國 立 交 通 大 學

應用數學系
碩 士 論 文

對於弦環式網路上直徑的研究

On the Study of the Diameter of Chordal Ring Networks

研 究 生：劉士慶
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研 究 生：劉士慶　　Student: Shih-Ching Liu

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摘　要

環狀網路是最簡單的網路架構。由於環狀網路的可靠度低、傳輸延遲高，因此，以環狀網路為基礎的混合式環狀網路結構相繼被提出，以提高其可靠度與降低其傳輸延遲（亦即直徑）。弦環式網路是在環狀網路的結構中增加弦，使得其可靠度提高、直徑降低。Arden 和 Lee 在[1]中對於弦環式網路發表了一個公式來計算其直徑；此直徑公式包含了三個狀況，且第三個狀況中還包含了五個子狀況：子狀況 3.1、3.2、3.3、3.4 和 3.5。Huang 和 Chen 在[8]中指出 Arden 和 Lee 的直徑公式中的第三個狀況是錯的。在本論文中，我們執行電腦程式以得到直徑公式中的每個狀況所佔有的百分比、和第三個狀況中五個子狀況錯誤的比例。我們特別觀察到直徑公式在子狀況 3.4 是完全正確的，而且符合子狀況 3.4 的 N 有特殊形式。

關鍵詞：連接網路、環狀網路、弦環式網路、直徑。

中 華 民 國 九 十 九 年 一 月

On the Study of the Diameter of Chordal Ring Networks

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Abstract

One of the simplest topologies for interconnection networks is the ring network. Since the ring network has poor reliability and high transmission delay, hybrid topologies utilizing the ring network as a basis for synthesizing richer interconnection schemes have been proposed to improve the reliability and reduce the transmission delay (the diameter). The chordal ring network is a commonly used extension for the ring network and it is considered to be obtained by adding chords to a ring network so that the diameter can be reduced and the reliability can be increased. In [1], Arden and Lee proposed a formula for computing the diameter of a given chordal ring network; this diameter formula consists of three cases and case 3 of it consists of five subcases: subcases 3.1, 3.2, 3.3, 3.4 and 3.5. In [4], Huang and Chen pointed out that case 3 of the diameter formula is incorrect. In this thesis, we run a computer program to obtain the percentage of each of the three cases of the diameter formula, the percentage of each of the five subcases of case 3, and the faulty percentages for the five subcases of case 3. In particular, we observe that subcase 3.4 is never incorrect and the N that satisfying subcase 3.4 is of a specific form.

Keywords: interconnection network, ring network, chordal ring network, diameter.

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Contents

Abstract (in Chinese) i

Abstract (in English) ii

Acknowledgement iii

Contents iv

List of Figures v

List of tables v

1 Introduction 1

2 Some preliminaries and statistic data 4

3 Properties of subcase 3.4 8

4 Concluding remarks 12

A Appendix: the form of N for subcase 3.4 14

B Appendix: the source code of our program 24

C Appendix: the output of our program 29

List of Figures

1 $CR(26, 11)$: the chordal ring network with $N = 26$ and $w = 11$. The number inside parentheses is the distance to node 0. 2

2 The breadth first search (BFS) tree of $CR(26, 11)$ rooted at node 0: this tree shows that the Diameter Formula is incorrect. The number inside parentheses is the distance to node 0. 3

3 $CR(28, 9)$: the chordal ring network with $N = 28$ and $w = 9$. The numbers inside parentheses are the distances to node 0. 5

4 (a) The percentages of cases 1, 2 and 3 in the Diameter Formula. (b) The percentages of subcases of 3.1, 3.2, 3.3, 3.4 and 3.5 in case 3 of the Diameter Formula. 6

5 The value in each cell represents the diameter obtained from the Diameter Formula. The cells with a * represent the situation that subcase 3.4 occurs. The colored cells represent the faulty subcases of case 3 in the Diameter Formula, in which cells depicted with color red are for subcase 3.1, with green are for subcase 3.2, with color yellow are for subcase 3.3, and with color blue are for subcase 3.5. 8

List of Tables

1 The faulty percentage of each subcase of case 3. 6

2 Writing N in subcase 3.4 in the form $k \cdot w + 1$ for $28 \leq N \leq 134$. 7

1 Introduction

The ring network is one of the simplest topologies for interconnection networks. It has many attractive properties such as simplicity, extendibility, low degree, and ease of implementation. It has drawbacks as well: it has poor reliability (any failure in an interface or communication link destroys the function of the network) and it has high transmission delay. As a result, a lot of hybrid topologies utilizing the ring network as a basis for synthesizing richer interconnection schemes have been proposed to improve the reliability and reduce the transmission delay [1, 2, 3, 5, 6, 8]. One example of the commonly used extensions for the ring network is the *multi-loop network*, which was first proposed by Wong and Coppersmith in [8] for organizing multi-module memory services. Another example of the commonly used extensions for the ring network is the *chordal ring network* and is formally defined below.

In this thesis, G denotes a graph with vertex set $V(G)$ and edge set $E(G)$. A computer network is usually modeled by a graph in such a way that vertices in the graph represent nodes in the network and two vertices in the graph are connected by an edge if and only if their corresponding nodes can communicate with each other. Thereafter, vertices of a graph are also called nodes. The *distance* $d_G(u, v)$ between two nodes $u, v \in V(G)$ is the length of a shortest path joining u, v. The *eccentricity of a vertex* v in G is the distance between v and a farthest node from v. The *diameter* of G is the maximum eccentricity among all nodes of G and is denoted by $\mathcal{D}(G)$.

The chordal ring network was proposed by Arden and Lee in [1] and it can be viewed as the network obtained by adding chords to a a ring network. More precisely, a *chordal ring network* $CR(N, w)$, where N is a positive even integer and w is a positive odd integer such that $w \leq N/2$, is a graph with N nodes $0, 1, \ldots, N-1$ and $3N/2$ links of the form:

$$\begin{aligned} &(i, (i+1) \bmod N), \quad i = 0, 1, 2, \ldots, N-1, \\ &(i, (i+w) \bmod N), \quad i = 1, 3, 5, \ldots, N-1. \end{aligned}$$

See Figure 1 for an example. It is not difficult to see that a chordal ring network is

vertex-transitive; consequently,

$$\mathcal{D}(G) = \max\{d_G(0, v) \mid v \in V(G)\}.$$

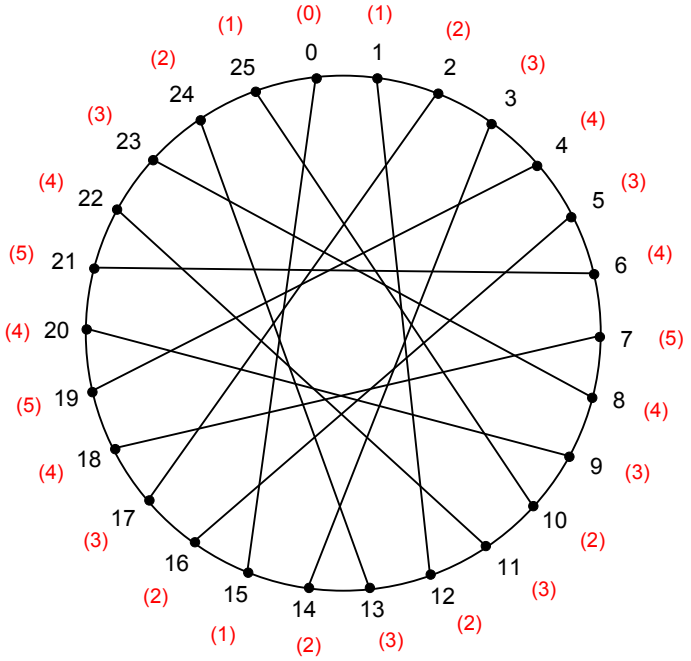


Figure 1: $CR(26, 11)$: the chordal ring network with $N = 26$ and $w = 11$. The number inside parentheses is the distance to node 0.

In [1], Arden and Lee proposed a formula to compute the diameter of a chordal ring network. For convenience, called this formula the *Diameter Formula*. See the following.

Theorem 1. [1] **(The Diameter Formula)**

Suppose $G = CR(N, w)$, $i = \lceil \frac{N}{2(w+1)} \rceil$, *and* $\triangle = \frac{N}{2} \bmod (w + 1)$. *Then*

Case 1. *When* $i \geq \frac{w-1}{2}$, $\mathcal{D}(G) = \begin{cases} 2i - 1 & \text{if } \triangle = 1, \\ 2i & \text{if } 2 \leq \triangle \leq \frac{w+3}{2}, \\ 2i + 1 & \text{if } \triangle = 0 \text{ or } \frac{w+3}{2} \leq \triangle \leq w. \end{cases}$

Case 2. *When* $i = \frac{w-3}{2}$, $\mathcal{D}(G) = \begin{cases} w - 3 & \text{if } 1 \leq \triangle \leq 2, \\ w - 2 & \text{if } \triangle = 0 \text{ or } 3 \leq \triangle \leq w. \end{cases}$

Case 3. *When* $i \leq \frac{w-5}{2}$, $\mathcal{D}(G) = \begin{cases} i + \frac{w-3}{2} & \text{if } 1 \leq \triangle \leq \frac{w+1}{2} - i \text{ or } \frac{w+5}{2} - i \leq \triangle \leq w - i, \\ i + \frac{w-1}{2} & \text{if } \triangle = 0 \text{ or } \triangle = \frac{w+3}{2} - i \text{ or } w - i + 1 \leq \triangle \leq w. \end{cases}$

Unfortunately, case 3 of the Diameter Formula is incorrect. We now give a counterexample. Consider $CR(26, 11)$ shown in Figure 1. In Figure 2, we give the breadth first search (BFS) tree of $CR(26, 11)$; from this figure, we obtain $\mathcal{D}(CR(26, 11)) = 5$. However, the Diameter Formula will obtain $\mathcal{D}(CR(26, 11)) = 6$. In [4], Huang and Chen tried to make a correction for the Diameter Formula. In particular, they showed that cases 1 and 2 of the Diameter Formula are correct and only case 3 is incorrect. They further obtained the correct diameter for the following two types of chordal ring networks:

- $N = 2w + 4$, $w \geq 11$, and $w = 12t + p$, where t is a positive integer and p is an odd integer such that $-1 \leq p \leq 9$.
- $N = 2w + 6$, $w \geq 11$, and $w = 8t + p$, where t is a positive integer and p is an odd integer such that $-1 \leq p \leq 5$.

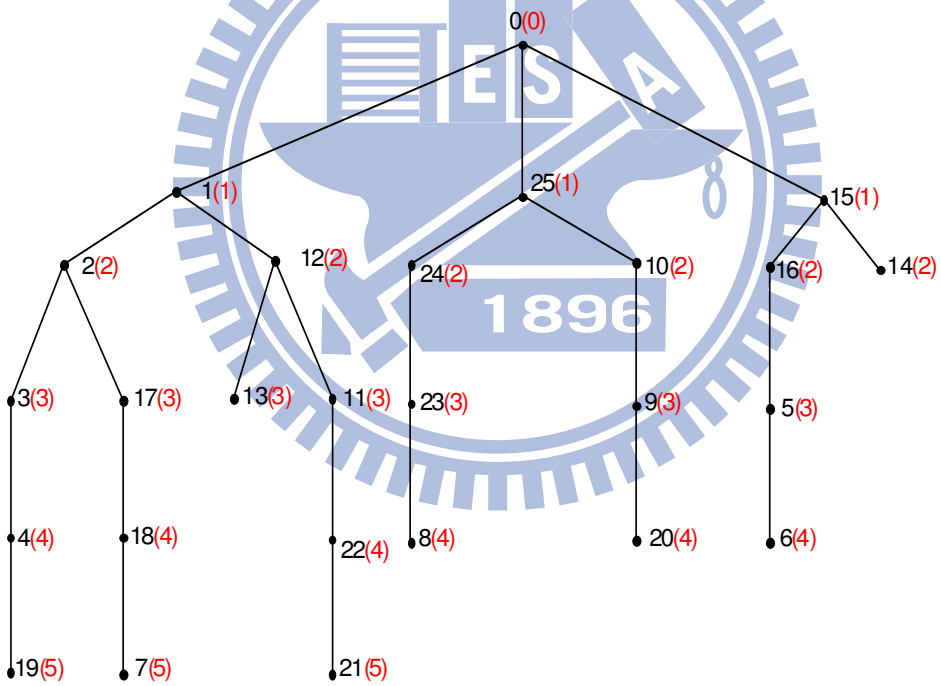


Figure 2: The breadth first search (BFS) tree of $CR(26, 11)$ rooted at node 0: this tree shows that the Diameter Formula is incorrect. The number inside parentheses is the distance to node 0.

Before going further, we divide case 3 of Diameter Formula five subcases; see the following.

case 3. When $i \leq \frac{w-5}{2}$,

$$\mathcal{D}(G) = \begin{cases} i + \frac{w-3}{2} & \text{if } 1 \leq \triangle \leq \frac{w+1}{2} - i \quad \textbf{subcase 3.1} \\ i + \frac{w-3}{2} & \text{if } \frac{w+5}{2} - i \leq \triangle \leq w - i \quad \textbf{subcase 3.2} \\ i + \frac{w-1}{2} & \text{if } \triangle = 0 \quad \textbf{subcase 3.3} \\ i + \frac{w-1}{2} & \text{if } \triangle = \frac{w+3}{2} - i \quad \textbf{subcase 3.4} \\ i + \frac{w-1}{2} & \text{if } w - i + 1 \leq \triangle \leq w \quad \textbf{subcase 3.5} \end{cases}$$

We will also show the computer statistics of the Diameter Formula and the results we found out from the computer statistics of the five subcases.

This paper is organized as follows. In Section 2, we present some preliminaries and statistic data obtained from a computer program. In Section 3, we present the properties found from the statistic data in Section 2. In Section 4, we give the concluding remarks. In Appendix A, we list the form of N for the subcase 3.4 for $6 \leq N \leq 2150$; in Appendix B, we give the source code of our program; and finally, in Appendix C, we list the output of our program.

2 Some preliminaries and statistic data

Throughout this thesis, nodes in a graph are assumed taken modulo N. As an example, $u + v$ is the node $(u + v)$ mod N and $u - v$ is the node $(u - v)$ mod N. An edge between nodes u and v is denoted by (u, v). The following definitions will be used throughout this thesis. For each odd-numbered node i, we call a chordal traversal from node i to node $i+w$ a *clockwise chordal traversal*. Similarly, for each even-numbered node j, we call a chordal traversal from node j to node $j - w$ a *counterclockwise chordal traversal*. A path from node u to node v is called a *clockwise path* if it consists of clockwise chordal traversals (possibly zero such traversals) plus appropriate ring-edge traversals. Similarly, a path from node u to node v is called a *counterclockwise path* if it consists of counterclockwise chordal traversals (possibly zero such traversals) plus appropriate ring-edge traversals. Let $\text{dist}_{G,R}(u, v)$ (resp., $\text{dist}_{G,L}(u, v)$) denote the length of a shortest clockwise (resp., counterclockwise) path from u to v in G. In [1], Arden and Lee had proven that

$$d_G(u, v) = \min\{\text{dist}_{G,R}(u, v), \text{dist}_{G,L}(u, v)\}.$$

We take $CR(28, 9)$ in Figure 3 as an example. Then $\text{dist}_{G,R}(0, 14) = 6$ and two clockwise paths with such a distance are $0 \to 1 \to 10 \to 11 \to 12 \to 13 \to 14$ and $0 \to 27 \to 8 \to 7 \to 16 \to 15 \to 14$. Also, $\text{dist}_{G,L}(0, 14) = 6$; two counterclockwise paths with such a distance are $0 \to 1 \to 2 \to 21 \to 22 \to 13 \to 14$ and $0 \to 27 \to 26 \to 17 \to 16 \to 15 \to 14$. From the above, $d_G(0, 14) = 6$.

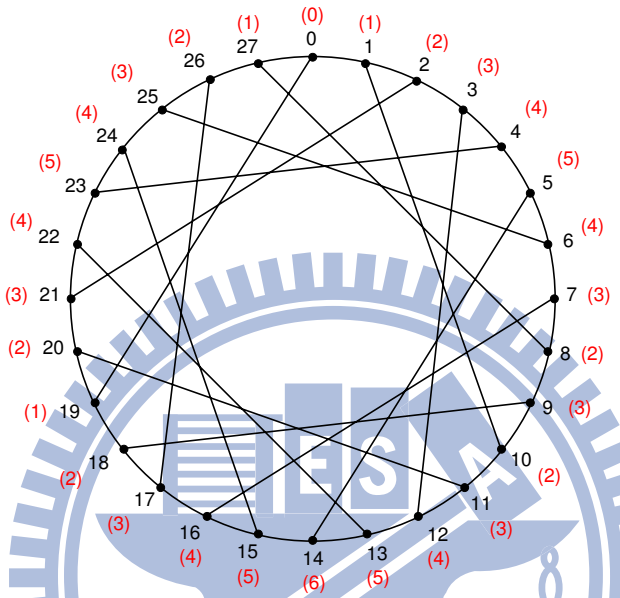


Figure 3: $CR(28, 9)$: the chordal ring network with $N = 28$ and $w = 9$. The numbers inside parentheses are the distances to node 0.

As was mentioned in the introduction, case 3 of the Diameter Formula is incorrect. For convenience, we divide case 3 into subcases 3.1, 3.2, 3.3, 3.4 and 3.5. To know how bad the incorrect situation of case 3 is, we have run a computer program to obtain the percentages of cases 1, 2 and 3, and the percentages of subcases 3.1, 3.2, 3.3, 3.4 and 3.5 for $N = 6, \ldots, 5000$; see Figure 4. And we obtain the faulty percentage of each subcase of case 3 for $N = 6, \ldots, 5000$; see Table 1. By Figure 4 and Table 1, we observe that:

- case 3 almost covers all the cases of the Diameter Formula;
- subcases 3.1 and 3.2 almost cover all the subcases of case 3;

- the faulty percentages of subcases 3.1 and 3.2 are 99.6% and 98.5%, respectively.

Besides, we get a surprising observation that:

- Among the five subcases of case 3, subcase 3.4 is never incorrect.

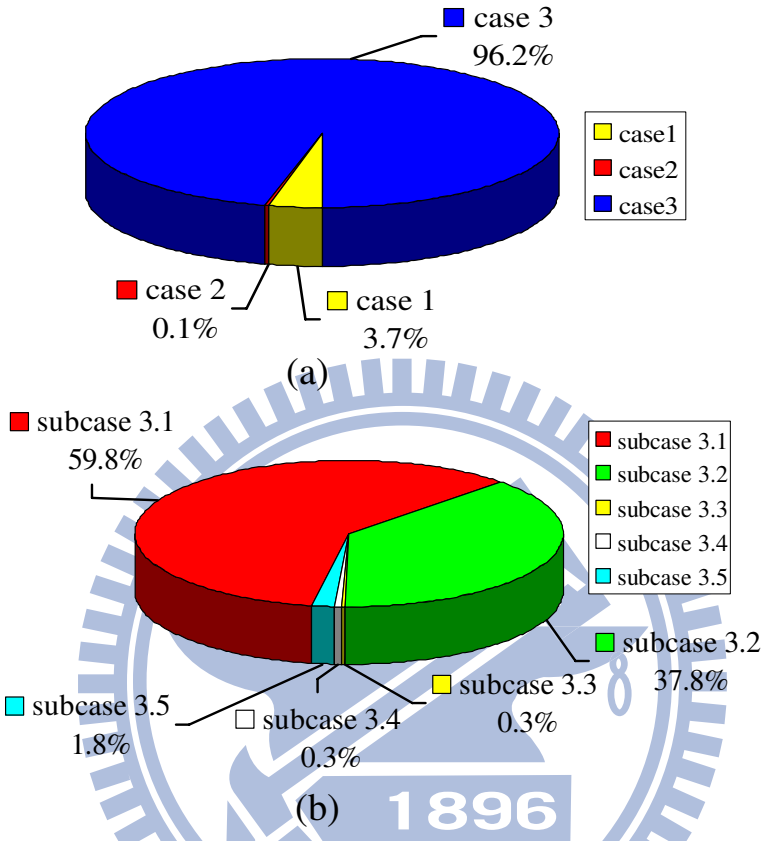


Figure 4: (a) The percentages of cases 1, 2 and 3 in the Diameter Formula. (b) The percentages of subcases of 3.1, 3.2, 3.3, 3.4 and 3.5 in case 3 of the Diameter Formula.

subcase of case 3	faulty percentage
3.1	99.6 %
3.2	98.5 %
3.3	70.7 %
3.4	0.0 %
3.5	68.2 %

Table 1: The faulty percentage of each subcase of case 3.

We use four colors to illustrate the four faulty subcases of case 3; see Figure 5 for an example. We observe two phenomenons.

1. There are some holes (uncolored cells) formed by the correct subcases of case 3 and these holes appear regularly.

2. The faulty subcases also have some regularities.

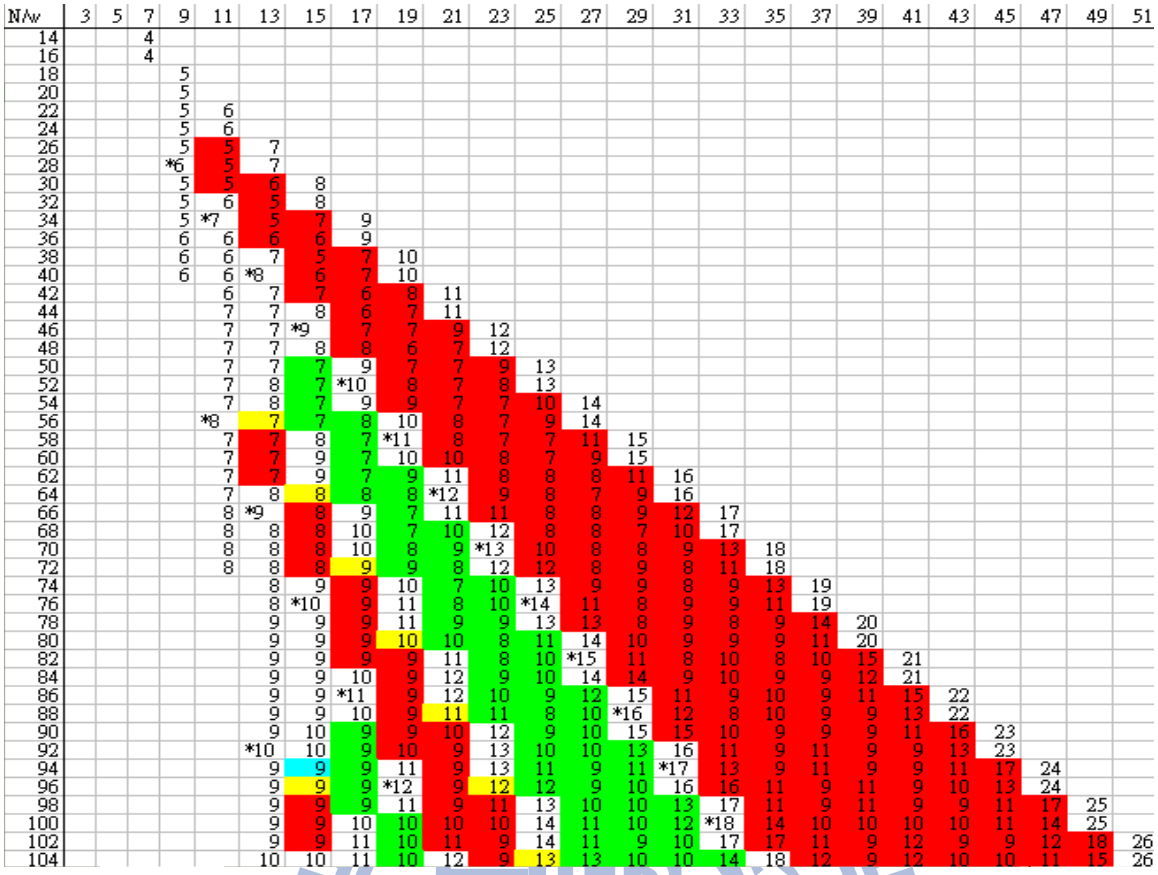
Take Figure 5 as an example. We observe that the Diameter Formula is correct when $N = 2w$ and $N = 2w + 2$. In particular, when $N = 2w$, subcase 3.5 occurs and when $N = 2w + 2$, subcase 3.3 occurs. For each $w \geq 13$, the first N such that $CR(N, w)$ satisfies subcase 3.3 and the Diameter Formula is false is $N = 4 \cdot w + 4$. Specially, $CR(56, 13)$, $CR(64, 15)$, $CR(72, 17)$, $CR(80, 19)$ satisfy subcase 3.3 and the Diameter Formula is incorrect. We also have several observations for those cells that satisfy subcase 3.4. One of them is that those N that satisfy subcase 3.4 are in the form $N = k \cdot w + 1$, where k is an odd number and $k \geq 3$. For further observing, we run a computer program to obtain the form of N for each $CR(N, w)$ that satisfies subcase 3.4; see Table 2. A complete table of the form of N for each $CR(N, w)$ that satisfies subcase 3.4 for $6 \leq N \leq 2150$ is given in the Appendix A. By the table in Appendix A, we have several observations and these observed results will be proposed and proven in next section.

N	w	form of N
28	9	$3 \cdot w + 1$
34	11	$3 \cdot w + 1$
40	13	$3 \cdot w + 1$
46	15	$3 \cdot w + 1$
52	17	$3 \cdot w + 1$
56	11	$5 \cdot w + 1$
58	19	$3 \cdot w + 1$
64	21	$3 \cdot w + 1$
66	13	$5 \cdot w + 1$
70	23	$3 \cdot w + 1$

N	w	form of N
76	15	$5 \cdot w + 1$
76	25	$3 \cdot w + 1$
82	27	$3 \cdot w + 1$
86	17	$5 \cdot w + 1$
88	29	$3 \cdot w + 1$
92	13	$7 \cdot w + 1$
94	31	$3 \cdot w + 1$
96	19	$5 \cdot w + 1$
100	33	$3 \cdot w + 1$
106	15	$7 \cdot w + 1$

N	w	form of N
106	21	$5 \cdot w + 1$
106	35	$3 \cdot w + 1$
112	37	$3 \cdot w + 1$
116	23	$5 \cdot w + 1$
118	39	$3 \cdot w + 1$
120	17	$7 \cdot w + 1$
124	41	$3 \cdot w + 1$
126	25	$5 \cdot w + 1$
130	43	$3 \cdot w + 1$
134	19	$7 \cdot w + 1$

Table 2: Writing N in subcase 3.4 in the form $k \cdot w + 1$ for $28 \leq N \leq 134$.



N/w	3	5	7	9	11	13	15	17	19	21	23	25	27	29	31	33	35	37	39	41	43	45	47	49	51
14			4																						
16			4																						
18				5																					
20				5																					
22				5	6																				
24				5	6																				
26				5	5	7																			
28				*6	5	7																			
30				5	5	6	8																		
32				5	6	5	8																		
34				5	*7	5	7	9																	
36				6	6	6	6	9																	
38				6	6	7	5	7	10																
40				6	6	*8	6	7	10																
42					6	7	7	6	8	11															
44					7	7	8	6	7	11															
46					7	7	*9	7	7	9	12														
48					7	7	8	8	6	7	12														
50					7	7	7	9	7	7	9	13													
52					7	8	7	*10	8	7	8	13													
54					7	8	7	9	9	7	7	10	14												
56					*8	7	7	8	10	8	7	9	14												
58					7	7	8	7	*11	8	7	7	11	15											
60					7	7	9	7	10	10	8	7	9	15											
62					7	7	9	7	9	11	8	8	8	11	16										
64					7	8	8	8	8	*12	9	8	7	9	16										
66					8	*9	8	9	7	11	11	8	8	9	12	17									
68					8	8	8	10	7	10	12	8	8	7	10	17									
70					8	8	8	10	8	9	*13	10	8	8	9	13	18								
72					8	8	8	9	9	8	12	12	8	9	8	11	18								
74						8	9	9	10	7	10	13	9	9	8	9	13	19							
76						8	*10	9	11	8	10	*14	11	8	9	9	11	19							
78						9	9	9	11	9	9	13	13	8	9	8	9	14	20						
80						9	9	9	10	10	8	11	14	10	9	9	9	11	20						
82						9	9	9	9	11	8	10	*15	11	8	10	8	10	15	21					
84						9	9	10	9	12	9	10	14	14	9	10	9	9	12	21					
86						9	9	*11	9	12	10	9	12	15	11	9	10	9	11	15	22				
88						9	9	10	9	11	11	8	10	*16	12	8	10	9	9	13	22				
90						9	10	9	9	10	12	9	10	15	15	10	9	9	9	11	16	23			
92						*10	10	9	10	9	13	10	10	13	16	11	9	11	9	9	13	23			
94						9	9	9	11	9	13	11	9	11	*17	13	9	11	9	9	11	17	24		
96						9	9	9	*12	9	12	12	9	10	16	16	11	9	11	9	10	13	24		
98						9	9	9	11	9	11	13	10	10	13	17	11	9	11	9	9	11	17	25	
100						9	9	10	10	10	10	14	11	10	12	*18	14	10	10	10	10	11	14	25	
102						9	9	11	10	11	9	14	11	9	10	17	17	11	9	12	9	9	12	18	26
104						10	10	11	10	12	9	13	13	10	10	14	18	12	9	12	10	10	11	15	26

Figure 5: The value in each cell represents the diameter obtained from the Diameter Formula. The cells with a * represent the situation that subcase 3.4 occurs. The colored cells represent the faulty subcases of case 3 in the Diameter Formula, in which cells depicted with color red are for subcase 3.1, with green are for subcase 3.2, with color yellow are for subcase 3.3, and with color blue are for subcase 3.5.

3 Properties of subcase 3.4

Do notice that even though case 3 of the Diameter Formula is incorrect, our computer output shows that subcase 3.4 of the Diameter Formula is always correct. Hence in this section, we will focus on subcase 3.4. In particular, we will prove several properties; these properties are first observed from the statistic data in Section 2.

Recall that in the Diameter Formula, i and $\triangle$ are defined to be $i = \lceil \frac{N}{2(w+1)} \rceil$ and $\triangle = \frac{N}{2} \bmod (w+1)$. We first prove a lemma.

Lemma 2. *For a chordal ring network $CR(N, w)$, we have*

$$\frac{N}{2} = p \cdot (w+1) + \triangle$$

where

$$\begin{cases} p = i, & \text{if } \triangle = 0 \\ p = i - 1, & \text{if } \triangle > 0 \end{cases}.$$

Proof. This lemma follows from the definition of i and $\triangle$. ∎

We have run a computer program to check with the relation between N and w in $CR(N, w)$ for $N = 6, 8, \ldots, 5000$. We found that when subcase 3.4 occurs, we always have $N = k \cdot w + 1$, where $k \geq 3$ and k is an odd number. We now prove this property.

Theorem 3. *Suppose subcase* 3.4 *occurs. Then* $N \geq 28$, $w \geq 9$, *and* N *is of the form* $N = k \cdot w + 1$, *where* $k \geq 3$ *and* k *is an odd number.*

Proof. By the assumption that subcase 3.4 occurs, $\triangle = \frac{w+3}{2} - i$. When subcase 3.3 occurs, $\triangle = 0$. Hence when subcase 3.4 occurs, $\triangle > 0$. Therefore, by Lemma 2, we have

$$\frac{N}{2} = p \cdot (w + 1) + \triangle \tag{1}$$

where

$$\triangle = \frac{w+3}{2} - i \text{ and } p = i - 1.$$

It follows that $i \geq 1$. When case 3 occurs, $i \leq \frac{w-5}{2}$. Thus when subcase 3.4 occurs,

$$1 \leq i \leq \frac{w-5}{2}. \tag{2}$$

Since $i \geq 1$, it implies that $w \geq 7$. By (2), when $w = 7$, i must be 1 and $N = 8$ must hold, which contradicts the fact that the chord length must satisfy $w \leq \frac{N}{2}$. By (2), when $w = 9$, either $i = 1$ or $i = 2$. If $i = 1$, then $N = 10$, which contradicts with the fact that $w \leq \frac{N}{2}$. If $i = 2$, then $N = 28$. By the definition of $\triangle$, $\triangle = 4$. Since $4 = \frac{w+3}{2} - i$, the condition $\triangle = \frac{w+3}{2} - i$ holds and therefore subcase 3.4 occurs. Hence when subcase 3.4 occurs, $w \geq 9$. Since $w \geq 9$, by (2), we have $N \geq 28$ when subcase 3.4 occurs. Besides, $i \leq \frac{w-5}{2}$ and $w \geq 9$ together imply that when subcase 3.4 occurs, we have $i \geq 2$. By substituting $p = i - 1$ and $\triangle = \frac{w+3}{2} - i$ into (1), we have

$$\frac{N}{2} = (i - 1)(w + 1) + (\frac{w+3}{2} - i).$$

So $N = 2iw - w + 1 = (2i-1)w + 1$. Hence N is of the form $N = k \cdot w + 1$. Since $i \geq 2$, it follows that $k \geq 3$ and k is an odd number. ∎

For example the first and second pairs of (N, w) that satisfy subcase 3.4 are $(28, 9)$ and $(34, 11)$, in which $28 = 3 \cdot 9 + 1$ and $34 = 3 \cdot 11 + 1$. From now on, for each $CR(N, w)$ which satisfies subcase 3.4, we will write $N = k \times w + 1$, where $k \geq 3$ and k is an odd number.

Theorem 4. *Suppose $CR(N, w)$ satisfies subcase 3.4. Then the following two statements are equivalent:*

(i) *For each $k \geq 3$ where k is odd, i in the Diameter Formula is of the form $\frac{k+1}{2}$, the smallest $\triangle$ in the Diameter Formula is 4, and the set of all possible $\triangle$'s is $\{4, 5, 6, \ldots\}$.*

(ii) *For each $k \geq 3$ where k is odd, the smallest w of $CR(N, w)$ is $k + 6$ and the set of all possible w's is $\{k + 6, k + 8, k + 10, \ldots\}$.*

Proof. By the assumption that subcase 3.4 occurs, $\triangle = \frac{w+3}{2} - i$. When subcase 3.3 occurs, $\triangle = 0$. Hence when subcase 3.4 occurs, $\triangle > 0$. Therefore, by Lemma 2, we have

$$\frac{N}{2} = p \cdot (w + 1) + \triangle$$

where

$$\triangle = \frac{w+3}{2} - i \text{ and } p = i - 1.$$

Suppose (i) holds. Then $\frac{k \cdot w + 1}{2} = \frac{N}{2} = (i-1)(w+1) + \triangle \overset{i = \frac{k+1}{2}}{=} (\frac{k-1}{2})(w+1) + \triangle$ which implies that $w = k - 2 + 2 \cdot \triangle$. Since $\triangle \geq 4$ and $\triangle = \{4, 5, 6, \ldots\}$ for each k, we have $w \geq k + 6$ and $w = \{k + 6, k + 8, k + 10, ...\}$ for each k.

Suppose (ii) holds. Since $N = k \cdot w + 1$ and $w \geq k + 6$, we have

$$\frac{N}{2} = \frac{k-1}{2} \cdot (w + 1) + \frac{w-k+2}{2}$$

where

$$0 \leq w - k + 2 < w + 1$$

Thus $i = \lceil \frac{N}{2(w+1)} \rceil = \frac{k-1}{2} + 1 = \frac{k+1}{2}$ and $\triangle = \frac{w-k+2}{2} \geq \frac{(k+6)-k+2}{2} = 4$. Since $w = \{k+6, k+8, k+10, \ldots\}$ for each k, we have $\triangle = \{4, 5, 6, \ldots\}$ for each k. ∎

Theorem 5. *$CR(N, w)$ satisfies subcase 3.4 if and only if $N = k \cdot w + 1$ and $w \in \{k+6, k+8, \ldots\}$ for some odd number $k \geq 3$.*

Proof. The necessity of this theorem follows from Theorems 3 and 4. We now prove the sufficiency of this theorem. Assume that $N = k \cdot w + 1$ and $w \in \{k+6, k+8, \ldots\}$ for some odd number $k \geq 3$. From the Diameter Formula , $i = \lceil \frac{N}{2(w+1)} \rceil$ and $\triangle = \frac{N}{2} \bmod (w+1)$. Since $\frac{N}{2} = \frac{k \cdot w + 1}{2} = \frac{k-1}{2} \cdot (w+1) + \frac{w-k+2}{2}$, it follows that $i = \frac{k-1}{2} + 1 = \frac{k+1}{2}$ and $\triangle = \frac{w-k+2}{2}$. By the assumption that $w \in \{k+6, k+8, \ldots\}$, we have $k \leq w - 6$. This implies that $i \leq \frac{w-5}{2}$ and $\triangle = \frac{w+3}{2} - i$. Hence $CR(N, w)$ satisfies subcase 3.4. ∎

From our computer output, we observe that when $k = 3$, the set of all possible N's such that $CR(N, w)$ satisfies subcase 3.4 is $\{28, 34, 40, \ldots\}$ and when $k = 5$, the set of all possible N's such that $CR(N, w)$ satisfies subcase 3.4 is $\{56, 66, 76, \ldots\}$. We now prove that for a fixed k, the N's such that $CR(N, w)$ satisfies subcase 3.4 is of a specific form.

Theorem 6. *Given $k \geq 3$ and k is an odd number, let N_k be the first N such that $CR(N, w)$ satisfies subcase 3.4, i.e., $N = k \cdot w + 1$. Then all the N's such that $CR(N, w)$ satisfies subcase 3.4 are of the form $N = N_k + (2k) \cdot q$, where q is a nonnegative integer.*

Proof. By Theorem 5, $N_k = k \cdot w + 1$. Since k is fixed, by Theorem 5 and by the fact that the chord length must be an odd number, all the N's such that $CR(N, w)$ satisfies subcase 3.4 are of the form $N = k \cdot (w + 2q) + 1$, where q is a nonnegative integer. Hence $N = N_k + (2k) \cdot q$ and we have this theorem. ∎

4 Concluding remarks

In [1], Arden and Lee proposed a formula to compute the diameter of a chordal ring network; unfortunately, case 3 of the Diameter Formula is incorrect. In the thesis, we run a computer program to observe how bad the incorrect situation is and discuss the

observed properties from the subcase 3.4, which is the only correct subcase of case 3 in Diameter Formula. In particular, we find that the N that satisfies subcase 3.4 is of the specific form $N = k \cdot w + 1$, where k is an odd number, $k \geq 3$, and w is the chord length of the chordal ring network. We also conjecture that $\mathcal{D}(CR(N, w)) = \frac{k+w}{2}$ if $CR(N, w)$ satisfies subcase 3.4.

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A Appendix: the form of N for subcase 3.4

N	w	form of N
28	9	$3 \cdot w + 1$
34	11	$3 \cdot w + 1$
40	13	$3 \cdot w + 1$
46	15	$3 \cdot w + 1$
52	17	$3 \cdot w + 1$
56	11	$5 \cdot w + 1$
58	19	$3 \cdot w + 1$
64	21	$3 \cdot w + 1$
66	13	$5 \cdot w + 1$
70	23	$3 \cdot w + 1$
76	15	$5 \cdot w + 1$
76	25	$3 \cdot w + 1$
82	27	$3 \cdot w + 1$
86	17	$5 \cdot w + 1$
88	29	$3 \cdot w + 1$
92	13	$7 \cdot w + 1$
94	31	$3 \cdot w + 1$
96	19	$5 \cdot w + 1$
100	33	$3 \cdot w + 1$
106	15	$7 \cdot w + 1$
106	21	$5 \cdot w + 1$
106	35	$3 \cdot w + 1$
112	37	$3 \cdot w + 1$
116	23	$5 \cdot w + 1$
118	39	$3 \cdot w + 1$
120	17	$7 \cdot w + 1$
124	41	$3 \cdot w + 1$
126	25	$5 \cdot w + 1$
130	43	$3 \cdot w + 1$
134	19	$7 \cdot w + 1$
136	15	$9 \cdot w + 1$
136	27	$5 \cdot w + 1$
136	45	$3 \cdot w + 1$
142	47	$3 \cdot w + 1$
146	29	$5 \cdot w + 1$
148	21	$7 \cdot w + 1$
148	49	$3 \cdot w + 1$
154	17	$9 \cdot w + 1$
154	51	$3 \cdot w + 1$
156	31	$5 \cdot w + 1$
160	53	$3 \cdot w + 1$
162	23	$7 \cdot w + 1$
166	33	$5 \cdot w + 1$
166	55	$3 \cdot w + 1$
172	19	$9 \cdot w + 1$

N	w	Form of N
172	57	$3 \cdot w + 1$
176	25	$7 \cdot w + 1$
176	35	$5 \cdot w + 1$
178	59	$3 \cdot w + 1$
184	61	$3 \cdot w + 1$
186	37	$5 \cdot w + 1$
188	17	$11 \cdot w + 1$
190	21	$9 \cdot w + 1$
190	27	$7 \cdot w + 1$
190	63	$3 \cdot w + 1$
196	39	$5 \cdot w + 1$
196	65	$3 \cdot w + 1$
202	67	$3 \cdot w + 1$
204	29	$7 \cdot w + 1$
206	41	$5 \cdot w + 1$
208	23	$9 \cdot w + 1$
208	69	$3 \cdot w + 1$
210	19	$11 \cdot w + 1$
214	71	$3 \cdot w + 1$
216	43	$5 \cdot w + 1$
218	31	$7 \cdot w + 1$
220	73	$3 \cdot w + 1$
226	25	$9 \cdot w + 1$
226	45	$5 \cdot w + 1$
226	75	$3 \cdot w + 1$
232	21	$11 \cdot w + 1$
232	33	$7 \cdot w + 1$
232	77	$3 \cdot w + 1$
236	47	$5 \cdot w + 1$
238	79	$3 \cdot w + 1$
244	27	$9 \cdot w + 1$
244	81	$3 \cdot w + 1$
246	35	$7 \cdot w + 1$
246	49	$5 \cdot w + 1$
248	19	$13 \cdot w + 1$
250	83	$3 \cdot w + 1$
254	23	$11 \cdot w + 1$
256	51	$5 \cdot w + 1$
256	85	$3 \cdot w + 1$
260	37	$7 \cdot w + 1$
262	29	$9 \cdot w + 1$
262	87	$3 \cdot w + 1$
266	53	$5 \cdot w + 1$
268	89	$3 \cdot w + 1$
274	21	$13 \cdot w + 1$

N	w	Form of N
274	39	$7 \cdot w + 1$
274	91	$3 \cdot w + 1$
276	25	$11 \cdot w + 1$
276	55	$5 \cdot w + 1$
280	31	$9 \cdot w + 1$
280	93	$3 \cdot w + 1$
286	57	$5 \cdot w + 1$
286	95	$3 \cdot w + 1$
288	41	$7 \cdot w + 1$
292	97	$3 \cdot w + 1$
296	59	$5 \cdot w + 1$
298	27	$11 \cdot w + 1$
298	33	$9 \cdot w + 1$
298	99	$3 \cdot w + 1$
300	23	$13 \cdot w + 1$
302	43	$7 \cdot w + 1$
304	101	$3 \cdot w + 1$
306	61	$5 \cdot w + 1$
310	103	$3 \cdot w + 1$
316	21	$15 \cdot w + 1$
316	35	$9 \cdot w + 1$
316	45	$7 \cdot w + 1$
316	63	$5 \cdot w + 1$
316	105	$3 \cdot w + 1$
320	29	$11 \cdot w + 1$
322	107	$3 \cdot w + 1$
326	25	$13 \cdot w + 1$
326	65	$5 \cdot w + 1$
328	109	$3 \cdot w + 1$
330	47	$7 \cdot w + 1$
334	37	$9 \cdot w + 1$
334	111	$3 \cdot w + 1$
336	67	$5 \cdot w + 1$
340	113	$3 \cdot w + 1$
342	31	$11 \cdot w + 1$
344	49	$7 \cdot w + 1$
346	23	$15 \cdot w + 1$
346	69	$5 \cdot w + 1$
346	115	$3 \cdot w + 1$
352	27	$13 \cdot w + 1$
352	39	$9 \cdot w + 1$
352	117	$3 \cdot w + 1$
356	71	$5 \cdot w + 1$
358	51	$7 \cdot w + 1$
358	119	$3 \cdot w + 1$

N	w	Form of N
364	33	$11 \cdot w + 1$
364	121	$3 \cdot w + 1$
366	73	$5 \cdot w + 1$
370	41	$9 \cdot w + 1$
370	123	$3 \cdot w + 1$
372	53	$7 \cdot w + 1$
376	25	$15 \cdot w + 1$
376	75	$5 \cdot w + 1$
376	125	$3 \cdot w + 1$
378	29	$13 \cdot w + 1$
382	127	$3 \cdot w + 1$
386	35	$11 \cdot w + 1$
386	55	$7 \cdot w + 1$
386	77	$5 \cdot w + 1$
388	43	$9 \cdot w + 1$
388	129	$3 \cdot w + 1$
392	23	$17 \cdot w + 1$
394	131	$3 \cdot w + 1$
396	79	$5 \cdot w + 1$
400	57	$7 \cdot w + 1$
400	133	$3 \cdot w + 1$
404	31	$13 \cdot w + 1$
406	27	$15 \cdot w + 1$
406	45	$9 \cdot w + 1$
406	81	$5 \cdot w + 1$
406	135	$3 \cdot w + 1$
408	37	$11 \cdot w + 1$
412	137	$3 \cdot w + 1$
414	59	$7 \cdot w + 1$
416	83	$5 \cdot w + 1$
418	139	$3 \cdot w + 1$
424	47	$9 \cdot w + 1$
424	141	$3 \cdot w + 1$
426	25	$17 \cdot w + 1$
426	85	$5 \cdot w + 1$
428	61	$7 \cdot w + 1$
430	33	$13 \cdot w + 1$
430	39	$11 \cdot w + 1$
430	143	$3 \cdot w + 1$
436	29	$15 \cdot w + 1$
436	87	$5 \cdot w + 1$
436	145	$3 \cdot w + 1$
442	49	$9 \cdot w + 1$
442	63	$7 \cdot w + 1$
442	147	$3 \cdot w + 1$

N	w	Form of N
446	89	$5 \cdot w + 1$
448	149	$3 \cdot w + 1$
452	41	$11 \cdot w + 1$
454	151	$3 \cdot w + 1$
456	35	$13 \cdot w + 1$
456	65	$7 \cdot w + 1$
456	91	$5 \cdot w + 1$
460	27	$17 \cdot w + 1$
460	51	$9 \cdot w + 1$
460	153	$3 \cdot w + 1$
466	31	$15 \cdot w + 1$
466	93	$5 \cdot w + 1$
466	155	$3 \cdot w + 1$
470	67	$7 \cdot w + 1$
472	157	$3 \cdot w + 1$
474	43	$11 \cdot w + 1$
476	25	$19 \cdot w + 1$
476	95	$5 \cdot w + 1$
478	53	$9 \cdot w + 1$
478	159	$3 \cdot w + 1$
482	37	$13 \cdot w + 1$
484	69	$7 \cdot w + 1$
484	161	$3 \cdot w + 1$
486	97	$5 \cdot w + 1$
490	163	$3 \cdot w + 1$
494	29	$17 \cdot w + 1$
496	33	$15 \cdot w + 1$
496	45	$11 \cdot w + 1$
496	55	$9 \cdot w + 1$
496	99	$5 \cdot w + 1$
496	165	$3 \cdot w + 1$
498	71	$7 \cdot w + 1$
502	167	$3 \cdot w + 1$
506	101	$5 \cdot w + 1$
508	39	$13 \cdot w + 1$
508	169	$3 \cdot w + 1$
512	73	$7 \cdot w + 1$
514	27	$19 \cdot w + 1$
514	57	$9 \cdot w + 1$
514	171	$3 \cdot w + 1$
516	103	$5 \cdot w + 1$
518	47	$11 \cdot w + 1$
520	173	$3 \cdot w + 1$
526	35	$15 \cdot w + 1$
526	75	$7 \cdot w + 1$

N	w	Form of N
526	105	$5 \cdot w + 1$
526	175	$3 \cdot w + 1$
528	31	$17 \cdot w + 1$
532	59	$9 \cdot w + 1$
532	177	$3 \cdot w + 1$
534	41	$13 \cdot w + 1$
536	107	$5 \cdot w + 1$
538	179	$3 \cdot w + 1$
540	49	$11 \cdot w + 1$
540	77	$7 \cdot w + 1$
544	181	$3 \cdot w + 1$
546	109	$5 \cdot w + 1$
550	61	$9 \cdot w + 1$
550	183	$3 \cdot w + 1$
552	29	$19 \cdot w + 1$
554	79	$7 \cdot w + 1$
556	37	$15 \cdot w + 1$
556	111	$5 \cdot w + 1$
556	185	$3 \cdot w + 1$
560	43	$13 \cdot w + 1$
562	33	$17 \cdot w + 1$
562	51	$11 \cdot w + 1$
562	187	$3 \cdot w + 1$
566	113	$5 \cdot w + 1$
568	27	$21 \cdot w + 1$
568	63	$9 \cdot w + 1$
568	81	$7 \cdot w + 1$
568	189	$3 \cdot w + 1$
574	191	$3 \cdot w + 1$
576	115	$5 \cdot w + 1$
580	193	$3 \cdot w + 1$
582	83	$7 \cdot w + 1$
584	53	$11 \cdot w + 1$
586	39	$15 \cdot w + 1$
586	45	$13 \cdot w + 1$
586	65	$9 \cdot w + 1$
586	117	$5 \cdot w + 1$
586	195	$3 \cdot w + 1$
590	31	$19 \cdot w + 1$
592	197	$3 \cdot w + 1$
596	35	$17 \cdot w + 1$
596	85	$7 \cdot w + 1$
596	119	$5 \cdot w + 1$
598	199	$3 \cdot w + 1$
604	67	$9 \cdot w + 1$

N	w	Form of N
604	201	$3 \cdot w + 1$
606	55	$11 \cdot w + 1$
606	121	$5 \cdot w + 1$
610	29	$21 \cdot w + 1$
610	87	$7 \cdot w + 1$
610	203	$3 \cdot w + 1$
612	47	$13 \cdot w + 1$
616	41	$15 \cdot w + 1$
616	123	$5 \cdot w + 1$
616	205	$3 \cdot w + 1$
622	69	$9 \cdot w + 1$
622	207	$3 \cdot w + 1$
624	89	$7 \cdot w + 1$
626	125	$5 \cdot w + 1$
628	33	$19 \cdot w + 1$
628	57	$11 \cdot w + 1$
628	209	$3 \cdot w + 1$
630	37	$17 \cdot w + 1$
634	211	$3 \cdot w + 1$
636	127	$5 \cdot w + 1$
638	49	$13 \cdot w + 1$
638	91	$7 \cdot w + 1$
640	71	$9 \cdot w + 1$
640	213	$3 \cdot w + 1$
646	43	$15 \cdot w + 1$
646	129	$5 \cdot w + 1$
646	215	$3 \cdot w + 1$
650	59	$11 \cdot w + 1$
652	31	$21 \cdot w + 1$
652	93	$7 \cdot w + 1$
652	217	$3 \cdot w + 1$
656	131	$5 \cdot w + 1$
658	73	$9 \cdot w + 1$
658	219	$3 \cdot w + 1$
664	39	$17 \cdot w + 1$
664	51	$13 \cdot w + 1$
664	221	$3 \cdot w + 1$
666	35	$19 \cdot w + 1$
666	95	$7 \cdot w + 1$
666	133	$5 \cdot w + 1$
668	29	$23 \cdot w + 1$
670	223	$3 \cdot w + 1$
672	61	$11 \cdot w + 1$
676	45	$15 \cdot w + 1$
676	75	$9 \cdot w + 1$
676	135	$5 \cdot w + 1$
676	225	$3 \cdot w + 1$
680	97	$7 \cdot w + 1$
682	227	$3 \cdot w + 1$
686	137	$5 \cdot w + 1$
688	229	$3 \cdot w + 1$
690	53	$13 \cdot w + 1$
694	33	$21 \cdot w + 1$
694	63	$11 \cdot w + 1$
694	77	$9 \cdot w + 1$
694	99	$7 \cdot w + 1$
694	231	$3 \cdot w + 1$
696	139	$5 \cdot w + 1$
698	41	$17 \cdot w + 1$
700	233	$3 \cdot w + 1$
704	37	$19 \cdot w + 1$
706	47	$15 \cdot w + 1$
706	141	$5 \cdot w + 1$
706	235	$3 \cdot w + 1$
708	101	$7 \cdot w + 1$
712	79	$9 \cdot w + 1$
712	237	$3 \cdot w + 1$
714	31	$23 \cdot w + 1$
716	55	$13 \cdot w + 1$
716	65	$11 \cdot w + 1$
716	143	$5 \cdot w + 1$
718	239	$3 \cdot w + 1$
722	103	$7 \cdot w + 1$
724	241	$3 \cdot w + 1$
726	145	$5 \cdot w + 1$
730	81	$9 \cdot w + 1$
730	243	$3 \cdot w + 1$
732	43	$17 \cdot w + 1$
736	35	$21 \cdot w + 1$
736	49	$15 \cdot w + 1$
736	105	$7 \cdot w + 1$
736	147	$5 \cdot w + 1$
736	245	$3 \cdot w + 1$
738	67	$11 \cdot w + 1$
742	39	$19 \cdot w + 1$
742	57	$13 \cdot w + 1$
742	247	$3 \cdot w + 1$
746	149	$5 \cdot w + 1$
748	83	$9 \cdot w + 1$
748	249	$3 \cdot w + 1$
750	107	$7 \cdot w + 1$
754	251	$3 \cdot w + 1$
756	151	$5 \cdot w + 1$
760	33	$23 \cdot w + 1$
760	69	$11 \cdot w + 1$
760	253	$3 \cdot w + 1$
764	109	$7 \cdot w + 1$
766	45	$17 \cdot w + 1$
766	51	$15 \cdot w + 1$
766	85	$9 \cdot w + 1$
766	153	$5 \cdot w + 1$
766	255	$3 \cdot w + 1$
768	59	$13 \cdot w + 1$
772	257	$3 \cdot w + 1$
776	31	$25 \cdot w + 1$
776	155	$5 \cdot w + 1$
778	37	$21 \cdot w + 1$
778	111	$7 \cdot w + 1$
778	259	$3 \cdot w + 1$
780	41	$19 \cdot w + 1$
782	71	$11 \cdot w + 1$
784	87	$9 \cdot w + 1$
784	261	$3 \cdot w + 1$
786	157	$5 \cdot w + 1$
790	263	$3 \cdot w + 1$
792	113	$7 \cdot w + 1$
794	61	$13 \cdot w + 1$
796	53	$15 \cdot w + 1$
796	159	$5 \cdot w + 1$
796	265	$3 \cdot w + 1$
800	47	$17 \cdot w + 1$
802	89	$9 \cdot w + 1$
802	267	$3 \cdot w + 1$
804	73	$11 \cdot w + 1$
806	35	$23 \cdot w + 1$
806	115	$7 \cdot w + 1$
806	161	$5 \cdot w + 1$
808	269	$3 \cdot w + 1$
814	271	$3 \cdot w + 1$
816	163	$5 \cdot w + 1$
818	43	$19 \cdot w + 1$
820	39	$21 \cdot w + 1$
820	63	$13 \cdot w + 1$
820	91	$9 \cdot w + 1$
820	117	$7 \cdot w + 1$

N	w	Form of N
820	273	$3 \cdot w + 1$
826	33	$25 \cdot w + 1$
826	55	$15 \cdot w + 1$
826	75	$11 \cdot w + 1$
826	165	$5 \cdot w + 1$
826	275	$3 \cdot w + 1$
832	277	$3 \cdot w + 1$
834	49	$17 \cdot w + 1$
834	119	$7 \cdot w + 1$
836	167	$5 \cdot w + 1$
838	93	$9 \cdot w + 1$
838	279	$3 \cdot w + 1$
844	281	$3 \cdot w + 1$
846	65	$13 \cdot w + 1$
846	169	$5 \cdot w + 1$
848	77	$11 \cdot w + 1$
848	121	$7 \cdot w + 1$
850	283	$3 \cdot w + 1$
852	37	$23 \cdot w + 1$
856	45	$19 \cdot w + 1$
856	57	$15 \cdot w + 1$
856	95	$9 \cdot w + 1$
856	171	$5 \cdot w + 1$
856	285	$3 \cdot w + 1$
862	41	$21 \cdot w + 1$
862	123	$7 \cdot w + 1$
862	287	$3 \cdot w + 1$
866	173	$5 \cdot w + 1$
868	51	$17 \cdot w + 1$
868	289	$3 \cdot w + 1$
870	79	$11 \cdot w + 1$
872	67	$13 \cdot w + 1$
874	97	$9 \cdot w + 1$
874	291	$3 \cdot w + 1$
876	35	$25 \cdot w + 1$
876	125	$7 \cdot w + 1$
876	175	$5 \cdot w + 1$
880	293	$3 \cdot w + 1$
886	59	$15 \cdot w + 1$
886	177	$5 \cdot w + 1$
886	295	$3 \cdot w + 1$
890	127	$7 \cdot w + 1$
892	33	$27 \cdot w + 1$
892	81	$11 \cdot w + 1$
892	99	$9 \cdot w + 1$
892	297	$3 \cdot w + 1$
894	47	$19 \cdot w + 1$
896	179	$5 \cdot w + 1$
898	39	$23 \cdot w + 1$
898	69	$13 \cdot w + 1$
898	299	$3 \cdot w + 1$
902	53	$17 \cdot w + 1$
904	43	$21 \cdot w + 1$
904	129	$7 \cdot w + 1$
904	301	$3 \cdot w + 1$
906	181	$5 \cdot w + 1$
910	101	$9 \cdot w + 1$
910	303	$3 \cdot w + 1$
914	83	$11 \cdot w + 1$
916	61	$15 \cdot w + 1$
916	183	$5 \cdot w + 1$
916	305	$3 \cdot w + 1$
918	131	$7 \cdot w + 1$
922	307	$3 \cdot w + 1$
924	71	$13 \cdot w + 1$
926	37	$25 \cdot w + 1$
926	185	$5 \cdot w + 1$
928	103	$9 \cdot w + 1$
928	309	$3 \cdot w + 1$
932	49	$19 \cdot w + 1$
932	133	$7 \cdot w + 1$
934	311	$3 \cdot w + 1$
936	55	$17 \cdot w + 1$
936	85	$11 \cdot w + 1$
936	187	$5 \cdot w + 1$
940	313	$3 \cdot w + 1$
944	41	$23 \cdot w + 1$
946	35	$27 \cdot w + 1$
946	45	$21 \cdot w + 1$
946	63	$15 \cdot w + 1$
946	105	$9 \cdot w + 1$
946	135	$7 \cdot w + 1$
946	189	$5 \cdot w + 1$
946	315	$3 \cdot w + 1$
950	73	$13 \cdot w + 1$
952	317	$3 \cdot w + 1$
956	191	$5 \cdot w + 1$
958	87	$11 \cdot w + 1$
958	319	$3 \cdot w + 1$
960	137	$7 \cdot w + 1$
964	107	$9 \cdot w + 1$
964	321	$3 \cdot w + 1$
966	193	$5 \cdot w + 1$
970	51	$19 \cdot w + 1$
970	57	$17 \cdot w + 1$
970	323	$3 \cdot w + 1$
974	139	$7 \cdot w + 1$
976	39	$25 \cdot w + 1$
976	65	$15 \cdot w + 1$
976	75	$13 \cdot w + 1$
976	195	$5 \cdot w + 1$
976	325	$3 \cdot w + 1$
980	89	$11 \cdot w + 1$
982	109	$9 \cdot w + 1$
982	327	$3 \cdot w + 1$
986	197	$5 \cdot w + 1$
988	47	$21 \cdot w + 1$
988	141	$7 \cdot w + 1$
988	329	$3 \cdot w + 1$
990	43	$23 \cdot w + 1$
994	331	$3 \cdot w + 1$
996	199	$5 \cdot w + 1$
1000	37	$27 \cdot w + 1$
1000	111	$9 \cdot w + 1$
1000	333	$3 \cdot w + 1$
1002	77	$13 \cdot w + 1$
1002	91	$11 \cdot w + 1$
1002	143	$7 \cdot w + 1$
1004	59	$17 \cdot w + 1$
1006	67	$15 \cdot w + 1$
1006	201	$5 \cdot w + 1$
1006	335	$3 \cdot w + 1$
1008	53	$19 \cdot w + 1$
1012	337	$3 \cdot w + 1$
1016	35	$29 \cdot w + 1$
1016	145	$7 \cdot w + 1$
1016	203	$5 \cdot w + 1$
1018	113	$9 \cdot w + 1$
1018	339	$3 \cdot w + 1$
1024	93	$11 \cdot w + 1$
1024	341	$3 \cdot w + 1$
1026	41	$25 \cdot w + 1$
1026	205	$5 \cdot w + 1$
1028	79	$13 \cdot w + 1$
1030	49	$21 \cdot w + 1$

N	w	Form of N
1030	147	$7 \cdot w + 1$
1030	343	$3 \cdot w + 1$
1036	45	$23 \cdot w + 1$
1036	69	$15 \cdot w + 1$
1036	115	$9 \cdot w + 1$
1036	207	$5 \cdot w + 1$
1036	345	$3 \cdot w + 1$
1038	61	$17 \cdot w + 1$
1042	347	$3 \cdot w + 1$
1044	149	$7 \cdot w + 1$
1046	55	$19 \cdot w + 1$
1046	95	$11 \cdot w + 1$
1046	209	$5 \cdot w + 1$
1048	349	$3 \cdot w + 1$
1054	39	$27 \cdot w + 1$
1054	81	$13 \cdot w + 1$
1054	117	$9 \cdot w + 1$
1054	351	$3 \cdot w + 1$
1056	211	$5 \cdot w + 1$
1058	151	$7 \cdot w + 1$
1060	353	$3 \cdot w + 1$
1066	71	$15 \cdot w + 1$
1066	213	$5 \cdot w + 1$
1066	355	$3 \cdot w + 1$
1068	97	$11 \cdot w + 1$
1072	51	$21 \cdot w + 1$
1072	63	$17 \cdot w + 1$
1072	119	$9 \cdot w + 1$
1072	153	$7 \cdot w + 1$
1072	357	$3 \cdot w + 1$
1074	37	$29 \cdot w + 1$
1076	43	$25 \cdot w + 1$
1076	215	$5 \cdot w + 1$
1078	359	$3 \cdot w + 1$
1080	83	$13 \cdot w + 1$
1082	47	$23 \cdot w + 1$
1084	57	$19 \cdot w + 1$
1084	361	$3 \cdot w + 1$
1086	155	$7 \cdot w + 1$
1086	217	$5 \cdot w + 1$
1090	99	$11 \cdot w + 1$
1090	121	$9 \cdot w + 1$
1090	363	$3 \cdot w + 1$
1096	73	$15 \cdot w + 1$
1096	219	$5 \cdot w + 1$
1096	365	$3 \cdot w + 1$
1100	157	$7 \cdot w + 1$
1102	367	$3 \cdot w + 1$
1106	65	$17 \cdot w + 1$
1106	85	$13 \cdot w + 1$
1106	221	$5 \cdot w + 1$
1108	41	$27 \cdot w + 1$
1108	123	$9 \cdot w + 1$
1108	369	$3 \cdot w + 1$
1112	101	$11 \cdot w + 1$
1114	53	$21 \cdot w + 1$
1114	159	$7 \cdot w + 1$
1114	371	$3 \cdot w + 1$
1116	223	$5 \cdot w + 1$
1120	373	$3 \cdot w + 1$
1122	59	$19 \cdot w + 1$
1126	45	$25 \cdot w + 1$
1126	75	$15 \cdot w + 1$
1126	125	$9 \cdot w + 1$
1126	225	$5 \cdot w + 1$
1126	375	$3 \cdot w + 1$
1128	49	$23 \cdot w + 1$
1128	161	$7 \cdot w + 1$
1132	39	$29 \cdot w + 1$
1132	87	$13 \cdot w + 1$
1132	377	$3 \cdot w + 1$
1134	103	$11 \cdot w + 1$
1136	227	$5 \cdot w + 1$
1138	379	$3 \cdot w + 1$
1140	67	$17 \cdot w + 1$
1142	163	$7 \cdot w + 1$
1144	127	$9 \cdot w + 1$
1144	381	$3 \cdot w + 1$
1146	229	$5 \cdot w + 1$
1148	37	$31 \cdot w + 1$
1150	383	$3 \cdot w + 1$
1156	55	$21 \cdot w + 1$
1156	77	$15 \cdot w + 1$
1156	105	$11 \cdot w + 1$
1156	165	$7 \cdot w + 1$
1156	231	$5 \cdot w + 1$
1156	385	$3 \cdot w + 1$
1158	89	$13 \cdot w + 1$
1160	61	$19 \cdot w + 1$
1162	43	$27 \cdot w + 1$
1162	129	$9 \cdot w + 1$
1162	387	$3 \cdot w + 1$
1166	233	$5 \cdot w + 1$
1168	389	$3 \cdot w + 1$
1170	167	$7 \cdot w + 1$
1174	51	$23 \cdot w + 1$
1174	69	$17 \cdot w + 1$
1174	391	$3 \cdot w + 1$
1176	47	$25 \cdot w + 1$
1176	235	$5 \cdot w + 1$
1178	107	$11 \cdot w + 1$
1180	131	$9 \cdot w + 1$
1180	393	$3 \cdot w + 1$
1184	91	$13 \cdot w + 1$
1184	169	$7 \cdot w + 1$
1186	79	$15 \cdot w + 1$
1186	237	$5 \cdot w + 1$
1186	395	$3 \cdot w + 1$
1190	41	$29 \cdot w + 1$
1192	397	$3 \cdot w + 1$
1196	239	$5 \cdot w + 1$
1198	57	$21 \cdot w + 1$
1198	63	$19 \cdot w + 1$
1198	133	$9 \cdot w + 1$
1198	171	$7 \cdot w + 1$
1198	399	$3 \cdot w + 1$
1200	109	$11 \cdot w + 1$
1204	401	$3 \cdot w + 1$
1206	241	$5 \cdot w + 1$
1208	71	$17 \cdot w + 1$
1210	39	$31 \cdot w + 1$
1210	93	$13 \cdot w + 1$
1210	403	$3 \cdot w + 1$
1212	173	$7 \cdot w + 1$
1216	45	$27 \cdot w + 1$
1216	81	$15 \cdot w + 1$
1216	135	$9 \cdot w + 1$
1216	243	$5 \cdot w + 1$
1216	405	$3 \cdot w + 1$
1220	53	$23 \cdot w + 1$
1222	111	$11 \cdot w + 1$
1222	407	$3 \cdot w + 1$
1226	49	$25 \cdot w + 1$
1226	175	$7 \cdot w + 1$
1226	245	$5 \cdot w + 1$

N	w	Form of N
1228	409	$3 \cdot w + 1$
1234	137	$9 \cdot w + 1$
1234	411	$3 \cdot w + 1$
1236	65	$19 \cdot w + 1$
1236	95	$13 \cdot w + 1$
1236	247	$5 \cdot w + 1$
1240	59	$21 \cdot w + 1$
1240	177	$7 \cdot w + 1$
1240	413	$3 \cdot w + 1$
1242	73	$17 \cdot w + 1$
1244	113	$11 \cdot w + 1$
1246	83	$15 \cdot w + 1$
1246	249	$5 \cdot w + 1$
1246	415	$3 \cdot w + 1$
1248	43	$29 \cdot w + 1$
1252	139	$9 \cdot w + 1$
1252	417	$3 \cdot w + 1$
1254	179	$7 \cdot w + 1$
1256	251	$5 \cdot w + 1$
1258	419	$3 \cdot w + 1$
1262	97	$13 \cdot w + 1$
1264	421	$3 \cdot w + 1$
1266	55	$23 \cdot w + 1$
1266	115	$11 \cdot w + 1$
1266	253	$5 \cdot w + 1$
1268	181	$7 \cdot w + 1$
1270	47	$27 \cdot w + 1$
1270	141	$9 \cdot w + 1$
1270	423	$3 \cdot w + 1$
1272	41	$31 \cdot w + 1$
1274	67	$19 \cdot w + 1$
1276	51	$25 \cdot w + 1$
1276	75	$17 \cdot w + 1$
1276	85	$15 \cdot w + 1$
1276	255	$5 \cdot w + 1$
1276	425	$3 \cdot w + 1$
1282	61	$21 \cdot w + 1$
1282	183	$7 \cdot w + 1$
1282	427	$3 \cdot w + 1$
1286	257	$5 \cdot w + 1$
1288	39	$33 \cdot w + 1$
1288	99	$13 \cdot w + 1$
1288	117	$11 \cdot w + 1$
1288	143	$9 \cdot w + 1$
1288	429	$3 \cdot w + 1$

N	w	Form of N
1294	431	$3 \cdot w + 1$
1296	185	$7 \cdot w + 1$
1296	259	$5 \cdot w + 1$
1300	433	$3 \cdot w + 1$
1306	45	$29 \cdot w + 1$
1306	87	$15 \cdot w + 1$
1306	145	$9 \cdot w + 1$
1306	261	$5 \cdot w + 1$
1306	435	$3 \cdot w + 1$
1310	77	$17 \cdot w + 1$
1310	119	$11 \cdot w + 1$
1310	187	$7 \cdot w + 1$
1312	57	$23 \cdot w + 1$
1312	69	$19 \cdot w + 1$
1312	437	$3 \cdot w + 1$
1314	101	$13 \cdot w + 1$
1316	263	$5 \cdot w + 1$
1318	439	$3 \cdot w + 1$
1324	49	$27 \cdot w + 1$
1324	63	$21 \cdot w + 1$
1324	147	$9 \cdot w + 1$
1324	189	$7 \cdot w + 1$
1324	441	$3 \cdot w + 1$
1326	53	$25 \cdot w + 1$
1326	265	$5 \cdot w + 1$
1330	443	$3 \cdot w + 1$
1332	121	$11 \cdot w + 1$
1334	43	$31 \cdot w + 1$
1336	89	$15 \cdot w + 1$
1336	267	$5 \cdot w + 1$
1336	445	$3 \cdot w + 1$
1338	191	$7 \cdot w + 1$
1340	103	$13 \cdot w + 1$
1342	149	$9 \cdot w + 1$
1342	447	$3 \cdot w + 1$
1344	79	$17 \cdot w + 1$
1346	269	$5 \cdot w + 1$
1348	449	$3 \cdot w + 1$
1350	71	$19 \cdot w + 1$
1352	193	$7 \cdot w + 1$
1354	41	$33 \cdot w + 1$
1354	123	$11 \cdot w + 1$
1354	451	$3 \cdot w + 1$
1356	271	$5 \cdot w + 1$
1358	59	$23 \cdot w + 1$

N	w	Form of N
1360	151	$9 \cdot w + 1$
1360	453	$3 \cdot w + 1$
1364	47	$29 \cdot w + 1$
1366	65	$21 \cdot w + 1$
1366	91	$15 \cdot w + 1$
1366	105	$13 \cdot w + 1$
1366	195	$7 \cdot w + 1$
1366	273	$5 \cdot w + 1$
1366	455	$3 \cdot w + 1$
1372	457	$3 \cdot w + 1$
1376	55	$25 \cdot w + 1$
1376	125	$11 \cdot w + 1$
1376	275	$5 \cdot w + 1$
1378	51	$27 \cdot w + 1$
1378	81	$17 \cdot w + 1$
1378	153	$9 \cdot w + 1$
1378	459	$3 \cdot w + 1$
1380	197	$7 \cdot w + 1$
1384	461	$3 \cdot w + 1$
1386	277	$5 \cdot w + 1$
1388	73	$19 \cdot w + 1$
1390	463	$3 \cdot w + 1$
1392	107	$13 \cdot w + 1$
1394	199	$7 \cdot w + 1$
1396	45	$31 \cdot w + 1$
1396	93	$15 \cdot w + 1$
1396	155	$9 \cdot w + 1$
1396	279	$5 \cdot w + 1$
1396	465	$3 \cdot w + 1$
1398	127	$11 \cdot w + 1$
1402	467	$3 \cdot w + 1$
1404	61	$23 \cdot w + 1$
1406	281	$5 \cdot w + 1$
1408	67	$21 \cdot w + 1$
1408	201	$7 \cdot w + 1$
1408	469	$3 \cdot w + 1$
1412	83	$17 \cdot w + 1$
1414	157	$9 \cdot w + 1$
1414	471	$3 \cdot w + 1$
1416	283	$5 \cdot w + 1$
1418	109	$13 \cdot w + 1$
1420	43	$33 \cdot w + 1$
1420	129	$11 \cdot w + 1$
1420	473	$3 \cdot w + 1$
1422	49	$29 \cdot w + 1$

N	w	Form of N
1422	203	$7 \cdot w + 1$
1426	57	$25 \cdot w + 1$
1426	75	$19 \cdot w + 1$
1426	95	$15 \cdot w + 1$
1426	285	$5 \cdot w + 1$
1426	475	$3 \cdot w + 1$
1432	53	$27 \cdot w + 1$
1432	159	$9 \cdot w + 1$
1432	477	$3 \cdot w + 1$
1436	41	$35 \cdot w + 1$
1436	205	$7 \cdot w + 1$
1436	287	$5 \cdot w + 1$
1438	479	$3 \cdot w + 1$
1442	131	$11 \cdot w + 1$
1444	111	$13 \cdot w + 1$
1444	481	$3 \cdot w + 1$
1446	85	$17 \cdot w + 1$
1446	289	$5 \cdot w + 1$
1450	63	$23 \cdot w + 1$
1450	69	$21 \cdot w + 1$
1450	161	$9 \cdot w + 1$
1450	207	$7 \cdot w + 1$
1450	483	$3 \cdot w + 1$
1456	97	$15 \cdot w + 1$
1456	291	$5 \cdot w + 1$
1456	485	$3 \cdot w + 1$
1458	47	$31 \cdot w + 1$
1462	487	$3 \cdot w + 1$
1464	77	$19 \cdot w + 1$
1464	133	$11 \cdot w + 1$
1464	209	$7 \cdot w + 1$
1466	293	$5 \cdot w + 1$
1468	163	$9 \cdot w + 1$
1468	489	$3 \cdot w + 1$
1470	113	$13 \cdot w + 1$
1474	491	$3 \cdot w + 1$
1476	59	$25 \cdot w + 1$
1476	295	$5 \cdot w + 1$
1478	211	$7 \cdot w + 1$
1480	51	$29 \cdot w + 1$
1480	87	$17 \cdot w + 1$
1480	493	$3 \cdot w + 1$
1486	45	$33 \cdot w + 1$
1486	55	$27 \cdot w + 1$
1486	99	$15 \cdot w + 1$

N	w	Form of N
1486	135	$11 \cdot w + 1$
1486	165	$9 \cdot w + 1$
1486	297	$5 \cdot w + 1$
1486	495	$3 \cdot w + 1$
1492	71	$21 \cdot w + 1$
1492	213	$7 \cdot w + 1$
1492	497	$3 \cdot w + 1$
1496	65	$23 \cdot w + 1$
1496	115	$13 \cdot w + 1$
1496	299	$5 \cdot w + 1$
1498	499	$3 \cdot w + 1$
1502	79	$19 \cdot w + 1$
1504	167	$9 \cdot w + 1$
1504	501	$3 \cdot w + 1$
1506	43	$35 \cdot w + 1$
1506	215	$7 \cdot w + 1$
1506	301	$5 \cdot w + 1$
1508	137	$11 \cdot w + 1$
1510	503	$3 \cdot w + 1$
1514	89	$17 \cdot w + 1$
1516	101	$15 \cdot w + 1$
1516	303	$5 \cdot w + 1$
1516	505	$3 \cdot w + 1$
1520	49	$31 \cdot w + 1$
1520	217	$7 \cdot w + 1$
1522	117	$13 \cdot w + 1$
1522	169	$9 \cdot w + 1$
1522	507	$3 \cdot w + 1$
1526	61	$25 \cdot w + 1$
1526	305	$5 \cdot w + 1$
1528	509	$3 \cdot w + 1$
1530	139	$11 \cdot w + 1$
1534	73	$21 \cdot w + 1$
1534	219	$7 \cdot w + 1$
1534	511	$3 \cdot w + 1$
1536	307	$5 \cdot w + 1$
1538	53	$29 \cdot w + 1$
1540	57	$27 \cdot w + 1$
1540	81	$19 \cdot w + 1$
1540	171	$9 \cdot w + 1$
1540	513	$3 \cdot w + 1$
1542	67	$23 \cdot w + 1$
1546	103	$15 \cdot w + 1$
1546	309	$5 \cdot w + 1$
1546	515	$3 \cdot w + 1$

N	w	Form of N
1548	91	$17 \cdot w + 1$
1548	119	$13 \cdot w + 1$
1548	221	$7 \cdot w + 1$
1552	47	$33 \cdot w + 1$
1552	141	$11 \cdot w + 1$
1552	517	$3 \cdot w + 1$
1556	311	$5 \cdot w + 1$
1558	173	$9 \cdot w + 1$
1558	519	$3 \cdot w + 1$
1562	223	$7 \cdot w + 1$
1564	521	$3 \cdot w + 1$
1566	313	$5 \cdot w + 1$
1570	523	$3 \cdot w + 1$
1574	121	$13 \cdot w + 1$
1574	143	$11 \cdot w + 1$
1576	45	$35 \cdot w + 1$
1576	63	$25 \cdot w + 1$
1576	75	$21 \cdot w + 1$
1576	105	$15 \cdot w + 1$
1576	175	$9 \cdot w + 1$
1576	225	$7 \cdot w + 1$
1576	315	$5 \cdot w + 1$
1576	525	$3 \cdot w + 1$
1578	83	$19 \cdot w + 1$
1582	51	$31 \cdot w + 1$
1582	93	$17 \cdot w + 1$
1582	527	$3 \cdot w + 1$
1586	317	$5 \cdot w + 1$
1588	69	$23 \cdot w + 1$
1588	529	$3 \cdot w + 1$
1590	227	$7 \cdot w + 1$
1592	43	$37 \cdot w + 1$
1594	59	$27 \cdot w + 1$
1594	177	$9 \cdot w + 1$
1594	531	$3 \cdot w + 1$
1596	55	$29 \cdot w + 1$
1596	145	$11 \cdot w + 1$
1596	319	$5 \cdot w + 1$
1600	123	$13 \cdot w + 1$
1600	533	$3 \cdot w + 1$
1604	229	$7 \cdot w + 1$
1606	107	$15 \cdot w + 1$
1606	321	$5 \cdot w + 1$
1606	535	$3 \cdot w + 1$
1612	179	$9 \cdot w + 1$

N	w	Form of N
1612	537	$3 \cdot w + 1$
1616	85	$19 \cdot w + 1$
1616	95	$17 \cdot w + 1$
1616	323	$5 \cdot w + 1$
1618	49	$33 \cdot w + 1$
1618	77	$21 \cdot w + 1$
1618	147	$11 \cdot w + 1$
1618	231	$7 \cdot w + 1$
1618	539	$3 \cdot w + 1$
1624	541	$3 \cdot w + 1$
1626	65	$25 \cdot w + 1$
1626	125	$13 \cdot w + 1$
1626	325	$5 \cdot w + 1$
1630	181	$9 \cdot w + 1$
1630	543	$3 \cdot w + 1$
1632	233	$7 \cdot w + 1$
1634	71	$23 \cdot w + 1$
1636	109	$15 \cdot w + 1$
1636	327	$5 \cdot w + 1$
1636	545	$3 \cdot w + 1$
1640	149	$11 \cdot w + 1$
1642	547	$3 \cdot w + 1$
1644	53	$31 \cdot w + 1$
1646	47	$35 \cdot w + 1$
1646	235	$7 \cdot w + 1$
1646	329	$5 \cdot w + 1$
1648	61	$27 \cdot w + 1$
1648	183	$9 \cdot w + 1$
1648	549	$3 \cdot w + 1$
1650	97	$17 \cdot w + 1$
1652	127	$13 \cdot w + 1$
1654	57	$29 \cdot w + 1$
1654	87	$19 \cdot w + 1$
1654	551	$3 \cdot w + 1$
1656	331	$5 \cdot w + 1$
1660	79	$21 \cdot w + 1$
1660	237	$7 \cdot w + 1$
1660	553	$3 \cdot w + 1$
1662	151	$11 \cdot w + 1$
1666	45	$37 \cdot w + 1$
1666	111	$15 \cdot w + 1$
1666	185	$9 \cdot w + 1$
1666	333	$5 \cdot w + 1$
1666	555	$3 \cdot w + 1$
1672	557	$3 \cdot w + 1$

N	w	Form of N
1674	239	$7 \cdot w + 1$
1676	67	$25 \cdot w + 1$
1676	335	$5 \cdot w + 1$
1678	129	$13 \cdot w + 1$
1678	559	$3 \cdot w + 1$
1680	73	$23 \cdot w + 1$
1684	51	$33 \cdot w + 1$
1684	99	$17 \cdot w + 1$
1684	153	$11 \cdot w + 1$
1684	187	$9 \cdot w + 1$
1684	561	$3 \cdot w + 1$
1686	337	$5 \cdot w + 1$
1688	241	$7 \cdot w + 1$
1690	563	$3 \cdot w + 1$
1692	89	$19 \cdot w + 1$
1696	113	$15 \cdot w + 1$
1696	339	$5 \cdot w + 1$
1696	565	$3 \cdot w + 1$
1702	63	$27 \cdot w + 1$
1702	81	$21 \cdot w + 1$
1702	189	$9 \cdot w + 1$
1702	243	$7 \cdot w + 1$
1702	567	$3 \cdot w + 1$
1704	131	$13 \cdot w + 1$
1706	55	$31 \cdot w + 1$
1706	155	$11 \cdot w + 1$
1706	341	$5 \cdot w + 1$
1708	569	$3 \cdot w + 1$
1712	59	$29 \cdot w + 1$
1714	571	$3 \cdot w + 1$
1716	49	$35 \cdot w + 1$
1716	245	$7 \cdot w + 1$
1716	343	$5 \cdot w + 1$
1718	101	$17 \cdot w + 1$
1720	191	$9 \cdot w + 1$
1720	573	$3 \cdot w + 1$
1726	69	$25 \cdot w + 1$
1726	75	$23 \cdot w + 1$
1726	115	$15 \cdot w + 1$
1726	345	$5 \cdot w + 1$
1726	575	$3 \cdot w + 1$
1728	157	$11 \cdot w + 1$
1730	91	$19 \cdot w + 1$
1730	133	$13 \cdot w + 1$
1730	247	$7 \cdot w + 1$

N	w	Form of N
1732	577	$3 \cdot w + 1$
1736	347	$5 \cdot w + 1$
1738	193	$9 \cdot w + 1$
1738	579	$3 \cdot w + 1$
1740	47	$37 \cdot w + 1$
1744	83	$21 \cdot w + 1$
1744	249	$7 \cdot w + 1$
1744	581	$3 \cdot w + 1$
1746	349	$5 \cdot w + 1$
1750	53	$33 \cdot w + 1$
1750	159	$11 \cdot w + 1$
1750	583	$3 \cdot w + 1$
1752	103	$17 \cdot w + 1$
1756	45	$39 \cdot w + 1$
1756	65	$27 \cdot w + 1$
1756	117	$15 \cdot w + 1$
1756	135	$13 \cdot w + 1$
1756	195	$9 \cdot w + 1$
1756	351	$5 \cdot w + 1$
1756	585	$3 \cdot w + 1$
1758	251	$7 \cdot w + 1$
1762	587	$3 \cdot w + 1$
1766	353	$5 \cdot w + 1$
1768	57	$31 \cdot w + 1$
1768	93	$19 \cdot w + 1$
1768	589	$3 \cdot w + 1$
1770	61	$29 \cdot w + 1$
1772	77	$23 \cdot w + 1$
1772	161	$11 \cdot w + 1$
1772	253	$7 \cdot w + 1$
1774	197	$9 \cdot w + 1$
1774	591	$3 \cdot w + 1$
1776	71	$25 \cdot w + 1$
1776	355	$5 \cdot w + 1$
1780	593	$3 \cdot w + 1$
1782	137	$13 \cdot w + 1$
1786	51	$35 \cdot w + 1$
1786	85	$21 \cdot w + 1$
1786	105	$17 \cdot w + 1$
1786	119	$15 \cdot w + 1$
1786	255	$7 \cdot w + 1$
1786	357	$5 \cdot w + 1$
1786	595	$3 \cdot w + 1$
1792	199	$9 \cdot w + 1$
1792	597	$3 \cdot w + 1$

N	w	Form of N
1794	163	$11 \cdot w + 1$
1796	359	$5 \cdot w + 1$
1798	599	$3 \cdot w + 1$
1800	257	$7 \cdot w + 1$
1804	601	$3 \cdot w + 1$
1806	95	$19 \cdot w + 1$
1806	361	$5 \cdot w + 1$
1808	139	$13 \cdot w + 1$
1810	67	$27 \cdot w + 1$
1810	201	$9 \cdot w + 1$
1810	603	$3 \cdot w + 1$
1814	49	$37 \cdot w + 1$
1814	259	$7 \cdot w + 1$
1816	55	$33 \cdot w + 1$
1816	121	$15 \cdot w + 1$
1816	165	$11 \cdot w + 1$
1816	363	$5 \cdot w + 1$
1816	605	$3 \cdot w + 1$
1818	79	$23 \cdot w + 1$
1820	107	$17 \cdot w + 1$
1822	607	$3 \cdot w + 1$
1826	73	$25 \cdot w + 1$
1826	365	$5 \cdot w + 1$
1828	63	$29 \cdot w + 1$
1828	87	$21 \cdot w + 1$
1828	203	$9 \cdot w + 1$
1828	261	$7 \cdot w + 1$
1828	609	$3 \cdot w + 1$
1830	59	$31 \cdot w + 1$
1834	47	$39 \cdot w + 1$
1834	141	$13 \cdot w + 1$
1834	611	$3 \cdot w + 1$
1836	367	$5 \cdot w + 1$
1838	167	$11 \cdot w + 1$
1840	613	$3 \cdot w + 1$
1842	263	$7 \cdot w + 1$
1844	97	$19 \cdot w + 1$
1846	123	$15 \cdot w + 1$
1846	205	$9 \cdot w + 1$
1846	369	$5 \cdot w + 1$
1846	615	$3 \cdot w + 1$
1852	617	$3 \cdot w + 1$
1854	109	$17 \cdot w + 1$
1856	53	$35 \cdot w + 1$
1856	265	$7 \cdot w + 1$

N	w	Form of N
1856	371	$5 \cdot w + 1$
1858	619	$3 \cdot w + 1$
1860	143	$13 \cdot w + 1$
1860	169	$11 \cdot w + 1$
1864	69	$27 \cdot w + 1$
1864	81	$23 \cdot w + 1$
1864	207	$9 \cdot w + 1$
1864	621	$3 \cdot w + 1$
1866	373	$5 \cdot w + 1$
1870	89	$21 \cdot w + 1$
1870	267	$7 \cdot w + 1$
1870	623	$3 \cdot w + 1$
1876	75	$25 \cdot w + 1$
1876	125	$15 \cdot w + 1$
1876	375	$5 \cdot w + 1$
1876	625	$3 \cdot w + 1$
1882	57	$33 \cdot w + 1$
1882	99	$19 \cdot w + 1$
1882	171	$11 \cdot w + 1$
1882	209	$9 \cdot w + 1$
1882	627	$3 \cdot w + 1$
1884	269	$7 \cdot w + 1$
1886	65	$29 \cdot w + 1$
1886	145	$13 \cdot w + 1$
1886	377	$5 \cdot w + 1$
1888	51	$37 \cdot w + 1$
1888	111	$17 \cdot w + 1$
1888	629	$3 \cdot w + 1$
1892	61	$31 \cdot w + 1$
1894	631	$3 \cdot w + 1$
1896	379	$5 \cdot w + 1$
1898	271	$7 \cdot w + 1$
1900	211	$9 \cdot w + 1$
1900	633	$3 \cdot w + 1$
1904	173	$11 \cdot w + 1$
1906	127	$15 \cdot w + 1$
1906	381	$5 \cdot w + 1$
1906	635	$3 \cdot w + 1$
1910	83	$23 \cdot w + 1$
1912	49	$39 \cdot w + 1$
1912	91	$21 \cdot w + 1$
1912	147	$13 \cdot w + 1$
1912	273	$7 \cdot w + 1$
1912	637	$3 \cdot w + 1$
1916	383	$5 \cdot w + 1$

N	w	Form of N
1918	71	$27 \cdot w + 1$
1918	213	$9 \cdot w + 1$
1918	639	$3 \cdot w + 1$
1920	101	$19 \cdot w + 1$
1922	113	$17 \cdot w + 1$
1924	641	$3 \cdot w + 1$
1926	55	$35 \cdot w + 1$
1926	77	$25 \cdot w + 1$
1926	175	$11 \cdot w + 1$
1926	275	$7 \cdot w + 1$
1926	385	$5 \cdot w + 1$
1928	47	$41 \cdot w + 1$
1930	643	$3 \cdot w + 1$
1936	129	$15 \cdot w + 1$
1936	215	$9 \cdot w + 1$
1936	387	$5 \cdot w + 1$
1936	645	$3 \cdot w + 1$
1938	149	$13 \cdot w + 1$
1940	277	$7 \cdot w + 1$
1942	647	$3 \cdot w + 1$
1944	67	$29 \cdot w + 1$
1946	389	$5 \cdot w + 1$
1948	59	$33 \cdot w + 1$
1948	177	$11 \cdot w + 1$
1948	649	$3 \cdot w + 1$
1954	63	$31 \cdot w + 1$
1954	93	$21 \cdot w + 1$
1954	217	$9 \cdot w + 1$
1954	279	$7 \cdot w + 1$
1954	651	$3 \cdot w + 1$
1956	85	$23 \cdot w + 1$
1956	115	$17 \cdot w + 1$
1956	391	$5 \cdot w + 1$
1958	103	$19 \cdot w + 1$
1960	653	$3 \cdot w + 1$
1962	53	$37 \cdot w + 1$
1964	151	$13 \cdot w + 1$
1966	131	$15 \cdot w + 1$
1966	393	$5 \cdot w + 1$
1966	655	$3 \cdot w + 1$
1968	281	$7 \cdot w + 1$
1970	179	$11 \cdot w + 1$
1972	73	$27 \cdot w + 1$
1972	219	$9 \cdot w + 1$
1972	657	$3 \cdot w + 1$

N	w	Form of N
1976	79	$25 \cdot w + 1$
1976	395	$5 \cdot w + 1$
1978	659	$3 \cdot w + 1$
1982	283	$7 \cdot w + 1$
1984	661	$3 \cdot w + 1$
1986	397	$5 \cdot w + 1$
1990	51	$39 \cdot w + 1$
1990	117	$17 \cdot w + 1$
1990	153	$13 \cdot w + 1$
1990	221	$9 \cdot w + 1$
1990	663	$3 \cdot w + 1$
1992	181	$11 \cdot w + 1$
1996	57	$35 \cdot w + 1$
1996	95	$21 \cdot w + 1$
1996	105	$19 \cdot w + 1$
1996	133	$15 \cdot w + 1$
1996	285	$7 \cdot w + 1$
1996	399	$5 \cdot w + 1$
1996	665	$3 \cdot w + 1$
2002	69	$29 \cdot w + 1$
2002	87	$23 \cdot w + 1$
2002	667	$3 \cdot w + 1$
2006	401	$5 \cdot w + 1$
2008	223	$9 \cdot w + 1$
2008	669	$3 \cdot w + 1$
2010	49	$41 \cdot w + 1$
2010	287	$7 \cdot w + 1$
2014	61	$33 \cdot w + 1$
2014	183	$11 \cdot w + 1$
2014	671	$3 \cdot w + 1$
2016	65	$31 \cdot w + 1$
2016	155	$13 \cdot w + 1$
2016	403	$5 \cdot w + 1$
2020	673	$3 \cdot w + 1$
2024	119	$17 \cdot w + 1$
2024	289	$7 \cdot w + 1$
2026	75	$27 \cdot w + 1$
2026	81	$25 \cdot w + 1$
2026	135	$15 \cdot w + 1$
2026	225	$9 \cdot w + 1$
2026	405	$5 \cdot w + 1$
2026	675	$3 \cdot w + 1$
2032	677	$3 \cdot w + 1$
2034	107	$19 \cdot w + 1$
2036	55	$37 \cdot w + 1$

N	w	Form of N
2036	185	$11 \cdot w + 1$
2036	407	$5 \cdot w + 1$
2038	97	$21 \cdot w + 1$
2038	291	$7 \cdot w + 1$
2038	679	$3 \cdot w + 1$
2042	157	$13 \cdot w + 1$
2044	227	$9 \cdot w + 1$
2044	681	$3 \cdot w + 1$
2046	409	$5 \cdot w + 1$
2048	89	$23 \cdot w + 1$
2050	683	$3 \cdot w + 1$
2052	293	$7 \cdot w + 1$
2056	137	$15 \cdot w + 1$
2056	411	$5 \cdot w + 1$
2056	685	$3 \cdot w + 1$
2058	121	$17 \cdot w + 1$
2058	187	$11 \cdot w + 1$
2060	71	$29 \cdot w + 1$
2062	229	$9 \cdot w + 1$
2062	687	$3 \cdot w + 1$
2066	59	$35 \cdot w + 1$
2066	295	$7 \cdot w + 1$
2066	413	$5 \cdot w + 1$
2068	53	$39 \cdot w + 1$
2068	159	$13 \cdot w + 1$
2068	689	$3 \cdot w + 1$
2072	109	$19 \cdot w + 1$
2074	691	$3 \cdot w + 1$
2076	83	$25 \cdot w + 1$
2076	415	$5 \cdot w + 1$
2078	67	$31 \cdot w + 1$
2080	63	$33 \cdot w + 1$
2080	77	$27 \cdot w + 1$
2080	99	$21 \cdot w + 1$
2080	189	$11 \cdot w + 1$
2080	231	$9 \cdot w + 1$
2080	297	$7 \cdot w + 1$
2080	693	$3 \cdot w + 1$
2086	139	$15 \cdot w + 1$
2086	417	$5 \cdot w + 1$
2086	695	$3 \cdot w + 1$
2092	51	$41 \cdot w + 1$
2092	123	$17 \cdot w + 1$
2092	697	$3 \cdot w + 1$
2094	91	$23 \cdot w + 1$

N	w	Form of N
2094	161	$13 \cdot w + 1$
2094	299	$7 \cdot w + 1$
2096	419	$5 \cdot w + 1$
2098	233	$9 \cdot w + 1$
2098	699	$3 \cdot w + 1$
2102	191	$11 \cdot w + 1$
2104	701	$3 \cdot w + 1$
2106	421	$5 \cdot w + 1$
2108	49	$43 \cdot w + 1$
2108	301	$7 \cdot w + 1$
2110	57	$37 \cdot w + 1$
2110	111	$19 \cdot w + 1$
2110	703	$3 \cdot w + 1$
2116	141	$15 \cdot w + 1$
2116	235	$9 \cdot w + 1$
2116	423	$5 \cdot w + 1$
2116	705	$3 \cdot w + 1$
2118	73	$29 \cdot w + 1$
2120	163	$13 \cdot w + 1$
2122	101	$21 \cdot w + 1$
2122	303	$7 \cdot w + 1$
2122	707	$3 \cdot w + 1$
2124	193	$11 \cdot w + 1$
2126	85	$25 \cdot w + 1$
2126	125	$17 \cdot w + 1$
2126	425	$5 \cdot w + 1$
2128	709	$3 \cdot w + 1$
2134	79	$27 \cdot w + 1$
2134	237	$9 \cdot w + 1$
2134	711	$3 \cdot w + 1$
2136	61	$35 \cdot w + 1$
2136	305	$7 \cdot w + 1$
2136	427	$5 \cdot w + 1$
2140	69	$31 \cdot w + 1$
2140	93	$23 \cdot w + 1$
2140	713	$3 \cdot w + 1$
2146	55	$39 \cdot w + 1$
2146	65	$33 \cdot w + 1$
2146	143	$15 \cdot w + 1$
2146	165	$13 \cdot w + 1$
2146	195	$11 \cdot w + 1$
2146	429	$5 \cdot w + 1$
2146	715	$3 \cdot w + 1$
2148	113	$19 \cdot w + 1$
2150	307	$7 \cdot w + 1$

B Appendix: the source code of our program

```
//File Name: Diameter_Of_Chordal_Ring_Network
//Author: 劉士慶
//Email address: maruco.am96g@g2.nctu.edu.tw
//Description: Using breadth first search (BFS) to obtain the diameter of a chordal ring network CR(N,w)
//Input: None
//Output: The minimum diameter among all chordal ring networks with N nodes, the diameter of a particular
//        chordal ring network CR(N,w), and the incorrect cases in the Diameter Formula

#include <iostream>
#include <iomanip>
#include <cmath>
#include <fstream>
using namespace std;

int f(int i, int N);
//compute the value of i mod N

int diameter1(int N, int w, int& thm_i_case, int& thm_delta_case);
//compute the diameter of CR(N,w) using the ceiling of i

int diameter2(int N, int w);
//compute the diameter of CR(N,w) using the floor of i

const int MAX_N = 100; //the maximum number of nodes

int main()
{

    int k;                  //a variable used to control the iteration of a for-loop
    int N, w ;              //N: number of nodes in CR(N,w), w: the chord length of CR(N,w)

    char t;                 //a variable used to determine if diameter, diameter1, and diameter2 are equivalent

    int candidate;          //the first int from the queue

    int neb1, neb2, neb3;   //the three neighbors of a node

    int d[MAX_N];           //d[i] is the distance from node 0 to node i

    int Q[MAX_N];           //the queue used in BFS
    int Q_end;              //the tail of the queue

    int diameter;           //the diameter obtained by BFS
    int c_diameter;         //the diameter obtained by using the ceiling of i in the Diameter Formula
    int f_diameter,         //the diameter obtained by using the floor of i in the Diameter Formula
    int thm_i_case;         //the i used in the Diameter Formula
    int thm_delta_case;     //the delta used in the Diameter Formula

    int min_diameter;       //the minimum diameter among all chordal ring networks with N nodes
    int number_of_Nw_pairs;     //the number of (N,w) pairs
    int number_of_all_fault;    //the number of all faulty (N,w) pairs

    double percentage_of_fault;  //faulty percentage of all case in Diameter Formula

    ofstream out_stream;

    out_stream.open("output.txt");
```

```
number_of_Nw_pairs = 0;
number_of_all_fault = 0;

for(N=6; N<=MAX_N; N+=2)
{
	out_stream<<setw(3)<<"N"<< setw(4) << "w" <<setw(6)
	<<"diam"<<setw(8)<<"c_diam"<<setw(8)<<"f_diam"<<setw(7)<<"c/f/!"<<endl;

	out_stream << "-----------------------------------------------------" << endl;

	min_diameter = N-1;
	for(w=3; w<=N/2; w+=2)
	{
		cout << N << " " << w << endl;

		number_of_Nw_pairs++;

		Q[0]=0;
		Q_end=0;
		d[0]= 0;

		for(k=1;k<N;k++)
			d[k]=-1;

		//compute the distance from 0 to each node by using BFS
		while (Q_end>=0)
		{
			candidate = Q[0];

			//remove candidate from Q
			for (k=1;k<=Q_end;k++)
			{
				Q[k-1]=Q[k];
			}
			Q_end--;

			//find the three neighbors of candidate
			neb1 = f(candidate + 1, N);
			neb2 = f(candidate - 1, N);

			//if candidate is even, the neighbor linked by chord is candidate+w;
			//if candidate is odd, the neighbor linked by chord is candidate-w
			if ((candidate%2)==1)
				neb3 = f(candidate + w, N);
			else
				neb3 = f(candidate - w, N);

			//if a neighbor is never visited before, then its distance to 0 is d[candidate]+1
			if (d[neb1]==-1)
			{
				Q_end++;
				Q[Q_end] = neb1;
				d[neb1] = d[candidate]+1;
			}
			if (d[neb2]==-1)
			{
				Q_end++;
				Q[Q_end] = neb2;
```

```
                        d[neb2] = d[candidate]+1;
                    }
                    if (d[neb3]==-1)
                    {
                        Q_end++;
                        Q[Q_end] = neb3;
                        d[neb3] = d[candidate]+1;
                    }
                }

                //find the farthest distance to node 0 and set this distance to be the diameter
                diameter=d[0];
                for(k=1; k<N; k++)
                    if (d[k]>diameter)
                        diameter=d[k];

                //obtain the minimum diameter
                if (diameter < min_diameter)
                        min_diameter = diameter;

                c_diameter = diameter1(N,w,thm_i_case,thm_delta_case);
                f_diameter = diameter2(N,w);
                if((diameter==c_diameter) && (c_diameter==f_diameter)) t='c';
                if((diameter==c_diameter) && (diameter!=f_diameter)) t='c';
                if((diameter!=c_diameter) && (diameter==f_diameter)) t='f';
                if((diameter!=c_diameter) && (diameter!=f_diameter)) t='!';
                if (t =='f' || t == '!') number_of_all_fault++;

                out_stream<<setw(3)<< N << setw(5) << w << setw(6) <<diameter
                    <<setw(8)<<c_diameter<<setw(8)<<f_diameter<< setw(6)<<t;

                if (t =='f' || t == '!')
                {
                    out_stream << " ##" ;
                    out_stream << "i-case = " << thm_i_case << " delta-case = " << thm_delta_case;
                }
                 out_stream <<endl;
            }

                out_stream << endl;

            out_stream << "min diameter = " << min_diameter << endl;
            out_stream << "-----------------------------------------------------" << endl;
        }

        percentage_of_fault = number_of_all_fault;
        percentage_of_fault = (percentage_of_fault/number_of_Nw_pairs)*100;
        out_stream << "total faulty percentage of Case 3 = " << percentage_of_fault << endl;

        return 0;
}

int f(int i, int N)
{
        while( i<0)
            i = i + N;
        return i%N;
}
```

```
//by using the Diameter Formula and taking i's ceiling to obtain the diameter
int diameter1(int N, int w, int& thm_i_case, int& thm_delta_case)
{
	int i, delta, d;

	i = ceil(static_cast<double>(N)/static_cast<double>(2*(w+1)));

	delta = (N/2) % (w+1);

	if (i >= (w-1)/2)
	{
		thm_i_case = 1;
		if (delta == 1)
		{
			d = 2*i-1;
			thm_delta_case = 1;
		}
		if (2 <= delta && delta <= (w+3)/2)
		{
			d = 2*i;
			thm_delta_case = 2;
		}
		if (delta == 0 || ((w+5)/2 <= delta && delta <= w))
		{
			d = 2*i+1;
			thm_delta_case = 3;
		}
	}

	if (i == (w-3)/2)
	{
		thm_i_case = 2;
		if (1 <= delta && delta <= 2)
		{
			d = w-3;
			thm_delta_case = 1;
		}
		if (delta == 0 || (3 <= delta && delta <= w))
		{
			d = w-2;
			thm_delta_case = 2;
		}
	}

	if (i < (w-3)/2)
	{
		thm_i_case = 3;
		if (1 <= delta && delta <= (w+1)/2-i)
		{
			d = i+(w-3)/2;
			thm_delta_case = 1;
		}
		if ((w+5)/2-i <= delta && delta <= w-i)
		{
			d = i+(w-3)/2;
			thm_delta_case = 2;
		}
```

```
        if (delta == 0)
        {
            d = i+(w-1)/2;
            thm_delta_case = 3;
        }
        if (delta == (w+3)/2-i)
        {
            d = i+(w-1)/2;
            thm_delta_case = 4;
        }
        if (w-i+1 <= delta && delta <= w)
        {
            d = i+(w-1)/2;
            thm_delta_case = 5;
        }
    }

    return d;
}

//by using the Diameter Formula and taking i's floor to obtain the diameter
int diameter2(int N, int w)
{
    int i, delta, d;

    i = floor(static_cast<double>(N)/static_cast<double>(2*(w+1)));

    delta = (N/2) % (w+1);

    if (i >= (w-1)/2)
    {
        if (delta == 1)
            d = 2*i-1;
        if (2 <= delta && delta <= (w+3)/2)
            d = 2*i;
        if (delta == 0 || ((w+5)/2 <= delta && delta <= w))
            d = 2*i+1;
    }

    if (i == (w-3)/2)
    {
        if (1 <= delta && delta <= 2)
            d = w-3;
        if (delta == 0 || (3 <= delta && delta <= w))
            d = w-2;
    }

    if (i < (w-3)/2)
    {
        if ((1 <= delta && delta <= (w+1)/2-i) || ((w+5)/2-i <= delta && delta <= w-i))
            d = i+(w-3)/2;
        if (delta == 0 || (delta == (w+3)/2-i) || (w-i+1 <= delta && delta <= w))
            d = i+(w-1)/2;
    }

    return d;
}
```

C Appendix: the output of our program

N	w	diam	c_diam	f_diam	c/f/!
6	3	2	2	1	c

min diameter = 2

N	w	diam	c_diam	f_diam	c/f/!
8	3	3	3	3	c

min diameter = 3

N	w	diam	c_diam	f_diam	c/f/!
10	3	3	3	1	c
10	5	3	3	1	c

min diameter = 3

N	w	diam	c_diam	f_diam	c/f/!
12	3	4	4	2	c
12	5	3	3	3	c

min diameter = 3

N	w	diam	c_diam	f_diam	c/f/!
14	3	4	4	2	c
14	5	3	3	2	c
14	7	4	4	2	c

min diameter = 3

N	w	diam	c_diam	f_diam	c/f/!
16	3	5	5	5	c
16	5	4	4	2	c
16	7	4	4	4	c

min diameter = 4

N	w	diam	c_diam	f_diam	c/f/!
18	3	5	5	3	c
18	5	4	4	3	c
18	7	4	4	3	c
18	9	5	5	3	c

min diameter = 4

N	w	diam	c_diam	f_diam	c/f/!
20	3	6	6	4	c
20	5	4	4	3	c
20	7	4	4	3	c
20	9	5	5	5	c

min diameter = 4

N	w	diam	c_diam	f_diam	c/f/!
22	3	6	6	4	c
22	5	5	5	3	c
22	7	5	5	3	c
22	9	5	5	4	c
22	11	6	6	4	c

min diameter = 5

N	w	diam	c_diam	f_diam	c/f/!
24	3	7	7	7	c
24	5	5	5	5	c
24	7	5	5	4	c
24	9	5	5	4	c
24	11	6	6	6	c

min diameter = 5

N	w	diam	c_diam	f_diam	c/f/!
26	3	7	7	5	c
26	5	5	5	3	c
26	7	5	5	3	c
26	9	5	5	4	c
26	11	5	6	5	f ## i-case = 3 delta-case = 1
26	13	7	7	5	c

min diameter = 5

N	w	diam	c_diam	f_diam	c/f/!
28	3	8	8	6	c
28	5	6	6	4	c
28	7	5	5	3	c
28	9	6	6	4	c
28	11	5	6	5	f ## i-case = 3 delta-case = 1
28	13	7	7	7	c

min diameter = 5

N	w	diam	c_diam	f_diam	c/f/!
30	3	8	8	6	c
30	5	6	6	4	c
30	7	5	5	4	c
30	9	5	5	5	c
30	11	5	6	5	f ## i-case = 3 delta-case = 1
30	13	6	7	6	f ## i-case = 3 delta-case = 1
30	15	8	8	6	c

min diameter = 5

N	w	diam	c_diam	f_diam	c/f/!
32	3	9	9	9	c
32	5	6	6	4	c
32	7	5	5	5	c
32	9	5	5	4	c
32	11	6	6	5	c
32	13	5	7	6	! ## i-case = 3 delta-case = 1
32	15	8	8	8	c

min diameter = 5

N	w	diam	c_diam	f_diam	c/f/!
34	3	9	9	7	c
34	5	7	7	5	c
34	7	5	5	4	c
34	9	5	5	4	c
34	11	7	7	5	c
34	13	5	7	6	! ## i-case = 3 delta-case = 1
34	15	7	8	7	f ## i-case = 3 delta-case = 1
34	17	9	9	7	c

min diameter = 5

N	w	diam	c_diam	f_diam	c/f/!
36	3	10	10	8	c
36	5	7	7	7	c
36	7	6	6	4	c
36	9	6	6	4	c
36	11	6	6	6	c
36	13	6	7	6	f ## i-case = 3 delta-case = 1
36	15	6	8	7	! ## i-case = 3 delta-case = 1
36	17	9	9	9	c

min diameter = 6

N	w	diam	c_diam	f_diam	c/f/!
38	3	10	10	8	c
38	5	7	7	5	c
38	7	6	6	5	c
38	9	6	6	5	c
38	11	6	6	5	c
38	13	7	7	6	c
38	15	5	8	7	! ## i-case = 3 delta-case = 1
38	17	7	9	8	! ## i-case = 3 delta-case = 1
38	19	10	10	8	c

min diameter = 5

N	w	diam	c_diam	f_diam	c/f/!
40	3	11	11	11	c
40	5	8	8	6	c
40	7	6	6	5	c
40	9	6	6	6	c
40	11	6	6	5	c
40	13	8	8	6	c

40	15	6	8	7	! ## i-case = 3 delta-case = 1
40	17	7	9	8	! ## i-case = 3 delta-case = 1
40	19	10	10	10	c

min diameter = 6

N	w	diam	c_diam	f_diam	c/f/!
42	3	11	11	9	c
42	5	8	8	6	c
42	7	6	6	5	c
42	9	6	6	5	c
42	11	6	6	5	c
42	13	7	7	7	c
42	15	7	8	7	f ## i-case = 3 delta-case = 1
42	17	6	9	8	! ## i-case = 3 delta-case = 1
42	19	8	10	9	! ## i-case = 3 delta-case = 1
42	21	11	11	9	c

min diameter = 6

N	w	diam	c_diam	f_diam	c/f/!
44	3	12	12	10	c
44	5	8	8	6	c
44	7	7	7	5	c
44	9	6	6	5	c
44	11	7	7	5	c
44	13	7	7	6	c
44	15	8	8	7	c
44	17	6	9	8	! ## i-case = 3 delta-case = 1
44	19	7	10	9	! ## i-case = 3 delta-case = 1
44	21	11	11	11	c

min diameter = 6

N	w	diam	c_diam	f_diam	c/f/!
46	3	12	12	10	c
46	5	9	9	7	c
46	7	7	7	5	c
46	9	7	7	5	c
46	11	7	7	6	c
46	13	7	7	6	c
46	15	9	9	7	c
46	17	7	9	8	! ## i-case = 3 delta-case = 1
46	19	7	10	9	! ## i-case = 3 delta-case = 1
46	21	9	11	10	! ## i-case = 3 delta-case = 1
46	23	12	12	10	c

min diameter = 7

N	w	diam	c_diam	f_diam	c/f/!
48	3	13	13	13	c
48	5	9	9	9	c
48	7	7	7	7	c
48	9	7	7	6	c
48	11	7	7	7	c
48	13	7	7	6	c
48	15	8	8	8	c
48	17	8	9	8	f ## i-case = 3 delta-case = 1
48	19	6	10	9	! ## i-case = 3 delta-case = 1
48	21	7	11	10	! ## i-case = 3 delta-case = 1
48	23	12	12	12	c

min diameter = 6

N	w	diam	c_diam	f_diam	c/f/!
50	3	13	13	11	c
50	5	9	9	7	c
50	7	7	7	5	c
50	9	7	7	5	c
50	11	7	7	6	c
50	13	7	7	6	c
50	15	7	8	7	f ## i-case = 3 delta-case = 2
50	17	9	9	8	c
50	19	7	10	9	! ## i-case = 3 delta-case = 1
50	21	7	11	10	! ## i-case = 3 delta-case = 1
50	23	9	12	11	! ## i-case = 3 delta-case = 1
50	25	13	13	11	c

min diameter = 7

N	w	diam	c_diam	f_diam	c/f/!
52	3	14	14	12	c
52	5	10	10	8	c
52	7	8	8	6	c
52	9	7	7	5	c
52	11	7	7	6	c
52	13	8	8	6	c
52	15	7	8	7	f ## i-case = 3 delta-case = 2
52	17	10	10	8	c
52	19	8	10	9	! ## i-case = 3 delta-case = 1
52	21	7	11	10	! ## i-case = 3 delta-case = 1
52	23	8	12	11	! ## i-case = 3 delta-case = 1
52	25	13	13	13	c

min diameter = 7

N	w	diam	c_diam	f_diam	c/f/!
54	3	14	14	12	c
54	5	10	10	8	c
54	7	8	8	6	c
54	9	7	7	5	c
54	11	7	7	6	c
54	13	8	8	7	c
54	15	7	8	7	f ## i-case = 3 delta-case = 2
54	17	9	9	9	c
54	19	9	10	9	f ## i-case = 3 delta-case = 1
54	21	7	11	10	! ## i-case = 3 delta-case = 1
54	23	7	12	11	! ## i-case = 3 delta-case = 1
54	25	10	13	12	! ## i-case = 3 delta-case = 1
54	27	14	14	12	c

min diameter = 7

N	w	diam	c_diam	f_diam	c/f/!
56	3	15	15	15	c
56	5	10	10	8	c
56	7	8	8	6	c
56	9	7	7	6	c
56	11	8	8	6	c
56	13	7	8	8	! ## i-case = 3 delta-case = 3
56	15	7	8	7	f ## i-case = 3 delta-case = 2
56	17	8	9	8	f ## i-case = 3 delta-case = 2
56	19	10	10	9	c
56	21	8	11	10	! ## i-case = 3 delta-case = 1
56	23	7	12	11	! ## i-case = 3 delta-case = 1
56	25	9	13	12	! ## i-case = 3 delta-case = 1
56	27	14	14	14	c

min diameter = 7

N	w	diam	c_diam	f_diam	c/f/!
58	3	15	15	13	c
58	5	11	11	9	c
58	7	8	8	6	c
58	9	7	7	6	c
58	11	7	7	7	c
58	13	7	8	7	f ## i-case = 3 delta-case = 1
58	15	8	8	7	c
58	17	7	9	8	! ## i-case = 3 delta-case = 2
58	19	11	11	9	c
58	21	8	11	10	! ## i-case = 3 delta-case = 1
58	23	7	12	11	! ## i-case = 3 delta-case = 1
58	25	7	13	12	! ## i-case = 3 delta-case = 1
58	27	11	14	13	! ## i-case = 3 delta-case = 1
58	29	15	15	13	c

min diameter = 7

N	w	diam	c_diam	f_diam	c/f/!
60	3	16	16	14	c
60	5	11	11	11	c
60	7	9	9	7	c
60	9	7	7	7	c
60	11	7	7	6	c
60	13	7	8	7	f ## i-case = 3 delta-case = 1
60	15	9	9	7	c
60	17	7	9	8	! ## i-case = 3 delta-case = 2
60	19	10	10	10	c
60	21	10	11	10	f ## i-case = 3 delta-case = 1
60	23	8	12	11	! ## i-case = 3 delta-case = 1
60	25	7	13	12	! ## i-case = 3 delta-case = 1
60	27	9	14	13	! ## i-case = 3 delta-case = 1
60	29	15	15	15	c

min diameter = 7

N	w	diam	c_diam	f_diam	c/f/!
62	3	16	16	14	c
62	5	11	11	9	c
62	7	9	9	7	c
62	9	7	7	6	c
62	11	7	7	6	c
62	13	7	8	7	f ## i-case = 3 delta-case = 1
62	15	9	9	8	c
62	17	7	9	8	! ## i-case = 3 delta-case = 2
62	19	9	10	9	f ## i-case = 3 delta-case = 2
62	21	11	11	10	c
62	23	8	12	11	! ## i-case = 3 delta-case = 1
62	25	8	13	12	! ## i-case = 3 delta-case = 1
62	27	8	14	13	! ## i-case = 3 delta-case = 1
62	29	11	15	14	! ## i-case = 3 delta-case = 1
62	31	16	16	14	c

min diameter = 7

N	w	diam	c_diam	f_diam	c/f/!
64	3	17	17	17	c
64	5	12	12	10	c
64	7	9	9	9	c
64	9	8	8	6	c
64	11	7	7	6	c
64	13	8	8	7	c
64	15	8	9	9	! ## i-case = 3 delta-case = 3
64	17	8	9	8	f ## i-case = 3 delta-case = 2
64	19	8	10	9	! ## i-case = 3 delta-case = 2
64	21	12	12	10	c
64	23	9	12	11	! ## i-case = 3 delta-case = 1
64	25	8	13	12	! ## i-case = 3 delta-case = 1
64	27	7	14	13	! ## i-case = 3 delta-case = 1
64	29	9	15	14	! ## i-case = 3 delta-case = 1
64	31	16	16	16	c

min diameter = 7

N	w	diam	c_diam	f_diam	c/f/!
66	3	17	17	15	c
66	5	12	12	10	c
66	7	9	9	7	c
66	9	8	8	7	c
66	11	8	8	6	c
66	13	9	9	7	c
66	15	8	9	8	f ## i-case = 3 delta-case = 1
66	17	9	9	8	c
66	19	7	10	9	! ## i-case = 3 delta-case = 2
66	21	11	11	11	c
66	23	11	12	11	f ## i-case = 3 delta-case = 1
66	25	8	13	12	! ## i-case = 3 delta-case = 1
66	27	8	14	13	! ## i-case = 3 delta-case = 1
66	29	9	15	14	! ## i-case = 3 delta-case = 1
66	31	12	16	15	! ## i-case = 3 delta-case = 1
66	33	17	17	15	c

min diameter = 7

N	w	diam	c_diam	f_diam	c/f/!
68	3	18	18	16	c
68	5	12	12	10	c
68	7	10	10	8	c
68	9	8	8	7	c
68	11	8	8	7	c
68	13	8	8	8	c
68	15	8	9	8	f ## i-case = 3 delta-case = 1
68	17	10	10	8	c
68	19	7	10	9	! ## i-case = 3 delta-case = 2
68	21	10	11	10	f ## i-case = 3 delta-case = 2
68	23	12	12	11	c
68	25	8	13	12	! ## i-case = 3 delta-case = 1
68	27	8	14	13	! ## i-case = 3 delta-case = 1
68	29	7	15	14	! ## i-case = 3 delta-case = 1
68	31	10	16	15	! ## i-case = 3 delta-case = 1
68	33	17	17	17	c

min diameter = 7

N	w	diam	c_diam	f_diam	c/f/!
70	3	18	18	16	c
70	5	13	13	11	c
70	7	10	10	8	c
70	9	8	8	7	c
70	11	8	8	7	c
70	13	8	8	7	c
70	15	8	9	8	f ## i-case = 3 delta-case = 1
70	17	10	10	9	c
70	19	8	10	9	! ## i-case = 3 delta-case = 2
70	21	9	11	10	! ## i-case = 3 delta-case = 2
70	23	13	13	11	c
70	25	10	13	12	! ## i-case = 3 delta-case = 1
70	27	8	14	13	! ## i-case = 3 delta-case = 1
70	29	8	15	14	! ## i-case = 3 delta-case = 1
70	31	9	16	15	! ## i-case = 3 delta-case = 1
70	33	13	17	16	! ## i-case = 3 delta-case = 1
70	35	18	18	16	c

min diameter = 8

N	w	diam	c_diam	f_diam	c/f/!
72	3	19	19	19	c
72	5	13	13	13	c
72	7	10	10	8	c
72	9	8	8	7	c
72	11	8	8	8	c
72	13	8	8	7	c
72	15	8	9	8	f ## i-case = 3 delta-case = 1
72	17	9	10	10	! ## i-case = 3 delta-case = 3
72	19	9	10	9	f ## i-case = 3 delta-case = 2
72	21	8	11	10	! ## i-case = 3 delta-case = 2
72	23	12	12	12	c
72	25	12	13	12	f ## i-case = 3 delta-case = 1
72	27	8	14	13	! ## i-case = 3 delta-case = 1
72	29	9	15	14	! ## i-case = 3 delta-case = 1
72	31	8	16	15	! ## i-case = 3 delta-case = 1
72	33	11	17	16	! ## i-case = 3 delta-case = 1
72	35	18	18	18	c

min diameter = 8

N	w	diam	c_diam	f_diam	c/f/!
74	3	19	19	17	c
74	5	13	13	11	c
74	7	10	10	8	c
74	9	9	9	7	c
74	11	8	8	7	c
74	13	8	8	7	c
74	15	9	9	8	c
74	17	9	10	9	f ## i-case = 3 delta-case = 1
74	19	10	10	9	c
74	21	7	11	10	! ## i-case = 3 delta-case = 2
74	23	10	12	11	! ## i-case = 3 delta-case = 2
74	25	13	13	12	c
74	27	9	14	13	! ## i-case = 3 delta-case = 1
74	29	9	15	14	! ## i-case = 3 delta-case = 1
74	31	8	16	15	! ## i-case = 3 delta-case = 1
74	33	9	17	16	! ## i-case = 3 delta-case = 1
74	35	13	18	17	! ## i-case = 3 delta-case = 1
74	37	19	19	17	c

min diameter = 7

N	w	diam	c_diam	f_diam	c/f/!
76	3	20	20	18	c
76	5	14	14	12	c
76	7	11	11	9	c
76	9	9	9	7	c
76	11	8	8	7	c
76	13	8	8	7	c
76	15	10	10	8	c
76	17	9	10	9	f ## i-case = 3 delta-case = 1
76	19	11	11	9	c
76	21	8	11	10	! ## i-case = 3 delta-case = 2
76	23	10	12	11	! ## i-case = 3 delta-case = 2
76	25	14	14	12	c
76	27	11	14	13	! ## i-case = 3 delta-case = 1
76	29	8	15	14	! ## i-case = 3 delta-case = 1
76	31	9	16	15	! ## i-case = 3 delta-case = 1
76	33	9	17	16	! ## i-case = 3 delta-case = 1
76	35	11	18	17	! ## i-case = 3 delta-case = 1
76	37	19	19	19	c

min diameter = 8

N	w	diam	c_diam	f_diam	c/f/!
78	3	20	20	18	c
78	5	14	14	12	c
78	7	11	11	9	c
78	9	9	9	7	c
78	11	9	9	7	c
78	13	9	9	7	c
78	15	9	9	9	c
78	17	9	10	9	f ## i-case = 3 delta-case = 1
78	19	11	11	10	c
78	21	9	11	10	! ## i-case = 3 delta-case = 2
78	23	9	12	11	! ## i-case = 3 delta-case = 2
78	25	13	13	13	c
78	27	13	14	13	f ## i-case = 3 delta-case = 1
78	29	8	15	14	! ## i-case = 3 delta-case = 1
78	31	9	16	15	! ## i-case = 3 delta-case = 1
78	33	8	17	16	! ## i-case = 3 delta-case = 1
78	35	9	18	17	! ## i-case = 3 delta-case = 1
78	37	14	19	18	! ## i-case = 3 delta-case = 1
78	39	20	20	18	c

min diameter = 8

N	w	diam	c_diam	f_diam	c/f/!
80	3	21	21	21	c
80	5	14	14	12	c
80	7	11	11	11	c
80	9	9	9	9	c
80	11	9	9	8	c
80	13	9	9	8	c
80	15	9	9	8	c
80	17	9	10	9	f ## i-case = 3 delta-case = 1
80	19	10	11	11	! ## i-case = 3 delta-case = 3
80	21	10	11	10	f ## i-case = 3 delta-case = 2
80	23	8	12	11	! ## i-case = 3 delta-case = 2
80	25	11	13	12	! ## i-case = 3 delta-case = 2
80	27	14	14	13	c
80	29	10	15	14	! ## i-case = 3 delta-case = 1
80	31	9	16	15	! ## i-case = 3 delta-case = 1
80	33	9	17	16	! ## i-case = 3 delta-case = 1
80	35	9	18	17	! ## i-case = 3 delta-case = 1
80	37	11	19	18	! ## i-case = 3 delta-case = 1
80	39	20	20	20	c

min diameter = 8

N	w	diam	c_diam	f_diam	c/f/!
82	3	21	21	19	c
82	5	15	15	13	c
82	7	11	11	9	c
82	9	9	9	7	c
82	11	9	9	7	c
82	13	9	9	8	c
82	15	9	9	8	c
82	17	9	10	9	f ## i-case = 3 delta-case = 1
82	19	9	11	10	! ## i-case = 3 delta-case = 1
82	21	11	11	10	c
82	23	8	12	11	! ## i-case = 3 delta-case = 2
82	25	10	13	12	! ## i-case = 3 delta-case = 2
82	27	15	15	13	c
82	29	11	15	14	! ## i-case = 3 delta-case = 1
82	31	8	16	15	! ## i-case = 3 delta-case = 1
82	33	10	17	16	! ## i-case = 3 delta-case = 1
82	35	8	18	17	! ## i-case = 3 delta-case = 1
82	37	10	19	18	! ## i-case = 3 delta-case = 1
82	39	15	20	19	! ## i-case = 3 delta-case = 1
82	41	21	21	19	c

min diameter = 8

N	w	diam	c_diam	f_diam	c/f/!
84	3	22	22	20	c
84	5	15	15	15	c
84	7	12	12	10	c
84	9	10	10	8	c
84	11	9	9	7	c
84	13	9	9	9	c
84	15	9	9	8	c
84	17	10	10	9	c
84	19	9	11	10	! ## i-case = 3 delta-case = 1
84	21	12	12	10	c
84	23	9	12	11	! ## i-case = 3 delta-case = 2
84	25	10	13	12	! ## i-case = 3 delta-case = 2
84	27	14	14	14	c
84	29	14	15	14	f ## i-case = 3 delta-case = 1
84	31	9	16	15	! ## i-case = 3 delta-case = 1
84	33	10	17	16	! ## i-case = 3 delta-case = 1
84	35	9	18	17	! ## i-case = 3 delta-case = 1
84	37	9	19	18	! ## i-case = 3 delta-case = 1
84	39	12	20	19	! ## i-case = 3 delta-case = 1
84	41	21	21	21	c

min diameter = 9

N	w	diam	c_diam	f_diam	c/f/!
86	3	22	22	20	c
86	5	15	15	13	c
86	7	12	12	10	c
86	9	10	10	8	c
86	11	9	9	7	c
86	13	9	9	8	c
86	15	9	9	8	c
86	17	11	11	9	c
86	19	9	11	10	! ## i-case = 3 delta-case = 1
86	21	12	12	11	c
86	23	10	12	11	! ## i-case = 3 delta-case = 2
86	25	9	13	12	! ## i-case = 3 delta-case = 2
86	27	12	14	13	! ## i-case = 3 delta-case = 2
86	29	15	15	14	c
86	31	11	16	15	! ## i-case = 3 delta-case = 1
86	33	9	17	16	! ## i-case = 3 delta-case = 1
86	35	10	18	17	! ## i-case = 3 delta-case = 1
86	37	9	19	18	! ## i-case = 3 delta-case = 1
86	39	11	20	19	! ## i-case = 3 delta-case = 1
86	41	15	21	20	! ## i-case = 3 delta-case = 1
86	43	22	22	20	c

min diameter = 9

N	w	diam	c_diam	f_diam	c/f/!
88	3	23	23	23	c
88	5	16	16	14	c
88	7	12	12	10	c
88	9	10	10	8	c
88	11	9	9	7	c
88	13	9	9	8	c
88	15	9	9	8	c
88	17	10	10	10	c
88	19	9	11	10	! ## i-case = 3 delta-case = 1
88	21	11	12	12	! ## i-case = 3 delta-case = 3
88	23	11	12	11	f ## i-case = 3 delta-case = 2
88	25	8	13	12	! ## i-case = 3 delta-case = 2
88	27	10	14	13	! ## i-case = 3 delta-case = 2
88	29	16	16	14	c
88	31	12	16	15	! ## i-case = 3 delta-case = 1
88	33	8	17	16	! ## i-case = 3 delta-case = 1
88	35	10	18	17	! ## i-case = 3 delta-case = 1
88	37	9	19	18	! ## i-case = 3 delta-case = 1
88	39	9	20	19	! ## i-case = 3 delta-case = 1
88	41	13	21	20	! ## i-case = 3 delta-case = 1
88	43	22	22	22	c

min diameter = 8

N	w	diam	c_diam	f_diam	c/f/!
90	3	23	23	21	c
90	5	16	16	14	c
90	7	12	12	10	c
90	9	10	10	8	c
90	11	9	9	8	c
90	13	9	9	8	c
90	15	10	10	8	c
90	17	9	10	9	f ## i-case = 3 delta-case = 2
90	19	9	11	10	! ## i-case = 3 delta-case = 1
90	21	10	12	11	! ## i-case = 3 delta-case = 1
90	23	12	12	11	c
90	25	9	13	12	! ## i-case = 3 delta-case = 2
90	27	10	14	13	! ## i-case = 3 delta-case = 2
90	29	15	15	15	c
90	31	15	16	15	f ## i-case = 3 delta-case = 1
90	33	10	17	16	! ## i-case = 3 delta-case = 1
90	35	9	18	17	! ## i-case = 3 delta-case = 1
90	37	9	19	18	! ## i-case = 3 delta-case = 1
90	39	9	20	19	! ## i-case = 3 delta-case = 1
90	41	11	21	20	! ## i-case = 3 delta-case = 1
90	43	16	22	21	! ## i-case = 3 delta-case = 1
90	45	23	23	21	c

min diameter = 9

N	w	diam	c_diam	f_diam	c/f/!
92	3	24	24	22	c
92	5	16	16	14	c
92	7	13	13	11	c
92	9	10	10	8	c
92	11	9	9	8	c
92	13	10	10	8	c
92	15	10	10	9	c
92	17	9	10	9	f ## i-case = 3 delta-case = 2
92	19	10	11	10	f ## i-case = 3 delta-case = 1
92	21	9	12	11	! ## i-case = 3 delta-case = 1
92	23	13	13	11	c
92	25	10	13	12	! ## i-case = 3 delta-case = 2
92	27	10	14	13	! ## i-case = 3 delta-case = 2
92	29	13	15	14	! ## i-case = 3 delta-case = 2
92	31	16	16	15	c
92	33	11	17	16	! ## i-case = 3 delta-case = 1
92	35	9	18	17	! ## i-case = 3 delta-case = 1
92	37	11	19	18	! ## i-case = 3 delta-case = 1
92	39	9	20	19	! ## i-case = 3 delta-case = 1
92	41	9	21	20	! ## i-case = 3 delta-case = 1
92	43	13	22	21	! ## i-case = 3 delta-case = 1
92	45	23	23	23	c

min diameter = 9

N	w	diam	c_diam	f_diam	c/f/!
94	3	24	24	22	c
94	5	17	17	15	c
94	7	13	13	11	c
94	9	11	11	9	c
94	11	9	9	8	c
94	13	9	9	9	c
94	15	9	10	9	f ## i-case = 3 delta-case = 5
94	17	9	10	9	f ## i-case = 3 delta-case = 2
94	19	11	11	10	c
94	21	9	12	11	! ## i-case = 3 delta-case = 1
94	23	13	13	12	c
94	25	11	13	12	! ## i-case = 3 delta-case = 2
94	27	9	14	13	! ## i-case = 3 delta-case = 2
94	29	11	15	14	! ## i-case = 3 delta-case = 2
94	31	17	17	15	c
94	33	13	17	16	! ## i-case = 3 delta-case = 1
94	35	9	18	17	! ## i-case = 3 delta-case = 1
94	37	11	19	18	! ## i-case = 3 delta-case = 1
94	39	9	20	19	! ## i-case = 3 delta-case = 1
94	41	9	21	20	! ## i-case = 3 delta-case = 1
94	43	11	22	21	! ## i-case = 3 delta-case = 1
94	45	17	23	22	! ## i-case = 3 delta-case = 1
94	47	24	24	22	c

min diameter = 9

N	w	diam	c_diam	f_diam	c/f/!
96	3	25	25	25	c
96	5	17	17	17	c
96	7	13	13	13	c
96	9	11	11	9	c
96	11	9	9	9	c
96	13	9	9	8	c
96	15	9	10	10	! ## i-case = 3 delta-case = 3
96	17	9	10	9	f ## i-case = 3 delta-case = 2
96	19	12	12	10	c
96	21	9	12	11	! ## i-case = 3 delta-case = 1
96	23	12	13	13	! ## i-case = 3 delta-case = 3
96	25	12	13	12	f ## i-case = 3 delta-case = 2
96	27	9	14	13	! ## i-case = 3 delta-case = 2
96	29	10	15	14	! ## i-case = 3 delta-case = 2
96	31	16	16	16	c
96	33	16	17	16	f ## i-case = 3 delta-case = 1
96	35	11	18	17	! ## i-case = 3 delta-case = 1
96	37	9	19	18	! ## i-case = 3 delta-case = 1
96	39	11	20	19	! ## i-case = 3 delta-case = 1
96	41	9	21	20	! ## i-case = 3 delta-case = 1
96	43	10	22	21	! ## i-case = 3 delta-case = 1
96	45	13	23	22	! ## i-case = 3 delta-case = 1
96	47	24	24	24	c

min diameter = 9

N	w	diam	c_diam	f_diam	c/f/!
98	3	25	25	23	c
98	5	17	17	15	c
98	7	13	13	11	c
98	9	11	11	9	c
98	11	9	9	8	c
98	13	9	9	8	c
98	15	9	10	9	f ## i-case = 3 delta-case = 1
98	17	9	10	9	f ## i-case = 3 delta-case = 2
98	19	11	11	11	c
98	21	9	12	11	! ## i-case = 3 delta-case = 1
98	23	11	13	12	! ## i-case = 3 delta-case = 1
98	25	13	13	12	c
98	27	10	14	13	! ## i-case = 3 delta-case = 2
98	29	10	15	14	! ## i-case = 3 delta-case = 2
98	31	13	16	15	! ## i-case = 3 delta-case = 2
98	33	17	17	16	c
98	35	11	18	17	! ## i-case = 3 delta-case = 1
98	37	9	19	18	! ## i-case = 3 delta-case = 1
98	39	11	20	19	! ## i-case = 3 delta-case = 1
98	41	9	21	20	! ## i-case = 3 delta-case = 1
98	43	9	22	21	! ## i-case = 3 delta-case = 1
98	45	11	23	22	! ## i-case = 3 delta-case = 1
98	47	17	24	23	! ## i-case = 3 delta-case = 1
98	49	25	25	23	c

min diameter = 9

N	w	diam	c_diam	f_diam	c/f/!
100	3	26	26	24	c
100	5	18	18	16	c
100	7	14	14	12	c
100	9	11	11	11	c
100	11	10	10	8	c
100	13	9	9	8	c
100	15	9	10	9	f ## i-case = 3 delta-case = 1
100	17	10	10	9	c
100	19	10	11	10	f ## i-case = 3 delta-case = 2
100	21	10	12	11	! ## i-case = 3 delta-case = 1
100	23	10	13	12	! ## i-case = 3 delta-case = 1
100	25	14	14	12	c
100	27	11	14	13	! ## i-case = 3 delta-case = 2
100	29	10	15	14	! ## i-case = 3 delta-case = 2
100	31	12	16	15	! ## i-case = 3 delta-case = 2
100	33	18	18	16	c
100	35	14	18	17	! ## i-case = 3 delta-case = 1
100	37	10	19	18	! ## i-case = 3 delta-case = 1
100	39	10	20	19	! ## i-case = 3 delta-case = 1
100	41	10	21	20	! ## i-case = 3 delta-case = 1
100	43	10	22	21	! ## i-case = 3 delta-case = 1
100	45	11	23	22	! ## i-case = 3 delta-case = 1
100	47	14	24	23	! ## i-case = 3 delta-case = 1
100	49	25	25	25	c

min diameter = 9

Total faulty percentage of case 3 = 42.3333 %