Solution for first-order design of a two-conjugate zoom system

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Abstract. The first-order solution is analyzed with particularly initial conditions for a two-conjugate zoom system that consists of three lenses. We find the solution ranges of system parameters with the conditions in which the object/image and pupil magnifications of the middle lens are taken to be 1 and −1 or −1 and 1, and the system magnification is 1 or −1 at the mean position of zooming. Several examples are given to demonstrate this analysis. [S0091-3286(97)03608-8]

Subject terms: two-conjugate zoom lenses.

Paper 19126 received Dec. 11, 1996; revised manuscript recieved Mar. 11, 1997; accepted for publication Mar. 11, 1997.

1 Introduction

A two-conjugate zoom system is one in which not only the object and image but also the entrance and exit pupils are fixed during zooming. Such a zoom system needs at least three lenses, which move separately. Hopkins[1] and Shiue[2] discussed two special symmetrical two-conjugate zoom systems with different initial conditions at the mean position of zooming. We have presented the general analysis for the first-order design of a two-conjugate zoom system.[3] In that paper, we analyzed the possible solution areas for the two pairs of conjugate positions under two particularly initial conditions. These solution areas are constrained by the positive interlens separations.

In many practical designs, the zoom system has an overall magnification that varies from $|M|=\sqrt{R}$ to $|M|=1/\sqrt{R}$ with a zoom ratio R. In this case, the initial condition is usually specified at the mean position of zooming, in which the system magnification is 1 or −1. With this initial condition, the design of a two-conjugate zoom system can be significantly simplified. In this paper, we analyze the solutions for the two-conjugate zoom system of three lenses under the initial conditions in which the object/image and pupil magnifications of the middle lens are taken to be $M_2=1$ and $\overline{M}_2=-1$ or $M_2=-1$ and $\overline{M}_2=1$, as used in our previous paper,[3] and the object/image magnification of the system is 1 or −1. We find the solution ranges of the object/image positions and the entrance/exit pupil positions that make both the interlens separations positive. Five examples are presented to demonstrate this analysis.

2 Theory

The notation used in this paper is shown in Fig. 1. The object O is imaged at O' with a object/image magnification M, and the entrance pupil E is imaged at the exit pupil E' with a pupil magnification $\overline{M}$. Here l and l' are the distances from the first principal plane H of the system to the object and from the second principal plane H' of the system to the image, respectively. Similarly, $\overline{l}$ and $\overline{l'}$ are the distances from the first principal plane to the entrance pupil and from the second principal plane to the exit pupil. The quantity $L=(OE)$ is the distance from object to entrance pupil of the system, and $L'=(O'E')$ is the distance from image to exit pupil. In this paper the distance to the right of a reference point is positive; that to the left is negative. In this analysis, we take the paraxial and thin-lens approximations.

In the following, we discuss the solutions with two initial cases in which $M_2=1$ and $\overline{M}_2=-1$ or $M_2=-1$ and $\overline{M}_2=1$, and the system magnifications $M=1$ or −1 at the mean position.

2.1 *Case 1: $M_2=1$, $\overline{M}_2=-1$, and $M=1$ or −1*

As shown in Fig. 2, the marginal ray is through the center of the middle lens. The distances d_{12} and d_{23} are the interlens separations; l_1 and $\overline{l}_1$ are the object and entrance pupil distances from the first lens; l_3' and $\overline{l}_3'$ are the image and exit pupil distances from the third lens. From Gaussian optics, we have[3]

$$F_2=-\frac{M_1-\overline{M}_1}{2}\,F_1, \tag{1}$$

$$F_3=-\frac{2}{\dfrac{1}{M_3}-\dfrac{1}{\overline{M}_3}}\,F_2, \tag{2}$$

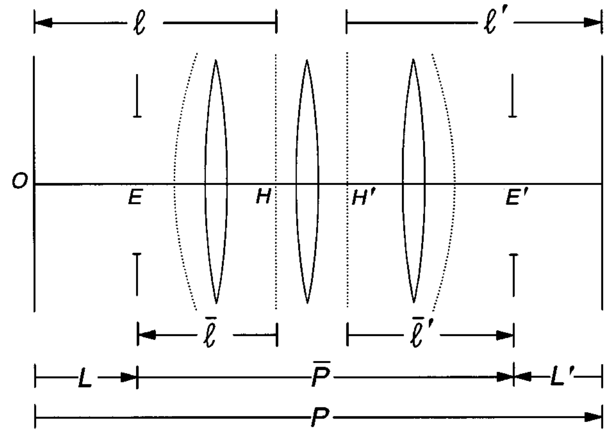

Fig. 1 Diagram of the Gaussian optics. Here P and $\bar{P}$ are the distance from object to image and the distance from entrance to exit pupil, respectively.

$$d_{12}=(1-M_1)F_1, \tag{3}$$

$$d_{23}=-\left(\frac{1}{M_3}-1\right)F_3, \tag{4}$$

$$F_3=\frac{P-(2-M_1-1/M_1)F_1}{2-M_3-1/M_3}, \tag{5}$$

where M_1 and $\overline{M_1}$ are the object/image and pupil magnifications of the first lens, respectively, and P is the distance from object to image (OO').

Assuming the object/image magnification of system is 1 at the mean position of zooming, we have $M_3=1/M_1$. Eq. (5) becomes

$$F_3=\frac{P}{2-M_1-1/M_1}-F_1=\left[-\frac{P(l_1+F_1)}{l_1^2}-1\right]F_1. \tag{6}$$

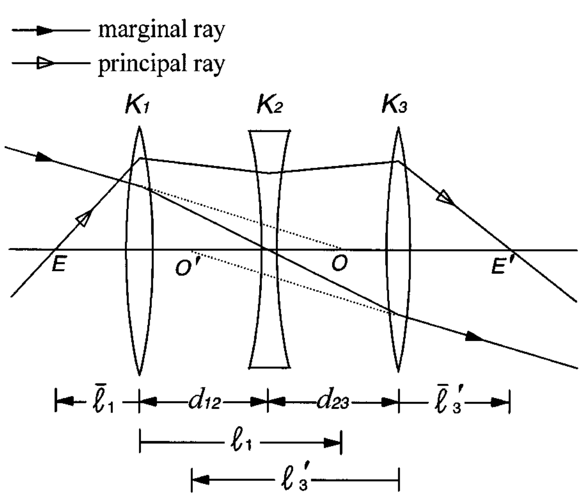

Fig. 2 Three-lens zoom system with the initial conditions $M_2=1$ and $\bar{M}_2=-1$. The marginal ray is through the center of the middle lens.

Table 1 Solution ranges of object position $\bar{l}_1$ for different types of F_1 with the initial conditions $M_2=1$ and $\bar{M}_2=-1$.

Type of F_1	Solution range of l_1
$F_1>0$	$l_1+F_1>F_1$ or $l_1+F_1<0$
$F_1<0$	$F_1<l_1+F_1<0$

Referring to Table 1, taken from Ref. 3, we have the solution ranges of l_1+F_1 for a positive d_{12}. Using Eq. (6), we can obtain the solution ranges of P and l_1 for different combinations of F_1 and F_3.

Because $M_3=1/M_1$, the image distance l_3' can be expressed as follows:

$$l_3'=\left(1-\frac{1}{M_1}\right)F_3=-\frac{F_3}{F_1}\ l_1, \tag{7}$$

or

$$l_3'-F_3=-(l_1+F_1)\ \frac{F_3}{F_1}. \tag{8}$$

The value of $l_3'-F_3$ is also required to be located in the solution ranges for a positive d_{23} listed in Table 2, which is also taken from Ref. 3. This requirement provides another constraint on the solution ranges of P and l_1 with Eq. (8). From above analysis, we can list the solution ranges of P, l_1, and l_3' for different combinations of F_1 and F_3 in Table 3(a).

If the object/image magnification of the system is -1 at the mean position, we get $M_3=-1/M_1$. Equations (6) to (8) become

$$F_3=\frac{P-(2-M_1-1/M_1)F_1}{2+M_1+1/M_1}, \tag{9}$$

$$l_3'=\left(1+\frac{1}{M_1}\right)F_3=\frac{l_1+2F_1}{F_1}\ F_3, \tag{10}$$

or

$$l_3'-F_3=(l_1+F_1)\ \frac{F_3}{F_1}. \tag{11}$$

Using the same method as before, we get the results listed in Table 3(b).

Table 2 Solution ranges of image position $\bar{l}_3'$ for different types of F_3 with the initial conditions $M_2=1$ and $\bar{M}_2=-1$.

Type of F_3	Solution range of l_3'
$F_3>0$	$l_3'-F_3>0$ or $l_3'-F_3<-F_3$
$F_3<0$	$-F_3>l_3'-F_3>0$

Table 3 Solution ranges of P, l_1, and l'_3 for different combinations of F_1 and F_3 with the initial conditions $M_2=1$ and $\bar{M}_2=-1$.

Types of lenses	Range of P	Range of l_1	Range of l'_3
	(a) $M=1,\ M_1=1/M_3$		
$F_1>0,\ F_3>0$	$P>-\frac{l_1^2}{l_1+F_1}>0$	$l_1<-F_1$	$l'_3>F_3$
	$P<-\frac{l_1^2}{l_1+F_1}<0$	$l_1>0$	$l'_3<0$
$F_1>0,\ F_3<0$	No solution	No solution	No solution
$F_1<0,\ F_3>0$	No solution	No solution	No solution
$F_1<0,\ F_3<0$	$P>-\frac{l_1^2}{l_1+F_1}>0$	$0<l_1<-F_1$	$F_3<l'_3<0$
	(b) $M=-1,\ M_1=-1/M_3$		
$F_1>0,\ F_3>0$	$P>-\frac{l_1^2}{l_1+F_1}$	$l_1>0$	$l'_3>2F_3$
	$P<-\frac{l_1^2}{l_1+F_1}$	$l_1<-2F_1$	$l'_3<0$
$F_1>0,\ F_3<0$	$P>-\frac{l_1^2}{l_1+F_1}>0$	$-2F_1<l_1<-F_1$	$F_3<l'_3<0$
$F_1<0,\ F_3>0$	$P>-\frac{l_1^2}{l_1+F_1}>0$	$0<l_1<-F_1$	$F_3<l'_3<2F_3$
$F_1<0,\ F_3<0$	No solution	No solution	No solution

As discussed in Ref. 3, the solutions for $\bar{l}_1$ and $\bar{l}'_3$ are acceptable only if l_1 and l'_3 are located in their solution ranges. So in practical design, P, l_1, and F_1 are given according to the system requirements. The values of F_3 and l'_3 are then calculated from Eqs. (6) and (7) for $M=1$ or from Eqs. (9) and (10) for $M=-1$. Referring to the solution areas in Fig. 3 of Ref. 3, we select $\bar{l}_1$ and then can determine $\bar{M}_1$ and F_2 with $\bar{l}_1=(1/\bar{M}_1-1)F_1$ and Eq. (1). The interlens separations, d_{12} and d_{23}, are obtained from Eqs. (3) and (4). Finally, $\bar{l}'_3$ is given by the following equations:

$$\bar{M}_3=\frac{F_3}{L-P+(2-\bar{M}_1-1/\bar{M}_1)F_1+4F_2+(2-M_3)F_3} \quad (12)$$

and

$$\bar{l}'_3=(1-\bar{M}_3)F_3, \quad (13)$$

where $L=\bar{l}_1-l_1$.

Of course, if the initial structure is not satisfactory, the input parameters should be adjusted. In zooming, we obtain the interlens separations, d_{12} and d_{23}, and the object and image distances, l_1 and l'_3, with the same procedures as described at Sec. 2.3 in Ref. 3.

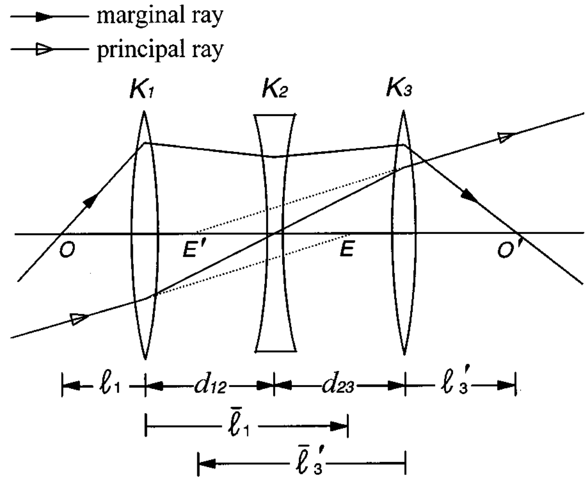

Fig. 3 Three-lens zoom system with the initial conditions $M_2=-1$ and $\bar{M}_2=1$. The principal ray is through the center of the middle lens.

2.2 *Case 2: $M_2=-1$, $\bar{M}_2=1$, and $M=1$ or -1*

In this case, the principal ray is through the center of the middle lens as shown in Fig. 3. Similar to case 1, we have the related equations as follows:

$$F_2=\frac{M_1-\bar{M}_1}{2}F_1, \quad (14)$$

$$F_3=\frac{2}{\frac{1}{M_3}-\frac{1}{\bar{M}_3}}F_2, \quad (15)$$

$$d_{12}=(1-\bar{M}_1)F_1, \quad (16)$$

$$d_{23}=-\left(\frac{1}{\bar{M}_3}-1\right)F_3, \quad (17)$$

$$F_3=\frac{P-(2-M_1-1/M_1)F_1-4F_2}{(2-M_3-1/M_3)} \quad (18)$$

$$=\frac{\bar{P}-(2-\bar{M}_1-1/\bar{M}_1)F_1}{2-\bar{M}_3-1/\bar{M}_3}, \quad (19)$$

$$l'_3=(1-M_3)F_3, \quad (20)$$

$$\bar{M}_3=\frac{F_3}{L-P+(2-\bar{M}_1-1/\bar{M}_1)F_1+(2-M_3)F_3}, \quad (21)$$

$$\bar{l}'_3=(1-\bar{M}_3)F_3, \quad (22)$$

where $\bar{P}$ is the distance from entrance pupil to exit pupil (EE').

In the same way as we did in case 1, we analyze the two cases corresponding to the object/image magnification

Table 4 Solution ranges of entrance pupil position $\bar{l}_1$ for different types of F_1 with the initial conditions $M_2=-1$ and $\bar{M}_2=1$.

Type of F_1	Solution range of $\bar{l}_1$
$F_1>0$	$\bar{l}_1+F_1>F_1$ or $\bar{l}_1+F_1<0$
$F_1<0$	$F_1<\bar{l}_1+F_1<0$

$M=1$ or $\bar{M}=-1$ at the mean position of zooming. The solutions for l_1 and l_3' are acceptable if $\bar{l}_1$ and $\bar{l}_3'$ are in their solution ranges given in Tables 4 and 5 under the initial conditions[3] $M_2=-1$ and $\bar{M}_2=1$. Using the relations $l_1=(1/M_1-1)F_1$ and $M_3=-1/M_1$ for $M=1$ or $M_3=1/M_1$ for $M=-1$, we can find reasonable solutions for P, l_1, and l_3' with Eqs. (18) and (20). The results are listed in Table 6. According to system requirements and the solution areas in Fig. 6 of Ref. 3, we give the parameters l_1, $\bar{l}_1$, P, M, and F_1, so we have M_1 and $\bar{M}_1$ from $l_1=(1/M_1-1)F_1$ and $\bar{l}_1=(1/\bar{M}_1-1)F_1$, and then can calculate F_2 and F_3 with Eqs. (14) and (18), where we use $\bar{M}_3=-1/\bar{M}_1$ or $1/\bar{M}_1$. The values of d_{12}, d_{23}, l_3', and $\bar{l}_3'$ are then obtained from Eqs. (16), (17), (20), and (22). In this case, the results for $l_3'-F_3$ and $\bar{l}_3'-F_3$ may not fall in the solution areas in Fig. 7 of Ref. 3. This means that the separation d_{23} is not positive and the input parameters, especially $\bar{l}_1$, should be adjusted.

In order to make sure both d_{12} and d_{23} are positive, we can assume two special situations of pupil magnifications: $M=1$ with $M_1=1/M_3$, and $\bar{M}=-1$ with $\bar{M}_1=-1/\bar{M}_3$. Using Eq. (19) and the solution ranges for $\bar{l}_1$ and $\bar{l}_3'$ in Tables 4 and 5, we can find the solution ranges of P, $\bar{l}_1$, and $\bar{l}_3'$ listed in Table 7. In design, l_1, $\bar{l}_1$, M, $\bar{M}$, and F_1 are given. So we know M_1, $\bar{M}_1$, M_3, and $\bar{M}_3$. Note that the combinations $M=1$ and $\bar{M}=1$ or $M=-1$ and $\bar{M}=-1$ cannot be taken, since both of them will result in $L=L'=0$. The values of F_2 and F_3 are calculated from Eqs. (14) and (15). If the system with $M=-1$ and $\bar{M}=1$ is used, we have the results $F_1=F_3$ and $L=-L'$, which were described by Hopkins.[1] Likewise, we have the results $F_1=-F_3$ and $L=-L'$ in the case of $M=1$ and $\bar{M}=-1$. As before, d_{12}, d_{23}, l_3', and $\bar{l}_3'$ are then given by the related equations. After we get the initial structure, d_{12}, d_{23}, l_1, and l_3' during zooming are calculated with the same procedures as that in Sec. 2.1.

Table 5 Solution ranges of exit pupil position $\bar{l}_3'$ for different types of F_3 with the initial conditions $M_2=-1$ and $\bar{M}_2=1$.

Type of F_3	Solution range of $\bar{l}_3'$
$F_3>0$	$\bar{l}_3'-F_3>0$ or $\bar{l}_3'-F_3<-F_3$
$F_3<0$	$-F_3>\bar{l}_3'-F_3>0$

Table 6 Solution ranges of P, l_1, and l_3' for different combinations of F_1 and F_3 with the initial conditions $M_2=-1$ and $\bar{M}_2=1$.

Types of lenses	Range of P	Range of l_1	Range of l_3'
(a) $M=1$, $M_1=-1/M_3$			
$F_1>0$, $F_3>0$	$P>-\frac{l_1^2}{l_1+F_1}+4F_2$	$l_1>-F_1$	$l_3'>F_3$
or			
$F_1<0$,$F_3<0$	$P<-\frac{l_1^2}{l_1+F_1}+4F_2$	$l_1<-F_1$	$l_3'<F_3$
$F_1>0$, $F_3<0$	$P>-\frac{l_1^2}{l_1+F_1}+4F_2$	$l_1<-F_1$	$l_3'>F_3$
or			
$F_1<0$, $F_3>0$	$P<-\frac{l_1^2}{l_1+F_1}+4F_2$	$l_1>-F_1$	$l_3'<F_3$
(b) $M=-1$, $M_1=1/M_3$			
$F_1>0$, $F_3>0$	$P>-\frac{l_1^2}{l_1+F_1}+4F_2$	$l_1<-F_1$	$l_3'>F_3$
or			
$F_1<0$, $F_3<0$	$P<-\frac{l_1^2}{l_1+F_1}+4F_2$	$l_1>-F_1$	$l_3'<F_3$
$F_1>0$, $F_3<0$	$P>-\frac{l_1^2}{l_1+F_1}+4F_2$	$l_1>-F_1$	$l_3'>F_3$
or			
$F_1<0$, $F_3>0$	$P<-\frac{l_1^2}{l_1+F_1}+4F_2$	$l_1<-F_1$	$l_3'<F_3$

3 Examples

With the method described above, we design five different types of zoom systems. Here as examples we take the systems that consist of two positive lenses and a negative lens in between. The zoom system has the overall magnification M, which varies from $|M|=\sqrt{R}$ to $|M|=1/\sqrt{R}$.

3.1 *Example 1: Given $l_1=85$, $\bar{l}_1=-120$, and $M=1$ at the Mean Position and $P=-110$, $F_1=50$*

In this example, the object is virtual and the entrance pupil is real at the mean position. Using the solution range in the second row of Table 3(a), we have $M_2=1$, $\bar{M}_2=-1$, and $M_3=1/M_1$. A positive F_3 and a negative l_3' are predicted. Following the calculating procedures described at case 1 in Sec. 2.1, we obtain $F_2=-27.1164$, $F_3=52.7682$, $l_3'=-89.706$, $\bar{l}_3'=133.038$, $M_1=0.370370$, $M_3=2.7$, $d_{12}=31.481$, and $d_{23}=33.224$ at the mean position. In this case, L is negative and L' is positive. During zooming, the result is shown in Fig. 4, with the natural logarithm of $|M|$ as ordinate. The system has a zoom ratio of 16 : 1, and the magnification M is from 4 to 0.25.

Table 7 Solution ranges of $\bar{P}$, $\bar{l}_1$, and $\bar{l}'_3$ for different combinations of F_1 and F_3 with the initial conditions $M_2=-1$ and $\bar{M}_2=1$.

Types of lenses	Range of $\bar{P}$	Range of $\bar{l}_1$	Range of $\bar{l}'_3$
	(a) $\bar{M}=1$, $\bar{M}_1=1/\bar{M}_3$		
$F_1>0$, $F_3>0$	$\bar{P}>-\frac{\bar{l}_1^2}{\bar{l}_1+F_1}>0$	$\bar{l}_1<-F_1$	$\bar{l}'_3>F_3$
	$\bar{P}<-\frac{\bar{l}_1^2}{\bar{l}_1+F_1}<0$	$\bar{l}_1>0$	$\bar{l}'_3<0$
$F_1>0$, $F_3<0$	No solution	No solution	No solution
$F_1<0$, $F_3>0$	No solution	No solution	No solution
$F_1<0$, $F_3<0$	$\bar{P}>-\frac{\bar{l}_1^2}{\bar{l}_1+F_1}>0$	$0<\bar{l}_1<-F_1$	$F_3<\bar{l}'_3<0$
	(b) $\bar{M}=-1$, $\bar{M}_1=-1/\bar{M}_3$		
$F_1>0$, $F_3>0$	$\bar{P}>-\frac{\bar{l}_1^2}{\bar{l}_1+F_1}$	$\bar{l}_1>0$	$\bar{l}'_3>2F_3$
	$\bar{P}<-\frac{\bar{l}_1^2}{\bar{l}_1+F_1}$	$\bar{l}_1<-2F_1$	$\bar{l}'_3<0$
$F_1>0$, $F_3<0$	$\bar{P}>-\frac{\bar{l}_1^2}{\bar{l}_1+F_1}>0$	$-2F_1<\bar{l}_1<-F_1$	$F_3<\bar{l}'_3<0$
$F_1<0$, $F_3>0$	$\bar{P}>-\frac{\bar{l}_1^2}{\bar{l}_1+F_1}>0$	$0<\bar{l}_1<-F_1$	$F_3<\bar{l}'_3<2F_3$
$F_1<0$, $F_3<0$	No solution	No solution	No solution

3.2 *Example 2: Given $l_1=120$, $\bar{l}_1=-160$, and $M=-1$ at the Mean Position and $P=180$, $F_1=50$*

Here we use the solution range in the first row of Table 3(b) and have $M_2=1$, $\bar{M}_2=-1$, and $M_3=-1/M_1$. Both positive F_3 and l'_3 are predicted. Following the same design procedures as that in example 1, we obtain $F_2=-18.7166$, $F_3=46.4876$, $l'_3=204.545$, $\bar{l}'_3=88.774$, $M_1=0.294118$, $M_3=-3.4$, $d_{12}=35.294$, and $d_{23}=60.160$ at the mean position. In this case, both L and L' are negative. The zoom loci are shown in Fig. 5. The system has a zoom ratio of 16 : 1, and the magnification M is from -4 to -0.25.

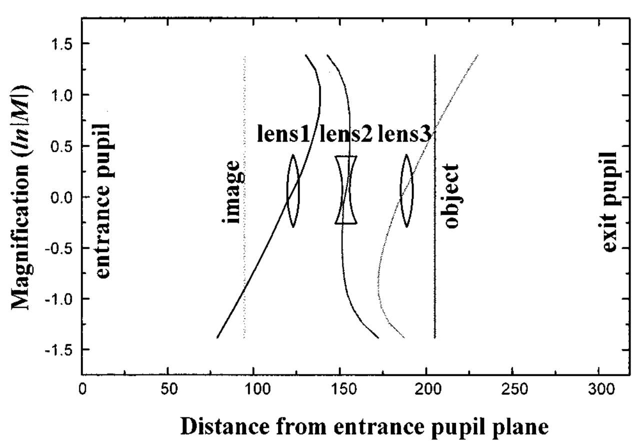

Fig. 4 Loci of three lenses in zooming with $F_1=50$, $F_2=-27.1164$, $F_3=52.7682$, and zoom ratio 16. The distance P from object to image is -110, the distance $\bar{P}$ from entrance to exit pupil is 317.744, and $L=-205$ and $L'=222.744$.

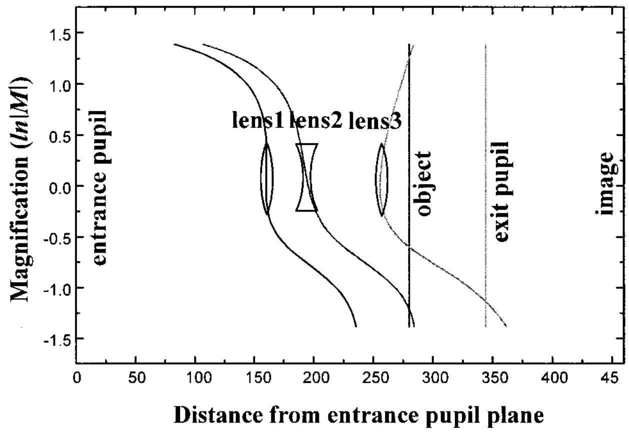

Fig. 5 Loci of three lenses in zooming with $F_1=50$, $F_2=-18.7166$, $F_3=46.4876$, and zoom ratio 16. The distance P from object to image is 180, the distance $\bar{P}$ from entrance to exit pupil is 344.229, and $L=-280$ and $L'=-115.771$.

3.3 *Example 3: Given $l_1=-130$, $\bar{l}_1=100$, and $M=-1$ at the Mean Position and $P=340$, $F_1=50$*

In this case, we use the solution range in the first row of Table 6(b) with $M_2=-1$, $\bar{M}_2=1$, and $M_3=1/M_1$. Both the positive F_3 and l'_3 are also predicted. Following the calculating procedures at case 2 in Sec. 2.2, we get $F_2=-23.9583$, $F_3=53.1558$, $l'_3=138.205$, $\bar{l}'_3=-139.133$, $M_1=-0.625$, $M_3=-1.6$, $d_{12}=33.333$, and $d_{23}=38.462$ at the mean position. The values of l'_3-F_3 and $\bar{l}'_3-F_3$ are located in the solution area marked (IVb) in Fig. 7(b) of Ref. 3. In zooming, the result is shown in Fig. 6. The system has a zoom ratio of 16 : 1, and the magnification M is from -4 to -0.25.

3.4 *Example 4: Given $l_1=-130$, $\bar{l}_1=100$, $M=-1$, and $\bar{M}=1$ at the Mean Position and $F_1=50$*

Instead of P in the previous example, we have $\bar{M}=1$ as an input parameter. We use the solution range in the second row of Table 7(a) for $\bar{l}_1$ and $\bar{l}'_3$. Following the procedures on the condition of $\bar{M}=1$ described in case 2 of Sec. 2.2, we obtain $F_2=-23.9583$, $F_3=50$, $l'_3=130$, $\bar{l}'_3=-100$, $M_1=-0.625$, $M_3=-1.6$, $\bar{M}_1=0.333333$, $\bar{M}_3=3$, $d_{12}=33.333$, and $d_{23}=33.333$ at the mean position. We have the results $F_1=F_3$, $d_{12}=d_{23}$, $M_1=1/M_3$, $\bar{M}_1=1/\bar{M}_3$, and $L=-L'$ as derived by Hopkins.[1] The zoom loci are shown in Fig. 7. The system has a zoom ratio of 16 : 1, and the magnification M is from -4 to -0.25.

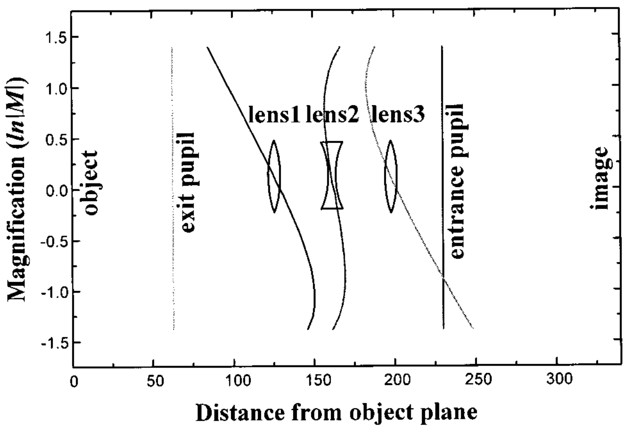

Fig. 6 Loci of three lenses in zooming with $F_1=50$, $F_2=-23.9583$, $F_3=53.1558$, and zoom ratio 16. The distance P from object to image is 340, the distance $\bar{P}$ from entrance to exit pupil is -167.338, and $L=230$ and $L'=-277.338$.

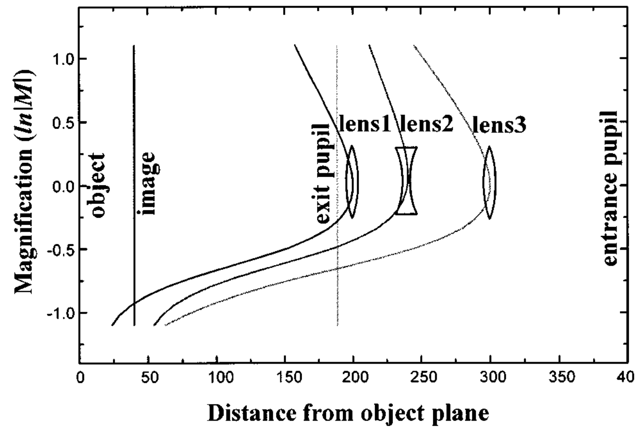

Fig. 8 Loci of three lenses in zooming with $F_1=50$, $F_2=-13.3333$, $F_3=129.9997$, and zoom ratio 9. The distance P from object to image is 40, the distance $\bar{P}$ from entrance to exit pupil is -211.428, and $L=400$ and $L'=148.572$.

3.5 *Example 5: Given $l_1=-200$, $\bar{l}_1=200$, and $M=1$ at the Mean Position and $P=40$, $F_1=50$*

We use the solution range in the second row of Table 6(a) with $M_2=-1$, $\bar{M}_2=1$, and $M_3=-1/M_1$. Following the same procedures as in example 3, we get $F_2=-13.3333$, $F_3=129.9997$, $l'_3=-259.9993$, $\bar{l}'_3=-111.428$, $M_1=-0.333333$, $M_3=3$, $d_{12}=40$, and $d_{23}=60$ at the mean position. The values of l'_3-F_3 and $\bar{l}'_3-F_3$ are in the solution area marked (III) in Fig. 7(b) of Ref. 3. The zooming result is shown in Fig. 8. The system has a zoom ratio of 9 : 1 and the magnification M is from 3 to 0.333333.

4 Discussion and Conclusion

We have analyzed the solutions with the particular initial conditions in which $M_2=1$ and $\bar{M}_2=-1$ or $M_2=-1$ and $\bar{M}_2=1$, and the system magnification $M=1$ or -1 at the mean position. We found the solutions for related system parameters that make both d_{12} and d_{23} positive. In case 1, where $M_2=1$, $\bar{M}_2=-1$, and $M=1$ or -1, the solution ranges are easily found and are listed in Table 1. But in case 2, where $M_2=-1$, $\bar{M}_2=1$, and $M=1$ or -1, we take the pupil magnification to be 1 or -1 as an additional condition at the mean position to make both the interlens separations positive. Tables 3, 6, and 7 show the solution ranges of system parameters, corresponding to the object/image and pupil positions, with different lens combinations and different initial conditions. From Tables 3 and 7, we find that only the F_1 and F_3 with the same sign are reasonable under the condition of $M=1$ in case 1 or $\bar{M}=1$ in case 2, and both the negative F_1 and F_3 are unacceptable under the condition of $M=-1$ in case 1 or $\bar{M}=-1$ in case 2. In examples 1 and 2, we use the same initial conditions on M_2 and $\bar{M}_2$, but different values of M. We find that the moving ranges of three lenses during zooming in example 1 are more compact than that in example 2. A similar situation occurs in examples 3 and 5.

In general, the first-order design of a two-conjugate zoom system is more difficult than that of an ordinary zoom system, since both the entrance and exit pupils are fixed during zooming. In our previous paper,[3] we have analyzed the general solutions for two pairs of the parameters $(l_1,\bar{l}_1)$ and $(l'_3,\bar{l}'_3)$ under two initial conditions. In this paper, with the extra conditions $M=1$ or -1 at the mean position, we can find the solutions that make both the d_{12} and d_{23} positive. With these analyses, it should be much easier to solve for the initial structure of the two-conjugate zoom lens.

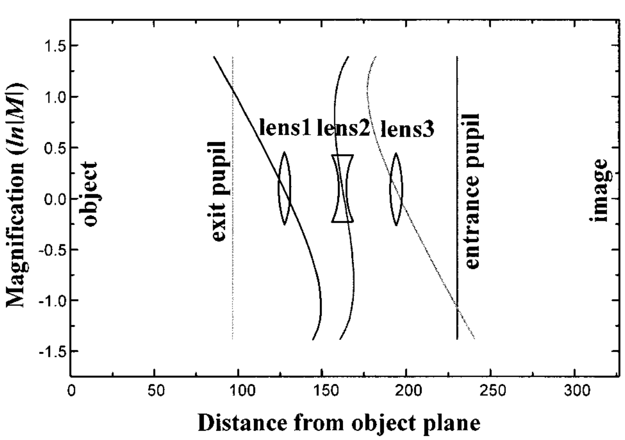

Fig. 7 Loci of three lenses in zooming with $F_1=50$, $F_2=-23.9583$, $F_3=50$, and zoom ratio 16. The distance P from object to image is 326.667, the distance $\bar{P}$ from entrance to exit pupil is -133.333, and $L=230$ and $L'=-230$.

Acknowledgments

This project was supported by the National Science Council of the Republic of China under grant No. NSC-85-2215-E-009-004.

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