國立交通大學

環境工程研究所

碩士論文

含井膚層水層在有限邊界條件下
井緣流量解與洩降解之研究

A study on Wellbore Flow-rate Solution and Drawdown Solution for a Finite Confined Aquifer with Considering the Effect of Skin Zone

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中 華 民 國 九 十 八 年 四 月

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摘要

工程上，定水頭試驗及定流量試驗的數據通常被用來推估含水層的參數。定水頭試驗是藉由注水或抽水產生固定水頭，進而量測井緣流量，通常是應用在低透水性的水層；定流量試驗則是藉由固定抽水量，在觀測井量測洩降的分佈值，此試驗適合用於透水性高的水層。在過去的文獻中，已有單層或含井膚層水層的解析解，得知當時間越久或觀測井距離試驗井越遠的情況下，井膚層的效應很小可忽略不計。但是，當外邊界為有限值的情況下，此問題對於井緣流量解和洩降解的影響，較少被討論。另一方面，由於此問題相關的解析解，形式複雜且不易計算數值，因此，近似解是值得探討的議題，過去的文獻顯示，利用拉普拉斯域變數與時間域變數成反比的關係，可自拉普拉斯解求得近似解。本論文目的為討論有限邊界的定流量試驗及定水頭試驗問題，利用拉普拉斯轉換，分別求得洩降及井緣流量的半解析解，再利用 Crump 數值逆轉方法，求得時間域的數值。此外，本文探討有限邊界對於井緣流量解及洩降解的影響，也考慮於遠處邊界為無

限或有限值的情況下，分別推導得含井膚層水層在長時間條件下的洩降及井緣流量近似解。所得結果顯示，當遠處邊界為無限時，井緣流量解、洩降解及此兩者的近似解經過簡化，可分別得到先前單層含水層的解。

關鍵詞：地下水，半解析解，定流量試驗，定水頭試驗，拉普拉斯轉換，有限邊界

A study on wellbore flow-rate solution and drawdown solution for a finite confined aquifer with considering well skin effect

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Abstract

The constant-head test and constant-flux test are commonly employed for estimating the aquifer parameters in engineering practice. The constant-head test injects or pumps water with a variable flow rate for maintaining a constant hydraulic head in a low-permeability aquifer while the constant-flux test keeps a constant flow rate to record the drawdown distribution from the observation well of a high-permeability aquifer. The solutions for the wellbore flow rate and drawdown at a well with a finite radius in an infinite confined aquifer with or without a skin zone have been reported in the groundwater literature. The effects of well radius and skin zone are negligible if the test period is very long and/or the distance between the observation well and test well is large. However, little attention has been paid to the effect of a finite boundary on the flow-rate and drawdown solutions in the

groundwater community. The main objectives of this thesis are first to develop new semi-analytical solutions for exploring the effect of finite boundary on the wellbore flow-rate and drawdown solutions in a confined aquifer where a finite skin zone is present. These solutions are then calculated by the modified Crump algorithm. The Laplace-domain solution can reduce to the existing infinite-domain solution in some special cases. In addition, an approximate solution for small- or large-time condition is useful if the analytical solution is very complicated and not easy to evaluate accurately. The second objective of this thesis is to derive approximate solutions with considering the effect of skin zone in a finite or infinite confined aquifer based on the relationship between the Laplace variable and time. An approximate solution for an infinite confined aquifer with a skin zone can reduce to the solution without a skin zone if the skin is absent. The large-time solution is equal to the steady-state solution for a finite confined aquifer with a skin zone. In addition, this solution can reduce to Thiem's equation if the skin zone is absent.

Keywords: Groundwater, constant-flux test, constant-head test, Laplace transform, finite confined aquifer.

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NOTATION

h	Hydraulic head
$\bar{h}$	Hydraulic head in Laplace domain
h_w.	Hydraulic head along the wellbore
$I_0(u), K_0(u)$	Modified Bessel function of the first and second kinds of order zero
$I_1(u), K_1(u)$	Modified Bessel function of the first and second kinds of order one
p	Laplace variable
q	$\sqrt{pS/T}$
$Q(r_w)$	Constant flow rate into or out wellbore
$\bar{Q}_{DW}$	Dimensionless constant flow rate in Laplace domain
$\bar{Q}(r_w)$	Constant flow rate in Laplace domain
r	Radial distance from the central line of the test well
r_1	Thickness of the skin zone
r_w	Well radius
R	Radial distance from the center of the well to the finite extended boundary
s	Drawdown
$\bar{s}$	Drawdown in Laplace domain
S	Storage coefficient

t	Time from the begin of test
T	Transmissivity
α	T_2 / T_1
β	S_2 / S_1
Φ_1	$\psi\sqrt{\frac{S_2T_2}{S_1T_1}}K_0(q_1r_1)K_0(q_2r_1)-K_1(q_1r_1)K_0(q_2r_1)$
Φ_2	$\psi\sqrt{\frac{S_2T_2}{S_1T_1}}I_0(q_1r_1)K_0(q_2r_1)+I_1(q_1r_1)K_0(q_2r_1)$
ϕ_1	$\sqrt{\frac{S_2T_2}{S_1T_1}}K_0(q_1r_1)K_1(q_2r_1)-K_1(q_1r_1)K_0(q_2r_1)$
ϕ_2	$\sqrt{\frac{S_2T_2}{S_1T_1}}I_0(q_1r_1)K_1(q_2r_1)+I_1(q_1r_1)K_0(q_2r_1)$
$\Gamma(x)$	Gamma function
γ	0.5772... Euler's constant
η	λt
λ	$T_2(r_1/r_2)^{2T_2/T_1} / r_w^2 S_2$
ρ	r / r_w
ρ_1	r_1 / r_w
ρ_R	R / r_w
τ	Dimensionless time
τ_c	Time criterion

ϖ $\left[-K_0(q_2R)I_0(q_2r)+K_0(q_2r)I_0(q_2R)\right]/\left[I_0(q_2R)K_0(q_2r_1)-I_0(q_2r_1)K_0(q_2R)\right]$

ξ(3) Riemann Zeta function

Subscripts

D Dimensionless

1 skin zone

2 Formation zone

w Pumping well

CHAPTER 1 INTRODUCTION

1.1 Background

The constant-head and constant-flux tests are normally utilized to obtain wellbore flow-rate or drawdown data for estimating the hydraulic properties (e.g., transmissivity and storage coefficient). The constant-head test, which injects or pumps water with a variable flow rate by keeping a constant hydraulic head at the test well, is commonly performed in low-permeability aquifers. On the other hand, the constant-flux test maintains a constant well discharge rate to observe the various drawdown distributions in a confined aquifer with high-permeability.

1.2 Literature Review

A finite thickness of the skin zone may exist near the well due to well drilling, installation, and development. Several researchers had addressed the issue of constant-head test for aquifers without skin and presented mathematical models for engineering applications. Yang and Yeh (2002) proposed a closed-form flow-rate solution at the wellbore for describing the flow rate with considering the effects of skin zone and finite well radius. They mentioned that the formation zone and skin zone may significantly affect the magnitude of the flow rate at the wellbore.

Yeh and Wang (2007) presented a new large-time wellbore flow-rate solution for the problem of constant-head tests based on the relationship of small Laplace-domain variable p versus large time-domain variable t (hereinafter referred to as SPLT). Their approximate solution was obtained for the confined aquifer of homogeneous, isotropic, infinitely extended in lateral, and without considering the skin zone. Furthermore, Wang and Yeh (2008) gave a review on the constant-head solution. They mentioned that the finite-domain drawdown solution rather than the infinite-domain drawdown solution can converge to the Thiem equation when the time becomes infinitely large. Nevertheless, to our knowledge, the issue of the effect of finite boundary on the wellbore flow-rate solution has not been address so far. Theis equation (Theis 1935) is the most famous solution for estimating the aquifer drawdown or determining the aquifer parameters for the constant-flux test. Theis equation is applicable to an aquifer with an infinitesimal diameter well when the observation well is located far away from the test well and/or the test period is very large. However, this equation is not applicable when the skin zone is present. The drawdown equation for the test well with a finite radius can be derived based on the analogy of the heat flow equation presented in Carslaw and Jaeger (1959). Moreover, this drawdown equation can be simplified to the Theis equation if the well radius is negligible. Yeh et al. (2003) proposed a closed-form drawdown solution for flow toward a well in an infinite confined aquifer under pumping by taking into account the effects of well radius and skin zone. They concluded that the effect of skin zone may be negligible at short and long pumping periods.

For the constant-flux test, Chen (1984) proposed a modified Theis equation for describing the drawdown distributions with considering the effect of boundary in a finite extended confined aquifer. He gave a boundary-effect time criterion for applying the Theis equation in a finite extended confined aquifer. In other words, the Theis equation is applicable before the influence of pump-induced drawdown reaching the finite extended boundary. Similar to the development of Chen (1984), the drawdown solution with considering both finite well radius and finite extended boundary can also be obtained according to Carslaw and Jaeger (1959, p.334). The issue of the effect of finite boundary on the drawdown solution so far has attracted little attention in the groundwater literature.

1.3 Objectives

This thesis develops two new solutions for the constant-head and constant-flux tests with the finite skin zone in a finite confined aquifer. The objectives of this thesis consist of the following:

(1) To extend previous works of Yang and Yeh (2002) and Yeh et al. (2003) for finite extended confined aquifers.

(2) To study the effect of finite boundary on the solution in a confined aquifer with a

skin zone

(3) To present the time criteria for using the infinite-domain solution to approximate the finite-domain solution.

(4) To derive large-time solutions for aquifers with considering the well radius and skin zone as well as finitely and infinitely extended boundaries.

CHAPTER 2 METHEMATICL MODEL

This chapter presents the mathematical models for the constant-head and constant-flux tests in a finite extended confined aquifer.

2.1 The Mathematical Model for the Constant-head Test

This section establishes the mathematical model for the constant-head test. The assumptions for model development are made as: (1) the aquifer is homogeneous, isotropic, and finite extended; (2) the test well is fully penetrated with a finite well radius; (3) the skin has a finite thickness with properties differing from those of the formation zone ; and (4) the water level (or hydraulic head) at the test well is kept constant throughout the whole test period. Figure 1 demonstrates the schematic diagram of the aquifer for the constant-head test.

The governing equation describing the hydraulic head $h(r, t)$ for the skin zone and the formation zone are, respectively

$$\frac{\partial^2 h_1}{\partial r^2} + \frac{1}{r}\frac{\partial h_1}{\partial r} = \frac{S_1}{T_1}\frac{\partial h_1}{\partial t} \qquad r_w \le r < r_1 \tag{1}$$

and

$$\frac{\partial^2 h_2}{\partial r^2} + \frac{1}{r}\frac{\partial h_2}{\partial r} = \frac{S_2}{T_2}\frac{\partial h_2}{\partial t} \qquad r_1 \le r \le R \tag{2}$$

where subscripts 1 and 2 respectively denote the skin zone and the formation zone, the variable r is the radial distance from the center of the test well, r_w is the well radius, r_1 is the radial distance from the central line of the test well to outer skin zone adjacency,

R is the radial distance from the center of the well to the finite extended boundary, t is the test time, S indicates the storage coefficient, and T is the transmissivity.

The initial hydraulic head is assumed as zero everywhere. Therefore, the initial conditions for Equations 1 and 2 are

$$h_1(r,0) = h_2(r,0) = 0 \tag{3}$$

In addition to the initial conditions, the hydraulic head is also assumed to maintain zero when r approaches to the finite distance R from the test well while the hydraulic head along the wellbore is maintained constant and denoted as h_w. Thus, the outer and inner boundary conditions can be expressed, respectively, as

$$h_2(R,t) = 0 \tag{4}$$

and

$$h_1(r_w,t) = h_w \tag{5}$$

The continuity conditions for the hydraulic head and flow rate at the interface between the skin zone and formation zone, respectively, require

$$h_1(r_1,t) = h_2(r_1,t) \tag{6}$$

and

$$T_1 \frac{\partial h_1(r_1,t)}{\partial r} = T_2 \frac{\partial h_2(r_1,t)}{\partial r} \tag{7}$$

2.2 The Hydraulic Head Solution for the Constant-head Test in a Finite Confined Aquifer

The detailed derivation of the hydraulic head solution for the constant-head test in a finite confined aquifer is given in Appendix A. The results are

$$\bar{h}_1 = \frac{h_w}{p}\left[\frac{\Phi_1 I_0(q_1 r) - \Phi_2 K_0(q_1 r)}{\Phi_1 I_0(q_1 r_w) - \Phi_2 K_0(q_1 r_w)}\right] \tag{8}$$

and

$$\bar{h}_2 = \frac{h_w \varpi}{p}\left[\frac{\Phi_1 I_0(q_1 r) - \Phi_2 K_0(q_1 r)}{\Phi_1 I_0(q_1 r_w) - \Phi_2 K_0(q_1 r_w)}\right] \tag{9}$$

with

$$\varpi = \frac{K_0(q_2 r) I_0(q_2 R) - K_0(q_2 R) I_0(q_2 r)}{I_0(q_2 R) K_0(q_2 r_1) - I_0(q_2 r_1) K_0(q_2 R)} \tag{10}$$

where p is the Laplace variable, $q_1 = \sqrt{pS_1/T_1}$, $q_2 = \sqrt{pS_2/T_2}$, I_0 and K_0 are the modified Bessel functions of the first and second kinds of order zero, respectively, and I_1 and K_1 are the modified Bessel functions of the first and second kinds of order one, respectively. The variables Φ_1 and Φ_2 in Equations (8) and (9) are, respectively,

$$\Phi_1 = \psi\sqrt{\frac{S_2 T_2}{S_1 T_1}} K_0(q_1 r_1) K_0(q_2 r_1) - K_1(q_1 r_1) K_0(q_2 r_1) \tag{11}$$

and

$$\Phi_2 = \psi\sqrt{\frac{S_2 T_2}{S_1 T_1}} I_0(q_1 r_1) K_0(q_2 r_1) + I_1(q_1 r_1) K_0(q_2 r_1) \tag{12}$$

with

$$\psi = \frac{I_0(q_2R)K_1(q_2r_1)+I_1(q_2r_1)K_0(q_2R)}{I_0(q_2R)K_0(q_2r_1)-I_0(q_2r_1)K_0(q_2R)} \tag{13}$$

The wellbore flow-rate solution in Laplace-domain can be obtained by applying Darcy's law and allowing r=r_w as

$$\overline{Q}(r_w,p)=2\pi r_wT_1\frac{q_1h_w}{p}\left[\frac{\Phi_1I_1(q_1r_w)+\Phi_2K_1(q_1r_w)}{\Phi_2K_0(q_1r_w)-\Phi_1I_0(q_1r_w)}\right] \tag{14}$$

The ψ in Equation (13) and ϖ in Equation (10) may be simplified as $K_1(q_2r_1)/K_0(q_2r_1)$ and $K_0(q_2r)/K_0(q_2r_1)$, respectively, if the finite extended boundary R becomes infinite. Equation (14) then reduces to the solution presented in Yang and Yeh (2002,) as follows

$$\overline{Q}(r_w,p)=2\pi r_wT_1\frac{q_1h_w}{p}\left[\frac{\phi_1I_1(q_1r_w)+\phi_2K_1(q_1r_w)}{\phi_2K_0(q_1r_w)-\phi_1I_0(q_1r_w)}\right] \tag{15}$$

with

$$\phi_1=\sqrt{\frac{S_2T_2}{S_1T_1}}K_0(q_1r_1)K_1(q_2r_1)-K_1(q_1r_1)K_0(q_2r_1) \tag{16}$$

$$\phi_2=\sqrt{\frac{S_2T_2}{S_1T_1}}I_0(q_1r_1)K_1(q_2r_1)+I_1(q_1r_1)K_0(q_2r_1) \tag{17}$$

which is identical to the equation for the wellbore flux in an aquifer of infinite extend. Clearly, the infinite-domain solution is much easier to compute than the finite-domain solution. In addition, when the time is less than a specific time, called the boundary-effect time criterion or simply time criterion and defined as $\tau_c = T_2t_c/S_2r_w^2$, the difference between the dimensionless finite-domain solution and dimensionless infinite-domain solution is less than 10^{-5}. In other words, the boundary has no effect on the solution before time reaches the time criterion.

2.3 Dimensionless Wellbore Flow-rate Solution for the Constant-head Test

Define the dimensionless variables as $\alpha = T_2/T_1$, $\beta = S_2/S_1$, $\kappa = \sqrt{\beta/\alpha}$, $\tau = T_2 t/S_2 r_w^2$, $\rho = r/r_w$, $\rho_1 = r_1/r_w$, $\rho_R = R/r_w$, $\overline{Q}_{DW} = \overline{Q}_W /(2\pi T_2 h_w)$. Note that variable α represents the skin type and the aquifer has a negative skin when α <1 and positive skin when α >1. The dimensionless wellbore flow-rate solution in the Laplace-domain Equation, (18), can be expressed as

$$\overline{Q}_{DW} = \sqrt{\frac{1}{\alpha\beta p}} \frac{\Phi_{1D} I_1(\sqrt{p}/\kappa) + \Phi_{2D} K_1(\sqrt{p}/\kappa)}{\Phi_{2D} K_0(\sqrt{p}/\kappa) - \Phi_{1D} I_0(\sqrt{p}/\kappa)} \tag{18}$$

The variables Φ_{1D} and Φ_{2D} are

$$\Phi_{1D} = \overline{\psi}_D \sqrt{\alpha\beta} K_0\left(\sqrt{p}\rho_1/\kappa\right) K_0\left(\rho_1\sqrt{p}\right) - K_1\left(\sqrt{p}\rho_1/\kappa\right) K_0\left(\rho_1\sqrt{p}\right) \tag{19}$$

and

$$\Phi_{2D} = \overline{\psi}_D \sqrt{\alpha\beta} I_0\left(\sqrt{p}\rho_1/\kappa\right) K_0\left(\rho_1\sqrt{p}\right) + I_1\left(\sqrt{p}\rho_1/\kappa\right) K_0\left(\rho_1\sqrt{p}\right) \tag{20}$$

with

$$\overline{\psi}_D = \frac{I_0\left(\rho_R\sqrt{p}\right) K_1\left(\rho_1\sqrt{p}\right) + I_1\left(\rho_1\sqrt{p}\right) K_0\left(\rho_R\sqrt{p}\right)}{I_0\left(\rho_R\sqrt{p}\right) K_0\left(\rho_1\sqrt{p}\right) - I_0\left(\rho_1\sqrt{p}\right) K_0\left(\rho_R\sqrt{p}\right)} \tag{21}$$

The time-domain wellbore flow rate for an aquifer with a finite extended boundary can be evaluated from Equation (18) by using the modified Crump algorithm (de Hoog et al., 1982), a numerical Laplace inversion method. The results obtained after numerical inversion can be compared with the infinite-domain flow-rate solutions presented by Yang and Yeh (2002) in which they considered the aquifer had an infinite extended boundary. Note that Equation (21) reduces to $K_1\left(\rho_1\sqrt{p}\right)/K_0\left(\rho_1\sqrt{p}\right)$ if $\rho_R \to \infty$. The dimensionless flow-rate solution for an infinite domain aquifer in Laplace-domain given by Yang and Yeh (2002) was expressed as

$$\overline{Q}_{DW} = \sqrt{\frac{1}{\alpha p}} \frac{\phi_{1D} I_1(\sqrt{p/\kappa}) + \phi_{2D} K_1(\sqrt{p/\kappa})}{\phi_{2D} K_0(\sqrt{p/\kappa}) - \phi_{1D} I_0(\sqrt{p/\kappa})} \tag{22}$$

with

$$\phi_{1D} = \sqrt{\alpha\beta} K_0\left(\sqrt{p}\rho_1/\kappa\right) K_1\left(\rho_1\sqrt{p}\right) - K_1\left(\sqrt{p}\rho_1/\kappa\right) K_0\left(\rho_1\sqrt{p}\right) \tag{23}$$

and

$$\phi_{2D} = \sqrt{\alpha\beta} I_0\left(\sqrt{p}\rho_1/\kappa\right) K_1\left(\rho_1\sqrt{p}\right) + I_1\left(\sqrt{p}\rho_1/\kappa\right) K_0\left(\rho_1\sqrt{p}\right) \tag{24}$$

Obviously, the flow-rate solution for an aquifer with an infinite-domain is in a simpler form and much easier to evaluate than the one with a finite-domain.

2.4 The Large-time Solution for the Constant-head Test

2.4.1 In a Finite Confined Aquifer

The large-time flow-rate solution at the wellbore can be evaluated from Equation (14) by utilizing the SPLT technique. The limiting forms of the Bessel functions for small arguments used for computing Equation (25) are $I_0(x) \sim 1/\Gamma(1)$, $I_1(x) \sim x/2\Gamma(2)$, $K_0(x) \sim -\ln(x)$, and $K_1(x) \sim 1/x$ where $\Gamma(x)$ is the gamma function (Abramowitz and Stegun 1970, p.375). Applying L'Hospital's rule to Equation (14) with p approaching zero, the Laplace-domain wellbore flow rate for small p gives

$$\bar{Q}(r_w, p) = \frac{2\pi T_1 h_w}{p} \frac{-1}{\ln\left(\frac{r_1}{r_w}\right) + \frac{T_1}{T_2}\ln\left(\frac{R}{r_1}\right)} \tag{25}$$

where the negative sign in Equation (25) expresses withdrawal at the test well.

Accordingly, the large-time wellbore flow-rate solution can be easily obtained by taking the inverse Laplace transform to Equation (25) as

$$Q(r_w, t) = 2\pi T_1 h_w \frac{-1}{\ln\left(\frac{r_1}{r_w}\right) + \frac{T_1}{T_2}\ln\left(\frac{R}{r_1}\right)} \tag{26}$$

Equation (26) is independent of time and naturally is a steady-state solution. By applying the Tauberian theory to Equation (14) (Sneddon, 1972), one can also obtain Equation (26). This result implies that the wellbore flow rate does approach steady state at large time condition for an aquifer with a finite-domain (Wang and Yeh, 2008). In addition, Equation (26) can be simplified to the Thiem equation if the skin zone is absent (i.e., r_1 equals r_w).

2.4.2 In an Infinite Confined Aquifer

If an infinite extended boundary is considered, i.e., $R \to \infty$, Equation (14) therefore is equivalent to Equation 15. Again, the Laplace-domain wellbore flow rate for small p in an infinite confined aquifer can be obtained as

$$\overline{Q}(r_w, p) = \frac{-4\pi T_2 h_w}{p} \frac{1}{\ln(p/\lambda)} \tag{27}$$

where $\lambda = T_2 (r_1/r_2)^{2T_2/T_1} / r_w^2 S_2$. If there is no skin zone, then $T_2 = T_1$ and $S_2 = S_1$; Equation (27) is, therefore, identical to the one of Yeh and Wang (2007, Equation 3).

The large-time wellbore flow-rate solution is obtained after taking the inverse Laplace transform to Equation (27) as (Yeh and Wang, 2007)

$$Q(r_w, t) = 4\pi T_2 h_w \left[\frac{1}{\ln\eta} - \frac{\gamma}{(\ln\eta)^2} + \frac{\gamma^2 - \frac{\pi^2}{6}}{(\ln\eta)^3} - \frac{\gamma^3 - \frac{\pi^2}{2}\gamma + 2\xi(3)}{(\ln\eta)^4} \right] \tag{28}$$

where $\gamma = 0.5772...$ is Euler's constant and the Riemann Zeta function $\xi(3)$ = 1.2020569032. The numerators of the right-hand side terms of Equation (28) are all constants and the denominators are a function of time since $\eta = \lambda t = T_2 (r_1/r_2)^{2T_2/T_1} t / r_w^2 S_2$. The value of $\ln\eta$ tends to infinity as t approaches infinity; consequently, Equation (28) becomes zero. It means that the steady-state wellbore flow rate in an infinite confined aquifer is zero implying the constant head has reached to the infinite extended boundary.

When neglecting the skin zone, Equation (28) can reduce to the solution of Yeh and Wang (2007, Equation 6). Table 1 provides a list for comparing Equations (26) and (28) with two existing solutions.

2.5 Drawdown Solution for Constant-flux Test

This section presents the mathematical model for the constant-flux test. The assumptions for the constant-flux test are the same as those for the constant-head test except that there is a constant discharge rate, rather than a constant head, maintained at the wellbore through out the entire pumping test. Therefore, the mathematical model describing the constant-flux test is identical to the constant-head test except the boundary condition specified at the wellbore. The boundary condition specified for a constant pumping with the flow rate Q can be expressed as

$$\left.\frac{dh_1}{dr}\right|_{r=r_w} = \frac{Q}{2\pi r_w T_1} \tag{29}$$

Equations (6) and (7) representing the continuity requirements of the hydraulic head and the flux at the interface of the skin zone and the formation zone are also applicable to the constant-flux test. Figure 2 illustrates the schematic diagram of an aquifer for the constant-flux test.

2.6 The Drawdown Solution for the Constant-flux Test in a Finite Confined Aquifer

The obtained hydraulic head solution are

$$\overline{h_1} = \frac{Q}{4\pi T_2}\left[\frac{1}{p}\frac{2T_2}{r_w T_1 q_1}\frac{\Phi_1 I_0(q_1 r) - \Phi_2 K_0(q_1 r)}{\Phi_1 I_1(q_1 r_w) + \Phi_2 K_1(q_1 r_w)}\right] \tag{30}$$

$$\overline{h_2} = \frac{Q}{4\pi T_2}\left[\frac{1}{p}\frac{2T_2\varpi}{r_w T_1 q_1}\frac{\Phi_1 I_0(q_1 r_1) - \Phi_2 K_0(q_1 r_1)}{\Phi_2 K_1(q_1 r_w) + \Phi_1 I_1(q_1 r_w)}\right] \tag{31}$$

Appendix B lists the derivation for the drawdown solution for the constant-flux test in a finite confined aquifer. The drawdown can be represented by $s = h_0 - h$, so the drawdown solution can be written as

$$\bar{s}_1 = \frac{-Q}{4\pi T_2}\left[\frac{1}{p}\frac{2T_2}{r_w T_1 q_1}\frac{\Phi_1 I_0(q_1 r) - \Phi_2 K_0(q_1 r)}{\Phi_1 I_1(q_1 r_w) + \Phi_2 K_1(q_1 r_w)}\right] \tag{32}$$

and

$$\bar{s}_2 = \frac{-Q}{4\pi T_2}\left[\frac{1}{p}\frac{2T_2\varpi}{r_w T_1 q_1}\frac{\Phi_1 I_0(q_1 r_1) - \Phi_2 K_0(q_1 r_1)}{\Phi_2 K_1(q_1 r_w) + \Phi_1 I_1(q_1 r_w)}\right] \tag{33}$$

If $R \to \infty$, Equations (32) and (33) can reduce the equations given in Yeh et al. (2003) as follows

$$\bar{s}_1 = \frac{-Q}{4\pi T_2}\left[\frac{1}{p}\frac{2T_2}{r_w T_1 q_1}\frac{\phi_1 I_0(q_1 r) - \phi_2 K_0(q_1 r)}{\phi_2 K_1(q_1 r_w) + \phi_1 I_1(q_1 r_w)}\right] \tag{34}$$

$$\bar{s}_2 = \frac{-Q}{4\pi T_2}\left[\frac{1}{p}\frac{2T_2}{r_w T_1 q_1}\frac{\left[\phi_1 I_0(q_1 r_1) - \phi_2 K_0(q_1 r_1)\right] K_0(q_2 r)}{\left[\phi_2 K_1(q_1 r_w) + \phi_1 I_1(q_1 r_w)\right] K_0(q_2 r_1)}\right] \tag{35}$$

2.7 Dimensionless Drawdown Solution for the Constant-flux Test

The dimensionless drawdown is defined as $\bar{s}_D = \bar{s}(4\pi T_2)/Q$. Equations (32) and (33) can then be respectively written as

$$\bar{s}_{1D} = \frac{1}{p}\left[\frac{2\alpha}{\sqrt{p/\kappa}}\frac{\Phi_{2D} K_0\left(\rho\sqrt{p/\kappa}\right) + \Phi_{1D} I_0\left(\rho\sqrt{p/\kappa}\right)}{\Phi_{2D} K_1\left(\sqrt{p/\kappa}\right) - \Phi_{1D} I_1\left(\sqrt{p/\kappa}\right)}\right] \tag{36}$$

and

$$\bar{s}_{2D} = \frac{1}{p}\left[\frac{2\alpha\varpi_D}{\sqrt{p/\kappa}}\frac{\Phi_{2D} K_0\left(\rho_1\sqrt{p/\kappa}\right) + \Phi_{1D} I_0\left(\rho_1\sqrt{p/\kappa}\right)}{\Phi_{2D} K_1\left(\sqrt{p/\kappa}\right) - \Phi_{1D} I_1\left(\sqrt{p/\kappa}\right)}\right] \tag{37}$$

with

$$\varpi_D = \frac{K_0\left(\rho\sqrt{p}\right)I_0\left(\rho_R\sqrt{p}\right)-K_0\left(\rho_R\sqrt{p}\right)I_0\left(\rho\sqrt{p}\right)}{I_0\left(\rho_R\sqrt{p}\right)K_0\left(\rho_1\sqrt{p}\right)-I_0\left(\rho_1\sqrt{p}\right)K_0\left(\rho_R\sqrt{p}\right)} \tag{38}$$

These two solutions can also be inverted to time domain numerically by the modified Crump algorithm (de Hoog et al. 1982). The dimensionless Laplace-domain drawdown solutions for the aquifer with an infinite domain provided by Yeh et al. (2003) can be expressed as

$$\bar{s}_{1D} = \frac{1}{p}\left[\frac{2\alpha}{\sqrt{p/\kappa}}\frac{\phi_{2D}K_0\left(\rho\sqrt{p/\kappa}\right)-\phi_{1D}I_0\left(\rho\sqrt{p/\kappa}\right)}{\phi_{2D}K_1\left(\sqrt{p/\kappa}\right)+\phi_{1D}I_1\left(\sqrt{p/\kappa}\right)}\right] \tag{39}$$

$$\bar{s}_{2D} = \frac{1}{p}\left[\frac{2\alpha}{\sqrt{p/\kappa}}\frac{\left[\phi_{2D}K_0\left(\rho_1\sqrt{p/\kappa}\right)-\phi_{1D}I_0\left(\rho_1\sqrt{p/\kappa}\right)\right]K_0\left(\rho\sqrt{p}\right)}{\left[\phi_{2D}K_1\left(\sqrt{p/\kappa}\right)+\phi_{1D}I_1\left(\sqrt{p/\kappa}\right)\right]K_0\left(\rho_1\sqrt{p}\right)}\right] \tag{40}$$

2.8 The Large-time Drawdown Solution for the Constant-flux Test

2.8.1 In Finite Confined Aquifers

The drawdown solution at late time can be obtained by applying the SPLT technique to Equations (32) and (33). Based on the L'Hospital rule, the Laplace-domain solutions for drawdowns in skin zone and the formation zone when p is small can be obtained respectively as

$$\overline{s_1}(r,p) = \frac{Q}{2\pi T_1 p}\left(\ln\frac{r_1}{r}+\frac{T_1}{T_2}\ln\frac{R}{r_1}\right) \tag{41}$$

$$\overline{s_2}(r,p) = \frac{Q}{2\pi T_2 p}\ln\frac{R}{r} \tag{42}$$

The large-time drawdown solutions are then obtained by taking the inverse Laplace transform to Equations (41) and (42) as

$$s_1(r,t) = \frac{Q}{2\pi T_1}\left(\ln\frac{r_1}{r} + \frac{T_1}{T_2}\ln\frac{R}{r_1}\right) \tag{43}$$

$$s_2(r,t) = \frac{Q}{2\pi T_2}\ln\frac{R}{r} \tag{44}$$

Equations (43) and (44) are independent of time and equal to the steady-state solution, which can also be obtained by applying the method of Tauberian theory (Sneddon 1972) to Equations (32) and (33). Wang and Yeh (2008) also showed that the drawdowns can reach steady state if aquifers have finite domain. In addition, Equations (43) and (44) can reduce to the Thiem equation if the skin zone is absent.

2.8.2 In Infinite Confined Aquifers

Equations (43) and (44) are the large-time solutions of Equations (32) and (33), respectively, for finite-domain confined aquifers. By applying the SPLT relationship to Equations (34) and (35), the Laplace-domain drawdown solutions for small p in skin zone and formation zone can be obtained respectively as

$$\overline{s_1}(r,p) = \frac{Q}{2\pi T_1 p}\left[\ln\frac{r_1}{r} + \frac{T_1}{T_2}\ln\left(\frac{1}{q_2 r_1}\right)\right] \tag{45}$$

$$\overline{s_2}(r,p) = \frac{Q}{2\pi T_2 p}\ln\frac{1}{q_2 r} \tag{46}$$

Finally, the large-time drawdown solutions in time domain can be obtained after employing the inverse Laplace transform as

$$s_1(r,t) = \frac{Q}{4\pi T_1}\left[2\ln\frac{r_1}{r} + \frac{T_1}{T_2}\ln\left(\frac{CT_2}{S_2 r_1^2}t\right)\right] \tag{47}$$

$$s_2(r,t) = \frac{Q}{4\pi T_2}\ln\left(\frac{CT_2}{S_2 r^2}t\right) \tag{48}$$

where $C = \exp(\gamma)$ and γ = 0.5772… is Euler's constant. Note Equation (47) and Equation (48) can reduce to cooper and Jacob equation (1946) if neglecting the effect of skin zone. In addition, this large-time drawdown solution for an aquifer with infinite extended boundary is equal to the heat flow solution at late time presented by Carslaw and Jaeger (1959, p.339) if the limiting form of the Bessel function is $K_0(x) \sim -[\ln(x/2)+\gamma]$ instead of $K_0(x) \sim -\ln(x)$.

Equations (45) and (46) are function of time. Equations (47) and (48) approach infinite when t reaches infinity due to the term $\ln t$ approaches infinity. This phenomenon indicates that the drawdown solution of the infinite extended boundary increases with time because the infinitely extended boundary can not provide enough groundwater to maintain the constant pumping rate at wellbore. In addition, Equations (47) and (48) decrease with increasing r indicating that the condition of zero drawdown at the outer boundary condition can be held. Table 2 provides a list of comparison between the large-time drawdown and steady state solutions in finitely and infinitely extended confined aquifers.

CHAPTER 3 RESULTS AND DISCUSSION

3.1 Comparison of Infinite-Domain Solution to Finite-Domain Solution for Constant-head Test

3.1.1 Effects of skin zone and Finite Domain

Figure 3 illustrates that the curve of the wellbore flow-rate as a function of time at ρ = 1 for ρ_1 = 3 and α = 0.1, 1, and 10. For α = 0.1 and 1, the finite-domain flow-rate matches with the infinite-domain flow-rate in small time (i.e., 10 < τ < 100). These results indicate that the infinite-domain flow-rate solution can approximate the finite-domain flow-rate solution if the test time is less than the time criterion. In other words, the boundary distance *R* has no effect on the wellbore flow-rate as the test time is shorter than the time criterion. However, these two solutions deviate for one another in the period of moderate time (100 < τ < 1000). The finite-domain flow-rate solution approaches its asymptotic limit, i.e., the steady state solution, when the test time is large while the infinite-domain flow-rate solution continuously declines with dimensionless time.

The dimensionless flow rate with a positive skin is smaller than that with a negative skin (Yang and Yeh, 2002) at a specific time. This result indicates that the effect of finite boundary on the flow-rate may be negligible for a high value of α (say, e.g., 10).

3.1.2 Time Criterion for Using the Infinite-Domain Solution

Figure 3 shows that the time criterion increases with ρ_R $(=R/r_w)$. At $\rho_1 = 3$ and α = 0.1, the values of time criterion τ_c are 26, 67, and 204 for ρ_R = 20, 30, and 50, respectively, as shown in Figure 3. When $\rho_1 = 3$ and $\alpha = 1$, τ_c is equal to 32, 77, and 231 for ρ_R = 20, 30, and 50, respectively. In addition, τ_c increases to 45, 101, and 234 for ρ_R = 20, 30, and 50, respectively, at $\rho_1 = 3$ and $\alpha = 10$. Those data represent that τ_c increases with α if ρ_R is fixed and τ_c increases with ρ_R if α is fixed.

3.2Comparision of the Infinite-Domain Solution to the Finite-Domain Solution for Constant-flux Test

3.2.1 Effects of Skin Type, Skin Thickness, and Finite-Domain

Figure 4 shows the curves of dimensionless drawdown at wellbore for aquifers with finite-domain and infinite-domain with $\rho_1 = 3$ and α = 0.1, 1, 5, and 10. The drawdown curves for both aquifers match exactly at early pumping time ($1 < \tau < 10$). However, the curves gradually deviate from one another in the middle time period ($10 < \tau < 100$) due to boundary effect. It indicates that the infinite-domain drawdown solution can approximate the finite-domain solution when the time is less than the time criterion. Finially, the finite-domain drawdown solution tends to an asymptotic limit, the steady state solution, while the infinite-domain drawdown continuously increases with dimensionless time. These results reflect that the pumped water comes from the remote constant-head boundary for the finite-domain aquifer and is from the aquifer storage for the infinite-domain aquifer. Figure 4 also shows the effect of skin type on the drawdown distribution. The drawdown increases with α indicating a larger α has a larger drawdown value.

Figure 5 demonstrates the curves of dimensionless drawdown at wellbore for aquifers with finite-domain and infinite-domain with $\rho_1 = 10$ and $\alpha = 0.1$, 1, 5, and 10. This figure has a similar pattern of temporal drawdown distribution to Figure 4 except that the time criterion relating to the boundary effect is delayed. Yeh et al. (2003) also mentioned that the thickness of the wellbore ρ_1 may influence the magnitude of the dimensionless drawdown.

Figures 6 and 7 show the dimensionless drawdown curves for $\rho_1 = 3$ and 10, respectively, with $\rho = 10$, $\alpha = 0.1$, 1, 5, and 10. There two figures indicate that the effect of skin type on the drawdown in the formation zone contrasts to that in the skin. The drawdown of the formation zone is less sensitive for a positive skin aquifer.

3.2.2 Time Criterion for Various Values of Skin Type, Skin Thickness and Radial Distance

The time criterion increases with the dimensionless boundary distance ρ_R as illustrated in Figures 4 - 7. Figure 8 shows the time criterion versus ρ_R for varying

α, ρ_1, and ρ. This figure indicates that the time criterion τ_c increases with α (skin type) and ρ_1 (skin thickness) but decreases with ρ (dimensionless radial distance from the center of the test well) if ρ_R is fixed. Additionally, τ_c also increases with ρ_R if α is fixed.

CHAPTER 4 CONCLUSIONS

The wellbore flow-rate solution of the constant-head test and the drawdown solution of the constant-flux test are generally employed to analyze measuring data for estimating the aquifer properties. This thesis develops mathematical models to describe the hydraulic head distribution for the constant-head test and the drawdown distribution for constant-flux test at a finite-domain confined aquifer. The hydraulic head solutions and the drawdown solutions in Laplace domain for skin zone and formation zone are derived using the Laplace transforms for both tests. In addition, the solution of wellbore flow rate for the constant-head test is derived based on the hydraulic head solution and Darcy' law. The time-domain results of wellbore flow rate for the constant-head test and the drawdown for the constant-flux test are evaluated by the modified Crump algorithm. The results show that the dimensionless wellbore flow-rate solution and drawdown solution for a finite-domain aquifer are significantly different from the one of an infinite-domain aquifer at late pumping times.

For the constant-head test, the infinite-domain solution will underestimate the flow rate at the wellbore in a finite-domain aquifer when time is fairly large. The effect of finite boundary on the flow rate appears to be considerable less for a positive skin. On the other hand, the infinite-domain solution for the constant-flux test may overestimate the drawdown for a finite-domain aquifer at large time. The analysis for the effect of skin type on the drawdown shows that the drawdown is significant in the skin zone and insignificant in the formation zone for a positive skin.

For both the constant head test and constant-flux test, the infinite-domain solution can be used to determine the drawdown distribution, wellbore flow rate, or aquifer parameters if coupled with an optimization algorithm when the time is less than the boundary-effect time criterion. The infinite-domain solutions are generally in simpler forms and much easier to evaluate than the finite-domain solutions. Therefore, time criteria provide a good reference for adopting the infinite-domain solution to calculate the drawdown or wellbore flow rate of finite-domain aquifers.

This thesis also derives large-time solutions for the constant head test and constant-flux test based on the relationship of small Laplace-domain variable p versus large time-domain variable t. It is found that the large-time solutions in finite-domain aquifers are equal to the steady-state solutions obtained from the Tauberian theorem. In addition, the large-time solutions can reduce to Thiem equation if neglecting the skin zone for finite-domain aquifers and approach infinity as the time goes infinitely large for infinite-domain aquifers.

APPENDIXES

Appendix A: Derivation of Equations (8) and (9)

Appendix B: Derivation of Equations (32) and (33)

Appendix A: Derivation of Equations (8) and (9)

The solutions for the hydraulic head distribution in the skin zone and formation zone are derived by taking the Laplace Transform to the governing equation of Equations (1) and (2), initial condition (Equations (3)), and boundary conditions (Equations (4), (5), (6), and (7) (Carslaw and Jaeger, 1959, p332) The results are

$$\overline{h_1} = C_1 I_0(q_1 r) + C_2 K_0(q_1 r) \tag{A1}$$

$$\overline{h_2} = D_1 I_0(q_2 r) + D_2 K_0(q_2 r) \tag{A2}$$

The constant coefficients in Eqs. (A1) and (A2) can be solved with the boundary conditions of Equations (4) and (5) and continuity requirements of Equations (6) and (7) as

$$C_1 = \frac{h_w}{p}\left[\frac{\Phi_1}{\Phi_1 I_0(q_1 r_w) - \Phi_2 K_0(q_1 r_w)}\right] \tag{A3}$$

$$C_2 = \frac{h_w}{p}\left[\frac{-\Phi_2}{\Phi_1 I_0(q_1 r_w) - \Phi_2 K_0(q_1 r_w)}\right] \tag{A4}$$

$$D_1 = \frac{h_w}{p}\frac{\zeta K_0(q_2 R)}{\chi I_0(q_2 R)} \tag{A5}$$

$$D_2 = \frac{-h_w}{p}\frac{\zeta}{\chi} \tag{A6}$$

with

$$\zeta = \frac{-\Phi_1 I_0(q_1 r_1) + \Phi_2 K_0(q_1 r_1)}{\Phi_1 I_0(q_1 r_w) - \Phi_2 K_0(q_1 r_w)} \tag{A7}$$

$$\chi = \frac{I_0\left(q_2 R\right) K_0\left(q_2 r_1\right) - I_0\left(q_2 r_1\right) K_0\left(q_2 R\right)}{I_0\left(q_2 R\right)} \tag{A8}$$

Consequently, the hydraulic head solutions in the skin zone and formation zone can be obtained by substituting the constants of Equations (A3) - (A6) into Equations (A1) and (A2) as Equations (8) and (9), respectively.

Appendix B Derivation of Equations (32) and (33)

The solutions for the hydraulic head distribution in the skin zone and formation zone are derived by taking the Laplace Transform to the governing equation of Equations (1) and (2), initial condition (Equation (3)), and boundary conditions Equations (5), (6), (7) and (29) (Carslaw and Jaeger, 1959, p332). The results can be expressed as

$$\overline{h_1} = C_1^{'} I_0(q_1 r) + C_2^{'} K_0(q_1 r) \tag{B1}$$

$$\overline{h_2} = D_1^{'} I_0(q_2 r) + D_2^{'} K_0(q_2 r) \tag{B2}$$

Accordingly, the drawdown solution can be written as

$$\overline{s_1} = C_1 I_0(q_1 r) + C_2 K_0(q_1 r) \tag{B3}$$

$$\overline{s_2} = D_1 I_0(q_2 r) + D_2 K_0(q_2 r) \tag{B4}$$

The constant coefficients in Equations (B3) and (B4) can be solved with the boundary conditions of Equations (5) and (29) and continuity requirements, i.e., Equations (6) and (7). as

$$C_1 = \frac{-Q}{2\pi r_w T_1 p q_1}\left[\frac{\Phi_1}{\Phi_1 I_1(q_1 r_w) + \Phi_2 K_1(q_1 r_w)}\right] \tag{B5}$$

$$C_2 = \frac{Q}{2\pi r_w T_1 p q_1}\left[\frac{\Phi_2}{\Phi_1 I_1(q_1 r_w) + \Phi_2 K_1(q_1 r_w)}\right] \tag{B6}$$

$$D_1 = \frac{Q}{2\pi r_w T_1 p q_1}\frac{\zeta K_0(q_2 R)}{\chi I_0(q_2 R)} \tag{B7}$$

$$D_2 = \frac{-Q}{2\pi r_w T_1 p q_1}\frac{\zeta}{\chi} \tag{B8}$$

The drawdown solution in the skin zone and the formation zone can therefore be obtained by substituting Equations (B5) - (B8) into Equations (B3) and (B4) as Equations (32) and (33).

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Table 1 List of Equations (26), (28), and two existing solutions for constant-head test

	Outer boundary	Solutions	
		Large-time solution	Steady-state solution
skin zone is absent	R is finite	Thiem equation	Thiem equation
	$R \to \infty$	$Q(r_w,t) = 4\pi T h_w \left[\frac{1}{\ln\eta} - \frac{\gamma}{(\ln\eta)^2} + \frac{\gamma^2 - \frac{\pi^2}{6}}{(\ln\eta)^3} - \frac{\gamma^3 - \frac{\pi^2}{2}\gamma + 2\xi(3)}{(\ln\eta)^4} \right]$ where $\eta = Tt/(r_w^{\ 2} S)$ [Yeh and Wang (2007), Equation (6)]	
skin zone is present	R is finite	Equation (26)	Equation (26)
	$R \to \infty$	Equation (28)	

Table 2 List of the large-time and steady-state drawdown solutions for the constant-flux test

Outer boundary	Solutions	
	Large-time	Steady-state
R is finite	Equations (43) and (44)	Equations (43) and (44)
R is infinite	Equations (47) and (48)	No solution

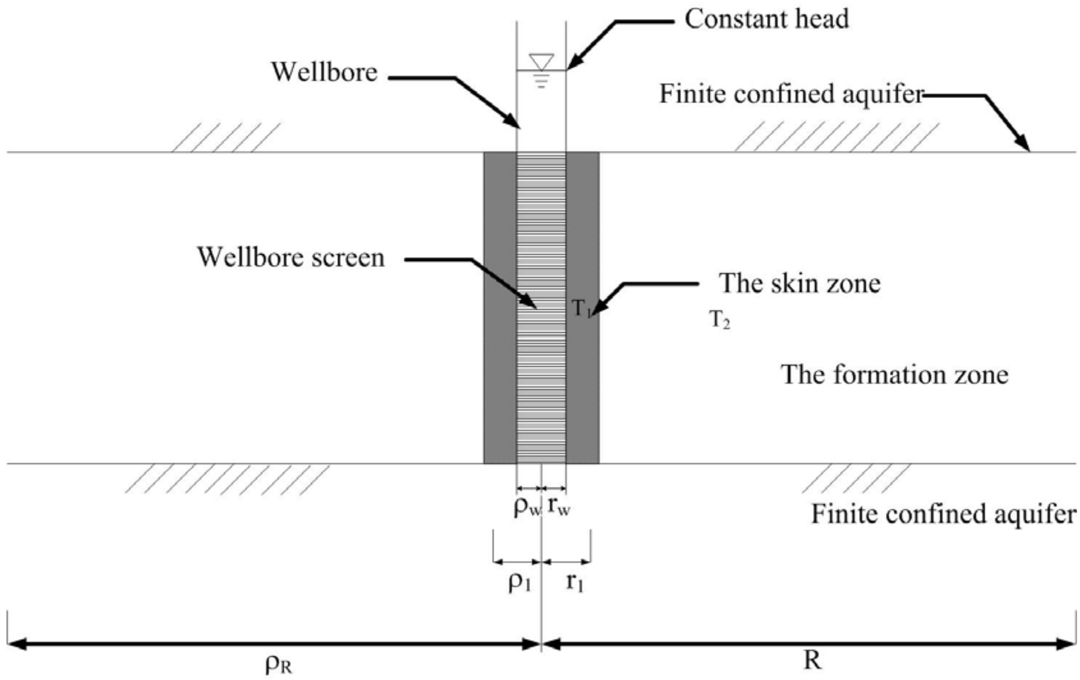

Figure 1 Schematic diagram for a constant-head test at a finite diameter well in a finite-domain confined aquifer.

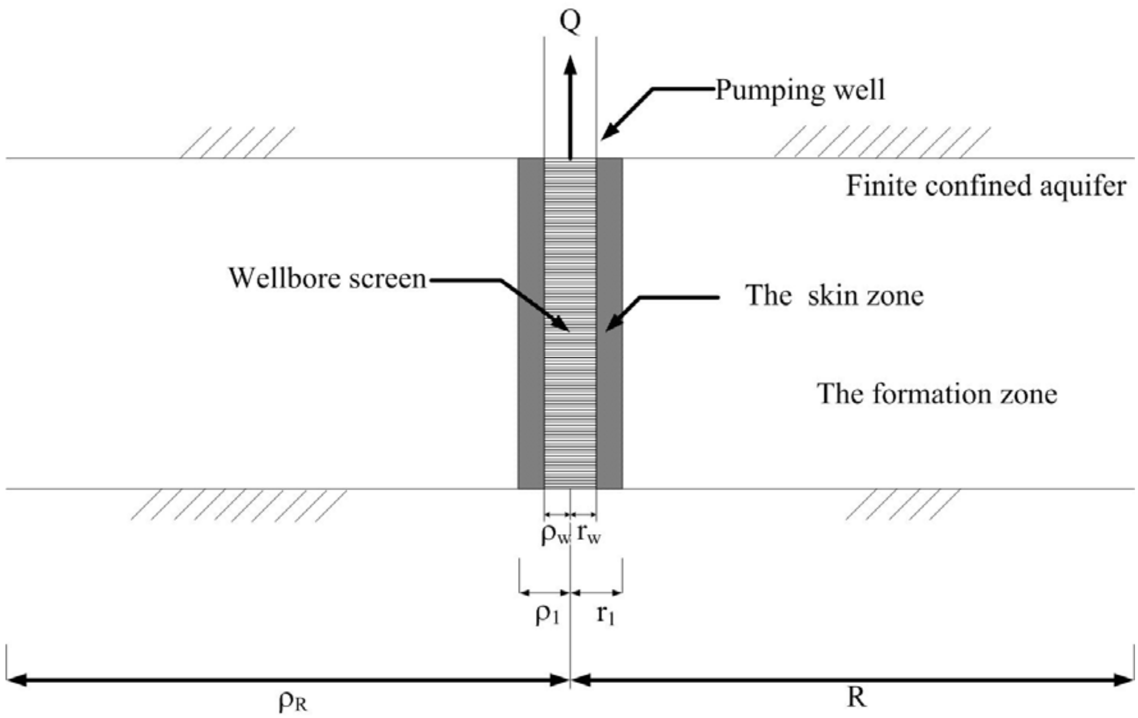

Figure 2 Schematic diagram of the pumping test in a finite-domain confined aquifer.

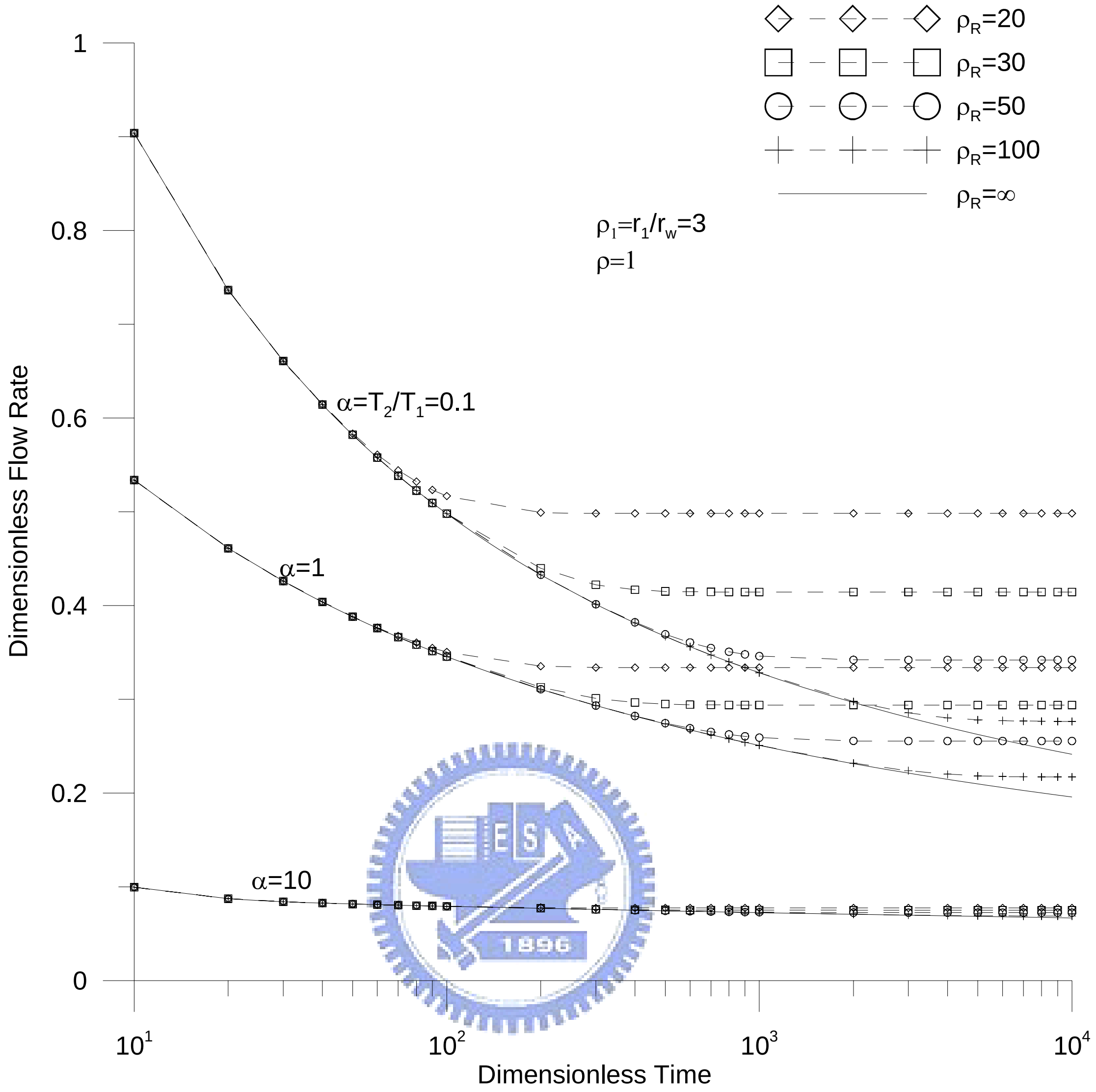

Figure 3 Dimensionless flow rate versus dimensionless pumping time for $\rho_1 = 3$ at $\rho = 1$ and $\alpha =$ 0.1, 1 and 10. The solid line presents the flow-rate solution of infinite-domain aquifers and the dash line present the flow-rate solution of finite-domain aquifers.

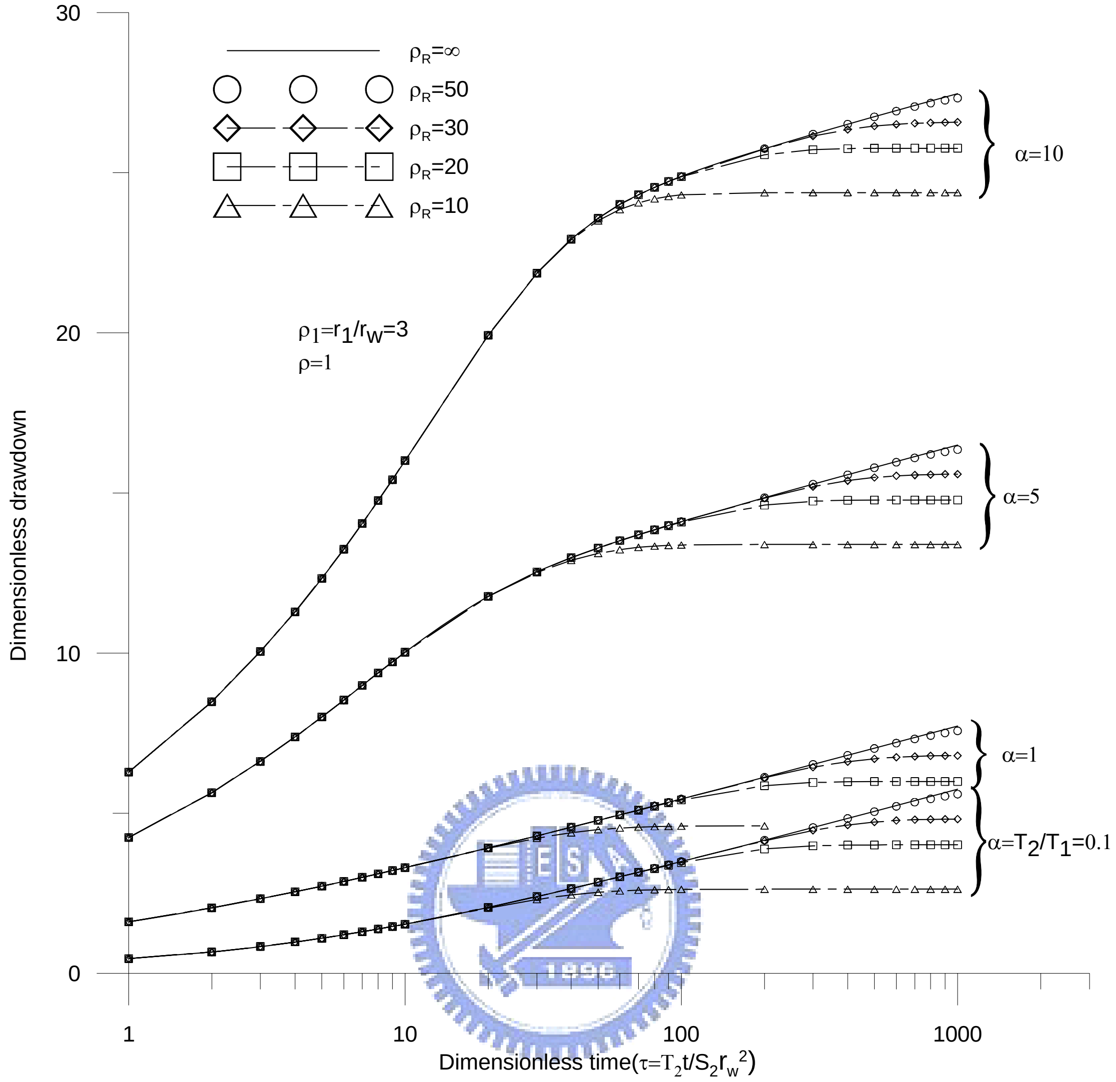

Figure 4 Comparison of the drawdown of a finite-domain aquifer to the drawdown of an infinite-domain aquifer for $\rho_1 = 3$ at $\rho = 1$ and α = 0.1, 1, 5, and 10. The solid line presents the infinite-domain drawdown solution and the dash line represent the finite-domain drawdown solution.

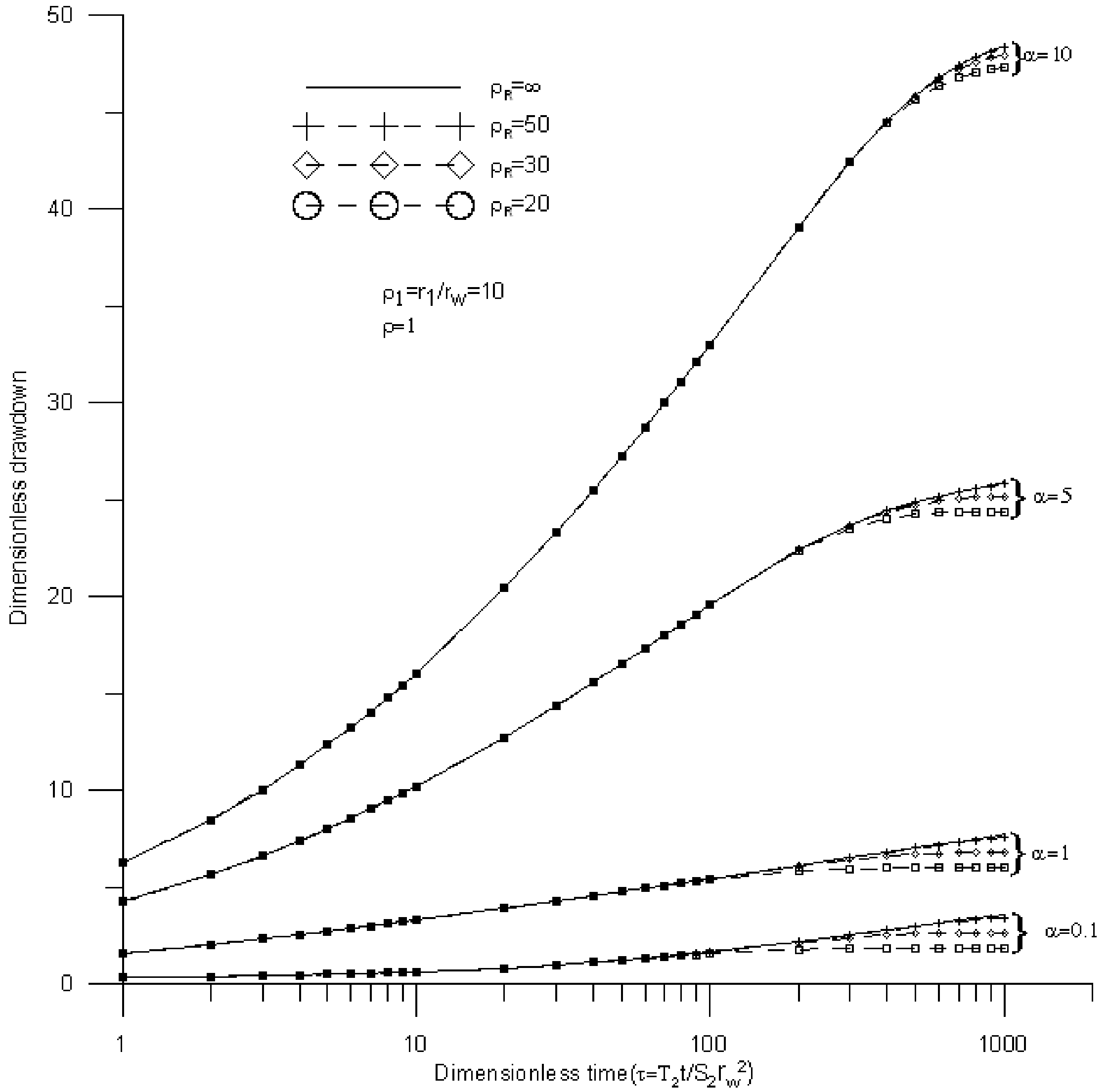

Figure 5 Comparison of the drawdown of a finite-domain aquifer to the drawdown of an infinite-domain aquifer for $\rho_1 = 10$ at $\rho = 1$ and $\alpha = 0.1$, 1, 5, and 10. The solid line presents the infinite-domain drawdown solution and the dash line present the finite-domain drawdown solution.

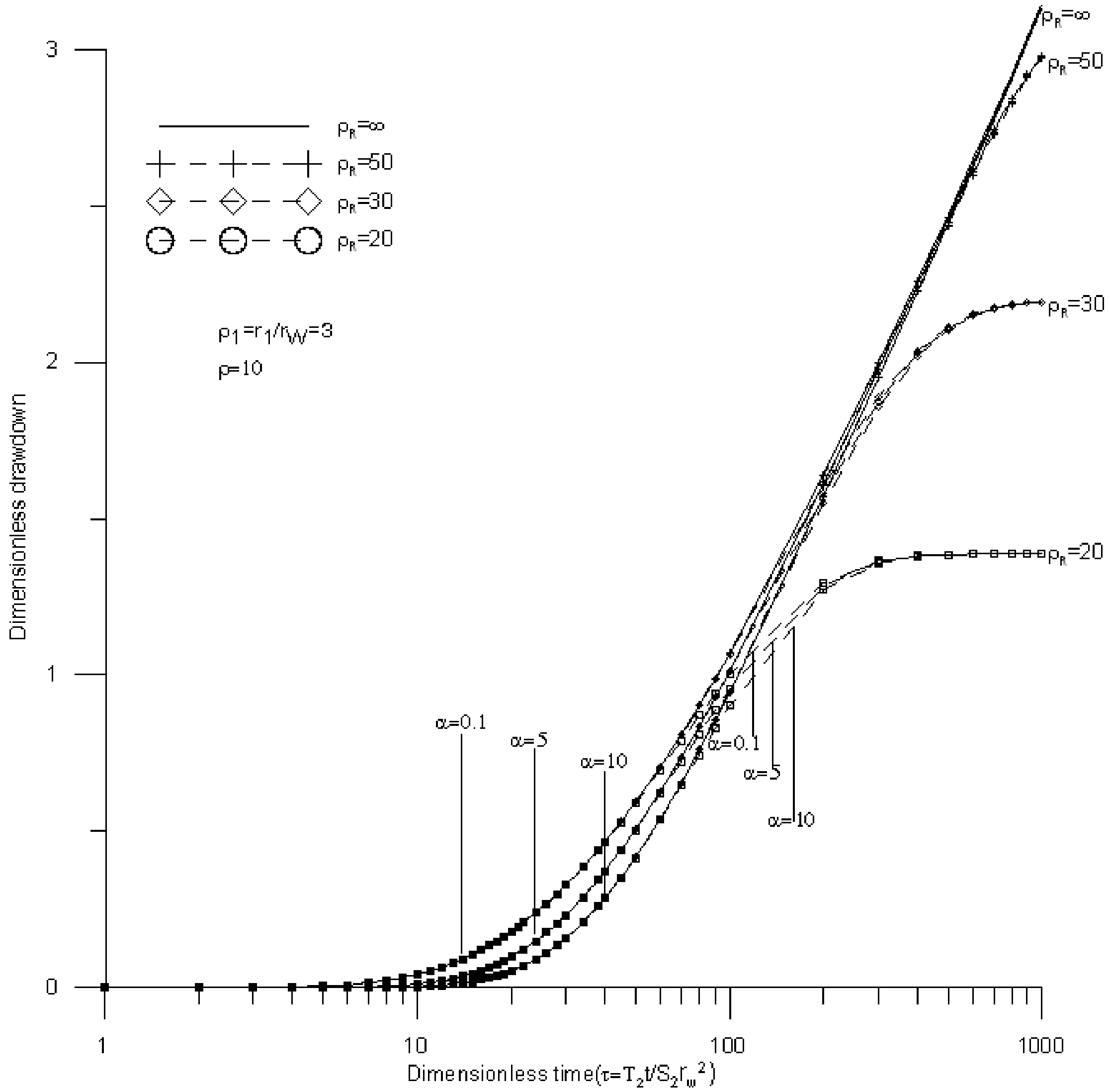

Figure 6 Comparison of the drawdown of a finite-domain aquifer to the drawdown of an infinite-domain aquifer for $\rho_1 = 3$ at $\rho = 10$ and α =0.1, 1, 5, and 10. The solid line presents the infinite-domain drawdown solution and the dash line present the finite-domain drawdown solution.

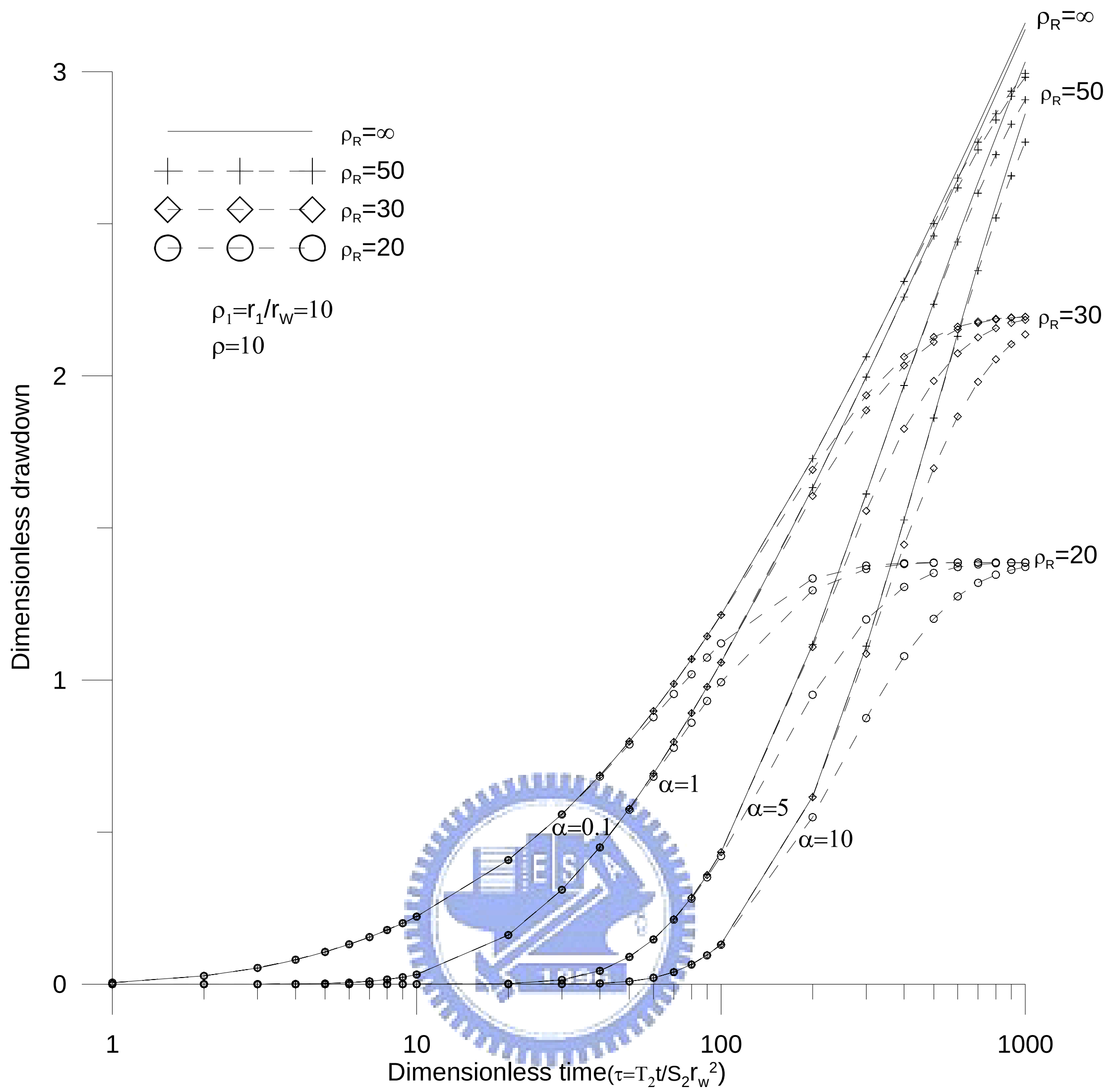

Figure 7 Comparison of the drawdown of a finite-domain aquifer to the drawdown of an infinite-domain aquifer for ρ_1 = 10 at ρ = 10 and α = 0.1, 1, 5, and 10. The solid line presents the infinite-domain drawdown solution and the dash line present the finite-domain drawdown solution.

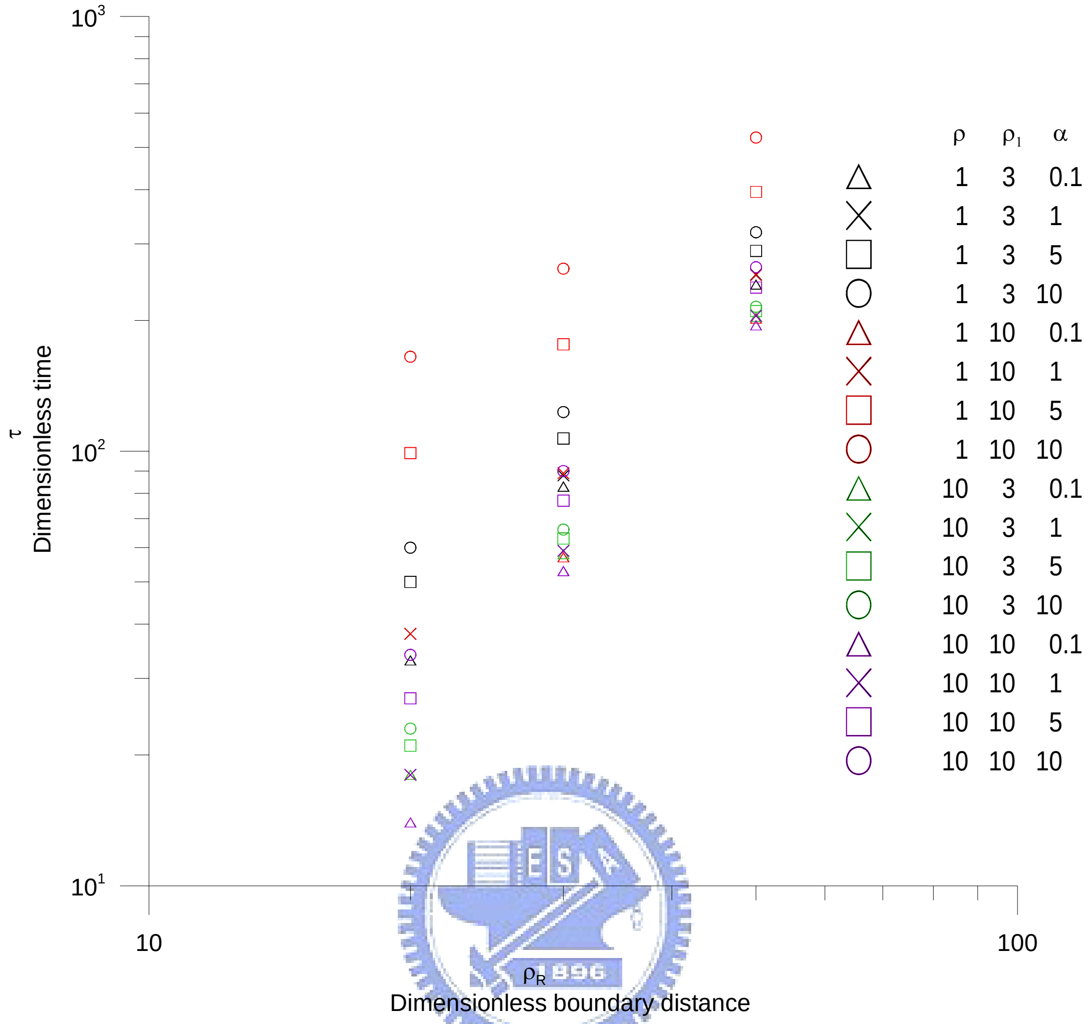

Figure 8 Time criterion τ_c versus dimensionless boundary distance ρ_R for various values of ρ, ρ_1, and α.