_{p-i})|\boldsymbol{\theta})], \tag{3.11}$$

$$\text{s.t. } 0 \leq b_{ji}^*(\theta_{s_j}, \theta_{p_i}, \boldsymbol{\theta}_{p-i}), \forall s_j, \forall(\theta_{s_j}, \boldsymbol{\theta}_{p-i}) \in \Theta_{m,i}(h^t), \tag{3.12}$$

$$W_i \geq \sum_j b_{ji}^*(\theta_{s_j}, \theta_{p_i}, \boldsymbol{\theta}_{p-i}), \forall(\boldsymbol{\theta}_s, \boldsymbol{\theta}_{p-i}) \in \Theta_{M,i}(h^t), \tag{3.13}$$

$$\mathbf{p}_j^{s*}(\theta_{p_i}, \boldsymbol{\theta}_{p-i}) = \nabla_{\mathbf{b}_j^s} E_{\boldsymbol{\theta}_{s_{-j}},\boldsymbol{\theta}_p}[u_{s_j}(\mathbf{b}_j^s|\boldsymbol{\theta})]|_{\mathbf{b}_j^s=\mathbf{b}_j^{s*}(\theta_{s_j},\theta_{p_i},\boldsymbol{\theta}_{p-i})}, \forall s_j, \forall\theta_{s_j} \in \Pi_{s_j}(h^t), \forall\boldsymbol{\theta}_{p_{-i}} \in \Pi_{p_{-i}}(h^t), \tag{3.14}$$

$$\forall\theta_{p_i} \in \Pi_{p_i}(h^t), \forall p_i \in \mathcal{I}_p$$

It is clear that if the constraint (3.14) is affine and the price profile $\mathbf{p}_{-i}^{p*}(\boldsymbol{\theta}_{p_{-i}})$ for all type profiles is known, then the KKT condition is sufficient and necessary for solving the convex optimization problem. However, finding the optimal strategy profile $\mathbf{p}_{-i}^{p*}(\boldsymbol{\theta}_{p_{-i}})$ for all possible $\boldsymbol{\theta}_{p_{-i}}$ needs the information of $\mathbf{p}_i^{p*}(\theta_{p_i})$ for all possible θ_{p_i}. It follows that each player has to jointly solve all PS's KKT conditions simultaneously. The joint KKT conditions are given by

$$\begin{cases} -b^*_{ji}(\theta_{s_j}, \boldsymbol{\theta}_p) & \leq 0, \forall(\theta_{s_j}, \boldsymbol{\theta}_{p_{-i}}) \in \Theta_{m,i}(h^t), \forall s_j \in \mathcal{I}_s \\ \sum_{j=1}^{M} b^*_{ji}(\theta_{s_j}, \boldsymbol{\theta}_p) - W_i & \leq 0, \forall(\boldsymbol{\theta}_s, \boldsymbol{\theta}_{p_{-i}}) \in \Theta_{M,i}(h^t) \\ \mathcal{K}_{jk}(\theta_{s_j}) & = p_{jk}(\theta_{p_k}), \forall \theta_{s_j} \in \Pi_{s_j}(h^t), \\ & \qquad \forall \boldsymbol{\theta}_{p_{-i}} \in \Pi_{p_{-i}}(h^t), \forall s_j \in \mathcal{I}_s, \forall p_k \in \mathcal{I}_p \\ \lambda_{i,j,\theta_{s_j},\boldsymbol{\theta}_p} & \geq 0, \forall(\theta_{s_j}, \boldsymbol{\theta}_{p_{-i}}) \in \Theta_{m,i}(h^t), \forall s_j \in \mathcal{I}_s \\ \nu_{i,\boldsymbol{\theta}_s,\boldsymbol{\theta}_p} & \geq 0, \forall(\boldsymbol{\theta}_s, \boldsymbol{\theta}_{p_{-i}}) \in \Theta_{M,i}(h^t) \\ \lambda_{i,j,\theta_{s_j},\boldsymbol{\theta}_p} b^*_{ji}(\theta_{s_j}, \boldsymbol{\theta}_p) & = 0, \forall(\theta_{s_j}, \boldsymbol{\theta}_{p_{-i}}) \in \Theta_{m,i}(h^t), \forall s_j \in \mathcal{I}_s \\ \nu_{i,\boldsymbol{\theta}_s,\boldsymbol{\theta}_p}(\sum_{j=1}^{M} b^*_{ji}(\theta_{s_j}, \boldsymbol{\theta}_p) - W_i) & = 0, \forall(\boldsymbol{\theta}_s, \boldsymbol{\theta}_{p_{-i}}) \in \Theta_{M,i}(h^t) \\ \nabla_{\mathbf{p}^p_i} \mathcal{L}_i & = \mathbf{0} \end{cases}$$

$$\forall \theta_{p_i} \in \Pi_{p_i}(h^t), \forall p_i \in \mathcal{I}_p. \tag{3.15}$$

where $\lambda_{i,j,\theta_{s_j},\boldsymbol{\theta}_{p_{-i}}}$, $\nu_{i,\boldsymbol{\theta}_s,\boldsymbol{\theta}_{p_{-i}}}$ and $\eta_{k,j,\theta_{s_j},\boldsymbol{\theta}_{p_{-i}}}$ are Lagrange multipliers. $\mathcal{K}_{jk}(\theta_{s_j})$ represents righthand part of equation (3.14), which is

$$\mathcal{K}_{jk}(\theta_{s_j}) \equiv \frac{\partial E_{\boldsymbol{\theta}_{s_{-j}},\boldsymbol{\theta}_p}[u_{s_j}(\mathbf{b}^s_j|\boldsymbol{\theta})]|_{\mathbf{b}^s_j=\mathbf{b}^{s*}_j(\theta_{s_j},\boldsymbol{\theta}_p)}}{\partial b_{jk}}, \forall p_k \in \mathcal{I}_p \tag{3.16}$$

$\mathcal{L}_i$ is Lagrangian function of PS i of type θ_{p_i}, which is

$$\nabla_{\mathbf{p}^p_i} \mathcal{L}_i = \nabla_{\mathbf{p}^p_i} E_{\boldsymbol{\theta}_s,\boldsymbol{\theta}_{p_{-i}}}[\mathcal{P}_{p_i}(\mathbf{p}^p_i, \mathbf{b}^{p*}_i(\boldsymbol{\theta})|\boldsymbol{\theta})] \tag{3.17}$$

$$- \sum_{s_j \in \mathcal{I}_s, (\theta_{s_j}, \boldsymbol{\theta}_{p_{-i}}) \in \Theta_{m,i}(h^t)} \lambda_{i,j,\theta_{s_j},\boldsymbol{\theta}_p} \nabla_{\mathbf{p}^p_i}(-b^*_{ji}(\theta_{s_j}, \boldsymbol{\theta}_p)) \tag{3.18}$$

$$- \sum_{(\boldsymbol{\theta}_s, \boldsymbol{\theta}_{p_{-i}}) \in \Theta_{M,i}(h^t)} \nu_{i,\boldsymbol{\theta}_s,\boldsymbol{\theta}_p} \nabla_{\mathbf{p}^p_i} \left(\sum_{j=1}^{M} b^*_{ji}(\theta_{s_j}, \boldsymbol{\theta}_p) - W_i \right) \tag{3.19}$$

$$- \sum_{p_k \in \mathcal{I}_p, s_j \in \mathcal{I}_s, \theta_{s_j} \in \Pi_{s_j}(h^t), \boldsymbol{\theta}_{p_{-i}} \in \Pi_{p_{-i}}(h^t)} \eta_{k,j,\theta_{s_j},\boldsymbol{\theta}_p} \nabla_{\mathbf{p}^p_i}[p_{jk}(\theta_{p_k}) - \mathcal{K}_{jk}(\theta_{s_j})] \tag{3.20}$$

The solution of joint KKT specifies $\mathbf{p}^{p*}_i(\theta_{p_i})$ for all possible θ_{p_i} for all p_i.

3.4 Explicit System For Multiple Sellers and Multiple Buyers

3.4.1 Utility Model

In this part, we adopt and modify the utility models in [8]. The profit function of the ith PS is given by

$$\mathcal{P}_{p_i}(\mathbf{p}_i^p, \mathbf{b}_i^p|\theta_{p_i}) = (\mathbf{p}_i^p)^T \mathbf{b}_i^p + c_1\theta_{p_i} - c_2\theta_{p_i}\left(\mathrm{B}_i^{\mathrm{req}} - k_i^{(p)}\frac{W_i - \sum_{j=1}^{M} b_{ji}}{\theta_{p_i}}\right)^2, \quad (3.21)$$

where c_1 and c_2 are constant weights, $\mathrm{B}_i^{\mathrm{req}}$ is the bandwidth requirement for a primary connection, $k_i^{(p)} = \log_2\left(1 + \frac{1.5\gamma_i^p}{\ln(0.2/\mathrm{BER}_i^{\mathrm{tar}})}\right)$ denotes the spectral efficiency of wireless transmission for the ith PS with γ_i^p being the signal-to-noise ratio (SNR) at the ith PS's receivers and $\mathrm{BER}_i^{\mathrm{tar}}$ being the target bit-error-rate (BER) for the ith PS's local connection [18]. The private information θ_{p_i}, taking values in the set Θ_p, represents the number of connections in the ith PS. The first term in righthand side of (3.21) is the monetary gain of selling bandwidths. The second term is the revenue of maintaining primary connections that is proportional to θ_{p_i}. The third term is the cost of sharing the spectrum with SS's, the square term could be interpreted as magnification of the difference between required throughput and actual serving throughput per terminal user of ith PS. Instead of single SS scenario in [8], the profit function (3.21) considers multiple SS's.

The profit function of SS j is given by

$$\mathcal{P}_{s_j}(\mathbf{p}_j^s, \mathbf{b}_j^s|\theta_{s_j}) = \frac{1}{\theta_{s_j}}\left[\sum_{i}^{N} b_{ji}k_i^{(s_j)} - \frac{1}{2}\left((\mathbf{b}_j^s)^T\mathbf{b}_j^s + 2\xi_j\sum_{k\neq i} b_{jk}b_{ji}\right)\right] - (\mathbf{p}_j^s)^T\mathbf{b}_j^s, \quad (3.22)$$

where $\xi_j \in [-1.0, 1.0]$ is jth SS's spectrum substitutability is defined as follows. When $\xi_j = 1$, SS j could switch among the spectrum rent from all PS's freely. When $\xi_j = 0$, SS j can't switch among the operating spectrum. If $\xi_j < 0$, spectrum sharing by SS j is complementary, that is, it will need to buy one or more additional spectrum simultaneously. We consider $0 \leq \xi_j \leq 1$ for the rest of the thesis, the other case $-1 \leq \xi_j \leq 0$ is straightforward. $k_i^{(s_j)} = \log_2\left(1 + \frac{1.5\gamma_{ji}^s}{\ln(0.2/\mathrm{BER}_j^{\mathrm{tar}})}\right)$ denotes the the spectral efficiency acquired by jth SS's secondary user on the band W_i owned by PS i. The first two term in righthand side of (3.22) are QoS satisfaction function of SS j, which is modeled as a concave function of $\mathbf{b}_j^s$. The last term is the payment for buying bandwidths from all PS's. Compared with the utility in [8], we introduce the private information θ_{s_j} of jth SS in this paper to represent the factor leveraging the weighting between QoS and the spectrum trading expense. This weighting factor is implicitly related to the number of active connections within SS. When there is no connections requested by the cognitive users in jth SS, jth SS must have zero profit in terms of QoS and the corresponding θ_{s_j} is ∞.

3.4.2 Solving for Perfect Bayesian Equilibrium

To obtain the optimal strategy of jth SS of type $\widehat{\theta}_{s_j}$, the KKT condition of the maximization of jth SS's profit function is

$$\nabla_{\mathbf{b}_j^s} E_{\boldsymbol{\theta}_{s_{-j}},\boldsymbol{\theta}_p}[\mathcal{P}_{s_j}(\mathbf{p}_j^s(\hat{\boldsymbol{\theta}}_p),\mathbf{b}_j^s|\widehat{\theta}_{s_j})]|_{\mathbf{b}_j^s=\mathbf{b}_j^{s*}} = \nabla_{\mathbf{b}_j^s}\mathcal{P}_{s_j}(\mathbf{p}_j^s(\hat{\boldsymbol{\theta}}_p),\mathbf{b}_j^s|\widehat{\theta}_{s_j})|_{\mathbf{b}_j^s=\mathbf{b}_j^{s*}} = \mathbf{0}, \quad (3.23)$$

In this example, the close form solution of the best demand from jth SS to ith PS is obtained as follows

$$b^*_{ji}(\hat{\theta}_{s_j},\hat{\boldsymbol{\theta}}_p) = \mathcal{D}_{ji}(\mathbf{p}_j^s(\widehat{\boldsymbol{\theta}}_p),\widehat{\theta}_{s_j}) = D_{1,ji}(\mathbf{p}^s_{j-i}(\widehat{\boldsymbol{\theta}}_{p-i}),\widehat{\theta}_{s_j}) - \widehat{\theta}_{s_j} p_{ji}(\widehat{\theta}_{p_i}) D_{2,j}, \quad (3.24)$$

where $\mathbf{p}^s_{j-i}(\widehat{\boldsymbol{\theta}}_{p-i})$ is $\mathbf{p}^s_j(\widehat{\boldsymbol{\theta}}_p)$ with the exclusion of $p_{ji}(\widehat{\theta}_{p_i})$ and

$$D_{1,ji}(\mathbf{p}^s_{j-i}(\widehat{\boldsymbol{\theta}}_{p-i}),\widehat{\theta}_{s_j}) = \frac{C_{ji}}{A_j} + \frac{\xi_j\widehat{\theta}_{s_j}\sum_{k\neq i} p_{jk}(\widehat{\theta}_{p_k})}{A_j} \quad (3.25)$$

$$D_{2,j} = \frac{(\xi_j(N-2)+1)}{A_j} > 0, \text{if } 0 \leq \xi_j \leq 1 \quad (3.26)$$

with $A_j = (1-\xi_j)(\xi_j(N-1)+1) \geq 0$, $C_{ji} = k_i^{(s_j)}(\xi_j(N-2)+1) - \xi_j\sum_{k\neq i} k_k^{(s_j)}$.

We observe that $\mathcal{D}_{ji}(\mathbf{p}_j^s(\widehat{\boldsymbol{\theta}}_p),\widehat{\theta}_{s_j})$ is an affine function of $\mathbf{p}_j^s$. It would increase as $p_{jk}(\widehat{\theta}_{p_k})$ increases for all $p_k \in \mathcal{I}_p$, $p_k \neq p_i$ and would decrease as $p_{ji}(\widehat{\theta}_{p_i})$ increases. The minimum of $\mathcal{D}_{ji}(\mathbf{p}_j^s(\widehat{\boldsymbol{\theta}}_p),\widehat{\theta}_{s_j})$ happens when $p_{ji}(\widehat{\theta}_{p_i})$ is highest and $\mathbf{p}^s_{j-i}(\widehat{\boldsymbol{\theta}}_{p-i})$ is lowest. However, the dependency on $\widehat{\theta}_{s_j}$ is not clear, which also depends on $\mathbf{p}^s_{j-i}(\widehat{\boldsymbol{\theta}}_{p-i})$ and $p_{ji}(\widehat{\theta}_{p_i})$. Similar reasoning could be applied for the maximum of $\mathcal{D}_{ji}(\mathbf{p}_j^s(\widehat{\boldsymbol{\theta}}_p),\widehat{\theta}_{s_j})$. Hence, the minimal set for bandwidth constraint (3.8) and (3.9) are

$$\Theta_{m,i,j}(h^t) = \{(\theta_{s_j}^m, \boldsymbol{\theta}_{p_{-i}}^m), (\theta_{s_j}^M, \boldsymbol{\theta}_{p_{-i}}^m)\} \tag{3.27}$$

$$\Theta_{M,i}(h^t) = \{(\boldsymbol{\theta}_s^c, \boldsymbol{\theta}_{p_{-i}}^M) \big| (\theta_s^c)_j = \theta_{s_j}^m \text{ or } \theta_{s_j}^M, \forall s_j \in \mathcal{I}_s\} \tag{3.28}$$

where $\theta_{s_j}^m$ is minimum of θ_{s_j} with $\mu(\theta_{s_j}^m|h^t) > 0$, $\theta_{s_j}^M$ is maximum of θ_{s_j} with $\mu(\theta_{s_j}^M|h^t) > 0$, $\boldsymbol{\theta}_{p_{-i}}^m$ is elementwise minimum of $\boldsymbol{\theta}_{p_{-i}}$ with $\mu(\boldsymbol{\theta}_{p_{-i}}^m|h^t) > 0$, $\boldsymbol{\theta}_{p_{-i}}^M$ is elementwise maximum of $\boldsymbol{\theta}_{p_{-i}}$ with $\mu(\boldsymbol{\theta}_{p_{-i}}^M|h^t) > 0$.

Then, we examine the objective function of PS's. $\mathcal{P}_{p_i}(\mathbf{p}_i^p, \mathbf{b}_i^{p*}(\boldsymbol{\theta}_s, \theta_{p_i}, \boldsymbol{\theta}_{p_{-i}})|\boldsymbol{\theta})$ is not a concave function of $(\mathbf{p}_i^p, \mathbf{b}_i^p)$ in this explicit case, but with $\mathbf{b}_i^{p*}(\boldsymbol{\theta}_s, \theta_{p_i}, \boldsymbol{\theta}_{p_{-i}})$ being replaced with $\mathcal{D}_i^p(\mathbf{p}(\boldsymbol{\theta}_p), \boldsymbol{\theta}_s)$, the new function $\mathcal{P}_{p_i}(\mathbf{p}_i^p, \mathcal{D}_i^p(\mathbf{p}(\boldsymbol{\theta}_p), \boldsymbol{\theta}_s)|\theta_{p_i})$ is concave of $\mathbf{p}_i^p$, where $\mathcal{D}_i^p(\mathbf{p}(\boldsymbol{\theta}_p), \boldsymbol{\theta}_s) = (\mathcal{D}_{1i}(\mathbf{p}_1^s(\boldsymbol{\theta}_p), \theta_{s_1}), \cdots, \mathcal{D}_{ji}(\mathbf{p}_j^s(\boldsymbol{\theta}_p), \theta_{s_j}), \cdots, \mathcal{D}_{Mi}(\mathbf{p}_M^s(\boldsymbol{\theta}_p), \theta_{s_M}))^T$. Together with $b_{ji}^*(\theta_{s_j}, \theta_{p_i}, \boldsymbol{\theta}_{p_{-i}})$ in (3.12) and (3.13) being replaced with $\mathcal{D}_{ji}(\mathbf{p}_j^s(\boldsymbol{\theta}_p), \theta_{s_j})$, we can drop the equation (3.14), and the equation (3.11)- (3.13) becomes,

$$\mathbf{p}_i^{p*}(\theta_{p_i}) = arg \max_{\mathbf{p}_i^p} E_{\boldsymbol{\theta}_s, \boldsymbol{\theta}_{p_{-i}}}[\mathcal{P}_{p_i}(\mathbf{p}_i^p, \mathcal{D}_i^p(\mathbf{p}(\boldsymbol{\theta}_p), \boldsymbol{\theta}_s)|\theta_{p_i})], \tag{3.29}$$

$$\text{s.t. } 0 \le \mathcal{D}_{ji}(\mathbf{p}_j^s(\boldsymbol{\theta}_p), \theta_{s_j}), \forall s_j \in \mathcal{I}_s, \forall(\theta_{s_j}, \boldsymbol{\theta}_{p_{-i}}) \in \Theta_{m,i,j}(h^t), \tag{3.30}$$

$$W_i \ge \sum_{j=1}^{M} \mathcal{D}_{ji}(\mathbf{p}_j^s(\boldsymbol{\theta}_p), \theta_{s_j}), \forall(\boldsymbol{\theta}_s, \boldsymbol{\theta}_{p_{-i}}) \in \Theta_{M,i}(h^t), \tag{3.31}$$

$$\forall \theta_{p_i} \in \Pi_{p_i}(h^t), \forall p_i \in \mathcal{I}_p.$$

The optimization of PS's in the explicit system is a convex optimization problem. And the joint KKT condition now becomes

$$\begin{cases} \begin{aligned} -\mathcal{D}_{ji}(\mathbf{p}_j^s(\boldsymbol{\theta}_p), \theta_{s_j}) &\le 0, \forall s_j \in \mathcal{I}_s, \forall(\theta_{s_j}, \boldsymbol{\theta}_{p_{-i}}) \in \Theta_{m,i,j}(h^t) \\ \textstyle\sum_{j=1}^M \mathcal{D}_{ji}(\mathbf{p}_j^s(\boldsymbol{\theta}_p), \theta_{s_j}) - W_i &\le 0, \forall(\boldsymbol{\theta}_s, \boldsymbol{\theta}_{p_{-i}}) \in \Theta_{M,i}(h^t) \\ \lambda_{i,j,\theta_{s_j},\boldsymbol{\theta}_p} &\ge 0, \forall s_j \in \mathcal{I}_s, \forall(\theta_{s_j}, \boldsymbol{\theta}_{p_{-i}}) \in \Theta_{m,i,j}(h^t) \\ \nu_{i,\boldsymbol{\theta}_s,\boldsymbol{\theta}_p} &\ge 0, \forall(\boldsymbol{\theta}_s, \boldsymbol{\theta}_{p_{-i}}) \in \Theta_{M,i}(h^t) \\ \lambda_{i,j,\theta_{s_j},\boldsymbol{\theta}_p} \mathcal{D}_{ji}(\mathbf{p}_j^s(\boldsymbol{\theta}_p), \theta_{s_j}) &= 0, \forall s_j \in \mathcal{I}_s, \forall(\theta_{s_j}, \boldsymbol{\theta}_{p_{-i}}) \in \Theta_{m,i,j}(h^t) \\ \nu_{i,\boldsymbol{\theta}_s,\boldsymbol{\theta}_p} \left(\textstyle\sum_{j=1}^M \mathcal{D}_{ji}(\mathbf{p}_j^s(\boldsymbol{\theta}_p), \theta_{s_j}) - W_i\right) &= 0, \forall(\boldsymbol{\theta}_s, \boldsymbol{\theta}_{p_{-i}}) \in \Theta_{M,i}(h^t) \\ \nabla_{\mathbf{p}_i^p} \mathcal{L}_i(\theta_{p_i}) &= \mathbf{0} \end{aligned} \end{cases}$$

$$\forall \theta_{p_i} \in \Pi_{p_i}(h^t), \forall p_i \in \mathcal{I}_p, \tag{3.32}$$

where $\mathcal{L}_i(\theta_{p_i})$ is the Lagrangian function for maximization of ith PS of type θ_{p_i}, and the n-th element of $\nabla_{\mathbf{p}_i^p}\mathcal{L}_i(\theta_{p_i})$ is

$$\begin{aligned}\left[\nabla_{\mathbf{p}_i^p}\mathcal{L}_i(\theta_{p_i})\right]_n &= \frac{\partial \mathcal{L}_i(\theta_{p_i})}{\partial p_{ni}} = \frac{\partial E\left[\mathcal{P}_{p_i}(\mathbf{p}_i^p, \mathcal{D}_i^p(\mathbf{p}^s(\boldsymbol{\theta}_p),\boldsymbol{\theta}_s)|\theta_{p_i})\right]}{\partial p_{ni}} \\ &\quad - \lambda_{i,n,\theta_{s_n}^m,\boldsymbol{\theta}_{p_{-i}}^m}\theta_{s_n}^m D_{2,ni} - \lambda_{i,n,\theta_{s_n}^M,\boldsymbol{\theta}_{p_{-i}}^m}\theta_{s_n}^M D_{2,ni} + \sum_{(\boldsymbol{\theta}_s,\boldsymbol{\theta}_{p_{-i}})\in\Theta_{M,i}(h^t)} \theta_{s_n} D_{2,ni}\nu_{i,\boldsymbol{\theta}_s,\boldsymbol{\theta}_p}\end{aligned}$$

Since $\nabla_{\mathbf{p}_i^p} E_{\boldsymbol{\theta}_s,\boldsymbol{\theta}_{p_{-i}}}[\mathcal{P}_{p_i}] = E_{\boldsymbol{\theta}_s,\boldsymbol{\theta}_{p_{-i}}}[\nabla_{\mathbf{p}_i^p}\mathcal{P}_{p_i}]$, we compute

$$\begin{aligned}&\left[\nabla_{\mathbf{p}_i^p}\mathcal{P}_{p_i}(\mathbf{p}_i^p, \mathcal{D}_i^p(\mathbf{p}^s(\boldsymbol{\theta}_p),\boldsymbol{\theta}_s)|\theta_{p_i})\right]_n = \frac{\partial \mathcal{P}_{p_i}(\mathbf{p}_i^p, \mathcal{D}_i^p(\mathbf{p}^s(\boldsymbol{\theta}_p),\boldsymbol{\theta}_s)|\theta_{p_i})}{\partial p_{ni}} \\ &= \underbrace{\left[\frac{C_{ni}}{A_n} + 2c_2 k_i^{(p)}\theta_{s_n}D_{2,ni}\left(\mathrm{B}_i^{\mathrm{req}} - k_i^{(p)}\frac{W_i - \sum_{j=1}^M \frac{C_{ji}}{A_j}}{\theta_{p_i}}\right)\right]}_{E_{n,i}(\theta_{p_i},\theta_{s_n})} \\ &- \underbrace{\left[2\theta_{s_n}D_{2,ni} + \frac{2c_2(k_i^{(p)}\theta_{s_n}D_{2,ni})^2}{\theta_{p_i}}\right]}_{G_{n,i}(\theta_{p_i},\theta_{s_n})} p_{ni}(\theta_{p_i}) - \underbrace{\frac{2c_2(k_i^{(p)})^2\theta_{s_n}D_{2,ni}}{\theta_{p_i}}}_{H_{n,i}(\theta_{p_i},\theta_{s_n})}\sum_{j\neq n}\theta_{s_j}p_{ji}(\theta_{p_i})D_{2,j} \\ &+ \underbrace{\left[\frac{\xi_n\theta_{s_n}}{A_n} + \frac{2c_2(k_i^{(p)}\theta_{s_n})^2 D_{2,ni}\xi_n}{\theta_{p_i}A_n}\right]}_{F_{n,i}(\theta_{p_i},\theta_{s_n})}\sum_{k\neq i}p_{nk}(\theta_{p_k}) + \underbrace{\frac{2c_2(k_i^{(p)})^2\theta_{s_n}D_{2,ni}}{\theta_{p_i}}}_{I_{n,i}(\theta_{p_i},\theta_{s_n})}\sum_{j\neq n}\frac{\xi_j\theta_{s_j}}{A_j}\sum_{k\neq i}p_{jk}(\theta_{p_k}) \\ &= E_{n,i}(\theta_{p_i},\theta_{s_n}) - G_{n,i}(\theta_{p_i},\theta_{s_n})p_{ni}(\theta_{p_i}) - H_{n,i}(\theta_{p_i},\theta_{s_n})\sum_{j\neq n}\theta_{s_j}p_{ji}(\theta_{p_i})D_{2,j} \\ &+ F_{n,i}(\theta_{p_i},\theta_{s_n})\sum_{k\neq i}p_{nk}(\theta_{p_k}) + I_{n,i}(\theta_{p_i},\theta_{s_n})\sum_{j\neq n}\frac{\xi_j\theta_{s_j}}{A_j}\sum_{k\neq i}p_{jk}(\theta_{p_k}), \forall s_n \in \mathcal{I}_s.\end{aligned}$$

Therefore,

$$\begin{aligned}E_{\boldsymbol{\theta}_s,\boldsymbol{\theta}_{p_{-i}}}\left[\frac{\partial \mathcal{P}_{p_i}(\mathbf{p}_i^p, \mathcal{D}_i^p(\mathbf{p}^s(\boldsymbol{\theta}_p),\boldsymbol{\theta}_s)|\theta_{p_i})}{\partial p_{ni}}\right] &= \overline{E_{n,i}} - \overline{G_{n,i}}p_{ni}(\theta_{p_i}) - \overline{H_{n,i}}\sum_{j\neq n}\overline{\theta_{s_j}}\cdot\overline{p_{ji}(\theta_{p_i})}D_{2,j} \\ &\quad + \overline{F_{n,i}}\sum_{k\neq i}\overline{p_{nk}(\theta_{p_k})} + \overline{I_{n,i}}\sum_{j\neq n}\frac{\xi_j\overline{\theta_{s_j}}}{A_j}\sum_{k\neq i}\overline{p_{jk}(\theta_{p_k})}.\end{aligned}$$

3.4.3 Algorithm for Solving Joint KKT Condition

The joint KKT conditions can be solved by active-set method [19], which is summarized in Algorithm 1.

Algorithm 1 Active-set method for solving joint KKT condition

0: **Define**: $\mathcal{S} \triangleq \bigsqcup_{i,j} \Theta_{m,i,j}(h^t) \cup \Theta_{M,i}(h^t)$, and $\mathcal{W}$ is the working set.

1: **Initialize**: Set $\mathcal{W} = \emptyset$.

2: **Repeat**: Solve the joint KKT conditions with that $\lambda_{i,j,\theta_{s_j},\boldsymbol{\theta}_p} = 0$ and $\nu_{i,\boldsymbol{\theta}_s,\boldsymbol{\theta}_p} = 0$ for those constraints $\notin \mathcal{W}$.

3: Condition 1: Check whether equation (3.30) is satisfied for $\theta_{p_i}^M, \forall p_i \in \mathcal{I}_p$

4: Condition 2: Check whether equation (3.31) is satisfied for $\theta_{p_i}^m, \forall p_i \in \mathcal{I}_p$

5: Condition 3: Check whether $\lambda_{i,j,\theta_{s_j},\boldsymbol{\theta}_p} \geq 0$ and $\nu_{i,\boldsymbol{\theta}_s,\boldsymbol{\theta}_p} \geq 0$ for those constraints $\in \mathcal{W}$,

6: **If** conditions 1, 2, and 3 all are satisfied, **then** we obtain the optimal $\mathbf{p}_i^{p*}(\theta_{p_i})$ for all $\theta_{p_i} \in \Pi_{p_i}(h^t)$ and for all p_i. We finish.

7: **Else** choose another $\mathcal{W} \subset \mathcal{S}$.

8: **End repeat**

The complexity of this algorithm depends on two factors, one is how you choose next working set, and the other is how you solve the linear equations. If the simplest working set choosing, *i.e.* linear choosing, is implemented, then the worst case searching number would be 2^{2MN+N2^M}. It's because there're $2MN + N2^M$ constraints in total, therefore 2^{2MN+N2^M} combinations of working set are possible. The number of linear equations for given working set $\mathcal{W}$ is $(N * M * |\Theta_p| + |\mathcal{W}|)$, where $|\mathcal{W}|$ is the number of active constraints, which ranges from 0 to 2^{2MN+N2^M}.

To make this algorithm more practical, we can reduce the complexity by quantizing Θ_p. For instance, if now $\Theta_p \equiv \{1, 2, \cdots, 10\}$, then we can quantize it into 2 subsets, the upper set and the lower set, and let 8 be the representative element for upper set, and 3 be the representative element for lower set. For all elements greater than 5, they are viewed 8; for all elements less or equal to 5, they are viewed 3. Now the algorithm is performed with the quantized type space $\Theta_p^q \equiv \{3, 8\}$. After the current period game is finished and the opponents' type are classified into either upper set or lower set, the upper or lower set could be further quantized for the next period game. In this way, the type space is now of size 2 for every time calculation, so the complexity is greatly reduced.

3.5 Convergence of Beliefs and Actions

In this section, we discuss the convergence of beliefs and actions. We'll conclude that 1. the belief update always tends to lead to a correct one, but may not converge; 2. although the belief may not converge, the action would converge to the one of actual type.

Proposition 1 *The belief of player $x \neq y$ about the actual type of player z at stage t would be greater or equal to the belief at stage t' if $t > t'$.*

$$\mu_i(\widehat{\theta}_j|h^t) \geq \mu_i(\widehat{\theta}_j|h^{t'}) \tag{3.33}$$

Proof 1

$$\mu_i(\widehat{\theta}_j|h^{t'+1}) = \frac{\mu_i(\widehat{\theta}_j|h^{t'})\delta(\mathbf{a}_j^{t*}(\widehat{\theta}_j) - \mathbf{a}_j^{t*}(\widehat{\theta}_j))}{\sum_{\theta'_j:\mathbf{a}_j^{t*}(\theta'_j)=\mathbf{a}_j^{t*}(\widehat{\theta}_j)} \mu_i(\theta'_j|h^{t'})} \tag{3.34}$$

$$= \frac{\mu_i(\widehat{\theta}_j|h^{t'})}{\sum_{\theta'_j:\mathbf{a}_j^{t*}(\theta'_j)=\mathbf{a}_j^{t*}(\widehat{\theta}_j)} \mu_i(\theta'_j|h^{t'})} \geq \mu_i(\widehat{\theta}_j|h^{t'}) \tag{3.35}$$

According to the above statement, the updating of belief is never a misleading updating. But it doesn't address about whether the updating converges to the actual one or not, perhaps the improvement stops before converging to the actual one. Fortunately, even the belief may not converge to actual profile, the action profile taken by all players converges, and it would converge to the action profile same as the one taken in the complete information game. The reasoning is as follows.

Given that $p_{j-i}(\boldsymbol{\theta}_{p_{-i}})$ are taken by joint KKT method, PS i knows that the optimal demand from SS j of type θ_{s_j} is $b^*_{ji}(\theta_{s_j}, \hat{\theta}_{p_i}, \boldsymbol{\theta}_{p_{-i}})$ by solving (3.10). The optimal pricing $p^{*\text{unc}}_{ji}(\hat{\theta}_{p_i})$ of (3.7) without constraint (3.8) and (3.9) may result in feasible or infeasible demand $b^{*\text{unc}}_{ji}(\theta_{s_j}, \hat{\theta}_{p_i}, \boldsymbol{\theta}_{p_{-i}})$. However, the demand must be feasible. If $p^{*\text{unc}}_{ji}(\hat{\theta}_{p_i})$ makes the i-th demand negative, then by joint KKT condition, $b^*_{ji}(\theta_{s_j}, \hat{\theta}_{p_i}, \boldsymbol{\theta}_{p_{-i}})$ would be fixed to 0, and that would reversely generate new optimal i-th pricing $p^*_{ji}(\hat{\theta}_{p_i})$ by solving the following equations

$$p_{ji}^*(\hat{\theta}_{p_i}) = \frac{\partial E_{\boldsymbol{\theta}_p}[u_{s_j}(\mathbf{b}_j^s|\boldsymbol{\theta})]|_{\mathbf{b}_j^s=\mathbf{b}_j^{s*}(\boldsymbol{\theta})}}{\partial b_{ji}}, \tag{3.36}$$

$$p_{jk}(\theta_{p_k}) = \frac{\partial E_{\boldsymbol{\theta}_p}[u_{s_j}(\mathbf{b}_j^s|\boldsymbol{\theta})]|_{\mathbf{b}_j^s=\mathbf{b}_j^{s*}(\boldsymbol{\theta})}}{\partial b_{jk}}, \forall k \neq i, \tag{3.37}$$

where the i-th term of $\mathbf{b}_j^{s*}(\boldsymbol{\theta})$ is $b_{ji}^*(\theta_{s_j}, \hat{\boldsymbol{\theta}}_{p_i}, \boldsymbol{\theta}_{p_{-i}}) = 0$. Note that since $b_{ji}^*(\theta_{s_j}, \hat{\boldsymbol{\theta}}_{p_i}, \boldsymbol{\theta}_{p_{-i}}) = 0$, Solving (3.37) obtains b_{j-i}^* for given $p_{j-i}(\boldsymbol{\theta}_{p_{-i}})$, which means that b_{j-i}^* is irrelevant to $p_{ji}^*(\hat{\theta}_{p_i})$ if $p_{ji}^{*\text{unc}}(\hat{\theta}_{p_i})$ gives negative demand. Then, since $b_{ji}^*(\boldsymbol{\theta}_s, \hat{\theta}_{p_i}, \boldsymbol{\theta}_{p_{-i}}) = 0$, b_{j-i}^* is determined by $p_{j-i}(\boldsymbol{\theta}_{p_{-i}})$ solely, and $p_{ji}^*(\hat{\theta}_{p_i})$ is determined by b_{j-i}^* completely. The relation between the newly generated optimal pricing $p_{ji}^*(\hat{\theta}_{p_i})$ and type $\hat{\theta}_{p_i}$ lies on $p_{ji}^{*\text{unc}}(\hat{\theta}_{p_i})$. If $p_{ji}^{*\text{unc}}(\hat{\theta}_{p_i})$ results in feasible $b_{ji}^{*\text{unc}}(\theta_{s_j}, \hat{\theta}_{p_i}, \boldsymbol{\theta}_{p_{-i}})$, then $p_{ji}^*(\hat{\theta}_{p_i}) = p_{ji}^{*\text{unc}}(\hat{\theta}_{p_i})$, which depends on $\hat{\theta}_{p_i}$. If $p_{ji}^{*\text{unc}}(\hat{\theta}_{p_i})$ results in negative $b_{ji}^{*\text{unc}}(\theta_{s_j}, \hat{\theta}_{p_i}, \boldsymbol{\theta}_{p_{-i}})$, then $p_{ji}^*(\hat{\theta}_{p_i})$ is determined by $p_{j-i}(\boldsymbol{\theta}_{p_{-i}})$ completely, which is independent of θ_{p_i}. Here, we define $\Theta_{j,i,neg} \equiv \{\theta_{p_i}$ except $\hat{\theta}_{p_i}|p_{ji}^{*\text{unc}}(\theta_{p_i})$ results in negative demand$\}$ to proceed the discussion. For those $\theta_{p_i} \in \Theta_{j,i,neg}$, the i-th demand $b_{ji}^*(\theta_{s_j}, \theta_{p_i}, \boldsymbol{\theta}_{p_{-i}}) = 0$ by joint KKT and $p_{ji}^*(\theta_{p_i})$ will also be constrained as (3.36). Following the same reasoning, $p_{j-i}(\boldsymbol{\theta}_{p_{-i}})$ determines $p_{ji}^*(\theta_{p_i})$ completely, and the constrained pricing $p_{ji}^*(\theta_{p_i})$ is independent of θ_{p_i}. Therefore, if $p_{ji}^{*\text{unc}}(\hat{\theta}_{p_i})$ results in negative demand, then $p_{ji}^*(\hat{\theta}_{p_i})$ is the same as $p_{ji}^*(\theta_{p_i})$ for $\theta_{p_i} \in \Theta_{j,i,neg}$ given the same $p_{j-i}(\boldsymbol{\theta}_{p_{-i}})$ (hence for the same $\boldsymbol{\theta}_{-i}$). Clearly, if $\Theta_{j,i,neg}$ is nonempty, then PS i's opponents couldn't tell what the actual type PS i is since the best strategy for those type are the same, but we should note that the best strategy still corresponds to the actual type.

It's similar to apply the reasoning for the case that the demand more than W_i, then by joint KKT condition, $b_{ji}^*(\theta_{s_j}, \hat{\theta}_{p_i}, \boldsymbol{\theta}_{p_{-i}})$ would be fixed to W_i, and that would reversely generate new optimal i-th pricing $p_{ji}^*(\hat{\theta}_{p_i})$ by (3.36) with the i-th term of $b^*(\boldsymbol{\theta})$ is $b_{ji}^*(\theta_{s_j}, \hat{\theta}_{p_i}, \boldsymbol{\theta}_{p_{-i}}) = W_i$.

To sum up, although the belief may not converge to the actual type, the actions always converge to the actual value.

Chapter 4

Simulations

4.1 Simulation Setup

The explicit model developed in Section 3.4 is adopted for simulation. In the first section, we show the effectiveness of the proposed joint KKT method for several cases and compare it with other existing work. In the second section, we examine the players' actions and the belief about players' type as time evolves and numerically analyze the result.

The type space of PS's is set to be $\Theta_P = \{10, 11, 12\}$, and the type space of SS's is set as $\Theta_S = \{1, 2, 3\}$. The initial beliefs are assumed uniformly distributed over the type space, $\mu(\theta_{p_i}|h^0) = \frac{1}{3}$ for all p_i and $\mu(\theta_{s_j}|h^0) = \frac{1}{3}$ for all s_j. The constants in the PS's utility are chosen as $c_1 = 2$ and $c_2 = 2$, and the spectrum substitutability ξ_j is 0.4 for all s_j. Note that some parameters may change depending on different simulation scenarios, and the remaining parameters will be specified in each simulation scenario.

4.2 Numerical Results

4.2.1 Effectiveness of The Joint KKT Conditions

In the section, we simulate the multistage game with complete information, *i.e.* $\mu_x(\widehat{\theta}_y) = 1$ for all x, y, and compare the results of the proposed joint KKT conditions with those in [8] that corresponds to the unconstrained (unc) spectrum sharing to observe the effectiveness with effectiveness of joint KKT conditions.

2 PS vs. 1 SS

First, we simulate the game with 2 PS and 1 SS with complete information, and compare the results of the proposed joint KKT conditions with those in [8] that correspond to unconstrained (unc) game. Since there's only one SS, b_i denotes b_{1i} for simplicity, and p_i denotes p_{1i}. Note that the constraint for b_i in (3.8) is denoted here as $f_{i,1}$, and that in (3.9) is denoted here as $f_{i,2}$. We show the best responses (BR), Nash equilibrium (NE) and the corresponding feasible regions in both Fig. 4.1 and Fig. 4.3. The intersection of the best responses is the NE which is the result of sequential rationality when the information is complete.

In Fig. 4.1, with parameters $W_1 = 15$ MHz and $W_2 = 15$ MHz, $\mathrm{B}_1^{\mathrm{req}} = 2$ Mbps and $\mathrm{B}_2^{\mathrm{req}} = 2$ Mbps, $\hat{\theta}_{p_1} = 10$ and $\hat{\theta}_{p_2} = 10$, $\hat{\theta}_{s_1} = 1$, and the received SNR's $\gamma_1^p = 15$ dB, $\gamma_2^p = 15$ dB, $\gamma_{11}^s = 22$ dB, and $\gamma_{12}^s = 22$ dB, the unc solution satisfies the bandwidth constraints, so it agrees with the solution of the proposed joint KKT conditions. Fig. 4.2(a) shows the profit function of PS1 given PS2 acting equilibrium strategy obtained by solving joint KKT condition and SS taking best demand. Fig. 4.2(b) shows the profit function of PS2 on similar condition. In this case, we observe that the feasible region on each PS's profit function cover the unconstrained best response point. Fig. 4.2(c) shows the contour plot of the profit of SS given that PS1 and PS2 act equilibrium strategy obtained by solving joint KKT condition, and it shows that SS's highest profit lies in strictly feasible region.

In Fig. 4.3, with parameters $W_1 = 5$ MHz, $W_2 = 5$ MHz, $\mathrm{B}_1^{\mathrm{req}} = 2$ Mbps and $\mathrm{B}_2^{\mathrm{req}} = 2$ Mbps, $\hat{\theta}_{p_1} = 10$ and $\hat{\theta}_{p_2} = 10$, $\hat{\theta}_{s_1} = 1$, and the received SNR's $\gamma_1^p = 15$ dB, $\gamma_2^p = 15$ dB, $\gamma_{11}^s = 22$ dB, $\gamma_{12}^s = 10$ dB, the unc solution lies outside the bandwidth constraints, while the optimal strategies $b_1^* = 0$ and $b_2^* = 0$ of the joint KKT conditions satisfy the constraint. Fig. 4.4(a) shows the profit function of PS1 given that PS2 acting equilibrium strategy obtained by solving joint KKT condition and SS taking best demand. Fig. 4.4(b) shows the profit function of PS2 on similar condition. Fig. 4.4(c) shows the contour plot of the profit of SS given that PS1 and PS2 act equilibrium strategy obtained

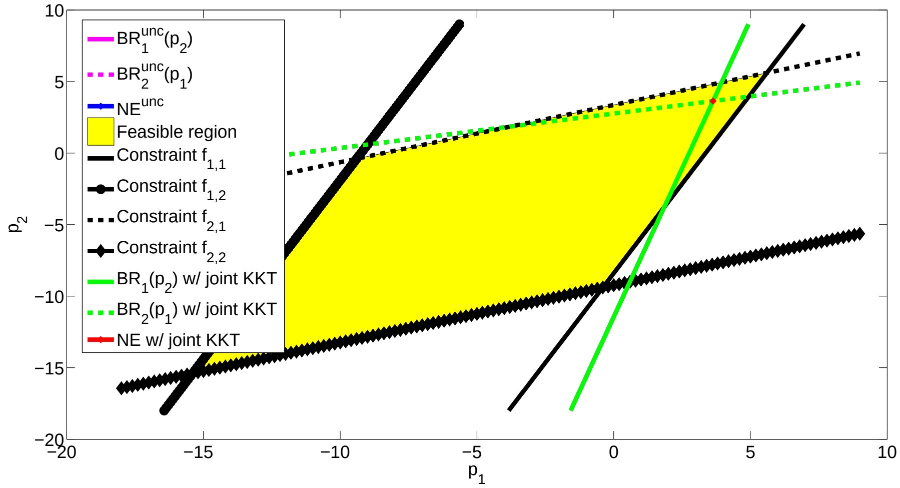


Figure 4.1: The best response, Nash equilirium and feasible region for PS's with $W_1 = 15$ MHz and $W_2 = 15$ MHz, $\gamma_1^p = 15$ dB, $\gamma_2^p = 15$ dB, $\gamma_{11}^s = 22$ dB, and $\gamma_{12}^s = 22$ dB.

by solving joint KKT condition. In this case, we observe that the feasible region on each PS's profit function is exactly one point which is also the mutual best response point. Correspondingly on Fig. 4.4(c), SS's profit is highest when $b_1 = 0$, $b_2 = 0$.

2 PS vs. 3 SS and 2 PS vs. 4 SS

Secondly, we look the case with 2 PS's and 3 SS's, and the parameters are $\hat{\theta}_{p_1} = 10$, $\hat{\theta}_{p_2} = 10$, $\hat{\theta}_{s_1} = \hat{\theta}_{s_2} = \hat{\theta}_{s3} = 1$, $\gamma_1^p = \gamma_2^p = 15$, $\gamma_{ji}^s = 22$ for all p_i, s_j, $\mathrm{B}_1^{\mathrm{req}} = 0.5$ Mbps and $\mathrm{B}_2^{\mathrm{req}} = 0.5$ Mbps, $W_1 = W_2 = 6$ MHz, and $\xi_1 = \xi_2 = \xi_3 = 0$. Due to the difficulty of drawing picture with a dimension more than 3, the actions evolving with time are plotted instead of the feasible region and best response curves. Basically, since all PS's have the same parameters, they would ask same price to each SS, and all SS's would ask same demand to each PS. Under these parameters, as shown in Fig. 4.5(a) and 4.5(b), the demand and unconstrained demand are the same for the case of 3 SS's, and also the sum of those demand are affordable for each PS, which is strictly inside the feasible region.

The case of 2 PS's and 4 SS's is also shown in Fig. 4.5(a) and 4.5(b), with the parameters $\hat{\theta}_{p_1} = 10$, $\hat{\theta}_{p_2} = 10$, $\hat{\theta}_{s_1} = \hat{\theta}_{s_2} = \hat{\theta}_{s3} = \hat{\theta}_{s4} = 1$, $\gamma_1^p = \gamma_2^p = 15$, $\gamma_{ji}^s = 22$ for all p_i, s_j,

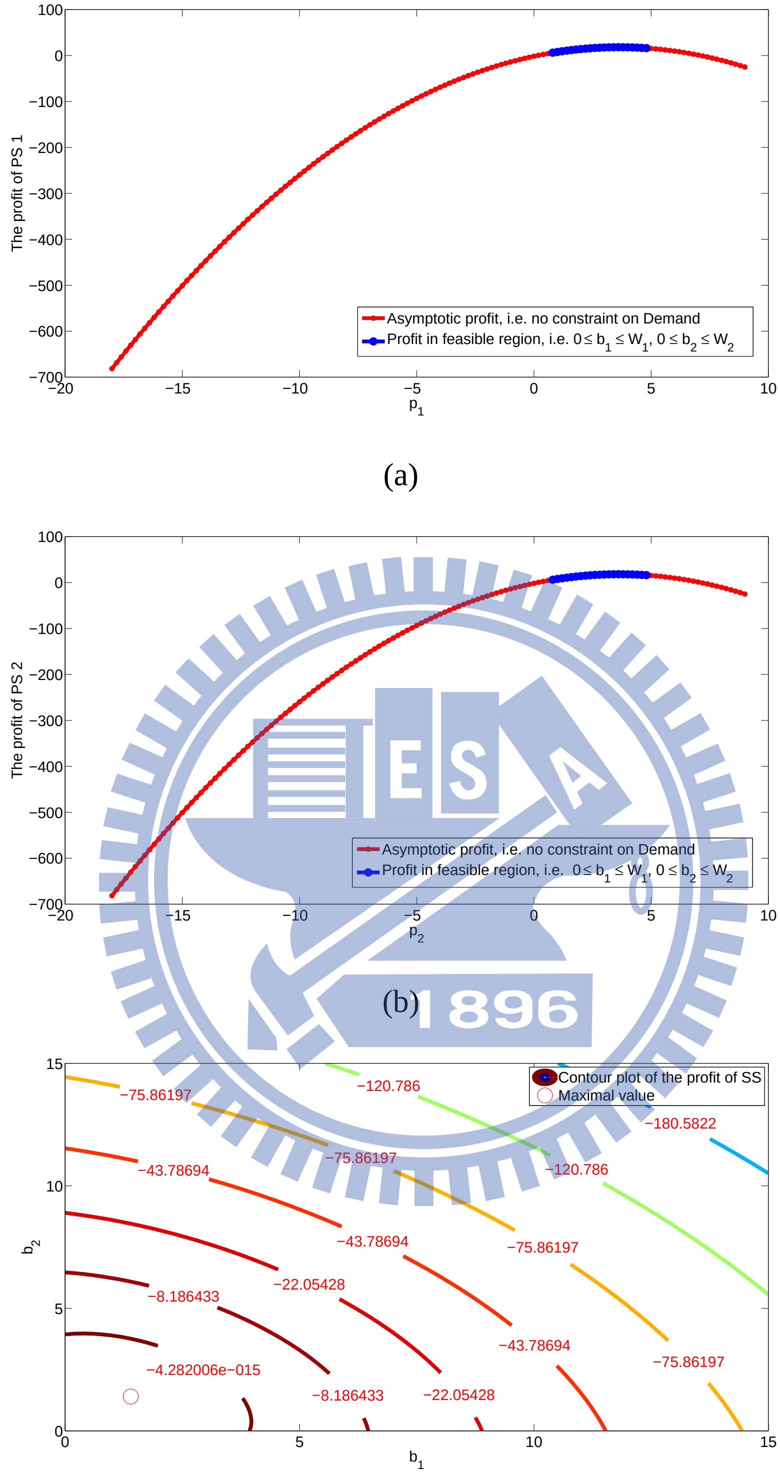


Figure 4.2: Profit function given that the opponents act equilibrium strategy with $W_1 = 15$ MHz and $W_2 = 15$ MHz, $\gamma_1^p = 15$ dB, $\gamma_2^p = 15$ dB, $\gamma_{11}^s = 22$ dB, and $\gamma_{12}^s = 22$ dB. (a) Of PS1. (b) Of PS2. (c) Of SS.

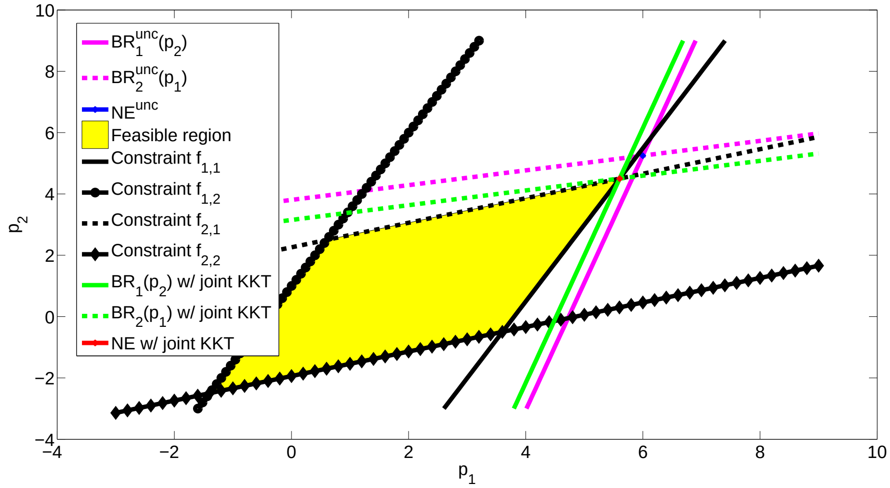
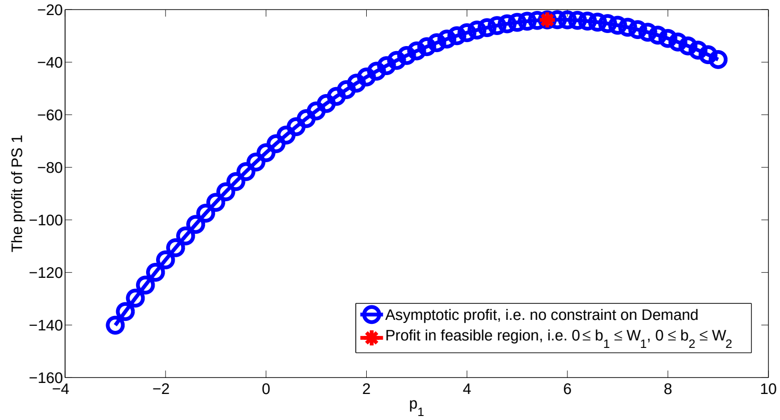


Figure 4.3: The best response, Nash equilirium and feasible region for PS's with with $W_1 = 5$ MHz and $W_2 = 5$ MHz, $\gamma_1^p = 15$ dB, $\gamma_2^p = 15$ dB, $\gamma_{11}^s = 22$ dB, $\gamma_{12}^s = 10$ dB.

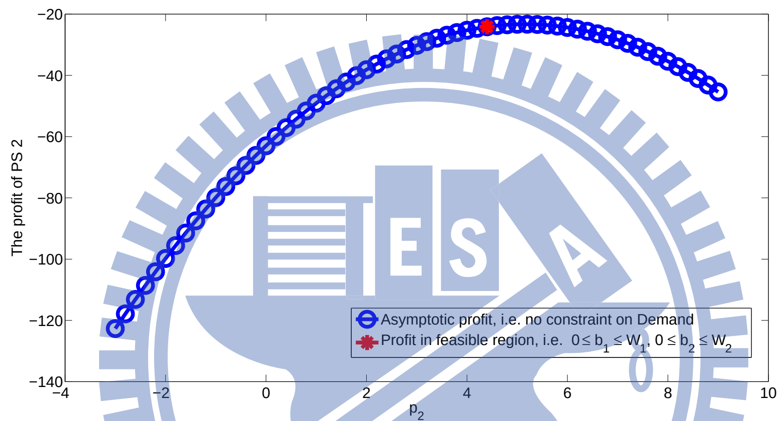
$\mathrm{B}_1^{\mathrm{req}} = 0.5$ Mbps and $\mathrm{B}_2^{\mathrm{req}} = 0.5$ Mbps, $W_1 = W_2 = 6$ MHz, and $\xi_1 = \xi_2 = \xi_3 = \xi_4 = 0$. Since the sum of the unconstrained demands exceeds the bandwidth available for each PS, each PS would ask a price such that the demand obtained by joint KKT method shrinks to meet bandwidth requirement. While some might wonder that whether the infeasible equilibrium strategy of [8] can be modified into a feasible one by directly setting excessive total demands into affordable demand, the answer is negative. Even if the total demands were set into affordable one, the pricing strategy of [8] aren't PBE strategy and are still different from our result. It is because when the demands meet some boundary conditions, the Lagrange multipliers corresponding to those active conditions start to function. It is the function of the Lagrange multipliers that differentiate the feasible equilibrium from the infeasible one.

Summary of Effectiveness of Joint KKT Conditions

To sum up, sometimes the parameters may intrinsically result in solution strictly inside the feasible region. In that case, both joint KKT method and the method in [8] have the same equilibrium strategy. Another case is, the parameters may result in equilibrium strategies on the boundary of nonnegative constraints. In this situation, joint KKT method gives



(a)



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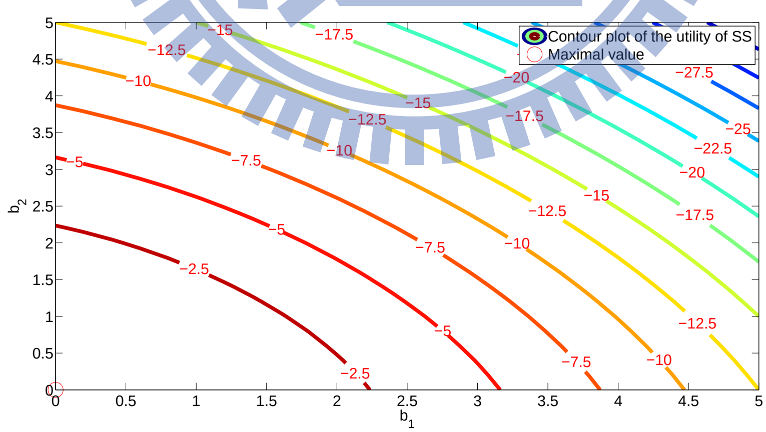
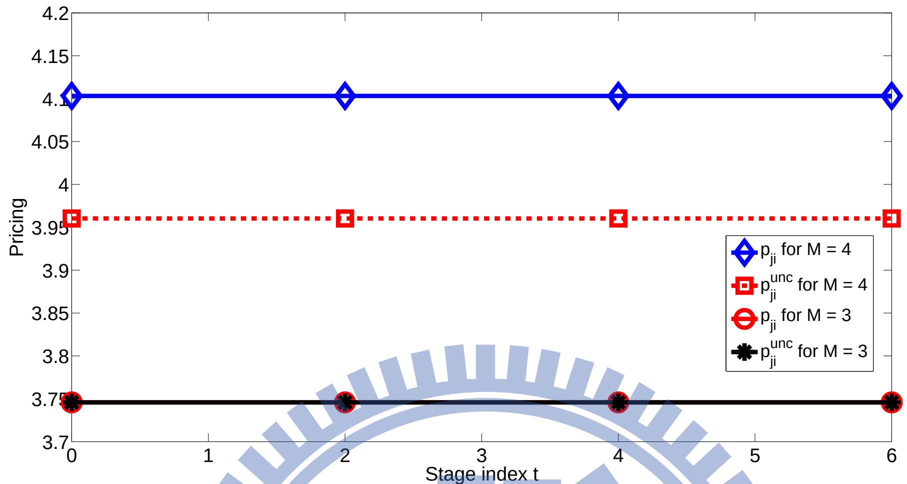
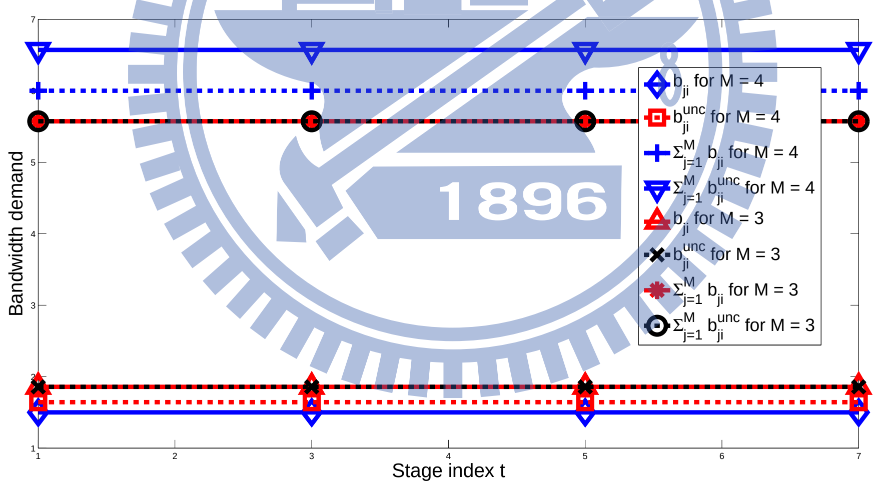


(c)

Figure 4.4: Profit function given that the opponents act equilibrium strategy with $W_1 = 5$ MHz and $W_2 = 5$ MHz, $\gamma_1^p = 15$ dB, $\gamma_2^p = 15$ dB, $\gamma_{11}^s = 22$ dB, $\gamma_{12}^s = 10$ dB. (a) Of PS1. (b) Of PS2. (c) Of SS.



(a)



(b)

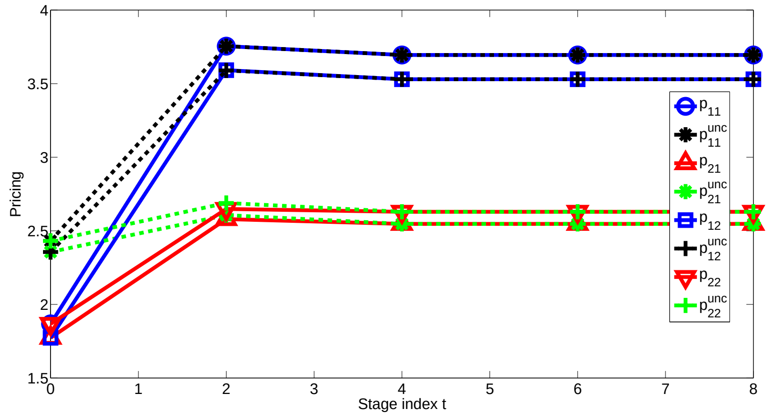
Figure 4.5: The equilibrium strategy over stages, a comparison between the scenario with M=3 and that with M=4. (a) Of PS's. (b) Of SS's.

feasible solution while the method in [8] has negative demand. While some might wonder that whether the infeasible equilibrium strategy of [8] can be modified into a feasible one by directly setting excessive total demands into affordable demand (or setting the negative demands into 0), the answer is negative. It is because when the demands meet some boundary conditions, the Lagrange multipliers corresponding to those active conditions start to function and make the solution feasible. Clearly, joint KKT method guarantees physically practical solution.

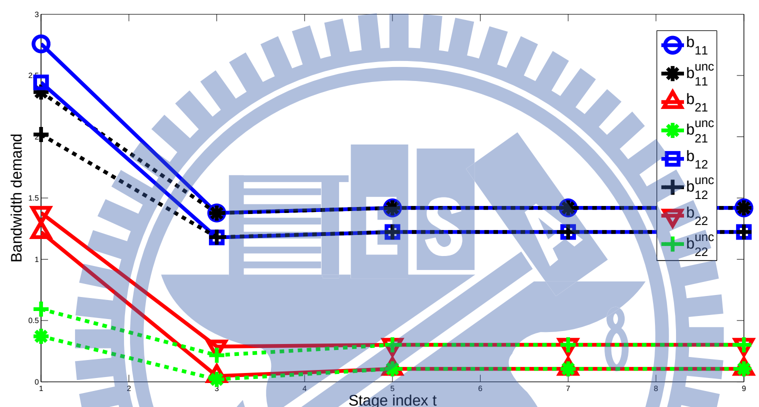
4.2.2 Evolutions of Beliefs and Actions

In this section, we simulate and show the evolution of action profile over stage. The simulation scenarios are classified into three different cases, they're 1. all action of all players profile are strictly feasible, which is shown in Fig. 4.6(a), 4.6(b), 4.6(c); 2. some action profile of some players are on the boundary of bandwidth constraints, which is shown in Fig. 4.7(a), 4.7(b), 4.7(c); 3. all action profile of some player are on the boundary of bandwidth constraints, which is shown in Fig. 4.8(a), 4.8(b), 4.8(c).

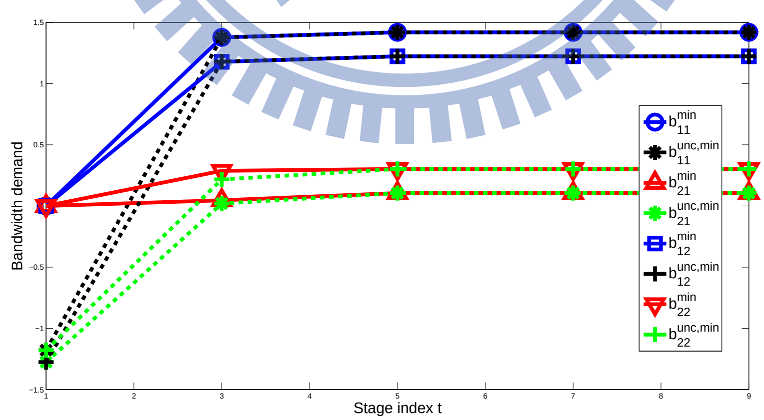
Fig. 4.6(a), 4.7(a) and 4.8(a) show the equilibrium pricing profile of the actual type corresponding to the one with proposed joint KKT condition and one which uses the same belief as the proposed one at each stage instant but without considering the constraint (unc). Fig. 4.6(b), 4.7(b) and 4.8(b) show the equilibrium demand profile of the actual type corresponding to the one with joint KKT condition and one which uses the same belief as the proposed one at each stage instant but without considering the constraint (unc). Fig. 4.6(c), 4.7(c) and 4.8(c) show the possible minimal equilibrium demand profile corresponding to the one with joint KKT condition and one which uses the same belief as the proposed one at each stage instant but without considering the constraint (unc). The belief update over stage about player of the three cases is also presented in TABLE 4.1, 4.2, 4.3, respectively. Numerical analysis about the action and the belief on the three cases are discussed.



(a)



(b)



(c)

Figure 4.6: The equilibrium strategies over stages of Case 1. (a) Of PS's. (b) Of SS's. (c) The possible minimal equilibrium strategies of SS's.

Case 1: When All Actions Are Strictly Inside Constraints

Next, we study the behavior of the sequence of equilibrium strategies as stage evolves under $W_1 = 15$MHz and $W_2 = 15$MHz, $\hat{\theta}_{p_1} = 10$, $\hat{\theta}_{p_2} = 10$, $\hat{\theta}_{s_1} = 1$, $\hat{\theta}_{s_2} = 2$, $\gamma_1^p = 15$ dB, $\gamma_2^p = 15$ dB, $\gamma_{11}^s = 22$, $\gamma_{12}^s = 18$, $\gamma_{21}^s = 18$, $\gamma_{22}^s = 22$, $\mathrm{B}_1^{\mathrm{req}} = 2$ Mbps and $\mathrm{B}_2^{\mathrm{req}} = 2$ Mbps. Fig.4.6(a) and Fig.4.6(b) show the equilibrium pricings and the equilibrium demands. Since $\gamma_{11}^s = 22$ is larger than $\gamma_{12}^s = 15$, SS1 demands more from PS1 than from PS2. Correspondingly, PS1 sets higher price to SS1 than PS2 does. On the other hand, SS2 demands more from PS2 than from PS1 since $\gamma_{22}^s = 22$ is larger than $\gamma_{21}^s = 15$. Therefore, PS2 sets higher price to SS2 than PS1 does. PS1 asks lower price to SS2 than to SS1 since γ_{11}^s is larger than γ_{21}^s, while with that γ_{22}^s is larger than γ_{12}^s, PS2 also asks lower price to SS2 than to SS1 since SS2 is with $\hat{\theta}_s = 2$ and puts less emphasis on the QoS satisfaction, or equivalently, is more concerned with the monetary expense. Therefore, both PS's set lower price to SS2 to stimulate the demand.

We also observe that the difference between utilizing joint KKT condition and without considering the constraints. Although the pricings and demands without considering the constraints evolve into the same value as those considering joint KKT condition after the belief update correctly since the solutions are strictly feasible, they are infeasible at the beginning. It's because the possible minimal demands of unconstrained case are negative in the beginning stage as Fig.4.6(c) shows, while the minimal demands with joint KKT condition are still feasible, being zero in this case.

The Bayesian game model allows the equilibrium strategies to update according to the beliefs (TABLE 4.1) of all players' private information. Since these actions are strictly inside feasible region, then the belief and hence the behavior converges in the end.

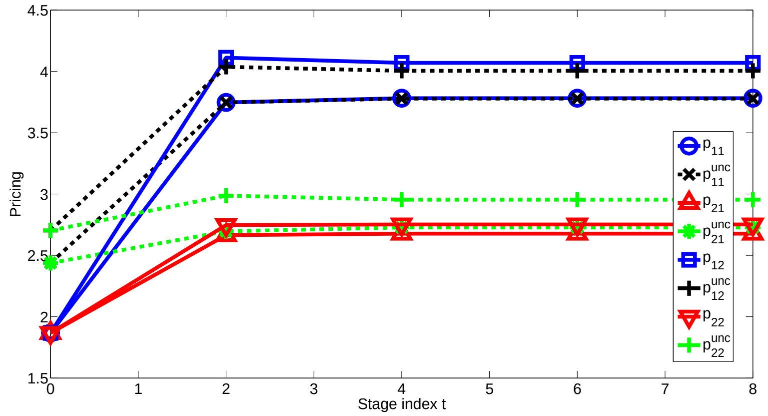
Table 4.1: Belief Updating versus Stage for Case 1

Belief about PS's	Stage t			
	0	1,2	3,4	$t>4$
$\mu(\theta_{p_1}=10\|h^t)$	$\frac{1}{3}$	1	1	1
$\mu(\theta_{p_1}=11\|h^t)$	$\frac{1}{3}$	0	0	0
$\mu(\theta_{p_1}=12\|h^t)$	$\frac{1}{3}$	0	0	0
$\mu(\theta_{p_2}=10\|h^t)$	$\frac{1}{3}$	1	1	1
$\mu(\theta_{p_2}=11\|h^t)$	$\frac{1}{3}$	0	0	0
$\mu(\theta_{p_2}=12\|h^t)$	$\frac{1}{3}$	0	0	0
Belief about SS's	Stage t			
	1	2,3	4,5	$t>5$
$\mu(\theta_{s_1}=1\|h^t)$	$\frac{1}{3}$	1	1	1
$\mu(\theta_{s_1}=2\|h^t)$	$\frac{1}{3}$	0	0	0
$\mu(\theta_{s_1}=3\|h^t)$	$\frac{1}{3}$	0	0	0
$\mu(\theta_{s_2}=1\|h^t)$	$\frac{1}{3}$	0	0	0
$\mu(\theta_{s_2}=2\|h^t)$	$\frac{1}{3}$	1	1	1
$\mu(\theta_{s_2}=3\|h^t)$	$\frac{1}{3}$	0	0	0

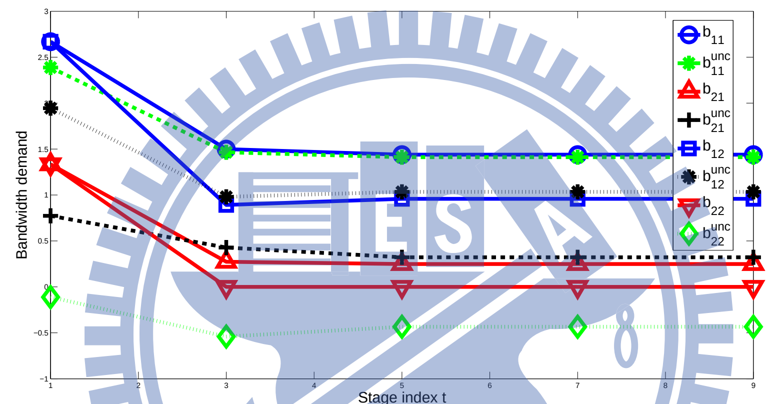
Case 2: When Some Actions of Some Players are on the Boundaries of Constraints

Next, we study the behavior of the sequence of equilibrium strategies as stage evolves under $W_1 = 15$MHz and $W_2 = 15$MHz, $\hat{\theta}_{p_1} = 10$, $\hat{\theta}_{p_2} = 12$, $\hat{\theta}_{s_1} = 1$, $\hat{\theta}_{s_2} = 2$, $\gamma_1^p = 15$ dB, $\gamma_2^p = 15$ dB, $\gamma_{11}^s = 22$ dB, $\gamma_{12}^s = 22$ dB, $\gamma_{21}^s = 22$ dB, $\gamma_{22}^s = 22$ dB, $\mathrm{B}_1^{\mathrm{req}} = 2$ Mbps and $\mathrm{B}_2^{\mathrm{req}} = 2$ Mbps.

Fig. 4.7(a) and Fig. 4.7(b) show that the equilibrium pricings and the equilibrium demands. Each PS asks lower price to SS2 than to SS1 since SS2 is with $\hat{\theta}_s = 2$ and so puts less emphasis on the QoS satisfaction, or equivalently, is more concerned with the monetary expense. The penalty, or the cost, of sharing spectrum for PS2 is higher than that for PS1 since PS2 with a higher volume of local connections is more reluctant to share the spectrum, in order to fulfill its primary users' QoS satisfaction. Consequently, PS2 would set a higher price, that yields lower b_{12} and b_{22}. Under this circumstance, SS1



(a)



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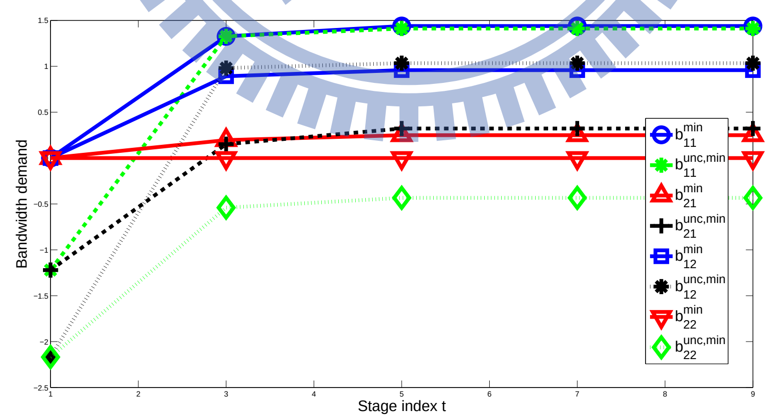


(c)

Figure 4.7: The equilibrium strategies over stages of Case 2. (a) Of PS's. (b) Of SS's. (c) The possible minimal equilibrium strategies of SS's.

and SS2 demands more b_{11} and b_{21} respectively to compensate the insufficiency of b_{12} and b_{22}.

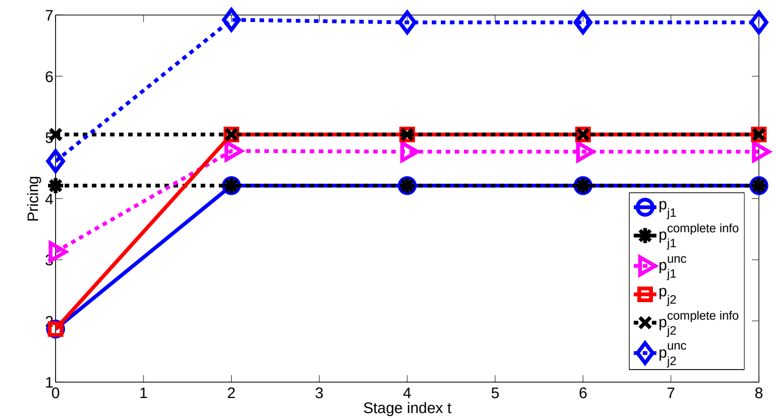
Under these parameter settings, the unconstrained demand b_{22}^{unc} would be negative even when the belief is updated to the correct one, while the proposed one is always feasible. Although b_{22} is 0, which is on the boundary of the nonnegative constraint of PS2, the belief about PS2 still converges to the actual one. It is because b_{12} still isn't on the boundary, the opponents could still update the belief about PS2. Since the beliefs (TABLE 4.2) could converge, the action profiles converge.

Table 4.2: Belief Updating versus Stage for Case 2

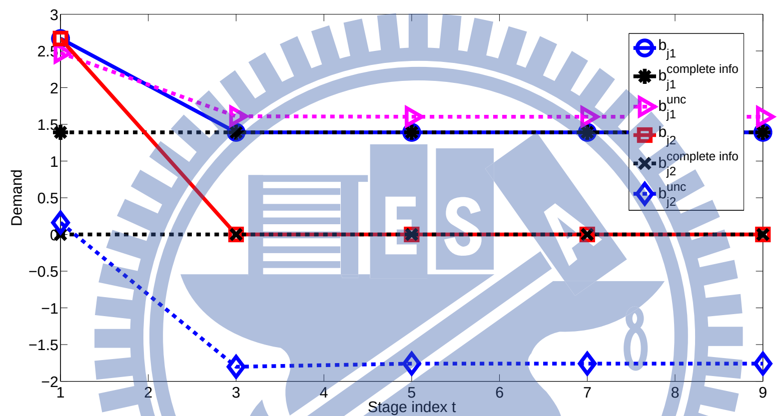
Belief about PS's	Stage t			
	0	1,2	3,4	$t > 4$
$\mu(\theta_{p_1} = 10\|h^t)$	$\frac{1}{3}$	1	1	1
$\mu(\theta_{p_1} = 11\|h^t)$	$\frac{1}{3}$	0	0	0
$\mu(\theta_{p_1} = 12\|h^t)$	$\frac{1}{3}$	0	0	0
$\mu(\theta_{p_2} = 10\|h^t)$	$\frac{1}{3}$	0	0	0
$\mu(\theta_{p_2} = 11\|h^t)$	$\frac{1}{3}$	0	0	0
$\mu(\theta_{p_2} = 12\|h^t)$	$\frac{1}{3}$	1	1	1
Belief about SS's	Stage t			
	1	2,3	4,5	$t > 5$
$\mu(\theta_{s_1} = 1\|h^t)$	$\frac{1}{3}$	1	1	1
$\mu(\theta_{s_1} = 2\|h^t)$	$\frac{1}{3}$	0	0	0
$\mu(\theta_{s_1} = 3\|h^t)$	$\frac{1}{3}$	0	0	0
$\mu(\theta_{s_2} = 1\|h^t)$	$\frac{1}{3}$	0	0	0
$\mu(\theta_{s_2} = 2\|h^t)$	$\frac{1}{3}$	1	1	1
$\mu(\theta_{s_2} = 3\|h^t)$	$\frac{1}{3}$	0	0	0

Case 3: When All Actions of Some player are on the Boundaries of Constraints

Next, we study the behavior of the sequence of equilibrium strategies as stage evolves under $W_1 = 15$MHz and $W_2 = 15$MHz, $\hat{\theta}_{p_1} = 10$, $\hat{\theta}_{p_2} = 10$, $\hat{\theta}_{s_1} = 1$, $\hat{\theta}_{s_2} = 1$, $\gamma_1^p = 15$



(a)



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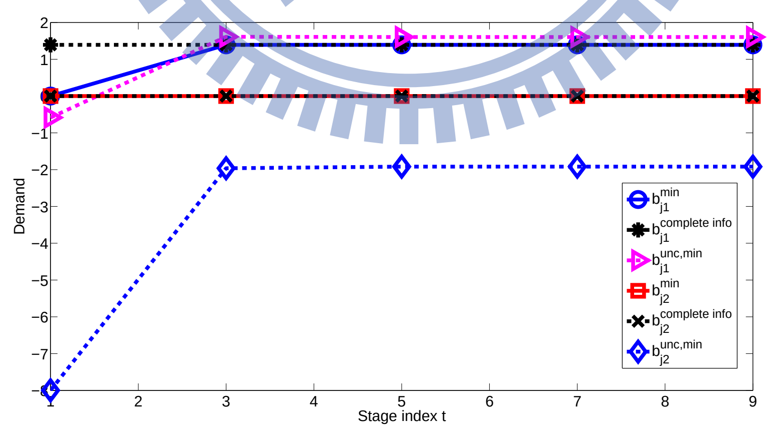

(c)

Figure 4.8: The equilibrium strategies over stages of Case 3. (a) Of PS's. (b) Of SS's. (c) The possible minimal equilibrium strategies of SS's.

dB, $\gamma_2^p = 15$ dB, $\gamma_{11}^s = 22$, $\gamma_{12}^s = 22$, $\gamma_{21}^s = 22$, $\gamma_{22}^s = 22$, $\mathrm{B}_1^{\mathrm{req}} = 2$ Mbps and $\mathrm{B}_2^{\mathrm{req}} = 4$ Mbps. Basically, the parameters of SS1 and SS2 are the same, so PS1 would set the same price to both SS's, and so does PS2. Likewise, both SS's would demand the same size of bandwidth from the same PS. Therefore, we let b_{ji} denote b_{1i} and b_{2i} and p_{ji} denote p_{1i} and p_{2i}.

As Fig. 4.8(a) and Fig. 4.8(b) show, since PS2 has higher bandwidth requirement for local connection, it asks high price to both SS's than PS1 does, which makes the demands from both SS's be 0. The action of PS2 makes the opponents difficult to update the belief about PS2 (TABLE 4.3), but the actions of PS2 converge to those of the actual type of PS2 which is shown in the curves of complete information scenario. It justifies that even thought the belief cannot converge to the actual one, the action profile still converges to the one with correct belief, *i.e.* complete information, and thus it doesn't influence the result of the game.

Table 4.3: Belief Updating versus Stage for Case 3

Belief about PS's	Stage t			
	0	2	4	$t > 4$
$\mu(\theta_{p_1} = 10 \mid h^t)$	$\frac{1}{3}$	1	1	1
$\mu(\theta_{p_1} = 11 \mid h^t)$	$\frac{1}{3}$	0	0	0
$\mu(\theta_{p_1} = 12 \mid h^t)$	$\frac{1}{3}$	0	0	0
$\mu(\theta_{p_2} = 10 \mid h^t)$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
$\mu(\theta_{p_2} = 11 \mid h^t)$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
$\mu(\theta_{p_2} = 12 \mid h^t)$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
Belief about PS's	Stage t			
	1	2,3	4,5	$t > 5$
$\mu(\theta_{s_1} = 1 \mid h^t)$	$\frac{1}{3}$	1	1	1
$\mu(\theta_{s_1} = 2 \mid h^t)$	$\frac{1}{3}$	0	0	0
$\mu(\theta_{s_1} = 3 \mid h^t)$	$\frac{1}{3}$	0	0	0
$\mu(\theta_{s_2} = 1 \mid h^t)$	$\frac{1}{3}$	1	1	1
$\mu(\theta_{s_2} = 2 \mid h^t)$	$\frac{1}{3}$	0	0	0
$\mu(\theta_{s_2} = 3 \mid h^t)$	$\frac{1}{3}$	0	0	0

Chapter 5

Conclusion and Future Work

5.1 Conclusion

We've studied the spectrum trading game with incomplete information in a sequential manner for a cognitive radio network. The incomplete information game is modeled as Bayesian game, which the incomplete information is viewed in Bayesian way. To ensure that the trading is physically practical, we constrain the trading bandwidth. To solve the optimization problem with bandwidth constraints in the multistage game, we've proposed using the KKT translation and joint KKT conditions to yield the perfect Bayesian equilibrium at each stage. We've demonstrated that the KKT translation technique provides a general rule that can be applied to optimization problems of multistage game theory. An algorithm for solving joint KKT condition is given, and the complexity of the algorithm is analyzed. In addition, we've studied the convergence behaviors of belief and action profiles, although belief profiles may not converge to the actual type, the action profiles converge to actual optimal strategy, which means the result is the same as that of complete information. In the simulations, we've justified the effectiveness of joint KKT condition, numerically study the convergence of belief and action profiles, and also how the parameters influences the action. Finally, we've concluded that the proposed multistage Baysian game model with bandwidth constraints is robust and capable of providing more reasonable strategy profiles for players.

5.2 Future Work

In this thesis, although we've derived the solution conditions, proposed an algorithm to solve the involved optimization problem for the constrained game, the efficiency and fairness of the game have not been analyzed. In game theory, there're several criteria for efficiency, for example, it can be the maximization of the summation of all players' utility or Pareto efficiency [16]. As for fairness, proportional fairness criterion is often utilized for resource allocation in wireless network. If the efficiency is not guaranteed, then we may look for other possible strategies in terms of repeated game to induce cooperative behaviors among PS's and among SS's for enhancing the efficiency. Also, the existence of the solution PBE for the considered game has not been proven yet.

In a practical system, channel conditions of each player are also unknown to others, or at least difficult to be obtained by others. The system model could be modified by considering the unknown and random nature of the channel. To account for that, future work could include concepts from stochastic game whose main feature is game with state. By defining the state as levels of channel conditions, we can build a more realistic game without requiring all players to know the exact channel state information. More specifically, we can apply finite state Markov channel proposed by [20] to divide the range of received SNR in spectral efficiency into finite discrete sets. It's worthy to note that the work in [20] has detail about how to partition SNR and what the corresponding transition probability is.

Another aspect for future work is to consider learning mechanisms in the constrained game. In this work, all solutions are obtained based on the assumption that all actions are observable and observed noiselessly by all players. Although the assumption is practically achievable, it would be more appreciated that the game can still attain the equilibrium without the assumption. Thus, we think it would be a good direction of future work to search learning algorithm guaranteeing that the constrained game converges to the equilibrium with as less information needed as possible. It should be noted that the difficulty lies mainly in "constraints." The work in [8] and [10] both propose learning algorithm in a game but fail to take constraints into account.

If both stochastic game and learning algorithm are combined, we think that learning using a hidden Markov model could be an interesting approach.

Bibliography

[1] Federal Communications Commission Spectrum Policy Task Force, "Report of the Spectrum Efficiency Working Group," Nov. 2002.

[2] M. C. Vuran I. F. Akyildiz, W. Y. Lee and S. Mohanty, "A Survey on Spectrum Management in Cognitive Radio Networks," *IEEE Communications Magazine*.

[3] S. Haykin, "Cognitive radio: brain-empowered wireless communications," *IEEE J. Select. Areas Commun.*, vol. 23, no. 2, pp. 201–220, Feb. 2005.

[4] J. Mitola, "Cognitive Radio: An Integrated Agent Architecture for Soft-ware Defined Radio," *Doctor of Technology, Royal Inst. Technol. (KTH), Stockholm, Sweden*, 2000.

[5] D. P. Palomar and M. Chiang, "A Tutorial on Decomposition Methods for Network Utility Maximization," *IEEE J. Select. Areas Commun.*, vol. 24, no. 8, pp. 1439–1451, Aug. 2006.

[6] Z. Ji and K. J. Ray Liu, "Dynamic spectrum sharing: a game theoretic overview," *IEEE Communications Magazine*, pp. 88–94, May 2007.

[7] Z. Ji and K. J. Ray Liu, "Multi-stage pricing game for collusion-resistant dynamic spectrum allocation," *IEEE J. Select. Areas Commun.*, vol. 26, no. 1, pp. 182–191, Jan. 2008.

[8] D. Niyato and E. Hossain, "Competitive pricing for spectrum sharing in cognitive radio networks: dynamic game, inefficiency of nash equilibrium, and collusion," *IEEE J. Select. Areas Commun.*, vol. 26, no. 1, pp. 192–202, Jan. 2008.

[9] D. Niyato and E. Hossain, “Competitive spectrum sharing in cognitive radio networks: a dynamic game approach,” *IEEE Trans. Wireless Commun.*, vol. 7, no. 7, pp. 2651–2660, July 2008.

[10] D. Niyato and E. Hossain, “Market-equilibrium, competitive, and cooperative pricing for spectrum sharing in cognitive radio networks: analysis and comparison,” *IEEE Trans. Wireless Commun.*, vol. 7, no. 11, pp. 4273–4283, 2008.

[11] D. Niyato and E. Hossain, “Dynamics of multiple-seller and multiple-buyer spectrum trading in cognitive radio networks: a game-theoretic modeling approach,” *IEEE Trans. Mobile Computing*, vol. 8, no. 8, pp. 1009–1022, 2009.

[12] C. Y. Lee and F. T. Chien, “Bandwidth-constrained multistage bayesian game for spectrum trading in cognitive radio networks,” in *Proc. IEEE PIMRC*, Sep. 2009.

[13] M. van der Schaar and F. Fu, “Spectrum access games and strategic learning in cognitive radio networks for delay-Critical applications,” *Proceeding of the IEEE*, vol. 97, no. 4, pp. 720–740, Apr. 2009.

[14] J. H. Reed A. B. MacKenzie and *et al*, “Cognitive Radio and Networking Research at Virginia Tech,” *Proceedings of the IEEE*, vol. 97, no. 4, pp. 660–688, Apr. 2009.

[15] M. Felegyhazi and J.-P. Hubaux, “Game Theory in Wireless Networks: A Tutorial,” .

[16] D. Fudenberg and J. Tirole, *Game Theory*, The MIT Press, 1991.

[17] S. Boyd and L. Vandenberghe, *Convex Optimization*, CAMBRIDGE UNIVERSITY PRESS, 2004.

[18] A. J. Goldsmith and S. G. Chua, “Variable rate variable power MQAM for fading channels,” *IEEE Trans. Commun.*, vol. 45, no. 10, pp. 1218–1230, Oct. 1997.

[19] J. Nocedal; S. J. Wright, *Numerical Optimization*, Springer, 2006.

[20] Q. Zhang and S. A. Kassam, “Finite-State Markov Model for Rayleigh Fading Channels,” *IEEE Trans. Commun.*, vol. 47, no. 11, pp. 1688–1692, Nov. 1999.

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