

A Novel Algorithm for Fast Codebook Search

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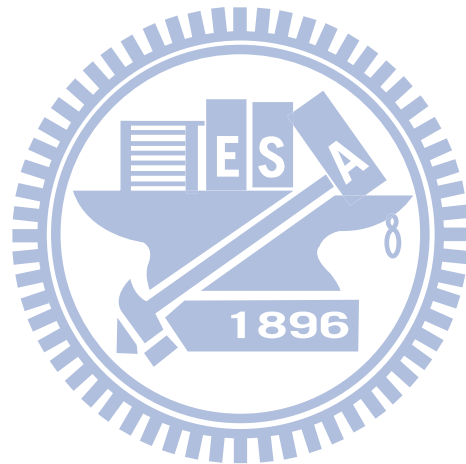
Abstract

In this thesis, we propose an algorithm to reduce the complexity to search the most suitable codeword for a given codebook. It is proven in the thesis that about half codewords are eliminated in each iteration. In addition, we derive two lower bounds for the proposed algorithm and show that they reach the actual SNR loss in high resolution codebook. Furthermore, the complexity analysis and simulations are given to see that the advantages of taking this algorithm are revealed in the scenario of large codebook size.

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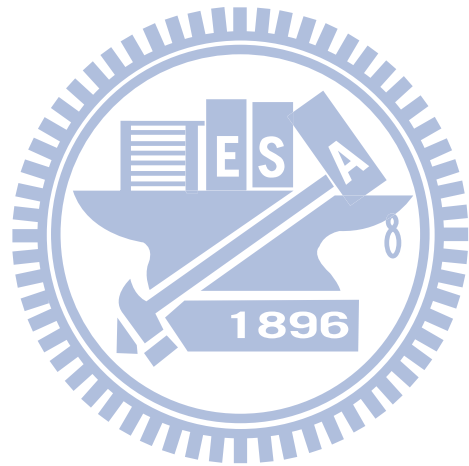
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Chapter 1

Introduction

1.1 Motivation and goal

Multiple-input and multiple-output (MIMO) system is a promising technology using multiple transmit and receive antennas to improve the performance of communication systems. The concept is first came up by A. R. Kaye, D. A. George (1970) and W. V Etten (1975, 1976). In 1994, A. Paulraj and T. Kailath issued a patent [12] on spatial multiplexing and G. J. Foschini in 1996 proposed a paper in [5] about a new configuration to improve the link throughput effectively.

There are various performance criteria for MIMO systems. The most popular criteria are SNR and channel capacity. SNR is an electrical measurement defined as the ratio of signal power and noise power. In [7], an upper bound for bit-error rate was offered and it was found that the SNR is related to the bit-error rate with the condition of the fixed signal constellation. Channel capacity is the information rate that can be reliably transmitted over the communication channel. The capacity for additive white Gaussian noise channels was first derived in [14] by C. E. Shannon. In [15], the author showed the computational procedures to derive the capacity over MIMO channels.

Beamforming and combining are general signal processing techniques for signal transmissions and receptions respectively. MIMO systems using beamforming and combining techniques detect the channel information and decide the beamforming vector based on the channel information in the receivers. The contents of

the beamforming vector are sent back to transmitters through a limited feedback channel. After receiving the information, the beamformer modifies the signal emitted from each antenna in transmitters and the information from different branches is combined in receivers. Given the fading channel and the transmit signal, the maximum ratio combining (MRC) [13] can maximize the post-processing SNR at the receivers.

In [9], a beamforming technique called maximum-ratio combination (MRT) was proposed to enhance the SNR value. Together with MRC, systems with MRT can have the maximum channel gain. The technique having beamforming vectors with uniform power in each antenna is called equal gain transmission (EGT). EGT has the advantage that the system does not need precise amplifiers since the power is equally allocated over the beamforming vector. With EGT, since only the phase information is concerned, the amount of feedback information reduces about half of that for MRT. In practical systems, the amount of feedback information is limited. Hence, we need to quantize the beamforming vector before sending it to transmitter. Therefore, the research has been directed in quantized beamforming recently.

The system with quantized beamforming has a given codebook known to both the transmitter and the receiver. The receiver decides the most suitable beamforming vector in the given codebook based on the performance criterion and sends back the index of the chosen vector to the transmitter. The transmitter keeps using the beamforming vector until the channel changes. An analytic method was given in [10] to judge the performance of a codebook.

Grassmannian beamforming is a vector quantized codebook, which is constructed in the Grassmannian space. In [4], the concepts to pack lines and an upper bound of the distance function in the Grassmannian space were introduced. A parameter inspired by [10] to judge the performance of a codebook in Grassmannian space was shown in [8].

Another vector quantization method is random vector quantization (RVQ). Just as the name called, the RVQ codebook is generated from random distribution. In [3], it was proven that the performance loss due to the quantization

vanishes when the number of quantization bits is large.

An EGT codebook consists codeword with uniform power in every element. In [7], it was proven that the EGT can achieve the full diversity. The author in [7] also proposed an algorithm to generate an EGT codebook using the discrete Fourier matrix. Scalar EGT is another EGT beamforming method which constructs the beamforming vector without using the codebook. In scalar quantization, the phase of each antenna is quantized equally. For instance, if the number b , bits per antenna is 2, the element of the codeword is either 0, $\frac{\pi}{2}$, π , and $\frac{3\pi}{2}$.

The optimal codebook is called the Lloyd book. In [11], an algorithm inspired by [6] was proposed to get the optimal EGT codeword by iterations. Inspiring by the general Lloyd's algorithm in [6] and [11], we propose a procedure to generate a Lloyd codebook for the vector quantization. First, we group training vectors into a given number of categories by the performance criterion and then use the characteristic of the group belonging to each codeword to generate the new codeword. By iterating the process numerous times, the proposed Lloyd book is done. The simulation results in Chapter 5 show that the Lloyd codebook is better than any other codebooks that we applied in this thesis.

Several methods can be used to decide the most suitable codeword. The most common way is the exhaustive search. Exhaustive search has to examine all the codewords in the codebook to decide the best codeword. It is apparent that it is not effective when the codebook size is large. Various algorithms were made to simplify the complexity. For EGT, a cyclic algorithm in [18] was proposed to find the optimal codeword quickly. With the condition of scalar EGT, [16] proposed a fast codebook search algorithm inspired by the binary search to get the most suitable codeword. Hierarchy-oriented search method in [2] is a fast codebook search algorithm using the training matrices and the concept of Voronoi region. [1] also proposed another search algorithm using the concept of the eigen space and the simplification of the search criterion to find the codeword quickly. However, although those methods above are great, they may be implemented with difficulties in the MIMO systems as these algorithms are constructed on

the condition of real-value codebook and different criterion. It is a challenge to design an algorithm to be both easily realized and efficient to find the most suitable codeword.

1.2 Contribution and feature

In this thesis, we propose an algorithm to reduce the search complexity to determine the most suitable codeword by adding one more unit called “eliminator”. It is proven that the number of codewords can be reduced about half in each elimination. Moreover, we also give two lower bounds for the SNR loss due to the elimination. We find that the gap between the actual SNR loss and the analytic loss becomes narrower as the codebook size increases. Furthermore, the simulation results are conducted for various number of transmit antennas, codebook sizes, numbers of stages, and kinds of codebooks to examine the effect of using the proposed algorithm. We find that the complexity can be reduced dramatically with acceptable SNR loss.

1.3 Organization

An outline of this thesis is as follows: Chapter 2 describes background including the system model, the performance criteria, and the concepts for beamforming and combining. In Chapter 3, the details of the proposed algorithm are introduced and a simple example is given to demonstrate how the proposed algorithm works. We have the performance analysis in Chapter 4 and the procedures to build the lower bounds together with a simulation results to see the lower bounds achieve the actual result in the situation of large codebook size. In Chapter 5, simulation results show the effects of the variations of some parameters. In Chapter 6, we summarize all the conclusions we declare in the thesis.

1.4 Notations

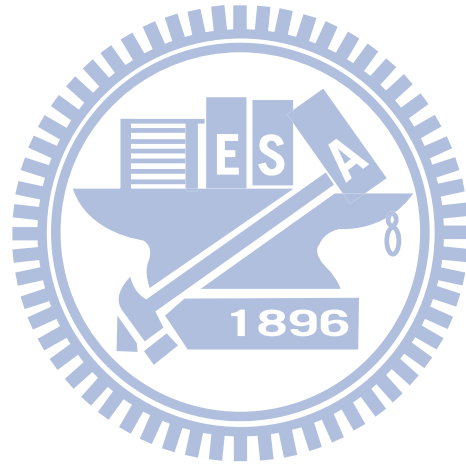
$E_s[a]$: expectation of a

$CN(\mu, \sigma)$: complex Gaussian distribution with mean μ and variance σ

\mathbf{A}^H : Hermitian matrix of \mathbf{A}

\mathbf{A}^t : transpose matrix of \mathbf{A}

$\angle \mathbf{A}$: phase of \mathbf{A} $\mathcal{P}[a]$: the probability that a occurs.



Chapter 2

Background: Beamforming with finite rate feedback

2.1 System model

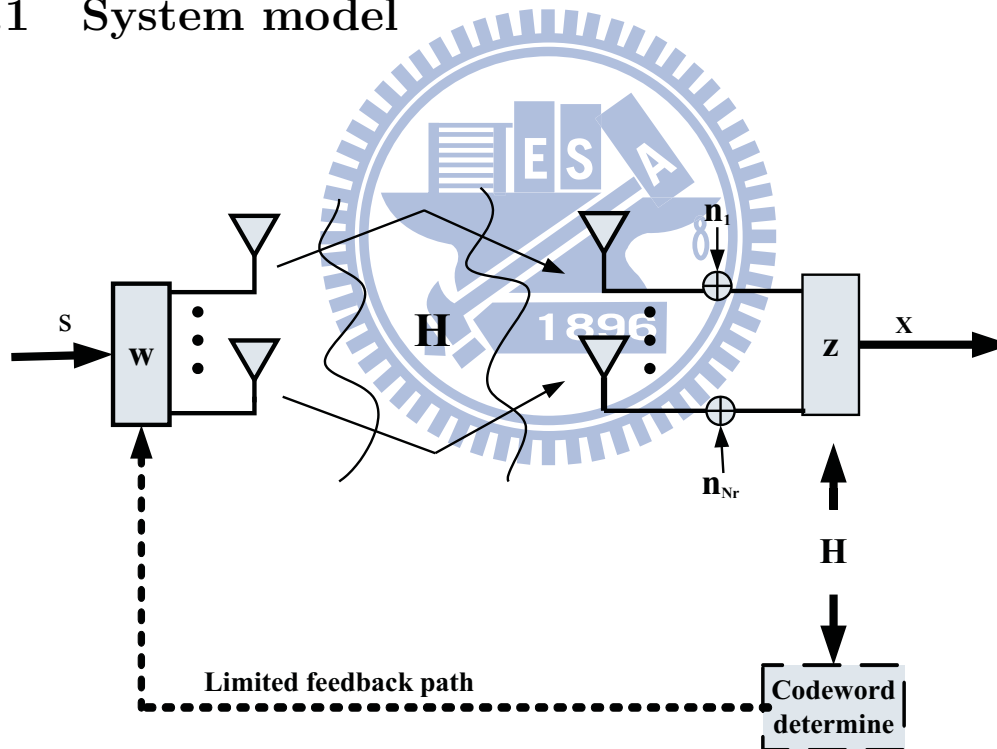


Figure 2.1: MIMO system

A MIMO system using beamforming and combining techniques with N_t transmitter antennas and N_r receiver antennas is illustrated as Fig 2.1. In one signal transmission, a modulated signal s is first attached to a beamforming vector \mathbf{w}

decided by the feedback information and then spread in channel \mathbf{H} by transmission antennas. The receiver combines data from different branches to estimate and detect symbol in the next step. At the same time, the receiver detects the channel information to choose the most suitable beamforming vector. Channel state information will be sent back to transmitter through the feedback channel but usually the amount of feedback information is limited.

Assume that the transmitted symbol s has symbol energy $E_s[|s|^2] = \mathcal{E}_t$. Channel can be modeled as a $N_r \times N_t$ matrix \mathbf{H} given by

$$\mathbf{H} = \begin{bmatrix} h_{11} & h_{12} & h_{13} & \dots & h_{1N_t} \\ h_{21} & h_{22} & h_{23} & \dots & h_{2N_t} \\ \vdots & \vdots & \ddots & \dots & \vdots \\ h_{N_r,1} & h_{N_r,2} & h_{N_r,3} & \dots & h_{N_r,N_t} \end{bmatrix} \quad (2.1)$$

and input/output relationship is given by

$$x = \mathbf{z}^H \mathbf{H} \mathbf{w} s + \mathbf{z}^H \mathbf{n} \quad (2.2)$$

The elements h_{ij} of \mathbf{H} is identical, independent, distributed according to $CN(0, 1)$. The complex vectors $\mathbf{z} = [z_1 \ z_2 \ \dots \ z_{N_r}]^t$ and $\mathbf{w} = [w_1 \ w_2 \ \dots \ w_{N_t}]^t$ are the combining and beamforming vectors respectively, and both vector norms are confined to unit. $\mathbf{n} = [n_1 \ n_2 \ \dots \ n_{N_r}]^t$ is the noise vector having elements distributed according to $CN(0, 1)$ with noise power $E_s[|n_i|^2] = N_0$, $1 \leq i \leq N_r$.

2.2 Various design criteria for MIMO system

There are various of performance parameters based on the purposes of the desired wireless communication systems. Here we present two terms generally used in most communication systems.

2.2.1 SNR (Signal-to-noise ratio)

In [7], the union bound on the symbol-detection error probability is

$$P_e \leq N_e Q \left(\sqrt{\frac{d_{min}^2 \gamma_r}{2}} \right) \quad (2.3)$$

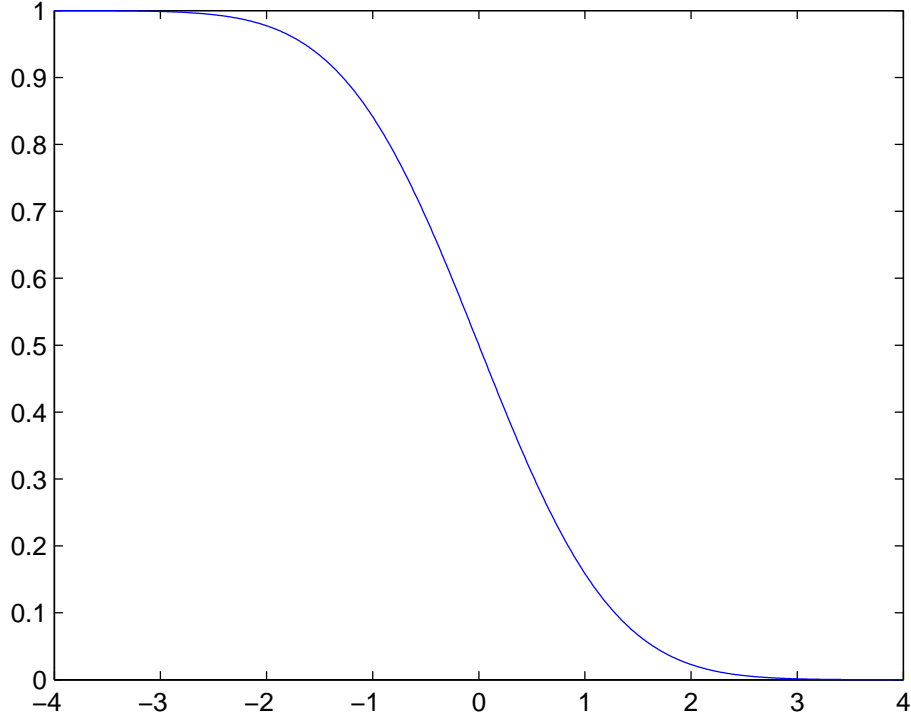


Figure 2.2: Q function

where N_e is the average number of the nearest neighbors per symbol, d_{min}^2 is the minimum distance of the transmit signal whose energy is normalized to unit, and γ_r is the receive SNR. Q is the Gaussian- Q function shown in Fig. 2.2 and it is defined as

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} \exp\left(-\frac{u^2}{2}\right) du \quad (2.4)$$

From Fig. 2.2, it is apparent that Q function is a monotonic decreasing function so (2.3) decreases with SNR increasing conditioned on a given d_{min}^2 .

SNR is a term of the power ratio between signal and noise. By (2.2), SNR in MIMO system is given as

$$\gamma_r = \frac{\mathcal{E}_t |\mathbf{z}^H \mathbf{H} \mathbf{w}|^2}{\|\mathbf{z}\|_2^2 N_0} = \frac{(\mathcal{E}_t \|\mathbf{w}\|_2^2) \left| \left(\frac{\mathbf{z}}{\|\mathbf{z}\|_2} \right)^H \mathbf{H} \left(\frac{\mathbf{w}}{\|\mathbf{w}\|_2} \right) \right|^2}{N_0} \quad (2.5)$$

Notice that we fix $\|\mathbf{z}\|_2^2 = \|\mathbf{w}\|_2^2 = 1$ and let \mathcal{E}_t be constant. Therefore, (2.5) becomes

$$\gamma_r = \frac{\mathcal{E}_t |\mathbf{z}^H \mathbf{H} \mathbf{w}|^2}{N_0} = \frac{\mathcal{E}_t \Gamma_r}{N_0} \quad (2.6)$$

where $\Gamma_r = |\mathbf{z}^H \mathbf{H} \mathbf{w}|^2$ is the effective precoding gain. Since in (2.3) improving SNR can decrease the symbol error rate, it is evident that applying suitable beamforming and combining vectors can enhance the effective precoding gain.

2.2.2 Channel Capacity

The maximum amount of information that can be reliably transmitted over the communication channel is called the channel capacity. From [8], channel capacity \mathbf{C} can be represented as follows

$$\mathbf{C} = \log_2 \left(1 + \frac{\mathcal{E}_t}{N_0} |\mathbf{z}^H \mathbf{H} \mathbf{w}|^2 \right) \text{ bps/Hz} \quad (2.7)$$

From (2.7), capacity is obviously limited by the signal power and the number of transmit and receive antennas. However, we can design the beamforming and combining vectors to increase the channel capacity.

Two statistic characterizations are used in the analysis of capacity. The first one is ergodic capacity $\bar{\mathbf{C}}$ defined as

$$\bar{\mathbf{C}} = \mathcal{E}_s \left[\log_2 \left(1 + \frac{\mathcal{E}_t}{N_0} |\mathbf{z}^H \mathbf{H} \mathbf{w}|^2 \right) \right] \text{ bps/Hz} \quad (2.8)$$

The significance of the ergodic capacity is that we can transmit signal at the ergodic rate reliably. The other characterization is the outage probability P_{out} given as

$$P_{out} = P(\mathbf{C} \leq \mathbf{C}_{out}) = q\% \quad (2.9)$$

where \mathbf{C}_{out} is outage probability. Outage probability is defined that the information rate is lower than outage probability by $q\%$ in each channel realization.

From (2.7), in high SNR condition, the capacity can be simplified to

$$\mathbf{C} \approx \log_2 \left(\frac{\mathcal{E}_t}{N_0} |\mathbf{z}^H \mathbf{H} \mathbf{w}|^2 \right) \text{ bps/Hz} \quad (2.10)$$

Since \log_2 is a monotonic increasing function, capacity in high SNR scheme increases with enhancement of SNR. It results in the same criterion as SNR criterion. Thus, we use SNR as our design criterion for MIMO system.

2.3 Beamforming and combining techniques

2.3.1 MRC combining

Beamforming and combining are signal processing techniques for signal transmission and reception. When the signal transmits, the beamformer controls the phase and magnitude of each antenna. Then receivers weight and combine information from multiple diversity branches.

Maximum-ratio-combining(MRC) [13] is a classical combining technique that the informations from each antenna are weighted and then combined. By this technique, the SNR is maximized under the constrain of assigned signal power and beamforming vector. Since to maximize the effective precoding gain is to maximize the SNR in (2.6), it has the norm inequality that

$$|\mathbf{z}^H \mathbf{H} \mathbf{w}|^2 \leq \|\mathbf{z}\|_2^2 \|\mathbf{H} \mathbf{w}\|_2^2 \quad (2.11)$$

We confine $\|\mathbf{z}\|_2^2=1$. Then (2.11) turns into

$$|\mathbf{z}^H \mathbf{H} \mathbf{w}|^2 \leq \|\mathbf{H} \mathbf{w}\|_2^2 \quad (2.12)$$

It is apparent that the MRC vector \mathbf{z} must be

$$\mathbf{z} = \frac{\mathbf{H} \mathbf{w}}{\|\mathbf{H} \mathbf{w}\|_2} \quad (2.13)$$

and the effective channel gain becomes

$$\Gamma_r = \|\mathbf{H} \mathbf{w}\|_2^2 \quad (2.14)$$

We assume that MRC is always applied in receiver.

2.3.2 Beamforming techniques

There are many beamforming techniques in classical wireless research. The most popular are MRT and EGT.

Maximum-ratio transmission(MRT). MRT is considered as the optimal beamforming vector . The concept of MRT arises from

$$\|\mathbf{H} \mathbf{w}\|_2^2 = |\mathbf{w}^H \mathbf{H}^H \mathbf{H} \mathbf{w}| \quad (2.15)$$

A singular-value decomposition of \mathbf{H} with $\text{rank}(\mathbf{H}) = k$ is given by

$$\mathbf{H} = \begin{bmatrix} \mathbf{s}_1 & \mathbf{s}_2 & \dots & \mathbf{s}_{N_r} \end{bmatrix} \begin{bmatrix} \sum_{k \times k} & \mathbf{0}_{k \times (N_t - k)} \\ \mathbf{0}_{(N_r - k) \times k} & \mathbf{0}_{(N_r - k) \times (N_t - k)} \end{bmatrix} \begin{bmatrix} \mathbf{d}_1 \\ \mathbf{d}_2 \\ \vdots \\ \mathbf{d}_{N_T} \end{bmatrix} \quad (2.16)$$

where

$$\sum_{k \times k} = \begin{bmatrix} v_1 & 0 & \dots & 0 \\ 0 & v_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & v_k \end{bmatrix} \quad (2.17)$$

is the diagonal singular value matrix with $v_1 > v_2 > \dots > v_k$.

Through (2.16), $\mathbf{H}^H \mathbf{H}$ becomes

$$\mathbf{H}^H \mathbf{H} = \begin{bmatrix} \mathbf{d}_1 \\ \mathbf{d}_2 \\ \vdots \\ \mathbf{d}_{N_T} \end{bmatrix}^H \begin{bmatrix} (\sum_{k \times k})^2 & \mathbf{0}_{k \times (N_t - k)} \\ \mathbf{0}_{(N_t - k) \times k} & \mathbf{0}_{(N_t - k) \times (N_t - k)} \end{bmatrix} \begin{bmatrix} \mathbf{d}_1 \\ \mathbf{d}_2 \\ \vdots \\ \mathbf{d}_{N_T} \end{bmatrix} \quad (2.18)$$

To have the largest $\|\mathbf{H}\mathbf{w}\|_2^2$, we use \mathbf{d}_1 as the MRT vector from (2.18) and \mathbf{s}_1 as the MRC vector, i.e., the MRC vector is the left singular vector with the largest singular value and the MRT vector is the right singular vector with the greatest singular value.

Equal gain transmission (EGT). EGT has the same power for all elements in the beamforming vector. It can be represented as

$$\mathbf{w} = \frac{1}{\sqrt{N_t}} \begin{bmatrix} e^{j\theta_1} & e^{j\theta_2} & \dots & e^{j\theta_{N_t}} \end{bmatrix} \quad (2.19)$$

where $\theta_i \in [0, 2\pi)$. It is obvious in (2.19) that in EGT, the vector energy is equally allocated in each element and thus EGT does not require the amplifiers to modify the magnitude of transmit signals. Comparing to MRT, EGT has more modest requirements and it is easier to be implemented in practice.

Though unquantized beamforming vectors such as MRT and EGT can improve the performance effectively, the amount of feedback information increases with N_t . For example, $2N_t$ in MRT and N_t in EGT information is sent back through feedback. As a result, the quantized beamformer is came up to reduce the complexity of the unquantized beamformer.

2.4 Categories of beamforming codebooks

The differences between unquantized and quantized beamforming lie in that there is a given codebook in both transmitters and receivers in the quantized beamforming. In the scheme of maximizing SNR, the receivers only need to decide the most suitable codeword by

$$\mathbf{w} = \arg \max_{\mathbf{x} \in \mathcal{F}} \|\mathbf{H}\mathbf{x}\|_2 \quad (2.20)$$

where $\arg \max$ returns the most suitable beamforming vector \mathbf{w} in a given codebook \mathcal{F} with booksize N determined by $B = \log_2 N$ bits. The codeword index then is sent back to the transmitters. Comparing to the unquantized beamforming, the quantized beamforming obviously decreases the amount of information dramatically though we sacrifice some precoding gain.

There are many categories of beamforming codebooks including RVQ codebook, Grassmannian codebook, EGT quantization, and Lloyd codebook. We introduce as follows.

Random vector quantization (RVQ). RVQ is a simple approach to generate a beamforming codebook. The codewords have elements generated from $CN(0, 1)$ and are confined to be unit norms. From [3], it is proven that the SNR gap between MRT/MRC and RVQ/MRC vanished while booksize N approaches to infinity, i.e. $N \rightarrow \infty$. Nevertheless, systems with RVQ codebooks still demand amplifiers to modify the magnitude.

Grassmannian codebook. Grassmannian codebook is a codebook constructed in Grassmannian space. Grassmannian line packing is an issue that optimally packs 1-dimension subspaces [4] and forms the codewords of Grassmannian codebook. Grassmannian manifold $\mathcal{G}(m, 1)$ is the set of all 1-dimension subspaces of the space \mathcal{C}^m . From [8], the density of packing lines for any N line packing in $\mathcal{G}(m, 1)$ is a parameter to judge the codebook and it can be shown as

$$\Delta(\mathbf{W}) = N(\delta(\mathbf{W})/2)^{2(m-1)} \quad (2.21)$$

where the distance function $\delta(\mathbf{W})$ is the sine of the smallest angle between any pairs of lines and can be written as [8]

$$\delta(\mathbf{W}) = \min_{1 \leq k < l \leq L} \sqrt{1 - |\mathbf{w}_k^H \mathbf{w}_l|^2} = \sin(\theta_{min}) \quad (2.22)$$

From (2.21), assuming that the number of lines N , i.e. the codebooksize, is given, we must have

$$|\mathbf{w}_k^H \mathbf{w}_l| \rightarrow 0. \quad (2.23)$$

to have greater $\delta(\mathbf{W})$ so that the performance enhances. The Grassmannian codebook used in the following chapters is designed by using (2.23) and can be obtained from [19].

Equal gain transmission(EGT) codebook. As we mentioned earlier, the beamforming vector whose antennas are weighted with $\frac{1}{\sqrt{N_t}}$ is called equal gain transmission (EGT). In EGT, only the phase information of codeword is quantized because the magnitude of each antenna is fixed. Various of ways can be used to a generate EGT codebooks. One simple method is scalar quantization. In scalar quantization, the phase of each antenna is quantized equally. For instance, if the number of b , bits per antenna is 2, we can use $0, \frac{\pi}{2}, \pi$, and $\frac{3\pi}{2}$ to generate the codewords.

In [1], a different way to generate the EGT codebook is proposed and the algorithm is given as follows

- 1. Set a constant R that $RN_t \geq 2^{b(N_t-1)}$ where b is the bits used in phase quantization of each antenna
- 2. Construct a matrix \mathcal{A} where \mathcal{A} consists of the first N_t rows and the $RN_t \times RN_t$ unitary DFT matrix. Scale each vector column by \sqrt{R} to guarantee unit vector columns
- 3. Construct a set of vectors \mathcal{W}_1 where the members of \mathcal{W}_1 are the columns of \mathcal{A} .
- 4. Let the set \mathcal{W}_2 be the columns of the $N_t \times N_t$ unitary DFT matrix

- 5. Choose the vector $\mathbf{w} \in \mathcal{W}_1 \setminus \mathcal{W}_2$ such that $\forall v \in \mathcal{W}_1 \setminus \mathcal{W}_2, f(\mathbf{v}) \geq f(\mathbf{w})$ where \mathbf{f} is defined as

$$f(\mathbf{w}) = \max_{\mathbf{x} \in \mathcal{W}} |\mathbf{x}^H \mathbf{w}| \quad (2.24)$$

Set $\mathcal{W}_2 = \mathcal{W}_2 \cup \{w\}$

- 6. Repeat 5 until $\text{card}((\mathcal{W})_2) = 2^{b(N_t-1)}$

where b is defined as $b = \frac{B}{N_t}$ bits, the average bits per antenna.

Both EGT codebooks made by scalar quantization and [1] will be used in simulations later.

Lloyd codebook. Among the beamforming codebooks, the codebook generating from Lloyd algorithm is the optimal. A Lloyd algorithm is introduced in Fig. 2.3 inspired by the general Lloyd's algorithm in [6] and Lloyd's algorithm for equal gain transmission in [11].

Lloyd's algorithm is an algorithm grouping training vectors into a given number of categories by (2.20). It starts by partitioning training vectors into initial N clusters belonged to individual codeword. The average of (2.14) can be represented as

$$E_s[\|\mathbf{H}\mathbf{w}\|^2] = E_s[\mathbf{w}^H \mathbf{H}^H \mathbf{H} \mathbf{w}] = \mathbf{w}^H E_s[\mathbf{H}^H \mathbf{H}] \mathbf{w}. \quad (2.25)$$

(2.25) reveals that the most suitable beamforming vector \mathbf{w} is the right singular vector with the largest singular value of $E_s[\mathbf{H}^H \mathbf{H}]$ and it replaces the original codeword. Then the new codebook generates. Each time the iteration performs, the effective precoding gain Γ_r is found that it enhances slightly. Therefore, a distortion value d is defined as

$$d = \frac{\Gamma_L - \Gamma_O}{\Gamma_O} \quad (2.26)$$

where Γ_O and Γ_L are the effective precoding gains for the original codebook and the modified codebook by the Lloyd's algorithm respectively.

When d is less than a standard distortion value d_s , i.e. when the changing of Γ_r is beyond our toleration, the iterations stop and we get the Lloyd's codebook.

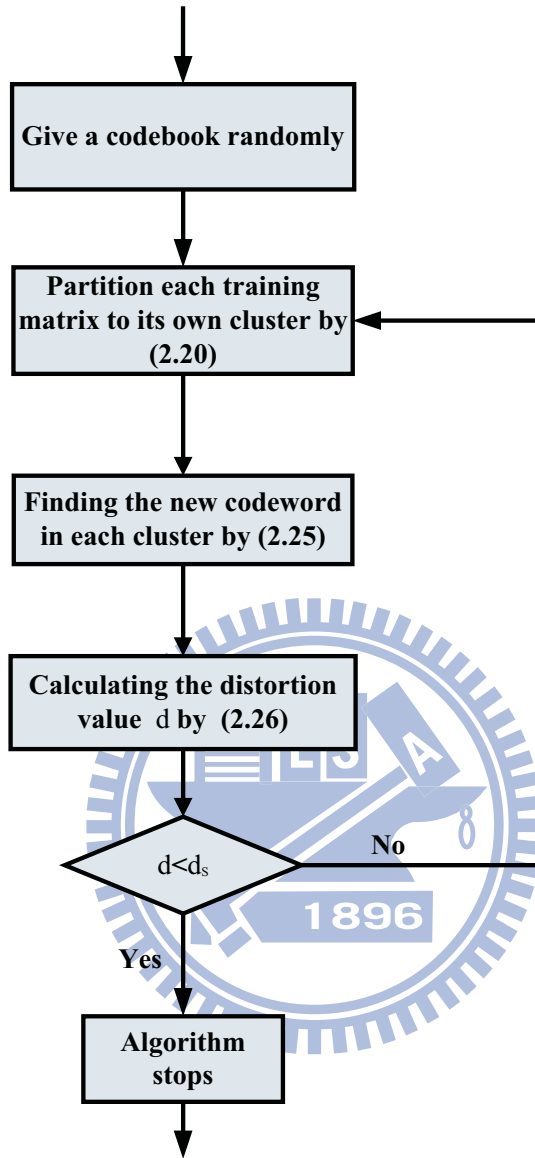


Figure 2.3: Lloyd's algorithm for MIMO codebook design

2.5 Complexity analysis to determine the most suitable codeword

Equal gain transmission. In [18], we know that the optimal beamforming vector in MISO systems is just the channel itself under the constrain of uniform

element power and can be shown as

$$\mathbf{w}_{EGT} = \frac{1}{\sqrt{N_t}} e^{-j\angle \mathbf{h}} \quad (2.27)$$

Nevertheless, there is no closed-form solution for beamforming vectors in MIMO systems. A cyclic algorithm [18] shown as follows is introduced to generate optimal unquantized EGT vectors by iterations.

- **Step 0:** Set the combining vector \mathbf{z} an initial value (e.g. the left singular vector of \mathbf{H} corresponding to its largest singular value).
- **Step 1:** Obtaining the beamformer \mathbf{w} that maximizes $|\mathbf{z}^H \mathbf{H} \mathbf{w}|$ for \mathbf{z} fixed at its most recent value. By taking $\mathbf{z}^H \mathbf{H}$ as the "effective MISO channel," this problem is equivalent to (2.27) for the MISO case. The optimal solution is

$$\mathbf{w} = \frac{1}{\sqrt{N_t}} e^{-j\angle \mathbf{z}^H \mathbf{H}} \quad (2.28)$$

- **Step 2:** Determine the combining vector \mathbf{z} that maximizes $|\mathbf{z}^H \mathbf{H} \mathbf{w}|$ for \mathbf{w} fixed at its most recent value. The optimal \mathbf{z} is the MRC and has the form as (2.13)

Iterate Steps 1 and 2 until a given stop criterion, e.g., the SNR or the capacity, is satisfied. After obtaining the optimal EGT vector, we can use some fast codebook search algorithms designed for EGT codebook to find the best codeword, e.g. [16].

For one iteration, We need $2N_t(2N_r+1)$ real-value multiplications and $N_t(5N_r-2)$ real-value additions in Step 1 and $2N_r(2N_t+1)$ real-value multiplications and $N_r(5N_t-2)$ real-value additions in Step 2. Assuming that \mathcal{T} iterations are carried out to find the most suitable codeword, $2\mathcal{T}(4N_tN_r+N_t+N_r)$ real-value multiplications and $2(5N_tN_r-N_t-N_r)$ real-value additions are required.

Noticed that this method is only effective in the condition that the codebook size is very huge. Since in large EGT codebook, even the fast codebook search like [16] is applied, it still costs a large number of complexity. As a result, for unquantized EGT, we can use cyclic algorithm to quickly find the optimal EGT codeword. For quantized EGT, we can use the cyclic algorithm to find the

category	number of operations
Real-value addition	$N(5N_tN_r - 1)$
Real-value multiplication	$N[2N_r(2N_t + 1)]$
Comparisons	$N - 1$

Table 2.1: Complexity for exhaustive search

optimal beamforming vector first and then use the fast codebook search designed for EGT codebook to reduce some computation. However, for scalar EGT, since the phases of antennas have a regular pattern, we only need to performance the cyclic algorithm to find the optimal EGT vector and then inspect which scalar EGT vector is the closest. In those methods, it is seeming that the complexity diminishes a lot by combining the cyclic algorithm and EGT fast codebook search.

Non-equal-gain precoder: For non-equal-gain precoder, the exhaustive search in (2.20) is generally used to find the best codeword. However, the complexity increases with the codebook size N . The complexity analysis of exhaustive search can be divided by two parts, one is calculating the effective precoding gain and the other is finding the maximum gain. Matrix multiplication and norm calculating are required in calculating the SNR value. In matrix multiplications, we need $4N_tN_r$ real-value multiplications and $N_r(5N_t - 2)$ real-value additions for each codeword. Norm calculating for a codeword requires $2N_r$ real-value multiplications and $2N_r - 1$ real-value additions. As a result, the computation of a codeword is $2N_r(2N_t + 1)$ real-value multiplications and $5N_tN_r - 1$ real-value additions. To find the maximum SNR value for a codebook with size N , $N - 1$ comparisons are required in evaluation. The total computational complexity is listed in Tab. 2.1. It is evident that the larger the codebook size is, the greater the complexity is. Therefore, the fast codebook search is an essential topic and the proposed algorithm in this thesis is to reduce the complexity of finding the best codeword for a given codebook.

Chapter 3

Proposed fast codebook search algorithm

3.1 System model of the proposed scheme

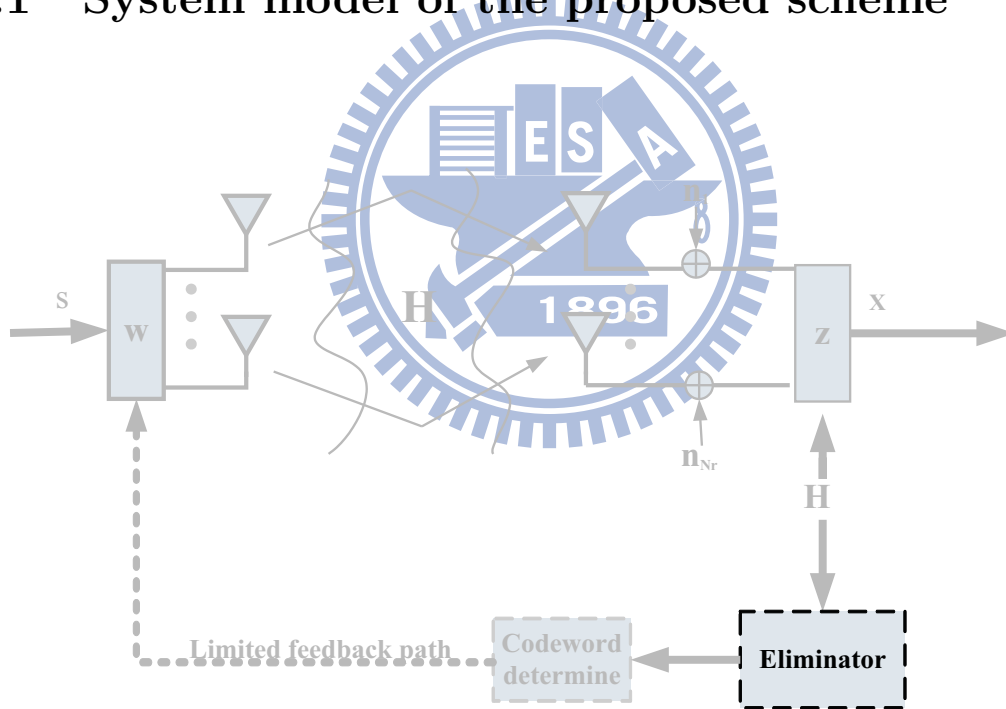


Figure 3.1: MIMO system with the eliminator built by the proposed algorithm

The proposed scheme is shown in Fig. 3.1, where we add one more unit called “eliminator” in front of the “codeword determine” unit. The eliminator can filter out unsuitable codewords in advance. Hence, the number of residual codewords

performing (2.20) to determine the best codeword is greatly reduced. As a result, the computational complexity decreases considerably.

Since the effective precoding gain can be represented as (2.15), $\mathbf{H}^H \mathbf{H}$ is given by matrix as follows

$$\mathbf{H}^H \mathbf{H} = \begin{bmatrix} \mathbf{h}_1^H \mathbf{h}_1 & \mathbf{h}_1^H \mathbf{h}_2 & \dots & \mathbf{h}_1^H \mathbf{h}_{N_t} \\ \mathbf{h}_2^H \mathbf{h}_1 & \mathbf{h}_2^H \mathbf{h}_2 & \dots & \mathbf{h}_2^H \mathbf{h}_{N_t} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{h}_{N_t}^H \mathbf{h}_1 & \mathbf{h}_{N_t}^H \mathbf{h}_2 & \dots & \mathbf{h}_{N_t}^H \mathbf{h}_{N_t} \end{bmatrix} \quad (3.1)$$

where $\mathbf{H} = [\mathbf{h}_1 \ \mathbf{h}_2 \ \dots \ \mathbf{h}_{N_t}]$, \mathbf{h}_i is the column vector of \mathbf{H} .

(3.1) can be expressed further as

$$\begin{bmatrix} \|\mathbf{h}_1\|_2^2 & \|\mathbf{h}_1\|_2 \|\mathbf{h}_2\|_2 e^{j\phi_{12}} & \dots & \|\mathbf{h}_1\|_2 \|\mathbf{h}_{N_t}\|_2 e^{j\phi_{1N_t}} \\ \|\mathbf{h}_2\|_2 \|\mathbf{h}_1\|_2 e^{-j\phi_{12}} & \|\mathbf{h}_2\|_2^2 & \dots & \|\mathbf{h}_2\|_2 \|\mathbf{h}_{N_t}\|_2 e^{j\phi_{2N_t}} \\ \vdots & \vdots & \ddots & \vdots \\ \|\mathbf{h}_{N_t}\|_2 \|\mathbf{h}_1\|_2 e^{-j\phi_{1N_t}} & \|\mathbf{h}_{N_t}\|_2 \|\mathbf{h}_2\|_2 e^{-j\phi_{2N_t}} & \dots & \|\mathbf{h}_{N_t}\|_2^2 \end{bmatrix} \quad (3.2)$$

where

$$e^{j\phi_{ij}} = \frac{\mathbf{h}_i^H \mathbf{h}_j}{\|\mathbf{h}_i\|_2 \|\mathbf{h}_j\|_2} \quad (3.3)$$

While the beamforming vector

$$\mathbf{w} = [w_1 \ w_2 \ \dots \ w_{N_t}]^t = [|w_1|e^{j\theta_1} \ |w_2|e^{j\theta_2} \ \dots \ |w_{N_t}|e^{j\theta_{N_t}}]^t \quad (3.4)$$

is put on, (2.14) becomes

$$\sum_{i=1}^{N_t} \|\mathbf{h}_i\|_2^2 |w_i|^2 + \sum_{j=1}^{N_t-1} \sum_{k=j+1}^{N_t} 2 \|\mathbf{h}_j\|_2 \|\mathbf{h}_k\|_2 |w_j| |w_k| \cos(\phi_{kj} + \theta_j - \theta_k) \quad (3.5)$$

From (3.5), we assume the effective channel gain may be reduced due to negative cosine values and the proposed algorithm is created based on this assumption. Since there are $\frac{N_t(N_t-1)}{2}$ cosine values in the right term of (3.5), the proposed algorithm has the parameter \mathcal{S} called "stage" conditioned that $\mathcal{S} \leq \frac{N_t(N_t-1)}{2}$ to decide how many cosine values are going to be evaluated.

Since knowing the number of stage \mathcal{S} , we proceed to decide which cosine values will be evaluated. From (3.5), we know that if $\|\mathbf{h}_i\|_2 > \|\mathbf{h}_j\|_2 > \|\mathbf{h}_k\|_2$, $\|\mathbf{h}_i\|_2 \|\mathbf{h}_j\|_2 |w_i| |w_j|$ may have greater influence on (2.14) than $\|\mathbf{h}_i\|_2 \|\mathbf{h}_k\|_2 |w_i| |w_k|$

does. According to the assumption, we build a sequence vector \mathbf{p} to save the sequence for the index of the cosine values by the following process.

Ordering the $\|\mathbf{h}_i\|$:

- **Step 0:** Given initial values $k = 1$ and $n = 1$, a $1 \times \frac{N_t(N_t-1)}{2}$ sequence vector \mathbf{p} , a $1 \times N_t$ norm-value sequence \mathbf{t} , and one channel realization $\mathbf{H} = [\mathbf{h}_1 \ \mathbf{h}_2 \ \dots \ \mathbf{h}_{N_t}]$.
- **Step 1:** Calculate and sort the norms of the column vectors \mathbf{h}_i of \mathbf{H} , and save the index of the norm value from the highest to the lowest in \mathbf{t} . For example, if $\|\mathbf{h}_3\|_2 > \|\mathbf{h}_1\|_2 > \|\mathbf{h}_2\|_2 > \|\mathbf{h}_4\|_2$, we save the sequence in \mathbf{t} as $[3 \ 1 \ 2 \ 4]$.
- **Step 2:** Set $m = k + 1$.
- **Step 3:** Have $\mathbf{p}(n) = (\mathbf{t}(k), \mathbf{t}(m))$, $n = n + 1$ and $m = m + 1$. Repeat Step 3 until $m > N_t$.
- **Step 4:** Have $k = k + 1$. Repeat Step 2 until $k = N_t$.

3.2 Proposed algorithm

With the given stage \mathcal{S} , an initial value $k = 1$, a $1 \times \frac{N_t(N_t-1)}{2}$ vector \mathbf{p} to save the evaluation sequence, and the candidate cluster \mathcal{U} equivalent to the codebook initially, the proposed algorithm to eliminate the codewords in advance can be interpreted as follows

- **Step 1:** Building the sequence vector \mathbf{p} using the ordering process for $\|\mathbf{h}_i\|$ mentioned in the previous section.
- **Step 2 :** Calculating the cosine value $\cos(\phi_{mn} + \theta_n - \theta_m)$ in (3.5) of each codeword in \mathcal{U} corresponding to the $\mathbf{p}(k) = (m, n)$.
- **Step 3 :** Evaluating the cosine values. If the cosine value of the codeword is negative, drop the codeword out off \mathcal{U} .

- **Step 4 :** Having $k = k + 1$. Repeat from Step 2 until $k > \mathcal{S}$

We mentioned in the previous that Step 1 decides the sequence of the cosine values in (3.5) going to be evaluated according to the norms of the column vectors. After Step 1, for all codewords in \mathcal{U} , we evaluate the cosine value in Step 2 and Step 3. If the value is negative, we drop the codeword out off the cluster \mathcal{U} . The algorithm stops until $k > \mathcal{S}$

However, it is possible that not all the elected cosine values are positive after several iterations. In this situation, we keep the residual codewords in the previous stage. Then, we stop the iterations and let the codewords in \mathcal{U} pass to the “codeword determine” unit, which performs the exhaustive search to find the best codeword.

Example 1: Stage $\mathcal{S}=1$ and 2. A codebook with 4 codewords $\mathcal{F} = [\mathbf{w}_1 \ \mathbf{w}_2 \ \mathbf{w}_3 \ \mathbf{w}_4]$ are given as

$$\mathcal{F} = \begin{bmatrix} -0.15 - 0.21j & 0.144 + 0.06j & 0.78 - 0.06j & 0.07 - 0.57j \\ -0.58 + 0.77j & -0.58 + 0.54j & -0.02 - 0.54j & -0.08 + 0.3j \\ 0.04 - 0.05j & 0.6 + 0.03j & 0.22 + 0.19j & 0.31 + 0.69j \end{bmatrix} \quad (3.6)$$

Assume a channel \mathbf{H} is given as

$$\mathbf{H} = \begin{bmatrix} -1.13 + 0.16j & -0.75 - 1.53j & -0.57 - 0.71j \\ 0.19 - 0.65j & 1 - 0.04j & 0.37 + 0.43j \end{bmatrix} \quad (3.7)$$

Follow by the steps mentioned earlier, first we find the maximum value of norm $\|\mathbf{h}_i\|_2$. Since

$$\|\mathbf{h}_2\|_2 > \|\mathbf{h}_1\|_2 > \|\mathbf{h}_3\|_2, \quad (3.8)$$

we first evaluate $\cos(\phi_{12} + \theta_2 - \theta_1)$, then $\cos(\phi_{32} + \theta_2 - \theta_3)$ and the final is $\cos(\phi_{31} + \theta_1 - \theta_3)$. The cosine values for the first two cosine values are given in Tab 3.1. From the Tab. 3.1, we know that \mathbf{w}_1 and \mathbf{w}_3 are kept in the first stage. In Stage 2, it is found that cosine values for both \mathbf{w}_1 and \mathbf{w}_3 are all negative. As a result, no codeword is dropped out in Stage 2 and hence \mathbf{w}_1 and \mathbf{w}_3 kept in Stage 1 will be passed to the “codeword determine” unit.

cosine	\mathbf{w}_1	\mathbf{w}_2	\mathbf{w}_3	\mathbf{w}_4
$\cos(\phi_{12} + \theta_2 - \theta_1)$	0.8121	-0.8953	0.9617	-0.1781
$\cos(\phi_{32} + \theta_2 - \theta_3)$	-0.9964	-0.6555	-0.7213	0.8101

Table 3.1: Cosine values for Example 1

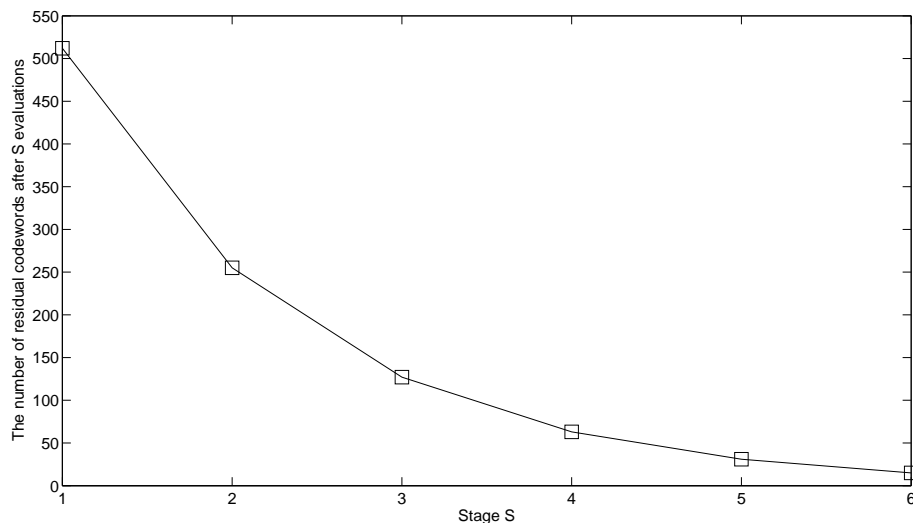


Figure 3.2: Residual codewords after S stage evaluations for four transmit antennas MISO system with $N = 1024$

3.3 The number of the residual codewords after using the proposed algorithm

As the phases of the channel coefficients have uniform distribution over $(-\pi, \pi)$, given θ_i and θ_j , $(\phi_{ji} + \theta_i - \theta_j)$ is also considered as uniform distributed over $(-\pi, \pi)$. Thus, in statistic, $\cos(\phi_{ji} + \theta_i - \theta_j)$ have 50% possibility to be negative for given θ_i and θ_j . Therefore, we assume about half of the codewords will be dropped out in each elimination. Fig. 3.2 shows the simulation for the residual codewords after each elimination for 4T2R MIMO system with RVQ codebook, which obviously supports this assumption. However, it is noticed that the result above is based on the assumption that the phase of the beamforming vector is designed to be equally

divided. Without the constrain, the number of residual codewords may not be half of that in the previous stage. Fortunately, the assumption that almost half codewords are filtered out in each elimination holds for most codebook including the Grassmannian codebook, the Lloyd codebook, the RVQ codebook with larger size, and EGT with the scalar quantization .

Please also noticed that when the number of stage is larger than $\log_2 N$, i.e., $\mathcal{S} > \log_2 N$, the algorithm may not operate through the whole stages. This is because that the number of the candidate codeword may be only one at the lower stage as a result that there is no need to proceed the rest stages.

3.4 Complexity analysis

In the proposed algorithm, extra computations are required. From (3.2), to get the element information of $\mathbf{H}^H \mathbf{H}$, N_t norm calculatings and \mathcal{S} vector multiplications are required. Norm calculatings of all the column vectors cost $2N_r N_t$ real-value multiplications and $N_t(2N_r - 1)$ real-value additions. In vector multiplications, $4N_r \mathcal{S}$ real-value multiplications and $\mathcal{S}(5N_r - 2)$ real-value additions are required. Based on Step 1 in the proposed algorithm, we need to perform $\frac{N_t(N_t-1)}{2} - 1$ comparisons.

After knowing all the information and deciding the elected cosine values, we start to evaluate the cosine values in all codewords. With the given stage number \mathcal{S} , the computational amount includes $4N[1 - (\frac{1}{2})^{\mathcal{S}}]$ real-value additions and $2N[1 - (\frac{1}{2})^{\mathcal{S}}]$ operations to determine $\cos(\phi_{jk} + \theta_k - \theta_j)$ is positive or negative. Various ways can be used to inspect the cosine value. One simple method is to check the sign bit of the cosine value. The residual $\frac{N}{2^{\mathcal{S}}}$ codewords are going to proceed the “codeword determine.” The total complexity of the eliminator is listed in Tab. 3.2

Example 2: complexity reduction using proposed algorithm. Assume a 4T2R MIMO system with total bits $B = 6$. For exhaustive search without the proposed fast codebook search, 2496 real-value additions, 2304 real-value multiplications, 63 comparisons are required. With the proposed method with

category	number of operations
Real-value addition	$N_t(2N_r - 1) + \mathcal{S}(5N_r - 2) + 4N[1 - (\frac{1}{2})^{\mathcal{S}}]$
Real-value multiplication	$2N_r(N_t + 2\mathcal{S})$
Comparison	$\frac{N_t(N_t-1)}{2} - 1$
Cosine value inspection	$2N[1 - (\frac{1}{2})^{\mathcal{S}}]$

Table 3.2: Complexity for the proposed algorithm

stage number $\mathcal{S} = 2$, the number of real-value additions decreases from 2496 to 656, the number of real-value multiplications decreases from 2304 to 796, and the number of comparisons decreases from 63 to 20. Nevertheless, extra 96 operations to determine that the cosine value is positive or negative are required. We can obviously see the computational complexity is greatly reduced due to the proposed algorithm. Later in the simulation results, we will show that for this example the SNR loss between the exhaustive search and the proposed fast codebook algorithm is around 0.1 dB.

Chapter 4

Performance analysis

In this chapter, we examine the SNR loss for the proposed algorithm. We define a parameter Γ_L as follows to analyze the effective precoding gain loss.

$$\Gamma_L = \frac{\mathcal{P}_d \Gamma_{max} + (1 - \mathcal{P}_d) \Gamma_{max} R_L}{\Gamma_{max}} \quad (4.1)$$

where Γ_{max} is the maximum SNR in each channel realization with a given codebook corresponding to the best codeword and detection rate \mathcal{P}_d is the probability that the best codeword is going to stay in the cluster after elimination. R_L is the loss ratio defined as that $R_L\%$ of the best gain Γ_{max} will be achieved while we miss the best codeword. (4.1) can be further shown as

$$\Gamma_L = \mathcal{P}_d + (1 - \mathcal{P}_d) R_L \quad (4.2)$$

From (4.2), it is obvious that the detection rate \mathcal{P}_d and the loss ratio R_L is related to the performance analysis. Consequently, we focus on how to get detection rate \mathcal{P}_d and the loss ratio R_L .

Detection rate \mathcal{P}_d : We use a probability value called “positive ratio” as follows to find the detection rate.

$$\mathcal{P}_i = \frac{\mathcal{P}[\cos(\phi_{jk} + \theta_k - \theta_j) > 0 | \mathbf{H} \in \mathbf{w}_i]}{\mathcal{P}[\mathbf{H} \in \mathbf{w}_i]} \quad (4.3)$$

The positive ratio is considered as the probability that the elected cosine value is positive when we choose \mathbf{w}_i as our beamforming vector.

After calculating all the positive value for each codeword, we can define the detection rate as follow with the assumption that the chance for each codeword

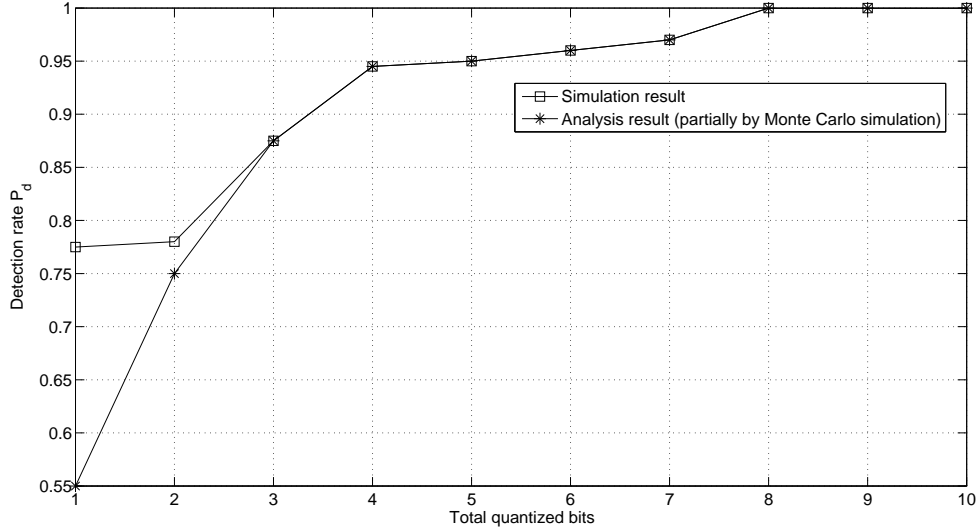


Figure 4.1: Detection rate for 4T1R MISO system using RVQ codebook with $\mathcal{S} = 1$

being chosen is equal.

$$\mathcal{P}_d = \frac{1}{N} \sum_{i=1}^N \mathcal{P}_i \quad (4.4)$$

Noticed that \mathcal{P}_i for all codewords converge to be equal when the codebook size grows larger.

However, since there are different characteristics among the codebooks, we use simulations to find \mathcal{P}_d for each codebook. Fig. 4.1 is a detection rate simulation result for 4T1R MISO system using RVQ codebook with $\mathcal{S} = 1$. The simulation result is the rate that the codeword decided by the proposed algorithm is the same as that decided by the exhaustive search. The analysis result is calculated by (4.4) and all the \mathcal{P}_i is obtained by simulations. It shows that the analysis probability obtained by the method mentioned in this section is very close to the simulation rate.

Loss ratio \mathbf{R}_L : It is found that the codeword with the second-best effective precoding gain has great chance to stay in the residual codewords when the best codeword is excluded by the eliminator. Fig. 4.2 shows the probability of choosing the codeword with x-th maximum SNR for a 4T1R MISO system using

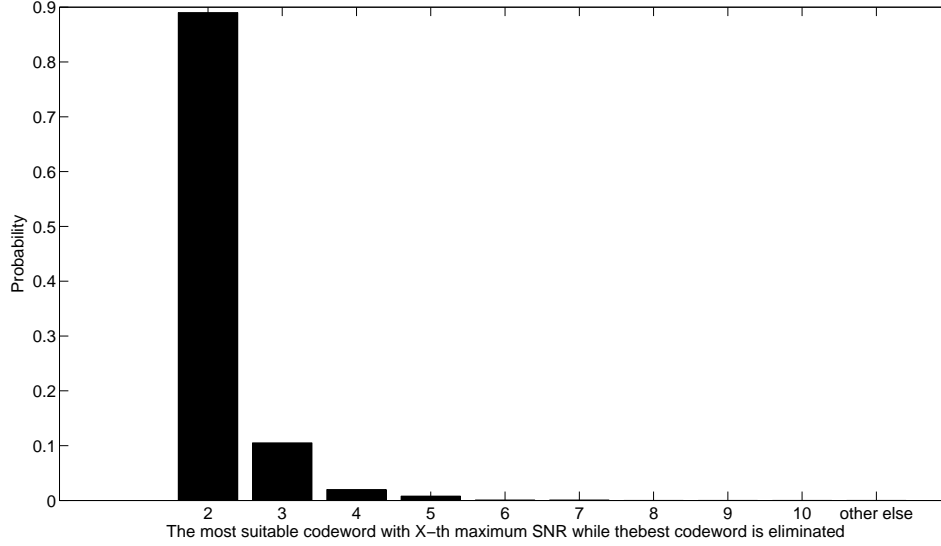


Figure 4.2: Probability of X-th maximum SNR for 4T1R MISO system using RVQ codebook with $N = 1024$ for $\mathcal{S} = 1$ when the best codeword is eliminated

RVQ codebook with $N = 1024$ for $\mathcal{S} = 1$ when the best codeword is filtered out. The results shown in Fig. 4.2 apparently support our assumption. While the most suitable beamforming vector in a given codebook \mathbf{w}_o is applied, by (2.18), (2.14) can be further represented as

$$\|\mathbf{H}\mathbf{w}_o\|_2^2 = \sum_{i=1}^k v_i^2 |\mathbf{d}_i \mathbf{w}_o|^2 \quad (4.5)$$

The effective precoding gain has an inequality that

$$v_1^2 |\mathbf{d}_1 \mathbf{w}_o|^2 \leq \|\mathbf{H}\mathbf{w}_o\|_2^2 \quad (4.6)$$

Noticed that both $\|\mathbf{d}_1\|_2$ and $\|\mathbf{w}_o\|_2$ are unit length. As we mentioned earlier, the eliminator has great chance to keep the second-best codeword \mathbf{w}_s when filtering out the best codeword. In this situation, the greatest loss happens in the case that $\mathbf{d}_1 = \mathbf{w}_o^H$ but we take \mathbf{w}_s as the beamforming vector. Therefore, (4.6) becomes

$$v_1^2 |\mathbf{w}_o^H \mathbf{w}_s|^2 \leq \|\mathbf{H}\mathbf{w}_o\|_2^2 \quad (4.7)$$

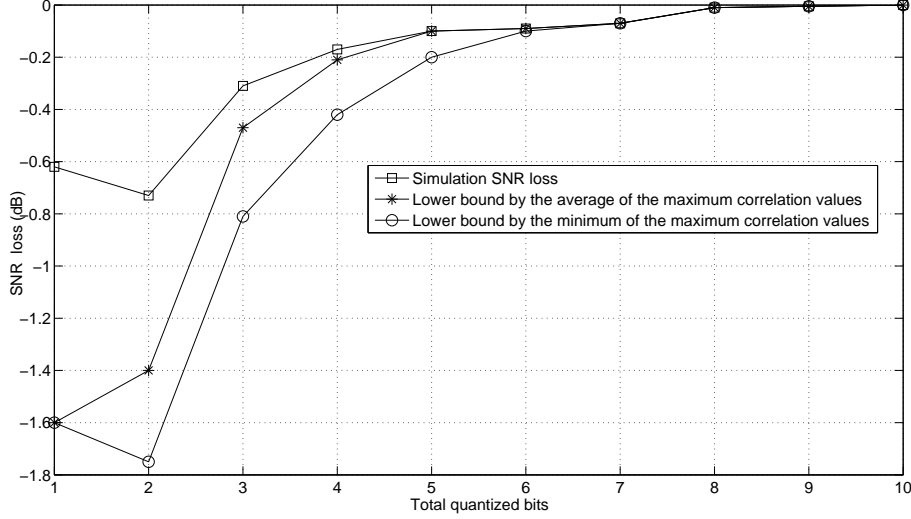


Figure 4.3: SNR loss for 5T2R MIMO system using RVQ codebook with $N = 1024$ for $\mathcal{S} = 1$

(4.7) suggests that we can build a lower bound by defining the loss ratio as the minimum of the maximum correlation between a codeword and the rest codewords . That is

$$R_L \geq \arg \min_{1 \leq i \leq N} \left(\max_{j \neq i, 1 \leq j \leq N} |\mathbf{w}_j^H \mathbf{w}_i| \right) \quad (4.8)$$

The loss ratio can also be approximated as the average of maximum correlation between a codeword and the rest codewords and can be shown as

$$R_L \geq \frac{1}{N} \sum_{i=1}^N \left(\max_{j \neq i, 1 \leq j \leq N} |\mathbf{w}_j^H \mathbf{w}_i| \right) \quad (4.9)$$

Figs. 4.3 and 4.4 show the SNR loss example for 5T1R and 6T1R MIMO system using RVQ codebook. When quantized bits B are more than 7 bits, the lower bound is close to the real SNR loss. Noticed that because the codebook is generated by randomly, it is not guaranteed that the codebook is well designed in the scenario with fewer quantized bits. Consequently, the SNR loss may be more with 2 quantized bits than that with 1 as shown in Fig. 4.4.

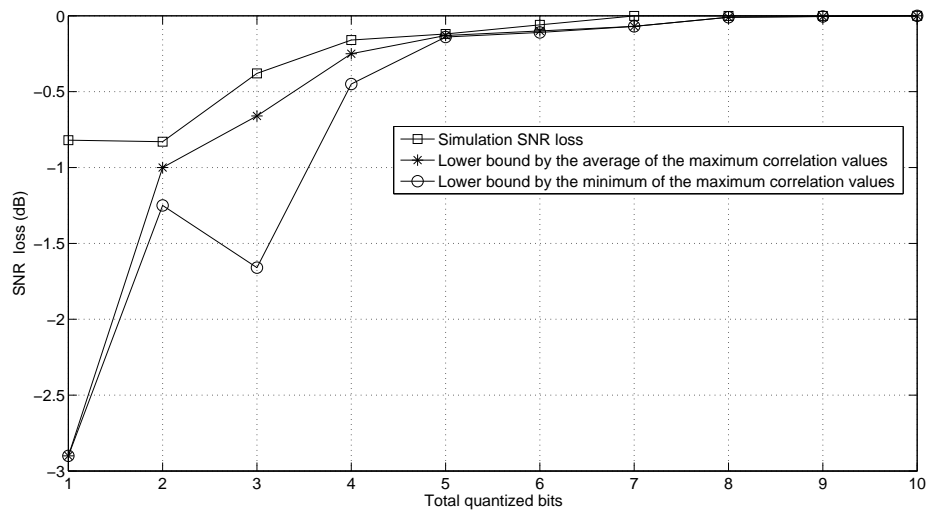


Figure 4.4: SNR loss for 6T2R MIMO system using RVQ codebook with $N = 1024$ for $\mathcal{S} = 1$

Chapter 5

Simulation result

In this chapter, we show simulation results to see the influence of the proposed algorithm on the bit-error-rate and capacity. Noticed that we assume that the signal is BPSK modulated and the probability model of channels is Rayleigh distributed, i.e. the elements of the channel matrix are generated according to $CN(0, 1)$. All of the codebooks used in simulations can be found in [20] and [19]. It is noticed that the 6 quantized bits Grassmannian codebook obtained from [19] is a EGT codebook.

Experiment 1: Various number of stage. In this experiment, we use simulations to see the influence of the stage number \mathcal{S} on the BER and capacity. We expect that the performance loss between systems with exhaustive search and that with one the proposed algorithm together with the exhaustive search increases with the stage number \mathcal{S} because the higher stage elimination consists of the cases that the the best codeword may be dropped out in the lower stage. Figs. 5.1 and 5.2 are simulation results for 4T1R MIMO system using RVQ codebook with $B=6$ bits and various number of stage. The results apparently supports our expectation.

As we mentioned earlier, the system with 4 antennas have at most 6 eliminations. From Fig 5.2, we can see that although we sacrifice about 0.7 dB after 6 eliminations, the complexity in codeword determine decreases dramatically even if we do not proceed the codeword determine since there might be only one codeword after 6 eliminations. From figure 5.1, we also know that even if the

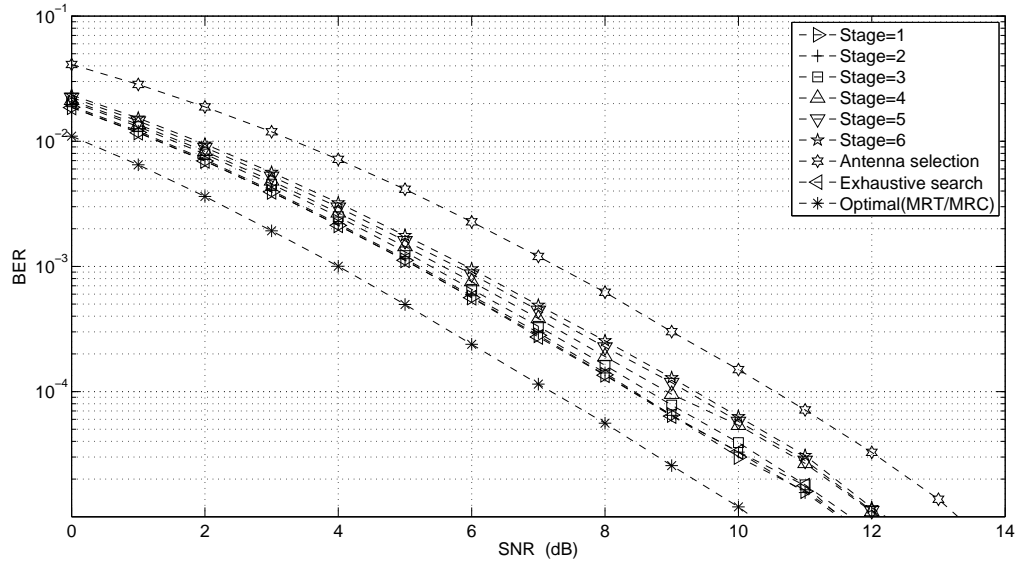


Figure 5.1: Comparisons for BER of 4T1R MISO system using RVQ codebook with $B = 6$ bits for various number of stage \mathcal{S}

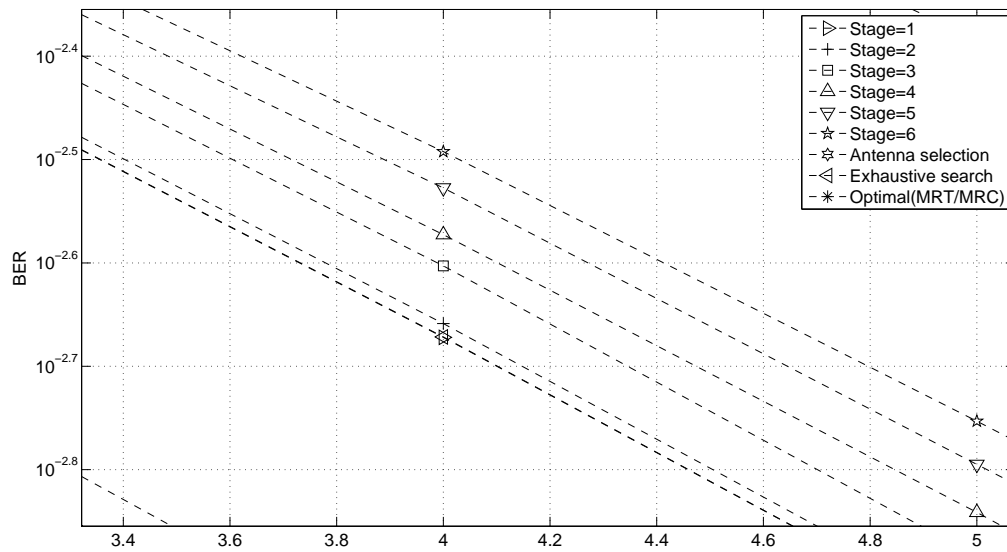


Figure 5.2: Zoom-in for Fig. 5.1

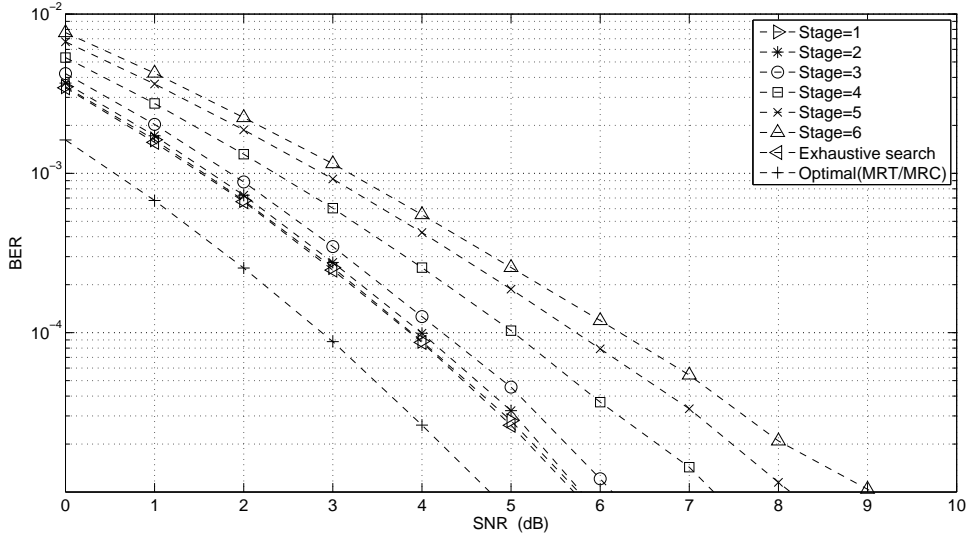


Figure 5.3: Comparisons for BER of 4T2R MIMO system using RVQ codebook with $B = 6$ bits for various stage of number \mathcal{S}

SNR loses about 0.7 dB in 6 eliminations, the MIMO system with the proposed algorithm in $\mathcal{S} = 6$ still has more 1.75 dB gain than that using antennas selection.

Fig. 5.3 is simulation result for 4T2R MIMO system using RVQ codebook with $B=6$ bits and various stae of number. In spite of around 1.4 dB loss in 5 eliminations, the number of codewords to perform exhaustive search decreases from 64 to about 2.

Comparing Figs. 5.1 and 5.3, it seems that the diversity is kept in MISO case but lost in MIMO case. Figs. 5.4 and 5.5 are simulation results for Lloyd codebook with $B=6$ bits and Figs 5.6 and 5.7 are simulation results for scalar EGT using $B=6$ bits. It is interesting to note that in vector quantization codebook, the diversity keeps in the MISO case but loses in the MIMO case using the proposed algorithm. However, in EGT with the scalar quantization, the diversity keep in both MISO and MIMO cases.

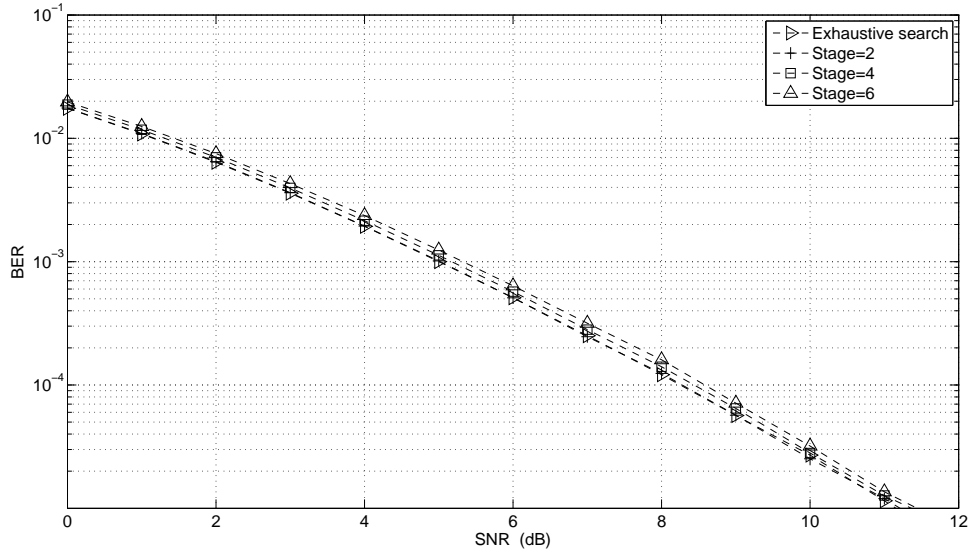


Figure 5.4: Comparisons for BER of 4T1R MISO system using Lloyd codebook with $B = 6$ bits for various number of stage \mathcal{S}

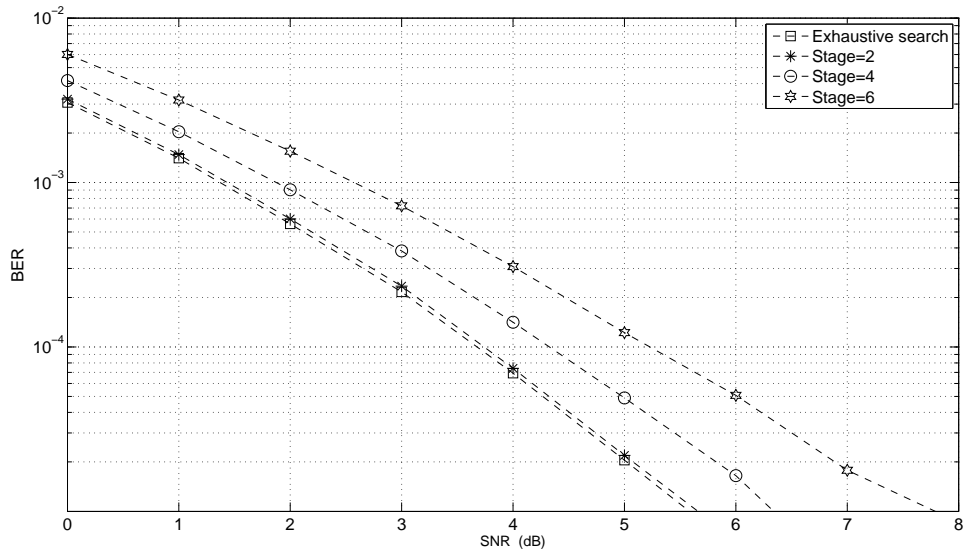


Figure 5.5: Comparisons for BER of 4T2R MIMO system using Lloyd codebook with $B = 6$ bits for various number of stage \mathcal{S}

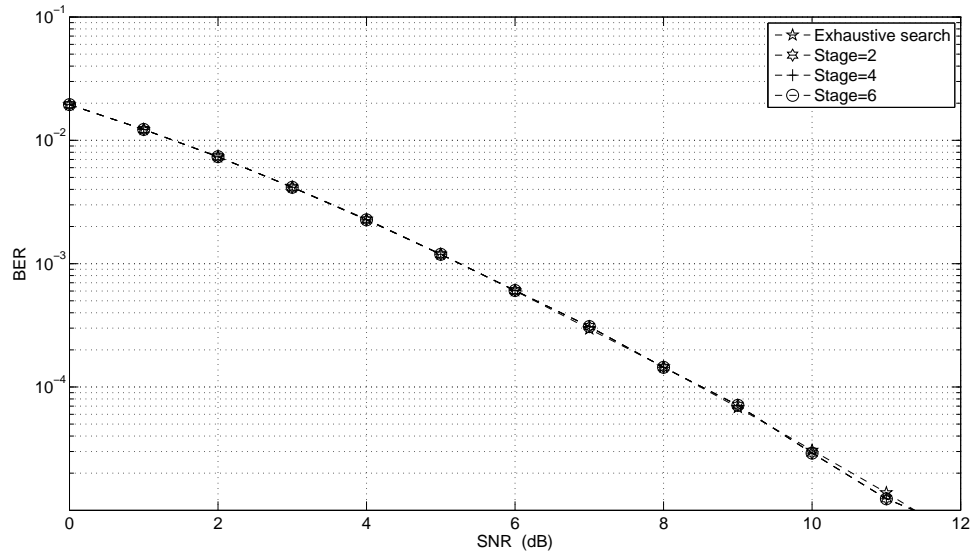


Figure 5.6: Comparisons for BER of 4T1R MISO system using scalar EGT with $B = 6$ bits for various number of stage \mathcal{S}

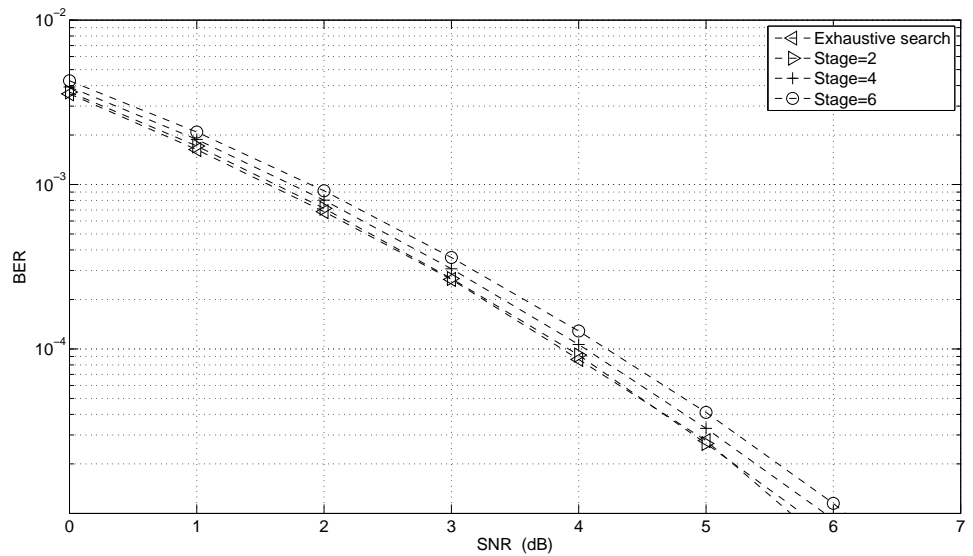


Figure 5.7: Comparisons for BER of 4T2R MIMO system using scalar EGT with $B = 6$ bits for various number of stage

Figs. 5.8 and 5.10 are capacity simulations for RVQ codebook with $B=6$ bits, 4T1R and 4T2R MIMO system respectively. It is seeming that the capacity decreases with the stage number. From the zoom-in Figs. 5.9 and 5.11, it is obvious that the capacity decreases less than 10% even after 6 times eliminations.

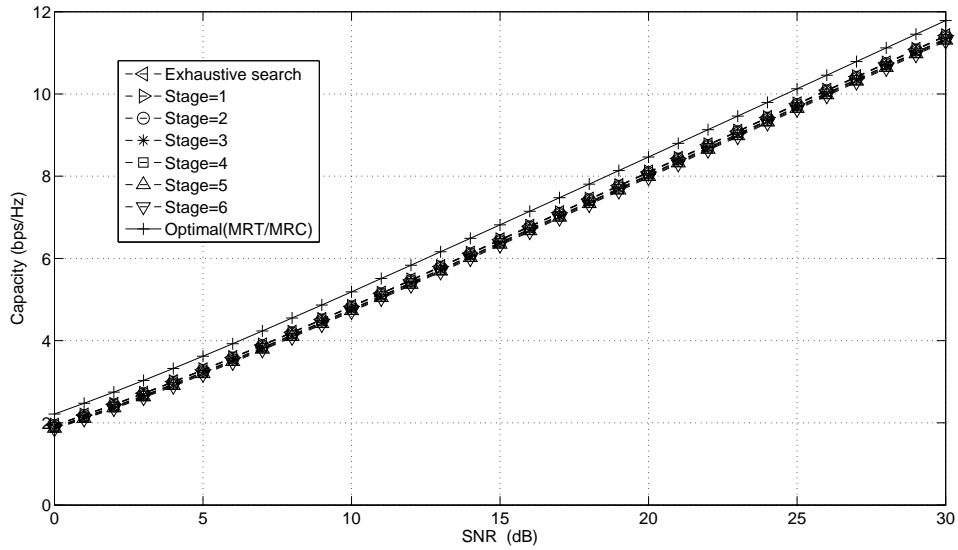


Figure 5.8: Comparisons for channel capacity of 4T1R MISO system using RVQ codebook with $B = 6$ bits for various number of stage \mathcal{S}

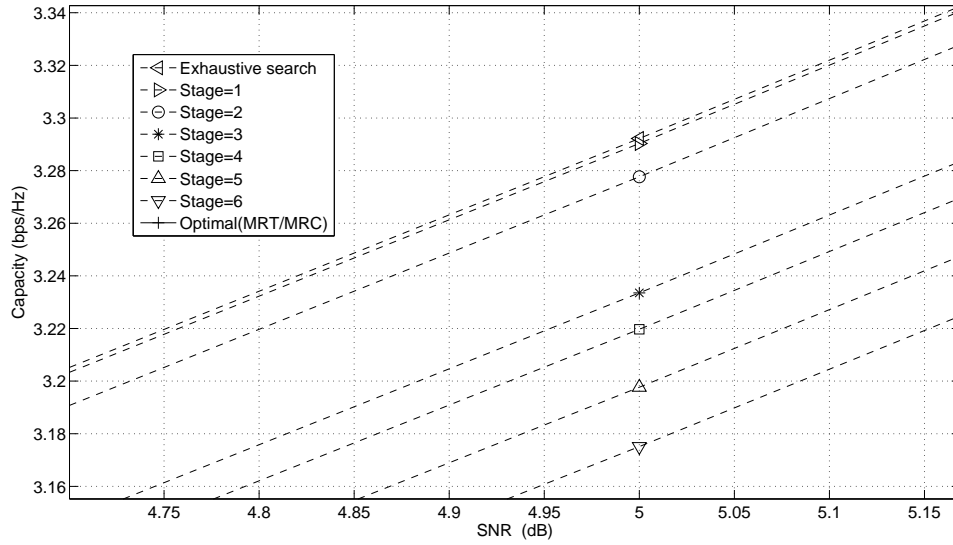


Figure 5.9: Zoom-in for Fig. 5.8

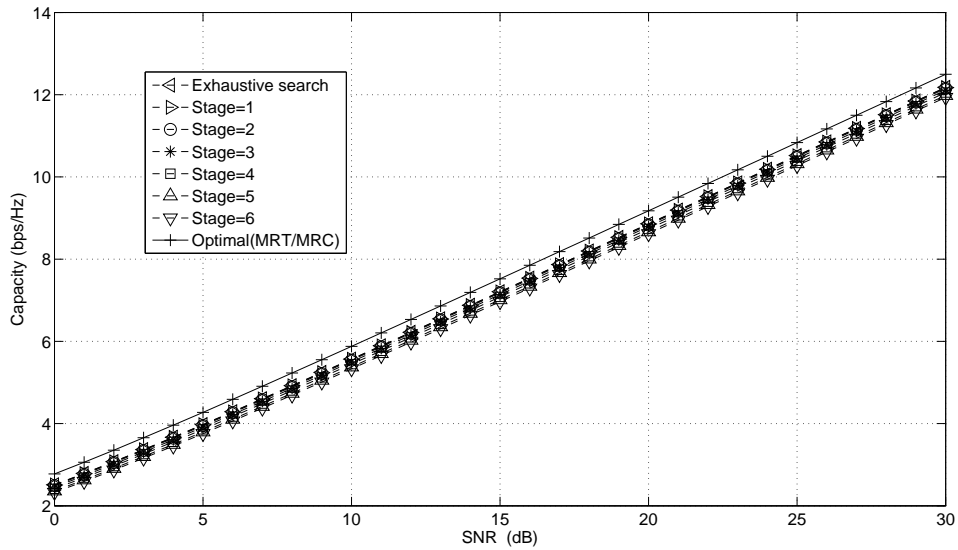


Figure 5.10: Comparisons for channel capacity of 4T2R MIMO system using RVQ codebook with $B = 6$ bits for various number of stage S

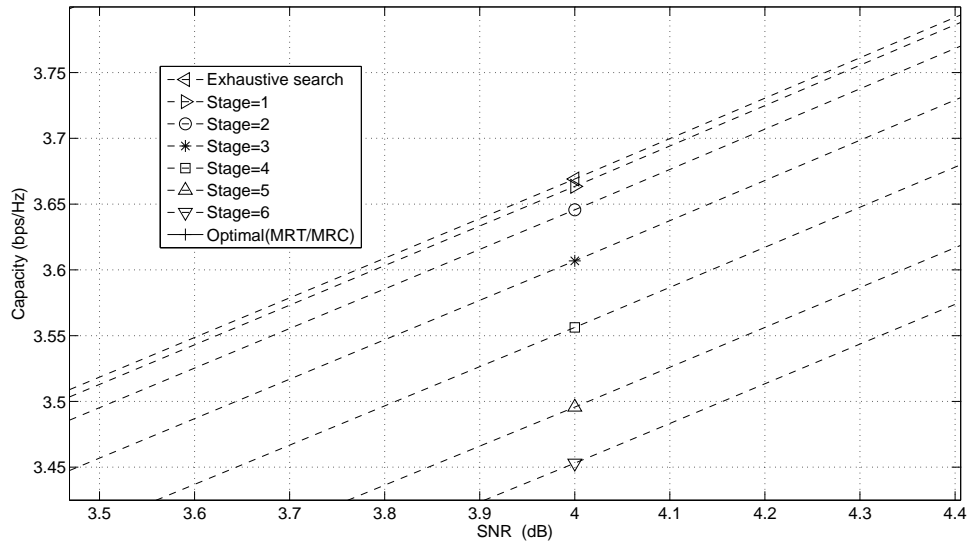


Figure 5.11: Zoom-in for Fig. 5.10

Experiment 2: Various number of transmit antennas. We compare the effect of different number of transmit antennas on the performance in this experiment. We assume that each element of the beamforming codeword is quantized by the equal amount of bits, i.e., we fix the number of quantized bits per antenna b . Fig. 5.12 is the simulation result for MISO system using RVQ codebook with $b = 1$ bit. The SNR loss is small in Fig. 5.12 but it apparently decreases with the numbers of the transmit antennas in Fig. 5.13, which shows the results for BER of 2 receive antennas MIMO system using RVQ codebook with $b = 1$ bit. The reason is explained as follows:

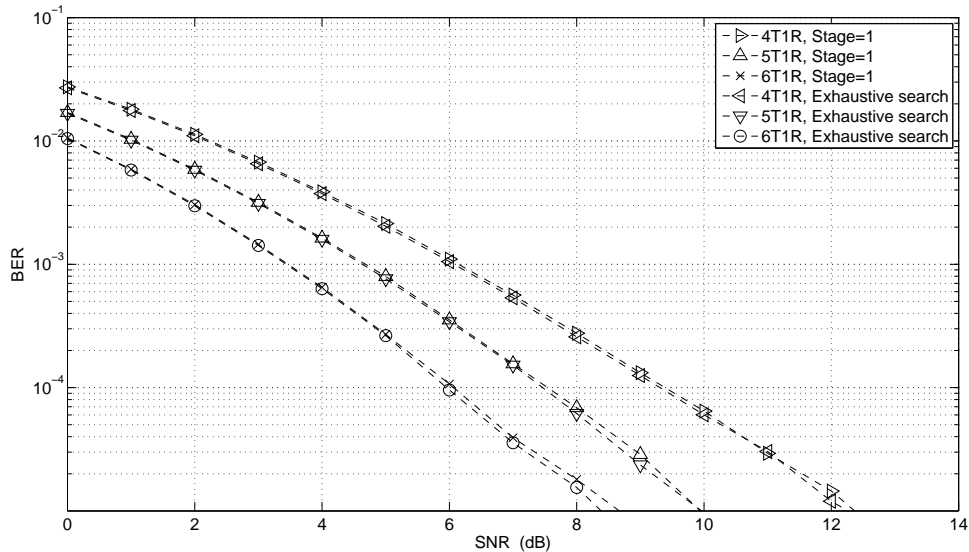


Figure 5.12: Comparisons for BER of MISO system using RVQ codebook with $b = 1$ bit for various number of transmit antennas

From (3.5), we know that the average value of $|w_i||w_k|$ with 4 transmit antennas is larger than the one with more transmit antennas under the condition that $\|\mathbf{w}\|_2 = 1$. In addition, there are more cosine values in (3.5) with more transmit antennas MIMO system. Therefore, the $|w_i||w_k|$ with smaller number of transmit antennas may play a major part in the SNR loss and thus the SNR loss decreases with the increasing number of the transmit antennas. Figs 5.15 and 5.17 are simulation for channel capacity of RVQ codebook $b = 1$ bit MISO

and MIMO systems respectively. The simulations support the result shown in BER simulation as well.

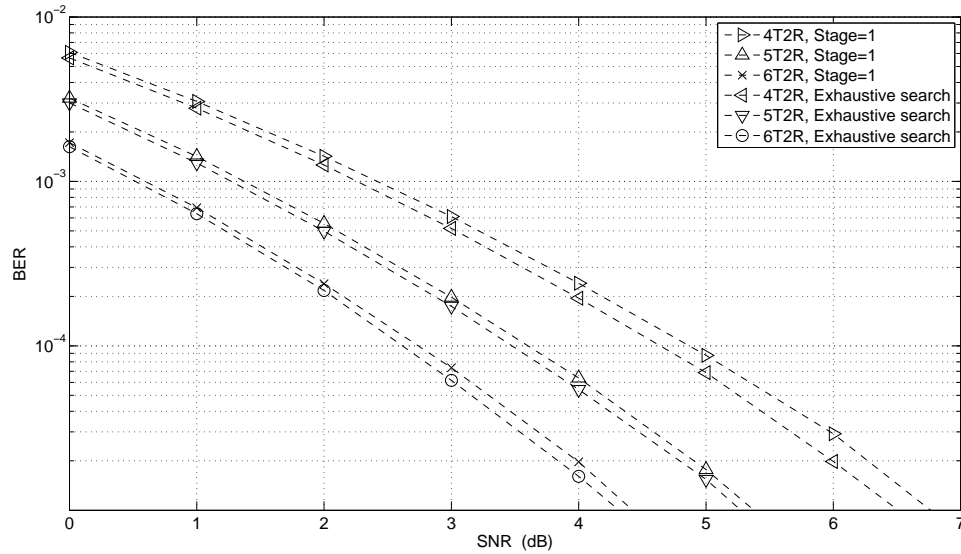


Figure 5.13: Comparisons for BER of two receive antennas MIMO system using RVQ codebook with $b = 1$ bit for various number of transmit antennas

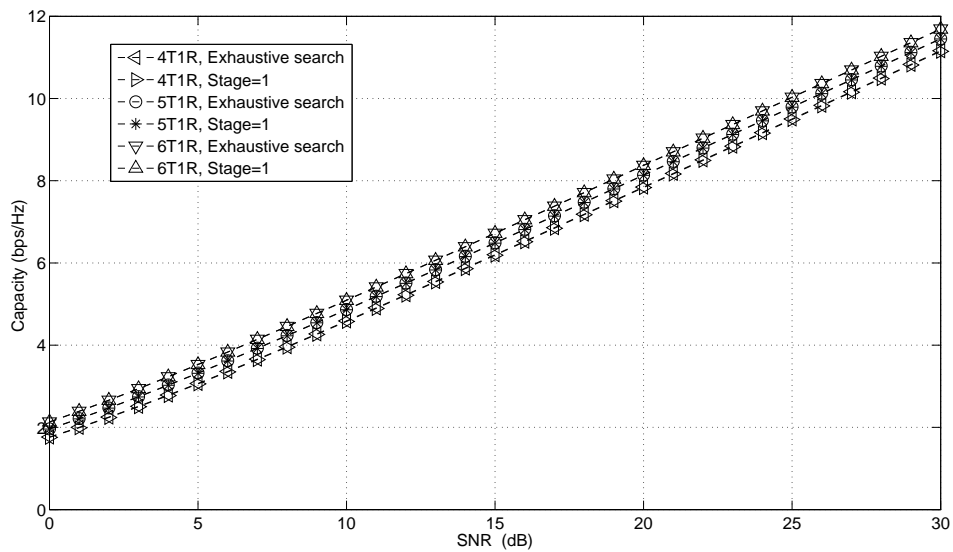


Figure 5.14: Comparisons for channel capacity of MISO system using RVQ codebook with $b = 1$ bit for various number of transmit antennas

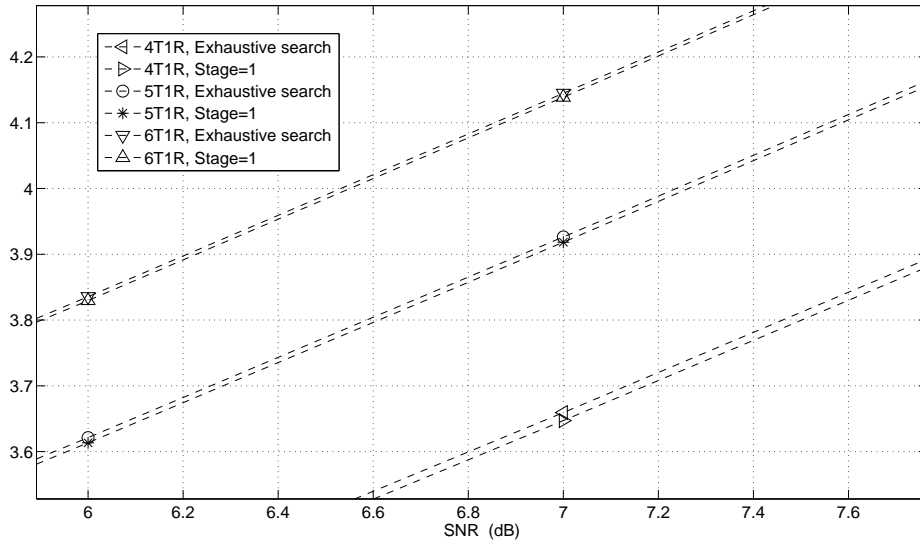


Figure 5.15: Zoom-in for Fig. 5.14

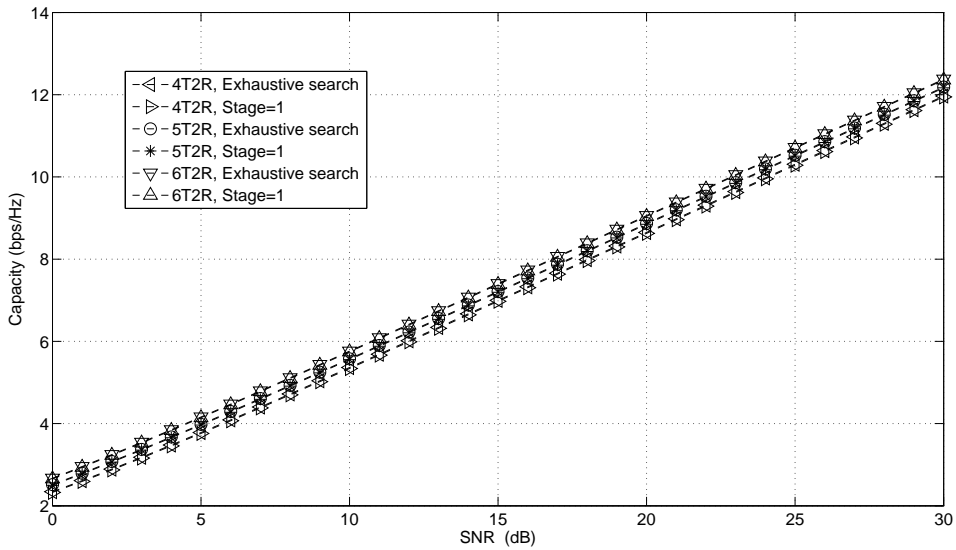


Figure 5.16: Comparisons for channel capacity of two receive antennas MIMO system using RVQ codebook $b = 1$ bit for various number of transmit antennas

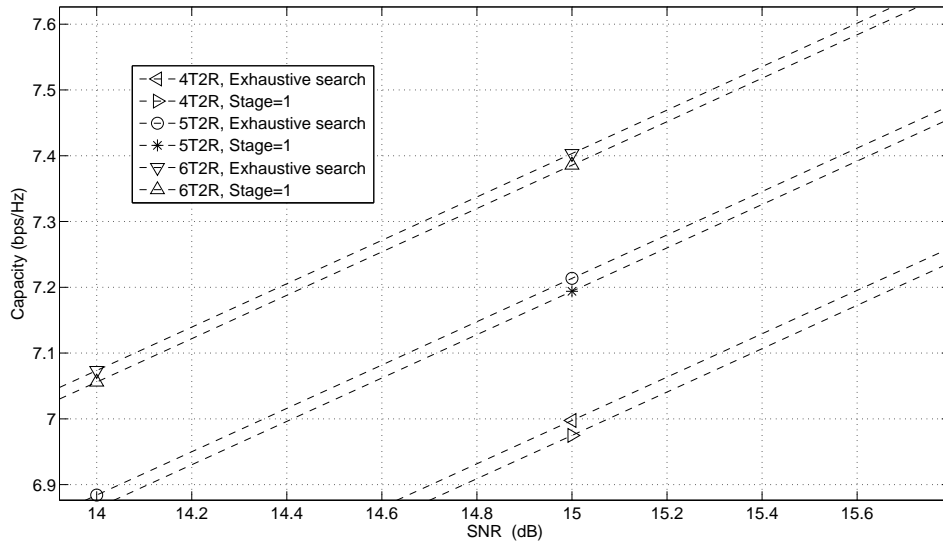


Figure 5.17: Zoom-in for Fig. 5.16

In the following simulation results, we fix the codebook size for systems with various numbers of transmit antennas. With the condition of the fixed codebook size, the average bits b to quantize the information of each element in the beamforming vector get fewer in systems with more transmit antennas. As a result, it is expected that the SNR loss increases with the number of transmit antennas. Figs. 5.18 and 5.19 show simulations for BER over RVQ codebook using $B = 5$ bits with various number of transmit antennas MISO and MIMO systems respectively and the results obviously fit our expectation.

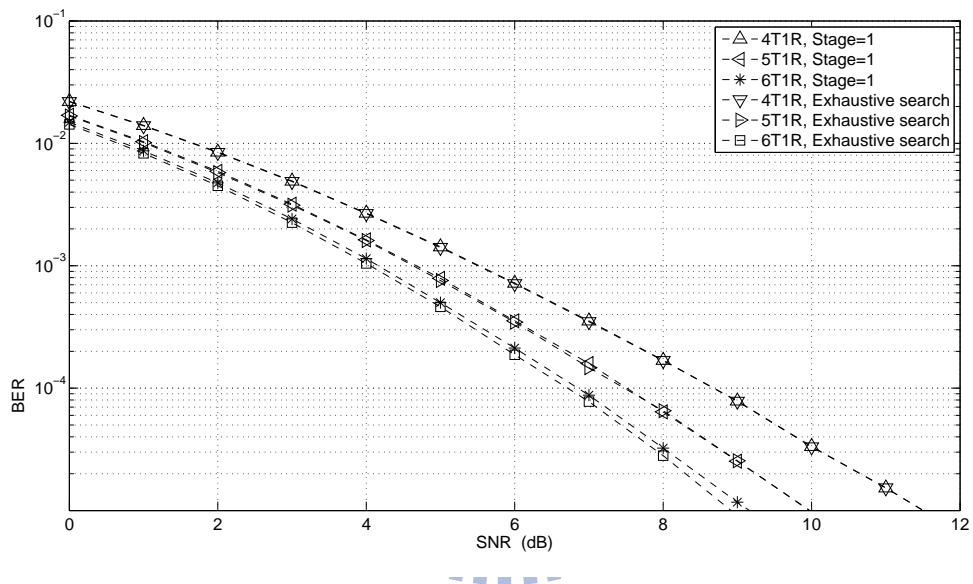


Figure 5.18: Comparisons for BER of MISO system using RVQ codebook with $B = 5$ bit for various number of transmit antennas

Figs. 5.20 and 5.22 show simulations for capacity over RVQ codebook using $B = 5$ bits with various number of transmit antennas MISO and MIMO systems respectively. From the zoom-in figures shown in Figs. 5.21 and 5.23, it is apparent that the result fits the conclusion we found in the BER simulation.

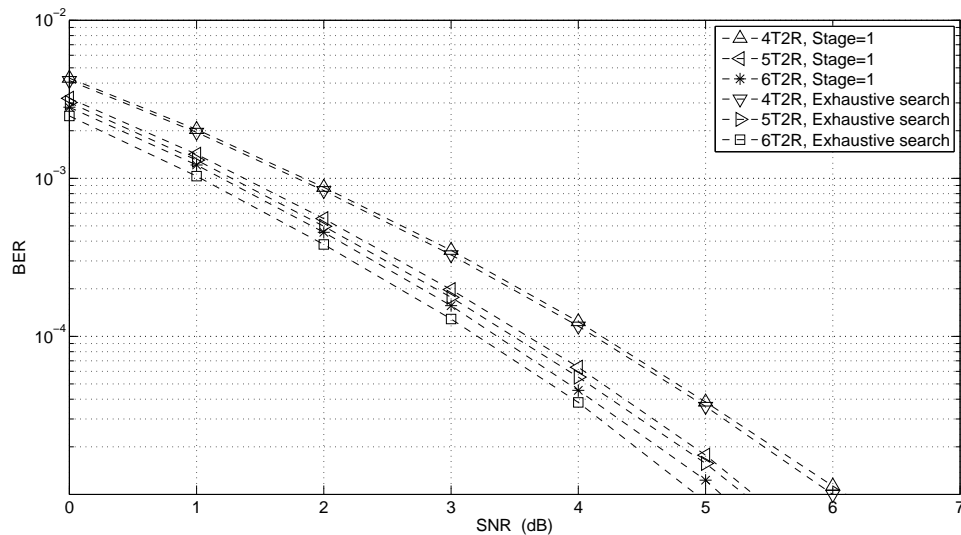


Figure 5.19: Comparisons for BER of two receive antennas MIMO system using RVQ codebook with $B = 5$ bit for various number of transmit antennas

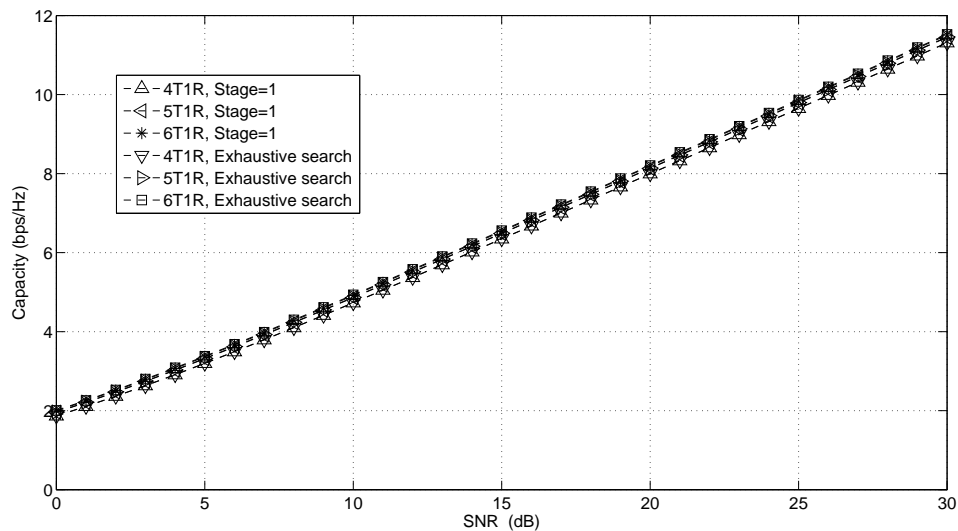


Figure 5.20: Comparisons for channel capacity of MISO system using RVQ codebook with $B = 5$ bit for various number of transmit antennas

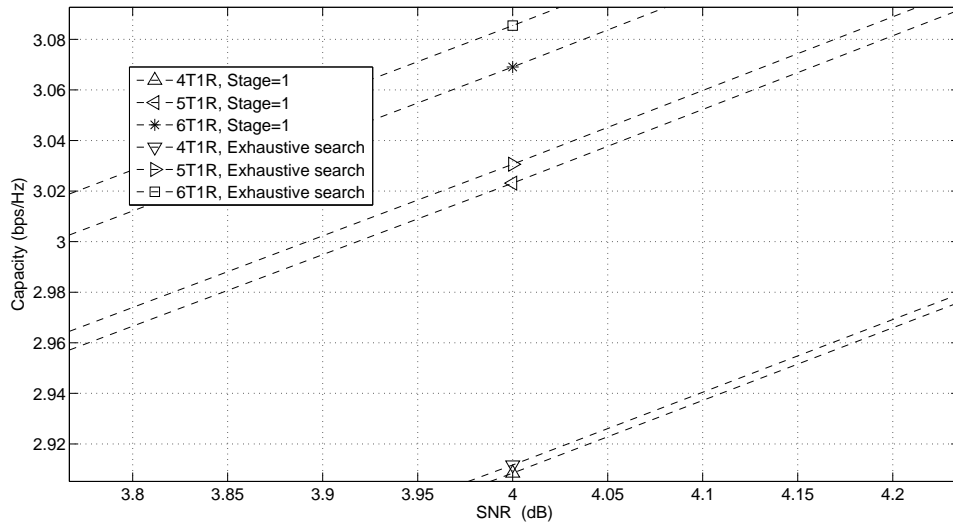


Figure 5.21: Zoom-in for Fig. 5.20

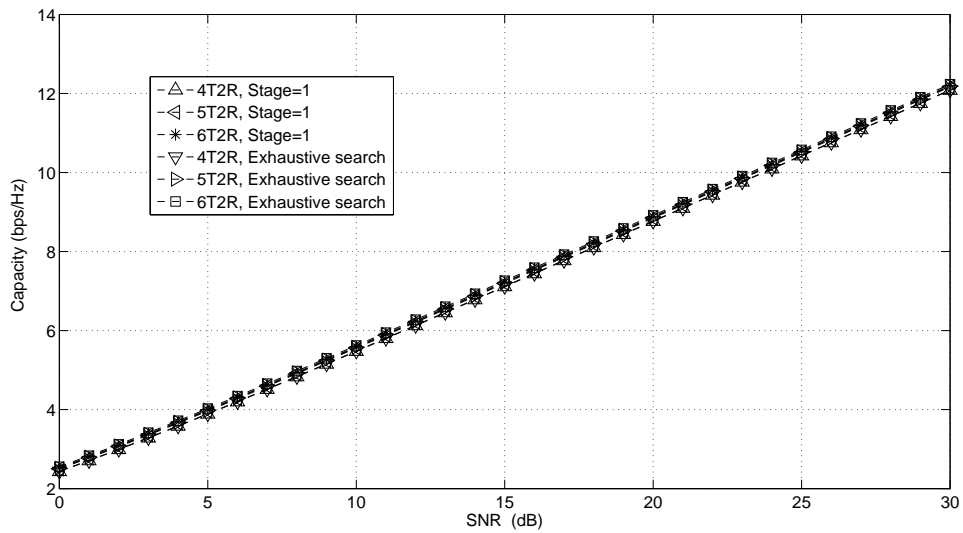


Figure 5.22: Comparisons for channel capacity of two receive antennas MIMO system using RVQ codebook $B = 5$ bit for various number of transmit antennas

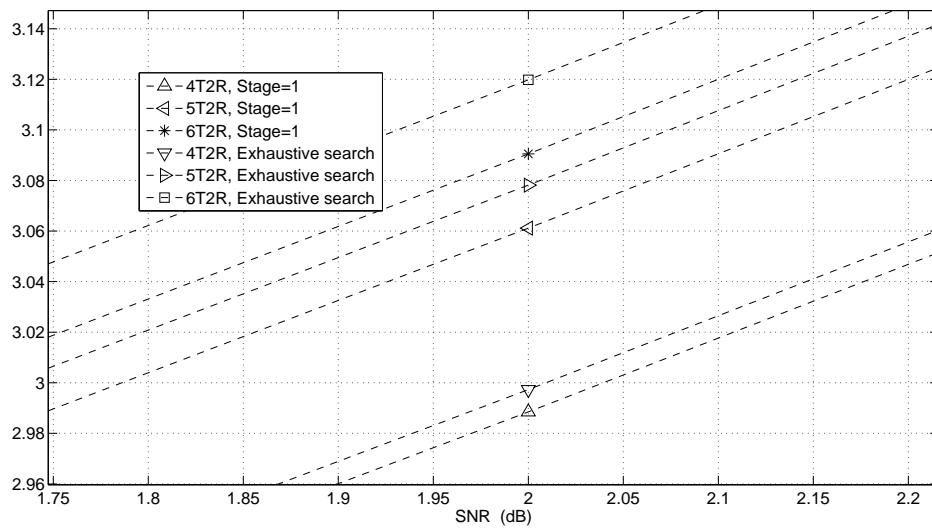


Figure 5.23: Zoom-in for Fig. 5.22

Experiment 3: Various kinds of codebooks. In this experiment, we would like to inspect the influences of different codebooks on the performance. Figs. 5.24 and 5.26 are the simulation results for 4T2R MIMO system having various codebooks with $B=6$ bits for $\mathcal{S} = 1$ and $\mathcal{S} = 2$ respectively. It is obvious that the Love's EGT codebook [7] is the worst because the codewords in it are highly correlated with each other in design. From the zoom-in figures in Figs. 5.25 and 5.27, we find that for most of the codebooks that we applied, the SNR loss is almost the same except the scalar EGT and Grassmannian codebook (note that the Grassmannian codebook for 4 transmit antennas with $B = 6$ bits is an EGT codebook). Since all the elements w_i have the same power in EGT codebook, we know that the complexity of exhaustive search can be simplified to find the most suitable phase combination. As a result, we expect the SNR loss for well designed EGT codebook is lower than the vector quantization codebook.

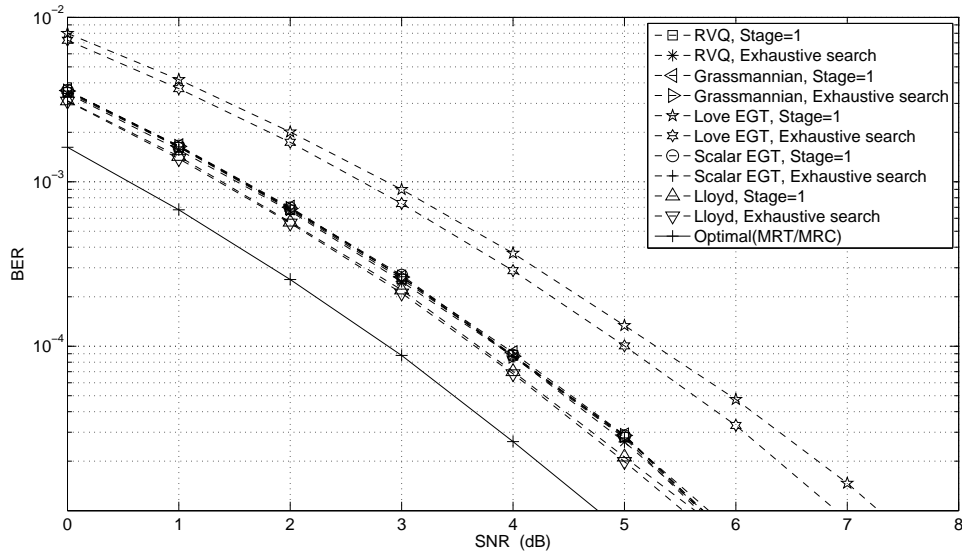


Figure 5.24: Comparisons for BER of 4T2R MIMO system using various kinds of codebooks with $B = 6$ bits for $\mathcal{S} = 1$

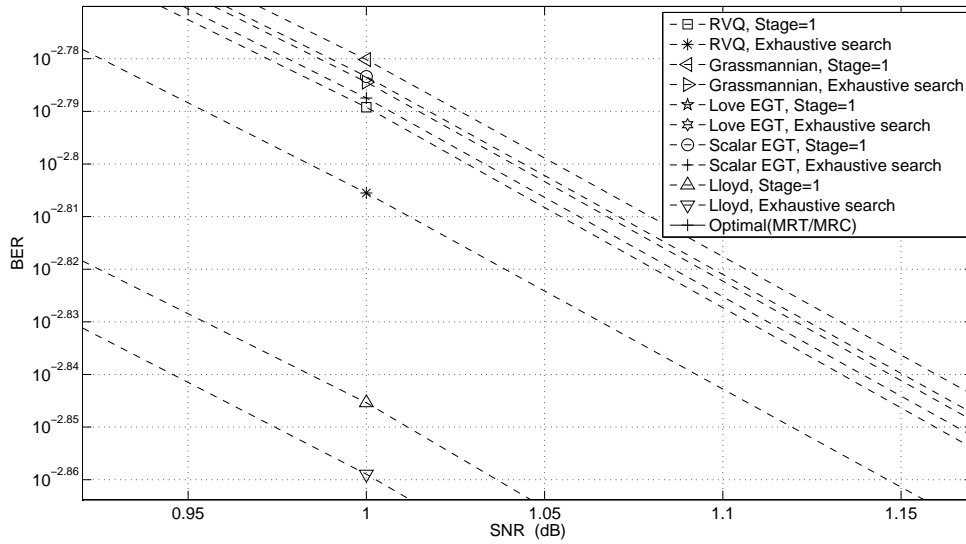


Figure 5.25: Zoom-in for Fig. 5.24

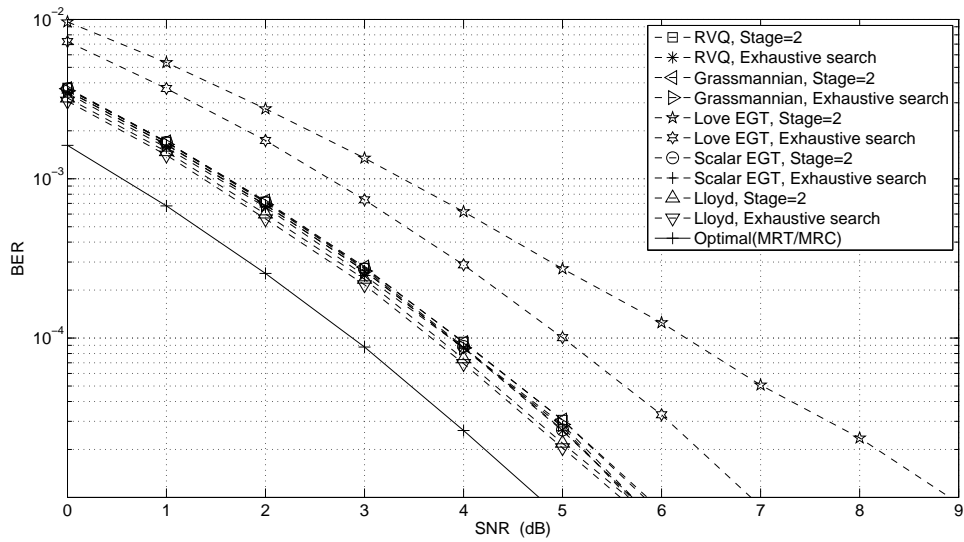


Figure 5.26: Comparisons for BER of 4T2R MIMO system using various kinds of codebooks using $B = 6$ bits for $S = 2$

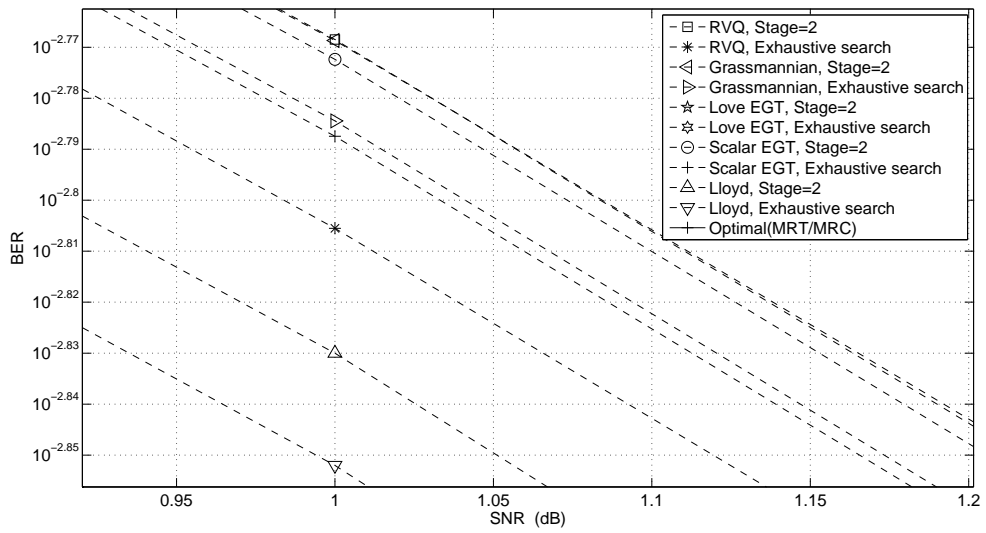


Figure 5.27: Zoom-in for Fig. 5.26

Figs. 5.28 and 5.29 are BER simulations for 4T2R systems using EGT and vector quantization with $B = 6$ bits in $\mathcal{S} = 6$ respectively. As we mentioned earlier, the SNR loss is much smaller in well designed EGT codebooks than in the vector quantization codebooks and the results support our expectation.

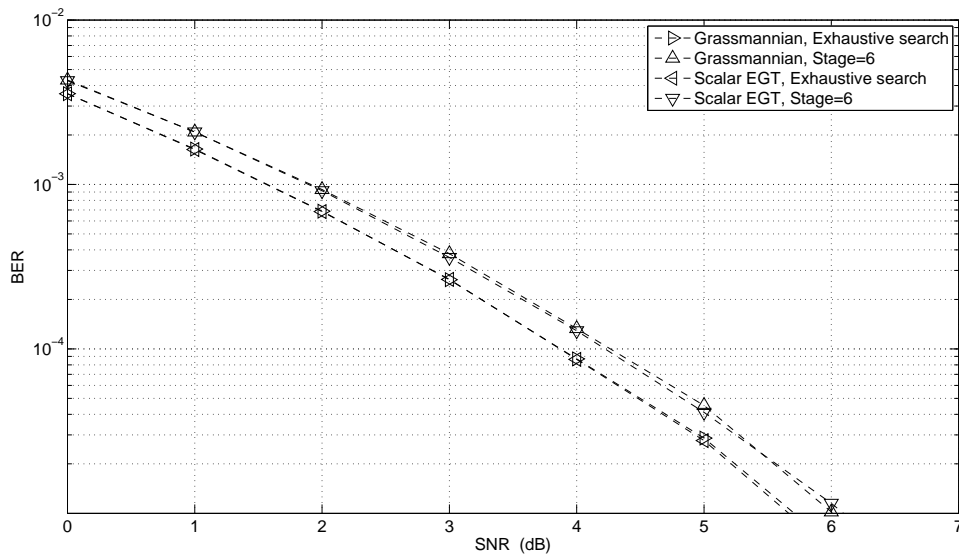


Figure 5.28: Comparisons for BER of 4T2R system using EGT quantization with $B = 6$ bits

Figs. 5.30 and figure 5.32 are channel capacity simulations for 4T2R MIMO systems using different codebooks with $B = 6$ bits. From the zoom in Figs. 5.31 and 5.33 (zoom in or Figs 5.30 and 5.32 respectively), we ensure that the performance loss is less in EGT codebooks than in other codebooks.

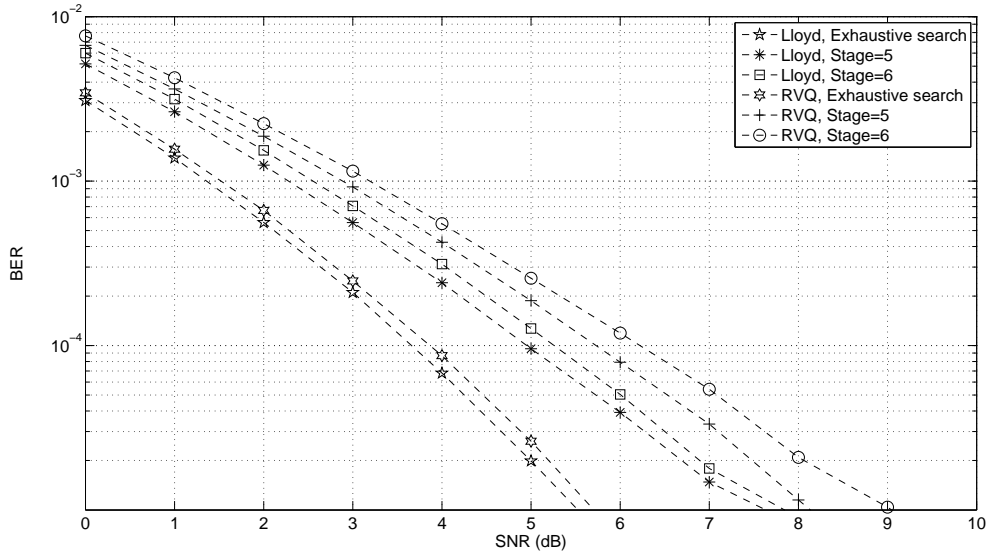


Figure 5.29: Comparisons for BER of 4T2R system using vector quantization codebooks with $B = 6$ bits

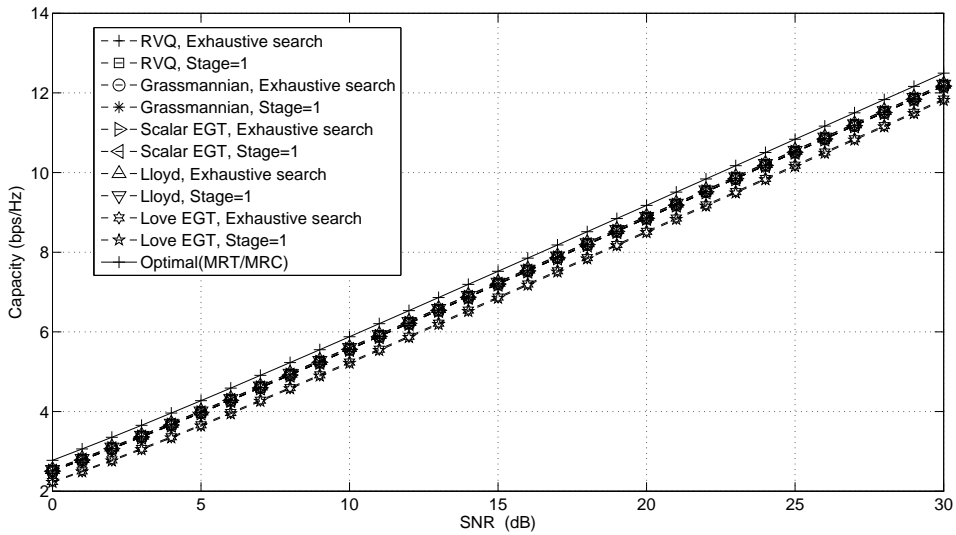


Figure 5.30: Comparisons for channel capacity of 4T2R MIMO system using the various kinds of codebooks with $B = 6$ bits for $\mathcal{S} = 1$

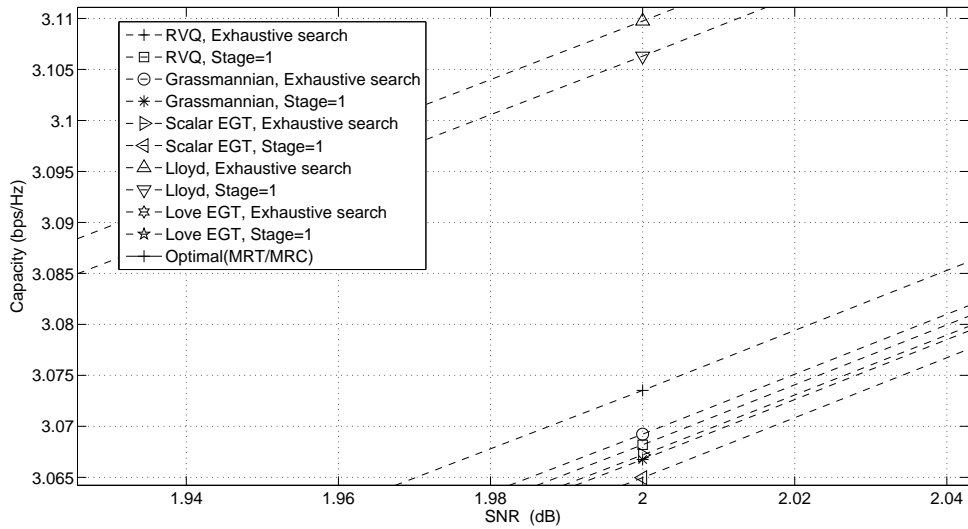


Figure 5.31: Zoom-in for Fig. 5.30

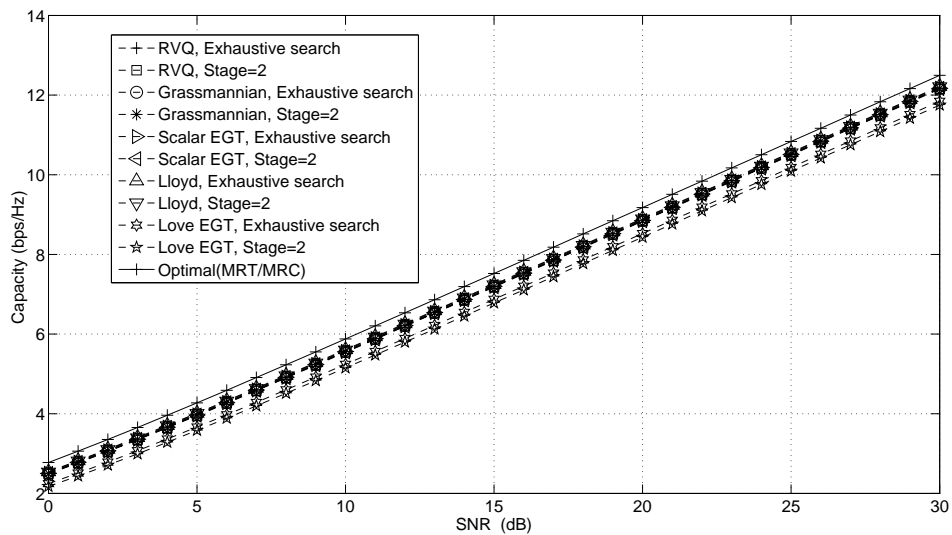


Figure 5.32: Comparisons for channel capacity of 4T2R MIMO system using various kinds of codebooks with $B = 6$ bits for $\mathcal{S} = 2$

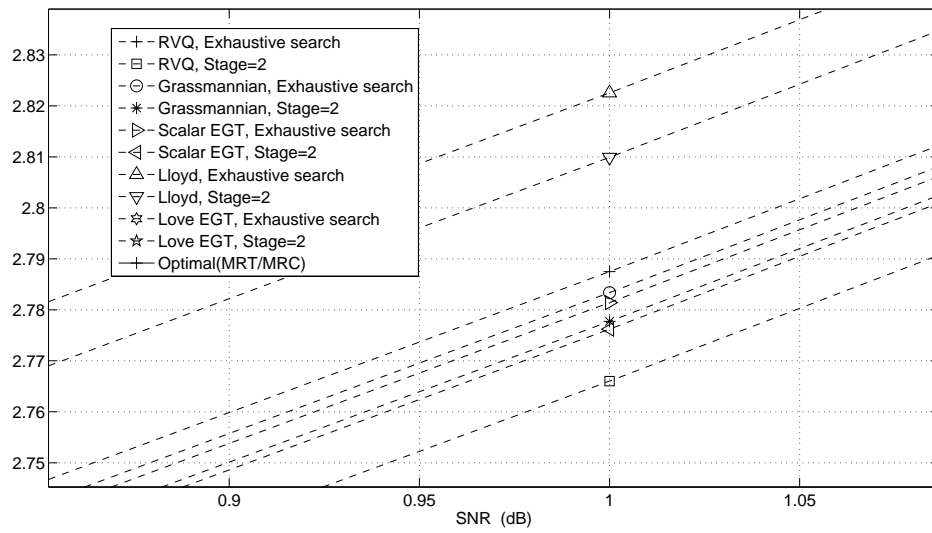


Figure 5.33: Zoom-in for Fig. 5.32

Experiment 4: Various codebook sizes As we know the fact that it has great probability that the second-best codeword will be kept while we miss the best codeword. Since in large codebook, the SNR value is very close between the best value and the second-best one. We expect that the SNR loss decreases with the the increasing of codebook size. Figs 5.34 and 5.35 are simulations for 4T2R MIMO system using RVQ codebook with various codebook sizes in $\mathcal{S} = 1$ and $\mathcal{S} = 2$ respectively and the results really support our expectation.

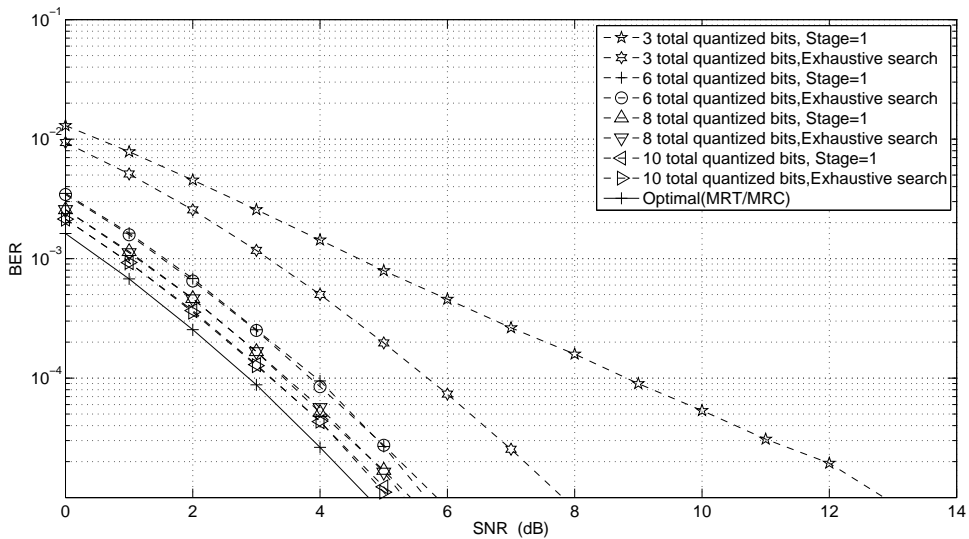


Figure 5.34: Comparisons for BER of 4T2R MIMO system using RVQ codebook with various codebook sizes for $\mathcal{S} = 1$

Fig. 5.36 is the simulation result for 4T2R MIMO system using RVQ codebook with various codebook sizes in $\mathcal{S} = 2$ bits. When the total quantized bits are 4 bits, the SNR loss is around 0.8dB. The SNR loss decreases with the total quantized bits and the gap is not manifest when the quantized bits are more than 4 bits.

Fig. 5.37 is simulation for capacity under the condition that 4T2R MIMO system using RVQ codebook with various codebook sizes in $\mathcal{S} = 2$. From the zoom-in Fig. 5.38 for Fig. 5.37, the capacity loss decreases with the booksize so the same result is revealed as the bit-error rate simulation.

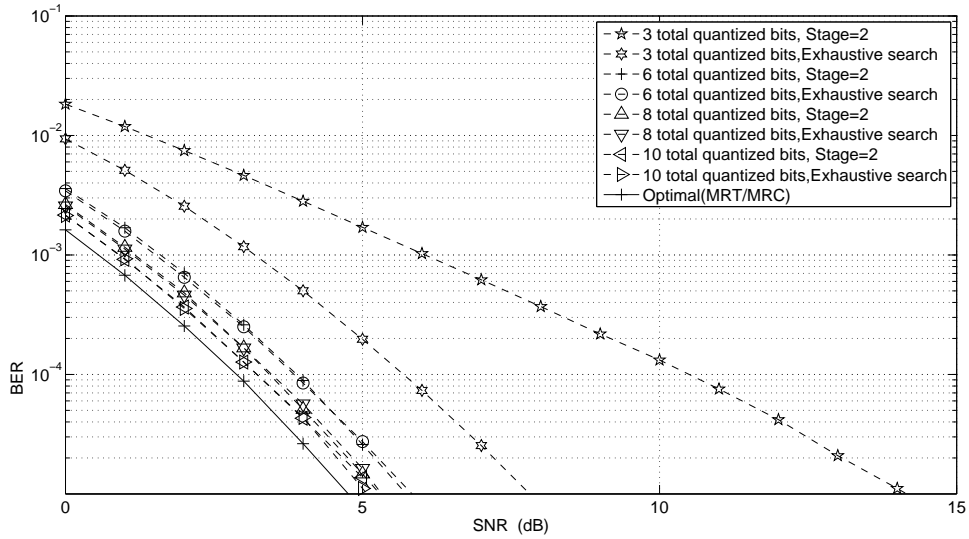


Figure 5.35: Comparisons for BER of 4T2R MIMO system using RVQ codebook with various codebook sizes for $S = 2$

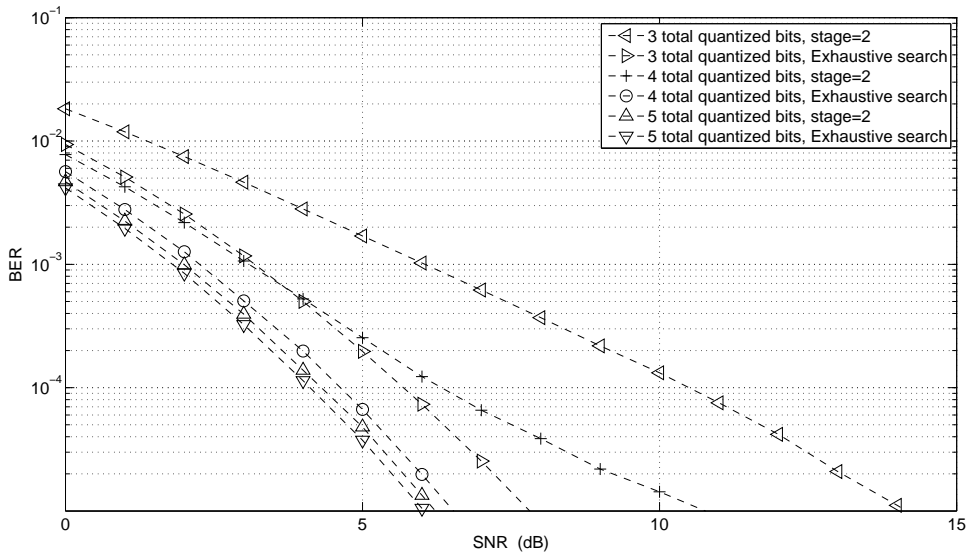


Figure 5.36: Comparisons for BER of 4T2R MIMO system using RVQ codebook with $B = 3, 4$ and 5 bits for $S = 2$

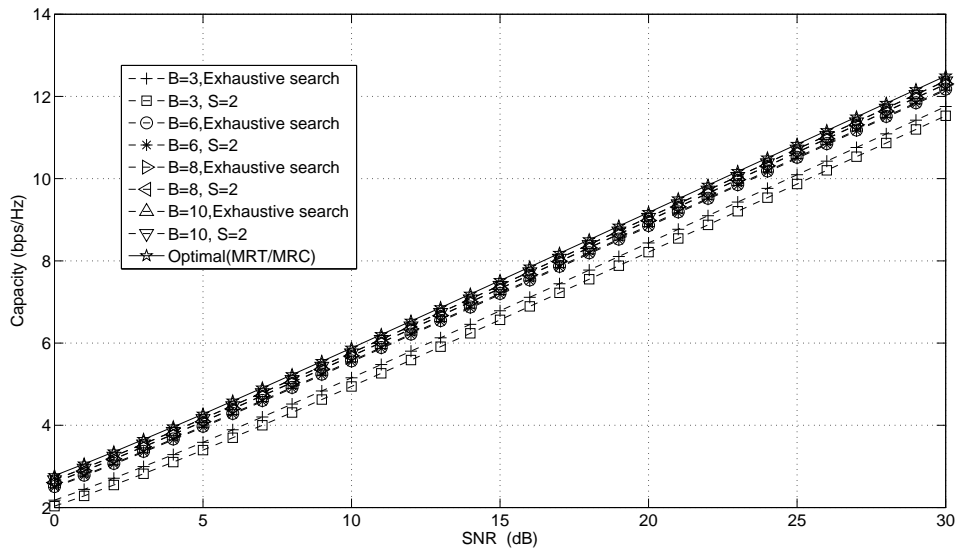


Figure 5.37: Comparisons for channel capacity of 4T2R MIMO system using RVQ codebook with various codebook sizes for $\mathcal{S} = 2$

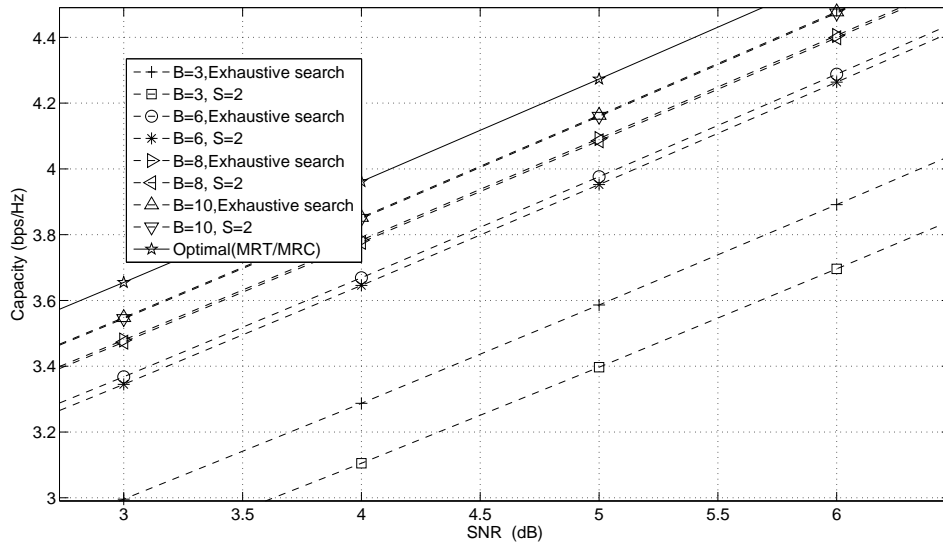


Figure 5.38: Zoom-in for Fig. 5.37

Chapter 6

Conclusion and Discussion

In this thesis, we propose an algorithm for fast codebook search. We add one more unit called “eliminator” built by the proposed algorithm in front of the “codeword determine” unit in Chapter 3. It is proven that about half codewords will be dropped off after each elimination for most codebooks. In Chapter 4, we learn the phenomenon that the second-best codeword has great chance to be chosen when the best codeword is filtered out by the eliminator. Based on the fact, we offer two lower bounds for the SNR loss by calculating the loss ratio R_L and the detection rate \mathcal{P}_d shown in (4.1). Figs. 4.3 and 4.4 shows that the lower bounds converge with the increasing of total quantized bits.

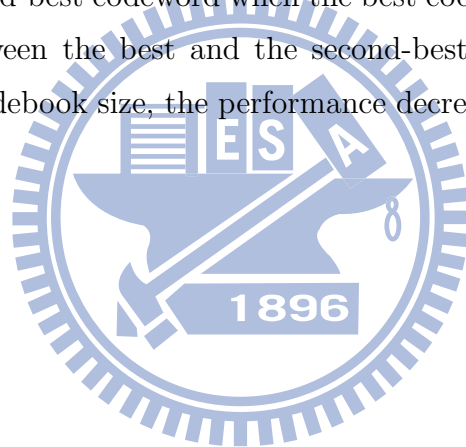
From the simulation results shown in Chapter 5, we find that the BER loss is more seriously effected than the capacity loss by the proposed algorithm. This is because that for BER, it may result in serious detection error with a channel realization when we use the worse codeword. However, for channel capacity, losing some SNR may not dominate the average capacity.

It is also found that the SNR loss increases with the number of \mathcal{S} . However, it is interesting to see that the diversity for well-designed EGT keeps in both MISO and MIMO systems but that for vector quantization loses in MIMO systems. From the Experiment 2 in Chapter 5, we verify the influence of the number of the transmit antennas on BER and capacity. From the simulation results, we find that the SNR loss decreases with number of transmit antennas. It is because that we confine the beamforming vector to be unit norm, the SNR value is divided by

more cosine values of (3.5). Therefore, the dominance of the elected cosine value for the SNR falls down in the case of more transmit antennas.

We analyze the effects of various kinds of codebooks on the SNR in the Experiment 3 as well. Both simulation results for BER and capacity show that the EGT quantization has less performance loss than the vector quantization. From concepts and procedures for the proposed algorithm in Chapter 3, it is obvious that the well-designed EGT codebooks definitely perform more excellent than the vector quantization codebooks as the magnitude of EGT codeword play a minor role on the codeword determine.

The Experiment 4 in Chapter 5 reveals that the performance loss decreases as the codebook size is large. As we shows in Fig. 4.2 of Chapter 4, it has great chance to select the second-best codeword when the best codeword is filtered out. Since the SNR loss between the best and the second-best codeword decreases with the increasing of codebook size, the performance decreases as the codebook is large.



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[20] <http://140.113.150.152/CASIC/>

