Residual Stresses in Cold-Bent Thick Steel Plates

By C. C. Weng[1] and R. N. White,[2] Member, ASCE

Abstract: An experimental investigation of the residual stresses in severely cold-bent thick high-strength steel plates is presented. Both the sectioning and the hole-drilling methods are used for residual stress measurement; the two sets of results are found to be in good agreement. Tension residual stresses on the inside surface of the bend range from 46% to 92% of the yield stress of the material. In addition, residual stresses through the plate thickness are measured using the sectioning method. A zigzag-type residual stress distribution pattern through the plate thickness is observed. The test results are then compared to the values predicted by equations proposed by Dat in 1980. The cold-bending behavior of the thick steel plates is also studied and the results are presented in a companion paper.

Introduction

Plate-bending is a fabricating process in which initially flat plates are permanently deformed into desired configurations by the use of rolls or dies. A typical bending operation using a punch and die for forming a steel plate to a 90° bend is shown in Fig. 1. The cold-bending of a steel plate involves loading the metal into the plastic range and is followed by an elastic unloading. Residual stresses are locked into the plate, and the elastic recovery produces a phenomenon known as springback. The inelastic loading stress, the elastic unloading stress, and the final state of residual stress for a typical cold-bent steel plate are shown in Fig. 2.

The zone of tensile residual stress on the inside surface of the bend is a possible source of fatigue cracking under cyclic loading, as well as being a region more susceptible to stress corrosion. Thus it is desirable to quantify the magnitude of residual stresses to a reasonable level of accuracy.

The experimental work includes the measurement of residual stresses in cold-bent HY-80 and HY-100 steel plates of 18 (457.2 mm) in. square with 1 (25.4 mm) and 1.5 (38.1 mm) in. thickness. A total of 18 steel plates were tested with bend radii of 1.5 (38.1 mm), 2.5 (63.5 mm), 3.5 (88.9 mm), and 5.5 (139.7 mm) in., and bend angles of θ = 90°, 120°, and 150° (see Fig. 3). Detailed results are given in a report prepared by the writers (1984).

Literature Review

Residual Stresses Due to Cold-Bending: Theory

In 1944, Sachs and Lubahn studied the problem of pure bending of an ideal plastic metal in plane strain. The stresses induced by the bending moment and the location of the neutral surface were found theoretically. Hill

[1]Assoc. Prof. of Civ. Engrg., Nat. Chiao-Tung Univ., 1001 Ta Hsueh Rd., Hsinchu, Taiwan, R.O.C.

[2]Prof. of Struct. Engrg., Cornell Univ., Ithaca, NY 14853.

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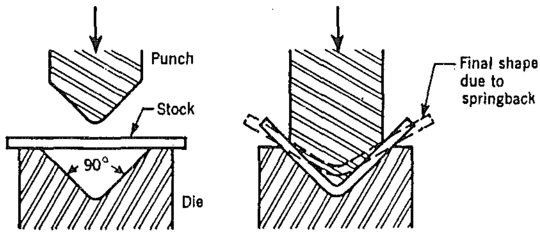

FIG. 1. Plate Bending Operation and Springback during Unloading

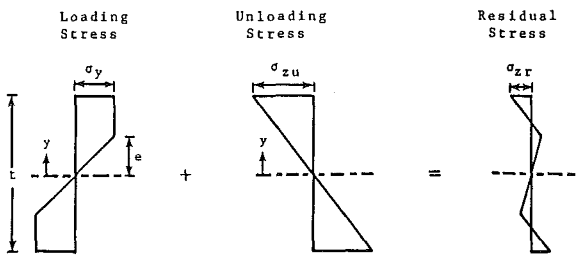

FIG. 2. Loading, Unloading, and Residual Stresses in Cold Bent Section

(1950) performed a more general analysis on the plane strain problem of wide plates under pure bending and under a combination of end moments and internal pressure. Analytical solutions were obtained for predicting the stress distribution in the plate caused by bending.

Alexander (1959) proposed both an analytical and a geometrical approach to solve the problem of moderate bending of a wide plate in plane strain. It was assumed that the outer fiber strain of the plate under bending was less than 2%. This problem was extended to include work-hardening material by Denton in 1966. Ingvarsson (1977) investigated residual stresses in cold-bent thin plates considering the presence of three kinds of external loadings: end moments, tension forces, and internal pressure. Due to the complexity of the problem, the solution was obtained by using a computerized incremental analysis.

In 1980, Dat developed equations for predicting the residual stresses caused by severe cold-bending. The equations are based on the results obtained by Hill (1950) and by Timoshenko and Goodier (1970), which give the loading stresses and unloading stresses in the bending of a plate, respectively. By summing the loading and unloading stresses, the residual stresses in the cold-bent plate are then found. The solutions assume that: (1) The material is elastic-perfect plastic; (2) bending occurs under plane strain condition; and (3) the small plastic region near the neutral axis can be neglected. The calculation of residual stresses is given as follows (the coordinate system used in Dat's investigation is shown in Fig. 4).

Loading stresses:

For $a \leq r \leq c$

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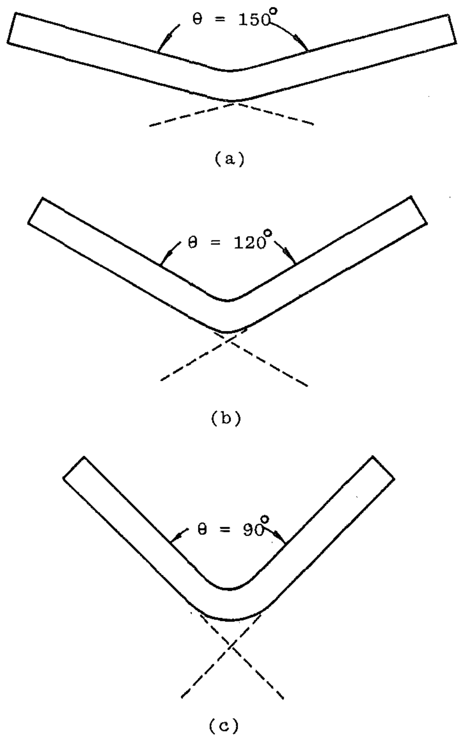

FIG. 3. Bend Angles of Steel Plates

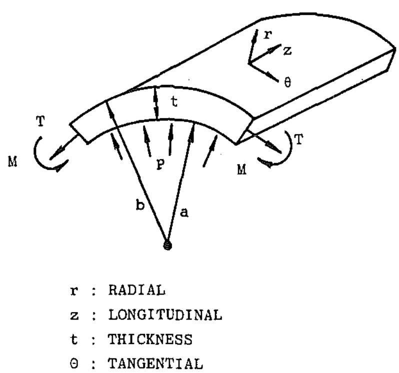

FIG. 4. Bent Plate Coordinate System

$$\sigma_r = 2K\left(-p - \ln\frac{r}{a}\right) \quad (1)$$

$$\sigma_\theta = 2K\left(-p - 1 - \ln\frac{r}{a}\right) \quad (2)$$

For $c < r \leq b$

$$\sigma_r = 2K \ln \frac{r}{b} \qquad (3)$$

$$\sigma_\theta = 2K\left(1 + \ln \frac{r}{b}\right) \qquad (4)$$

Elastic unloading stresses:

Bending unloading stresses

$$\sigma_{rb} = 2K\left(-\frac{4M}{a^2N}\right)\left(\frac{b^2}{r^2} \ln \frac{b}{a} + \frac{b^2}{a^2} \ln \frac{r}{b} + \ln \frac{a}{r}\right) \qquad (5)$$

$$\sigma_{\theta b} = 2K\left(-\frac{4M}{a^2N}\right)\left(-\frac{b^2}{r^2} \ln \frac{b}{a} + \frac{b^2}{a^2} \ln \frac{r}{b} + \ln \frac{a}{r} + \frac{b^2}{a^2} - 1\right) \qquad (6)$$

Pressure unloading stresses

$$\sigma_{rp} = 2K\left(\frac{-a^2p}{b^2 - a^2}\right)\left(1 - \frac{b^2}{r^2}\right) \qquad (7)$$

$$\sigma_{\theta p} = 2K\left(\frac{-a^2p}{b^2 - a^2}\right)\left(1 + \frac{b^2}{r^2}\right) \qquad (8)$$

Residual stresses:

$$\sigma_{r'} = \sigma_r + \sigma_{rp} + \sigma_{rb} \qquad (9)$$

$$\sigma_{\theta'} = \sigma_\theta + \sigma_{\theta p} + \sigma_{\theta b} \qquad (10)$$

$$\sigma_{z'} = 0.5(\sigma_r + \sigma_\theta) + 0.3(\sigma_{rp} + \sigma_{rb} + \sigma_{\theta b} + \sigma_{\theta p}) \qquad (11)$$

where $2K = \sigma_y$ for Tresca criterion, $(2\sigma_y)/\sqrt{3}$ for von Mises criterion; r = radius to an arbitrary point in the bend; a = radius to the inside of bend; b = radius to the outside of bend; c = radius to the neutral surface of the bend that can be found as $c = \sqrt{(abe^{-p})}$; p = internal pressure applied over the inner surface of the bend; $M = (a^2 + b^2 - 2abe^{-p})/4 - abp/2$; and $N = [(b^2/a^2) - 1]^2 - 4(b/a)^2 \cdot ([\ln (b/a)]^2$.

Residual Stresses Due to Cold-Bending: Experiment

Methods for determining residual stresses can be categorized into three groups: nondestructive (NDT) methods, semi-destructive methods, and destructive methods.

Nondestructive (NDT) Methods

Of the nondestructive techniques, X-ray diffraction (XRD) is the most frequently used method (see Donachie 1962; Norton 1973; Ruud 1979 and others). The XRD technique is based on the principle that the interplanar spacing of the atomic planes within a specimen is changed when subjected to stress. In some materials, such as cold-worked steel, interpretation of the results may be difficult because the cold working causes a change of the

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texture and grain size of the material, and results in a marked effect upon X-ray readings.

Some other nondestructive techniques, such as the electromagnetic method and the neutron diffraction method, are occasionally used for measuring residual stresses. The advantage of the neutron diffraction method is the much greater penetration of neutrons, usually several orders of magnitude greater than the X-rays. However, the application of this technique is usually limited due to the lack of the highly specialized equipment.

A recent innovation in the area of experimental residual stress determination is the ultrasonic technique. Ruud (1981) indicated that the ultrasonic technique holds great promise in that three-dimensional stresses within the volume of components might be measured. This method is based on the observation by Firestone and Frederick (1946) that residual stresses can cause changes in the velocity and attenuation of ultrasonic waves applied to the specimens. Unfortunately, many other characteristics of metals besides elastic strain affect the wave velocity or cause complicating noise. The technology for solving these problems is still under investigation.

Semi-Destructive Methods

Of the semi-destructive techniques, the hole-drilling method is most commonly used. It is described as "semi-destructive" because the hole is usually less than 1/8 in. both in diameter and depth, which may not noticeably impair the structural integrity of the part being tested. This method is based on the fact that drilling a hole in a stress-field locally relaxes the residual stresses, thus resulting in measurable deformations on the surface adjacent to the hole.

The concept of hole-drilling for measuring residual stresses was first introduced by Mathar in 1934. Later, Rendler and Vigness (1966) recognized the practical need to position accurately three strain gages on a known radius for measuring the strains released by hole-drilling. The hole-drilling method was used by Ross and Chen (1975) to measure the residual stresses in a circular tube. More recently, by using commercially available hole-drilling equipment such as that made by Measurements Group Inc., residual stresses have been measured more easily (1983). A standard procedure for the hole-drilling method is given by ASTM ("Determining" 1983). Other semi-destructive techniques, such as the indentation method, the trepaning method, and Gunnert's method, are occasionally used for measuring residual stresses.

Destructive Methods

Most of the mechanical methods for residual stress measurement are destructive, and involve removing portions of metal by cutting, drilling, grinding and etching, etc. The sectioning method (saw-cutting) is the most commonly used destructive technique. It has been used extensively for the past four decades for measuring residual stresses in wide-flange and welded box sections. Results of the measured residual stresses were reported by Huber and Beedle (1954), Beedle and Tall (1960), and Tebedge et al. (1973). Although the sectioning procedure itself is time-consuming and costly, it is one of the most reliable methods for measuring residual stresses. Other destructive techniques, such as the Sachs boring method, the layer removal method, and the chemical etching technique, are also used for measuring residual stresses.

Measurement of Residual Stresses

The test specimen geometries are summarized in Table 1, and material properties for the HY-80 and the HY-100 steel plates as obtained from the uniaxial tensile coupon tests are given in Table 2. Young's modulus and Poisson's ratio obtained for both HY-80 and HY-100 steels are about the same, which are E = 29,500 ksi (203.25 kN/mm^2) and μ = 0.28, respectively.

Residual stresses on the surface of the plate along the bend line were measured by sectioning and by hole-drilling. The purpose of using both methods was to obtain two sets of results from the same plate so that a comparison between them could be made. Residual stresses through the thickness of the plate were measured by sectioning.

Sectioning Method

In the sectioning method, electric resistance strain gages were used to measure the residual strains released by sectioning. One 90° strain gage rosette was applied on each side of the plate surface at the mid-width of the bend line. Figs. 5(a) and (b) show the arrangement of the strain gages and

TABLE 1. Cold-Bent Plate Geometries

Material (1)	Bend radius R (in.) (2)	Bend angle[a] θ° (3)	Number of specimens (4)
HY-80	1.5	90	2
(t = 1.0 in.)	1.5	120	1
	1.5	150	1
	2.5	90	2
	3.5	90	1
	3.5	120	1
	3.5	150	1
	5.5	90	1
	5.5	120	1
	5.5	150	1
HY-80	1.5	90	2
(t = 1.5 in.)	2.5	90	1
HY-100	1.5	90	2
(t = 1.0 in.)	3.5	90	1

[a]See Fig. 3 for definition of θ.
Note: 1 in. = 25.4 mm.

TABLE 2. Material Properties of Steel Plates

Steel (1)	σ_y (ksi) (2)	σ_u (ksi) (3)	E (10^3 ksi) (4)	Elongation (%) (in 2 in.) (5)
HY-80 (1.0 in.)	86.1	107.1	29.6	24.1
HY-80 (1.5 in.)	86.4	108.1	29.3	23.5
HY-100 (1.0 in.)	104.6	118.5	29.5	18.6

Note: 1 in. = 25.4 mm, 1 ksi = 6.89 N/mm^2.

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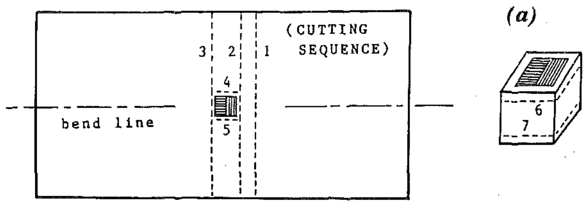

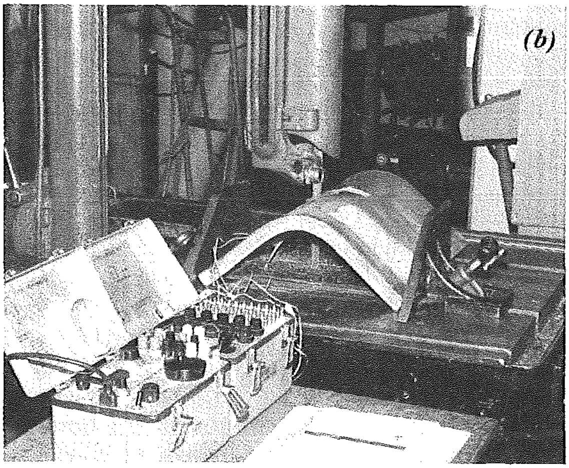

FIG. 5. Sectioning Method: (*a*) Saw-Cutting Sequence for Measuring Surface Residual Strains; (*b*) Sawing 90° Specimen

the saw-cutting of a bent plate for measuring surface residual strains. The sectioning was done by using an automatic-feed band saw with a slow cutting speed and continuous coolant to minimize heating.

In the measurement of residual strains through the plate thickness, strain gages were applied on a 1/2-in. wide strip that was cut out from the bent plate. As shown in Fig. 6, six strain gages were used on the 1-in.-thick plate, and eight on the 1.5-in.-thick plate. The strip was cut twice to release the locked-in residual strains in the tangential direction.

Hole-Drilling Method

For this investigation, an RS 200 high-speed air turbine hole-drilling machine and the 062RE-type strain gage rosette (made by Measurements Group, Inc.) were used. Fig. 7 shows the geometry of the three-gage strain rosette.

The diameter of the drill used in this investigation is 0.062 in. During the test, a drilling-depth increment of 0.005 in. was used. The strains released after each increment of drilling depth were recorded, and the hole diameter after drilling from which the residual stresses can be determined, was measured by a microscope. According to the ASTM standard E837-81 ("Determining" 1983), the maximum depth of the hole is 1.2 times the hole diameter, which results in a nearly complete relaxation of surface strain within the close vicinity of the hole.

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Residual Stresses Obtained by Using Sectioning Method

The residual stresses on the plate surfaces can be calculated from the residual strains measured with the strain gage rosettes by using the following equations:

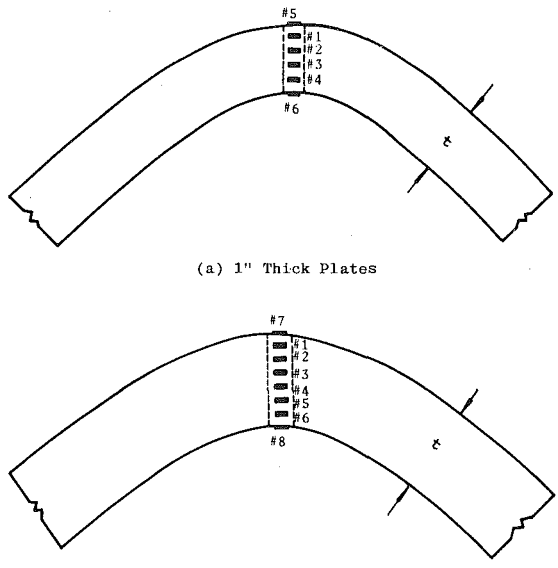

FIG. 6. Locations of Strain Gages for Measuring Residual Strains through Plate Thickness (1 in. = 25.4 mm)

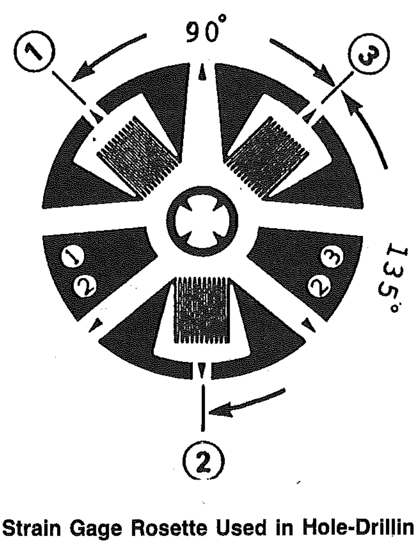

FIG. 7. Strain Gage Rosette Used in Hole-Drilling Method

TABLE 3. Residual Stresses on Plate Surfaces

Plate			RESIDUAL STRESS (ksi)			
			Tension Side		Compression Side	
Material (1)	R (in.) (2)	$\theta°$ (3)	σ_θ (4)	σ_z (5)	σ_θ (6)	σ_z (7)
HY-80	1.5	90(A)	−28.1	+16.7	+69.9	+8.0
(t = 1 in.)	1.5	90(B)	−25.3	+19.2	+66.3	+6.1
	1.5	120	−33.4	+17.5	+54.3	−5.8
	1.5	150	−41.2	+7.8	+52.7	−5.4
	2.5	90(A)	−31.2	+21.3	+62.2	+4.8
	2.5	90(B)	−30.1	+20.1	+61.1	+8.2
	3.5	90	−33.4	+21.5	+52.5	−2.5
	3.5	120	−32.5	+11.0	+50.9	−5.9
	3.5	150	−38.3	+8.8	+42.1	−11.3
	5.5	90	−31.2	+16.6	+46.9	−8.2
	5.5	120	−29.3	+11.4	+46.6	−1.7
	5.5	150	−36.3	+13.3	+39.8	−11.1
HY-80	1.5	90(A)	−32.0	+20.0	+79.2	+19.8
(t = 1.5 in.)	1.5	90(B)	−33.4	+21.7	+75.9	+14.7
	2.5	90	−35.1	+21.2	+70.8	+9.4
HY-100	1.5	90(A)	−27.0	+25.8	+83.3	−13.6
(t = 1 in.)	1.5	90(B)	−29.0	+24.4	+78.4	−12.7
	3.5	90	−33.3	+21.3	+60.1	−5.4

Note: 1 in. = 25.4 mm, 1 ksi = 6.89 N/mm^2.

$$\sigma_\theta = \lambda e + 2G\epsilon_\theta \quad (12)$$

$$\sigma_z = \lambda e + 2G\epsilon_z \quad (13)$$

where ϵ_θ, ϵ_z = measured residual strains in θ- and z-directions; $e = \epsilon_\theta + \epsilon_z$; $\lambda = \mu E/[(1 + \mu)(1 - 2\mu)]$; $G = E/[2(1 + \mu)]$; μ = Poisson's ratio; and E = Young's modulus.

The resulting residual stresses are given in Table 3. It is observed that the magnitude of the tangential residual stresses on the compression side are all larger than those on the tension side, often by a factor of 2 to 3. This observation is important because it indicates that a zone of high-tension residual stress exists in the inside surface of the bent plate, which is considered a possible source of failure due to fatigue cracking or stress corrosion. The calculated tension residual stresses on the inside surface of the bent plate ranged from $0.46\sigma_y$ to $0.92\sigma_y$; corresponding compression residual stresses on the outside surface varied from $0.26\sigma_y$ to $0.48\sigma_y$.

Fig. 8 shows the tangential residual stresses as a function of the bend angle, θ, and the R/t ratio. It is seen that the tension residual stresses on the compression side increase as the R/t ratio decreases, and decrease as the angle of bend increases.

In addition, it is observed that plates with higher yielding strength resulted in higher tension residual stresses on the inside surface of the bend. This can be seen from Table 3, in which the tangential residual stress on the compression side for plates bent to 90° with R = 1.5 in. (38.1 mm) are 69.9

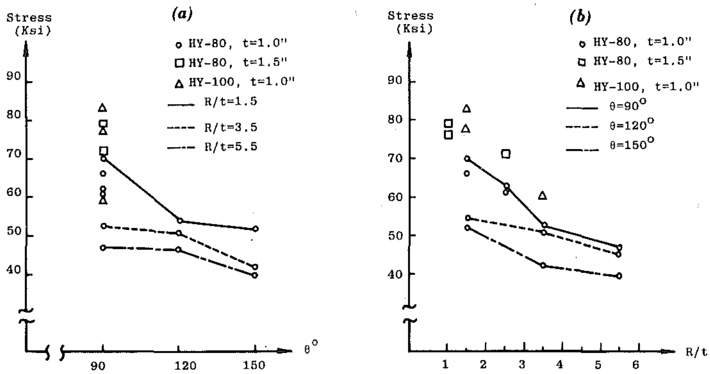

FIG. 8. Tension Residual Stresses on Surface of Compression Side of Bent Plates (1 in. = 25.4 mm, 1 ksi = 6.89 N/mm^2): (*a*) Residual Stress versus Angle of Bend; (*b*) Residual Stress versus *R/T* Ratio

ksi (481.6 N/mm^2) for HY-80 steel plate (t = 1.0 in.) (25.4 mm), and 83.3 ksi (573.9 N/mm^2) for HY-100 steel plate.

Residual Stresses Obtained by Using Hole-Drilling Method

Seven HY-80 steel plates were tested by both hole-drilling and sectioning methods. As shown in Fig. 7, gages 1 and 3 of the three-gage strain rosette are perpendicular to each other, and the angle between gages 2 and 3 is 135°. Since the directions of the principal stresses of the bent plate are known, the strain gage rosette was thus positioned to coincide with the principal directions; i.e., gages 1 and 3 were located in the Z- and θ- directions, respectively. Using the measured residual strains, the following equations ("Determining" 1983) were used to calculate residual stresses:

$$\sigma_\theta = \frac{\epsilon_1 + \epsilon_2}{4A} + \frac{\sqrt{2}}{4B}[(\epsilon_1 - \epsilon_2)^2 + (\epsilon_2 - \epsilon_3)^2]^{1/2} \quad (14)$$

$$\sigma_z = \frac{\epsilon_1 + \epsilon_2}{4A} - \frac{\sqrt{2}}{4B}[(\epsilon_1 - \epsilon_2)^2 + (\epsilon_2 - \epsilon_3)^2]^{1/2} \quad (15)$$

$$\beta = \frac{1}{2}\tan^{-1}\left(\frac{\epsilon_3 - 2\epsilon_2 + \epsilon_1}{\epsilon_3 - \epsilon_1}\right) \quad (16)$$

where ϵ_1, ϵ_2, ϵ_3 = measured strains from gages 1, 2, and 3; σ_θ, σ_z = residual stresses in θ- and Z-direction; and β = angle between the direction of principal stress, σ_θ, and gage 3

$$A = -\left(\frac{1 + \mu}{2E}\right)\cdot\frac{1}{\gamma^2} \quad (17)$$

$$B = -\left(\frac{1 + \mu}{2E}\right)\left(\frac{4}{1 + \mu}\cdot\frac{1}{\gamma^2} - \frac{3}{\gamma^4}\right) \quad (18)$$

TABLE 4. Surface Residual Strains and Stresses Obtained by Using Hole-Drilling Method

Plate (Material: HY-80; t = 1 in.)		RESIDUAL STRAIN ($\mu\epsilon$)						RESIDUAL STRESSES (ksi)					
		Tension Side			Compression Side			Tension Side			Compression Side		
R (in.) (1)	$\theta°$ (2)	ϵ_1 (3)	ϵ_2 (4)	ϵ_3 (5)	ϵ_1 (6)	ϵ_2 (7)	ϵ_3 (8)	σ_θ (9)	σ_z (10)	$\beta°$ (11)	σ_θ (12)	σ_z (13)	$\beta°$ (14)
1.5	120	−317	+78	+423	+227	−260	−661	−31.2	+13.8	−0.03	+62.7	+8.5	−0.05
3.5	90	−351	+72	+428	+308	−147	−638	−31.2	+18.0	−0.04	+60.4	−1.4	+0.02
3.5	120	−312	+101	+417	+219	−193	−562	−30.9	+13.7	−0.07	+56.2	+5.1	−0.03
3.5	150	−307	+83	+441	+312	−95	−491	−35.0	+12.1	−0.02	+40.6	−9.9	−0.01
5.5	90	−204	+87	+332	+250	−184	−492	−29.0	+6.1	−0.04	+44.4	−3.0	−0.08
5.5	120	−217	+41	+334	+252	−70	−464	−27.4	+7.6	+0.03	+42.5	−4.5	+0.05
5.5	150	−316	+94	+449	+264	−80	−424	−34.2	+12.4	−0.04	+36.8	−8.2	+0.00

Note: 1 in. = 25.4 mm, 1 ksi = 6.89 N/mm^2.

$\gamma = D/D_0$; D = diameter of gage circle; and D_0 = diameter of the drilled hole.

The measured residual strains and the corresponding residual stresses are given in Table 4. Since gages 3 and 1 were positioned along the directions of principal stresses, the calculated values of β are nearly zero.

Comparison of Results from Two Methods

The surface tangential residual stresses obtained by the sectioning method and by the hole-drilling method are compared in Table 5. The differences between the compression side tangential residual stresses obtained by these two methods ranged from −8.8% to +15.5%. When the magnitude of the

TABLE 5. Comparison between Tangential Surface Residual Stresses Obtained by Sectioning and Hole-Drilling Methods

Plate (Material: HY-80; t = 1 in.)		TANGENTIAL RESIDUAL STRESSES (ksi)					
		Sectioning		Hole-Drilling		Difference (%)	
R (in.) (1)	$\theta°$ (2)	T (3)	C (4)	T (5)	C (6)	T (7)	C (8)
1.5	120	−33.4	+54.3	−31.2	+62.7	−6.6	+15.5
3.5	90	−33.4	+52.5	−31.2	+60.4	−6.7	+15.1
3.5	120	−32.5	+50.9	−30.9	+56.2	−4.9	+10.4
3.5	150	−38.3	+42.1	−35.0	+40.6	−8.6	−3.7
5.5	90	−31.2	+46.9	−29.0	+44.4	−7.1	−5.3
5.5	120	−29.3	+46.6	−27.4	+42.5	−6.5	−8.8
5.5	150	−36.3	+39.8	−34.2	+36.8	−5.8	−7.5

Note: difference = $(S - H)/H \times 100\%$; S = residual stress obtained by sectioning method; H = residual stress obtained by hole-drilling method; T = tension side; C = compression side; 1 in. = 25.4 mm; 1 ksi = 6.89 N/mm^2.

tangential residual stress is less than 50 ksi (345 N/mm^2) (about 60% of the yield stress of the HY-80 steel), the results obtained by the hole-drilling method are smaller than those obtained by the sectioning method, and the differences between the results obtained by these two methods are less than 9%.

When the measured residual stresses are higher than about 60% of the yielding stress of the material, the results obtained by the hole-drilling method give higher values than those obtained by the sectioning method, and the differences ranged from 10% to 16%. The reason that the hole-drilling method gives higher values when residual stresses are larger than 60% of the yielding stress may be ascribed to the possible effect of plasticity around the area adjacent to the drilled hole due to the stress concentration.

Residual Stresses through Plate Thickness

Based on the results of the measured residual strains through the plate thickness, it was observed that the maximum released residual strain occurred either at the inside surface of the strip, or near the neutral surface of the plate. No measured strain is higher than the yield strain of the material. The distribution of the released residual strains across the plate thickness has a pattern similar to that shown in the sketch at the right in Fig. 2. Selected calculated residual stresses for HY-80 steel plates with 90° bends are given in Table 6. Typical residual stress distribution patterns through the plate thickness are shown in Figs. 9(*a–c*) for HY-80 steel plates with $R/t = 1.5$ and $\theta = 90°$, 120°, and 150°.

Using the calculated residual stresses through the thickness of the cold-bent plate, moment equilibrium was applied to see how well the required condition of zero net moment is met. The results show that the sum of the calculated internal positive and negative moments is not zero, and differences of 13.4%, 17.8%, and 10.2% were found for the plates shown in Figs. 9(*a–c*), respectively. The discrepancies between the calculated results and the required condition of net zero moment may be mainly due to the fact that only four strain gages were applied on the thickness direction of the

TABLE 6. Residual Stresses through Plate Thickness: Test Results versus Predicted Values (HY-80 Steel, t = 1.0 in., θ = 90°)

R (in.) (1)	θ (degrees) (2)	Type (3)	Residual Stresses (ksi) #1 (4)	#2 (5)	#3 (6)	#4 (7)	#5 (8)	#6 (9)
1.5	90(A)	Measured	+9.0	+45.2	−35.0	−28.7	−28.1	+69.9
1.5	90(A)	Calculated	+9.8	+43.1	−78.4	−8.0	−18.0	+81.0
2.5	90(A)	Measured	+16.5	+50.2	−37.9	−22.4	−31.2	+62.2
2.5	90(A)	Calculated	+10.4	+49.3	−70.4	−6.6	−24.1	+67.3
3.5	90	Measured	+8.0	+54.4	−35.7	−22.6	−33.4	+52.5
3.5	90	Calculated	+13.3	+55.0	−63.7	−3.4	−24.7	+63.9
5.5	90	Measured	+10.0	+36.5	−43.1	−11.7	−31.2	+46.9
5.5	90	Calculated	+15.1	+60.0	−58.9	−1.9	−26.7	+59.3

Note: Predicted values are calculated by using Dat's equations; 1 in. = 25.4 mm, 1 ksi = 6.89 N/mm^2.

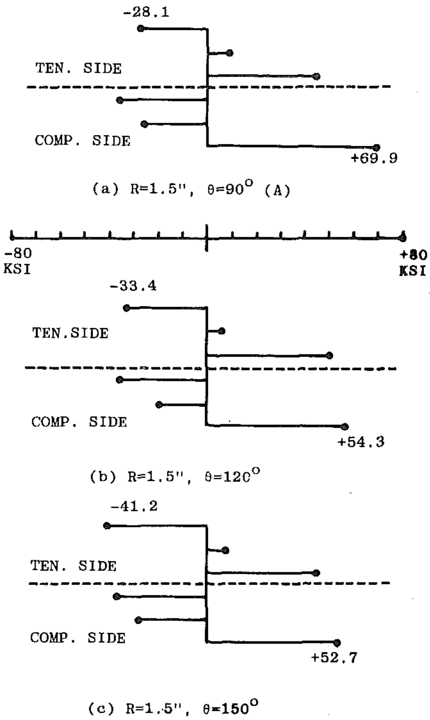

FIG. 9. Residual Stresses through Plate Thickness—HY-80 Steel, t = 1.0 in. (1 in. = 25.4 mm, 1 ksi = 6.89 N/mm^2)

plate (t = 1 in.), which gave released residual strains at those four points only, thus precluding an accurate prescription of the true stress distribution.

Comparison between Test Results and Predicted Values

Residual stresses through the plate thickness, as predicted by Dat's equations, are shown in Table 6 for HY-80 steel plates with $\theta = 90°$. Test results and the predicted values for tangential surface strain are then compared as shown in Table 7. The comparison shows that the surface residual stresses predicted by Dat's equations are generally higher on the compression side and smaller on the tension side than those obtained from the experiment. For plates bent to 90°, the differences of the surface residual stresses on the compression side ranged between 8% and 36%. Larger discrepancies are observed for plates bent to 120° and 150°.

Reviewing Dat's approach, it is noted that the effect of the strain hardening is not taken into account, and the bent portion is assumed to be fully plastic after bending. The former assumption is not appropriate for HY-80 and HY-100 steel, since both materials show significant strain hardening. The latter assumption is satisfactory for plates bent to 90°, but not for plates bent to 120° and 150°, which explains why larger discrepancies are observed in the cases of plates bent to larger angles. However, the pattern of the

TABLE 7. Tangential Residual Stresses on Plate Surfaces: Test Results versus Predicted Values

Plate			Test (ksi)		Predicted[a] (ksi)		Difference (%)	
Material (1)	R (in.) (2)	θ° (3)	T (4)	C (5)	T (6)	C (7)	T (8)	C (9)
HY-80	1.5	90(A)	−28.1	+69.9	−18.0	+81.0	−35.9	+15.9
(t = 1 in.)	1.5	90(B)	−25.3	+66.3	−18.3	+80.6	−27.7	+21.6
	1.5	120	−33.4	+54.3	−19.1	+79.4	−42.8	+46.2
	1.5	150	−41.2	+52.7	−22.3	+73.5	−45.9	+39.5
	2.5	90(A)	−31.2	+62.6	−24.1	+67.3	−22.8	+8.2
	2.5	90(B)	−30.1	+61.6	−23.1	+68.4	−23.3	+11.0
	3.5	90	−33.4	+52.5	−24.7	+63.9	−26.0	+21.7
	3.5	120	−32.5	+50.9	−26.8	+61.9	−17.5	+21.6
	3.5	150	−38.3	+42.1	−27.4	+61.2	−28.5	+45.4
	5.5	90	−31.2	+46.9	−26.7	+59.3	−14.4	+26.4
	5.5	120	−29.3	+46.6	−29.0	+57.5	−1.0	+23.4
	5.5	150	−36.3	+39.8	−30.1	+56.6	−17.1	+42.2
HY-80	1.5	90(A)	−32.0	+79.2	−9.1	+102.6	−71.6	+29.5
(t = 1.5 in.)	1.5	90(B)	−33.4	+75.9	−8.7	+103.2	−74.0	+36.0
	2.5	90	−35.1	+70.8	−15.6	+82.0	−55.6	+15.8
HY-100	1.5	90(A)	−27.0	+83.3	−21.7	+99.6	+19.6	+16.9
(t = 1 in.)	1.5	90(B)	−29.0	+78.4	−21.7	+99.6	+25.2	+27.0
	3.5	90	−33.3	+60.1	−31.1	+77.4	+6.6	+28.8

[a]Residual stress predicted by Dat's equations; T: surface residual stress on tension side; C: surface residual stress on compression side.
Note: 1 in. = 25.4 mm; 1 ksi = 6.89 N/mm^2.

distribution of the residual stress across the plate thickness is the same for both the test results and the predicted values.

The principal advantage of Dat's equation is its simplicity, since a closed-form solution is achieved. To account for the strain hardening effect and the influence of bend angle on the residual stresses in the cold-bent plate, a numerical approach such as the finite element technique is recommended.

Summary and Conclusions

The test results showed the following.

1. The magnitude of the measured tension residual stress on the inside surface of the bend ranged between 46% and 92% of the yield stress of the material. The highest value occurred when the plate was bent to 90° with a bend radius of 1.5 in.
2. The tension residual stress on the inside surface of the bend increases as the R/t ratio decreases, the angle of bend decreases, and the yield strength of the material increases.
3. A zigzag-type residual stress distribution through the plate thickness was obtained from the released residual strain measurements. The maximum residual stress was found either at the inside surface of the bend, or near the neutral surface of the plate.

4. The residual stresses obtained from the hole-drilling method and the sectioning method agreed well when the magnitude of the residual stress was less than about 60% of the yield stress of the material.

5. The experimental findings obtained from this research are useful for future investigations, particularly for the study of possible stress corrosion susceptibility and fatigue strength of cold-bent steel plates.

Appendix I. References

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Appendix II. Notation

The following symbols are used in this paper:

a = radius to inside surface of bend;
b = radius to outside surface of bend;
c = radius to neutral surface of bend;
D = diameter of strain gage circle;
D_0 = diameter of drilled hole;
E = Young's modulus;
G = shear modulus;
M = applied bending moment;
p = applied internal pressure;
R = radius of bend before springback;
r = radius to arbitrary point in bend;
t = plate thickness;
γ = D/D_0;
$\epsilon_\theta, \epsilon_z$ = residual strains in θ- and z-directions;
θ = angle of bend before springback;
μ = Poisson's ratio;
σ_u = yield stress of material; and
σ_y = tensile strength of material.

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