Raman cross talk of soliton collision in a lossless fiber

Sien Chi and Senfar Wen

Institute of Electro-optical Engineering, National Chiao Tung University, Hsinchu, Taiwan, China

The cross talk induced by soliton collision with a Raman effect in a lossless fiber is discussed. There are frequency-shift enhancements for the two colliding solitons in addition to the energy transfer. The maximum rate–distance product limited by energy depletion is obtained for the soliton-based wavelength-division-multiplexing system.

Since the optical soliton was proposed as a carrier for high-bit-rate signal transmission in fiber communication systems,[1] extensive studies have been carried out in this field.[2] Among these studies, wavelength division multiplexing (WDM) has been considered to increase the information capacity.[3] Because there are nonlinear couplings between the solitons with different carrier frequencies, the collisions of solitons due to different group velocities will affect their properties as information carriers. The collisions in a lossless fiber and in a lossy fiber with the loss compensated by Raman pumping have been considered.[3,4] In Ref. 3 the limitation of the WDM system caused by timing jitter is considered. There are two sources of the jitter. One is from the collison-induced frequency change in the Raman-pumped lossy fiber, and the other is from the random modulation of the frequency caused by the spontaneous emission from the Raman effect. Both effects limit the maximum product of the total bit rate and propagation distance. In this Letter we study the Raman cross talk between two colliding solitons in a lossless fiber by modifying the nonlinear Schrödinger equation with the Raman effect.[2,5] This effect is expected to become another limitation for the WDM system. This modified equation is first applied to describe the self-pumping effect of the soliton.[2,5,6] It can be shown that the equation also describes the effect of Raman cross talk between the colliding solitons.

The dimensionless model equation is[2,5,7]

$$i\frac{\partial q}{\partial \xi} + \frac{1}{2}\frac{\partial^2 q}{\partial \tau^2} + |q|^2 q - c_R \frac{\partial |q|^2}{\partial \tau} q = 0. \qquad (1)$$

The fourth term in the equation represents the Raman effect and leads to the self-pumping effect that causes the carrier frequency of the soliton to be downshifted. During the collision of the two solitons with different carrier frequencies, this term makes the soliton with a higher carrier frequency transfer its energy to the soliton with a lower carrier frequency. The variables and coefficient are related to real-world quantities by

$$\tau = \frac{1}{T}(t - k_0' z), \qquad \xi = \frac{|k_0''|}{T^2} z,$$

$$q = T\left(\frac{\omega_0 n_2}{2|k_0''|c}\right)^{1/2} \phi, \qquad (2)$$

and

$$c_R = \frac{1}{2(2\pi)^2}\frac{r_g}{T},$$

where $r_g = (\lambda_0/n_2)(\partial g/\partial \Delta\nu)$. In Eqs. (2), ϕ is the complex envelope of the electric field, t and z are the time and propagation distance, respectively, along the fiber, T is the time scale factor, k_0' and k_0'' are the derivatives of the propagation constant k with respect to ω at the angular frequency ω_0 with a corresponding wavelength of λ_0, and n_2 is the Kerr coefficient. In Eq. (3), $\partial g/\partial \Delta\nu$ is the differential Raman gain coefficient, where $g = g(\Delta\nu)$ is the Raman gain and $\Delta\nu$ is the frequency difference between the pump and Stokes waves. By the triangular Raman gain approximation, because the product $\lambda_0 g$ is independent of the wavelength λ_0 and has a peak value of 9.9×10^{-16} cm²/W at $\Delta\nu = 13.2$ THz (440 cm⁻¹),[5] we have $r_g = 0.234$ and $c_R = 0.00297/T$, where $n_2 = 3.2 \times 10^{-16}$ cm²/W is used and T is in picoseconds. By the propagating-beam or split-step Fourier method,[8] we solve Eq. (1) with c_R given above. The numerical results are ensured by doubling the number of the nodes and halving the distance per step in selected cases.

A fundamental soliton solution of the nonlinear Schrödinger equation [Eq. (1) with $c_R = 0$] is known to be

$$q = \text{sech}(\tau + \Omega\xi)\exp[i(-\Omega\tau + \varphi)], \qquad (4)$$

where $\varphi = (1 - \Omega^2)\xi/2$. The pulse width (FWHM) of the soliton is 1.76. Because the carrier wave is defined by $\exp[i(k_0 z - \omega_0 t)]$ when Eq. (1) is derived, where $k_0 = k(\omega_0)$, the soliton in Eq. (4) has a frequency $\Omega[=(\omega - \omega_0)T]$. Note that the soliton propagates with a velocity of $-\Omega$ in the τ, ξ frame. When $c_R \neq 0$, self-pumping occurs and the soliton experiences a frequency shift. Solving Eq. (1) with the initial condition given by Eq. (4) with $\xi = 0$ and $\varphi = 0$, we calculate the frequency shift as $\Delta\Omega = -\text{Im}[\int q^*(\partial/\partial\tau q d\tau)]/\int q^* q d\tau - \Omega$. The numerical results show that the soliton propagates down the fiber with a frequency shift of $\Delta\Omega \cong 0.53 c_R \xi$, which is independent of Ω. The numerical result agrees well with the approximate formula of the frequency shift derived by Gordon.[5]

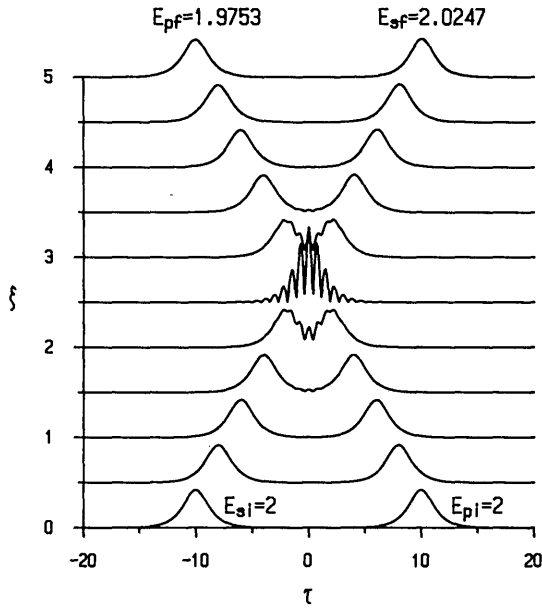To consider the Raman cross talk we use the following initial condition to solve Eq. (1):

Fig. 1. Envelope evolution of the collision of the two solitons with different carrier frequencies. The half-frequency separation is $\Omega_h = 4$, the time scale is $T = 1$ psec, E_{si} and E_{pi} are the energies of the Stokes and pump solitons, respectively, at $\xi = 0$, and E_{sf} and E_{pf} are the energies at $\xi = 5$.

$$q(\tau, \xi = 0) = \mathrm{sech}(\tau + 10)\exp(-i\Omega_s\tau)$$
$$+ \mathrm{sech}(\tau - 10)\exp(-\Omega_p\tau + i\theta), \quad (5)$$

where the carrier frequencies $\Omega_p = -\Omega_s = \Omega_h > 0$ and θ is a constant that represents the initial phase. First we consider the case with $\theta = 0$. In Eq. (5), the soliton with the lower frequency Ω_s is the Stokes soliton and the other one is the pump soliton.

To study the energy transfer, the energy E is defined as $E = \int_{\tau_1}^{\tau_2}|q|^2 d\tau$, where the interested soliton lies within τ_1 and τ_2. Figure 1 shows the case with $T = 1$ psec and $\Omega_h = 4$. After collision, the two solitons are only slightly changed. It is found that, in addition to the energy transfer from the pump solition to the Stokes soliton, as is expected, the frequency shifts of the two solitons are enhanced. The amounts of the energy transfer and frequency-shift enhancements decrease almost linearly with the time scale T. The energy-transfer ratio is defined by $r_E = (E_i - E_{pf})/E_i$, where $E_i = 2$ is the initial soliton energy, which is the same for both solitons given in Eq. (5), and E_{pf} is the energy of the pump soliton after collison. In Fig. 1, $r_E = 1.2\%$. Figure 2 shows r_E with respect to Ω_h for $T = 1, 5,$ and 10 psec. In the figure, r_E reaches a constant for $\Omega_h \geq 5$ and a given T. From the results, we have an approximate formula for $\Omega_h \geq 5$ and $T \geq 1$ psec:

$$r_E = 4c_R. \quad (6)$$

A larger Ω_h means a larger Raman gain and relative velocity for the two solitons. A larger gain enhances energy transfer, but a larger relative velocity decreases the interaction length during collision and reduces the energy transfer. It is surprising that, from Fig. 2, the two factors almost compensate each other for $\Omega_h \geq 5$. The frequency-shift enhancement is defined by $\Delta\Omega_{ej} =$

$\Delta\Omega_j - \Delta\Omega_0, j = s, p$. $\Delta\Omega_s$ is the frequency shift of the Stokes soliton after collison, and $\Delta\Omega_p$ is that of the pump soliton. $\Delta\Omega_0$ is the frequency shift of the soliton that propagates without collison. In Fig. 1, at $\xi = 5$, where the solitons separate well after collision, $\Delta\Omega_{es} = -0.0025$ and $\Delta\Omega_{ep} = -0.0022$, while $\Delta\Omega_0 = 0.0079$. Compared with $\Delta\Omega_0$, the frequency-shift enhancements $\Delta\Omega_{es}$ and $\Delta\Omega_{ep}$ are significant and the Stokes soliton experiences more frequency shift than the pump soliton. Figure 3 shows the frequency-shift enhancements with respect to Ω_h for $T = 1, 5,$ and 10 psec. It can be seen that, for $\Omega_h \geq 5$, $\Delta\Omega_{es}$ and $\Delta\Omega_{ep}$ are inversely proportional to Ω_h by some powers, and they approach the same value as Ω_h increases. The inversely proportional powers are close to -1. The contribu-
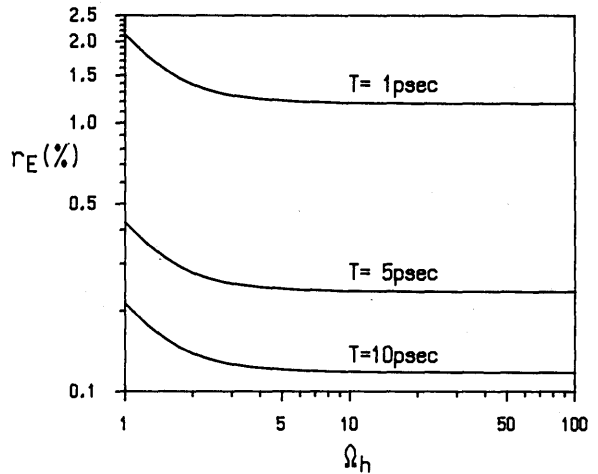

Fig. 2. Energy-transfer ratio r_E with respect to the half-frequency separation Ω_h for the time scales $T = 1, 5,$ and 10 psec.
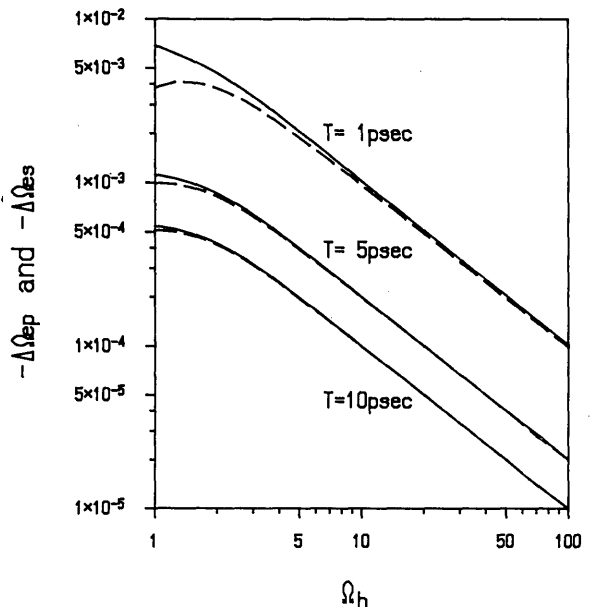

Fig. 3. Frequency-shift enhancements, $\Delta\Omega_{es}$ and $\Delta\Omega_{ep}$, with respect to the half-frequency separation Ω_h for the time scales $T = 1, 5,$ and 10 psec. The solid curves represent $\Delta\Omega_{es}$, and the dashed curves represent $\Delta\Omega_{ep}$.

tions of $\Delta\Omega_{es}$ and $\Delta\Omega_{ep}$ come from the nonlinear coupling during collision and the self-pumping after collision. For the cases of interest, the energy-transfer ratio is below approximately 1% and the contributions of the frequency-shift enhancements from the energy (or power intensity) changes of the two solitons after collision can be neglected. We can infer that the frequency-shift enhancements of both solitons occur mainly during the collision. There are two effects that contribute to the nonlinear coupling. During the collision, the two solitons overlap and their total power intensity enhances the frequency shift through the Raman effect. Meanwhile, the energy transfer between the two solitons takes place, and the cross-phase modulation changes their frequencies through the Kerr effect.[3,4] The contributions of both effects depend on the interaction length, which is inversely proportional to Ω_h. This is believed to be the reason that the frequency-shift enhancements are inversely proportional to Ω_h by some powers.

In a WDM system the initial solitons may have different phases. We have found that the nonzero initial phase θ in Eq. (5) does not change the numerical results shown above. Reference 7 deals with the limiting case with $\Omega_h = 0$, where the effect caused by the initial phases is important. For the solitons with the same carrier frequency, coalescence takes place instead of collision because of mutual interaction between neighboring solitons. However, the effect can be reduced by increasing the initial separation between neighboring solitons. For example, a 10-pulse-width separation is large enough to avoid the effect.

The energy transfer depletes the pump soliton, and the frequency-shift enhancements cause jittering. In the following, we only consider the limitation of the WDM system due to the energy depletion and leave the limitation due to the frequency shift to a future publication in which we will consider the Raman cross talk in the Raman-pumped fiber. The most serious depletion happens in the highest-carrier-frequency channel. We assume a WDM system with N channels and a channel spacing of $2\Omega_h$. The channels are labeled in order from the highest-frequency channel by a number i, where $i = 1$ represents the highest-frequency channel and $i = N$ represents the lowest-frequency channel. We also assume that the width of a time slot is 10 pulse widths and that the probability of a soliton in a time slot is 0.5. Then the soliton in the highest-carrier-frequency channel will collide $0.5 \times 2\Omega_h(i - 1)\xi/17.6$ times with the solitons in the ith channel after propagating a distance ξ. In total, it collides $n_c = \Omega_h\xi N(N - 1)/35.2$ times with the solitons in the other channels. For example, $n_c = 2253$ for a soliton-based WDM system with a 44-GHz total bit rate ($\Omega_h = 10.8$ N = 10) and a 5200-km propagation distance ($\xi = 81.6$).[3] In this example, a 20-nm total wavelength span and a 2-psec/km/nm dispersion parameter are taken. Note that the product of the total

bit rate and propagation distance of this example is limited by the two jittering effects considered in Ref. 3. The energy transfer after the first collision can be calculated from Eq. (6), but the energy transfer in the following collisions should decrease. If we assume that the average energy-transfer ratio is $\bar{r}_E = r_c r_E$, where r_c is a constant, the energy depletion of a soliton in the highest frequency channel is $\Delta E_d = \bar{r}_E n_c E_i$. Because the soliton would seriously disperse when its area is reduced to half, the limitation of the energy depletion should be $0.75E_i$. Then the product of the total bit rate R and the distance L is limited by

$$RL < \frac{(3.35 \times 10^4)T_w}{r_c r_g \Delta\lambda D} \text{ (GHz} \cdot \text{km)}, \qquad (7)$$

where $T_w = 1.76T$ is the pulse width, $\Delta\lambda$ is the total wavelength span of the WDM system in nanometers, and $D = -(2\pi c_0^2)k_0''$ is the dispersion parameter in picoseconds per kilometer per nanometer. In relation (7), $\Delta\lambda = (N - 1)\lambda_0^2\Omega_h/\pi cT$. Because the maximum product is inversely proportional to $\Delta\lambda$, it is better to choose a small channel spacing, which corresponds to a decrease in the pump depletion. Note that the maximum rate–distance product is independent of the channel number. For the example shown above, the maximum rate–distance product is 162,489 GHz · km. This shows that only a 3692-km propagation distance is permitted for this example. In comparing the rate–distance products in the examples shown in Ref. 3 with the numeric results obtained by relation (7) for the same parameters, we find that the limitation caused by energy depletion is more serious. However, they are of the same order of magnitude. These results indicate that further study of the combined effect of the energy transfer and timing jitter is required to obtain the limitation of the WDM system.

In conclusion, the cross talk induced by two-soliton collision with the Raman effect in a lossless fiber has been considered. It is shown that, in addition to the energy transfer, there are frequency-shift enhancements for the two solitons. The maximum rate–distance product limited by energy depletion is obtained for the soliton-based WDM system.

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