國 立 交 通 大 學

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碩 士 論 文

車輛方向盤於不同路面操控模擬

Simulation on Different Road Surface of Vehicle Steering Wheel

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中華民國九十八年七月

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摘要

本文內容包括直流馬達、馬達驅動電路和微電腦控制器以及車輛方向盤機構，嘗試利用直流馬達的特性，模擬車輛在不同道路上行駛轉向，車輛方向盤上所反映的力矩變化，利用車輛動力學，我們可以藉由車輛的特徵參數及車輛行駛狀態來模擬，車輛行駛在不同的道路上，車輛方向盤應的回饋力矩變化。

更可以透過更改微電腦控制器中車輛的特徵參數和車輛行駛狀態，模擬不同車輛在不同路面上轉向時車輛方向盤力矩變化。

未來可發展為線控轉向系統其中的力回饋方向盤系統，以取代傳統轉向系統，透過力回饋馬達將路面狀況回授給駕駛人。

關鍵字：直流馬達、車輛方向盤、回饋力矩、線控轉向

Simulation on Different Road Surface of Vehicle Steering Wheel

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Abstract

This article content including the DC motor, the DC motor driving circuit and the microcomputer controller as well as the vehicles steering wheel organization, the attempt use DC motor's characteristic, simulates the vehicles to go on the different path changes, on the vehicles steering wheel reflected the moment variation. Using the vehicles dynamics, we may simulates because of vehicles' characteristic parameter and the vehicles drive condition, the vehicles travel on the different path, the vehicles steering wheel should respond how many moment variation.

In this article, we can change microcomputer controller vehicles' characteristic parameter and vehicles drive condition, easier simulation different vehicles go the vehicles steering wheel moment variation which in different road surfaces.

Keywords: DC motor, steering wheel of vehicle, feedback torque, steering-by-wire (SBW)

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林志隆僅於 2009.08

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Chapter 1 Introduction

1.1.Motive and objectives

We want to build the steer-by-wire model to replace the traditional steering model. The steer-by-wire system used the electronic signal to connect the steering wheel and front wheel reduce the steering column on vehicle can made the vehicle cost down, weight light and faster. So the project's objective is build the steer by wire (SBW) system control structure. In future we control steering of vehicle from the control structure.

1.2.Literature Review

The feedback torque of steering wheel is from the steering system of vehicles when the vehicle corner. So we research the steering system of vehicles to build the model of vehicles. From the steering system we know the feedback torque of steering wheel produced from the force on the tire of front wheels.

From 1931 year, Bradley and Allen being to research the dynamic of vehicle tires. Now, we have several ways to build the model of tire.

1.2.1. Faila's model

Faila used the basis of Fromm's model obtain the equation of lateral force, aligning torque and slip angle.[1]

$$F = \frac{K_1 l^2}{2}\tan\beta - \frac{1}{8}\frac{K_1{}^2 l^3}{\mu p_m b}\tan^2\beta + \frac{1}{96}\frac{K_1{}^3 l^4}{\mu^2 p_m{}^2 b^2}\tan^3\beta \tag{1- 1}$$

$$M = \frac{K_1 l^3}{12}\tan\beta - \frac{1}{16}\frac{K_1{}^2 l^4}{\mu p_m b}\tan^2\beta + \frac{1}{64}\frac{K_1{}^3 l^5}{\mu^2 p_m{}^2 b^2}\tan^3\beta - \frac{1}{768}\frac{K_1{}^4 l^6}{\mu^3 p_m{}^3 b^3}\tan^4\beta \tag{1- 2}$$

$$K_1 = \frac{K_0}{1 + \frac{\alpha^3 l^3}{12k} K_0} \tag{1- 3}$$

$$K_0 = \frac{E}{2(1+\nu)} \frac{b}{d} \tag{1- 4}$$

Where:

F = lateral force.

M = Feedback torque.

E =Young's Modulus of tire surface stuff.

ν = Poisson ratio.

b =Width of contact ground surface.

d = Width of tire surface rubber.

k = Spring constant.

p_m = Max pressure of contact ground.

l = Length of contact ground surface.

μ = friction coefficient between ground and tire surface.

β = Slip angle of tire.

1.2.2. Magic Formula

Bakker and Pacejka obtain "Magic Formula" on the basis of experiment. Express the relationship of lateral force, traction force and feedback torque.[2][3][4]

$$Y(X) = y(x) + S_v \tag{1- 5}$$

$$y(x) = D\sin\left\{C\arctan\left[Bx - E\left(Bx - \arctan(Bx)\right)\right]\right\} \tag{1- 6}$$

$$x = X + S_h \tag{1-7}$$

Where:

Y(X)	= Generalized force.
X	= Generalized offset.
B	= Stiffness factor.
C	= Shape factor.
D	= Peak factor.
E	= Curvature factor.
S_v,S_h	= Initial offset of slip angle.

1.2.3. Mitschke's model

Mitschke's idea is the feedback torque includes two parts. One is the lateral force and tire trail to produce, which is in direct ratio with centrifugal acceleration. Another produced by the kingpin incline angle with offset, which unrelated with velocity of vehicle.[5]

$$M_Z = P_y\left(\xi' + \xi''\right) \tag{1-8}$$

$$P_y = m\frac{v^2}{R}\frac{b}{l} \tag{1-9}$$

$$R = \frac{\left(l + m\dfrac{k_2 b - k_1 a}{k_1 k_2 l}v^2\right)}{\delta_f} \tag{1-10}$$

Where:

M_z	= Moment produced by tire trail.
P_y	= Lateral force.

ξ' = Pneumatic trail.

ξ'' = Mechanical trail.

m = Mass of vehicle.

b = Longitudinal distance from center of gravity to rear axle.

l = Wheelbase.

R = Radius of turn.

v = Vehicle velocity.

a = Longitudinal distance from front axle to center of gravity.

k_1 = Cornering stiffness of front wheel.

k_2 = Cornering stiffness of rear wheel.

δ_f = Steer angle of front wheel.

The aligning torque produced by kingpin incline angle, which unrelated with velocity of vehicle. At the high speed has the same torque at low speed. This is the main source of feedback torque at low speed.

$$M_A = \frac{Q \cdot D}{2} \sin 2\beta' \sin \delta_f \tag{1- 11}$$

Where:

M_A = Moment produced by kingpin incline angle.

Q = The offset of kingpin.

D = Load of tire.

β' = Kingpin incline angle.

The feedback torque of tire is:

$$M_h = \frac{M_Z + M_A}{i_f} \tag{1- 12}$$

Where:

i_f = Gear ratio.

1.2.4. UniTire model

Guo and his research team obtain "UniTire model" on the basis of experiment. The physical model of a tire is simplified as shown in Figure1.1.[6][7]

$$\begin{aligned}
\overline{F} &= 1 - \exp\left[-\phi - E_1\phi^2 - \left(E_1^{\ 2} + \frac{1}{12}\right)\phi^3\right] \\
D_x &= \left(D_{x0} + D_e\right)\exp\left(-D_1\phi - D_2\phi^2\right) - D_e \\
F_x &= \overline{F}\frac{\phi_x}{\phi}\mu_x F_z \\
F_y &= \overline{F}\frac{\phi_y}{\phi}\mu_y F_z \\
M_z &= F_y\left(D_x + X_c\right) + F_x Y_c
\end{aligned} \tag{1- 13}$$

Where:

$\overline{F}$ = Total force.

F_x = Traction force.

F_y = lateral force.

E_1 = Curvature factor.

D_{x0}, D_e, D_1, D_2 = Structure parameter of D_x.

ϕ_x = Longitudinal slip ratio.

ϕ_y = Lateral slip ratio.

ϕ = Total slip ratio.

X_c = The longitudinal out of shape produced by F_x.

Y_c = The lateral out of shape produced by F_y.

μ_x = The friction coefficient on longitudinal surface.

μ_y = The friction coefficient on lateral surface.

1.2.5. The model of vehicles

From fundamentals of vehicles dynamics, we can analyze the feedback torque of steering wheel. We will detail illustration on next chapter.

1.3.Structure of The Steering Wheel

A feedback dynamic gear force is applied physically to the steering wheel hardware by a motor actuator. A command signal for a motor-force actuator is generated from a virtual vehicle handling dynamics model that is simulated in the real time by a very fast digital signal processor. The inputs to the real-time vehicle dynamic simulation model are front wheel steering angles driven through a steering system by a driver. Figure1.2 and Figure1.3 illustrate some of these.

Chapter 2 Vehicle Steering System

2.1.The Steering System

2.1.1. Introduction of the Steering System

The design of the steering system has an influence on the direction response behavior of a motor vehicle that is often not fully appreciated. The function of the steering system is to steer the front wheels response to driver command inputs in order to provide overall directional control of the vehicle. However, the actual steer angle achieved are modified by the geometry of the suspension system, the geometry and reactions within the steering system, and in the case of front-wheel-drive (FWD), the geometry and reactions from the drive train. These phenomena will be examined in this section first as a general analysis of a steering system and then by considering the influence of front-wheel-drive.

2.1.2. The steering linkages

The steering system used on motor vehicles vary widely in design [9][10][11], but are functionally quite similar. Figure2.1 illustrates some of these.

The steering wheel connects by shaft, universal joints, and vibration isolators to the steering gearbox whose purpose is to transform the rotary motion of the steering wheel to a translational motion appropriate for steering the wheels. The rack-and-pinion system consists of a linearly moving rack and pinion, mounted on the firewall or a forward cross member, which steers the left and right wheels directly by a tie-rod connection. The tie-rod linkage connects to steering arms on the wheels, thereby controlling the steer angle. With the tie-rod located ahead of the wheel center, as shown in Figure2.1, it is a forward-steer configuration.

The steering gear box is an alternative design used on passenger cars and light trucks. It differs from the rack-and-pinion in that a frame-mounted steering gearbox rotates a pitman arm which controls the steer angle of the left and right wheels through a series of relay linkages and tie rods, the specific configuration of which varies from vehicle to vehicle. A rear-steer configuration is show in the figure, identified by the fact that the tie-rod linkage connects to the steering arm behind the wheel center.

Between these two, the rack-and-pinion system has been growing in popularity for passenger cars because of the obvious advantage of reduced complexity, easier accommodation of front-wheel-drive systems, and adaptability to vehicle without frames. The primary functional difference in the steering systems used on heavy truck is the fact that the frame-mounted steering gearbox steers the left road wheel through a longitudinal drag line, and the right wheel is steered from the left wheel via a tie-rod linkage [9].

The gearbox is the primary means for numerical reduction between the rotational input from the steering wheel and the rotational output about the steer axis. The steering wheel to road wheel angle ratios normally vary with angle, but have nominal values on the order of 15 to 1 in passenger cars, and up to as much as 36 to 1 with some heavy trucks. Initially all rack-and-pinion gearboxes had a fixed gear ratio, in which case any variation in ratio with steer angle was achieved through the geometry of the linkages. Today, rack-and-pinion systems are available that vary their gear ratio directly with steer angle.

2.2.Tire Angle Geometry

2.2.1. Front wheel geometry

The important elements of a steering system consist not only of the visible linkages just described, but also the geometry associated with the steer rotation axis at the road wheel. The geometry determines the force and moment reaction in the steering system, affecting its overall performance. The important features of the geometry are shown in Figure2.2.

The steer angle is achieved by rotation of the wheel about a steer rotation axis. Historically, this axis has the name "kingpin" axis, although it may be established by ball joint or the upper mounting bearing on a strut. The axis is normally not vertical, but may be tipped outward at the bottom, producing a lateral inclination angle (kingpin inclination angle; K.P.I.) in the range of 10-15degrees on passenger cars.

It is common for the wheel to be offset laterally from the point where the steer rotation axis intersects the ground. The lateral distance from the ground intercept to the wheel centerline is the offset at the ground and is considered positive when the wheel is outboard of the ground intercept. Offset may be necessary to obtain packaging space for brakes, suspension, and steering components. At the same time, it adds "feel of the road" and reduces static steering efforts by allowing the tire to roll around an arc when it is turned [12].

Caster angle result when the steer rotation axis is inclined in the longitudinal plane. Positive caster places the ground intercept of the steer axis ahead of the center of tire contact. A similar effect is created by including a longitudinal offset between the steer axis and the spin axis of the wheel, although this is only infrequently used. Caster angle normally ranges from 0 to 5 degrees and may vary with suspension deflection.

2.2.2. Camber angle

Camber angle is the angle made by the wheel of an automobile; specifically, it is the angle between the vertical axis of the wheel and the vertical axis of the vehicle when viewed from the front or rear. It is used in the design of steering and suspension. If the top of the wheel is farther out than the bottom (that is, away from the axle), it is called positive camber; if the bottom of the wheel is farther out than the top, it is called negative camber. The camber angle can be recognized better in a front view, as shown in Figure2.3.

The different camber angle change the touch point and the pivot point of tire with ground, direct effect the traction and abrasion of tire. Then change the achieve force distribute on axle from vehicle weight, avoid axle produces exceptionally abrasion.

Besides, the camber can be cancel the angels change from vehicle weight on suspension system to become deformed and the gap of active surfaces. Camber also effects the car drive direction, like the motorcycle incline the body to corner. So the camber of the right and left wheels must equivalent can’t affect the cars straight-line after the balance of force. And combine toe angle, greater straight-line stability and avoid tire exceptionally abrasion. Increase the negative camber needs combine increase toe-out. Increase the positive camber needs combine increase toe-in.

2.2.3. Kingpin inclination angle(Lateral inclination angle)

The angle is in front elevation between the steering axis and the vertical. The K.P.I. angle can be recognized better in a front view, as shown in Figure2.4. The K.P.I. can make the vehicle weight uniform distribution on the axis; protect the bearing to impair to hard. The angle can make the cornering force uniform and steering easily. Otherwise, if the angle is zero, then the vehicle weight and reacting force from ground will produce more lateral shear force, make the bearing to impair to easy and hard to cornering. Besides, the angle is the source of the aligning torque after the front wheel corning. The angle usually can’t change from the vehicle suspension design started.

2.2.4. Toe angle

Toe is the symmetric angle that each wheel makes with the longitudinal axis of the vehicle, as a function of static geometry, and kinematic and compliant effects. The toe angle can be recognized better in a top view, as shown in Figure2.5. This can be contrasted with steer, which is the ant symmetric angle, i.e. both wheels point to the left or right, in parallel (roughly). Positive toe, or toe in, is the front of the wheel pointing in towards the centerline of the vehicle. Negative toe, or toe out, is the front of the wheel pointing away from the centerline of the vehicle. Toe can be measured in linear units, at the front of the tire, or as an angular deflection.

In a rear wheel drive car, increased front toe in (*i.e.* the fronts of the front wheels are closer together than the backs of the front wheels) provides greater straight-line stability at the cost of some sluggishness of turning response, as well as a little more tire wear as they are now driving a bit sideways. On front wheel drive cars, the situation is more complex.

Toe is always adjustable in production automobiles, even though caster angle and camber angle are often not adjustable. Maintenance of front end alignment, which used to involve all three adjustments, currently involves only setting the toe ; in most cases, even for a car in which caster or camber are adjustable, only the toe will need adjustment.

One related concept is that the proper toe for straight line travel of a vehicle will not be correct while turning, since the inside wheel must travel around a smaller radius than the outside wheel; to compensate for this, the steering linkage typically conforms more or less to Ackermann steering geometry, modified to suit the characteristics of the individual vehicle.

It should be noted that individuals who decide to adjust their car's static ride height, either by raising or lowering, should immediately have the car properly aligned. The common misconception is that Camber angle causes an increased rate of tire wear, when in fact camber's contribution to tire wear is usually only visible over the entire life of the tire.

2.2.5. Caster angle

Caster angle is the angular displacement from the vertical axis of the suspension of a steered wheel in a car, bicycle or other vehicle, measured in the longitudinal direction. It is the angle between the pivot line (in a car - an imaginary line that runs through the center of the upper ball joint to the center of the lower ball joint) and vertical. The caster angle can be recognized better in a lateral view, as shown in Figure2.6. Car racers sometimes adjust caster angle to optimize their car's handling characteristics in particular driving situations.

The pivot points of the steering are angled such that a line drawn through them intersects the road surface slightly ahead of the contact point of the wheel. The purpose of this is to provide a degree of self-centering for the steering - the wheel casters around so as to trail behind the axis of steering. This makes a car easier to drive and improves its straight line stability (reducing its tendency to wander). Excessive caster angle will make the steering heavier and less responsive, although, in racing, large caster angles are used to improve camber gain in cornering. Caster angles over 10 degrees with radial tires are common. Power steering is usually necessary to overcome the jacking effect from the high caster angle.

The steering axis (the dotted line in the diagram above) does not have to pass through the center of the wheel, so the caster can be set independently of the mechanical trail, which is the distance between where the steering axis hits the ground, in side view, and the point directly below the axle. The interaction between caster angle and trail is complex, but roughly speaking they both aid steering, caster tends to add damping, while trail adds 'feel', and return ability. In the extreme case of the shopping trolley (shopping cart in the US) wheel, the system is undammed but stable, as the wheel oscillates around the 'correct' path. The shopping trolley/cart setup has a great deal of trail, but no caster. Complicating this still further is that the lateral forces at the tire do not act at the center of the contact patch, but at a distance behind the nominal contact patch. This distance is called the pneumatic trail and varies with speed, load, steer angle, surface, tire type, tire pressure and time. A good starting point for this is 30 mm behind the nominal contact patch

2.2.6. Slip angle

Slip angle is the angle between a rolling wheel's actual direction of travel and the direction towards which it is pointing. This slip angle results in a force perpendicular to the wheel's direction of travel (the cornering force). This cornering force increases approximately linearly for the first few degrees of slip angle, and then increases non-linearly to a maximum before beginning to decrease.

A non-zero slip angle arises because of deformation in the tire carcass and tread. As the tire rotates, the friction between the contact patch and the road result in individual tread 'elements' (infinitely small sections of tread) remaining stationary with respect to the road. If a side-slip velocity u is introduced, the contact patch will be deformed. As a tread element enters the contact patch the friction between road and tire means that the tread element remains stationary, yet the tire continues to move laterally. This means that the tread element will be ‘deflected’ sideways. In reality it is the tire/wheel that is being deflected away from the stationary tread element, but convention is for the co-ordinate system to be fixed around the wheel mid-plane. The slip angle can be recognized better in a top view, as shown in Figure2.7.

Because the forces exerted on the wheels by the weight of the vehicle are not distributed equally, the slip angles of each tire will be different. The ratios between the slip angles will determine the vehicle's behavior in a given turn. If the ratio of front to rear slip angles is greater than 1:1, the vehicle will tend to understeer, while a ratio of less than 1:1 will produce oversteer. Actual instantaneous slip angles depend on many factors, including the condition of the road surface, but a vehicle's suspension can be designed to promote specific dynamic characteristics. A principal means of adjusting developed slip angles is to alter the relative roll couple (the rate at which weight transfers from the inside to the outside wheel in a turn) front to rear by varying the relative amount of front and rear lateral load transfer. This can be achieved by modifying the height of the Roll centers, or by adjusting roll stiffness, either through suspension changes or the addition of an anti-roll bar.

Because of asymmetries in the side-slip along the length of the contact patch. The resultant force of this side-slip occurs behind the geometric center of the contact patch, a distance described as the pneumatic trail, and so creates a torque on the tire.

2.2.7. Formulation of geometry

Before we analyze the geometry, we define the geometry meaning of each term in fundamental dynamics equation.

Nomenclature

d : Kingpin offset at the ground

F_{xr} : Traction force on right-front wheel

F_{xl} : Tractive force on left-front wheel

F_{yr} : Lateral force on right-front wheel

F_{yl} : Lateral force on left-front wheel

F_{zr} : Vertical force on right-front wheel

F_{zl} : Vertical force on left-front wheel

M_{zr} : Aligning torque on right-front wheel

M_{zl} : Aligning torque on left-front wheel

M_T : Moment produced by traction force

M_L : Moment produced by lateral force

M_V : Moment produced by vertical force

$M_{V,KPI}$: Moment produced by vertical force acting on kingpin inclination angle

$M_{V,caster}$: Moment produced by vertical force acting on caster angle

M_{AT} : Moment produced by aligning torque

T_s : The feedback torque of steering system

T_h : The torque of driver steering

r : Tire radius

R_w : Steering wheel radius

λ : Kingpin inclination angle

υ : Caster angle

δ : Tire steer angle

θ : Steering wheel angle

α : Slip angle

I_s : Moment of inertia of steering shaft

C_s : Damping coefficient of steering shaft

K_s : Elasticity coefficient of steering system

I_h : Moment of inertia of steering wheel

C_h : Damping coefficient of steering wheel

2.3.Steering System Forces and Moments

The forces and moments imposed on the steering system emanate from those generated at the tire-road interface. The SAE has selected a convention by which to describe the force on a tire, as shown in Figure2.8. Assume that the forces are measured at the center of the contact with the ground and provide a convenient basis by which to analyze steering reactions.

The ground reactions on the tire are described by three forces and moments, as follow:

1. Vertical force
2. Traction force
3. Lateral force
4. Aligning torque
5. Rolling resistance moment
6. Overturning moment

The reaction in the steering system is described by the moment produced on the steer axis, which must be resisted to control the wheel steer angle. Ultimately, the sum of moments from the left and right wheels acting through the steering linkages with their associated ratios and efficiencies account for the steering-wheel torque feedback to the driver.

Figure2.9 shows the three forces and moments acting on a right-hand road wheel. Each will be examined separately to illustrate its effect on the steering system.

2.3.1. Vertical force

The vertical load, F_z, acts vertically upward on the wheel and by SAE convention is considered a positive force. Because the steering axis is inclined, F_Z has a component acting to product a moment attempting to steer the wheel. The moment arises from both the caster and kingpin inclination angles. Assuming small angles and neglecting camber of the wheel as it steers, the total moment from the two can be approximated by:

$$M_V = -\left(F_{zl} + F_{zr}\right) d \sin\gamma \sin\delta + \left(F_{zl} - F_{zr}\right) d \sin\nu \cos\delta \qquad (2\text{-}1)$$

Where:

M_V = Total moment from left and right wheels.

F_{zl}, F_{zr} = Vertical load on left and right wheels.

d = Kingpin offset at the ground.

λ = Kingpin inclination angle.

δ = Steer angle.

ν = Caster angle.

The first expression on the right side of the above equation arises from kingpin inclination angle, and the last from caster angle. The source of each of these moments is most easily visualized by considering the effects of kingpin inclination angle and caster angle separately.

The vertical force acting on kingpin inclination angle, illustrated in Figure2.10, results in a sine angle force component, which normally acts lateral on the moment arm when the wheel is steered. The moment is zero at zero steer angle. When steering, both sides of the vehicle lift, an effect which is often described as the source of the centering moment.

$$M_{V,inclination} = -(F_{zl} + F_{zr}) \cdot d \cdot \sin\lambda \cdot \sin\delta \tag{2-2}$$

The caster angle results in a sin angle force component, which nominally acts forward on the moment arm as shown in Figure2.11. The moment on the left and right wheels are opposite in direction.

$$M_{V,caster} = (F_{zl} - F_{zr}) \cdot d \cdot \sin\nu \cdot \cos\delta \tag{2-3}$$

When steer angle, one side of the axle lift and the other drops, so that the net moment produce depends also on the roll stiffness of the front suspension as it influences the left and right wheel roads.

2.3.2. Traction force

The traction force, F_x, acts on the kingpin offset to produce a moment as shown in Figure2.12.

Then net moment is:

$$M_T = (F_{xl} - F_{xr})d \tag{2-4}$$

Where:

F_{xl},F_{xr} = Traction force on left and right wheels (positive forward).

The left and right moments are opposite in direction and tend to balance through the relay linkage. Imbalances, such as may occur with a tire blowout, brake malfunction, or split coefficient surfaces, will tend to produce a steering moment which is dependent on the lateral offset dimension.

2.3.3. Lateral force

The lateral force, F_y, acting at the tire center produces a moment through the longitudinal offset resulting from caster angle, as shown in Figure2.13. The net moment produced is:

$$M_L = \left(F_{yl} + F_{yr}\right) r \tan \nu \tag{2-5}$$

Where:

F_{yl},F_{yr} = Lateral force on left and right wheels (positive forward).

r = Tire radius.

The lateral force is generally dependent on the steer angle and cornering condition, and with positive caster produces a moment attempting to steer the vehicle out of the turn.

2.3.4. Aligning torque

The aligning torque, M, acts vertically and may be resolved into a component acting parallel to the steering axis. Since moments may be translated without a change in magnitude, the equation for the net moment is:

$$M_{AT} = \left(M_{zl} + M_{zr}\right) \cos \sqrt{\gamma^2 + \nu^2} \tag{2-6}$$

Where:

M_{yl}, M_{yr} = Aligning torque on left and right wheels.

Under normal driving conditions, the aligning torques always act to resist any turning motion, thus their effect is under steer. Only under high braking conditions do they act in a contrary fashion.

2.3.5. Rolling resistance moment and Overturning moment

These moments at most only have a sine angle component acting about the steer axis. They are second-order effects and are usually neglected in analysis of steering system torques.

2.4.Torque feedback on the steering wheel

These equations, (2-1),(2-4), (2-5), (2-6), are the moments of steering axis, we can add direct them to get the total moment, T_S, on the kingpin axis:

$$T_s = M_V + M_L + M_T + M_{AT} \tag{2-7}$$

Then we can through the steering system model, as shown Figure2.14, and calculate the force on the steering wheel by the way of the total moment. We can the steering system model to simplify, which equivalent that, as show Figure2.15.

$$I_s\ddot{\delta} + C_s\dot{\delta} + K_s\left(\delta - \theta\right) = T_s \tag{2-8}$$

Where:

I_s = Moment of inertia of steering shaft.

C_s = Damping coefficient of steering shaft.

K_s = Elasticity coefficient of steering system.

θ = Steering wheel angle.

δ = Steering angle

Chapter 3 Math Model of Steering System

3.1 Nomenclature of the math model

Before we analyze the model of steering system, we define the symbols meaning of each term in fundamental equation.

Symbol	**Description**
J_1	Steering wheel moment of inertia
L_1	Steering wheel motor inductance
R_1	Steering wheel motor resistances
$K_{\Phi 1}$	Steering wheel motor voltage constant
$K_{\tau 1}$	Steering wheel motor torque constant
J_2	Front wheel moment of inertia
B_2	Front wheel damping
K_2	Front wheel torsion stiffness
L_2	Front wheel motor inductance
R_2	Front wheel motor resistances
$K_{\Phi 2}$	Front wheel motor voltage constant
$K_{\tau 2}$	Front wheel motor torque constant
K	Steering column torsion stiffness
B	Steering column damping
n	Steering angle ratio
Q	Steering torque ratio
F_x	Longitudinal Force
F_y	Lateral Force

M_z	Moment of z axis
m	Vehicle's mass
V_x	Longitudinal velocity of vehicle
V_y	Lateral velocity of vehicle
V_z	Normal velocity of vehicle
$\dot{V}_x$	Longitudinal acceleration of vehicle
$\dot{V}_y$	Lateral acceleration of vehicle
q	Pitch velocity of vehicle's y axis
γ	Yaw velocity of vehicle's z axis
I_z	vehicle yaw moment of inertia
$C_{\alpha f}$	Front tire cornering stiffness coefficient
$C_{\alpha r}$	Rear tire cornering stiffness coefficient
α_f	Slip angles for front tires
α_r	Slip angles for rear tires
C_{sf}	Front tire longitudinal stiffness coefficient
C_{sr}	Rear tire longitudinal stiffness coefficient
s	Slip ratio
V_s	Slip velocity
μ_0	Road fiction coefficient
A_s	Adhesion reduction coefficient
z	Dugoff coefficient
R_p	Radius of pinion.

3.2 Mechanical steering system

We can obtain the math model as Figure3.2 from the mechanism as Figure3.1 of traditional steering system.

$$J_1 \frac{d\theta_1^2}{d^2t} + B_1 \frac{d\theta_2}{dt} = T_{hum} - B\left(\frac{d\theta_1}{dt} - \frac{d\theta_2}{dt}\right) - K(\theta_1 - \theta_2) \tag{3- 1}$$

$$J_2 \frac{d\theta_2^2}{d^2t} + B_2 \frac{d\theta_2}{dt} + K_2\theta_2 = T_{ext} + B\left(\frac{d\theta_1}{dt} - n\frac{d\theta_2}{dt}\right) + K(\theta_1 - \theta_2) \tag{3- 2}$$

And we cut off the steering column made the steering system become two subsystems as shown in Figure3.3. One of the subsystems is the steering wheel system to feedback the force of the tire on the road. Another of the subsystems is the front wheel system to steer the vehicle. We can obtain the equivalent equations.

$$J_1 \frac{d\theta_1^2}{d^2t} + B_1 \frac{d\theta_2}{dt} = T_{hum} + T_1 \tag{3- 3}$$

$$J_2 \frac{d\theta_2^2}{d^2t} + B_2 \frac{d\theta_2}{dt} + K_2\theta_2 = T_{ext} + T_2 \tag{3- 4}$$

Than we need the device to feedback the force of tire and to steer the front wheel. The math model of steering wheel and front wheel system is as shown in Figure3.4.

3.3 Steer-by-wire system

The steer-by-wire system is used the motor replace steering column as shown in Figure3.5. So we can obtain the equivalent equation of the math model of mechanical steering system.

$$J_1 \frac{d\theta_1^2}{d^2t} + B_1 \frac{d\theta_1}{dt} = K_{\tau 1} i_1 + T_{hum} \tag{3- 5}$$

$$L_1 \frac{di_1}{dt} + R_1 i_1 + K_{\emptyset 1} \frac{d\theta_1}{dt} = -B\left(\frac{d\theta_1}{dt} - n\frac{d\theta_2}{dt}\right) - K(\theta_1 - n\theta_2) \tag{3- 6}$$

$$J_2 \frac{d\theta_2^2}{d^2t} + B_2 \frac{d\theta_2}{dt} + K_2\theta_2 = K_{\tau 2} i_2 + T_{ext} \tag{3- 7}$$

$$L_2 \frac{di_2}{dt} + R_2 i_2 + K_{\emptyset 2} \frac{d\theta_2}{dt} = QB\left(\frac{d\theta_1}{dt} - n\frac{d\theta_2}{dt}\right) + QK(\theta_1 - n\theta_2) \tag{3- 8}$$

Control block diagram of SBW as show in Figure3.6. Torque human is the driver steering torque on steering wheel. It is the one of input of SBW system. Another of input of SBW system is the torque external. It is mean the total force on front wheel tire. Then we known the steering wheel angle and front wheel angle by angle senor on steering wheel and front wheel angle. The SBW system included steering wheel, steering wheel motor, front wheel motor and front wheel of steering system on vehicle. The ratio of front wheel angle between steering wheel angles is n. The ratio of torque human between external torques is Q.

3.4 Torque of the external environment

We feel the condition of the road through the tires, steering column and steering wheel from the external environment. Now we try to simulate actual driving conditions caused by the external environment torque. Before we analysis the vehicle dynamic, we defined the vehicle axis system as shown in Figure3.7and 2DOF of bicycle model as shown in Figure3.8.

First we used the dynamical equation of vehicle.

$$\sum F_x = m\left(\dot{V}_x + qV_z - \gamma V_y\right) \tag{3- 9}$$

$$\sum F_y = m\left(\dot{V}_y + \gamma V_x\right) \tag{3- 10}$$

$$\sum M_z = I_z \dot{\gamma} \tag{3- 11}$$

Assume the vehicle velocity V_x is constant and neglect the pitch angle's effect. Then we simplify these equations.

$$\sum F_x = -m\gamma V_y \tag{3- 12}$$

At the small slip angle, the lateral force at front and rear tire can be derived as

$$F_{yf} = -C_{\alpha f}\alpha_f \tag{3- 13}$$

$$F_{yr} = -C_{\alpha r}\alpha_r \tag{3- 14}$$

From definition of longitudinal stiffness of tire we have

$$F_{xf} = C_{sf}s_f \tag{3- 15}$$

$$F_{xr} = C_{sr}s_r \tag{3- 16}$$

We can obtain the dynamical equation.

$$\sum F_x = F_{xf} + F_{xr} = -m\gamma V_y = C_{sf}s_f + C_{sr}s_r \tag{3- 17}$$

$$\sum F_y = F_{yf} + F_{yr} = m(\dot{V}_y + \gamma V_x) = -C_{\alpha f}\alpha_f - C_{\alpha r}\alpha_r \tag{3- 18}$$

$$\sum M_z = F_{yf}L_f - F_{yr}L_r = I_z\dot{\gamma} = -C_{\alpha f}\alpha_f L_f + C_{\alpha r}\alpha_r L_r \tag{3- 19}$$

We assume $C_{sf} = C_{sr} = C_s$, $s_f = s_r = s$ to simplify that equation. So we rewrite these equations.

$$2C_s s = -m\gamma V_y \tag{3- 20}$$

$$-C_{\alpha f}\left(\frac{V_y + L_f\gamma}{V_x} - \delta\right) - C_{\alpha r}\left(\frac{V_y - L_r\gamma}{V_x}\right) = m(\dot{V}_y + \gamma V_x) \tag{3- 21}$$

$$-C_{\alpha f}L_f\left(\frac{V_y + L_f\gamma}{V_x} - \delta\right) + C_{\alpha r}L_r\left(\frac{V_y - L_r\gamma}{V_x}\right) = I_z\dot{\gamma} \tag{3- 22}$$

After finishing we obtain.

$$\begin{bmatrix}\dot{V}_y\\ \dot{\gamma}\end{bmatrix} = \begin{bmatrix} -\frac{C_{\alpha f}+C_{\alpha r}}{mV_x} & \frac{-L_fC_{\alpha f}+L_rC_{\alpha r}}{mV_x} - V_x \\ \frac{-L_fC_{\alpha f}+L_rC_{\alpha r}}{I_zV_x} & -\frac{L_f^2C_{\alpha f}+L_r^2C_{\alpha r}}{I_zV_x}\end{bmatrix}\begin{bmatrix}V_y\\ \gamma\end{bmatrix} + \begin{bmatrix}\frac{C_{\alpha f}}{m}\\ \frac{L_fC_{\alpha f}}{I_z}\end{bmatrix}\delta \tag{3- 23}$$

$$\begin{bmatrix}V_y\\ \gamma\end{bmatrix} = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}V_y\\ \gamma\end{bmatrix} + \begin{bmatrix}0\\ 0\end{bmatrix}\delta \tag{3- 24}$$

We can calculate the lateral velocity and yaw rate.

Second, we used the Dugoff tire model to calculate the road force exerted on the tire by the road.[23]

$$\alpha_f = \frac{V_y + L_f\gamma}{V_x} - \theta_2 \tag{3- 25}$$

$$s = \frac{-m\gamma V_y}{2C_s} \tag{3- 26}$$

$$V_s = V_x\sqrt{s^2 + \tan^2\alpha} \tag{3- 27}$$

$$\mu = \mu_0(1 - A_s V_s) \tag{3- 28}$$

$$z = \frac{\mu F_z(1-s)}{2\sqrt{(C_s s)^2 + (C_\alpha \tan\alpha)^2}} \tag{3- 29}$$

$$\text{if } z < 1 \quad f(z) = z(2 - z) \text{ else } f(z) = 1 \tag{3- 30}$$

$$F_x = \frac{C_s s f(z)}{1-s} \tag{3- 31}$$

$$F_y = \frac{-C_\alpha f(z)\tan\alpha}{1-s} \tag{3- 32}$$

$$F_r = F_x\sin\theta_2 + F_y\cos\theta_2 \tag{3- 33}$$

$$T_{reaction} = F_r R_p \tag{3- 34}$$

Simplified model of steering rack and pinion system is Figure3.9.

We can change the model parameter to calculate external torque of vehicle. We assume vehicle on different road to test and verify the model. The simulation result is Figure3.10. And we assume different vehicle velocity to test and verify the model. The simulation result is Figure3.11. From simulation results, we known the model can simulate the external torque by different road and vehicle velocity.

3.5 LQG controller

We have derived the LQ regular which has the deficiency of assuming all states are available for measurement, and we also have the development of the

Kalman filter, which produce an optimal estimation of the states. We can put them together and use the control law to obtain so-called LQG. The LQG control structure is Figure3.12. We used LQG controller to improve the SBW system response, made the system more stability.

Chapter 4 Simulation results

We used Table4.1, Table4.2, Table4.3 and equation to simulation. We used pole/zero map to analysis the systems. Figure4.1 is the pole/zero map of mechanical model. Figure4.2 is the red frame on pole/zero map of mechanical model. We know system response from the pole/zero maps. Figure4.3 is the pole/zero map of steering-by-wire model. Figure4.4 is the red frame on pole/zero map of steering-by-wire model. The SBW model is equivalent to mechanical model from their pole/zero maps. Figure4.5 is the pole/zero map of steering-by-wire model with LQG control. Figure4.6 is the red frame on pole/zero map of steering-by-wire model with LQG control. The reaction torque will change the K2 of model, so we change K2 to analysis pole/zeros changes as shown in Figure4.7 and Figure4.8.

The simulation of SBW model with LQG is Figure4.9 and Figure4.10. From the results of simulation we known the model with LQG control can filter out the measurement noise and made the model stability time to decrease.

We assume two cases to test and verify the response of SBW model with LQG better than SBW model.

4.1 Case 1: circular motion

Assume the vehicle drive on no wind and flat road, and the vehicle velocity is constant. One sec later we input the same torque human to the model to simulation the model on different road. And simulate the model of different vehicle velocity on the same road.

We knew the road friction coefficient larger and harder to steer. The simulation of SBW model on different road accords with real life as shown in Figure4.11 and Figure4.12.

We knew the vehicle velocity faster and harder to steer. The simulation of SBW model of different vehicle velocity accords with real life as shown in Figure4.13 and Figure4.14. Than we compare the result of the model with LQG control and without LQG control on different road. We obtain the model with LQG control faster to stability than model without LQG control as shown in Figure4.15 and Figure4.16. We compare the result of the model with LQG control and without LQG control of different vehicle velocity. We obtain the model with LQG control faster to stability than model without LQG control as shown in Figure4.17 and Figure4.18. We know the model with LQG control to improve the stability apparent.

4.2 Case 2: turn a corner

Assume the vehicle drive on no wind and flat road, and the vehicle velocity is constant. One sec later we input the same torque human to steer a corner until 90 degree of the model to simulation the model on different road. And simulate the model of different vehicle velocity on the same road.

We knew the road friction coefficient larger and harder to steer. The simulation of SBW model on different road accords with real life as shown in Figure4.19 and Figure4.20. Because the road friction coefficient large and the vehicle locus outside.

We knew the vehicle velocity faster and harder to steer. The simulation of SBW model of different vehicle velocity accords with real life as shown in

Figure4.21 and Figure4.22. Than we compare the result of the model with LQG control and without LQG control on different road. We obtain the model with LQG control faster to stability than model without LQG control as shown in Figure4.23 and Figure4.24. We compare the result of the model with LQG control and without LQG control of different vehicle velocity. We obtain the model with LQG control faster to stability than model without LQG control as shown in Figure4.25 and Figure4.26. We know the model with LQG control to improve the stability apparent.

4.3 Case 3 : change line

Assume the vehicle drive on no wind and flat road, and the vehicle velocity is 20 km/h. One sec later we input the same torque human to change line on asphalt road.

We compare the result of the model with LQG control and without LQG control on this case. We obtain the model with LQG control faster to stability than model without LQG control as shown in Figure4.27 and Figure4.28. The longitudinal distance with LQG less than without LQG. The LQG control can improve steering response.

4.4 Conclusion

The SBW model can simulate the mechanical model equivalent. The SBW model without steer column can decrease the vehicle weight and the cost of cars. The tire model can simulate the external torque of vehicle on different road. Future we can use the tire model and steering wheel system of SBW model to simulate the steering wheel reactive torque of the vehicle on different road.

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Figures

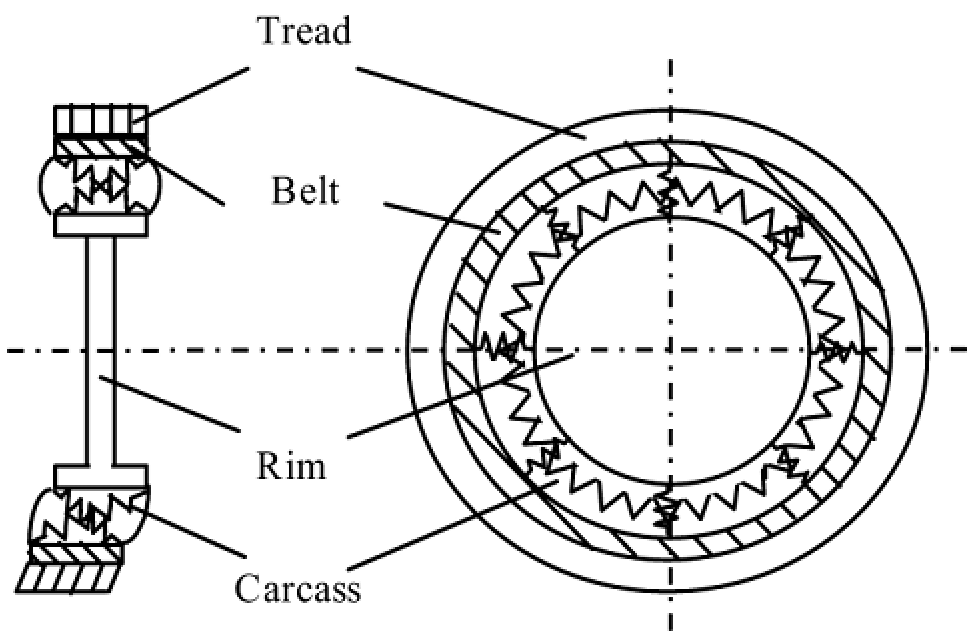

Figure1.1　The physical model of a tire

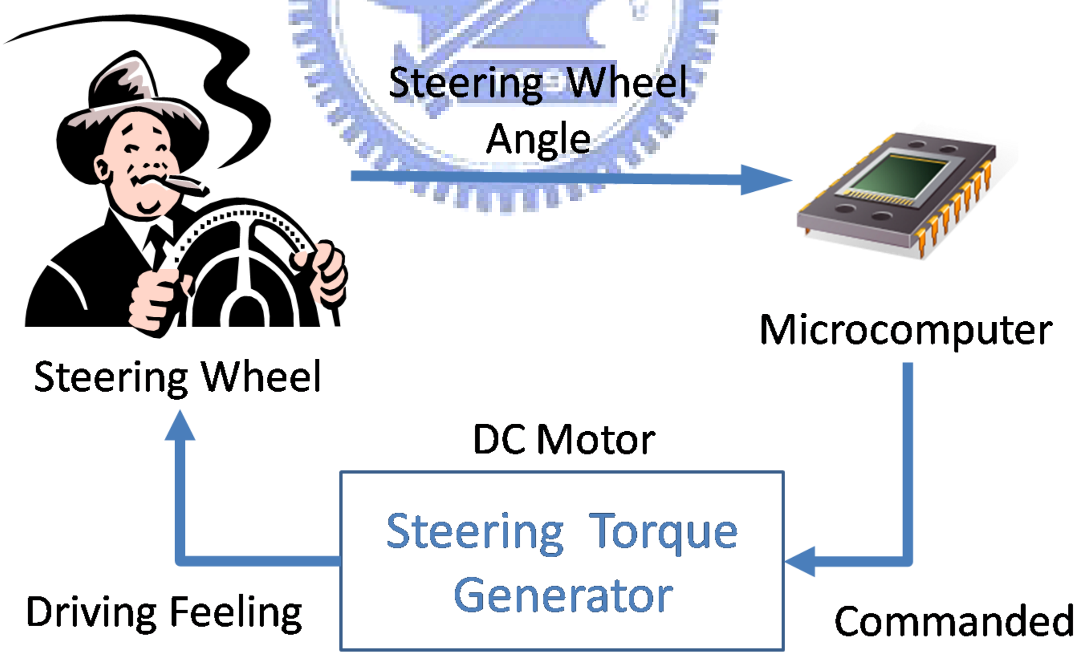

Figure1.2　Schematic diagram of the system

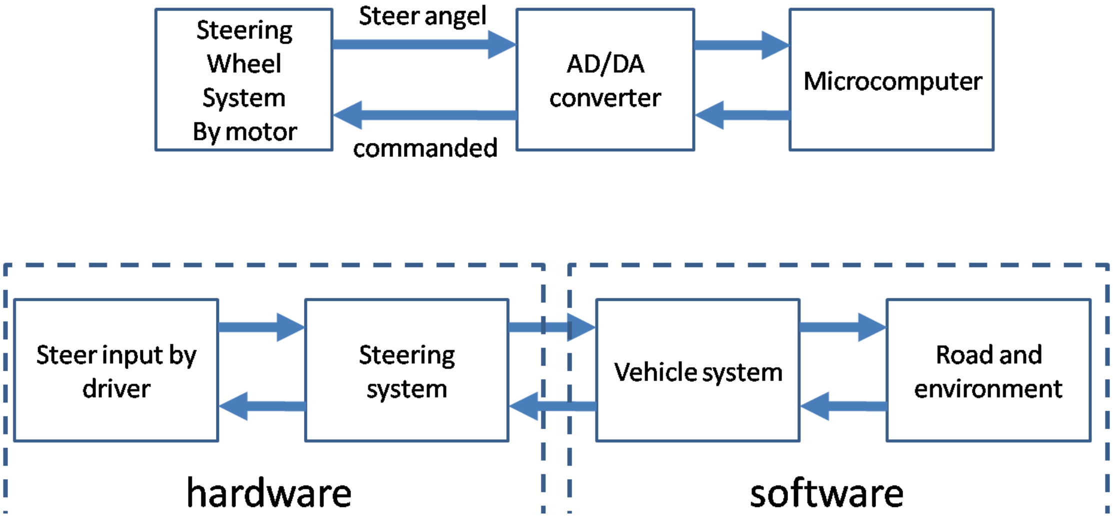

Figure1.3 Schematic block diagram of the system

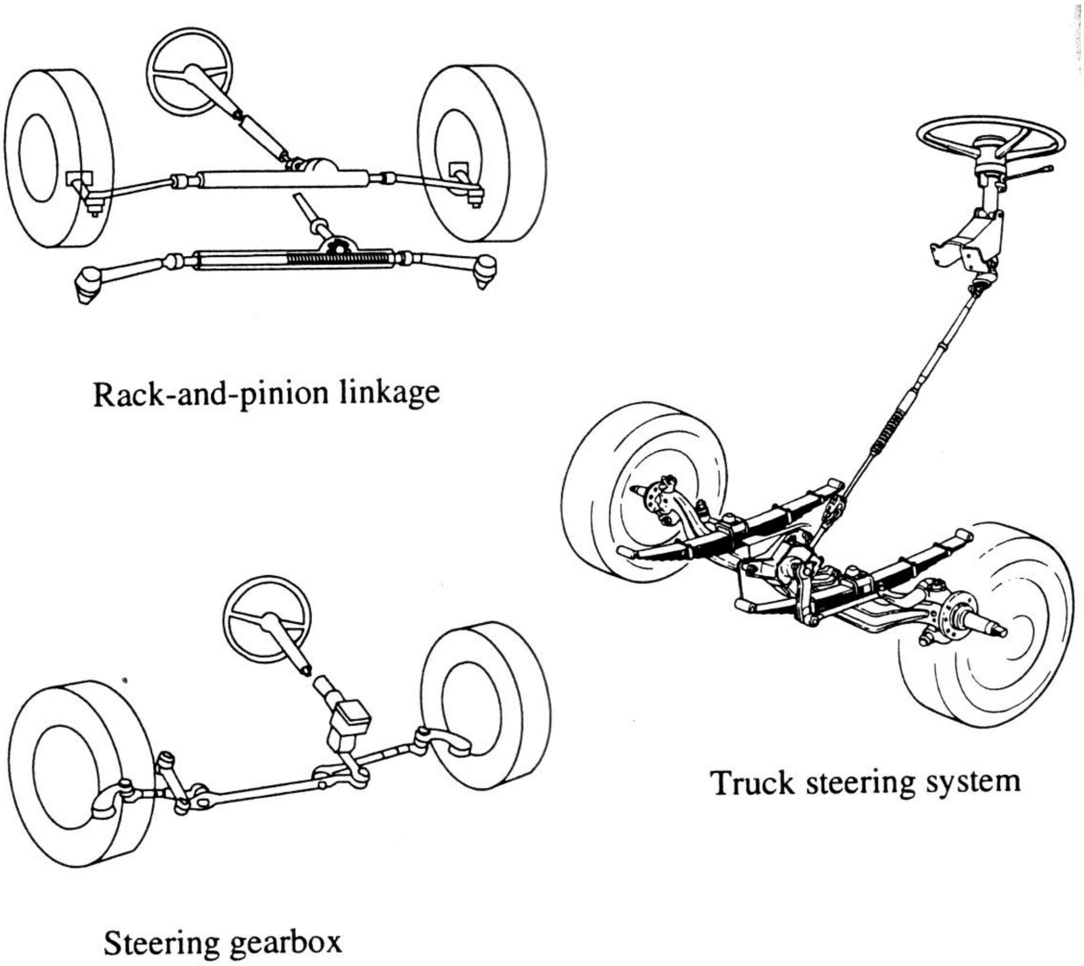

Figure2.1 Illustration of typical steering systems

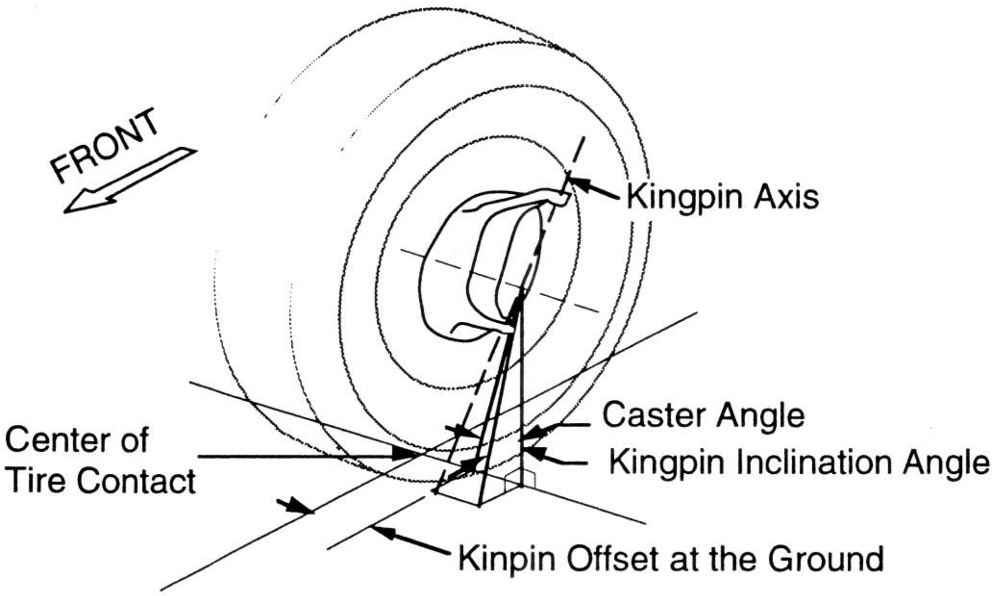

Figure2.2 Steer rotation geometry at the road wheel

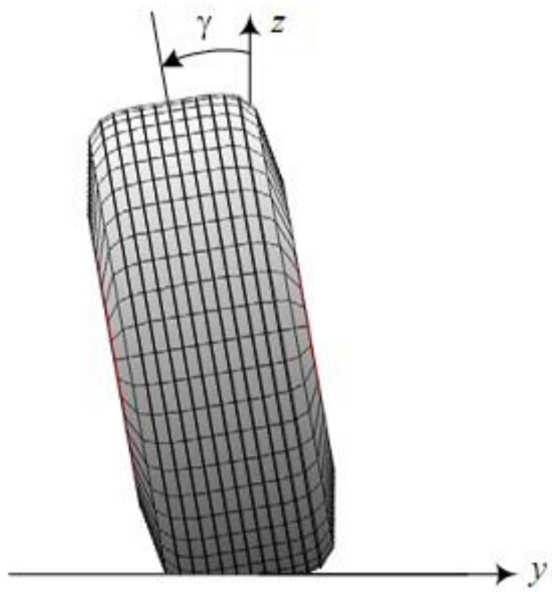

Figure2.3 Front view of a tire and measurement of the camber angle

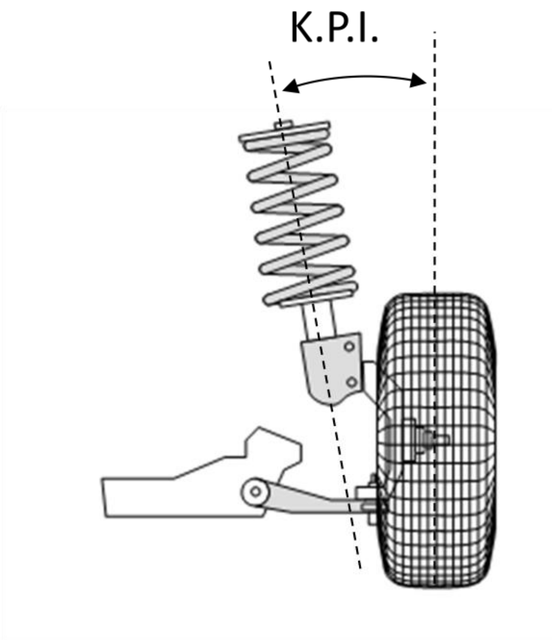

Figure2.4 Front view of a tire and measurement of the kingpin incline angle

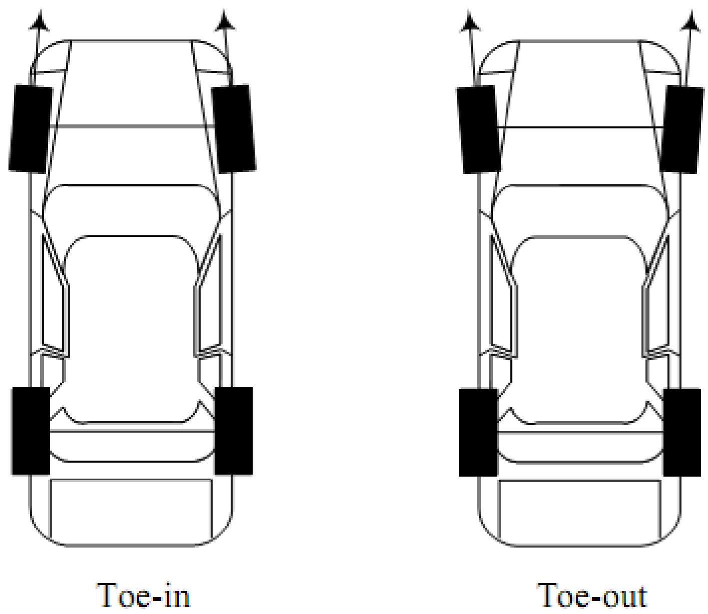

Figure2.5 Toe-in and toe-out configuration on the front wheels of a car

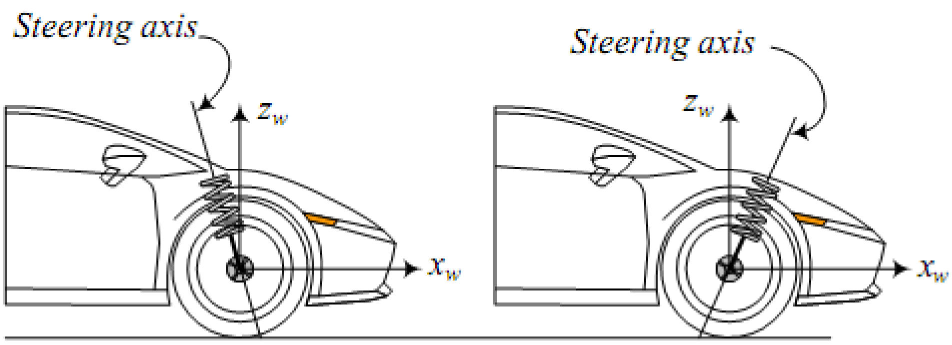

Figure2.6 A positive and negative caster configuration on front wheel of a car

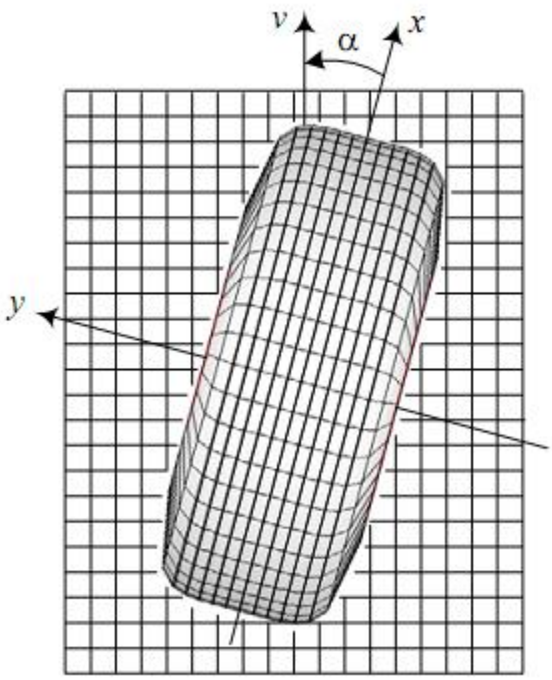

Figure2.7 Top view of a tire and measurement of the side slip angle

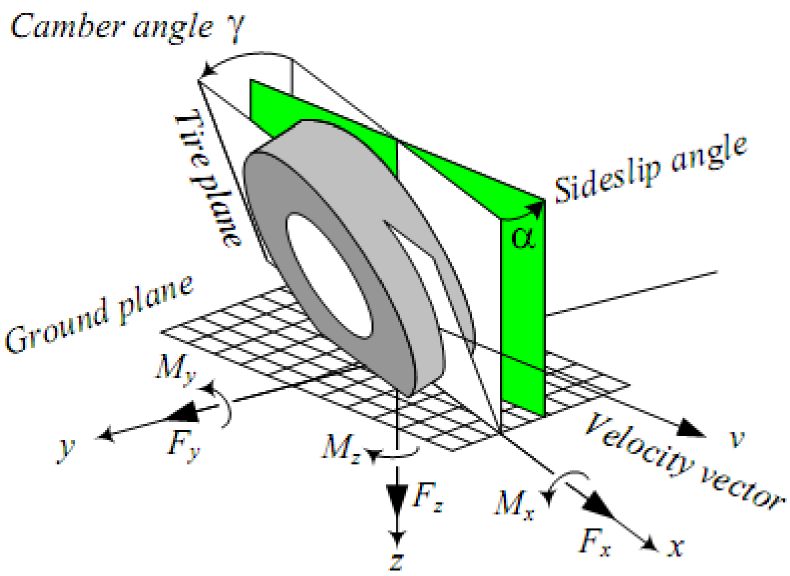

Figure2.8 SAE tire force and moment axis system

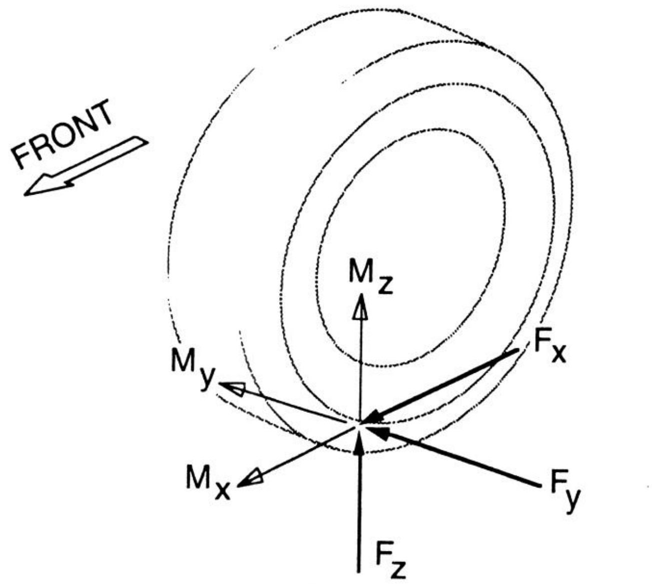

Figure2.9 Forces and moments acting on a right-hand road wheel

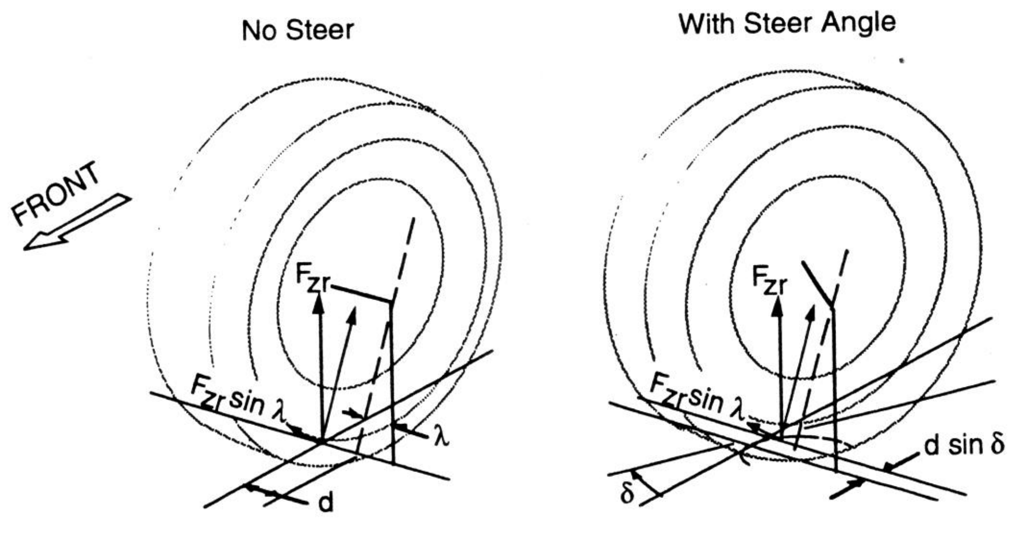

Figure2.10　Moment produced by vertical force acting on K.P.I. angle

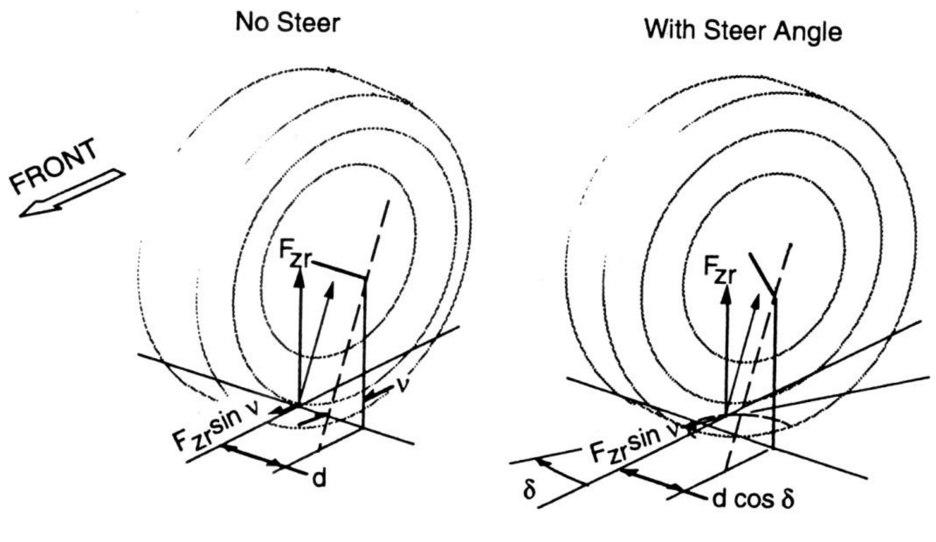

Figure2.11　Moment produced by vertical force acting on caster angle

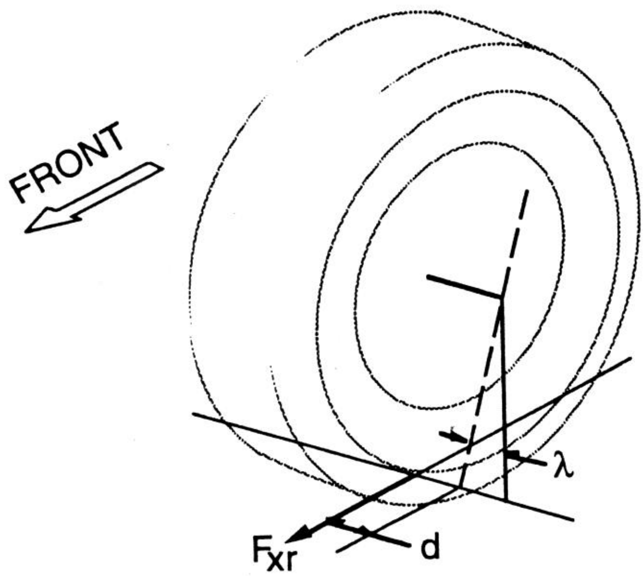

Figure2.12 Steering moment produced by traction force

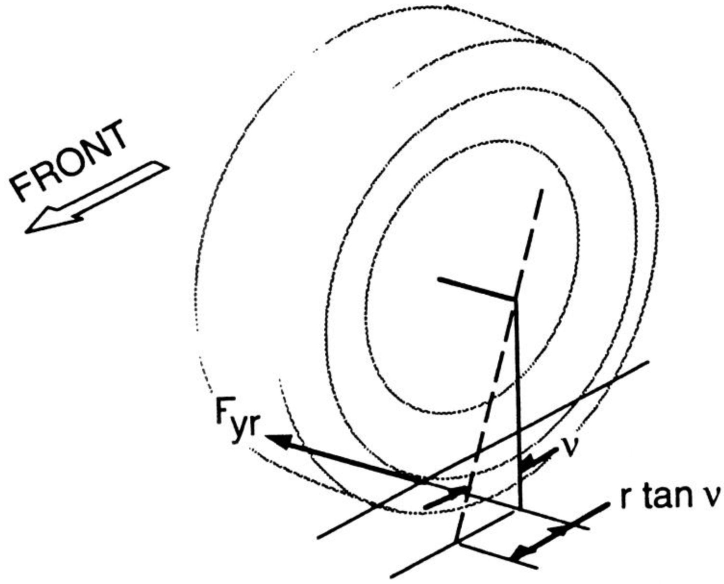

Figure2.13 Steering moment produced by lateral force

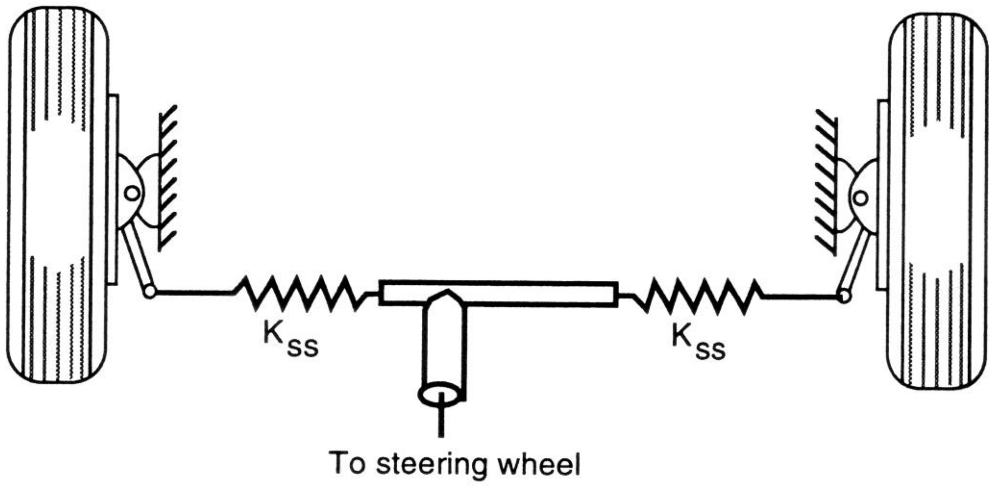

Figure2.14 Steering linkages model

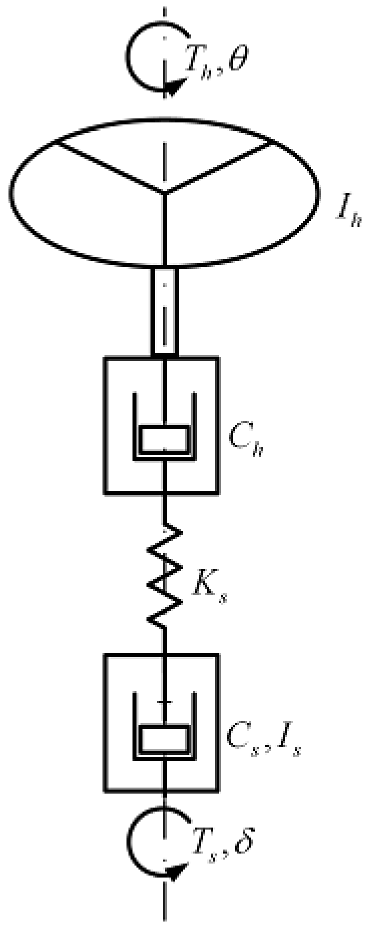

Figure2.15 Equivalent dynamic model of steering system

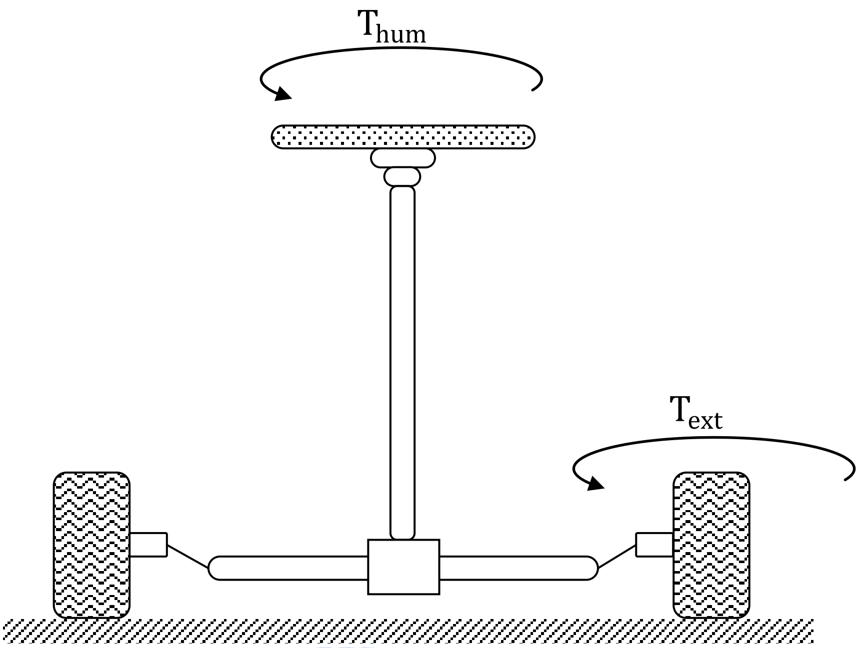

Figure3.1　Mechanism diagram of mechanical steering system

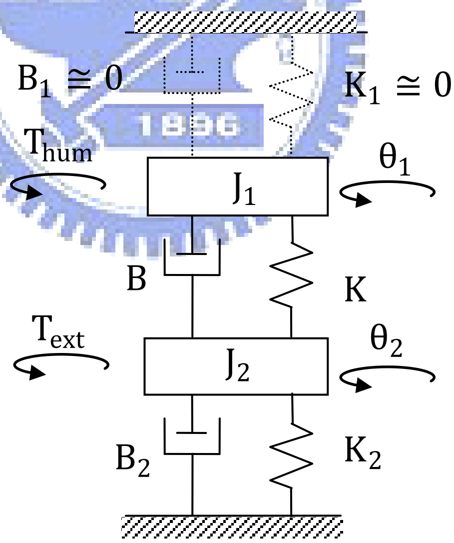

Figure3.2　Math model of the mechanical steering system

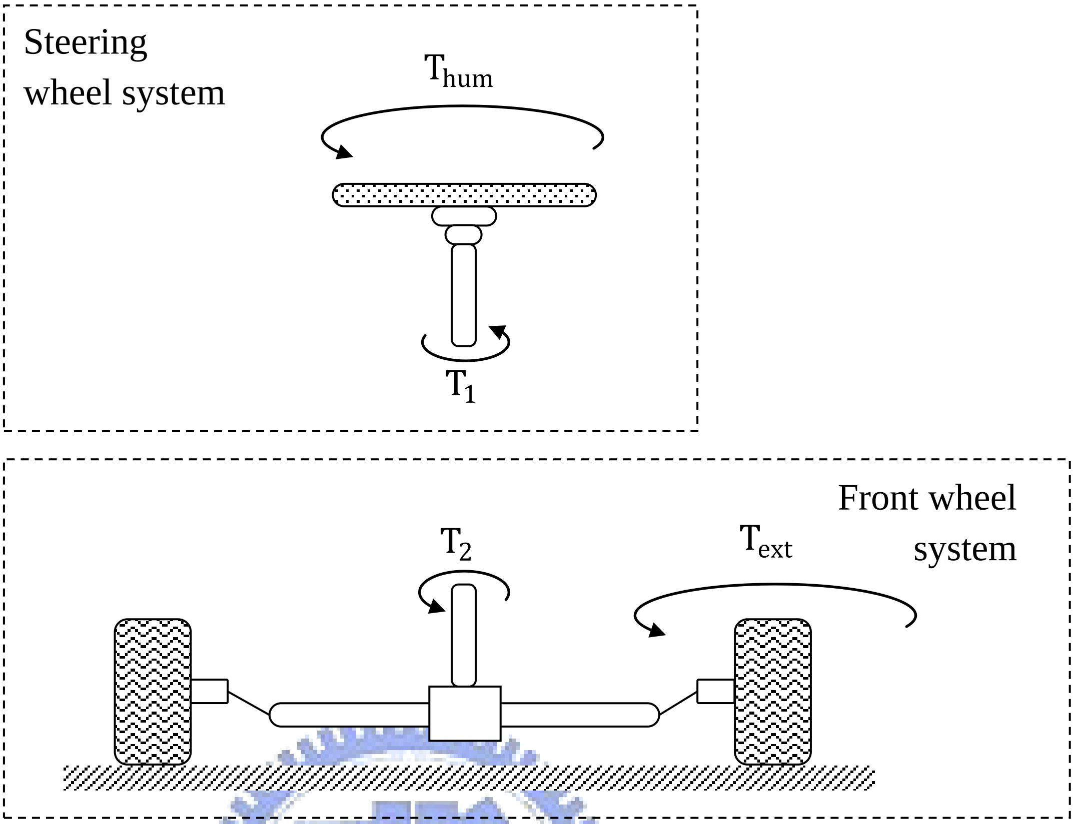

Figure3.3 Steering wheel system and front wheel system

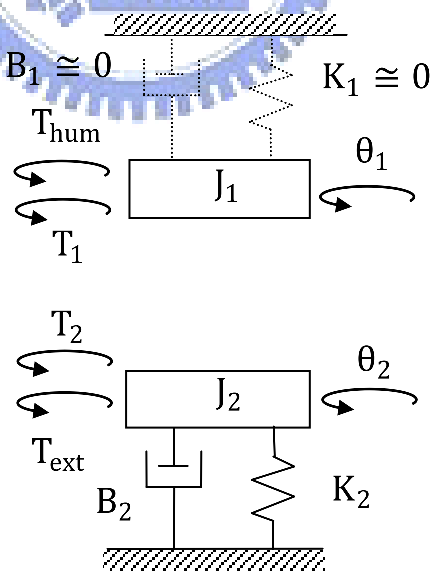

Figure3.4 Math model of steering system

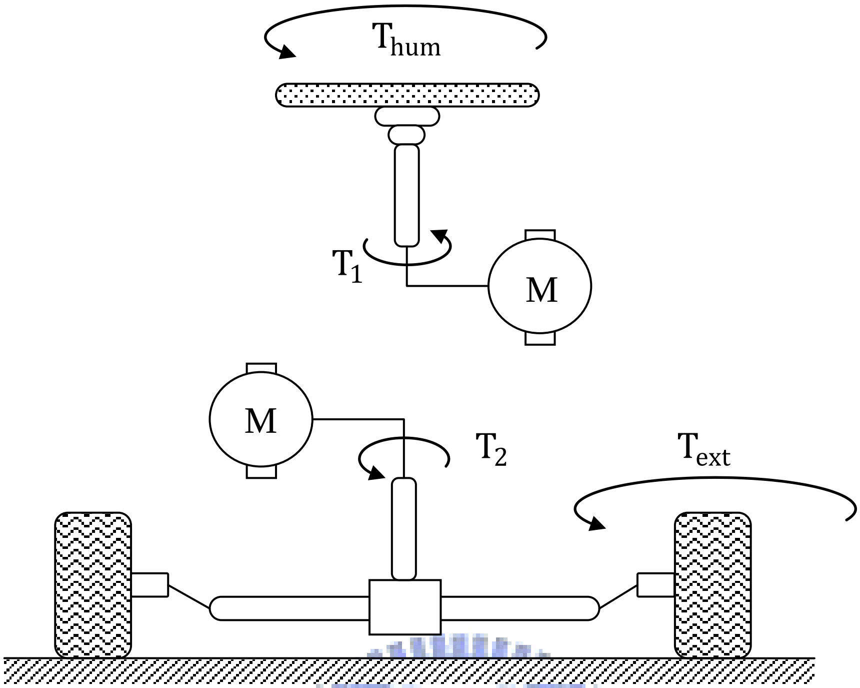

Figure3.5 Steer by wire system

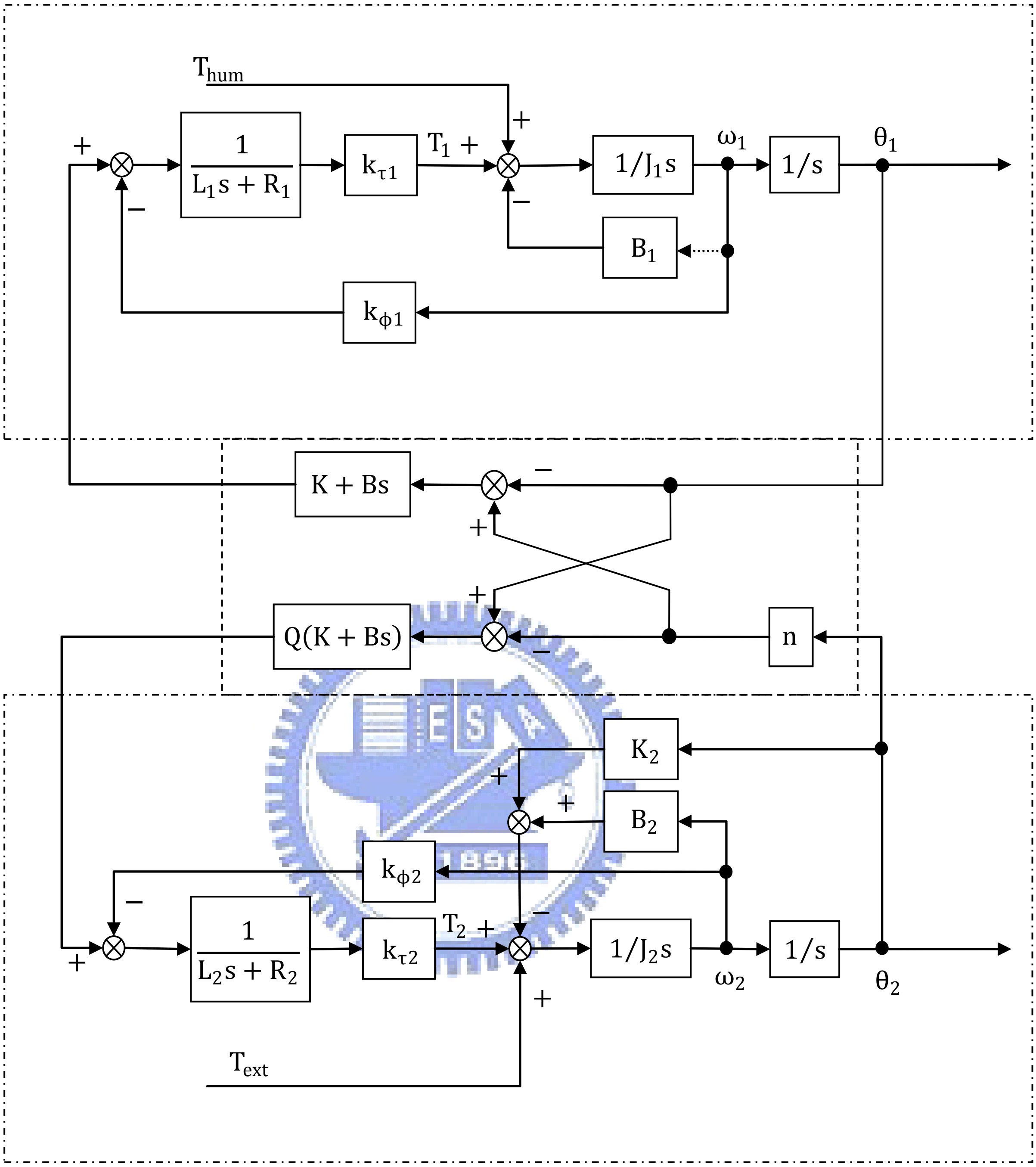

Figure3.6 Control block diagram of SBW

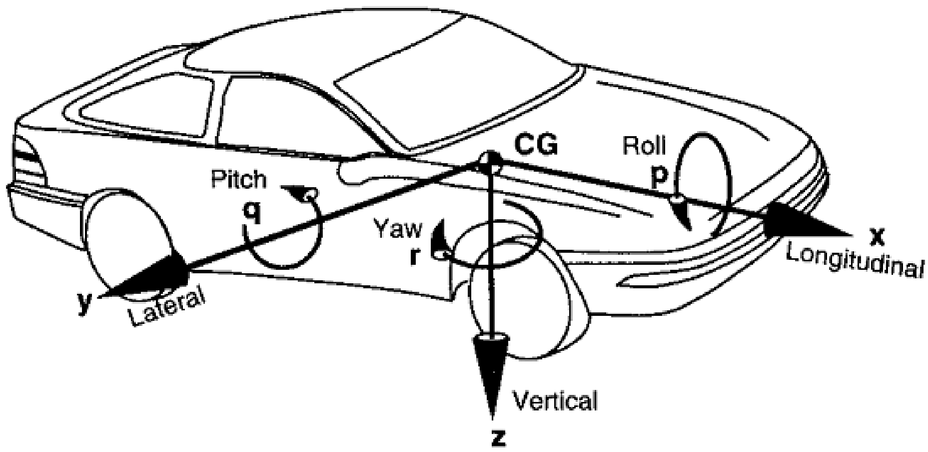

Figure3.7 SAE vehicle axis systems. [8]

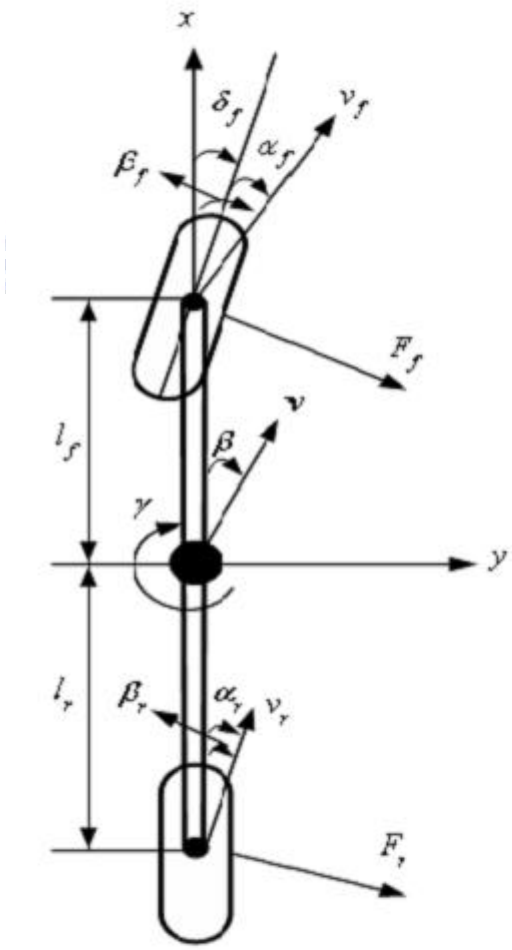

Figure3.8 2DOF of bicycle model [8]

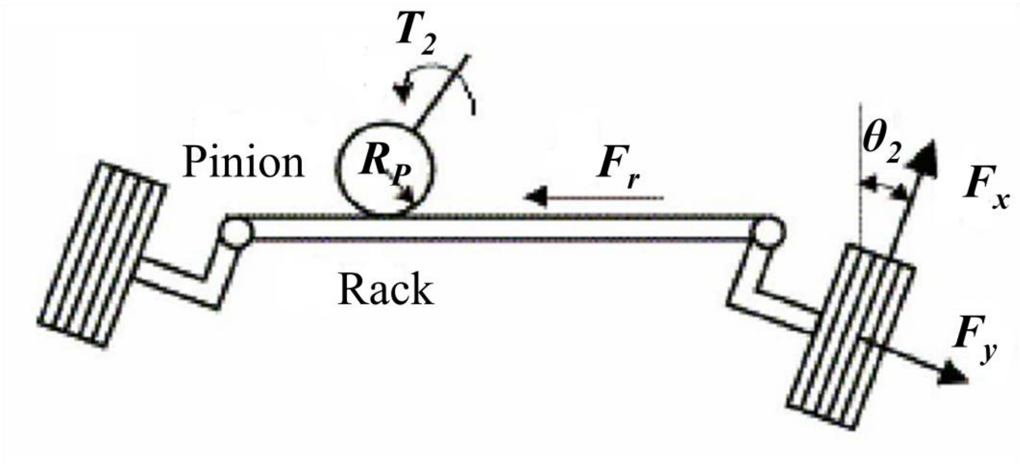

Figure3.9 Simplified model of steering system [22]

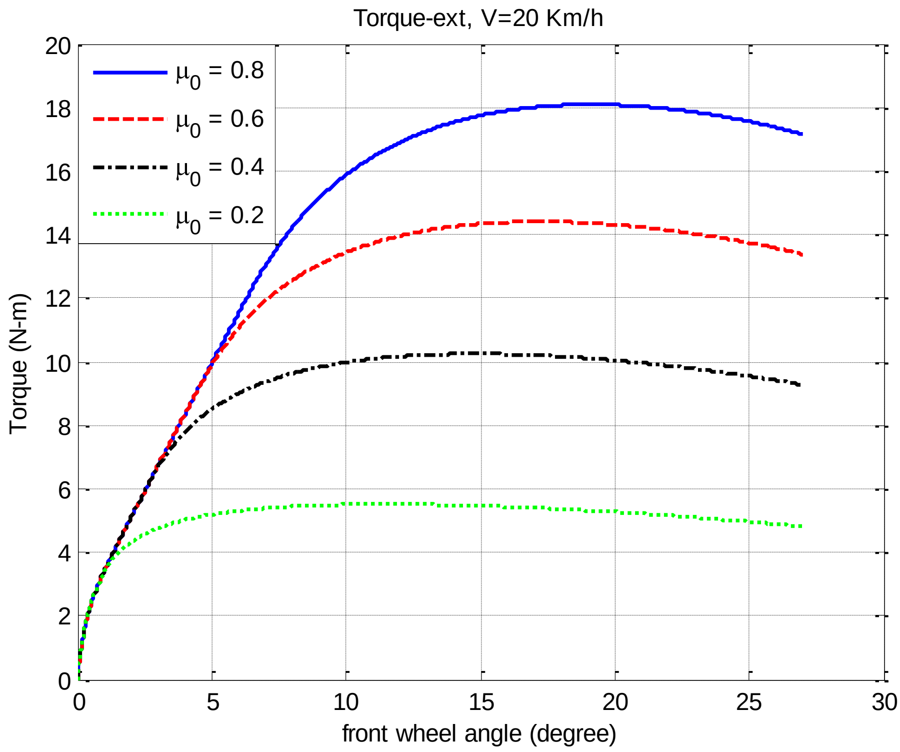

Figure3.10 Reaction torque on different road

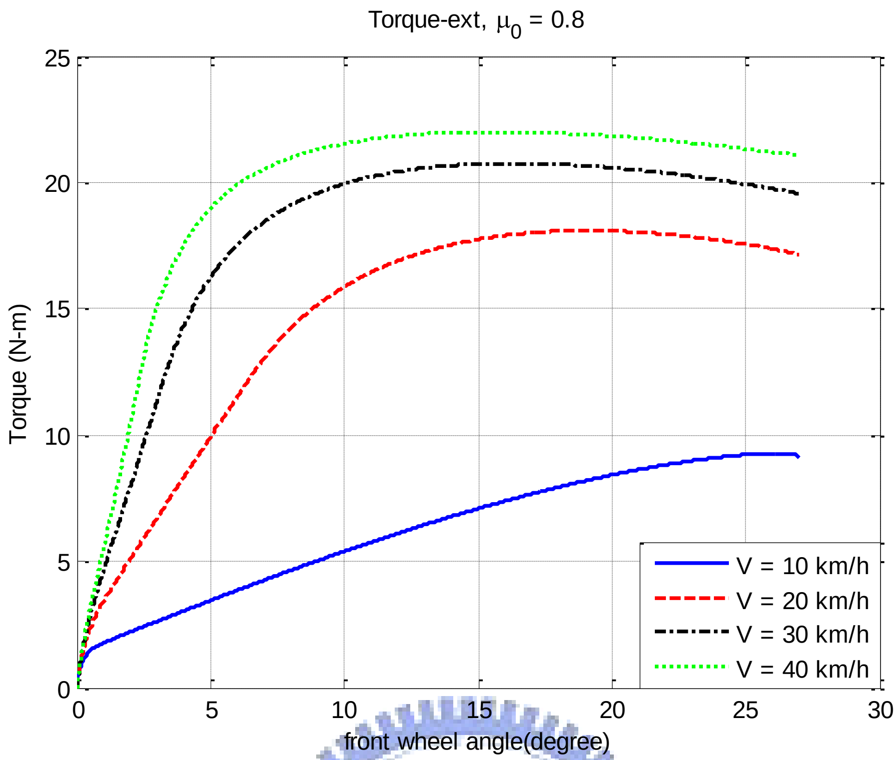

Figure3.11 Reaction torque of different vehicle velocity

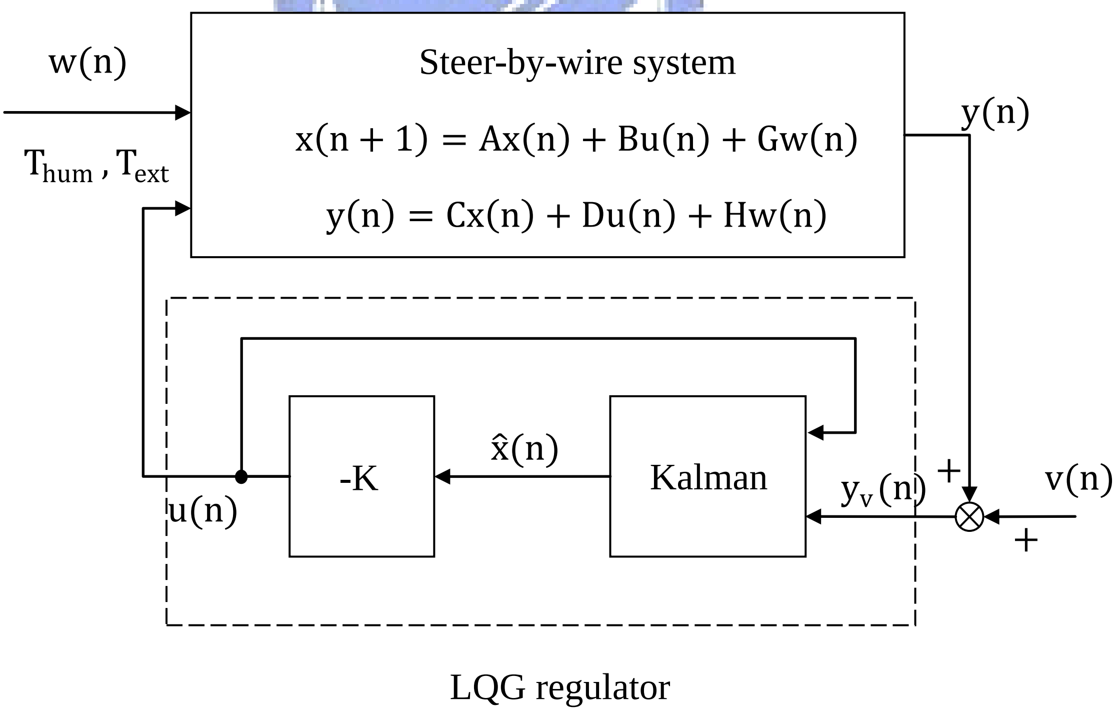

Figure3.12 LQG control structure

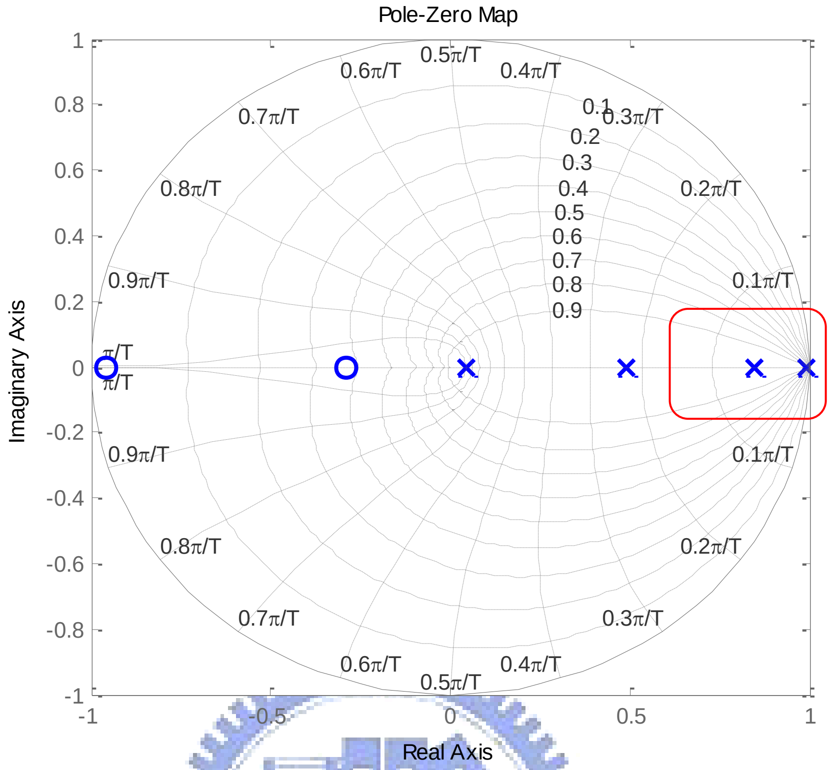

Figure4.1 Pole/Zero map of mechanical model

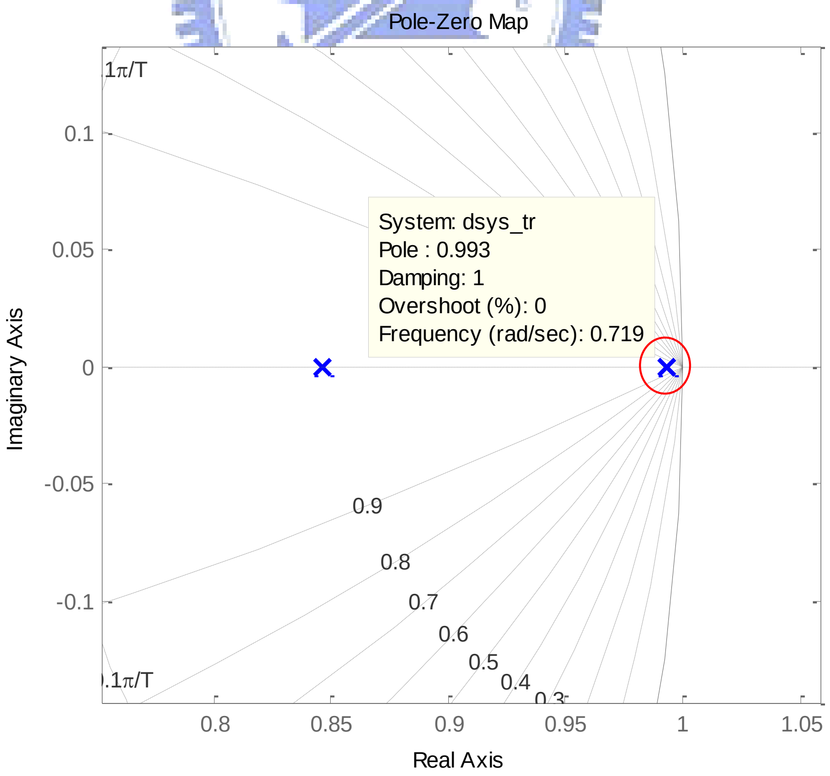

Figure4.2 Red frame on mechanical model

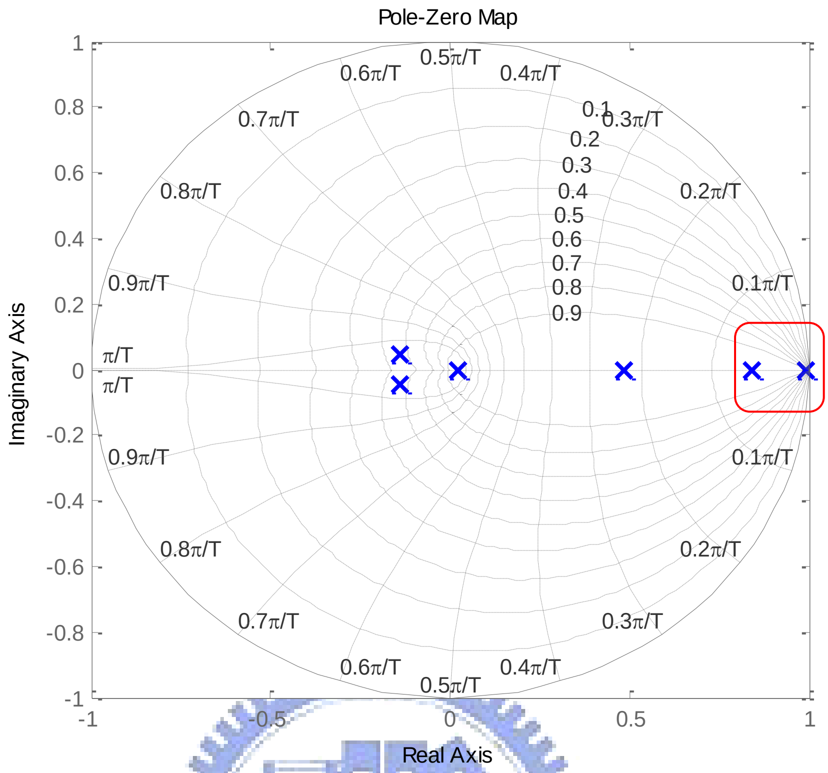

Figure4.3 Pole/Zero map of steer-by-wire models

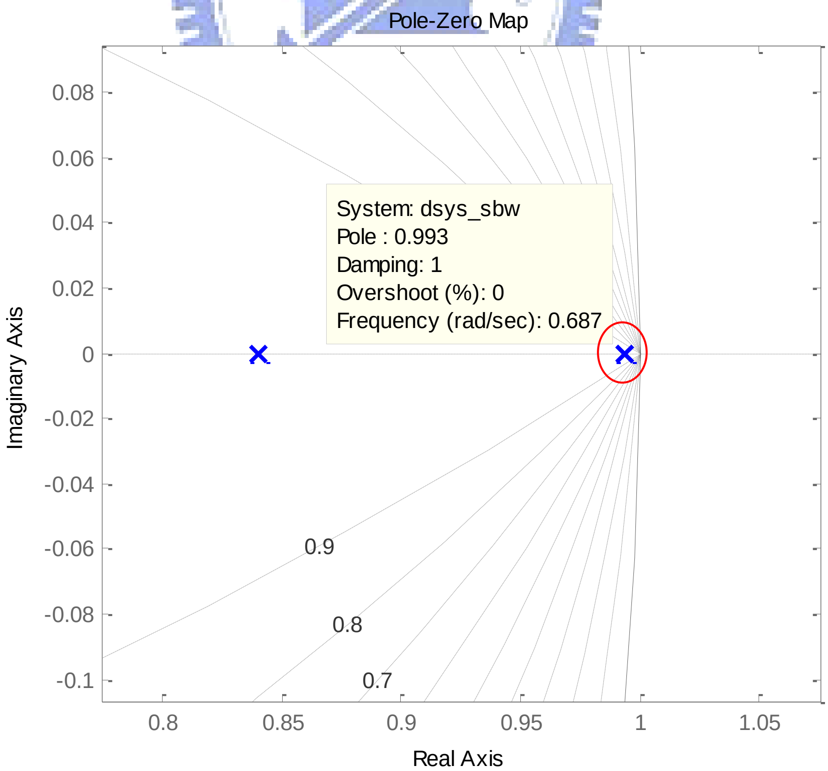

Figure4.4 Red frame on steer-by-wire models

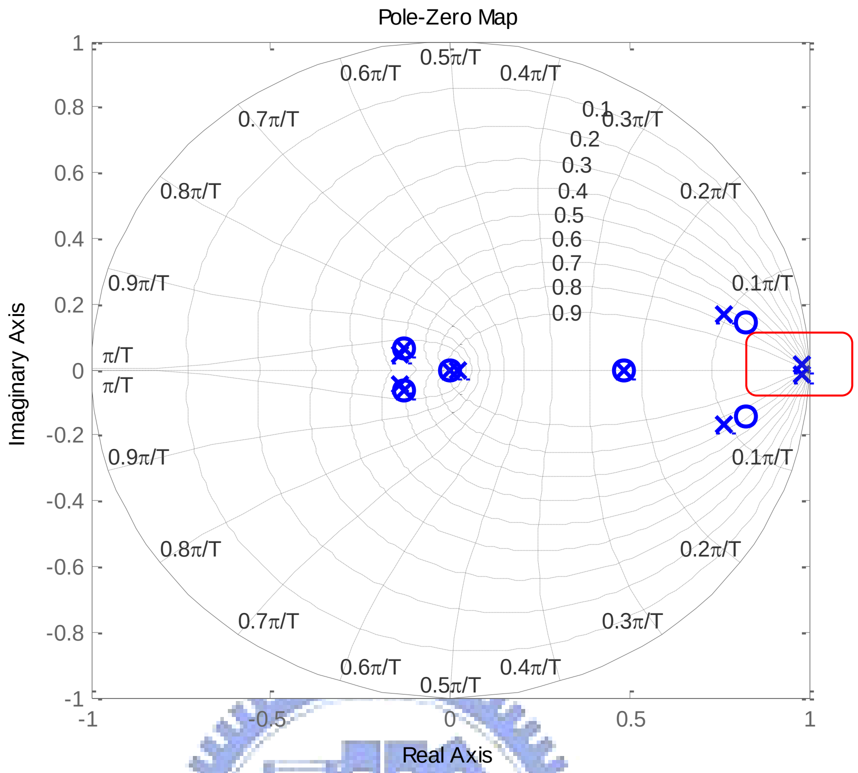

Figure4.5 Pole/Zero map of SBW model with LQG control

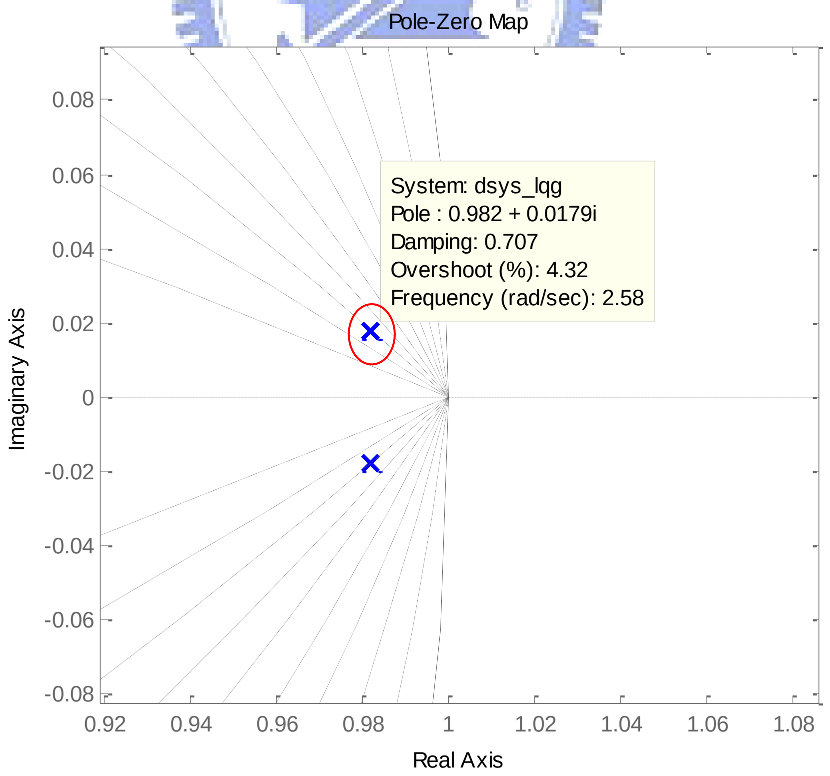

Figure4.6 Red frame on SBW model with LQG control

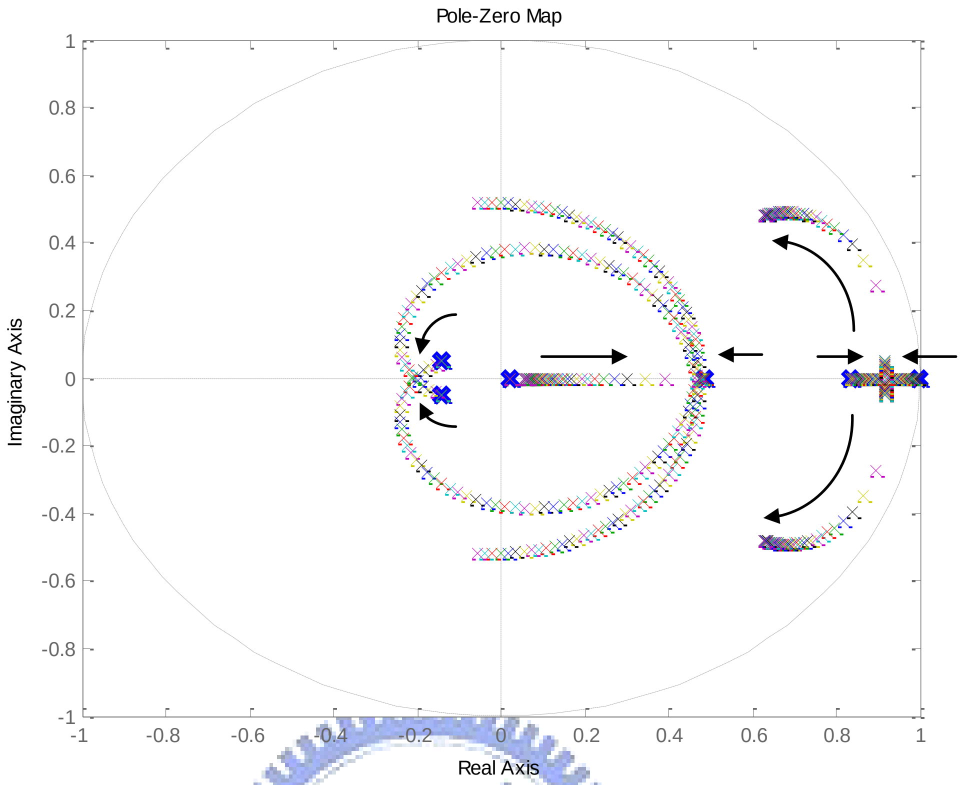

Figure4.7 SBW model root locus

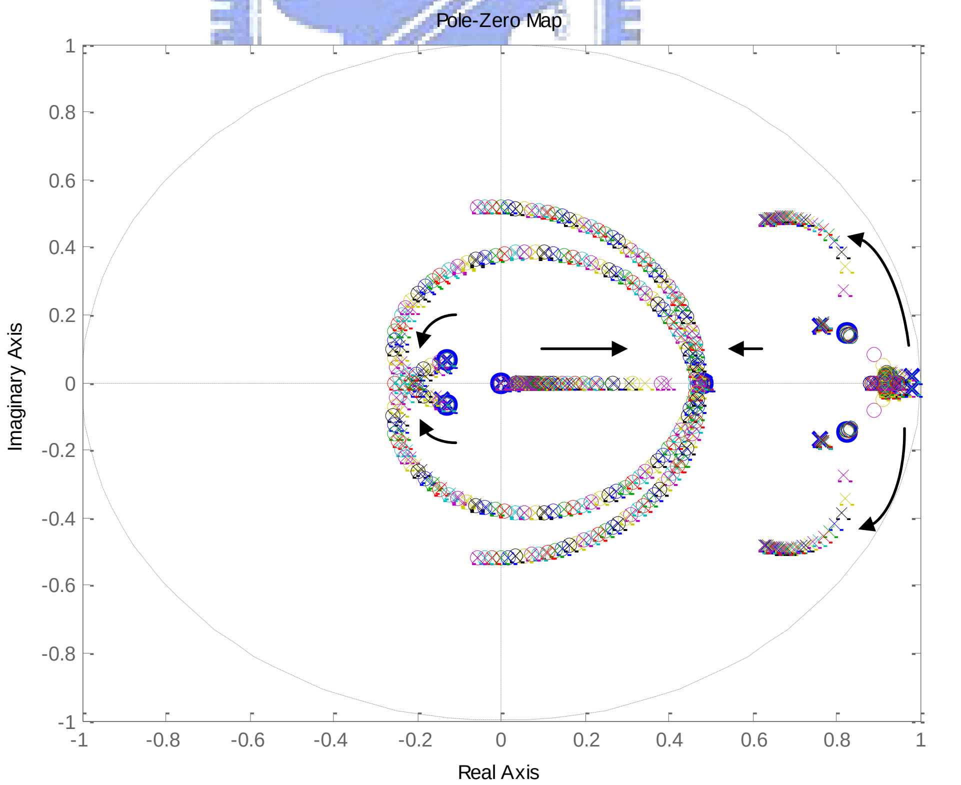

Figure4.8 SBW model with LQG root locus

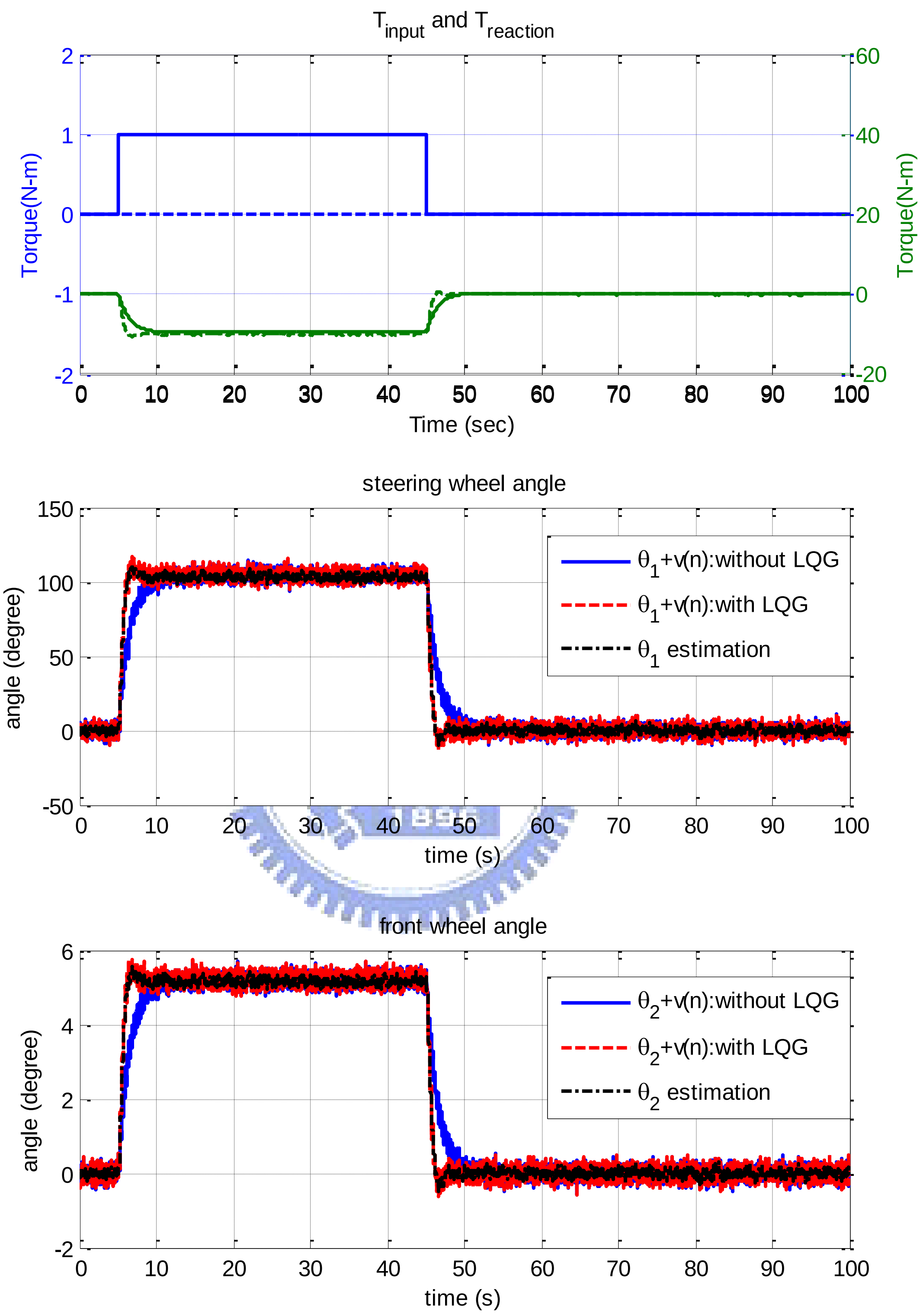

Figure4.9 Simulation with LQG, when input is torque human

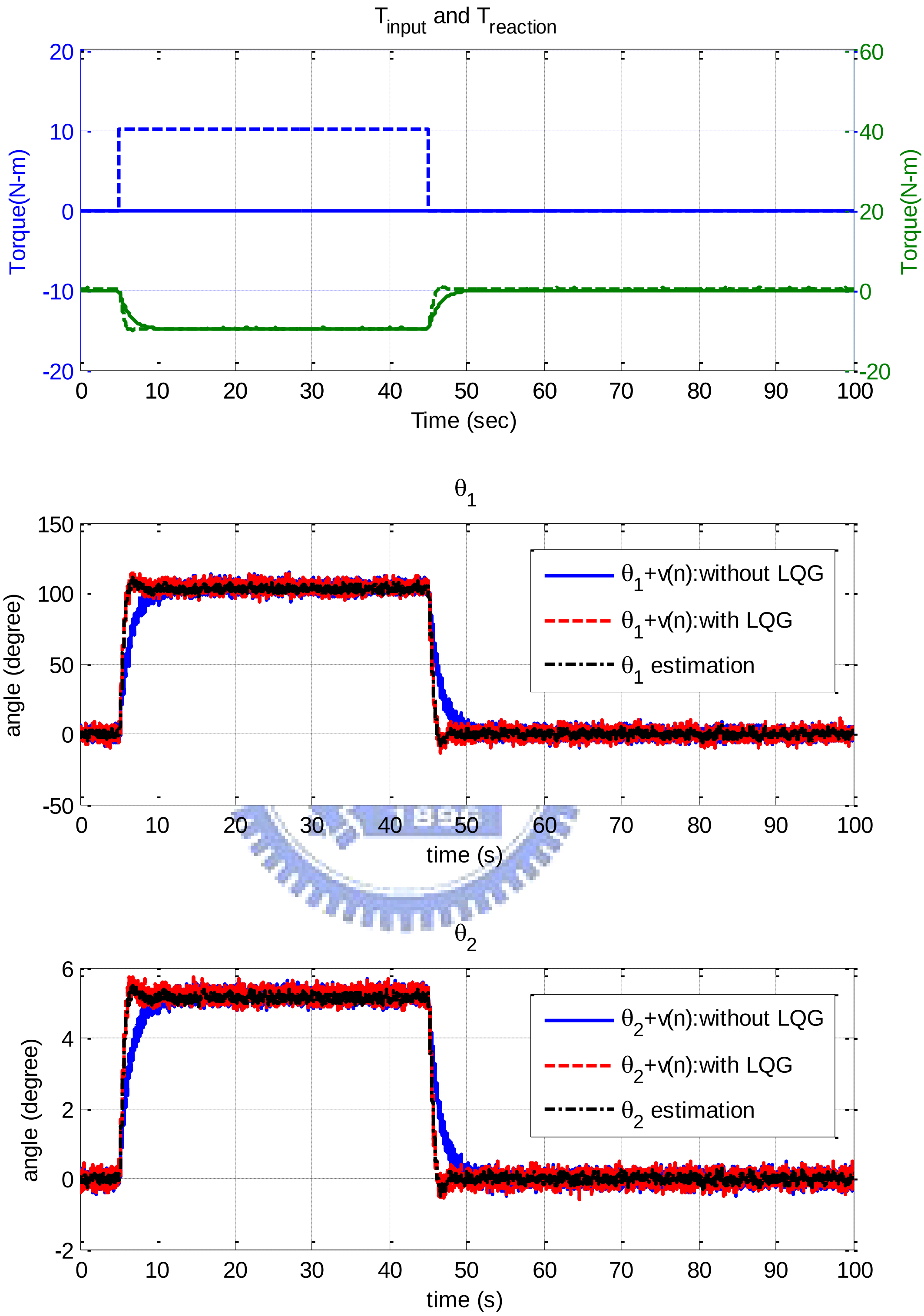

Figure4.10 Simulation with LQG, when input is external torque

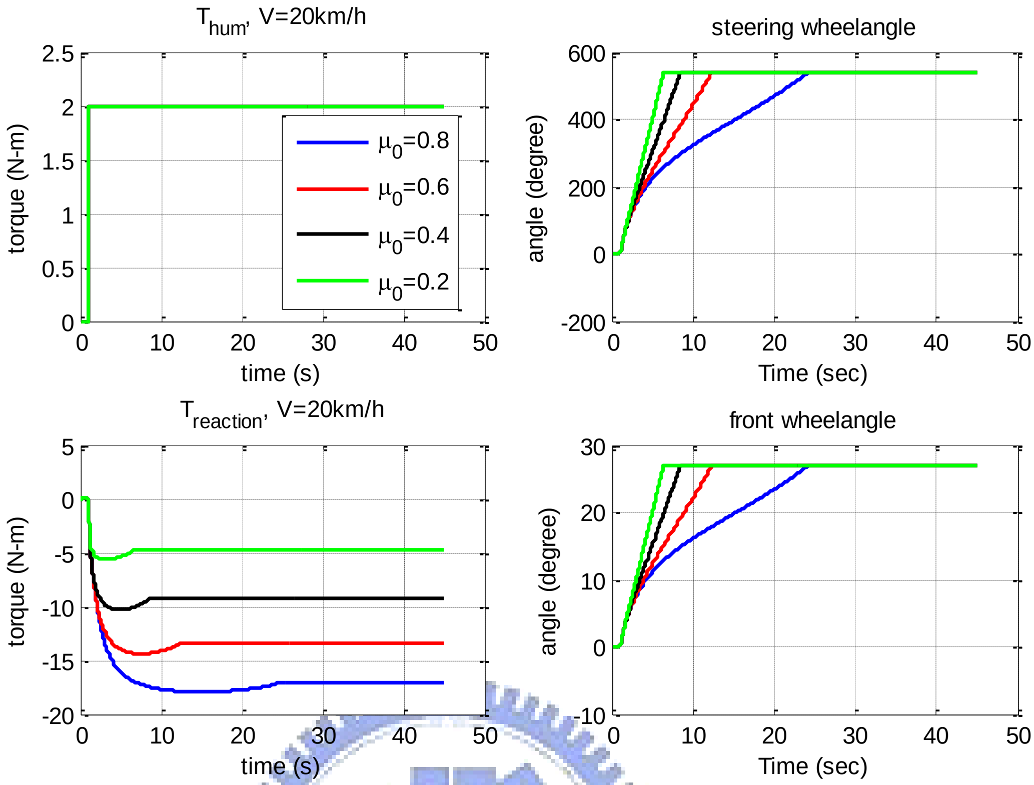

Figure4.11 Case 1: simulation on different road

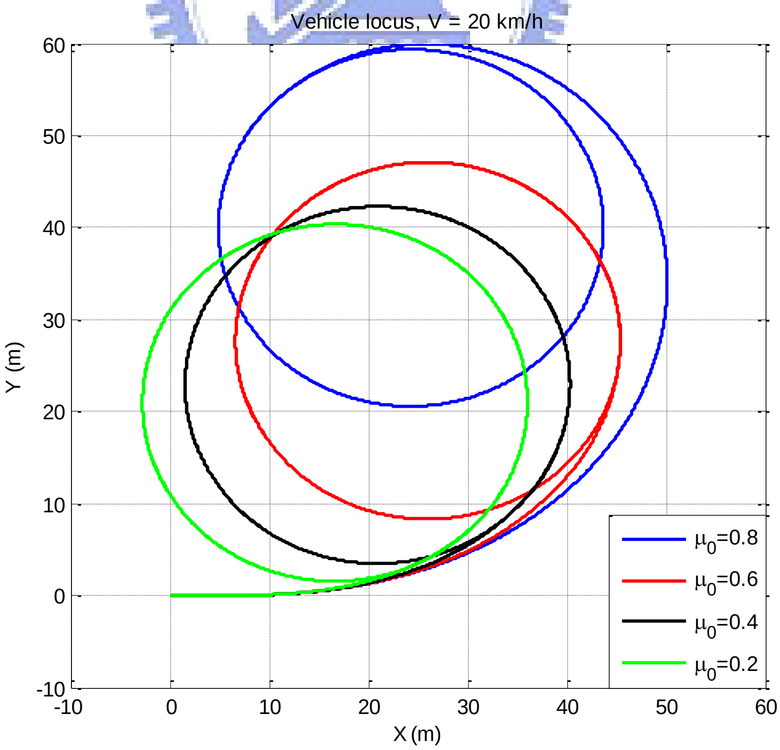

Figure4.12 Case1: vehicle locus on different road

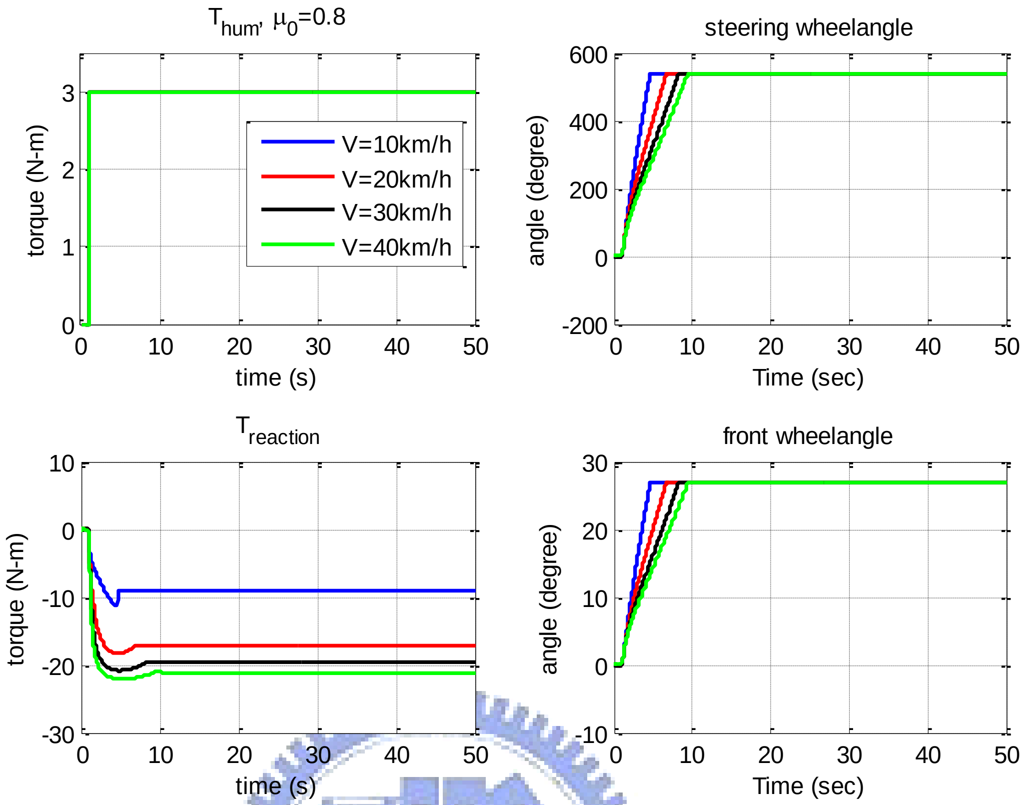

Figure4.13 Case 1: simulation of different vehicle velocity

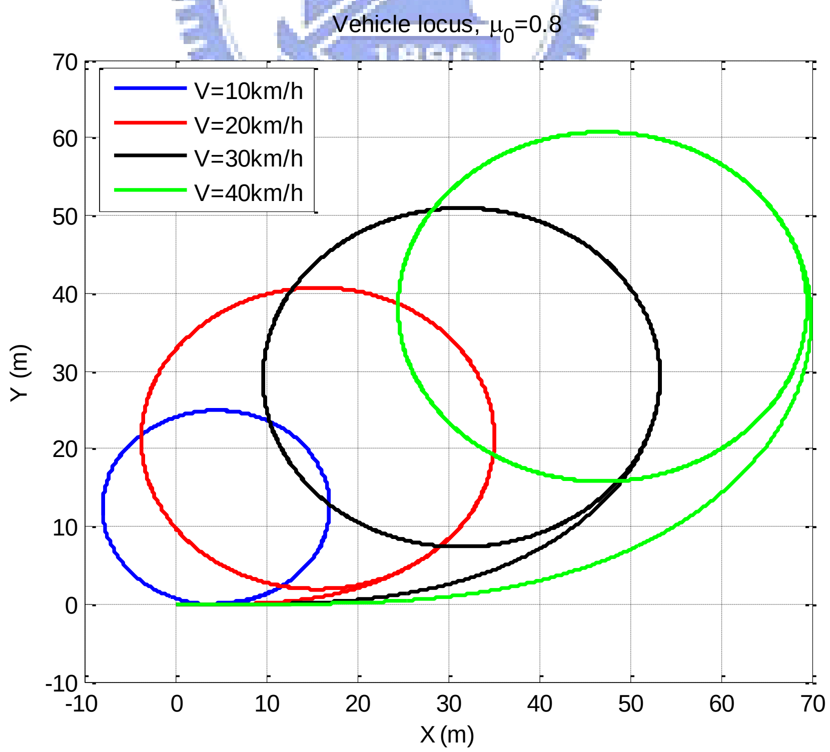

Figure4.14 Case1: vehicle locus of different vehicle velocity

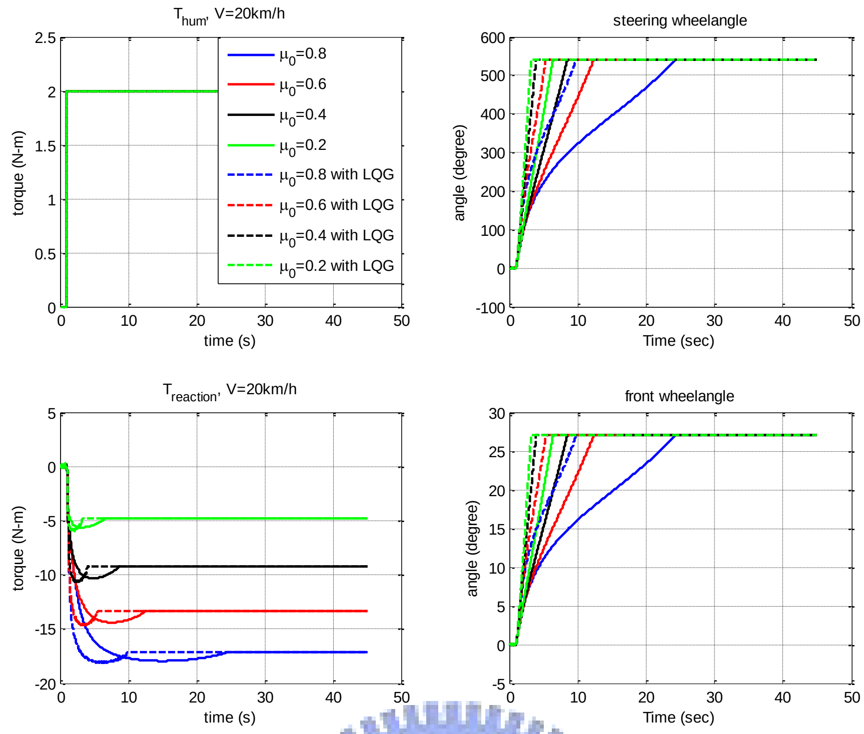

Figure4.15　Case 1: simulation with and without LQG control

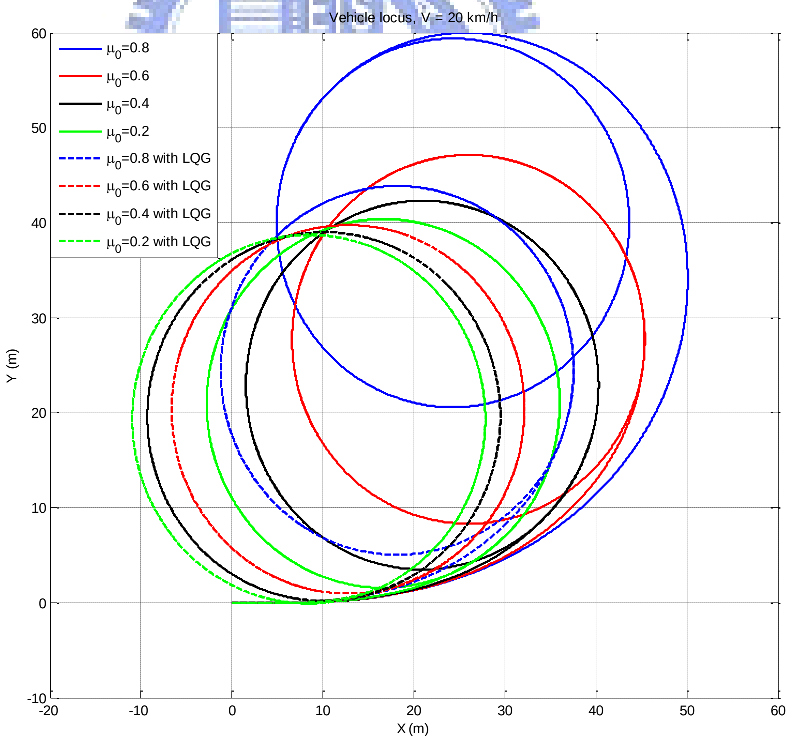

Figure4.16　Case 1: vehicle locus with and without LQG control

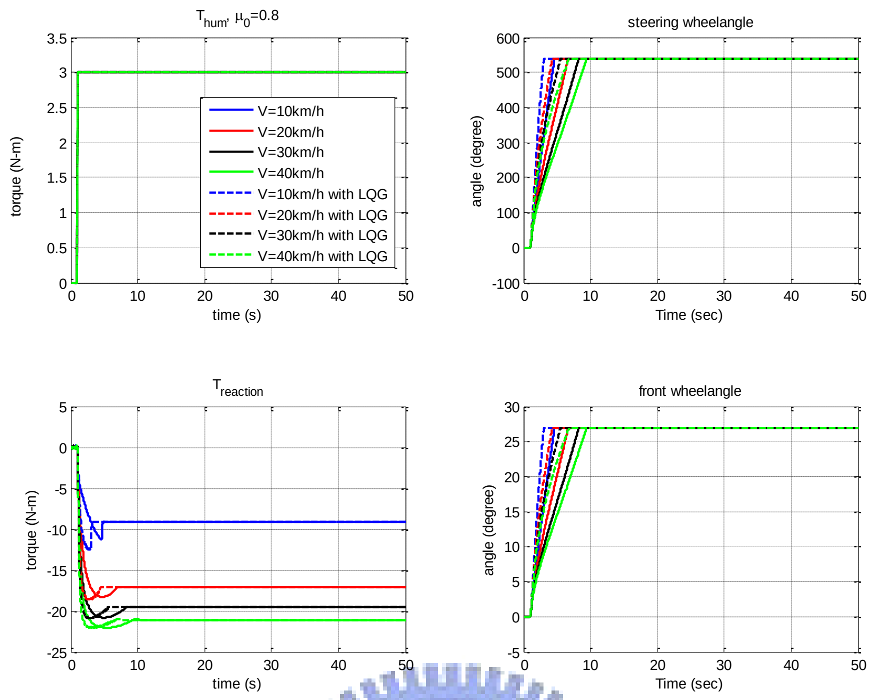

Figure4.17 Case 1: Simulation with and without LQG control

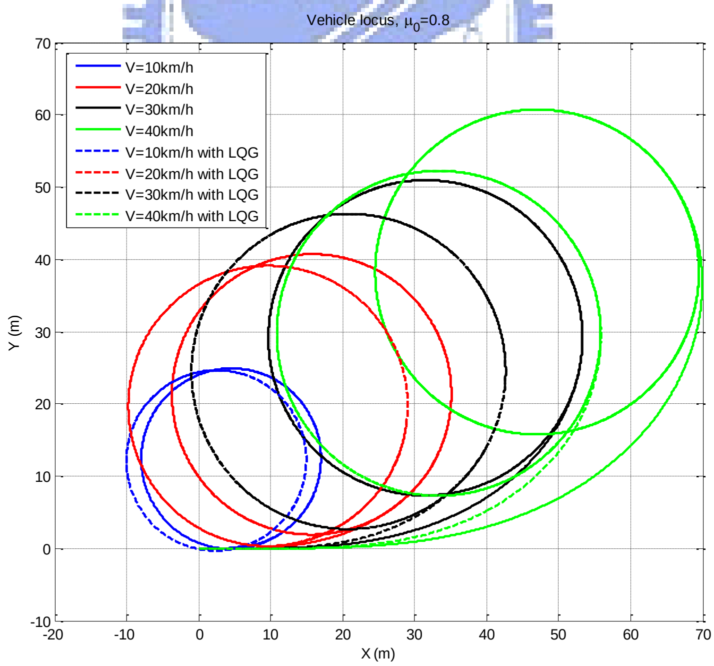

Figure4.18 Case 1: vehicle locus with and without LQG control

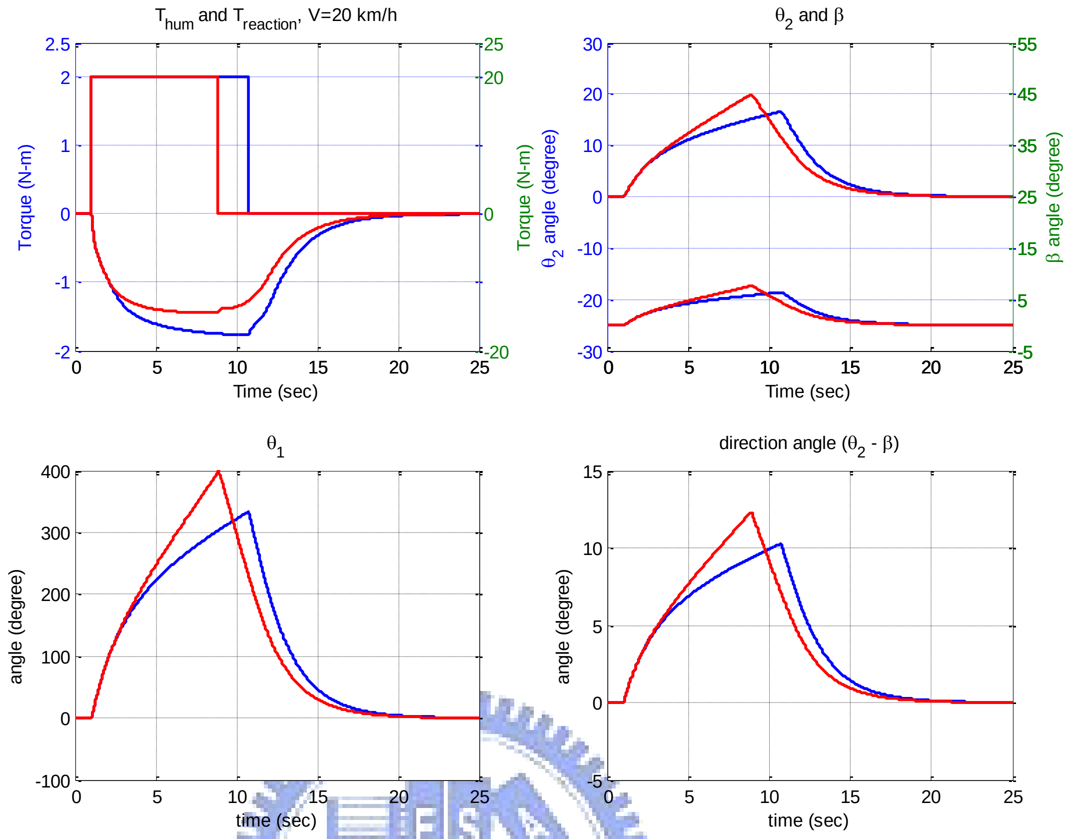

Figure4.19 Case 2: simulation on different road

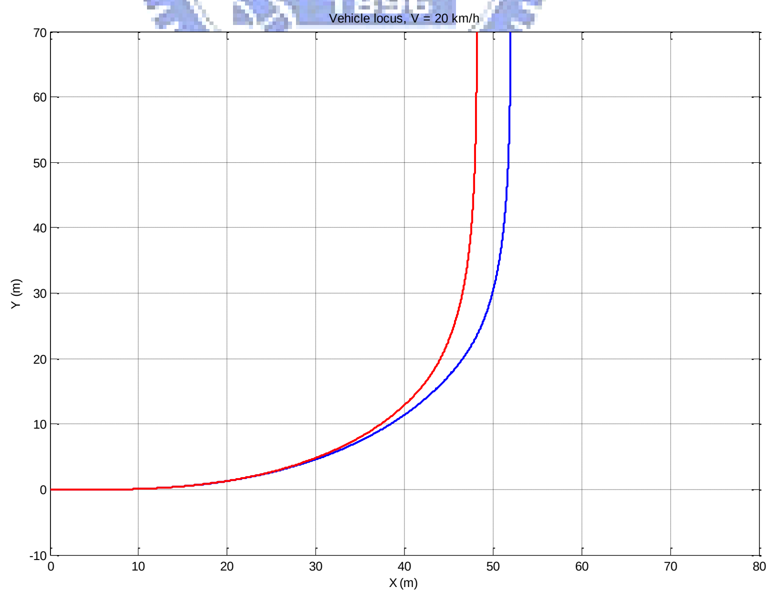

Figure4.20 Case 2: vehicle locus on different road

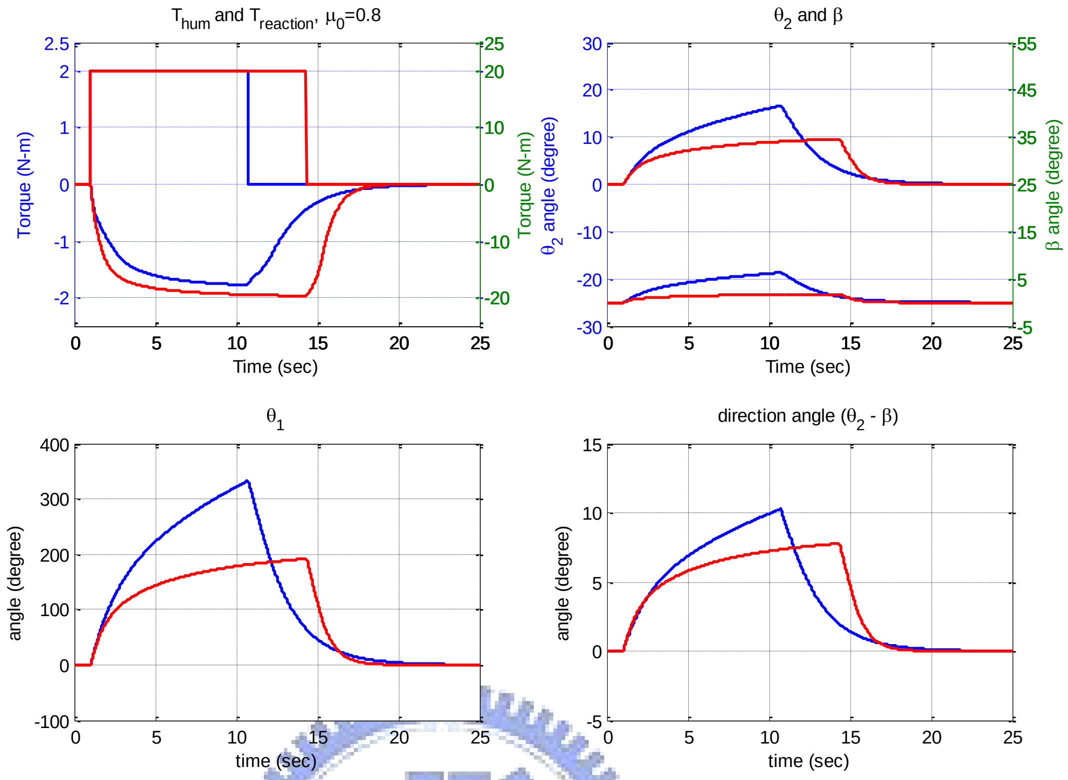

Figure4.21 Case 2: simulation of different vehicle velocity

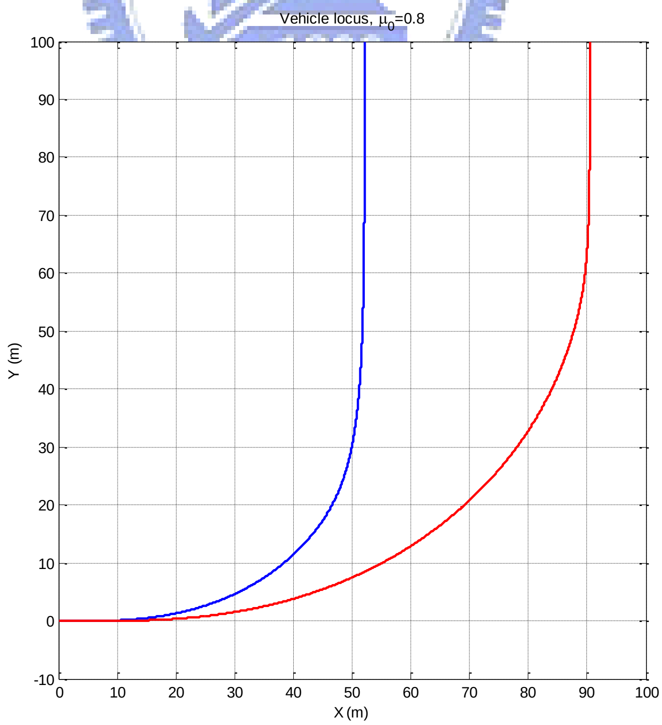

Figure4.22 Case 2: vehicle locus of different vehicle velocity

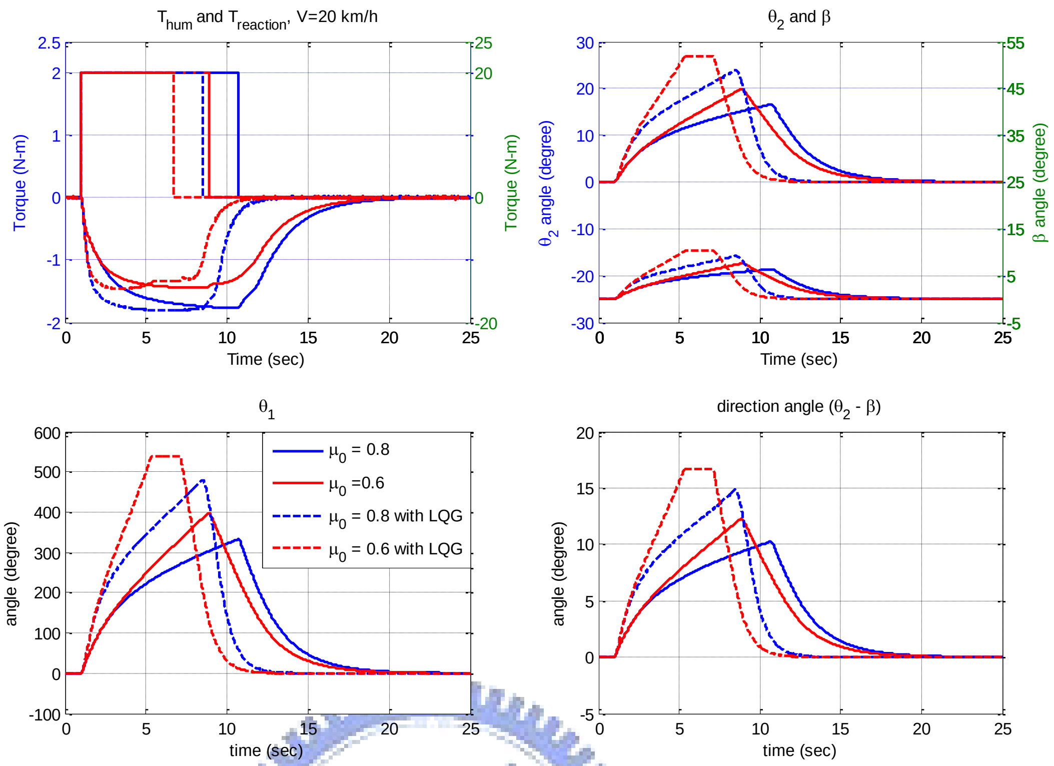

Figure4.23 Case 2: simulation with and without LQG control

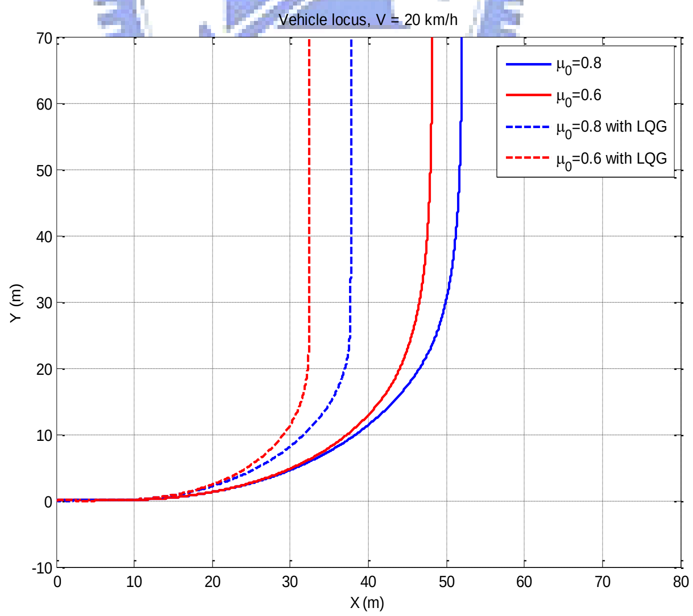

Figure4.24 Case 2: vehicle locus with and without LQG control

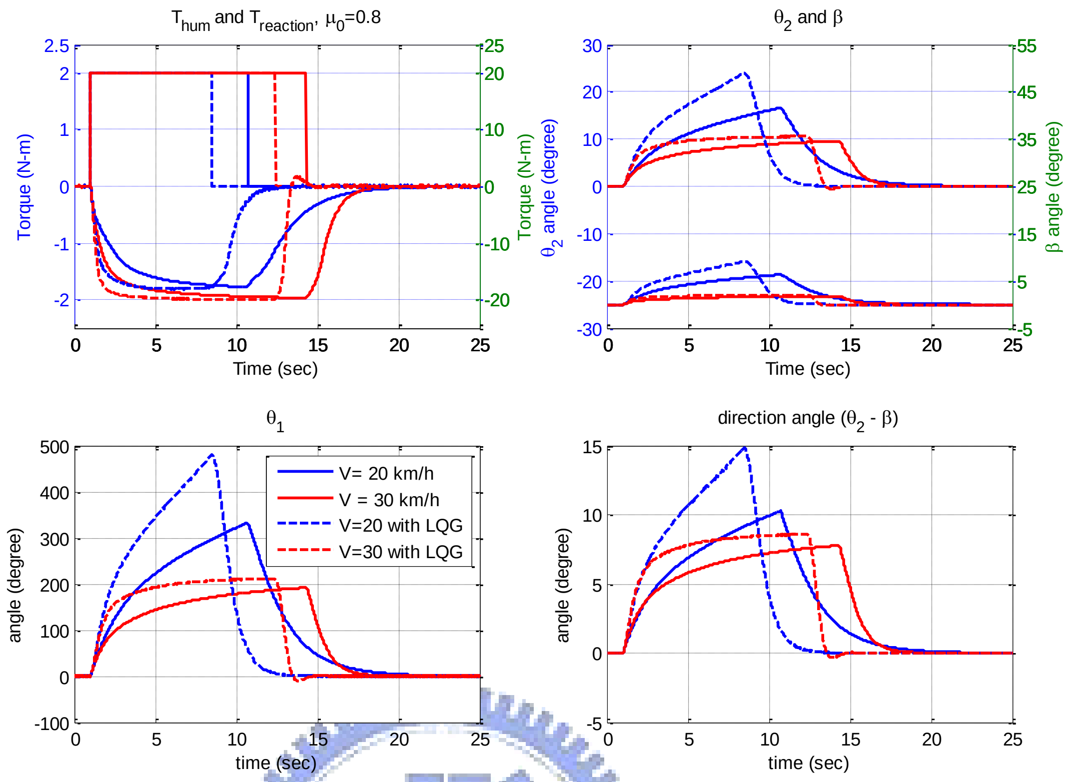

Figure4.25 Case 2: simulation with and without LQG control

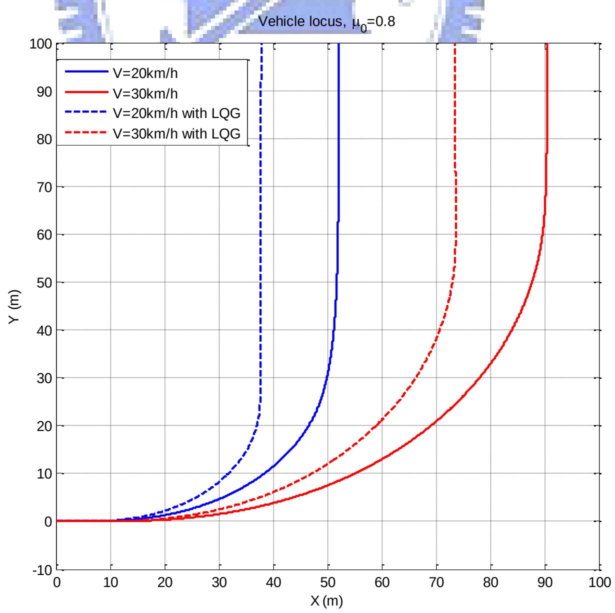

Figure4.26 Case 2: vehicle locus with and without LQG control

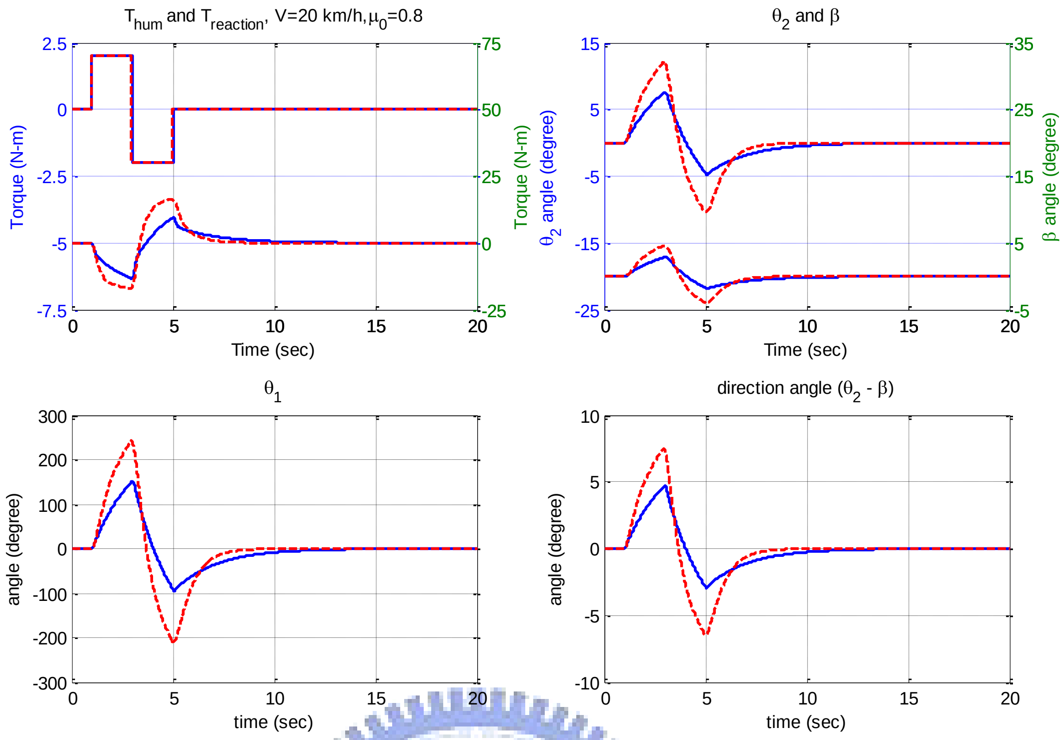

Figure4.27 Case 3: simulation with and without LQG

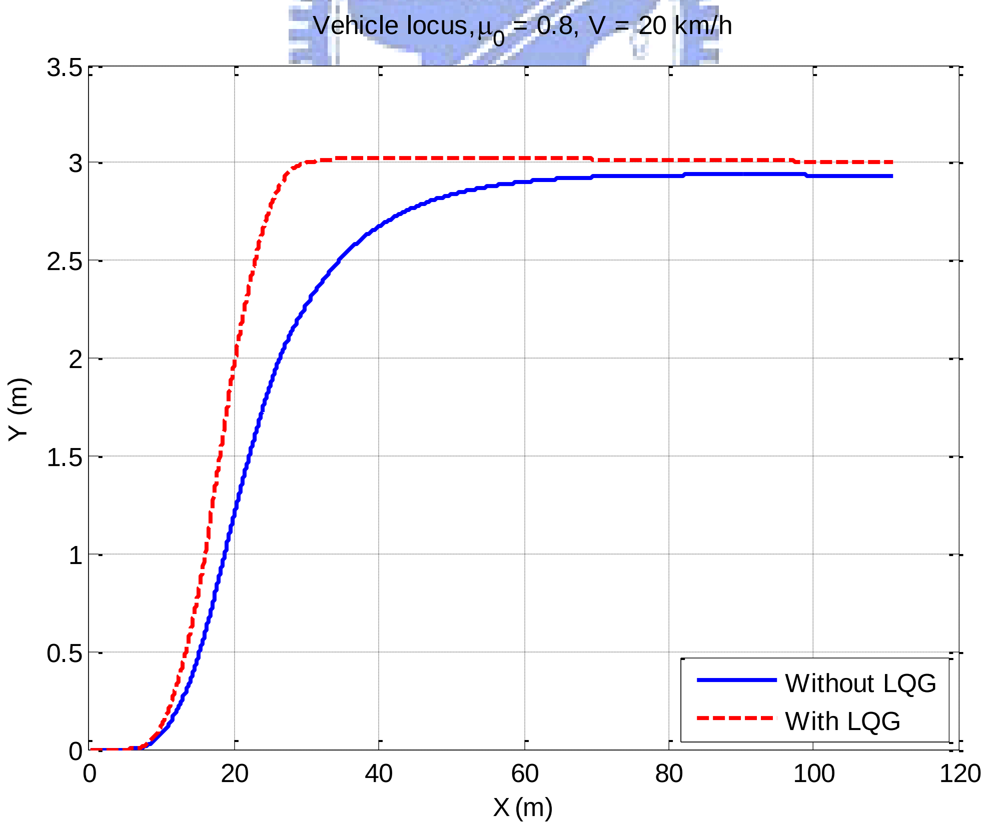

Figure4.28 Case 3: vehicle locus with and without LQG

Tables

Parameter	Symbol	Values	Units
Steering wheel moment of inertia	J_1	0.04	kg-m^2
Steering wheel motor inductance	L_1	9.06×10^{-5}	Henry
Steering wheel motor resistances	R_1	0.0345	Ohm
Steering wheel motor voltage constant	$K_{\Phi 1}$	0.0345	V/(rad/s)
Steering wheel motor torque constant	$K_{\tau 1}$	0.0345	Nm/A
Front wheel moment of inertia	J_2	1.8	kg-m^2
Front wheel damping	B_2	160	Nm/(rad/s)
Front wheel torsion stiffness	K_2	110	Nm/rad
Front wheel motor inductance	L_2	9.06×10^{-5}	Henry
Front wheel motor resistances	R_2	0.0345	Ohm
Front wheel motor voltage constant	$K_{\Phi 2}$	0.0345	V/(rad/s)

Front wheel motor torque constant	$K_{\tau 2}$	0.0345	Nm/A
Steering column torsion stiffness	K	172	Nm/rad
Steering column damping	B	2.25	Nm/(rad/s)
Steering angle ratio	n	20	
Steering torque ratio	Q	10	

Table4.1 Parameter of steer-by-wire model

Parameter	**Symbol**	**Values**	**Units**
Vehicle mass	M	1573	kg
Longitudinal distance from front axle to center of gravity	L_f	0.96	m
Longitudinal distance from center of gravity to rear axle	L_r	1.225	m
Longitudinal distance from rear axle to front axle	L	2.185	m
Vehicle yaw moment of inertia	I_Z	2868	kg-m^2
adhesion reduction coefficient	A_S	0.00353	
longitudinal stiffness of tire	C_S	45837	Nm/rad
Front wheel cornering stiffness	$C_{\alpha f}$	58000	Nm/rad
Rear wheel cornering stiffness	$C_{\alpha r}$	62812	Nm/rad
Pinion radius	R_p	0.008	m

Table4.2 Vehicle parameter

PORTLAND CEMENT				
	Polished or Glazed	Well-traveled	New, Coarse	
Dry	0.5~0.75	0.6~0.75	0.7~1.0	
Wet	0.35~0.6	0.45~0.7	0.5~0.8	
ASPHALT or TAR				
	Excess Tar, Bleeding	Polished or Glazed	Well-traveled, Smooth	New, Coarse
Dry	0.35~0.6	0.45~0.75	0.55~0.8	0.65~1.0
Wet	0.25~0.55	0.4~0.65	0.4~0.65	0.45~0.8
GRAVEL				
	Loose		Packed, Well-traveled	
	0.4~0.7		0.5~0.85	
SNOW			**ICE**	
	Cold, Loose	Cold, Packed	Cold, Frosted	Warm
Dry	0.1~0.25	0.25~0.55	0.1~0.25	
Wet	0.3~0.5	0.3~0.6		0.05~0.1

Table4.3 The road friction coefficient [24]