# 國立交通大學

# 電信工程學系

# 碩士論文

多天線-正交分頻多工存取無線網路之資源 配置 Resource Allocation for MIMO-OFDMA based Wireless Network

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中 華 民 國 九十八年 八月

## 多天線-正交分頻多工存取無線網路之資源配置

### Resource Allocation for MIMO-OFDMA based Wireless Network

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國立交通大學電信工程學系

碩士論文

### A Thesis

Submitted to Department of Communications Engineering

College of Electrical and Computer Engineering

National Chiao Tung University

in partial Fulfillment of the Requirements

for the Degree of

Master of Science

in

**Communications Engineering** 

August 2009

Hsinchu, Taiwan, Republic of China

中華民國九十八年八月

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#### 摘 要

在傳收兩端同時使用多根天線,配合預先編碼技術(precoding),我們可以 在每個子載波上面獲得多個不同的空間通道(spatial channel)。另一方面,正 交分頻多工存取(OFDMA)將一個寬頻帶切成許多窄頻子通道,以便平行傳送的訊 號只受到非頻率選擇性衰退(frequency nonselective fading),不但可以簡化 接收複雜度,且因子通道會隨著時間以及使用者的位置而變化,傳送端可利用這 些變化所造成的分集(diversity),視各子通道之增益適當地調整其傳輸功率與 調變訊號之階數,進而大大提高其頻寬使用效率。因此多天線-正交分頻多工存 取系統可以分配的資源就包含了空間、頻率以及使用者這三個範疇。

在本論文中我們將會探討如何有效的配置空間、頻率以及使用者這三個維度 的資源將用戶的傳輸功率與平均的位元錯誤率降到最低。我們首先提出了兩種以 奇異值分解(singular value decomposition)為基礎的預先編碼技術。我們首先 利用奇異值向量的線性組合來合成預先編碼向量以建立多個正交通道。我們提出 的第二個預先編碼技術則把通道矩陣的秩(rank)限制移除,以增加頻譜使用效 率。雖然如此一來將會讓空間通道間產生干擾,而讓資源配置的問題因需滿足某 些訊號干擾比之要求而更形複雜,但透過適當的設計,仍可將用戶間的干擾控制 在一定的範圍下。針對這兩個預先編碼技術我們分別提出不同的動態資源分配演 算法讓用戶的總傳輸功率降到最低。

除此之外,我們還針對實用的碼書(codebook)預先編碼技術之資源配置進行 探討,我們提出了動態的子載波分配及傳送功率調整等兩種基本方式及其組合來 讓用戶的平均位元錯誤率降到最低。

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## Resource Allocation for MIMO-OFDMA based Wireless Networks

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#### Abstract

By using multiple antennas at both the transmit and receive sides, one obtains multiple eigenmodes (eigenchannels) on the same carrier through beamforming (precoding). With each eigenchannel represents an equivalent SISO channel, array gain is obtained by using only the strongest eigenmode but the capacity is maximized by allocate the transmit power across subchannels according to the water-filling result. The orthogonal frequency division multiplexing (OFDM) scheme divide a wideband channel into parallel narrowband subchannels so that signals propagate through each subchannel suffer from frequency nonselective fading. The OFDM-based multiple access (OFDMA) scheme has been shown to be capable of achieving the maximum spectral efficiency with extremely high probability. A key ingredient of an OFDMA system is that it can exploit the diversity offered by the time-varying and user(location)-dependent nature of the subchannels via proper scheduling and power/subchannel allocation. Hence a MIMO-OFDMA system is expected to offer high capacity through an efficient use of the spectral, spatial and user domain resources.

The main design issue we try to solve in this thesis is the following. Given the users' rate requirements of a MIMO-OFDMA system, how to apportion the transmission resources in space, frequency, and user domains so that the total transmit power and each

user's average bit error rate are minimized. We first consider two orthogonal precoding schemes based on singular value decomposition (SVD). In the first design we construct orthogonal eigenchannels by performing linear operations on the channel matrix's singular vectors under the channel rank constraint. To improve spectral efficiency, we then remove the rank constraint on the number of users allowed on a subcarrier. Although the resulting co-channel interference may cause performance degradation, it is more than compensated for by the increased capacity through a proper RA plan that ensure the associated signal-to-interference ratios are within the tolerable limits. The proposed RA algorithms for both precoding scenarios are designed to minimize the total transmit power while satisfying the users' QoS constraints.

Finally, we examine the resource allocation (RA) issue for MIMO systems with limited feedback. More specifically, we consider the codebook based precoding scheme and suggest subcarrier assignment scheme based on the Lagrange multiplier method. For a given subcarrier assignment, we then present a power allocation method which minimizes the average bit error rate performance.

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### 誌 謝

時間過得好快,在交大的六年(大學四年、研究所兩年)眼看著即將進入尾聲,感謝學校和系上對我的栽培,讓我在這六年的求學生涯中過得充實而有意義,也讓我在電信這個領域裡學到了許多知識,我要特別感謝我的指導教授蘇育德博士,沒有他的指導,我的研究及論文不會那麼順利地完成,除了學業方面, 也要感謝蘇老師在做人處世上教導我們的一切,讓我在研究所的這兩年成熟了不少。

除此之外,還要感謝實驗室的林淵斌學長和林坤昌學長,謝謝你們在這兩年 在研究方面指導我,給我很多建議與經驗分享,幫助我論文的完成;還有實驗室 的同學們,謝謝大家這兩年的陪伴,感謝盧彥碩和廖俊傑(我的室友)這兩年陪我 經歷了很多事,希望未來大家都能有很好的發展。還要特別感謝實驗室的助理(淑 琪姊、昱岑姊),謝謝妳們幫實驗室處理好多大大小小的事情,幫我們解決好多 問題;還要感謝6號家族的林政翰學長與郭政錦學長,謝謝你們在我進大學之後 就一直照顧我,像我的哥哥一樣,謝謝你們。

感謝我的家人,謝謝我的爸爸媽媽,因為你們不求回報的付出,讓我可以沒 有後顧之憂的在求學路上朝自己的目標邁進,如果有一天能讓你們感到驕傲,我 的努力才有價值;感謝疼愛我的姊姊,對我的問題與要求總是有求必應,常常給 我很多關於生活上的意見與經驗;感謝我的好朋友徐志寧,在求學的過程中受到 你太多的幫助與指導,謝謝你。最後要謝謝我的女朋友,泳妍,謝謝妳一直陪在 我身邊,在我遇到挫折的時候鼓勵我,在我開心的時候與我一起分享,謝謝妳這 些日子和我一起度過、一起成長。

最後,對於每一個幫助過我的人,謹以此論文獻上我最深的的敬意,感謝你 們!

翁志倫謹誌 于新竹國立交通大學

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# Chapter 1 Introduction

Notwithstanding the recent worldwide financial crisis, the demand for high data rate multiuser multimedia wireless communications continues to grow. Future wireless communication systems are expected to provide even higher rate multimedia services with more varieties of QoS requirements. It has been shown that, for a given frequency band, the Orthogonal Frequency-Division Multiple Access (OFDMA) is the optimal multiple access (MA) scheme that provides the highest capacity in almost all cases if the channel information is perfectly known. The OFDMA refers to an Orthogonal Frequency-Division Multiplexing (OFDM) based MA scheme that assigns disjoint subsets of subcarriers to different users. Besides having anti-interference and anti-fading capabilities, OFDM offers another practical advantage for multimedia transmission due to its flexibility in allotting transmission resources to meet various media's bandwidth and performance requirements.

The conventional wireless capacity, however, is limited to make the most of time and frequency (or equivalent code) degrees of freedom only. The capacity can be greatly enhanced by exploiting the space domain through the use of multiple antennas at either or both ends of a wireless link. The the multiple antenna systems, now commonly known as multiple-input multiple-output (MIMO) systems, provide not only spatial and interference diversities and multiplexing gain but also makes possible space-division multiple access (SDMA). Hence it is only natural to incorporate the MIMO technique in an OFDMA system to achieve the maximum capacity.

Adaptive resource allocation (RA) is not only important for efficient resource usage but also mandatory if the theoretical channel capacity is to be achieved or approached. It is an effective means to mitigate interference and reduce the outage probability in a interference-limited system. With the spatial channels as part of the radio resource, a MIMO-OFDMA system enjoys larger degrees of freedom in distribute its radio resources (power, subcarriers, time-slots, and spatial channels). Adaptive RA methods for maximizing the capacity or throughput have been proposed [1]-[5]. Taking users' quality of service (QoS) requirements into account, [6]-[10] proposed adaptive RA algorithms that minimize the total transmit power. These earlier results either have the single-user-persubcarrier constraint or the single (strongest)-eigenmode-per-subcarrier constraint. In contrast, this thesis presents new RA schemes for MIMO-OFDM systems without the above constraints. Our schemes differ from these earlier results in that we propose two different precoding schemes that permit assigning eigenchannels on the same subcarrier to different users. We improve the "resolution" of the radio resources so that diversity gain and greater flexibility are obtained. It is also noted that the proposed precoding and the associated RA schemes can be used for uplink and downlink transmissions. A brief review of and comments on earlier works are given in **Table 1.1** and **Table 1.2**. Comparison with our work can also be found in the same table.

In order that multiple user signals over the same subcarrier can be decoupled at the receiver, we follow the conventional approach invoking the Singular Value Decomposition (SVD) to obtain the pre-processing and the post-processing vectors for different users through proper linear combinations of the singular vectors. Based on the above concept, we propose an orthogonal precoding scheme for the MIMO-OFDMA systems. The users transmitting data on the same subcarrier will cause no interference to each other. This feature will make resource allocation process more easily because once the eigen-channel assignment is done, users can do their own bit and power management individually

without consider other users' effects.

However, the user number on the same subcarrier of the orthogonal precoding scheme will be bounded by the rank of the MIMO channel matrix such that the spectrum efficiency may be constrained. In order to improve the efficiency of the radio resource utilization, we bring up another precoding scheme which not only provides orthogonal but also non-orthogonal eigenchannels for transmission. In this scheme, we allow more users to transmit data on the same subcarrier than the first scheme to improve spectrum efficiency. Nevertheless, The co-channel interference is no longer avoidable in this scheme. In this situation, the resource optimization problem will become more complicated.

For the first scheme, we propose two adaptive RA algorithms that take care of subcarrier assignment, pre-processing and post-processing vectors selection and bit allocation. They are designed to minimize the total power and meet each user's QoS requirement. It should be noted that although the power consumption problem is always considered as a critical issue especially for the uplink transmission due to the power-limited feature of the mobile devices, we still take both the downlink and uplink transmission into account in this thesis for generality. Similarly, we propose an adaptive RA algorithm for the non-orthogonal scheme according to its system structure. The adaptive algorithm also aims to minimize the total consuming power while each user's QoS requirement is guaranteed.

In addition to the two SVD based precoding schemes we design, we also take the precoder with limited feedback into account [11]-[12]. For such the codebook based precoding scheme, we perform subcarrier assignment and dynamic power loading in order to minimize the average bit error rate (BER).

The rest of this thesis is organized as follows. The ensuing chapter describes the orthogonal precoding scheme for the MIMO-OFDMA systems and the two proposed adaptive RA algorithms. In Chapter 3, we discuss the design of the non-orthogonal precoding scheme and the corresponding adaptive RA algorithm. In Chapter 4, the resource allocation for codebook based precoding scheme is investigated. The numerical simulation results are all given in the end of these chapters. Finally, we give a conclusion for the resource allocation we have done for the MIMO-OFDMA systems in the last chapter.



Source	Assumptions	Precoding	Criterion	Constraint	Proposed algorithm	Comments
[1]	Multiuser, downlink,	SVD, all eigenmodes	Maximize total	BER and power	Subcarrier assign-	Low complexity, but
	one user per subcar-	are used.	throughput	constraint	ment and power/bit	power/bit allocation
	rier				allocation	is not optimal.
[2]	Multiuser, downlink,	SVD, all eigenmodes	Maximize total	power constraint	Subcarrier assign-	Low complexity, bit
	one user per subcar-	are used.	throughput	and packet delay	ment and power/bit	allocation is not op-
	rier				allocation	timal.
[3]	Multiuser, downlink,	SVD, all eigenmodes	Maximize total	BER and power	Subcarrier assign-	Low complexity, but
	one user per subcar-	are used.	throughput	constraint	ment and power/bit	the assignment for
	rier	1	E		allocation	subcarriers is not
		89	S			clearly described.
[4]	Multiuser, downlink,	VBLAST VBLAST	Maximize total	Power constraint	Subcarrier assign-	Resource allocation
1	one user per subcar-		throughput	and proportional	ment and power/bit	for VBLAST system
	rier			data rate fair-	allocation	is considered.
				ness		
[5]	Multiuser, downlink,	SVD, all eigenmodes	Maximize total	BER constraint	Subcarrier as-	Low complexity, and
	one user per subcar-	are used.	throughput	and user fairness	signment and bit	the definition of the
	rier				allocation	user priority con-
						siders user through-
						put, buffer size, and
						packet delay.

Table 1.1: A brief review and comments of previous works I

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Table

[9]	Multiuser, downlink,		Minimize total	Data rate con-	Subcarrier assign-	Low complexity, but
	one user/subcarrier		transmit power	straint	ment and power/bit	the bit-loading algo-
					allocation	rithm is not optimal.
[2]	Multiuser, uplink, one	SVD, strongest eigen-	Minimize total	Data rate con-	Lagrange multiplier	The ZF receiver is
	user/subcarrier	mode is used, ZF re-	transmit power	straint	method is used to	restricted by the
		ceiver			get loading bits	rank of the channel
						matrix.
[8]	Multiuser, downlink,	Alamouti scheme,	Minimize total	Data rate and	Subcarrier assign-	Adaptive modu-
	one user/subcarrier		transmit power	BER constraint	ment	lation and power
						allocation is not
						discussed.
[6]	Multiuser, downlink,	Orthogonal beam-	Minimize total	Data rate and	User selection	Only consider the
	one user/subcarrier,	forming,	transmit power	BER constraint		case with one an-
	one antenna at the	18	s			tenna at the re-
	receiver	9				ceiver.
[10]	Single user, downlink,	SVD, strongest eigen-	Minimize total	Data rate and	Bit and power allo-	Only consider the
	one user/subcarrier	mode is used.	transmit power	BER constraint	$\operatorname{cation}$	single user case.
[a], [b]	Multiuser, down-	SVD, all eigenmodes	Minimize total	Data rate and	Eigenchannel assign-	Higher radio re-
	link/uplink, multiuser	can be used.	transmit power	BER constraint	ment and bit alloca-	source resolution,
	per subcarrier				tion	affordable complex-
						ity.
[c]	Multiuser, uplink, one	Codebook based pre-	Minimize aver-	Power constraint	Subcarrier assign-	Low complexity,
	user per subcarrier	coding	age BER	and proportional	ment and power	power allocation is
				data rate fair-	allocation	optimal.
				ness		
[a]:Ort	thogonal precoding schem	the sources of the corresponding	resource allocation	algorithms propos	ed in this thesis.	
oN:[d]	n-orthogonal precoding se	cheme and the correspondence	ding resource alloc	ation algorithm prc	posed in this thesis.	
[c]:Re	source allocation for code	book based precoding pre	oposed in this thesi	is.	ı	

## Chapter 2

# Resource Allocation for Orthogonally Precoded MIMO Systems

## 2.1 Background

In recent years, the MIMO technology has drawn much attention since it promises a capacity that is proportional to the smallest number of antennas used at the transmit and the receive sites [13]. Many novel MIMO transceiver designs have been proposed and verified in past decade. In [14], the BLAST (Bell Labs Layered Space-Time) architectures proposed exploits the capacity advantage of multiple antenna systems for multiplexing. A simple but ingenious transmit diversity technique-the Alamouti scheme [15], is designed to achieve diversity gain and has been adopted in the standards of many communication systems. In addition, SVD also can be used in MIMO systems as the beam patterns of the beamforming technology to improve the system performance. Some SVD based orthogonalization schemes have been proposed [16]-[17] for MIMO precoding such that the CCI can be minimized. A basic assumption used is that there exits enough orthogonal spatial channels that each user will have access to at least one of them, which, unfortunately, may not always be valid. Besides SVD based precoding,

there are also other precoding schemes such as lattice-reduction based precoding [18] or codebook based precoding [11]-[12]. In the design of our precoding scheme, we use the SVD to obtain the basis of the precoding vectors.

### 2.2 System Parameters and Transceiver Model

Consider a MIMO-OFDMA system with a single base station (BS) equipped with  $T_x$ antennas and K mobile station (MS) users, each equipped with  $R_x$  antennas. The frequency band used contains M subcarriers which are to be allocated to the K MS'. Besides orthogonal subcarriers, such a system provides additional spatial channels for transmission.

Let the kth MS' channel matrix for subcarrier m be denoted by the  $R_x \times T_x$  matrix  $\mathbf{H}_{mk}$ . Applying SVD to  $\mathbf{H}_{mk}$  gives

$$\mathbf{H}_{mk} = \mathbf{U}_{mk} \mathbf{\Lambda}_{mk} \mathbf{V}_{mk}^{\dagger}$$
(2.1)

where  $\mathbf{U}_{mk}$  contains the left singular vectors of  $\mathbf{H}_{mk}$  and  $\mathbf{U}_{m,k}$  contains the right singular vectors of  $\mathbf{H}_{mk}$ .  $\mathbf{\Lambda}_{mk}$  is the diagonal matrix with diagonal entries being the singular values (SVs). In order to separate the signals from different user perfectly the proposed scheme provides at most R eigen-channels on the same subcarrier where R is the rank of the MIMO channel matrix. (Here we assume that the channel matrices of all users are all full rank. Although there are still the case that the channel matrix may be rank-deficient due to the spatial correlation, we can still suppose the channel matrix be full rank with some neglectable eigenmode magnitudes.) It is well known that the right and left singular vectors can be used as the pre-processing and post-processing vectors such that the receiver can easily extract the data symbol without interference.

Define the eigenchannel coefficient  $A_{rmk}$  by  $A_{rmk} = 1$  if user k is to use the mth subcarrier's rth eigenchannel and  $A_{rmk} = 0$ , otherwise. Then the received signal corre-

sponds to subcarrier m of user k can be expressed as

$$\mathbf{y}_{mk} = \mathbf{H}_{mk} \sum_{i=1}^{R} \sum_{j=1}^{K} A_{imj} \sqrt{p_{imj}} \mathbf{t}_{imj} x_{imj} + \mathbf{n}_{mk}, \qquad (2.2)$$

where  $x_{imj}$  denotes the data symbol of user j carried by the *i*th eigenchannel of subcarrier m.  $\mathbf{T}_{imj}$  and  $p_{imj}$  are the pre-processing vector and transmit power, respectively. The entries of the  $R_x \times 1$  noise vector  $\mathbf{n}_{mk}$  are i.i.d. zero-mean complex Gaussian random variables with variance  $\sigma^2$ .  $x_{rmk}$ , the *k*th user's data symbol transmitted over the *r*th eigenchannel of subcarrier m, is pre-multiplied by the pre-processing vector  $\mathbf{T}_{rmk}$  to form the  $T_x$  data symbols which are then transmitted with power  $p_{rmk}$ . Pre-multiplying the received signal  $\mathbf{y}_{mk}$  by the post-processing vector  $\mathbf{W}_{rmk}$ , we obtain

$$\mathbf{r}_{rmk} = \mathbf{w}_{rmk}^{\dagger} \mathbf{y}_{mk}$$
$$= \mathbf{w}_{rmk}^{\dagger} \mathbf{H}_{mk} \sum_{i=1}^{R} \sum_{j=1}^{k} A_{imj} \sqrt{p_{imj}} \mathbf{t}_{imj} x_{imj} + \mathbf{w}_{rmk}^{\dagger} \mathbf{n}_{mk}$$

where † denotes conjugate transpose.

## 2.3 Spatial Channel Assignment and Related Signal Processing

We use a simple example to illustrate the basic idea of the proposed scheme. Assume  $T_x = R_x = 2, K = 2$  and the subcarrier *m* channel matrices for the two users,  $\mathbf{H}_{m1}$  and  $\mathbf{H}_{m2}$ , are of full rank and have the SVDs

$$\mathbf{H}_{m1} = \mathbf{U}_{m1} \mathbf{\Lambda}_{m1} \mathbf{V}_{m1}^{\dagger}$$
$$= \left[\mathbf{u}_{11}^{m} \mathbf{u}_{12}^{m}\right] \left[\begin{array}{c} s_{11}^{m} & 0\\ 0 & s_{12}^{m} \end{array}\right] \left[\begin{array}{c} \mathbf{v}_{11}^{m\dagger}\\ \mathbf{v}_{12}^{m\dagger} \end{array}\right]$$
(2.3)

and

$$\mathbf{H}_{m2} = \mathbf{U}_{m2} \mathbf{\Lambda}_{m2} \mathbf{V}_{m2}^{\dagger}$$
$$= \begin{bmatrix} \mathbf{u}_{21}^{m} \mathbf{u}_{22}^{m} \end{bmatrix} \begin{bmatrix} s_{21}^{m} & 0\\ 0 & s_{22}^{m} \end{bmatrix} \begin{bmatrix} \mathbf{v}_{21}^{m\dagger}\\ \mathbf{v}_{22}^{m\dagger} \end{bmatrix}$$
(2.4)

where  $s_{11}^m = \max\{s_{ij}^m : i, j = 1, 2\}$ . We assign the strongest eigenchannel to user 1 and allow user 2 to use one that is orthogonal to the strongest one. In other words, we use  $\mathbf{v}_{11}^m$  and  $\mathbf{u}_{11}^m$  as the pre-processing vectors and assume those for user 2 are of the form  $\bar{\mathbf{v}}_2^m = \alpha_1 \mathbf{v}_{21}^m + \alpha_2 \mathbf{v}_{22}^m$  and  $\bar{\mathbf{u}}_2^m = \beta_1 \mathbf{u}_{21}^m + \beta_2 \mathbf{u}_{22}^m$ , where  $\alpha$  and  $\beta$  are weighting coefficients to be determined. The corresponding received signal from user 1 is given by

$$\mathbf{y}_{m1} = \mathbf{H}_{m1}(\sqrt{p_{m1}}\mathbf{v}_{11}^m x_{m1} + \sqrt{p_{m2}}\bar{\mathbf{v}}_2^m x_{m2}) + \mathbf{n}_{m1},$$
(2.5)

which, after post-processing, becomes

$$\mathbf{r}_{1m1} = \mathbf{u}_{11}^{m\dagger} \mathbf{y}_{m1}$$

$$= \mathbf{u}_{11}^{m\dagger} \mathbf{H}_{m1} (\sqrt{p_{m1}} \mathbf{v}_{11}^m x_{m1} + \sqrt{p_{m2}} \bar{\mathbf{v}}_2^m x_{m2}) + \mathbf{n}_{m1}$$

$$= \sqrt{p_{m1}} s_{11}^m x_{m1} + s_{11}^m (\alpha_1 \mathbf{v}_{11}^{m\dagger} \mathbf{v}_{21}^m + \alpha_2 \mathbf{v}_{11}^{m\dagger} \mathbf{v}_{22}^m) \mathbf{u}_{11}^m + \mathbf{u}_{11}^{m\dagger} \mathbf{n}_{m1}.$$
(2.6)

To eliminate cochannel interference from user 2, we need

$$\alpha_1 = -\frac{\mathbf{v}_{11}^{m\dagger} \mathbf{v}_{22}^m}{\mathbf{v}_{11}^{m\dagger} \mathbf{v}_{21}^m} \alpha_2.$$
(2.7)

Similarly, to completely suppress interference into user 2's received waveform, we need

$$\beta_1 = -\frac{s_{22}^m \mathbf{v}_{22}^{m\dagger} \mathbf{v}_{11}^m}{s_{21}^m \mathbf{v}_{21}^{m\dagger} \mathbf{v}_{11}^m} \beta_2.$$
(2.8)

The resulting  $\alpha$ 's and  $\beta$ 's should then be normalized such that the norm of the processing vectors are all equal to unity.

The received signal-to-noise ratio (SNR) for the two eigenchannels are given by

$$SNR_{m1(1)} = \frac{(s_{11}^m)^2 p_{m1}}{\sigma^2},$$
 (2.9)

$$SNR_{m2(2)} = \frac{(s_{21}^m \alpha_1 \beta_1 + s_{22}^m \alpha_2 \beta_2)^2 p_{m2}}{\sigma^2}$$
(2.10)

where the numbers in the subscript brackets denote the indices of the users who have the access to the corresponding eigenchannels. To be more specific, if the BS wants to transmit the data to user k through the rth eigenchannel on subcarrier m, we should have

$$\mathbf{w}_{imj_i}^{\dagger} \mathbf{H}_{mj_i} \mathbf{t}_{rmk} \sqrt{p_{rmk}} x_{rmk} = 0 \qquad i = 1, \cdots, (r-1)$$
(2.11)

and

$$\mathbf{w}_{rmk}^{\dagger}\mathbf{H}_{mk}\mathbf{t}_{imj_i}\sqrt{p_{imj_i}}x_{imj_i} = 0 \qquad i = 1, \cdots, (r-1)$$
(2.12)

where  $j_i$  denotes the index of the user to whom the *i*th eigenchannel is assigned. Since  $\mathbf{t}_{rmk}$  and  $\mathbf{w}_{rmk}$  can be written as

$$\mathbf{t}_{rmk} = \sum_{l=1}^{r} \alpha_l \mathbf{v}_{kl}^m, \tag{2.13}$$

$$\mathbf{w}_{rmk} = \sum_{l=1}^{r} \beta_l \mathbf{u}_{kl}^m \tag{2.14}$$

(2.9) and (2.10) are equivalent to

$$\mathbf{w}_{imj_i}^{\dagger} \mathbf{H}_{mj_i} \left( \sum_{l=1}^{r} \alpha_l \mathbf{v}_{kl}^m \right) \sqrt{p_{rmk}} x_{rmk} = 0 \qquad i = 1, \cdots, (r-1)$$
(2.15)

and

$$\left(\sum_{l=1}^{r} \beta_l \mathbf{u}_{kl}^m\right)^{\dagger} \mathbf{H}_{mk} \mathbf{t}_{imj_i} \sqrt{p_{imj_i}} x_{imj_i} = 0 \qquad i = 1, \cdots, (r-1).$$
(2.16)

The above equations and the condition that the the precoding vectors should be normalized to render unity norm imply that the corresponding gain to noise ratio (GNR) is given by

$$GNR_{mr(k)} = \frac{\left|\sum_{i=1}^{r} \alpha_i \beta_i s_{ki}\right|^2}{\sigma^2}.$$
(2.17)

Similarly, if user k wants to transmit the data to the BS through the rth eigenchannel on subcarrier m, the following identities should be satisfied.

$$\left(\sum_{l=1}^{r} \beta_l \mathbf{u}_{kl}^m\right)^{\dagger} \mathbf{H}_{mk} \mathbf{t}_{imj_i} \sqrt{p_{rmk}} x_{rmk} = 0 \qquad i = 1, \cdots, (r-1)$$
(2.18)

and

$$\mathbf{w}_{imj_i}^{\dagger} \mathbf{H}_{mj_i} \left( \sum_{l=1}^{r} \alpha_l \mathbf{v}_{kl}^m \right) \sqrt{p_{imj_i}} x_{imj_i} = 0 \qquad i = 1, \cdots, (r-1).$$
(2.19)

Our design philosophy is force the user whose candidate transmit channels have weaker eigenmodes to "fit" the user(s) with stronger eigenmodes by transmitting over an eigenchannel which lies within the dual space of the space spanned by all previously selected eigenchannels. Each new eigenchannel is obtained by using proper processing vectors which are linear combinations of known eigenvectors. The process is similar to a Gram-Schmidt orthonormalization process except that the process follows the descending eigen-magnitude order. Hence, a precoder based on the above design procedure is henceforth referred to as a Gram-Schmidt (GS) precoder.

Once the assignment and the orthogonalizing weighting coefficients of the first r eigenchannels are determined, the corresponding GNR can be computed accordingly.

## 2.4 Problem Formulation

Now we are ready to recast in mathematical form the RA problem of assigning subcarriers, and the corresponding power and the number of bits loaded to users such that the total transmit power of the system is minimized while the QoS of each user is satisfied. Let  $R_k$  be the rate requirement for user k (bits/per OFDM symbol) and  $b_{rmk}$  the number of bits transmitted over the *m*th subcarrier using the *r*th eigenchannel.  $b_{max}$  denotes the maximum bit number (the highest modulation order) allowed to be carried by an eigenchannel. The RA problem can then be stated as

$$\arg\min_{A_{rmk}, p_{rmk}} \sum_{m=1}^{M} \sum_{r=1}^{R} \sum_{k=1}^{K} A_{rmk} p_{rmk}$$
(2.20)

subject to the constraints:

$$\sum_{r=1}^{R} \sum_{m=1}^{M} b_{rmk} = R_k \qquad \forall k$$
(2.20a)

$$\sum_{r=1}^{R} \sum_{k=1}^{K} A_{rmk} = R \qquad \forall m \qquad (2.20b)$$

$$A_{rmk} \in \{0, 1\} \qquad \forall r, m, k \tag{2.20c}$$

$$p_{rmk} \ge 0 \qquad \forall \ r, m, k \tag{2.20d}$$

$$b_{max} \ge b_{rmk} \ge 0 \qquad \forall r, m, k$$

$$(2.20e)$$

where  $p_{rmk} = f(BER_k, b_{rmk}, GNR_{mr(k)})$  if  $A_{rmk} = 1$  and  $p_{rmk} = 0$ , otherwise.  $BER_k$ represents user k's target BER and  $f(\cdot, \cdot, \cdot)$  usually has a closed-form expression. If an M-ary quadrature amplitude modulation (M-QAM) is employed, then  $f(\cdot, \cdot, \cdot)$  or  $p_{rmk}$ is given by [19]

$$p_{rmk} = \frac{1}{GNR_{mr(k)}} \ln\left(\frac{1}{5BER_k}\right) \frac{2^{b_{rmk}} - 1}{1.5}$$
(2.21)

It is noted that we only consider the case that  $\sum_{i=1}^{K} R_i \leq RMb_{bmax}$  since that if we have  $\sum_{i=1}^{K} R_i > RMb_{bmax}$ , the optimization problem will have no feasible solutions. The above optimization problem is a mixed-integer problem which is NP-hard. To find the optimal solution all transmission resources–subcarriers, eigenchannels, bits and power–should be jointly allocated, which, unfortunately requires very high computational complexity. Suboptimal but affordable-complexity solutions are perhaps more practical and desirable.

### 2.5 Resource Allocation Algorithms

In this section, we present two adaptive resource allocation algorithms for MIMO systems with the orthogonal precoding scheme described in previous sections. The first algorithm is an efficient space/frequency allocation algorithm (Algorithm I). It first determines the eigenchannel number for each user and then assigns eigenchannels to the users based on their eigenmode strengths. Once the eigenchannel assignment is done, we

use convention bit-loading algorithm to load bits on users' eigenchannel set and compute the corresponding transmitting power. The second algorithm is a constraint relaxation based greedy search algorithm (algorithm II). The algorithm contains total M stage and each stage, we will check all possible combinations of the eigenchannel assignment for certain subcarrier to decide which combination results in smallest increment of the transmitting power and repeat the process until all eigenchannels on all subcarriers are allocated.

### 2.5.1 An Space/Frequency Allocation Algorithm

We first determine the required eigenchannel number for each user and assign the eigenchannels to the users using a modified version of the two-phase algorithm of [20]-[21]. Then we use the conventional bit-loading algorithm to allocate bits over each user's eigenchannel subset and compute the required transmit power.



Figure 2.1: Flow Chart Description of Algorithm I.

In the first phase we compute the required eigenchannel number for each user according to the QoS and the *average channel condition*. For each subcarrier, say, the mth, we sort the maximum eigenmodes  $\lambda_{max}(k, m)$  of the channel matrices  $\mathbf{H}_{mk}, k = 1, 2, \cdots, K$ in descending order, i.e.,

$$\lambda_{max}(k_1, m) > \lambda_{max}(k_2, m) > \cdots > \lambda_{max}(k_K, m),$$

where

$$k_i = \arg \max_{k \in I_K \setminus \{k_1, k_2, k_{i-1}\}} \lambda_{max}(k, m), \text{ and } I_K = \{1, 2, \cdots, K\}$$

and set  $A_{rmk} = 1$  if  $k = k_r$ . If R < K we set  $A_{Rmk} = 1$  for those  $k = k_i, i > R$ . The computing of the weighting coefficients and the corresponding GNR follow that described in Section II-B. Define the *average channel condition* for user k by

$$T_k = \frac{1}{M} \sum_{i=1}^{M} \sum_{j=1}^{R} A_{jik} GNR_{ij(k)}.$$
 (2.22)

The minimum required eigenchannel number for user k is  $\lceil R_k/b_{max} \rceil$ . The actual eigenchannel number  $c_k$  is determined by iteratively verifying the relative reduction of the total transmit power after the allocation of an additional subcarrier. The detailed algorithm is given in **Table 2.1**.

Step 1: (assume 
$$T_k$$
 has been computed for all  $k$   
according to (3.1))  
(initialization)  $c_k^{max} = \lceil R_k/b_{min} \rceil$ ;  
 $c_k = c_k^{min} = \lceil R_k/b_{max} \rceil$  for each  $k$   
Step 2: while  $\sum_{k=1}^{K} c_k < RM$  and  $c_k < c_k^{max} \forall k$   
for  $k = 1 : K$   
if  $c_k < c_k^{max}$   
 $\bar{P}_k = c_k \cdot f(BER_k, \frac{R_k}{c_k}, T_k)$   
 $\bar{P}_k^{new} = (c_k + 1) \cdot f(BER_k, \frac{R_k}{(c_k+1)}, T_k)$   
 $\Delta P_k = \bar{P}_k - \bar{P}_k^{new}$   
end  
 $w = \arg \max_k \Delta P_k$   
 $c_w = c_w + 1$   
end

After determining  $c_k$ 's, we then assign the eigenchannels on all subcarriers to each user based on the eigenmode magnitudes of the channel matrix and GNRs as described in Table 2.1: Algorithm for computing the required eigenchannel number.

Section II-B. Let the kth eigenchannel of the mth subcarrier be represented by the two-tuple (k,m). The channel assignment follows the order  $(1,1) \rightarrow (1,2) \rightarrow \cdots \rightarrow \cdots$  $(1, M) \rightarrow (2, 1) \rightarrow (2, 2) \rightarrow \cdots \rightarrow (2, M) \rightarrow (3, 1) \rightarrow \cdots$ . That is, we assign the first eigenchannel on all subcarriers first, and then assign the second eigenchannel on all subcarriers, and so on. The ordering of subcarriers in the channel assignment process is important as once the eigenchannels of a subcarrier are assigned, no re-assignment is allowed. When assign rth eigenchannel on all subcarriers, we first sort the user on each subcarrier according to their  $GNR_{mr(k)}$  in descending order and denote the largest  $GNR_{mr(k)}$  on the *m*th subcarrier as  $Q_m$  and then sort subcarriers according to  $Q_m$  in descending order. Once the order of the subcarrier is determined, we assign the eigenchannel to the user with largest GNR; see Table 2.2 for details. After finishing channel assignment, we use the conventional bit-loading algorithm to allocate bits and compute the corresponding required transmit power for each user. This algorithm initially allocates zero bit to all subcarriers and then allocates bit by bit to the subcarrier which requires the least additional transmit power. The allocation process repeats until all data rate requirements are satisfied. The details of the bit-loading algorithm is given in **Table 2.3**. For a given set of assigned eigenchannels, the proposed bit-loading algorithm is optimal which we summarize below.

**Lemma 2.5.1.** For a fixed eigenchannel assignment, the bit allocation algorithm described by Table 2.3 is optimal, i.e., it results in minimum power consumption.

**Proof.** For the given BER and the GNR of the eigenchannel assigned to user k, define  $\Delta f(b_{rmk})$  as

$$\Delta f(b_{rmk}) = \begin{cases} f(BER_k, b_{rmk}, GNR_{mr(k)}) - f(BER_k, b_{rmk} - 1, GNR_{mr(k)}), & \text{if } b_{rmk} \ge 1\\ f(BER_k, b_{rmk}, GNR_{mr(k)}) - f(BER_k, 0, GNR_{mr(k)}), & \text{if } b_{rmk} < 1 \end{cases}$$

The author of [22] introduce necessary and sufficient conditions for a discrete bit allocation to be optimal:

1. 
$$\Delta f(b_{rmk}) \leq \Delta f(b_{r'm'k} + 1) \quad \forall r, r' = 1, 2, ..., R , \forall m, m' = 1, 2, ..., M$$
 (efficient)  
2.  $\sum_{r=1}^{R} \sum_{m=1}^{M} b_{rmk} = R_k$  (B-tight)

Any bit distribution that satisfies the above conditions will be the optimal solution. The second condition is clearly satisfied since the bit-loading algorithm will not stop loading bits until the loaded bits achieve the user data rate. As for the first condition, we first show that  $\Delta f(b_{rmk} + 1) > \Delta f(b_{rmk})$  for all r, m and k. Since we use a closed form expression to estimate the require power when QAM modulation is used (2.21), we have

$$\Delta f(b_{rmk} + 1) = f(BER_k, b_{rmk} + 1, GNR_{mr(k)}) - f(BER_k, b_{rmk}, GNR_{mr(k)})$$

$$= \frac{1}{GNR_{mr(k)}} \ln\left(\frac{1}{5BER_k}\right) \frac{2^{b_{rmk}+1} - 2^{b_{rmk}}}{1.5}$$

$$= \frac{1}{GNR_{mr(k)}} \ln\left(\frac{1}{5BER_k}\right) \frac{2^{b_{rmk}-1}}{1.5}$$

$$> \frac{1}{GNR_{mr(k)}} \ln\left(\frac{1}{5BER_k}\right) \frac{2^{b_{rmk}-1}}{1.5}$$

$$= \frac{1}{GNR_{mr(k)}} \ln\left(\frac{1}{5BER_k}\right) \frac{2^{b_{rmk}-1}}{1.5}$$

$$= \Delta f(b_{rmk}) \quad \text{for } b_{rmk} > 0$$
and

and

$$\begin{split} \Delta f(b_{rmk}+1) &= f\left(BER_k, b_{rmk}+1, GNR_{mr(k)}\right) - f\left(BER_k, b_{rmk}, GNR_{mr(k)}\right) \\ &= \frac{1}{GNR_{mr(k)}} \ln\left(\frac{1}{5BER_k}\right) \frac{2^{b_{rmk}+1} - 2^{b_{rmk}}}{1.5} \\ &= \frac{1}{GNR_{mr(k)}} \ln\left(\frac{1}{5BER_k}\right) \frac{2^{b_{rmk}}}{1.5} \\ &> 0 \\ &= \frac{1}{GNR_{mr(k)}} \ln\left(\frac{1}{5BER_k}\right) \frac{2^{b_{rmk}} - 0}{1.5} \\ &= \Delta f(b_{rmk}) \quad \text{for } b_{rmk} = 0 \end{split}$$

If there exist a  $b_{rmk}$  and a  $b_{r'm'k}$  in the result of the bit-loading algorithm such that  $\Delta f(b_{rmk}) > \Delta f(b_{r'm'k} + 1)$ , this will contradict the step 2 in bit-loading process because when deciding to increase the bits of the *r*th eigenchannel on the *m*th subcarrier from  $b_{rmk} - 1$  to  $b_{rmk}$ , the power increment of loading a bit to the *r*th eigenchannel on the *m*th subcarrier is less than the power increment of loading a bit to the *r*th eigenchannel on the *m*th subcarrier. Therefore, the result of the bit-loading process must satisfy the first condition.



Table 2.2: The eigenchannel assignment algorithm.

One of the advantages of this algorithm is its low computational complexity. We need only to perform bit-loading for each user once; the complexity analysis is discussed later. Another advantage of this algorithm is that it considers not only the fairness but also the efficiency of the resource utilization. In step 1, we insure that every user is assigned enough eigenchannels to transmit data so that outage will not occur. In step 2, we assign eigenchannels to the user who has the highest eigenmode magnitude, making the most of the available spatial resources.



 Table 2.3: The conventional bit-loading algorithm.

### 2.5.2 A Constraint-Relaxation based Greedy Search Algorithm

The second algorithm begins with a fair initial condition that gives all users the opportunity to access all its eigenchannels over all subcarriers. Each user uses a bit-loading algorithm to obtain the local power minimization solution based on the allocated eigenchannel subset. The proposed channel allocation process consists of a series of M-stage deletion decisions. At each stage, the eigenchannels associated with a subcarrier is given to the users who have the desired spatial channel condition. These eigenchannels are then removed from the serving channel subsets of all other users. The order of subcarriers assignment is the same as Algorithm I and the assignment process is carried out on a search tree whose root node has  $K^R$  outgoing branches to represent all possible assignments of subcarrier 1.



Figure 2.2: A search tree representing the multi-stage bit-loading procedure (Algorithm II).

Similarly, every node at any given level of the search tree, say the *t*th level, has  $K^R$  outgoing branches (to  $K^R$  child nodes), each represents a possible eigenchannel assignment (removal) decision and a tentative eigenchannels allocation. The resource allocation is tentative because only the eigenchannels for the first *t* subcarriers are assigned and those for the remaining M - t subcarriers are still unassigned. Given the initial fair channel allocation and the ultimate object of minimizing the required total power, the cost for a decision at any level should be the minimum required power for the corresponding tentative eigenchannel allocation. Repeating such a channel assignment and power allocation process for M times, we complete the search over the M-level tree and finish allocating all the eigenchannels and subcarriers; the corresponding power/rate allocations are accomplished simultaneously.

We first set  $A_{rmk} = 1$  for all r, m, k and re-index subcarriers by the same order described in Algorithm I. The channel-deletion process begins at the 1st subcarrier and continues until the last one. Let  $F_i^{(m)} = [f_1 \ f_2 \ \cdots \ f_R]^T$  be the *i*th eigenchannel assignment vector for the *m*th subcarrier that assign the *i*th eigenchannel to user  $f_i$ . There are  $R^k$  candidate eigenchannel assignment vectors in total, each represents a possible eigenchannel assignment. Reset  $A_{rmk}$  based on  $F_i^{(m)}$ . As for the eigenchannels on the M-1 unassigned subcarriers, each user treat these eigenchannels without considering other users' interference. That is, when an user loads bits to the *r*th eigenchannel on the unassigned subcarriers, the channel gain is exactly equal to the *r*th singular value. Use the bit-loading algorithm to allocate bits and compute the corresponding power. After we check all  $R^K$  candidate assignment options, we choose the assignment vector with the minimal transmit power and re-set  $A_{rmk}$  accordingly. The same procedure continues for the (m + 1)th subcarrier and repeat the deletion process until all eigenchannels for all subcarriers are assigned.

Note that a node of the, say, mth stage, represents a particular assignment of the eigenchannels associated with the mth subcarrier. The corresponding optimal bitloading scheme can be found by reconsidering only bit-loading on these eigenchannels with bit-loadings on all other eigenchannels remain intact. This fact is summarized as

**Lemma 2.5.2.** For the constraint-relaxation based greedy search algorithm, the required power associated with a node of the mth stage remains unchanged no matter one re-loads only the eigenchannels of the mth subcarrier or all spatial channels.

**Proof.** For user k, let  $b_{rnk}$   $(r = 1 \sim R, n = 1 \sim M, n \neq (m - 1))$  denote the bit distribution of eigenchannels on all subcarriers except subcarrier m - 1 on the m - 1 stage and  $B = \sum_{r=1}^{R} \sum_{n=1}^{M} b_{rnk} - \sum_{r=1}^{R} b_{r(m-1)k}$  denote the number of bits of user k which are allocated to the eigenchannels on all subcarriers except subcarrier m - 1. If the result of only re-allocating the bits on the eigenchannels which are assigned on the (m - 1)th stage is not the same as re-allocating all the bits on all the eigenchannels, this means that the result of only re-allocating the bits on the eigenchannels which are assigned on the (m - 1)th stage is not optimal. In other words, there must exist another bit distribution  $b'_{rnk}$   $(r = 1 \sim R, n = 1 \sim M, n \neq (m - 1))$  for these B bits with lower

power consumption than bit distribution  $b_{rnk}$ . But this will contradict the fact that the bit distribution of stage M - 1 is  $b_{rnk}$  since the power consumption of bit distribution  $b_{rnk}$  should be minimal. Therefore, it is impossible to have another bit distribution for these B bits with lower power consumption than bit distribution  $b_{rnk}$ . So we only need to re-allocate the bits on the eigenchannels which are assigned on the (m - 1)th stage instead of re-allocating all the bits on all the eigenchannels.

## 2.6 Complexity Analysis and Numerical Results

### 2.6.1 Computational Complexity Analysis

In this subsection, we analyze the complexity of the proposed resource allocation algorithms for the orthogonal precoding scheme.

First we check the complexity of the efficient space/frequency resource allocation algorithm. The efficient space/frequency resource allocation algorithm consists of three parts: compute eigenchannel number for each user, assign eigenchannels to the users and finally the bit-loading algorithm. The complexity of computing eigenchannel number for each user and assigning eigenchannels to the users are  $O(RM - R_kK/b_{max}) \leq O(RM)$ and  $O(R(Klog_2K + Mlog_2M) + RM)$ , respectively. The  $R(Klog_2K + Mlog_2M)$  term in eigenchannel assignment is the complexity of the sorting before assigning the *r*th eigenchannel on all subcarriers. And the complexity of the bit-loading algorithm is  $O(\sum_{k=1}^{K} R_kMR) \leq O(KR_{max}MR)$  where  $R_{max}$  is the maximum date rate of the users. So the overall complexity of the efficient space/frequency resource allocation algorithm is  $O(RM + R(Klog_2K + Mlog_2M) + RM + KR_{max}MR) \approx O(KR_{max}MR)$ .

Now we examine the complexity of the constraint-relaxation based greedy search algorithm. The the constraint-relaxation based greedy search algorithm contains total M stage and in each stage, we have to check  $K^R$  possible choices. Therefore, the computational complexity of the constraint-relaxation based greedy search algorithm is  $O(K^R R_{max} MR + (M-1)K^R KR b_{max} MR)$ . The first term means that on the first stage, we have to perform complete bit-loading algorithm for each user. But for the later stage, The bit-loading algorithm only need to reload the bits on the eigenchannels which are assigned on the last stage (second term). The complexity can be further approximated by  $O(M^2K^{R+1}R^2b_{max})$ .

### 2.6.2 Numerical Results

Selected simulated performance of the proposed RA algorithms are presented in this subsection. First we evaluate the performance of the proposed RA algorithms for the orthogonal precoding scheme in Fig.  $2.2 \sim 2.4$ . For Fig.  $2.5 \sim 2.10$ , we compare the performance of the proposed RA algorithm for the orthogonal precoding scheme and BD [16].

The performance of the proposed algorithms for uplink and downlink transmissions is shown in Fig. 2.2 and Fig. 2.3 respectively. In Fig. 2.4 we compare the performance of the proposed algorithms with that of the optimal solution. The average power is normalized by that of the single-user case, i.e., when a single user has access to all eigenchannels and all subcarriers. We define the average power ratio at BER=B as :

$$\mathbf{P}_B = 10 \log_{10} \left( \frac{P_{avg,B}}{P_{avg,10^{-5},single}} \right)$$
(2.23)

where  $P_{avg,B}$  represents the average transmit power for a given modulation scheme at BER=B and  $P_{avg,10^{-5},single}$  represents the average transmit power for the single user case at BER= $10^{-5}$ .

We assume each  $\mathbf{H}_{mk}$  is a 2 × 4 (4 antennas at the BS and 2 antennas at each MS) matrix with i.i.d. zero-mean, unit-variance complex Gaussian entries. The system has six different modulation modes, BPSK, QPSK, 8QAM, 16QAM, 32QAM, and 64 QAM, respectively. For simplicity, we assume that the required data rate and BER are the same for all users.

In Fig. 2.2, we compare the performance of the proposed algorithms with that of the adaptive zero-forcing MIMO-OFDM receiver proposed in [23] and its non-adaptive counterpart. Algorithm I (the efficient space/frequency resource allocation algorithm) is superior to the adaptive MIMO-OFDM ZF approach by a 2.5 dB margin and Algorithm II (the constraint-relaxation based greedy search algorithm) offers additional 0.5 dB performance gain. Both algorithms achieve more than 12 dB performance gain against the non-adaptive MIMO-OFDM ZF receiver.



Figure 2.3: Average power ratio per user for a MIMO-OFDM uplink; 32 subcarriers, 64 bits per OFDM symbol, 4 users.

In Fig. 2.3, we consider downlink transmission and compare the performance of the FDMA scheme and our algorithms. For the FDMA scheme (without bit-loading), the subcarriers are allocated to the users like the classical FDMA scheme according to their data rate requirements. Each user has access to all the eigenchannels on the corresponding subcarrier subset. Each required data rate (bits/transmission) is equally distributed among all eigenchannels available to a user. We also consider the FDMA scheme with bit-loading. Similar to the uplink case, Algorithm I outperforms the FDMA scheme with bit-loading by 2.5 dB and Algorithm II provides additional 0.5 dB gain.

Both algorithms have more than 10 dB advantage over the FDMA scheme without bit-loading.



Figure 2.4: Average power ratio per user for a MIMO-OFDM downlink; 32 subcarriers, 64 bits per OFDM symbol, 4 users.

Finally, Fig. 2.4 plots the performance of both the optimal solution and the proposed algorithms. It is found that Algorithm II yields performance almost identical to that of the optimal solution while Algorithm II suffers only minor degradation.

In Figs. 2.5–2.9, we compare the proposed orthogonal precoding scheme (using Algorithm I) and BD in downlink transmission. BD is a popular linear precoding technique for the multiple antenna multicast channel that involves transmission of multiple data streams to each receiver such that no multiuser interference is experienced at any of the receivers. In BD, each user data vector is multiplied by a precoding matrix to project the transmitted signal to the null space of the space spanned by all other users' spatial channels.

We examine the performance for the 2-user and 4-user cases. In the 2-user case, we


Figure 2.5: Average power ratio per user for a MIMO-OFDM downlink; 8 subcarriers, 16 bits per OFDM symbol, 2 users.

assume that  $T_x = 4$  and  $R_x = 2$  and for the 4-user case,  $T_x = 8$  and  $R_x = 2$ . As for the required user data rate, we consider three cases: 64 bits per OFDM symbol, 128 bits per OFDM symbol, and 256 bits per OFDM symbol, respectively. The system offers eight different modulation options ranging from from BPSK to 256 QAM.

First we examine the performance of the 2-user case in Figs. 2.5 and 2.6. It is found that the proposed precoding scheme outperforms the BD approach when desired data rate is 64 or 128 bits per OFDM symbol. However, Fig. 2.7 indicates that BD outperforms the proposed precoding scheme by almost 0.5 dB. The same behavior can be found in the 4-user case. In Figs. 2.8 and 2.9, the performance of the proposed precoding scheme is better than that of the BD precoder when desired data rate is 64 bits or 128 bits per OFDM symbol. However, when the data rate increases to 256 bits per OFDM symbol, BD is superior to our precoder by a 1.5 dB margin.

Such a performance trend can be explained by (2.21) which indicates that the trans-

mit power of an eigenchannel is a function of both GNR  $(GNR_{mr(k)})$  and the number of bits loaded  $(b_{rmk})$ . When the eigenchannel's GNR is large (strong eigenmode) or the number of loaded bits is small, the corresponding required transmit power is low. For the BD precoder, the number of eigenchannels per subcarrier for each user is  $T_x - (R_x * (K - 1))$  while that for the proposed precoder is  $R = \min(T_x, R_x)$ . For the 2-user case, the numbers of eigenchannel per subcarrier for the BD and GS precoders are 4 and 2. For the 4-user case, the number of eigenchannel per subcarrier for the BD precoder are 8 while that for the GS precoder remains to be 2. Moreover, both simulation and theoretical analysis show that the eigenmode magnitude suffer degradation by using BD (this will be proved in the last of this section). Define the channel loading coefficient  $\eta$  as

$$\eta = \frac{\sum_{i=1}^{K} R_k}{RMb_{max}}.$$
(2.24)

When  $\eta$  is more close to 1, this means that each eigenchannel must be loaded more bits since the user's data rate is close to the system limit such that the total consuming power is higher. When  $\eta$  is more close to 0, this means that the bits on each eigenchannel is less and the total consuming power is also less relatively. When the channel loading is light ( $\eta$  is close to 1), the bit number on each eigenchannel ( $b_{rmk}$ ) is small and the eigenmode magnitude ( $GNR_{mr(k)}$ ) will dominate the system performance. However, when we increase the user's data rate such that the channel loading is heavy, the exponential term  $b_{rmk}$  in (2.21) becomes an important factor that will affect the system performance.

The channel loading coefficient for BD and the proposed precoding scheme is listed as follows:

Precoding scheme/Users' data rate	64 bits	128 bits	256 bits	
BD	0.125	0.25	0.5	
Orthogoal precoding scheme	0.25	0.5	1	

Table 2.4:  $\eta$  for the 2 users case

Precoding scheme/Users' data rate	64 bits	128  bits	256 bits	
BD	0.0625	0.125	0.25	
Orthogoal precoding scheme	0.25	0.5	1	

Table 2.5:  $\eta$  for the 4 users case

Numerical results given in the above two tables indicate that the BD precoder has lower channel loading coefficients ( $\leq 0.5$ ) as it offers more eigenchannels. Therefore, when the data rate requirement is low, the performance of BD precoder is inferior to that of GS precoder due to the fact that BD precoding results in weaker eigenmodes; see *Lemma 2.6.2* below. But for high data rate requirements, the per eigenchannel loading for the BD precoder remains relatively low which then leads to better performance.



Figure 2.6: Average power ratio per user for a MIMO-OFDM downlink; 32 subcarriers, 64 bits per OFDM symbol, 2 users.

To prove the eigenmode degradation suffered in the BD scheme, we need

**Lemma 2.6.1.** (Weak Majorization Lemma) Let  $x_1, \ldots, x_n, y_1, \ldots, y_n$  be 2n given real numbers such that  $x_1 \ge \ldots \ge x_n, y_1 \ge \ldots \ge y_n$  and  $\sum_{i=1}^k y_i \ge \sum_{i=1}^k x_i, k = 1, \ldots, n$ .



Figure 2.7: Average power ratio per user for a MIMO-OFDM downlink; 32 subcarriers, 128 bits per OFDM symbol, 2 users.

Then for any real-valued function  $f(\cdot)$  which is increasing and convex on the interval  $[\min\{x_n, y_n\}, y_1], f(x_1) \ge \ldots \ge f(x_n), f(y_1) \ge \cdots \ge f(y_n), \text{ and } \sum_{i=1}^k f(y_i) \ge \sum_{i=1}^k f(x_i), k = 1, \ldots, n.$ 

Using the above lemma we can prove that the sum magnitude of the eigenmodes associated with the augmented matrix  $\mathbf{XY}$  is always less than the sum of magnitude product for the component matrixes  $\mathbf{X}$  and  $\mathbf{Y}$ .

**Lemma 2.6.2.** Consider the  $m \times p$  and  $p \times n$  matrices,  $\mathbf{X}$  and  $\mathbf{Y}$  and let  $\sigma_i(\mathbf{X})$  be the ith singular value of  $\mathbf{X}$ . Then  $\sum_{i=1}^{k} [\sigma_i(\mathbf{X})\sigma_i(\mathbf{Y})] \geq \sum_{i=1}^{k} [\sigma_i(\mathbf{X}\mathbf{Y})]$  for  $k = 1, \ldots, q$ , where  $q = \min\{m, p, n\}$ .

**Proof.** We first show that  $\prod_{i=1}^{k} [\sigma_i(\mathbf{X})\sigma_i(\mathbf{Y})] \ge \prod_{i=1}^{k} [\sigma_i(\mathbf{X}\mathbf{Y})]$  for k = 1, ..., q, where  $q = \min\{m, p, n\}$ . Performing SVD on  $\mathbf{X}\mathbf{Y}$  gives  $\mathbf{X}\mathbf{Y} = \mathbf{U}\mathbf{\Sigma} \mathbf{V}^{\dagger} = [\mathbf{U}_k \ \mathbf{U}_{\times}]\mathbf{\Sigma}[\mathbf{V}_k \ \mathbf{V}_{\times}]^{\dagger}$  where  $\mathbf{U}_k$  and  $\mathbf{V}_k$  both contain the first k columns of  $\mathbf{U}$  and  $\mathbf{V}$ , respectively. SVD



Figure 2.8: Average power ratio per user for a MIMO-OFDM downlink; 32 subcarriers, 256 bits per OFDM symbol, 2 users.

of  $\mathbf{Y}\mathbf{V}_k$  yields  $\mathbf{Y}\mathbf{V}_k = \widetilde{\mathbf{U}}\widetilde{\mathbf{\Sigma}}\widetilde{\mathbf{V}}^{\dagger} = [\widetilde{\mathbf{U}}_k \ \widetilde{\mathbf{U}}_{\times}] \begin{bmatrix} \widetilde{\mathbf{\Sigma}}_k \\ \mathbf{0} \end{bmatrix} \widetilde{\mathbf{V}}^{\dagger}$  where  $\widetilde{\mathbf{U}}_k$  consists of the first k columns of  $\widetilde{\mathbf{U}}$ . Then we have

$$\begin{split} \sigma_{1}(\mathbf{X}\mathbf{Y})\cdots\sigma_{k}(\mathbf{X}\mathbf{Y}) &= |\det(\mathbf{U}_{k}^{\dagger}\mathbf{X}\mathbf{Y}\mathbf{V}_{k})| \\ &= |\det(\mathbf{U}_{k}^{\dagger}\mathbf{X}\widetilde{\mathbf{U}}\widetilde{\mathbf{\Sigma}}\widetilde{\mathbf{V}}^{\dagger})| \\ &= |\det(\mathbf{U}_{k}^{\dagger}\mathbf{X}\widetilde{\mathbf{U}}_{k}\widetilde{\mathbf{V}}^{\dagger}\widetilde{\mathbf{V}}\widetilde{\mathbf{\Sigma}}_{k}\widetilde{\mathbf{V}}^{\dagger})| \\ &= |\det(\mathbf{U}_{k}^{\dagger}\mathbf{X}\widehat{\mathbf{U}})||\det(\widetilde{\mathbf{V}}\widetilde{\mathbf{\Sigma}}_{k}\widetilde{\mathbf{V}}^{\dagger})| \\ &\leq [\sigma_{1}(\mathbf{X})\cdots\sigma_{k}(\mathbf{X})][\sigma_{1}(\widetilde{\mathbf{\Sigma}}_{k})\cdots\sigma_{k}(\widetilde{\mathbf{\Sigma}}_{k})] \\ &= [\sigma_{1}(\mathbf{X})\cdots\sigma_{k}(\mathbf{X})][\sqrt{\sigma_{1}(\mathbf{V}_{k}^{\dagger}\mathbf{Y}^{\dagger}\mathbf{Y}\mathbf{V}_{k})}\cdots\sqrt{\sigma_{k}(\mathbf{V}_{k}^{\dagger}\mathbf{Y}^{\dagger}\mathbf{Y}\mathbf{V}_{k})} \ ] \\ &\leq [\sigma_{1}(\mathbf{X})\cdots\sigma_{k}(\mathbf{X})][\sqrt{\sigma_{1}(\mathbf{Y}^{\dagger}\mathbf{Y})}\cdots\sqrt{\sigma_{k}(\mathbf{Y}^{\dagger}\mathbf{Y}^{\dagger}\mathbf{Y}\mathbf{V}_{k})} \ ] \\ &= [\sigma_{1}(\mathbf{X})\cdots\sigma_{k}(\mathbf{X})][\sqrt{\sigma_{1}^{2}(\mathbf{Y})}\cdots\sqrt{\sigma_{k}^{2}(\mathbf{Y})}] \\ &= \prod_{i=1}^{k}[\sigma_{i}(\mathbf{X})\sigma_{i}(\mathbf{Y})]. \end{split}$$



Figure 2.9: Average power ratio per user for a MIMO-OFDM downlink; 64 subcarriers, 64 bits per OFDM symbol, 4 users.

It is noted that the inequality becomes equality when m = p = n and k = n. If we take log for both sides, we get  $\sum_{i=1}^{k} \ln(\sigma_i(\mathbf{X})\sigma_i(\mathbf{Y})) \ge \sum_{i=1}^{k} \ln(\sigma_i(\mathbf{XY}))$ . Let  $x_i = \ln(\sigma_i(\mathbf{XY}))$ ,  $y_i = \ln(\sigma_i(\mathbf{X})\sigma_i(\mathbf{Y}))$  and  $f(\cdot)$  is to take the exponential of the argument matrix. The Weak Majorization Lemma then implies  $\sum_{i=1}^{k} [\sigma_i(\mathbf{X})\sigma_i(\mathbf{Y})] \ge \sum_{i=1}^{k} [\sigma_i(\mathbf{XY})]$  for  $k = 1, \dots, q$ , where  $q = \min\{m, p, n\}$ .

Let  $\mathbf{X} = \mathbf{H}$  and  $\mathbf{Y}$  be the precoder matrix. For the BD precoder,  $\sigma_i(\mathbf{Y}) = 1$ ,  $\forall i$ . Hence, the above lemma tells us that BD precoding decreases the sum strength of all eigenchannels. Moreover, our simulation results show that not only the sum of the eigenmode magnitudes but also the individual eigenmode magnitude degrades after BD precoding.



Figure 2.10: Average power ratio per user for a MIMO-OFDM downlink; 64 subcarriers, 128 bits per OFDM symbol, 4 users.



Figure 2.11: Average power ratio per user for a MIMO-OFDM downlink; 64 subcarriers, 256 bits per OFDM symbol, 4 users.

## Chapter 3

# Resource Allocation for MIMO Systems with Non-Orthogonal Precoding

#### 3.1 System and Transceiver Models

In the previous chapter, we consider MIMO systems that use a orthogonal precoding scheme so that system users can transmit through distinct eigenchannels on the same subcarrier without causing interference to each other. For such a scheme, however, the maximum eigenchannel number is bounded by the rank of the MIMO channel matrix (R)and thus the spectrum efficiency may be constrained. To increase the spectrum efficiency, we allow more than R users to transmit over the eigenchannels on the same subcarrier. In this situation, the co-channel interference among users is no longer avoidable. Therefore, the associated optimization problem becomes more complicated due to the constraints on the tolerable inter-channel interference (ICI).

Similar to the previous system setup, we consider a MIMO-OFDMA system with a single base station (BS) equipped with  $T_x$  antennas and K mobile station (MS) users, each equipped with  $R_x$  antennas. The frequency band used contains M subcarriers which are to be allocated to the K MS'. Based on the GS precoder design, we provide R - 1

orthogonal eigenchannels for users with no interference and additional  $Q (R \sim R + Q - 1)$ eigenchannels with various tolerable interference levels.

For the R - 1 orthogonal eigenchannels, the way to choose the pre-processing and post-processing vectors is the same as that described in Chapter 2. That is, for the user to whom the *r*th eigenchannel is given, the pre-processing and post-processing vectors are the linear combinations of first *r* left and right singular vectors, respectively. In order that the *Q* non-orthogonal eigenchannels do not induce interference to the R - 1orthogonal eigenchannels, we require that the users who are allocated non-orthogonal eigenchannels to transmit over an eigenchannel which lies in the null space spanned by all R - 1 orthogonal eigenchannels. More specifically, they use linear combinations of *R* singular vectors as the processing vectors to project the transmitting signal to the null space of the R - 1 dimensional space spanned by orthogonal eigenchannels.

Although the non-orthoganal eigenchannels will not interfere with the R-1 orthogonal eigenchannels, the co-channel interference among the non-orthogonal eigenchannels is unavoidable. Here we define  $B_{mk} = 1$  if user k is to transmit on the mth subcarrier's non-orthogonal eigenchannel and  $B_{mk} = 0$ , otherwise. The GINR (gain to interference and noise ratio) for users k who is allowed to transmit data on the non-orthogonal eigen-channels can be expressed as:

$$GINR_{mk} = \frac{\left(\sum_{i=1}^{R} \alpha_{ki} \beta_{ki} s_{ki}\right)^2}{\sigma^2 + \sum_{i=1, i \neq k}^{K} \left| \mathbf{w}_{mk}^{\dagger} \mathbf{H}_{mk} \mathbf{B}_{mi} \mathbf{t}_{mi} \right|^2 p'_{mi}}$$
(3.1)

If we define  $\sum_{i=1}^{R} \alpha_{ki} \beta_{ki} s_{ki}$  as  $g_{mk}$  (the channel gain of user k) and  $\mathbf{w}_{mk}^{\dagger} \mathbf{H}_{mk} \mathbf{B}_{mi} \mathbf{t}_{mi}$ (the correlation between user k and user i) as  $\rho_{mki}$ , then (1) can be simplified as:

$$GINR_{mk} = \frac{|g_{mk}|^2}{\sigma^2 + \sum_{i=1, i \neq k}^{K} |\rho_{mki}|^2 p'_{mi}}$$
(3.2)

It is noted that if a user is assigned more than one non-orthogonal eigen-channels,

the interference will become too large since the correlation is unity. Therefore, we will assign one non-orthogonal eigenchannel to the same user at most.

#### 3.2 Problem Formulation

Let  $b'_{mk}$  and  $p'_{mk}$  be the number of bits and the corresponding power transmitted over the mth subcarrier using the non-orthogonal eigenchannel by user k. The resource allocation problem can then be reformulated as:

$$\min_{A_{rmk}, p_{rmk}, B_{mk}, p'_{mk}} \sum_{m=1}^{M} \sum_{r=1}^{R-1} \sum_{k=1}^{K} A_{rmk} p_{rmk} + \sum_{m=1}^{M} \sum_{k=1}^{K} B_{mk} p'_{mk}$$
(3.3)

subject to the following constraints:

$$\sum_{r=1}^{R-1} \sum_{m=1}^{M} b_{rmk} + \sum_{m=1}^{M} b'_{mk} = R_k \quad \forall k$$
(3.3a)

$$\sum_{r=1}^{R-1} \sum_{k=1}^{K} A_{rmk} = R - 1 \quad \forall m$$
(3.3b)

$$\sum_{k=1}^{K} B_{mk} = Q \quad \forall m \tag{3.3c}$$

$$A_{rmk} \in \{0, 1\} \qquad \forall r, m, k \tag{3.3d}$$

$$B_{mk} \in \{0, 1\} \qquad \forall \ m, k \tag{3.3e}$$

$$p_{rmk}, p'_{mk} \ge 0 \qquad \forall \ r, m, k \tag{3.3f}$$

$$b_{max} \ge b_{rmk}, b'_{mk} \ge 0 \qquad \forall \ r, m, k \tag{3.3g}$$

The above optimization problem is a mixed-integer problem which is NP-hard. In this case, we have to assign eigenchannels to users more carefully.

### 3.3 A ICI-Constrained Resource Allocation Algorithm

In this section, we propose a dynamic resource allocation algorithm for the non-orthogonal pre-coding scheme described above. The algorithm contains three steps. We assign the orthogonal eigenchannels and non-orthogonal eigenchannels in step one and step two, respectively. In the last step, we describe how to modify the conventional bit-loading algorithm such that it can be suitable for this case.

In step one, we determine how to assign R - 1 orthogonal eigenchannels to the users. Since the way to choose the pre-processing and post-processing vectors and the computation of the GNR for R - 1 orthogonal eigenchannels is the same as mentioned in section 1 of chapter 2, we will assign R - 1 orthogonal eigenchannels based on the proposed efficient space/frequency resource allocation algorithm described in section 3 of chapter 2.

In step two, since the user will be only assigned one non-orthogonal eigenchannel at most, there will be  $\frac{K!}{(K-Q)!}$  possible choices. We then choose the one with the largest sum GINR as the assignment set.

Once all eigenchannels are assigned to the users, we will start to load the bits to the eigenchannels for each user. However, the conventional bit-loading is not suitable for this case. The reason is that each user cannot be considered individually due to the co-channel interference among the non-orthogonal eigenchannels. Such scenario can be seen in digital subscriber line (DSL) systems. The author of [24] propose a multiuser discrete bit-loading process for such DSL systems. Thus, our bit-loading algorithm will based on conventional bit-loading algorithm and the method proposed in [24]. The modified bit-loading algorithm will consist of two parts. The first part is to check the power increment after loading a bit to the orthogonal eigenchannels. This part is just the same as the conventional bit-loading algorithm since there is no interference between the orthogonal eigenchannels. The second part is to check the power increment after loading a bit to the non-orthogonal eigenchannels. Let  $\mathbf{b}_m = [b'_{m1} \ b'_{m2} \ \cdots \ b'_{mK}]^T$ . By [19], the required SINR for user j to transmit  $b'_{mj}$  through the orthogonal eigenchannel on subcarrier m can be expressed as:

$$\gamma_j(BER_j, b'_{mj}) = \ln\left(\frac{1}{5BER_j}\right)\frac{2^{b'_{mj}-1}}{1.5}.$$
 (3.4)

The corresponding transmitting power  $p'_{mj}$  should satisfy

$$\frac{|g_{mj}|^2 p'_{mj}}{\sigma^2 + \sum_{i=1, i \neq j}^K |\rho_{mki}|^2 p'_{mi}} \ge \gamma_j (BER_j, b'_{mj}) \quad \forall \quad j, m$$
(3.5)

which can be rearranged in a matrix form:

$$(\mathbf{I} - \mathbf{C}_m)\mathbf{p}_m \succeq \mathbf{y}_m \quad \forall \quad m \tag{3.6}$$

where

$$\{\mathbf{C}_m\}_{i,j} = \begin{cases} \frac{\gamma_j (BER_j, b'_{mj}) |\rho_{mij}|^2}{|g_{mi}|^2}, & \text{for } i \neq j \\ 0, & \text{otherwise} \end{cases}$$
(3.7)

$$\mathbf{p}_{m} = [p'_{m1} \ p'_{m2} \ \cdots \ p'_{mK}]^{T}$$
(3.8)

$$\mathbf{y}_{m} = \left[\frac{\gamma_{1}(BER_{1}, b'_{m1})\sigma^{2}}{|g_{m1}|^{2}}, \frac{\gamma_{2}(BER_{2}, b'_{m2})\sigma^{2}}{|g_{m2}|^{2}}, \cdots, \frac{\gamma_{K}(BER_{K}, b'_{mK})\sigma^{2}}{|g_{mK}|^{2}}\right]^{T}.$$
 (3.9)

Here  $\mathbf{a} \succeq \mathbf{b}$  means the inequality holds element-wise. Then we can compute  $\mathbf{p}_m$  by

$$\mathbf{p}_m = (\mathbf{I} - \mathbf{C}_m)^{-1} \mathbf{y}_m \quad \forall \quad m.$$
(3.10)

If the solution  $\mathbf{p}_m$  is all-positive then it is a feasible solution that satisfies (3.6). Otherwise, no feasible solution exists. The authors of [24] also showed that if the solution of (3.6) exists and the elements of the solution vector are all positive, the Perron eigenvalue (that is, the largest positive eigenvalue) of  $\mathbf{C}_m$ , denoted as  $\lambda(\mathbf{C}_m)$ , must be less than 1. In addition,  $\mathbf{p}_m$  computed by (3.10) is the Pareto optimal solution to (3.6). In other words, any positive vector that satisfies (3.6) is greater than or equal to  $\mathbf{p}_m$  element-wise. The modified bit-loading algorithm is now described as follows:

- 1. Initialize  $\mathbf{b}_m = [0 \ 0 \ \cdots \ 0]^T$  and  $\mathbf{p}_m = [0 \ 0 \ \cdots \ 0]^T$  for  $m = 1, 2, \cdots, M$ .
- 2. Initialize  $b_{rmk} = 0$  and  $p_{rmk} = 0$  for all r, m, k.

3. For 
$$k = 1, 2, \cdots, K$$
, if  $\sum_{r=1}^{R} \sum_{m=1}^{M} b_{rmk} + \sum_{m=1}^{M} b'_{mk} < R_k$ 

- (a) check the smallest power increment after adding one bit among all orthogonal eigenchannels assigned to user k. Denote it as  $P_{1k}$  and record that it is belong to which orthogonal eigenchannel.
- (b) check the smallest power increment after adding one bit among all nonorthogonal eigenchannels assigned to user k by using (3.10). It is noted that we should confirm that the solution vector is positive by checking  $\lambda(\mathbf{C}_m)$  is less than 1 or not. Denote it as  $P_{2k}$  and record that it is belong to which non-orthogonal eigenchannel.
- (c) Let P(k) be the smaller one of  $P_{1k}$  and  $P_{2k}$  and record that the bit and the power increments are belong to which orthogonal (or non-orthogonal) eigenchannel.
- 4. Find the user k with the smallest P(k) and add one bit to the corresponding orthogonal (or non-orthogonal) eigenchannel. Go back to step 3 if

$$\sum_{r=1}^{R} \sum_{m=1}^{M} b_{rmk} + \sum_{m=1}^{M} b'_{mk} < R_k, \ \forall \ k$$

#### **3.4** Complexity Analysis and Simulation Results

#### 3.4.1 Computational Complexity Analysis

In this subsection, we analyze the complexity of the proposed resource allocation algorithm for the non-orthogonal precoding scheme.

The assignment of the R-1 orthogonal eigenchannels is the same as the low complexity algorithm described in chapter 2. Thus the complexity is O((R-1)M + (R-1))  $(Klog_2K + Mlog_2M) + (R - 1)M)$ . For assigning the Q non-orthogonal eigenchannels, we check all  $\frac{K!}{(K-Q)!}$  possible choices so the complexity is  $O\left(M\frac{K!}{(K-Q)!}\right)$ . Finally, the complexity of the bit-loading algorithm is  $O\left(\sum_{k=1}^{K} R_k M R\right) \leq O(KR_{max}MR)$ . Therefore, the total complexity is  $O\left((R - 1)M + (R - 1)(Klog_2K + Mlog_2M) + (R - 1)M + M\frac{K!}{(K-Q)!} + KR_{max}MR\right) \approx O(KR_{max}MR)$ .

#### 3.4.2 Simulation Results

In this subsection, we evaluate the performance of the non-orthogonal precoding scheme and compare it with the performance of the orthogonal precoding scheme (using the low complexity algorithm) described in chapter 2.

The performance of the non-orthogonal precoding scheme and the orthogonal precoding scheme for downlink transmissions with different channel matrix rank value is shown in Fig.  $3.1 \sim$  Fig. 3.3 respectively. The average power is normalized by that of the single-user case, i.e., when a single user has access to all eigenchannels and all subcarriers. We define the average power ratio at BER=*B* as :

$$\mathbf{P}_B = 10 \log_{10} \left( \frac{P_{avg,B}}{P_{avg,10^{-5},single}} \right)$$
(3.11)

where  $P_{avg,B}$  represents the average transmit power for a given modulation scheme at BER=B and  $P_{avg,10^{-5},single}$  represents the average transmit power for the single user case at BER= $10^{-5}$ .

We assume the number of the antenna at the BS and the MS are the same. The antenna number is from 3 to 5. Each entry of the channel matrix is i.i.d. zero-mean, unitvariance complex Gaussian The system has eight different modulation modes, BPSK, QPSK, 8QAM, 16QAM, 32QAM, 64QAM, 128QAM and 256 QAM, respectively. For simplicity, we assume that the required data rate and BER are the same for all users.

We can notice that when the rank of the channel matrix is 3, the performance of the non-orthogonal precoding scheme is about 1 dB worse than the orthogonal precoding

scheme. However, when we increase the rank to 4 and 5, the performance of the nonorthogonal precoding scheme is better than the orthogonal precoding scheme by 0.7 and 1.8 dB, correspondingly.

The reason for this phenomenon is that for the rank 3 case, we provide 2 orthogonal eigenchannels and 2 non-orthogonal eigenchannels on each subcarrier in the nonorthogonal precoding scheme. However, in the orthogonal precoding scheme, we provide 3 orthogonal eigenchannels on each subcarrier in total. This means that for the non-orthogonal precoding scheme, we "sacrifice" 33% of the orthogonal eigenchannels to get Q non-orthogonal eigenchannels (in this case, Q=2). However, the gain of the non-orthogonal eigenchannels is not enough to compensate the loss of the orthogonal eigenchannels. For rank 4 and rank 5 cases, 25% and 20% of the orthogonal eigenchannels are sacrificed. This implies that when the rank of the MIMO channel matrix is increased, the impact of sacrificing the orthogonal eigenchannels to users for the non-orthogonal pre-coding scheme. Therefore, when the rank of the MIMO channel matrix is increased, the advantage of the non-orthogonal pre-coding scheme will begin to appear.

In Fig. 3.4 and 3.5, the performance is not improved as we increase the value of Q. This contradicts the intuition that the larger Q provides more eigenchannels (with interference) than smaller Q. This is because the bit-loading algorithm used in the non-orthogonal precoding scheme is not guaranteed optimal. The optimalty is destroyed by loading the bits to the non-orthogonal eigenchannel. Therefore, the result of the bit-loading algorithm used in the non-orthogonal precoding scheme may be the local optimal solution instead of global optimal solution. Thus, increasing the value of Q will not insure the better performance.



Figure 3.1: Average power ratio per user for a MIMO-OFDM downlink; 32 subcarriers, 128 bits per OFDM symbol, 4 users, rank=3.



Figure 3.2: Average power ratio per user for a MIMO-OFDM downlink; 32 subcarriers, 192 bits per OFDM symbol, 4 users, rank=4.



Figure 3.3: Average power ratio per user for a MIMO-OFDM downlink; 32 subcarriers, 240 bits per OFDM symbol, 4 users, rank=5.



Figure 3.4: Average power ratio per user for a MIMO-OFDM downlink; 32 subcarriers, 144 bits per OFDM symbol, 4 users, rank=3.



Figure 3.5: Average power ratio per user for a MIMO-OFDM downlink; 32 subcarriers, 192 bits per OFDM symbol, 4 users, rank=4.

## Chapter 4

# Resource Allocation for MIMO Systems with Codebook-based Precoding

The precoding schemes we considered so far require complete channel state information to achieve full performance gain. In a frequency-division duplex system, however, full channel state information must be conveyed through a feedback channel. This is not very efficient and practical due to the number of channel coefficients that needed to be quantized and sent back to the transmitter over limited bandwidth control channels.

Precoding schemes for spatial multiplexing systems with limited feedback capacity is more feasible in real-world applications [11]-[12]. The basic idea is that the transmit precoder is chosen from a finite set of precoding matrices, called the codebook, known to both the receiver and the transmitter. The receiver chooses the optimal precoder from the codebook as a function of the current channel state information and sends the binary index of this (precoder) matrix to the transmitter over a feedback channel. In this chapter, we discuss the resource optimization problem for codebook-based MIMO-OFDMA systems.

#### 4.1 Transceiver Models and Precoding Criteria

#### 4.1.1 System parameters and transceiver model

Again, we consider the uplink of a MIMO-OFDMA system with  $R_x$  transmit antennas at the base station and  $T_x$  receiver antennas at mobile stations. The frequency band is divided into M subcarriers. For the kth MS on the mth subcarrier, a bit stream is sent into a vector encoder and modulator block where it is demultiplexed into N different substreams. Each of the N bit substreams is then modulated independently using the same constellation  $\mathcal{W}$ . This yields a symbol vector of  $\mathbf{s}_{mk} = [s_{mk}^1 \ s_{mk}^2 \ \dots \ s_{mk}^N]$ . For convenience, we will assume that  $E[\mathbf{s}_{mk}\mathbf{s}_{mk}^{\dagger}] = \mathbf{I}_N$ .

The symbol vector  $\mathbf{s}_{mk}$  is then multiplied by an  $T_x \times N$  precoding matrix  $\mathbf{F}_{mk}$  producing a length  $T_x$  vector  $\mathbf{x}_{mk} = \sqrt{\frac{E_m}{N}} \mathbf{F}_{mk} \mathbf{s}_{mk}$  where  $E_m$  is the total transmit energy on the subcarrier m,  $T_x$  is the number of transmit antennas, and  $T_x > N$ . We assume throughout the correspondence that  $R_x > M$ . Assuming perfect timing, synchronization, sampling, and a memoryless linear matrix channel, this formulation allows the baseband, discrete-time equivalent received signal to be written as

$$\mathbf{y}_{mk} = \sqrt{\frac{E_m}{N}} \mathbf{H}_{mk} \mathbf{F}_{mk} \mathbf{s}_{mk} + \mathbf{v}_{mk}$$
(4.1)

where **H** is the channel matrix and  $\mathbf{v}_{mk}$  is the noise vector. We assume that the the entries of  $\mathbf{v}_{mk}$  are independent and distributed according to  $\mathcal{CN}(0, N_o)$ . The received vector is then decoded by a vector decoder, assuming perfect knowledge of  $\mathbf{H}_{mk}\mathbf{F}_{mk}$ , that produces a hard decoded symbol vector  $\mathbf{s}_{mk}^{\hat{}}$ .

In this correspondence, the BS chooses a precoding matrix  $\mathbf{F}_{mk}$  from a finite set of possible precoding matrices  $\mathcal{F} = [\mathbf{F}_1 \ \mathbf{F}_2 \dots \ \mathbf{F}_L]$  and conveys the index of the chosen precoding matrix back to the transmitter over a limited capacity, zero-delay feedback link.

At the receiver side, we consider linear receivers such as Zero-forcing (ZF) receiver and MMSE receiver instead of the ML receiver due to the lower complexity of linear receivers. Linear receivers apply an  $N \times R_x$  matrix  $\mathbf{G}_{mk}$ , chosen according to some criterion, to produce  $\hat{\mathbf{s}}_{mk} = Q(\mathbf{G}_{mk}\mathbf{y}_{mk})$  where Q() is a function that performs single-dimensional ML decoding for each entry of a vector. For a ZF linear receiver,  $\mathbf{G}_{mk} = (\mathbf{H}_{mk}\mathbf{F}_{mk})^+$ . When a MMSE linear decoder is used,  $\mathbf{G}_{mk} = [\mathbf{F}_{mk}^{\dagger}\mathbf{H}_{mk}^{\dagger}\mathbf{H}_{mk}\mathbf{F}_{mk} + (NN_o/E_m)\mathbf{I}_N]^{-1}\mathbf{F}_{mk}^{\dagger}\mathbf{H}_{mk}^{\dagger}$ .

#### 4.1.2 Precoding Criteria

In this subsection, we introduce the criteria for choosing the precoding matrix from the predetermined codebook  $\mathcal{F}$ . The author of [12] propose several precoding criteria for minimizing average BER or maximizing the system capacity. In this thesis, we focus on the criteria for minimizing average BER.

In [25], it is shown that in order to minimize a bound on the average probability of a symbol vector error, the minimum substream SNR must be maximized. It was also shown in [25] that the SNR of the *n*th substream on the *m*th subcarrier of the user k is given by

$$SNR_{mnk}^{(ZF)} = \frac{\mathbf{E} \mathbf{E}_{E_m}}{NN_0 [\mathbf{F}_{mk}^{\dagger} \mathbf{H}_{mk}^{\dagger} \mathbf{H}_{mk} \mathbf{F}_{mk}]_{n,n}^{-1}}$$
(4.2)

for the ZF receiver and

$$SNR_{mnk}^{(MMSE)} = \frac{E_m}{NN_0 [\mathbf{F}_{mk}^{\dagger} \mathbf{H}_{mk}^{\dagger} \mathbf{H}_{mk} \mathbf{F}_{mk} + (NN_0/E_m) \mathbf{I}_N]_{n,n}^{-1}} - 1$$
(4.3)

for the MMSE decoder, where  $A_{n,n}$  is the entry (n, n) of A. Since the minimum substream SNR will dominate the BER performance, , when we are choosing the precoding matrix  $F_{mk}$  from the codebook  $\mathcal{F}$  for the user k on the *m*the subcarrier, we will choose the one with the maximum "minimum substream SNR". That is, we will choose the *i*th precoding matrix  $F_i$  such that

$$\mathbf{F}_{mk} = \arg \max_{\mathbf{F}_i \in \mathcal{F}} \min_{n=1 \sim N} SNR_{mnk}$$
(4.4)

where  $SNR_{mkn}$  is determined by (4.2) or (4.3).

#### 4.2 **Problem Formulation**

Instead of minimizing the total transmit power or maximizing the overall system capacity (throughput), we now choose to minimize the average BER performance with user peak power constraints and proportional subcarrier number fairness.

In order to avoid co-channel interference (CCI), we adopt the single-user-per-subcarrier policy, allowing each subcarrier to serve one user only. Define the subcarrier coefficient  $C_{mk}$  and let  $C_{mk} = 1$  if user k is to transmit on the mth subcarrier and  $C_{mk} = 0$ , otherwise. Denote the transmit power of the nth substream on the mth subcarrier of user k as  $p_{mnk}$ . Then RA is equivalent to solving the following optimization problem.

$$\arg\min_{C_{mk}, p_{mnk}} \frac{1}{MNK} \sum_{m=1}^{M} \sum_{n=1}^{N} \sum_{k=1}^{K} C_{mk} BER_{mnk}$$
(4.5)  
traints:

subject to the constraints:

$$\sum_{m=1}^{M} C_{m1} : \sum_{m=1}^{M} C_{m2} : \dots : \sum_{m=1}^{M} C_{mk} = R_1 : R_2 : \dots : R_K$$
(4.5a)

$$\sum_{k=1}^{K} C_{mk} = \mathbf{1}_{\mathbf{896}} \forall \ m$$
(4.5b)

$$C_{mk} \in \{0,1\} \qquad \forall \ m,k \tag{4.5c}$$

$$p_{mnk} \ge 0 \qquad \forall r, m, k$$

$$(4.5d)$$

$$\sum_{m=1}^{M} \sum_{n=1}^{N} C_{mk} p_{mnk} = \bar{P} \qquad \forall k$$
(4.5e)

where  $R_k$  denotes the date rate of user k and  $\overline{P}$  is the user's power constraint. The constraint (4.5b) means that the subcaarier numbers assigned to users are proportional to the user's data rates. As mentioned in chapter 2, if an *M*-ary quadrature amplitude modulation (M-QAM) is employed, then  $BER_{mnk}$  or  $p_{rmk}$  is given by [19]

$$BER_{mnk} = \frac{1}{5} \exp\left(-SNR_{mnk}\frac{1.5}{2^b - 1}\right) \tag{4.6}$$

where b is the number of transmit bits of each substream.

#### 4.3 **Resource Allocation Algorithms**

In this section, we propose two adaptive RA algorithms for both subcarrier assignment and power loading.

#### 4.3.1 The Subcarrier Assignment Algorithm

As mentioned in Section 4.2, in order to minimize average user's BER, we have to maximize the minimum substream SNR. Thus we will assign subcarriers based on the minimum substream SNR of each user.

First, the subcarrier number of each user will be determined by the user's data rate such that

$$c_1:c_2:\ldots:c_K = R_1:R_2:\ldots:R_K$$

$$(4.7)$$

where  $c_k$  is the subcarrier number of the user k. After determining the subcarrier number of each user, we then begin to assign subcarriers to the user. We assumed that the total power of user k is equally distributed to the all substreams on the subcarriers assigned to the user k. Thus SNR of the *n*th substream on the *m*th subcarrier of the user k is given by

$$SNR_{mnk}^{(ZF)} = \frac{E_m}{NN_0 [\mathbf{F}_{mk}^{\dagger} \mathbf{H}_{mk}^{\dagger} \mathbf{H}_{mk} \mathbf{F}_{mk}]_{n,n}^{-1}}$$
(4.8)

for the ZF receiver and

$$SNR_{mnk}^{(MMSE)} = \frac{E_m}{NN_0 [\mathbf{F}_{mk}^{\dagger} \mathbf{H}_{mk}^{\dagger} \mathbf{H}_{mk} \mathbf{F}_{mk} + (NN_0/E_m) \mathbf{I}_N]_{n,n}^{-1}} - 1$$
(4.9)

for the MMSE decoder, as described in section 4.2. The ordering of subcarriers in the subcarrier assignment process is important as once the subcarriers are assigned, no re-assignment is allowed. We first sort the user on each subcarrier according to their minimum substream SNR in descending order and denote the largest  $SNR_{mnk}$  on the mth subcarrier as  $Q_m$  and then sort subcarriers according to  $Q_m$  in descending order. Once the order of the subcarrier is determined, we assign the subcarrier to the user with largest minimum substream SNR. If that user has been assigned enough subcarriers, then the current subcarrier will assigned to the remained users with largest minimum substream SNR. The detail can be checked in **Table 4.1**.



Table 4.1: The subcarrier assignment algorithm.

#### 4.3.2 The Power Loading Scheme

In the previous subsection, we assume that the total power of user k is equally distributed to the all substreams on the subcarriers assigned to the user k and perform dynamic subcarrier assignment to extract the diversity gain of multiuser MIMO-OFDMA systems. In this subsection, we consider the dynamic power loading to further enhance the overall system performance.

As discussed in section 4.2, our goal is to minimize the average BER. In [26], the author had derived how to obtain the optimum power allocation for minimizing BER in muticarrier systems. Now we follow the method proposed in [26] to get the optimal power loading for the codebook based MIMO-OFDMA systems.

Since the subcarrier assignment has been done in previous subsection and we allow at most one user to transmit signals on each subcarrier, there is no co-channel interference and therefore the multiuser power loading is then decoupled into single user case. That is, we can deal with the power allocation for each user individually.

The BER for the *n*th substream on the *m*th subcarrier is generally a function of the corresponding power and GNR (gain to noise ratio), like (4.6). Because (4.6) is a convex function with respect to the power  $p_{mnk}$ , we can use the Lagrange multiplier method with the total power constraint. The Lagrangian function of user k may be expressed as

$$J(P_{k_11k}, P_{k_12k}, \dots, P_{k_{c_k}Nk}) = \frac{1}{c_k N} \sum_{t=1}^{c_k} \sum_{n=1}^N BER_{k_tnk} + \lambda_k \left(\sum_{t=1}^{c_k} \sum_{n=1}^N p_{k_tnk} - \bar{P}\right) \quad (4.10)$$

where  $k_1 \sim k_{c_k}$  is the subcarrier index assigned to the user k and  $\lambda_k$  denotes the Lagrange multiplier. By differentiating (4.10) with respect to  $p_{k_tn_k}$  and setting it to zero, we obtain a set of equations as

$$\frac{1}{c_k N} \frac{\partial BER_{k_t nk}}{\partial p_{k_t nk}} + \lambda_k = 0 \quad k = 1, 2, 3, \dots, K.$$

$$(4.11)$$

As mentioned before,  $BER_{ktnk}$  is the function of  $p_{ktnk}$  and  $GNR_{ktnk}$  (4.6). After some computation, we can get

$$p_{ktnk} = \frac{2^b - 1}{1.5GNR_{ktnk}} \ln\left(\frac{0.3GNR_{ktnk}}{c_k N\lambda_k (2^b - 1)}\right).$$
(4.12)

But (4.12) still depends on the Lagrange multiplier  $\lambda_k$ . So we take (4.12) into the user's

power constraint  $\sum_{t=1}^{c_k} \sum_{n=1}^{N} p_{k_t n k} = \bar{P}$  and then we can express  $\lambda_k$  as

$$\lambda_k = \exp\left(-\frac{\bar{P} - \sum_{t=1}^{c_k} \sum_{n=1}^{N} \frac{2^{b-1}}{1.5GNR_{k_tnk}} \ln(\frac{0.3GNR_{k_tnk}}{c_k N(2^{b}-1)})}{\sum_{t=1}^{c_k} \sum_{n=1}^{N} \frac{2^{b-1}}{1.5GNR_{k_tnk}}}\right).$$
(4.13)

Thus, the corresponding power can then be computed. It is noted that if some substreams' power is negative after the computation, it means that the GNRs of these substream are too low and these substreams should not be allocated any power in order not to deteriorate the overall performance. In such case, we should exclude these substreams and do the Lagrange multiplier method again until the power of all substreams are not negative.

# 4.4 Complexity Analysis and Numerical Results

#### 4.4.1 Computational Complexity Analysis

In this subsection, we analyze the complexity of the subcarrier assignment algorithm and the power loading algorithm. 1896

For the subcarrier assignment algorithm, the complexity of step 2 is  $O(KNlog_2N + MKlog_2K + Mlog_2M)$ , and for step 3 the complexity is O(MK), so the total complexity of the subcarrier assignment algorithm is  $O(KNlog_2N + MKlog_2K + Mlog_2M + MK) \approx O(MKlog_2K)$ . And for the power loading algorithm, the complexity is O(NM).

#### 4.4.2 Numerical Results

Selected simulated performance of the proposed RA algorithm are presented in this subsection. First the performance of the subcarrier assignment algorithm for the codebook based MIMO-OFDAM systems is shown in Fig.  $4.1 \sim 4.4$ . For Fig.  $4.5 \sim 4.6$ , we evaluate the performance of the proposed power-loading algorithm.

We assume each  $\mathbf{H}_{mk}$  is a 4 × 2 (4 antennas at the BS and 2 antennas at each MS) matrix for 2 substream codebook 4×3 (4 antennas at the BS and 3 antennas at each MS) matrix for 3 substream codebook with i.i.d. zero-mean, unit-variance complex Gaussian entries. The system's modulation mode is BPSK. For simplicity, we assume that the required data rate are the same for all users. The codebook used here is from 802.16e standard.

In Fig. 4.1 and Fig. 4.2, we compare the subcarrier assignment algorithm for the ZF receiver with fixed subcarrier assignment scheme. From the figures we can find that the performance of the dynamic subcarrier assignment is better than fixed subcarrier assignment by almost 4dB at  $BER=10^{-2}$  for the 2 substream case and more than 4dB at  $BER=10^{-2}$  for the 3 substream case. The same result can also be found when the MMSE receiver is used (Fig. 4.3 and Fig. 4.4). The performance of the dynamic subcarrier assignment is better than fixed subcarrier assignment by 3dB at  $BER=10^{-2}$  for the 2 substream case.

For the dynamic power loading algorithm, we compare the performance of it with the equally power distributed system. We consider three different scheme:fixed subcarrier assignment without codebook precoding, fixed subcarrier assignment with codebook precoding and dynamic subcarrier assignment with codebook precoding. Here we assume QPSK modulation is used. In Fig. 4.5, using the dynamic power loading algorithm will provide nearly 1.5dB gain at BER= $10^{-2}$  over the equally power distributed system in the fixed subcarrier assignment without codebook precoding environment. In Fig. 4.6, the dynamic power loading algorithm also achieves approximately 1dB gain at BER= $10^{-2}$  in the fixed subcarrier assignment with codebook precoding environment. Finally, in Fig.4.7, the dynamic power loading algorithm is superior to the equally power distributed system in the dynamic subcarrier assignment with codebook precoding environment by more than 1.5dB at BER= $10^{-4}$ .

These figures (Fig. 4.5  $\sim$  4.7) also show that the improvement of the dynamic powerloading algorithm is more obvious in the fixed subcarrier assignment without codebook precoding environment. This is because that without precoding, the variation of the channel condition is much larger. Simulation results show that the variance of GNR is reduced by almost 50% after precoding. Therefore, the performance gain offered by the power-loading algorithm is smaller in the other two cases.



Figure 4.1: Average BER performance for the ZF receiver ; 128 subcarriers, 8 users, 2 substreams.



Figure 4.2: Average BER performance for the ZF receiver ; 128 subcarriers, 8 users, 3 substreams.



Figure 4.3: Average BER performance for the MMSE receiver ; 128 subcarriers, 8 users, 2 substreams.



Figure 4.4: Average BER performance for the MMSE receiver ; 128 subcarriers, 8 users, 3 substreams.



Figure 4.5: Average BER performance for the ZF receiver; fixed subcarrier assignment without codebook precoding; 128 subcarriers, 16 users, 2 substreams.



Figure 4.6: Average BER performance for the ZF receiver ; fixed subcarrier assignment with codebook precoding ; 128 subcarriers, 16 users, 2 substreams.



Figure 4.7: Average BER performance for the ZF receiver ; dynamic subcarrier assignment with codebook precoding ; 128 subcarriers, 16 users, 2 substreams.

# Chapter 5 Conclusion

The allocation of radio resources in a MIMO-OFDMA system is critical in maximizing resource efficiency, system capacity, and mitigating interference. We have presented two SVD-based precoding schemes (orthogonal GS precoding and non-orthogonal precoding) that minimize the total consumed power while meeting various rate and SINR requirements. For the orthogonal (GS) precoding scheme, we propose two adaptive RA algorithms and provide simulation results that prove the effectiveness of both algorithms in both uplink and downlink scenarios. These two algorithms yield almost the same relative performance when compared with the optimal solution. To further increase the spectrum efficiency, we extend our concern to non-orthogonal precoding schemes that guarantee zero or limited cochannel interference. An adaptive RA algorithm is proposed and its numerical performance is given. It is found that the lift of the orthogonal constraint leads to improved performance when the rank of the channel matrix is sufficient.

We also consider the RA issue for spatial multiplexing systems with limited feedback (codebook based precoding) and present subcarrier assignment and power loading algorithms that minimize the average BER performance. The simulation results show that these dynamic RA methods do indeed yield low average BER performance.

Several remarks on possible extensions to our work are in order. Firstly, the proposed schemes can be further extended for use in a multi-cellular network with the transmit antennas distributed among several BS'. They are certainly viable candidate RA and interference control schemes for networked MIMO systems. Secondly, the only performance criterion we consider is total power or average BER minimization. These criteria are often more suitable for uplink design when the transmit power is limited. For downlink communications, it is more desirable to maximize the total throughput and take the fairness issue into account. Thirdly, since the resource unit considered in our work is a single space-frequency subchannel, transmitting the overall RA information would require a large control-channel bandwidth. Practical concern often implies a resource unit be made of multiple space-time-frequency subchannels. It would be desirable to provide modified RA solutions that take such a resource unit definition into account. Finally, the fairness concern is usually answered by implementing a proper scheduling. Such a scheduling has to consider the time-varying nature of the associated multiuser channel. Therefore, a complete RA design needs to consider the channels' time, frequency and space selectivities and invoke appropriate multiuser channel models accordingly.



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