# 國立交通大學

## 運輸科技與管理學系

## 碩 士 論 文

優勢零售商市場中差異性商品價格與銷量競爭之
比較

A Comparison of Price and Quantity Competition in a Retailer-Dominant Channel with Differentiated Products

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摘　要

在現今社會中，大部分消費性商品會透過通路零售商售予市場顧客，而這些零售商通常販售不同品牌的商品，甚至其中一些品牌、商品本身存在替代關係。故此商品批發商與通路零售商兩者的目標存在著一些衝突。

寡佔模式中，當選擇不同的決策變數其均衡價格、需求量會有所不同。過去關於上下游市場的研究傾向把批發商視為供應鏈中較具影響力的一方。而隨著科技進步、消費者行為等改變因素，供應鏈結構近十幾年來發生巨大變化，現今越來越多優勢零售商，如：WAL-MART, CARREFORE, COSTCO 等，其對於市場影響力已經超越批發商。

本文旨在應用賽局理論探討在優勢零售商主導的市場中，零售商與批發商應該採用價格抑或銷量為其決策變數，而不同的決策變數將如何影響上下游廠商之間的互動關係。推導結果發現無論在任何情況下，零售商採用價格或銷量作決策均不影響均衡結果；當商品互為替代品（互補品）時，批發商傾向選擇銷量（價格）為其決策變數。

關鍵字：賽局理論、價格競爭、銷量競爭

A Comparison of Price and Quantity Competition in a Retailer-Dominant Channel with Differentiated Products

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# ABSTRACT

Most consumer goods today are sold through independent retailers who also sell other competing brands. They normally have conflicting goals from those of the manufacturers.

Equilibrium prices, quantities, and profits in oligopoly models depend on the strategic variables adopted by firms. A distinction is commonly drawn between Bertrand (price) competition, where firms compete directly on prices, and Cournot (quantity) competition, where firms compete directly on quantities. Most of the previous channel studies have approached the problem from the manufacturers' perspective, and typically made assumptions that retailers are passive decision makers and manufacturers can induce their retailers' decisions through various incentives, pricing schedules and cooperation. However, the retailers nowadays are often much larger than many manufacturers, such as Wal-Mart, Tesco, Carrefour, etc. also known as "dominant retailers".

The purpose of this paper is to provide a precise understanding of the price and quantity competition models when differentiated products of different manufacturers sold through a common retailer.

Keywords: Game Theory; Price Competition; Quantity Competition

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# 1 INTRODUCTION

## 1.1 Motivation

Most consumer goods today are sold through independent retailers who also sell other competing brands. They normally have conflicting goals from those of the manufacturers.

Equilibrium prices, quantities, and profits in oligopoly models depend on the strategic variables adopted by firms. A distinction is commonly drawn between Bertrand (price) competition, where firms compete directly on prices, and Cournot (quantity) competition, where firms compete directly on quantities. In Bertrand- competition, an equilibrium is reached if no firm can improve its profit by unilaterally changing its price. In Cournot-competition, the equilibrium is reached if no firm can improve its profit by unilaterally changing its product quantity assuming that market clearing prices are determined by inverting the demand function. Bertrand and Cournot equilibria are, in general, different even though a one-to-one correspondence exists between prices and quantities.

Most of the previous channel studies have approached the problem from the manufacturers' perspective, and typically made assumptions that retailers are passive decision makers and manufacturers can induce their retailers' decisions through various incentives, pricing schedules and cooperation. However, the retailers nowadays are often much larger than many manufacturers, such as Wal-Mart, Tesco, Carrefour, etc. also known as "dominant retailers". For example, according to Forbes's special report *Global 2000* in 2007, Wal-Mart's sale rank is #1, its profit rank is #21, and it is the 17$^{th}$ biggest public company in the world in 2007. These dominant retailers are gaining more influence on how goods are distributed and at what price.

The purpose of this paper is to provide a precise understanding of the price and quantity competition models when differentiated products of different manufacturers sold through a common retailer.

## 1.2 Organization of the Paper

This paper is organized as follows. Literature reviews about the evolution of channel structure and comparison between price and quantity competition are elaborated in Chapter 2. In Chapter 3, we give a brief introduction to the methodology we apply among this paper. Model description will be contained in Chapter 3. Numerical examples are performed in chapter 4. Finally, in Chapter 5, we summarize the findings of this article.

# 2 LITERATURE REVIEW

## 2.1 Price and Quantity Competition with Differentiated Products

Oligopoly competition is the subject of extensive study in marketing (Choi, 1991; McGuire and Staelin 1983; Padmanabhan and Png, 1997). The role or oligoplolistic competition in pricing, revenue management, and supply chain models has been highlighted by a number of authors. Equilibrium prices, quantities, and profits in oligopoly models depend on the strategy variables adopted by firms. Two classical models in the theory of oligopoly are those of Cournot (1838) and Bertrand (1883). In both models the equilibrium concept is the non-cooperative equilibrium of Nash (1950).

In Bertrand (price) competition, the equilibrium is reached if no firm can improve its profit by unilaterally changing its price. In Cournot (quantity) competition, the equilibrium is reached if no firm can improve its profit by unilaterally changing its product quantity assuming that market clearing prices are determined by inverting the demand function.

Bertrand and Cournot equilibria are, in general, different even though a one-to-one correspondence exists between prices and quantities. In a non-cooperative profit maximization environment, the analysis of the one-shot duopoly game was studied in Singh and Vives' (1984) classical paper. It showed that Bertrand-competition is more efficient than Cournot-competition when the goods are differentiated, with the exception of independent goods in which case the Bertrand and the Cournot models yield equal welfare. Cournot equilibrium profits are higher (smaller) than Bertrand equilibrium profits when goods are substitutes (complements). Over the past two decades, there are array of extended and generalized researches of the analysis in the comparison of Bertrand and Cournot competition. Cheng (1985) presented a geometric analysis of the results obtained by Singh and Vives (1984). It showed that under fairly general and reasonable assumptions, Cournot equilibrium prices (quantities) are higher than Bertrand equilibrium prices (quantities) and, a quantity (price) strategy dominates a price (quantity) strategy if the goods are substitutes (complements). Boyer and Moreaux (1987) showed that in a differentiated product world, the relationship between those products (substitutes or complements) will be an important factor in the determination of the kind of strategic competition (Cournot-Bertrand, Mixed Nash, Stackelberg; through prices or quantities) between duopolists. Friedman (1988) has shown, under mild restrictions on the demand function, that the outcome of this two-stage quantity-price game is equivalent to the outcome of a one-stage Cournot game. If the sequence of decisions is reversed with pricing decisions preceding production decision then the outcome is that of a one-stage Bertrand game. Thus the Bertrand and Cournot models are applicable in situations where firms compete in both prices and quantities as long as these decisions are sequential. Dastidar (1997) concluded that Bertrand equilibrium prices may not be lower that Cournot equilibrium prices under the equal sharing rule with asymmetric costs. Hackner (2000) showed that the results developed in Singh and Vives (1984) are sensitive to the duopoly assumption. In his article he extended the model to the n-firm oligopoly structure allowing for vertical quality differences across horizontally differentiated goods. Under the assumption of exogenous an identical marginal costs across firms, it was shown that in the presence of large quality differences, high-quality firms may obtain higher profit under Bertrand competition when goods are substitutes. Amir and Jin (2001) assessed the view that Bertrand equilibrium is intrinsically more competitive than Cournot equilibrium in a differentiated oligopoly market under alternative definitions for the strategic nature of the games.

Another branch of the literature addresses the Bertrand-Cournot debate in different channel characteristics. Examples include the model analyzed by Holt and Scheffman (1987) who show that if firms use most-favored customer or meet-or-release contracts then the Bertrand game transforms into a Cournot game. Padmanabhan and Png (1997) show that if manufacturers use return policies then the Cournot game transforms into a Bertrand game. In a market of non-differentiated products, it is well known that Cournot competition yields higher prices, higher profits, and lower quantities than Bertrand competition. Miller and Pazgal (2001) modeled a two-stage differentiated-products oligopoly model with owners and managers. More recently, Correa-Lopez and Naylor (2004) compare Cournot and Bertrand equilibrium profit levels in a unionized duopoly model with substitutes. Correa-Lopez and Naylor (2004) show that Bertrand profits may exceed Cournot profits when decentralized bargaining over the labor cost is introduced. Moreover, Correa-Lopez and Naylor (2007) analyzed the non-cooperative game on the choice of strategic variable to set in duopoly in the presence of an upstream market for the input. Finally, Farahat and Perakis (2008) compare equilibrium under Bertrand and Cournot competition with an arbitrary number of firms offering gross substitute products in both affine demand case and multinomial logit demand case.

Clearly, firms in many industries compete in both prices and quantities. However, technological, marketing, or legislative characteristics of an industry often dictate the relative timing of pricing and quantity (or capacity) decisions and the relative ease by which these decisions may be adjusted. The significance of the Bertrand and Cournot models is that they provide equivalent one-stage reductions of more complex models. A prime example is the case when production and pricing decisions are sequential. Consider an industry where strategic production (or capacity build-up), decisions need to be made well in advance of the selling season and are later followed by tactical price competition (for instance, a make-to-stock setting). For oligopolies having an arbitrary number of firms offering differentiated products, Friedman (1988) has shown, under mild restrictions on the demand function, that the outcome of this two-stage quantity-price game is equivalent to the outcome of a one-stage Cournot game. If the sequence of decisions is reversed with pricing decisions preceding production decisions (for instance, a make-to-order setting) then the outcome is that of a one-stage Bertrand game. Thus the Bertrand and Cournot models are applicable in situations where firms compete in both prices and quantities as long as these decisions are sequential (see also Vives (1999) p. 132, Tirole (1988) p.217-218, and Kreps and Scheinkman (1983)).

These researches consider the horizontal competition between the channel members mostly with respect to the manufacturer view. In this research, we aim to compare these two competitions between manufacturers with a common retailer, which has its impact on how goods are distributed and at what price.

## 2.2 Evolution of Channel Structure

A supply chain which represents a network of organizations that are involved, through upstream and downstream linkages, in the different processes and activities that produce value in the form of products and services in the hands of the ultimate consumer. It is referred to as an integrated system which synchronizes a series of inter-related business processes in order to: (1) acquire raw materials and parts; (2) transform these raw materials and parts into finished products; (3) add value to these products; (4) distribute and promote these products to either retailers or customers; (5) facilitate information exchange among various business entities (e.g. suppliers, manufacturers, distributors, third-party logistics providers, and retailers). Its main objective is to enhance the operational efficiency, profitability and competitive position of a firm and its supply chain partner (H. Min and G. Zhou; 2002). A supply chain is characterized by a forward flow of goods and a backward flow of information as shown by Figure **1**. A channel structure refers to institutional, environmental, and physical factors that influence interactions of channel participants. In a channel, strategies and patterns of behavior that emerge in the interaction, and the performance, especially the profitability, for the channel as whole and for individual firms, are what the researchers focus on.

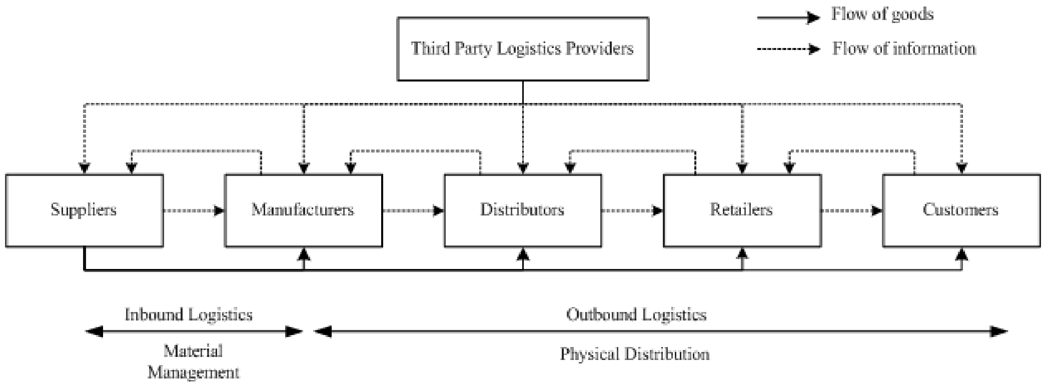



Figure 1. The supply chain process

(Source: H. Min and G. Zhou, 2002)

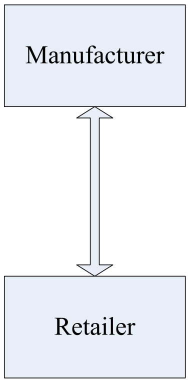
In the last few decades, the restructuring channel and the improving information technology have shifted the relations between manufacturers and retailers. Messinger and

Narasimhan (1995) documented that the major structural changes in the grocery channel are (1) fewer and bigger retailer, (2) increased product mix, (3) changes in consumer tastes and shopping habits, (4) increased concentration in retailers and manufacturers, (5) improving information technology, e.g. scan technology. All these facts lead the original channel power to ship from manufacturers to retailers. Emergence of the dominant retailers is a result of the shift of channel power from manufacturers to retailers.

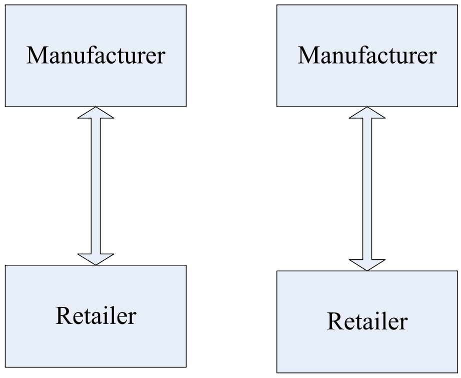
Being dominant retailers, they have the following characteristics. First, due to their ability to offer consumers the opportunity for one-stop shopping and to offer manufacturers effective promotional services, these dominant retailers command a large market share in the retail market. (Epstein, 1994; Zerrillo and Iacobucci, 1995; Wahl, 1992) Secondly, dominant retailers are frequently the largest distributors for manufacturers. For example, sales through Wal-Mart are a double-digit percentage for many manufacturers. (Useem, 2003) Furthermore, dominant retailers are frequently the price leader. (Stone, 1995; Weistein, 2000)

The theoretic literature in the manufacturer-retailer channel interactions mainly develop in the recent two decades. Most previous channel studies have approached the problem from the manufacturers' perspective. For example, these problems contain setting transfer price schedule such as quantity discounts (Dolan, 1987; Lal and Staelin, 1984) or two-part tariffs (Ingene and Parry, 1995), achieving cooperation among channel members via formal agreement or implicit understanding for maximum joint profit (Corghlan, 1985; Jeuland and Shugan, 1983; Shugan, 1985), and analyzing channel efficiency and stability (McGuire and Staelin, 1986). Choi (1991) first sets up a price competition model of two manufacturers which sell differentiated products and one single retailer, and discusses three different vertical relations between manufacturers and single retailer: (1) Manufacturer-Stackelberg (MS) game: each manufacturer chooses the wholesale price using the response function of the retailer, conditional on the observed wholesale price of the competitor's product. The retailer determines the price of each product so as to maximize total profit from both brands given the respective wholesale prices. (2) Vertical Nash (VN) game: each manufacturer chooses its wholesale price conditional on both th retailer's price on its own product and the observed retailer price of the competing brand. The retailer determines the price of each brand conditional on the respective wholesale prices. (3) Retailer-Stackelberg (RS) game: each manufacturer chooses its wholesale price conditional on both the retailer's price on its own product and the observed retail price of the competing brand. The retailer sets up the price of each brand using the reaction functions of both manufacturers in terms of respective wholesale prices. Researches in the channel pricing game may use different definitions of pricing decision variables.

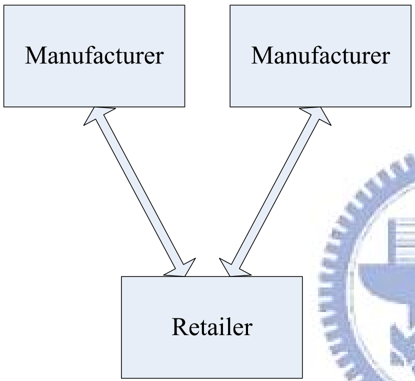
The latter researches upon interactions in channels begin to throw light on the effects of retailer power that stems from dealing multiple products when channels are not coordinated. Choi (1996) extends his research into a channel structure into a channel structure in which there are duopoly manufacturers and duopoly common retailers. Lee and Staelin (1997) also focus on vertical interactions that include four models as shown in Figure 2. Kadiyali et al. (2000) dicuss the manufacturer-retailer channel interactions based on Choi's research (1991) but replace the demand function form into logarithmic form and do an investigation of pricing by two real cases in a local market in which one case considers one retailer's private brand.



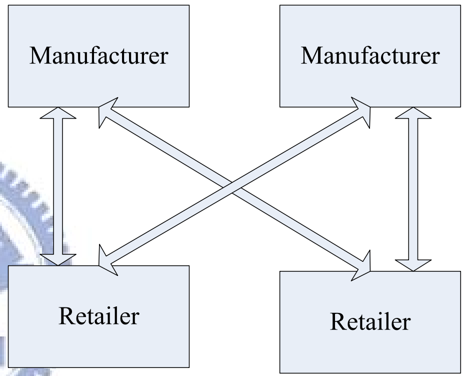

**Model 1**
Totally Differentiated Products;
Separate Retailer Markets




**Model 2**
Substitutable Products; Separate Retailer
Markets; No Product Line Pricing




**Model 3**
Substitute Products; Separate Retail
Markets; Product Line Pricing




**Model 3**
Substitute Products; Competing
Retailers; Product Line Pricing

Figure 2. Vertical pricing strategic interactions

(Source: Lee and Staelin, 1997)

# 3 METHODOLOGY

## 3.1Bilevel Programming

Bilevel programming problems are mathematical optimization problems where the set of all variables is partitioned between two vectors *x* and *y*, and *x* is to be chosen as an optimal solution of a second mathematical programming problem parameterized in *y*. Thus, the bilevel programming problem is hierarchical in the sense that its constraints are defined in part by a second optimization problem. Let this second problem be introduced first as follows:

$$\min_x \{ f(x,y) : g(x,y) \le 0, h(x,y) = 0 \} \tag{1}$$

Where $f : \Re^n \times \Re^m \to \Re$, $g : \Re^n \times \Re^m \to \Re^p$, $h : \Re^n \times \Re^m \to \Re^q$, $g(x,y) = (g_1(x,y),\ldots,g_p(x,y))^T$, $h(x,y) = (h_1(x,y),\ldots,h_q(x,y))^T$. This problem will also be referred to as the lower level or the follower's problem. Let $\Psi(y)$ denote the solution set of problem (1) for fixed $y \in \Re^m$. Then $\Psi$ is a so-called point-to-set mapping from $\Re^m$ into the power set of $\Re^n$ denoted by $\Psi : \Re^m \to 2^{\Re^n}$.

Denote some element of $\Psi(y)$ by $x(y)$ and assume for the moment that this choice is unique for all possible *y*. Then, the aim of the bilevel programming problem is to select that parameter vector y describing the "environmental data" for the lower level problem which is the optimal one in a certain sense. To be more precise, this selection of *y* is conducted so that certain (nonlinear) equality and /or inequality constraints

$$G(x(y), y) \le 0, H(x(y), y) = 0 \tag{2}$$

are satisfied and an objective function $F(x(y), y)$ is minimized, where $F : \Re^n \times \Re^m \to \Re$, $G : \Re^n \times \Re^m \to \Re^k$, $H : \Re^n \times \Re^m \to \Re^l$. Throughout we will assume that all functions *F, G, H, f, g, h,* are sufficiently smooth, i.e. that all the gradients and Hessian matrices of these functions exist and are smooth. Clearly this assumption can be weakened at many places but it is not our intention to present the results using the weakest differentiability assumption.

The problem of determining a best solution $y^*$ of parameters for the parametric optimization problem (1) which together with the response $x(y) \in \Psi(y)$ proves to satisfy the constraints (2) and to give the best possible function value for $F(x(y), y)$. That is

$$"\min_{y}"\{F(x(y),y) : G(x(y),y) \le 0, H(x(y),y) = 0, x(y) \in \Psi(y)\}. \tag{3}$$

This problem is the bilievel-programming problem or the leader's problem. The function $F$ is called the upper level objective and the functions $G$ and $H$ are called the upper level constraint functions. Strongly speaking, this definition of the bilevel programming problem is valid only in case when the lower level solution is uniquely determined for each possible $y$. The quotation marks have been used to express this uncertainty in the definition of the bilievel programming problem in case of non-uniquely determined lower level optimal solutions. If the lower level problem has at most one (global) optimal solution for all values of the parameter, the quotation marks can be dropped and the familiar notation of an optimization problem arises.

The bilevel programming problem demonstrates that applications in economics, in engineering, medicine, ecology etc. have often inspired mathematicians to develop new theories and to investigate new mathematical models. The bilevel programming problem in its original formulation goes back to H.v. Stackelberg (1934), introduced a special case of such problems when he investigated real market situations. This particular formulation is called a Stackelberg game which we will give a briefly state in the following part.

## 3.2Game Theory

Two classical models in the theory of oligopoly are those of Cournot (1838) and Bertrand (1883). In both models the equilibrium concept is the non-cooperative equilibrium of Nash (1950) in Game theory. Game theory (hereafter GT) is a powerful tool for analyzing situation in which the decisions of multiple agents affect each agent's payoff. As such, GT deals with interactive optimization problems. While many economists in the past few centuries have worked on what can be considered game-theoretic models, John von Neumann and Oskar Morgenstern are formally credited as the fathers of modern game theory. Their classic book "Theory of Games and Economic Behavior" written by von Neumann and Morgenstern (1944), summarizes the basic concepts existing at that time. GT has since enjoyed an explosion of developments, including the concept of equilibrium by Nash (1950), games with imperfect information by Kuhn (1953), cooperative games by Aumann (1959) and Shubik

(1962) and auctions by Vickrey (1961). Citing Shubik (2002), "By the late 1980s, game theory in the new industrial organization has taken over… game theory has proved its success in many disciplines."

The essential elements of a game are players, actions, payoffs, and information. These are collectively known as the rules of the game, and the modeler's objective is to describe a situation in terms of the rules of a game so as to explain what will happen in that situation. Trying to maximize their payoffs, the players will devise plans known as strategies that pick actions depending on the information that has arrived at each moment. The combination of strategies chosen by each player is known as the equilibrium. Given an equilibrium, the modeler can see what actions come out of the conjunction of all the players' plans, and this tells him the outcome of the game.

To predict the outcome of a game, the modeler focuses on the possible combination of strategy **s**, since it is the interaction of the different players' strategies that determines what happens. The distinction between strategy combinations, which are sets of strategies, and outcomes, which are sets of values of whichever variables are considered interesting, is a common source of confusion. Often different strategy combinations lead to the same outcome.

Predicting what happens consists of selecting one or more strategy combination as being the most rational behavior for all player i acting to maximize his payoff $\pi_i$. That is, an equilibrium $s^* = \left(s_1^*, s_2^*, \ldots, s_n^*\right)$ is a strategy combination consisting of a best strategy for each of the n players in the game. If no player has incentive to deviate from his strategy given that the other players do not deviate, the strategy combination $s^*$ is known as a Nash equilibrium. Formally,

$\forall i,\ \ \pi_i\left(s_i^*, s_{-i}^*\right) \geq \pi_i\left(s'_i, s_{-i}^*\right),\ \ \forall s'_i$ where $s_{-1}^*$ refers to the strategies chosen by the other players except player *i*.

### 3.2.1 Stackelberg Games

The investigation of bilevel programming problems is strongly motivated by (real world) applications. In his monograph about market economy, Stackelberg (1934) used by the first time an hierarchical model to describe real market situations. This model especially reflects the case that different decision makers try to realize best decisions on the market with respect to their own, generally different objectives and that they are often not able to realize their decisions independently but are forced to act according to a certain hierarchy. We will first consider the simplest case of such a situation where there are only two acting decision makers. Then, this hierarchy divides the two decision makers in one which can handle independently on the market (the so-called leader) and in the other who has to act in a dependent manner (the follower). A leader is able to dictate the selling prices or to overstock the market with his products but in choosing his selections he has to anticipate the possible reactions of the follower since his profit strongly depends not only on his own decision but also on the response of the follower. On the other hand, the choice of the leader influences the set of possible decisions as well as the objectives of the follower who thus has to react on the selection of the leader.

It seems to be obvious that, if one decision maker is able to take on an independent position (and thus to observe and utilize the reactions of the dependent decision maker on his decisions) then he will try to make good use of this advantage (in the sense of making higher profit). The problem he has to solve is the so-call *Stackelberg game*, which can be formulated as follows: Let $X$ and $Y$ denote the set of admissible strategies $x$ and $y$ of the follower and of the leader, respectively. Assume that the values of the choices are measured by means of the functions $f_L(x,y)$ and $f_F(x,y)$, denoting the utility functions of the leader resp. the follower. Then, knowing the selection $y$ of the leader the follower has to select his best strategy $x(y)$ such that his utility function is maximized on $X$:

$$x(y) \in \Psi(y) := Arg\max_{x}\{f_F(x,y) : x \in X\} \qquad (4)$$

Being aware of this selection, the leader solves the Stackelberg game for computing his best selection:

$$\text{"}\max_{x}\text{"}\{f_F(x,y) : y \in Y, x \in \Psi(y)\} \qquad (5)$$

If there are more than one person on one or both levels of the hierarchy, then these are assumed to search for an equilibrium (as e.g. a Nash or again a Stackelberg equilibria) between them.

# 4 THE MODEL OF CHANNEL COMPETITION GAME

The model we demonstrate is mentioned in N. Singh and X. Vives' (1984) paper and here we extend it into a two-echelon manufacturer-retailer market.

## 4.1Notations and Relationships

$q_i$ : demand quantity of product *i*

$p_i$ : price of product *i* sold by retailer

$w_i$ : wholesale price of product *i* sold by its manufacturer;

$\mu_i$ : manufacturing cost of product *i;*

$rm_i$ : retailer margin of product *i*;

$mm_i$ : manufacturer margin of product *i*;

$\Pi_R$ : profit of retailer;

$\Pi_{M_i}$ : profit of manufacturer *i*

Consider there are two manufacturers, each one produce a differentiated good and a competitive numeraire sector. Both of them sell their product through a common retailer to the market. There is a continuum of consumers of the same type with a utility function separable and linear in numeraire good. Therefore, we can neglect the income effect on the duopolistic sector, and we can perform partial equilibrium analysis. Each consumer maximizes his utility function as follow:

$$U(q_1, q_2) - \sum_{i=1}^{2} p_i q_i \tag{6}$$

where $q_i$ is the amount of good *i* and $p_i$ its price. *U* is assumed to be quadratic and strictly concave and,

$$U(q_1, q_2) = \alpha_1 q_1 + \alpha_2 q_2 - (\beta_1 q_1^2 + 2\gamma q_1 q_2 + \beta_2 q_2^2)/2 \tag{7}$$

where $\alpha_i$ and $\beta_i$ are positive, *i*=1, 2. With non-identical products, an absolute advantage in demand enjoyed by one of the firms will be reflected in higher $\alpha_i$ for it. The parameter $\beta_i$ refers to its own effects, while $\gamma$ measures the cross-price effects.

**Assumption 4.1** *Utility function U is assumed to be quadratic and strictly concave.*

**Assumption 4.2** *Parameters* $\alpha_i$ *and* $\beta_i$ *are positive, i=1, 2,* $\beta_1\beta_2 - \gamma > 0$ *, and* $\alpha_i\beta_j - \alpha_j\gamma > 0$.

Assumption 4.2 says that the direct effects dominate the cross effects. Since this has zero income effects on the duopoly industry, we can consider it in isolation. The inverse demands are the partial derivatives of the utility function *U*, given by:

$$p_1 = \alpha_1 - \beta_1 q_1 - \gamma q_2 \tag{8}$$

$$p_2 = \alpha_2 - \beta_2 q_2 - \gamma q_1 \tag{9}$$

Here in the region of quantity space where prices are positive. Letting $\delta = \beta_1\beta_2 - \gamma^2$, $a_i = (\alpha_i\beta_j - \alpha_j\gamma)/\delta$, $b_i = \beta_j/\delta$, $c = \gamma/\delta$ (note that $a_i$ and $b_i$ are positive because of our assumptions), we can write direct demands as follows:

$$q_1 = a_1 - b_1 p_1 + cp_2 \tag{10}$$

$$q_2 = a_2 - b_2 p_2 + cp_1 \tag{11}$$

provided that quantities are positive. The goods are substitutes, independent, or complements according to the sign of $\gamma$. Demand for good *i* is always downward sloping in its own price and increases (decreases) with increases in the price of the competitor if the goods are substitutes (complements). When $\alpha_1 = \alpha_2$ and $\beta_1 = \beta_2 = \gamma$, the goods are perfect substitutes. When $\alpha_1 = \alpha_2$, $\gamma^2/\beta_1\beta_2$ expresses the degree of product differentiation, ranging from zero when the goods are independent to one when the goods are the goods are perfect substitutes. When $\gamma$ is positive and $\gamma^2/\beta_1\beta_2$ approaches one, we are close to a homogeneous market.

Manufacturers have constant marginal costs, $\mu_1$ and $\mu_2$. Profits of retailer and manufacturers, $\Pi_R$ and $\Pi_{M_i}$, are given by:

$$\Pi_R = \sum_{i=1}^{2} rm_i \cdot q_i \tag{12}$$

$$\Pi_{M_i} = mm_i \cdot q_i \tag{13}$$

Where $rm_i = p_i - w_i$ and $mm_i = w_i - \mu_i$. Each member in the market aims to maximize its own profit.

## 4.2The Basic Framework

In this paper, we attempt to assess the channel competition game on the choice of strategic variable to set in a market channel with one dominant retailer and two manufacturers. We analyze the interaction between channel members by a non-coorperative two-stage game. In the first stage of their game, each member chooses the type of strategic variable: price or quantity. In the second stage, each member optimizes its profit by choosing it price and quantity. We construct two types of non-cooperative game to compare the different kinds of channel structure: Vertical Nash Game and Retailer Stackelberg Game (VN Game and RS Game, hereafter). Each member contained in the channel optimizes its profit by choosing different strategic variable according to diverse scenarios (see Figure 3.)

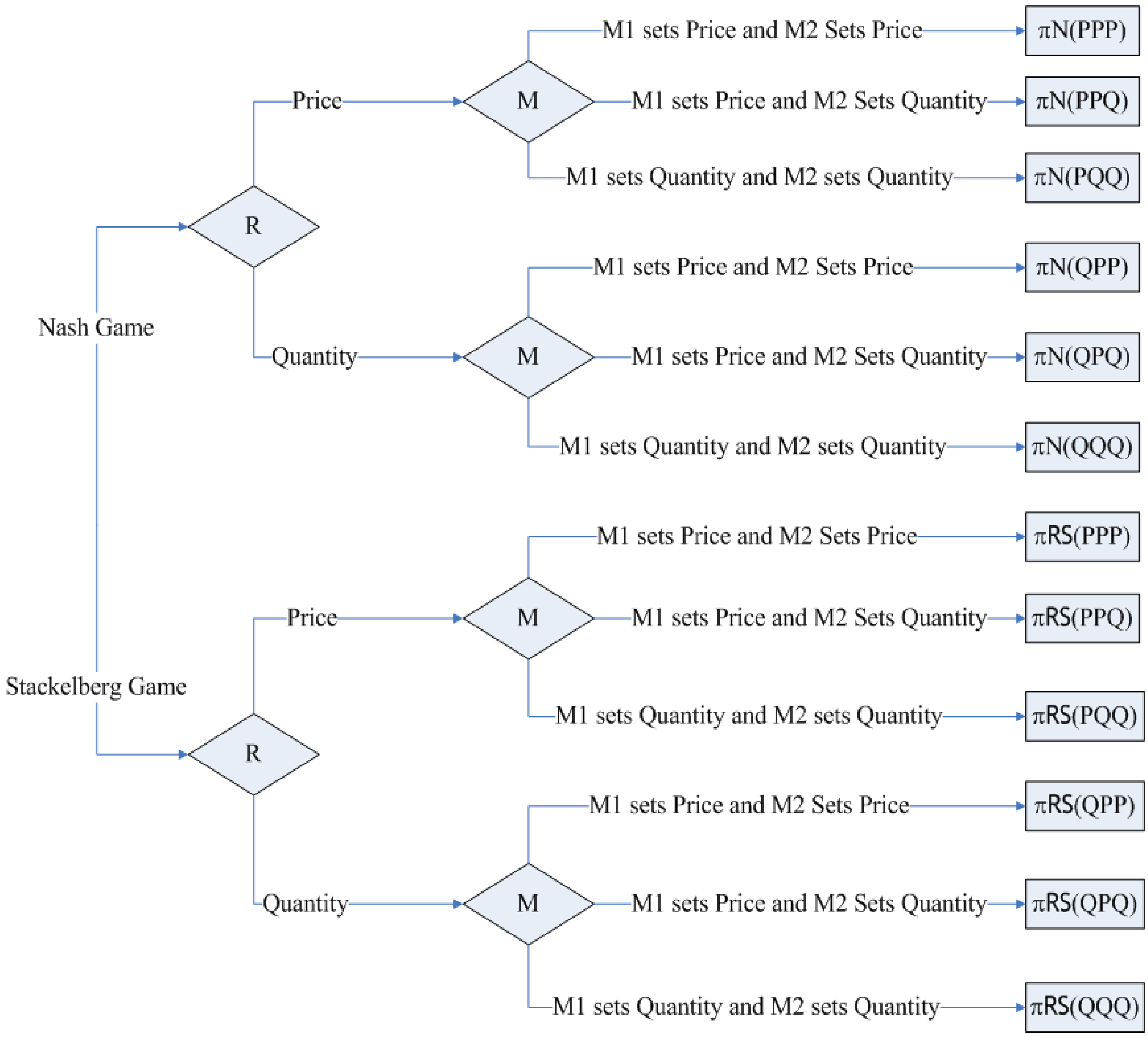



Figure 3. The channel competition game as a game tree

### 4.2.1 VN Game

Under the assumption of VN game, each manufacturer takes as given the competing brand's strategic variable and the margin on its own brand, whereas the retailer conditions it margins on both brands. We now demonstrate the procedure to derive the equilibrium solutions for the VN game under difference choices of the channel members. The equilibrium solutions in expression form can be found in the next section.

**Retailer sets price, manufacturers set price**

Assume that the channel members face the inverse demand functions as (8) and (9) when both of them set price as their strategic variable. The profit functions are given by (12) and (13). Each member optimizes its profit by its best response to any given quantity produced by the difference products. The best reaction function can be derived from the first-order condition as follows:

Reaction function of manufacturer *i*

$$\frac{\partial \Pi_{M_i}}{\partial mm_i} = \frac{\partial}{\partial mm_i}(w_i - \mu_i)\cdot q_i = w_i + q_i \cdot \frac{\partial p_i}{\partial q_i} - \mu_i = 0 \tag{14}$$

Reaction function of retailer

$$\frac{\partial \Pi_R}{\partial rm_i} = \frac{\partial}{\partial rm_i}\left(\sum_i^2 (rm_i)\cdot q_i\right) = q_i + \frac{\partial q_i}{\partial p_i}\cdot\frac{\partial p_i}{\partial rm_i}\cdot rm_i + \frac{\partial q_j}{\partial p_i}\cdot\frac{\partial p_i}{\partial rm_i}\cdot rm_j = 0 \tag{15}$$

Each member's Nash equilibrium is given by the unique intersection of the reaction functions above.

**Retailer sets quantity, manufacturers set quantity**

Assume that the channel members face the demand functions as (10) and (11). The best reaction function can be derived from the first-order condition as follows:

Reaction function of manufacturer *i*

$$\frac{\partial \Pi_{M_i}}{\partial q_i} = \frac{\partial}{\partial q_i}(w_i - \mu_i)\cdot q_i = w_i + q_i \cdot \frac{\partial p_i}{\partial q_i} - \mu_i = 0 \tag{16}$$

Reaction function of retailer

$$\frac{\partial \Pi_R}{\partial q_i} = (p_i - w_i) + \frac{\partial rm_i}{\partial p_i} \cdot \frac{\partial p_i}{\partial q_i} \cdot q_i + \frac{\partial rm_j}{\partial p_i} \cdot \frac{\partial p_i}{\partial q_i} \cdot q_j = 0 \qquad (17)$$

**Retailer sets price, manufacturers set quantity**

Assume that the retailer faces the inverse demand functions as (8) and (9), however, manufacturers look out upon the demand functions as (11) and (12). Thus, the best reaction function of retailer is as (15) and manufacturers' are as (16). Each member's Nash equilibrium is given by the unique intersection of these functions.

**Retailer sets quantity, manufacturers set price**

Assume that the retailer faces the demand functions as (10) and (11), on the other hand, manufacturers look out upon the demand functions as (8) and (9). Thus, the best reaction function of retailer is as (17) and manufacturers' are as (14).

**Retailer sets price, manufacturer1 sets price and manufacturer2 sets quantity**

Assume that the retailer faces the inverse demand functions as (8) and (9), manufacturer1 faces the inverse demand function as (8) and manufacturer2 faces the demand function as (11). Thus, the best reaction function of retailer is as (15), manufacturer1's is as (14) and manufacturer2's is as (16). This competition game is symmetrical. The equilibrium result of manufacturer1 sets quantity and manufacturer2 sets price is in the same manner.

**Retailer sets quantity, manufacturer1 sets price and manufacturer2 sets quantity**

Assume that the retailer faces the demand functions as (10) and (11), manufacturer1 faces the inverse demand function as (8) and manufacturer2 faces the demand function as (11). Thus, the best reaction function of retailer is as (17), manufacturer1's is as (14) and manufacturer2's is as (16). This competition game is symmetrical. Hence, the equilibrium result of manufacturer1 sets quantity and manufacturer2 sets price is in the same manner.

### 4.2.2 RS Game

Under the assumption RS, the retailer becomes the leader and the manufacturers the followers. In this situation, the leader takes the followers' reaction functions into account for its own decision. We now derive analytical equilibrium solutions for the RS game under difference choices of the channel members.

**Retailer sets price, manufacturers set price**

Assume that the channel members face the inverse demand functions as (8) and (9). Reaction function of manufacturer *i* is as (14), retailer's best reaction function can be derived from the first-order condition as follows:

$$\frac{\partial \Pi_R}{\partial rm_i} = \frac{\partial}{\partial rm_i}\left\{(p_i - w_i(\mathbf{p}))q_i(\mathbf{p}) + (p_j - w_j(\mathbf{p}))q_j(\mathbf{p})\right\} = 0 \tag{18}$$

Each member's RS equilibrium is given by the unique intersection of these simultaneous equations.

**Retailer sets quantity, manufacturers set quantity**

Assume that the channel members face the demand functions as (10) and (11). Reaction function of manufacturer *i* is as (16), retailer's best reaction function can be derived from the first-order condition as follows:

$$\frac{\partial \Pi_R}{\partial q_i} = \frac{\partial}{\partial q_i}\left\{(p_i(\mathbf{q}) - w_i(\mathbf{q}))q_i + (p_j(\mathbf{q}) - w_j(\mathbf{q}))q_j\right\} = 0 \tag{19}$$

**Retailer sets price, manufacturers set quantity**

Reaction function of manufacturer *i* is as (16), retailer's best reaction function can be derived from the first-order condition as follows:

$$\frac{\partial \Pi_R}{\partial rm_i} = \frac{\partial}{\partial rm_i}\left\{(p_i - w_i(\mathbf{q}))q_i(\mathbf{p}) + (p_j - w_j(\mathbf{q}))q_j(\mathbf{p})\right\} = 0 \tag{20}$$

**Retailer sets quantity, manufacturers set price**

Reaction function of manufacturer *i* is as (14), retailer's best reaction function can be derived from the first-order condition as follows:

$$\frac{\partial \Pi_R}{\partial q_i} = \frac{\partial}{\partial q_i}\left\{(p_i(\mathbf{q}) - w_i(\mathbf{p}))q_i + (p_j(\mathbf{q}) - w_j(\mathbf{p}))q_j\right\} = 0 \tag{21}$$

**Retailer sets price, manufacturer1 sets price and manufacturer2 sets quantity**

Reaction function of manufacturer1 is as (14) and manufacturer2's is as (16). Retailer's best reaction can be derived from the first-order condition as follows:

$$\frac{\partial \Pi_R}{\partial rm_i} = \frac{\partial}{\partial rm_i}\left\{(p_i - w_i(\mathbf{p}))\cdot q_i(\mathbf{p}) + \left(p_j - w_j(\mathbf{q})\right)\cdot q_j(\mathbf{p})\right\} = 0 \tag{22}$$

**Retailer sets quantity, manufacturer1 sets price and manufacturer2 sets quantity**

Reaction function of manufacturer1 is as (14) and manufacturer2's is as (16). Retailer's best reaction can be derived from the first-order condition as follows:

$$\frac{\partial \Pi_R}{\partial q_i} = \frac{\partial}{\partial q_i}\left\{(p_i(\mathbf{q}) - w_i(\mathbf{p}))\cdot q_i + \left(p_j(\mathbf{q}) - w_j(\mathbf{q})\right)\cdot q_j\right\} = 0 \tag{23}$$

## 4.3 Derivation of the Channel Competition Game Equilibria

In this appendix, we present the equilibrium results of the channel competition game in expression form. Equilibrium results of Vertical Nash game are contained in 4.3.1 and results of Retailer Stackelberg game are performed in 4.3.2.

### 4.3.1 Vertical Nash Game

**Retailer sets price, manufacturers set price**

$$w_1^{N(PPP)} = \frac{a_2 c + b_2(3a_1 + 6b_1\mu_1 + 2c\mu_2)}{9b_1 b_2 - c^2} \tag{24}$$

$$w_2^{N(PPP)} = \frac{a_1 c + b_1(3a_2 + 2c\mu_1 + 6b_2\mu_2)}{9b_1 b_2 - c^2} \tag{25}$$

$$p_1^{N(PPP)} = \frac{a_2 c(5b_1b_2 - c^2) - b_2[2a_1(c^2 - 3b_1b_2) + (c^2 - b_1b_2)(3b_1\mu_1 + c\mu_2)]}{9b_1^2 b_2^2 - 10b_1b_2c^2 + c^4} \tag{26}$$

$$p_2^{N(PPP)} = \frac{a_1 c(5b_1b_2 - c^2) - b_1[2a_2(c^2 - 3b_1b_2) + (c^2 - b_1b_2)(3b_2\mu_2 + c\mu_1)]}{9b_1^2 b_2^2 - 10b_1b_2c^2 + c^4} \tag{27}$$

**Retailer sets quantity, manufacturers set quantity**

$$w_1^{N(QQQ)} = \frac{\beta_1(3\alpha_1\beta_2 - 2\alpha_2\gamma - 3\beta_2\mu_1 + 2\gamma\mu_2)}{9\beta_1\beta_2 - 4\gamma^2} + \mu_1 \tag{28}$$

$$w_2^{N(QQQ)} = \frac{\beta_2(3\alpha_2\beta_1 - 2\alpha_1\gamma - 3\beta_1\mu_2 + 2\gamma\mu_1)}{9\beta_1\beta_2 - 4\gamma^2} + \mu_2 \tag{29}$$

$$q_1^{N(QQQ)} = \frac{-3\alpha_1\beta_2 + 2\alpha_2\gamma + 3\beta_2\mu_1 - 2\gamma\mu_2}{9\beta_1\beta_2 - 4\gamma^2} \tag{30}$$

$$q_2^{N(QQQ)} = \frac{-3\alpha_2\beta_1 + 2\alpha_1\gamma + 3\beta_1\mu_2 - 2\gamma\mu_1}{9\beta_1\beta_2 - 4\gamma^2} \tag{31}$$

**Retailer sets price, manufacturers set quantity**

$$w_1^{N(PQQ)} = \frac{a_1b_2(3b_1b_2 - 2c^2) + a_2b_1b_2c + 2(b_1b_2 - c^2)[(3b_1b_2 - 2c^2)\mu_1 + b_2c\mu_2]}{9b_1^2b_2^2 - 13b_1b_2c^2 + 4c^4} \tag{32}$$

$$w_2^{N(PQQ)} = \frac{a_2b_1(3b_1b_2 - 2c^2) + a_1b_1b_2c + 2(b_1b_2 - c^2)[(3b_1b_2 - 2c^2)\mu_2 + b_1c\mu_1]}{9b_1^2b_2^2 - 13b_1b_2c^2 + 4c^4} \tag{33}$$

$$p_1^{N(PQQ)} = \frac{3a_1b_2(2b_1b_2 - c^2) + a_2c(5b_1b_2 - 2c^2) + (b_1b_2 - c^2)[(3b_1b_2 - 2c^2)\mu_1 + b_2c\mu_2]}{9b_1^2b_2^2 - 13b_1b_2c^2 + 4c^4} \tag{34}$$

$$p_2^{N(PQQ)} = \frac{3a_2b_1(2b_1b_2 - c^2) + a_1c(5b_1b_2 - 2c^2) + (b_1b_2 - c^2)[(3b_1b_2 - 2c^2)\mu_2 + b_2c\mu_1]}{9b_1^2b_2^2 - 13b_1b_2c^2 + 4c^4} \tag{35}$$

**Retailer sets quantity, manufacturers set price**

$$w_1^{N(QPP)} = \frac{3\alpha_1\beta_1\beta_2 - \alpha_1\gamma^2 - 2\alpha_2\beta_1\gamma + 6\beta_1\beta_2\mu_1 + 2\beta_1\gamma\mu_2}{9\beta_1\beta_2 - \gamma^2} \tag{36}$$

$$w_2^{N(QPP)} = \frac{3\alpha_2\beta_1\beta_2 - \alpha_2\gamma^2 - 2\alpha_1\beta_2\gamma + 6\beta_1\beta_2\mu_2 + 2\beta_2\gamma\mu_1}{9\beta_1\beta_2 - \gamma^2} \tag{37}$$

$$q_1^{N(QPP)} = \frac{3\alpha_2\beta_1^2\beta_2 - 2\alpha_1\beta_1\beta_2\gamma - \alpha_2\beta_1\gamma^2 + 2\beta_1\beta_2\gamma\mu_1 + \beta_1\gamma^2\mu_2 - 3\beta_1^2\beta_2\mu_2}{(9\beta_1\beta_2 - \gamma^2)(\beta_1\beta_2 - \gamma^2)} \tag{38}$$

$$q_2^{N(QPP)} = \frac{3\alpha_1\beta_1^2\beta_2 - 2\alpha_2\beta_1\beta_2\gamma - \alpha_1\beta_2\gamma^2 + 2\beta_1\beta_2\gamma\mu_2 + \beta_2\gamma^2\mu_1 - 3\beta_2^2\beta_1\mu_1}{(9\beta_1\beta_2 - \gamma^2)(\beta_1\beta_2 - \gamma^2)} \tag{39}$$

**Retailer sets price, Manufacturer1 sets price and Manufacturer2 sets quantity**

$$w_1^{N(PPQ)} = \frac{-2a_1c^2 + a_2b_1c + 3a_1b_1b_2 - 4b_1c^2\mu_1 + 6b_1^2b_2\mu_1 + 2c\mu_2(b_1b_2 - c^2)}{b_1(9b_1b_2 - 7c^2)} \tag{40}$$

$$w_2^{N(PPQ)} = \frac{a_1 c + 3a_2 b_1 + 2b_1 c\mu_1 + 6\mu_2 (b_1 b_2 - c^2)}{9b_1 b_2 - 7c^2} \tag{41}$$

$$p_1^{N(PPQ)} = \frac{a_1\left(6b_1^2 b_2^2 - 6b_1 b_2 c^2 + c^4\right) + a_2 b_1 c\left(5b_1 b_2 - 4c^2\right) + \left(b_1 b_2 - c^2\right)\left(3b_1^2 b_2 \mu_1 - c^3 \mu_2 - b_1 c\left(2c\mu_1 - b_2 \mu_2\right)\right)}{b_1\left(9b_1 b_2 - 16b_1 b_2 c^2 + 7c^4\right)} \tag{42}$$

$$p_2^{N(PPQ)} = \frac{a_2 b_1\left(6b_1 b_2 - 5c^2\right) + a_1 c\left(5b_1 b_2 - 4c^2\right) + \left(b_1 b_2 - c^2\right)\left(b_1\left(3b_2 \mu_2 + c\mu_1\right) - 3c^2 \mu_2\right)}{9b_1^2 b_2^2 - 16b_1 b_2 c^2 + 7c^4} \tag{43}$$

**Retailer sets quantity, Manufacturer1 sets price and Manufacturer2 sets quantity**

$$w_1^{N(QPQ)} = \frac{\left(\beta_1 \beta_2 - \gamma^2\right)\left(3\alpha_1 \beta_2 - 2\alpha_2 \gamma + 2\gamma\mu_2\right) + 2\beta_2 \mu_1\left(3\beta_1 \beta_2 - 2\gamma^2\right)}{\beta_2\left(9\beta_1 \beta_2 - 7\gamma^2\right)} \tag{44}$$

$$w_2^{N(QPQ)} = \frac{\alpha_2\left(3\beta_1 \beta_2 - \gamma^2\right) - 2\alpha_1 \beta_2 \gamma + 2\beta_2 \gamma\mu_1 + 6\mu_2\left(\beta_1 \beta_2 - \gamma^2\right)}{9\beta_1 \beta_2 - 7\gamma^2} \tag{45}$$

$$q_1^{N(QPQ)} = \frac{3\alpha_1 \beta_2 - 2\alpha_2 \gamma - 3\beta_2 \gamma\mu_1 + 2\gamma\mu_2}{9\beta_1 \beta_2 - 7\gamma^2} \tag{46}$$

$$q_2^{N(QPQ)} = \frac{\alpha_2\left(3\beta_1 \beta_2 - \gamma^2\right) - 2\alpha_1 \beta_2 \gamma + 2\beta_2 \gamma\mu_1 + \mu_2\left(3\beta_1 \beta_2 - \gamma^2\right)}{\beta_2\left(9\beta_1 \beta_2 - 7\gamma^2\right)} \tag{47}$$

### 4.3.2 Retailer Stackelberg Game

**Retailer sets price, manufacturers set price**

$$w_1^{RS(PPP)} = \frac{1}{b_1}\left(a_1 - b_1 p_1 + cp_2\right) + \mu_1 \tag{48}$$

$$w_2^{RS(PPP)} = \frac{1}{b_2}\left(a_2 - b_2 p_2 + cp_1\right) + \mu_2 \tag{49}$$

$$p_1^{RS(PPP)} = \frac{a_2 c\left(5b_1 b_2 - 2c^2\right) + b_2\left(3a_1\left(b_1 b_2 - c^2\right) + \left(b_1 b_2 - c^2\right)\left(2b_1 \mu_1 + c\mu_2\right)\right)}{2\left(4b_1^2 b_2^2 - 5b_1 b_2 c^2 + c^4\right)} \tag{50}$$

$$p_2^{RS(PPP)} = \frac{a_1 c\left(5b_1 b_2 - 2c^2\right) + b_1\left(3a_2\left(b_1 b_2 - c^2\right) + \left(b_1 b_2 - c^2\right)\left(2b_2 \mu_2 + c\mu_1\right)\right)}{2\left(4b_1^2 b_2^2 - 5b_1 b_2 c^2 + c^4\right)} \tag{51}$$

**Retailer sets quantity, manufacturers set quantity**

$$w_1^{RS(QQQ)} = \beta_1 q_1 + \mu_1 \tag{52}$$

$$w_2^{RS(QQQ)} = \beta_2 q_2 + \mu_2 \tag{53}$$

$$q_1^{RS(QQQ)} = \frac{2\alpha_1\beta_2 - \alpha_2\gamma - 2\beta_2\mu_1 + \gamma\mu_2}{2(4\beta_1\beta_2 - \gamma^2)} \tag{54}$$

$$q_2^{RS(QQQ)} = \frac{2\alpha_2\beta_1 - \alpha_1\gamma - 2\beta_1\mu_2 + \gamma\mu_1}{2(4\beta_1\beta_2 - \gamma^2)} \tag{55}$$

**Retailer sets price, manufacturers set quantity**

$$w_1^{RS(PQQ)} = \frac{b_2}{b_1 b_2 - c^2}(a_1 - b_1 p_1 + c p_2) + \mu_1 \tag{56}$$

$$w_2^{RS(PQQ)} = \frac{b_1}{b_1 b_2 - c^2}(a_2 - b_2 p_2 + c p_1) + \mu_2 \tag{57}$$

$$p_1^{RS(PQQ)} = \frac{2a_1 b_2 (3b_1 b_2 - c^2) + a_2 c (5b_1 b_2 - c^2) + \mu_1 (c^4 - 3b_1 b_2 c^2 + 2b_1^2 b_2^2) + b_2 c \mu_2 (b_1 b_2 - c^2)}{2(4b_1^2 b_2^2 - 5b_1 b_2 c^2 + c^4)} \tag{58}$$

$$p_2^{RS(PQQ)} = \frac{2a_2 b_1 (3b_1 b_2 - c^2) + a_1 c (5b_1 b_2 - c^2) + \mu_2 (c^4 - 3b_1 b_2 c^2 + 2b_1^2 b_2^2) + b_1 c \mu_1 (b_1 b_2 - c^2)}{2(4b_1^2 b_2^2 - 5b_1 b_2 c^2 + c^4)} \tag{59}$$

**Retailer sets quantity, manufacturers set price**

$$w_1^{RS(QPP)} = \frac{\beta_1\beta_2 - \gamma^2}{\beta_2} q_1 + \mu_1 \tag{60}$$

$$w_2^{RS(QPP)} = \frac{\beta_1\beta_2 - \gamma^2}{\beta_1} q_2 + \mu_2 \tag{61}$$

$$q_1^{RS(QPP)} = \frac{\alpha_1\beta_2(2\beta_1\beta_2 - \gamma^2) - \alpha_2\beta_1\beta_2\gamma - \beta_2\mu_1(2\beta_1\beta_2 - \gamma^2) + \beta_1\beta_2\gamma\mu_2}{2(4\beta_1^2\beta_2^2 - 5\beta_1\beta_2\gamma^2 + \gamma^4)} \tag{62}$$

$$q_1^{RS(QPP)} = \frac{\alpha_2\beta_1\left(2\beta_1\beta_2-\gamma^2\right)-\alpha_1\beta_1\beta_2\gamma-\beta_1\mu_2\left(2\beta_1\beta_2-\gamma^2\right)+\beta_1\beta_2\gamma\mu_1}{2\left(4\beta_1^{\,2}\beta_2^{\,2}-5\beta_1\beta_2\gamma^2+\gamma^4\right)} \tag{63}$$

**Retailer sets price, Manufacturer1 sets price and Manufacturer2 sets quantity**

$$w_1^{RS(PPQ)} = \frac{1}{b_1}\left(a_1-b_1p_1+cp_2\right)+\mu_1 \tag{64}$$

$$w_2^{RS(PPQ)} = \frac{b_1}{b_1b_2-c^2}\left(a_2-b_2p_2+cp_1\right)+\mu_2 \tag{65}$$

$$\begin{aligned} p_1^{RS(PPQ)} = & \frac{1}{2b_1\left(4b_1^{\,2}b_2^{\,2}-7b_1b_2c^2+3c^4\right)}\{a_1\left(6b_1^{\,2}b_2^{\,2}-6b_1b_2c^2+c^4\right)+a_2b_1c\left(5b_1b_2-4c^2\right) \\ & +\left(b_1b_2-c^2\right)\left(b_1\mu_1\left(2b_1b_2-c^2\right)+c\mu_2\left(b_1b_2-c^2\right)\right)\} \end{aligned} \tag{66}$$

$$p_2^{RS(PPQ)} = \frac{a_2b_1\left(6b_1b_2-5c^2\right)+a_1c\left(5b_1b_2-4c^2\right)+\left(b_1b_2-c^2\right)\left(b_1\left(c\mu_1+2b_2\mu_2\right)-2c^2\mu_2\right)}{2\left(4b_1^{\,2}b_2^{\,2}-7b_1b_2c^2+3c^4\right)} \tag{67}$$

**Retailer sets quantity, Manufacturer1 sets price and Manufacturer2 sets quantity**

$$w_1^{RS(QPQ)} = \frac{\beta_1\beta_2-\gamma^2}{\beta_2}q_1+\mu_1 \tag{68}$$

$$w_2^{RS(QPQ)} = \beta_2q_2+\mu_2 \tag{69}$$

$$q_1^{RS(QPQ)} = \frac{3\alpha_1\beta_2-2\alpha_2\gamma-3\beta_2\mu_1+2\gamma\mu_2}{9\beta_1\beta_2-7\gamma^2} \tag{70}$$

$$q_2^{ES(QPQ)} = \frac{\alpha_2\left(3\beta_1\beta_2-\gamma^2\right)-2\alpha_2\beta_1\gamma+2\beta_2\gamma\mu_1-\left(3\beta_1\beta_2-\gamma^2\right)\mu_2}{\beta_2\left(9\beta_1\beta_2-7\gamma^2\right)} \tag{71}$$

In this section we have demonstrated the reaction function form of the strategic variables combination sets by each member in the channel. The equilibrium results will be illustrated in the following chapter.

# 5 NUMERICAL EXAMPLES

In this chapter, we present the equilibrium results for the non-cooperative game of our model. As in Singh and Vives (1984), the payoff matrices for the non-cooperative games are given by Table 1 and Table 2. In the following discussion, we use superscripts N and RS to denote the difference equilibrium of VN game and RS game. (P, Q, Q) refers to the strategic variable (price or quantity) chose by retailer and manufacturer 1, 2 respectively. Subscript R represents retailer and, $M_1$, $M_2$ represent manufacturer 1 and 2 respectively. The equilibrium outcomes obtained by different channel structure—Vertical Nash and Retailer-Stackelberg will be illustrated in section 5.1. For section 5.2, we focus on the retailer-dominant channel and perform the choice of strategic variable in the presence of product differentiation.

Table 1. Payoff matrix of the Vertical-Nash game

| Retailer set price | | | | Retailer set quantity | | | |
|---|---|---|---|---|---|---|---|
| | $M_2$ | | | | $M_2$ | | |
| $M_1$ | | Price | Quantity | $M_1$ | | Price | Quantity |
| | Price | $\Pi_R^{N(PPP)}$, $\Pi_{M_1}^{N(PPP)}$, $\Pi_{M_2}^{N(PPP)}$ | $\Pi_R^{N(PPQ)}$, $\Pi_{M_1}^{N(PPQ)}$, $\Pi_{M_2}^{N(PPQ)}$ | | Price | $\Pi_R^{N(QPP)}$, $\Pi_{M_1}^{N(QPP)}$, $\Pi_{M_2}^{N(QPP)}$ | $\Pi_R^{N(QPQ)}$, $\Pi_{M_1}^{N(QPQ)}$, $\Pi_{M_2}^{N(QPQ)}$ |
| | Quantity | $\Pi_R^{N(PQP)}$, $\Pi_{M_1}^{N(PQP)}$, $\Pi_{M_2}^{N(PQP)}$ | $\Pi_R^{N(PQQ)}$, $\Pi_{M_1}^{N(PQQ)}$, $\Pi_{M_2}^{N(PQQ)}$ | | Quantity | $\Pi_R^{N(QQP)}$, $\Pi_{M_1}^{N(QQP)}$, $\Pi_{M_2}^{N(QQP)}$ | $\Pi_R^{N(QQQ)}$, $\Pi_{M_1}^{N(QQQ)}$, $\Pi_{M_2}^{N(QQQ)}$ |

Table 2. Payoff matrix of the Retailer-Stackelberg game

| Retailer set price | | | | Retailer set quantity | | | |
|---|---|---|---|---|---|---|---|
| | $M_2$ | | | $M_2$ | | | |
| $M_1$ | | Price | Quantity | $M_1$ | | Price | Quantity |
| | Price | $\Pi_R^{RS(PPP)}$, $\Pi_{M_1}^{RS(PPP)}$, $\Pi_{M_2}^{RS(PPP)}$ | $\Pi_R^{RS(PPQ)}$, $\Pi_{M_1}^{RS(PPQ)}$, $\Pi_{M_2}^{RS(PPQ)}$ | | Price | $\Pi_R^{RS(QPP)}$, $\Pi_{M_1}^{RS(QPP)}$, $\Pi_{M_2}^{RS(QPP)}$ | $\Pi_R^{RS(QPQ)}$, $\Pi_{M_1}^{RS(QPQ)}$, $\Pi_{M_2}^{RS(QPQ)}$ |
| | Quantity | $\Pi_R^{RS(PQP)}$, $\Pi_{M_1}^{RS(PQP)}$, $\Pi_{M_2}^{RS(PQP)}$ | $\Pi_R^{RS(PQQ)}$, $\Pi_{M_1}^{RS(PQQ)}$, $\Pi_{M_2}^{RS(PQQ)}$ | | Quantity | $\Pi_R^{RS(QQP)}$, $\Pi_{M_1}^{RS(QQP)}$, $\Pi_{M_2}^{RS(QQP)}$ | $\Pi_R^{RS(QQQ)}$, $\Pi_{M_1}^{RS(QQQ)}$, $\Pi_{M_2}^{RS(QQQ)}$ |

## 5.1 Choice of Strategic Variable with Different Channel Structure

In this section, we explore the choice of strategic variable in different channel structure: Vertical-Nash game and Retailer-Stackelberg Game with respect to independent, complement and substitute products in section 5.1.1, 5.1.2 and 5.1.3 respectively.

### 5.1.1 Independent Product

First we assume the duopoly market with two independent products, i.e., $\gamma$ =0, and the consumer have symmetric utility on both product, $\alpha_1 = \alpha_2 = 500$ and $\beta_1 = \beta_2 = \beta$. Moreover we normalize the marginal cost of each product to 1. Figure 4 and Figure 5 depict the comparisons of equilibrium profits for alternative values for the direct-effective parameter $\beta$.

Since we assume products are homogenous and independent here, by setting different $\beta$, we can observe the change in direct-effect of the product in both VN and RS game. According to Figure 4 and Figure 5, it is easy to verify that: (i) choice of strategic variable is irrelevant to the outcome, (ii) retailer gains more profit by being a Stackelberg leader, and (iii) manufacturers suffer a loss when they are follower.

The result is not surprising since, by definition, the leader knows the followers' reaction function and exploits this information in its strategy formulation. The followers simply accept the leader's strategy as given, and maximize their own profit.

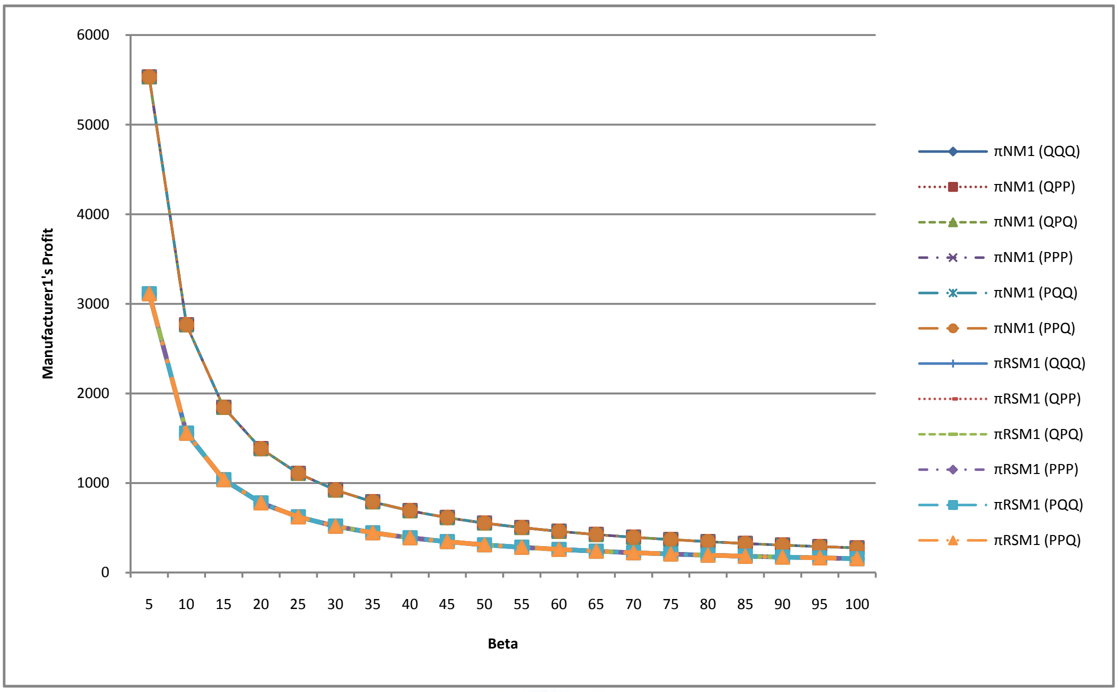



Figure 4. Manufacturers' profit in different channel structure (independent products)

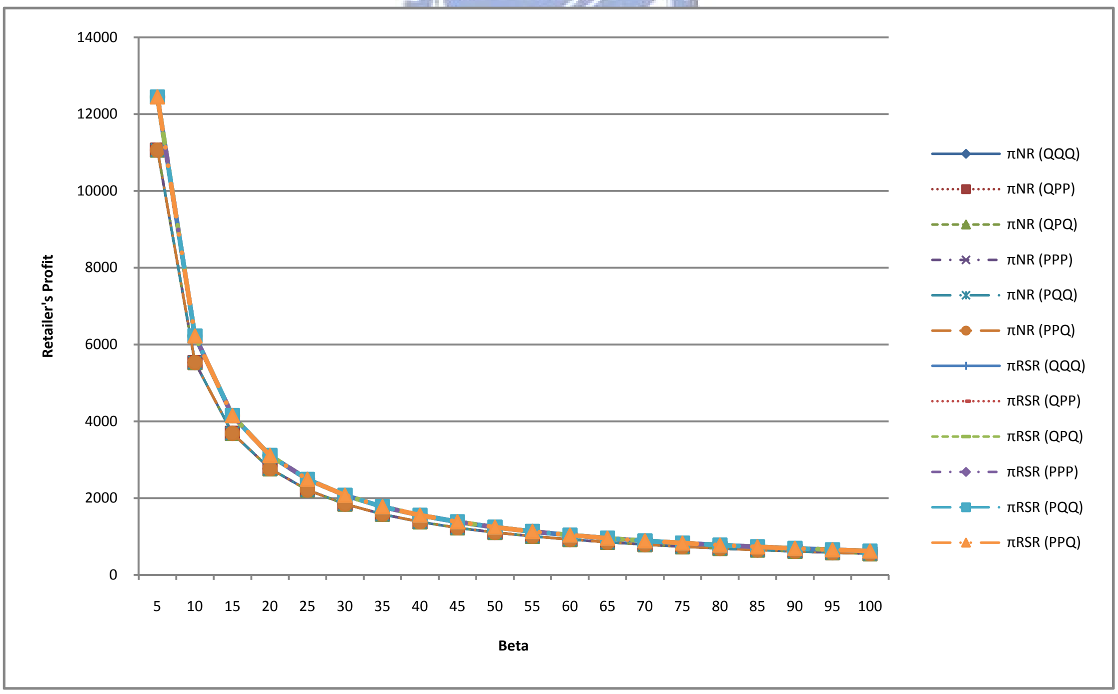



Figure 5. Retailer's profit in different channel structure (independent products)

### 5.1.2 Complement Products

In this section, we continue the comparison of channel structure. With consideration of the complementary characteristic, recall that $\gamma^2/\beta_1\beta_2$ express the degree of cross-effect. Letting $\gamma^2/\beta_1\beta_2$ as 0.3, 0.6 and 0.9, Figure 6 to Figure 8 depict the equilibrium profit of Manufacturer1 with distinct degree of cross-effect. Figure 9 to Figure 11 display the equilibrium profit of Manufacturer2. The equilibrium results of retailer's profit in different degree of cross-effect are shown in Figure 12 to Figure 14.

According to our results, we observed the following characteristics:

① $\gamma^2/\beta_1\beta_2$ =0.3

Manufacturer1's profit:

$$\Pi_{M_1}^{N(PQQ)} = \Pi_{M_1}^{N(QQQ)} > \Pi_{M_1}^{N(PPP)} = \Pi_{M_1}^{N(QPP)} > \Pi_{M_1}^{N(PPQ)} = \Pi_{M_1}^{N(QPQ)}$$
$$> \Pi_{M_1}^{RS(PPP)} = \Pi_{M_1}^{RS(QPP)} > \Pi_{M_1}^{RS(PQQ)} = \Pi_{M_1}^{RS(QQQ)} > \Pi_{M_1}^{RS(PPQ)} = \Pi_{M_1}^{RS(QPQ)}$$

Manufacturer2's profit:

$$\Pi_{M_2}^{N(PPQ)} = \Pi_{M_2}^{N(QPQ)} > \Pi_{M_2}^{N(PQQ)} = \Pi_{M_2}^{N(QQQ)} > \Pi_{M_2}^{N(PPP)} = \Pi_{M_2}^{N(QPP)}$$
$$> \Pi_{M_2}^{RS(PPP)} = \Pi_{M_2}^{RS(QPP)} > \Pi_{M_2}^{RS(PPQ)} = \Pi_{M_2}^{RS(QPQ)} > \Pi_{M_2}^{RS(PQQ)} = \Pi_{M_2}^{RS(QQQ)}$$

Retailer's profit:

$$\Pi_{R}^{RS(PPP)} = \Pi_{R}^{RS(QPP)} > \Pi_{R}^{RS(PPQ)} = \Pi_{R}^{(QPQ)} > \Pi_{R}^{N(PPP)} = \Pi_{R}^{N(QPP)}$$
$$> \Pi_{R}^{RS(PQQ)} = \Pi_{R}^{RS(QQQ)} > \Pi_{R}^{N(PPQ)} = \Pi_{R}^{N(QPQ)} > \Pi_{R}^{N(PQQ)} = \Pi_{R}^{N(QQQ)}$$

These inequalities interpret that when $\gamma^2/\beta_1\beta_2 = 0.3$ and under VN game, whatever manufacturer2 chooses, the best strategy of manufacturer1 is to set quantity, and vice versa. That is, to set quantity is the dominant strategy for manufacturers. For the case of retailer, no matter which strategic variable it chooses, it makes no effect on the equilibrium outcomes.

In terms of RS game, although we observe that both manufacturers set quantity still yield the highest profit. However, if manufacturer1 set price as its strategic variable, the best response of manufacturer2 is to set price. That means there is no dominant strategy here when the manufacturers play as Stackelberg follower.

② $\gamma^2/\beta_1\beta_2$ =0.6

Manufacturer1's profit:

$$\Pi_{M_1}^{N(PPP)} = \Pi_{M_1}^{(QPP)} > \Pi_{M_1}^{N(PQQ)} = \Pi_{M_1}^{N(QQQ)} > \Pi_{M_1}^{N(PPQ)} = \Pi_{M_1}^{N(QPQ)}$$
$$> \Pi_{M_1}^{RS(PPP)} = \Pi_{M_1}^{RS(QPP)} > \Pi_{M_1}^{RS(PQQ)} = \Pi_{M_1}^{RS(QQQ)} > \Pi_{M_1}^{RS(PPQ)} = \Pi_{M_1}^{RS(QPQ)}$$

Manufacturer2's profit:

$$\Pi_{M_2}^{N(PPQ)} = \Pi_{M_2}^{N(QPQ)} > \Pi_{M_2}^{N(PPP)} = \Pi_{M_2}^{N(QPP)} > \Pi_{M_2}^{N(PQQ)} = \Pi_{M_2}^{N(QQQ)}$$
$$> \Pi_{M_2}^{RS(PPP)} = \Pi_{M_2}^{RS(QPP)} > \Pi_{M_2}^{RS(PPQ)} = \Pi_{M_2}^{RS(QPQ)} > \Pi_{M_2}^{RS(PQQ)} = \Pi_{M_2}^{RS(QQQ)}$$

Retailer's profit:

$$\Pi_{R}^{RS(PPP)} = \Pi_{R}^{RS(QPP)} > \Pi_{R}^{N(PPP)} = \Pi_{R}^{N(QPP)} > \Pi_{R}^{RS(PPQ)} = \Pi_{R}^{RS(QPQ)}$$
$$> \Pi_{R}^{RS(PQQ)} = \Pi_{R}^{RS(QQQ)} > \Pi_{R}^{N(PPQ)} = \Pi_{R}^{N(QPQ)} > \Pi_{R}^{N(PQQ)} = \Pi_{R}^{N(QQQ)}$$

These inequalities interpret that when $\gamma^2 / \beta_1 \beta_2 = 0.6$ and under VN game, whatever manufacturer2 chooses, the best strategy of manufacturer1 is to set quantity, and vice versa. That is, to set quantity is the dominant strategy. But we have notice that, the best outcome it that both manufacturers set price. The competition game here can be considered as the "Prisoner's Dilemma"—individuals in a conflict that hurts them all.

In terms of RS game, although we observe that both manufacturers set quantity still yield the highest profit. However, if manufacturer1 set price as its strategic variable, the best response of manufacturer2 is to set price. That means there is no dominant strategy here when the manufacturers play as Stackelberg follower.

③ $\gamma^2 / \beta_1 \beta_2$ =0.9

Manufacturer1's profit:

$$\Pi_{M_1}^{N(PPP)} = \Pi_{M_1}^{N(QPP)} > \Pi_{M_1}^{RS(QPP)} = \Pi_{M_1}^{RS(PPP)} > \Pi_{M_1}^{N(PQQ)} = \Pi_{M_1}^{N(QQQ)}$$
$$> \Pi_{M_1}^{N(PPQ)} = \Pi_{M_1}^{N(PQP)} > \Pi_{M_1}^{RS(PQQ)} = \Pi_{M_1}^{RS(QQQ)} > \Pi_{M_1}^{RS(PPQ)} = \Pi_{M_1}^{RS(QPQ)}$$

Manufacturer2's profit:

$$\Pi_{M_2}^{N(PPP)} = \Pi_{M_2}^{N(QPP)} > \Pi_{M_2}^{N(PPQ)} = \Pi_{M_2}^{N(QPQ)} > \Pi_{M_2}^{RS(PPP)} = \Pi_{M_2}^{RS(QPP)}$$
$$> \Pi_{M_2}^{N(PQQ)} = \Pi_{M_2}^{N(QQQ)} > \Pi_{M_2}^{RS(PPQ)} = \Pi_{M_2}^{RS(QPQ)} > \Pi_{M_2}^{RS(PQQ)} = \Pi_{M_2}^{RS(QQQ)}$$

Retailer's profit:

$$\Pi_{R}^{RS(PPP)} = \Pi_{R}^{RS(QPP)} > \Pi_{R}^{N(PPP)} = \Pi_{R}^{N(QPP)} > \Pi_{R}^{RS(PPQ)} = \Pi_{R}^{RS(QPQ)}$$
$$> \Pi_{R}^{(PQQ)} = \Pi_{R}^{(QQQ)} > \Pi_{R}^{N(PPQ)} = \Pi_{R}^{N(QPQ)} > \Pi_{R}^{N(PQQ)} = \Pi_{R}^{N(QQQ)}$$

These inequalities interpret that when $\gamma^2 / \beta_1 \beta_2 = 0.9$, whatever manufacturer2 chooses, the best strategy of manufacturer1 is to set quantity, and vice versa. That is, to set quantity is the dominant strategy. Again, the best outcome of manufacturers occurred when both of them set price. This phenomenon holds in VN and RS game.

Moreover, as the result obtained from the independent products, retailer always benefits by playing the Stackelberg leader and manufacturers gain less profit when they are Stackelberg follower. When the product interaction is considered, choices of strategic variable by manufacturers do have influence on the equilibrium outcome whereas retailer's choice still makes no effect.

When the degree of cross-effect is relatively low, channel structure plays an important role. However, as the cross-effect become larger, manufacturers' choices of strategic variable become more critical. As the results we displayed above, the best strategy of manufacturers is depend on the value of $\gamma^2/\beta_1\beta_2$. The relationships between equilibrium profit of each member and $\gamma^2/\beta_1\beta_2$ are depicted in Figure 15, Figure 16 and Figure 17.

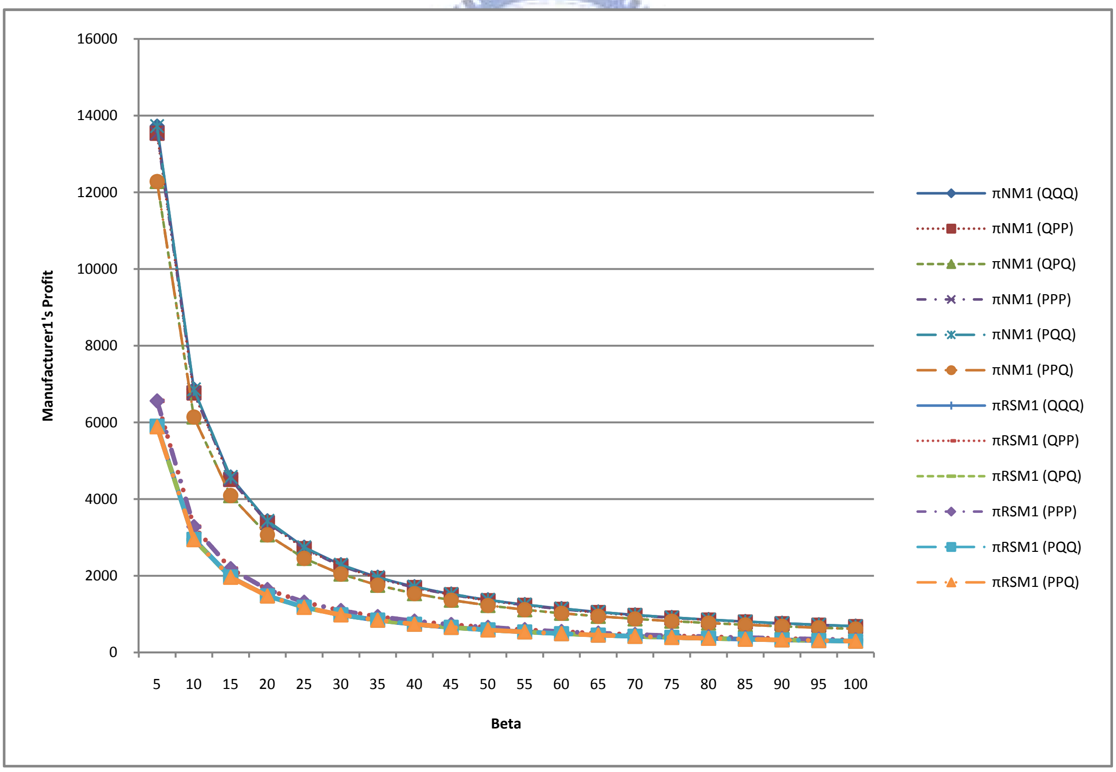



Figure 6. Manufacturer1's profit in different channel structure (complement products; $\gamma^2/\beta_1\beta_2 = 0.3$)

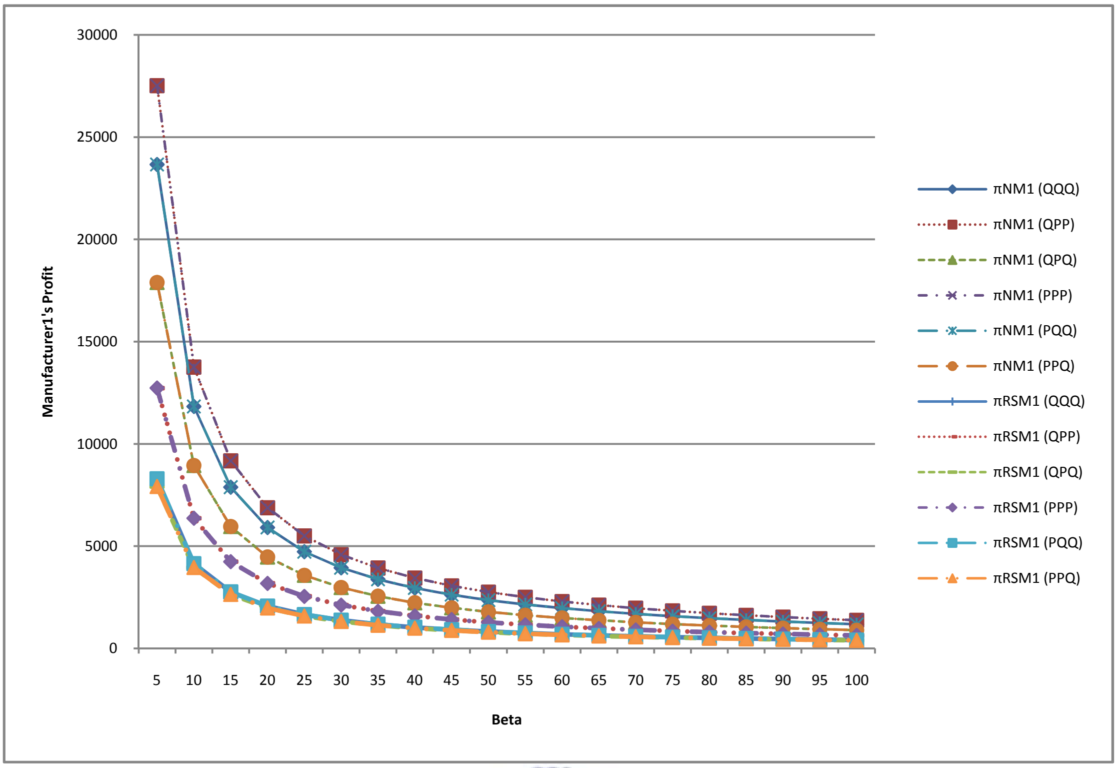



Figure 7. Manufacturer1's profit in different channel structure (complement products; $\gamma^2/\beta_1\beta_2 = 0.6$)

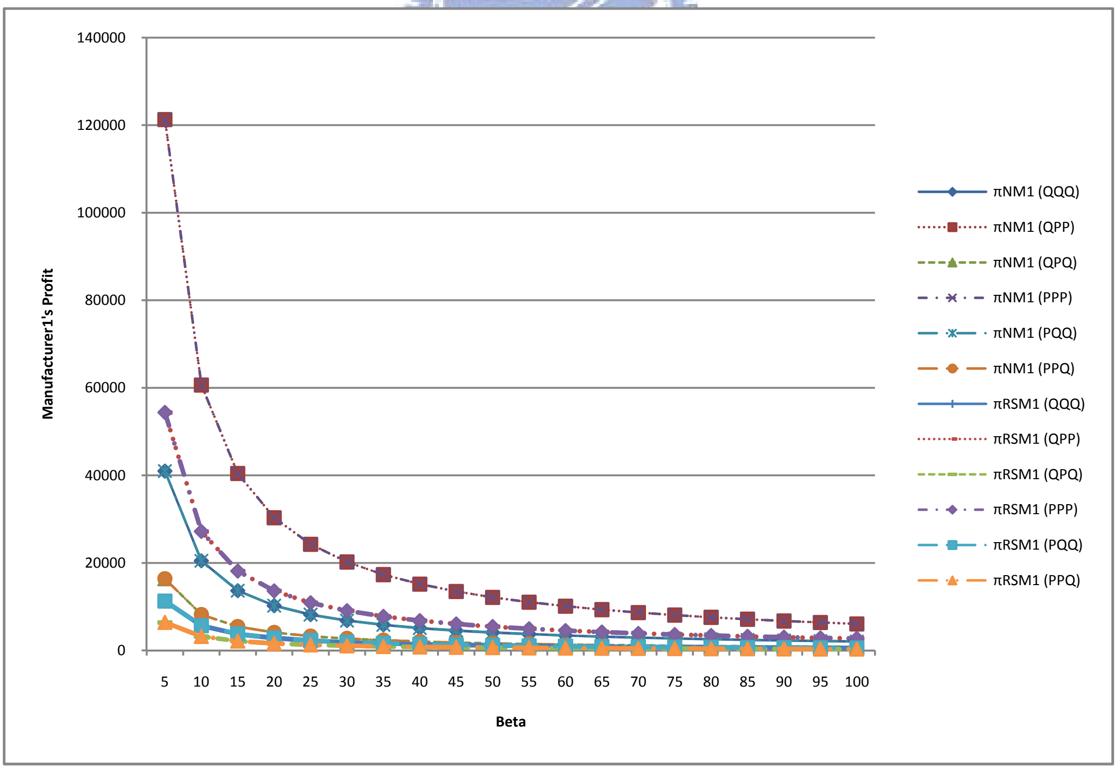



Figure 8. Manufacturer1's profit in different channel structure (complement products; $\gamma^2/\beta_1\beta_2 = 0.9$)

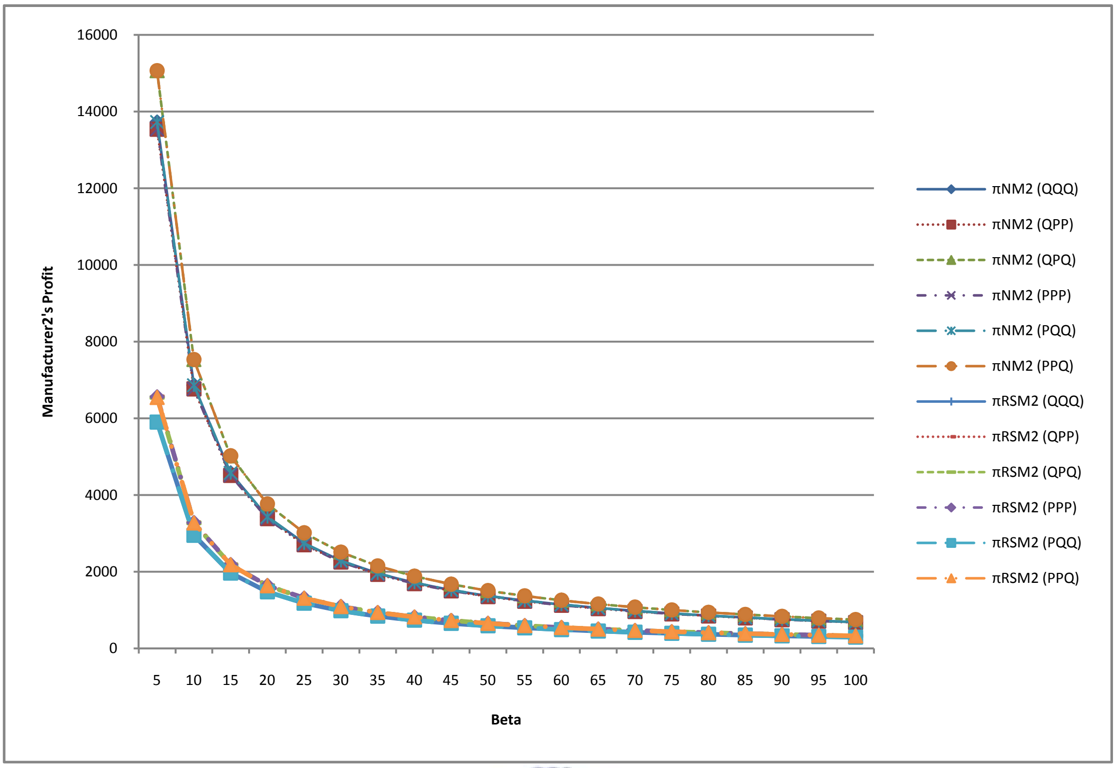



Figure 9. Manufacturer2's profit in different channel structure (complement products; $\gamma^2/\beta_1\beta_2 = 0.3$)

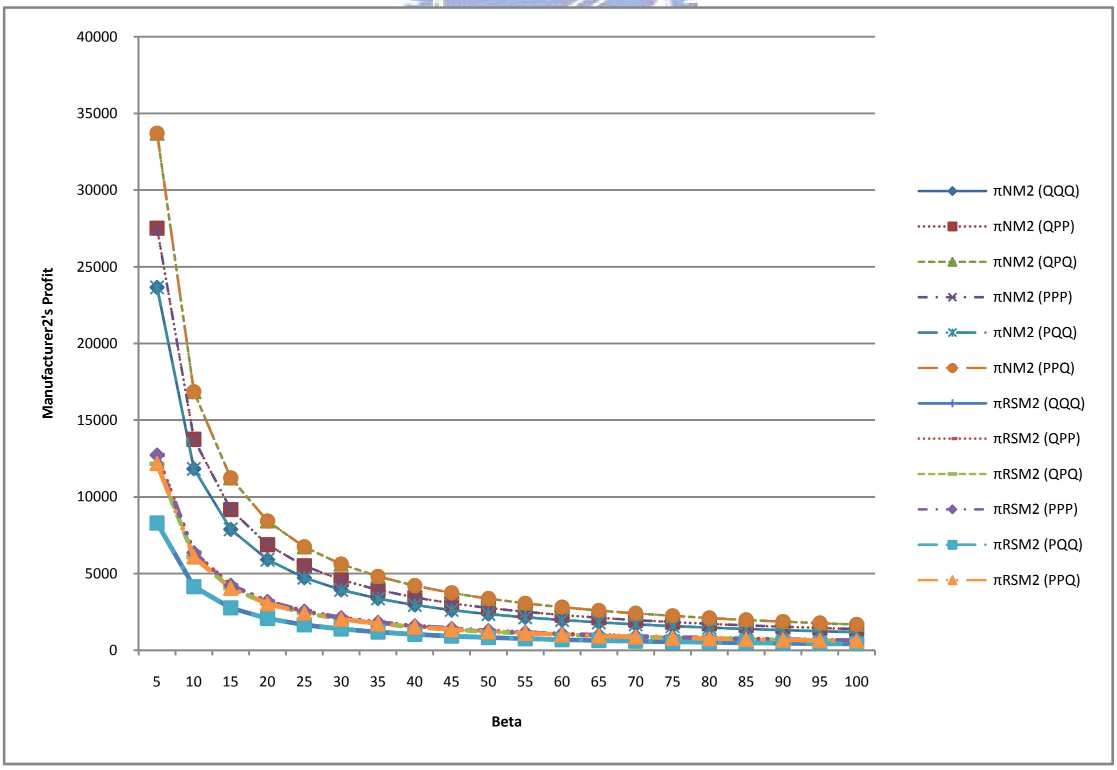



Figure 10. Manufacturer2's profit in different channel structure (complement products; $\gamma^2/\beta_1\beta_2 = 0.6$)

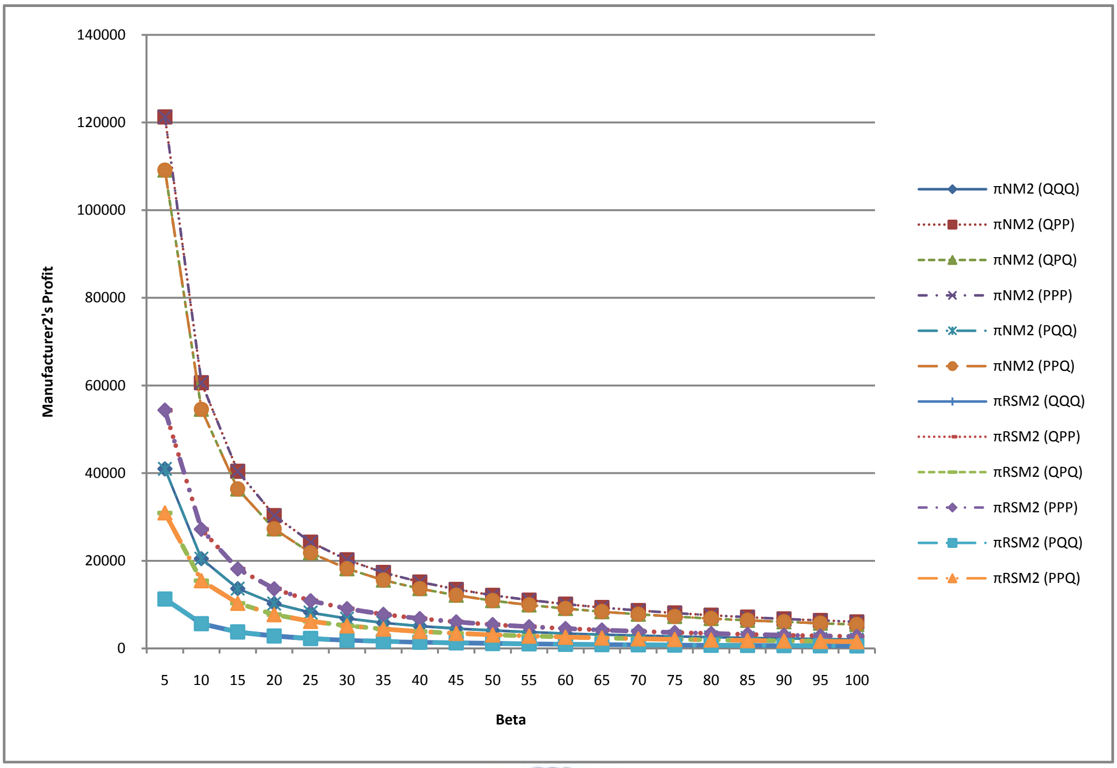



Figure 11. Manufacturer2's profit in different channel structure (complement products; $\gamma^2/\beta_1\beta_2 = 0.9$)

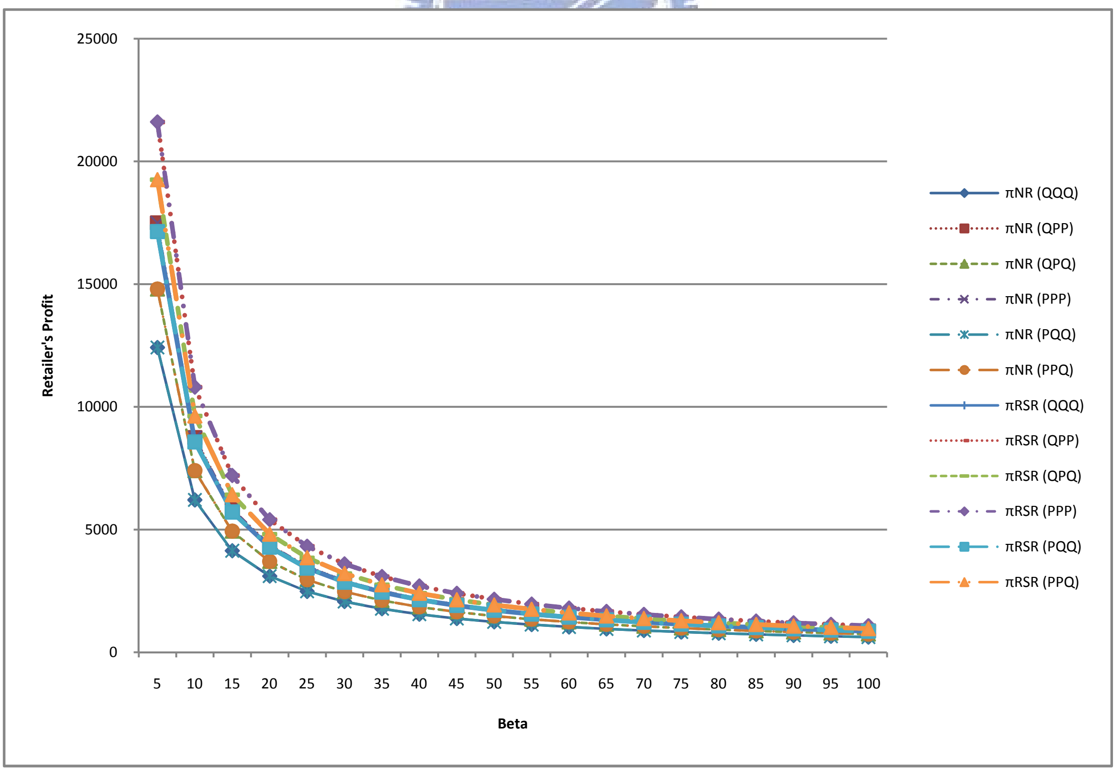



Figure 12. Retailer's profit in different channel structure (complement products; $\gamma^2/\beta_1\beta_2 = 0.3$)

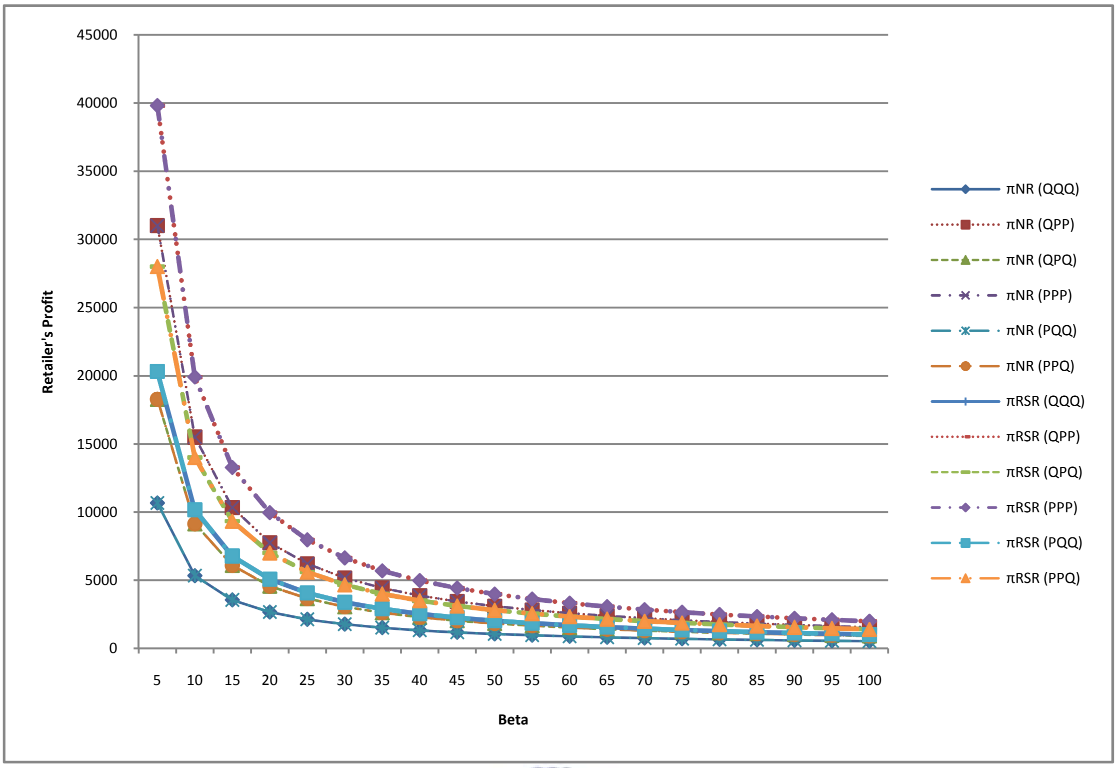



Figure 13. Retailer's profit in different channel structure (complement products; $\gamma^2 / \beta_1 \beta_2 = 0.6$)

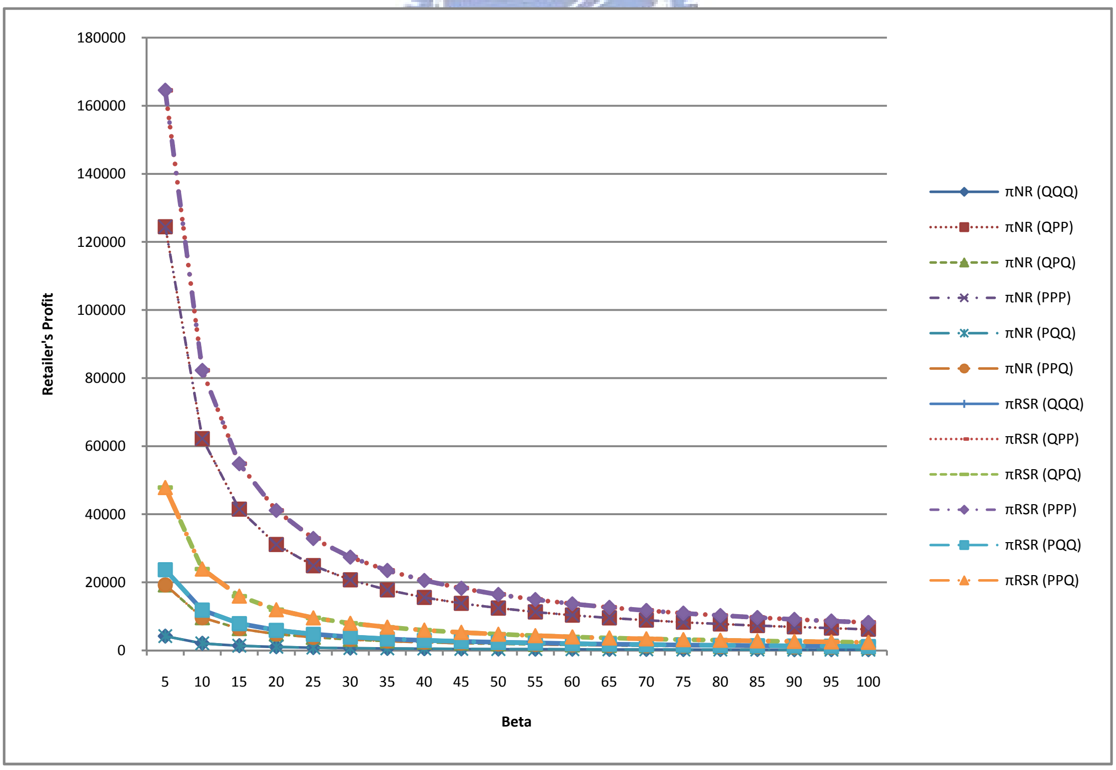



Figure 14. Retailer's profit in different channel structure (complement products; $\gamma^2 / \beta_1 \beta_2 = 0.9$)

Table 3. Corresponding parameters applied in Figure 15-17

| $\alpha_1$ | $\alpha_2$ | | $\beta_1$ | | $\beta_2$ | | $\mu_1$ | | $\mu_2$ | |
|---|---|---|---|---|---|---|---|---|---|---|
| 500 | 500 | | 5 | | 5 | | 1 | | 1 | |
| Degree of difference | 0.05 | 0.1 | 0.15 | 0.2 | 0.25 | 0.3 | 0.35 | 0.4 | 0.45 | 0.5 |
| $\gamma$ | -1.118 | -1.581 | -1.936 | -2.236 | -2.500 | -2.739 | -2.958 | -3.162 | -3.354 | -3.536 |
| Degree of difference | 0.55 | 0.6 | 0.65 | 0.7 | 0.75 | 0.8 | 0.85 | 0.9 | 0.95 | 0.99 |
| $\gamma$ | -3.708 | -3.873 | -4.031 | -4.183 | -4.330 | -4.472 | -4.610 | -4.743 | -4.873 | -4.975 |

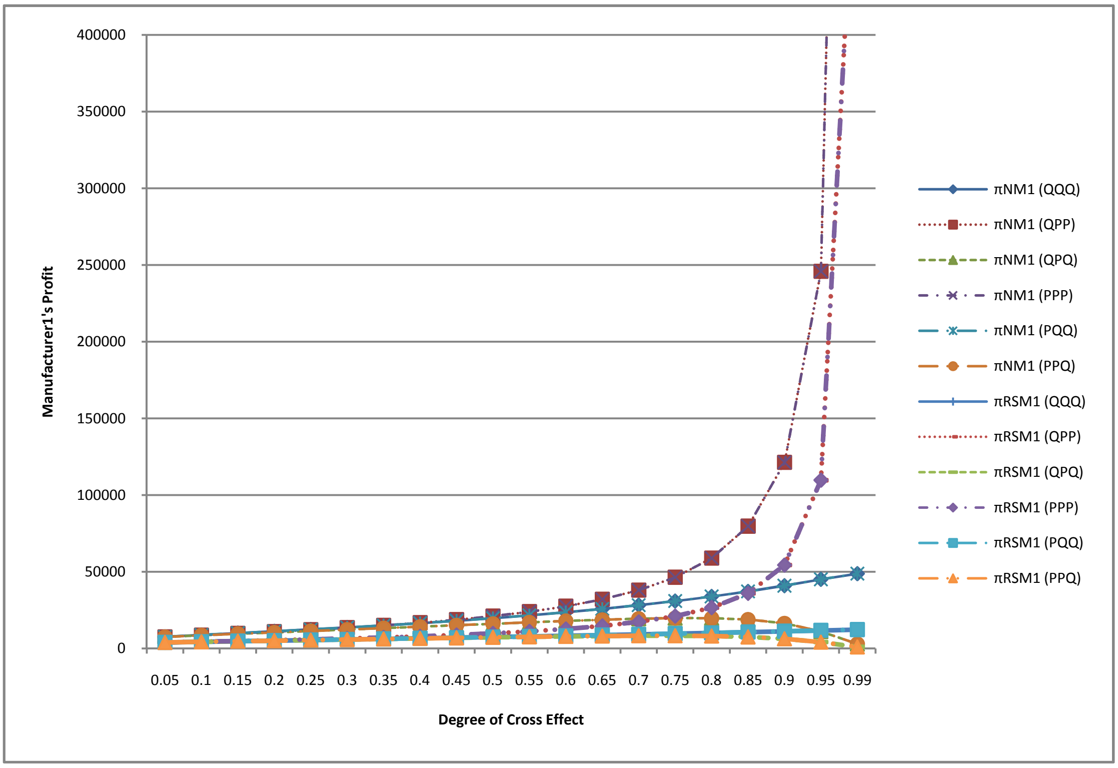



Figure 15. Manufacturer1's profit in different $\gamma^2 / \beta_1 \beta_2$ (complement products)

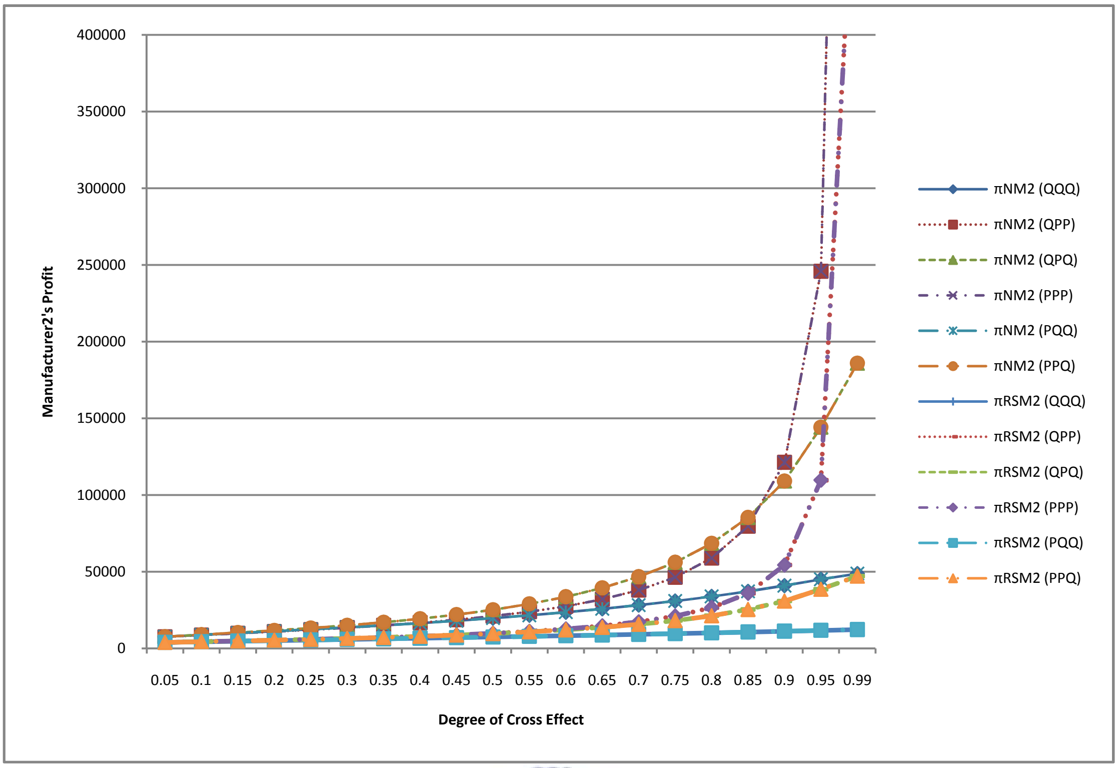



Figure 16. Manufacturer2's profit in different $\gamma^2/\beta_1\beta_2$ (complement products)

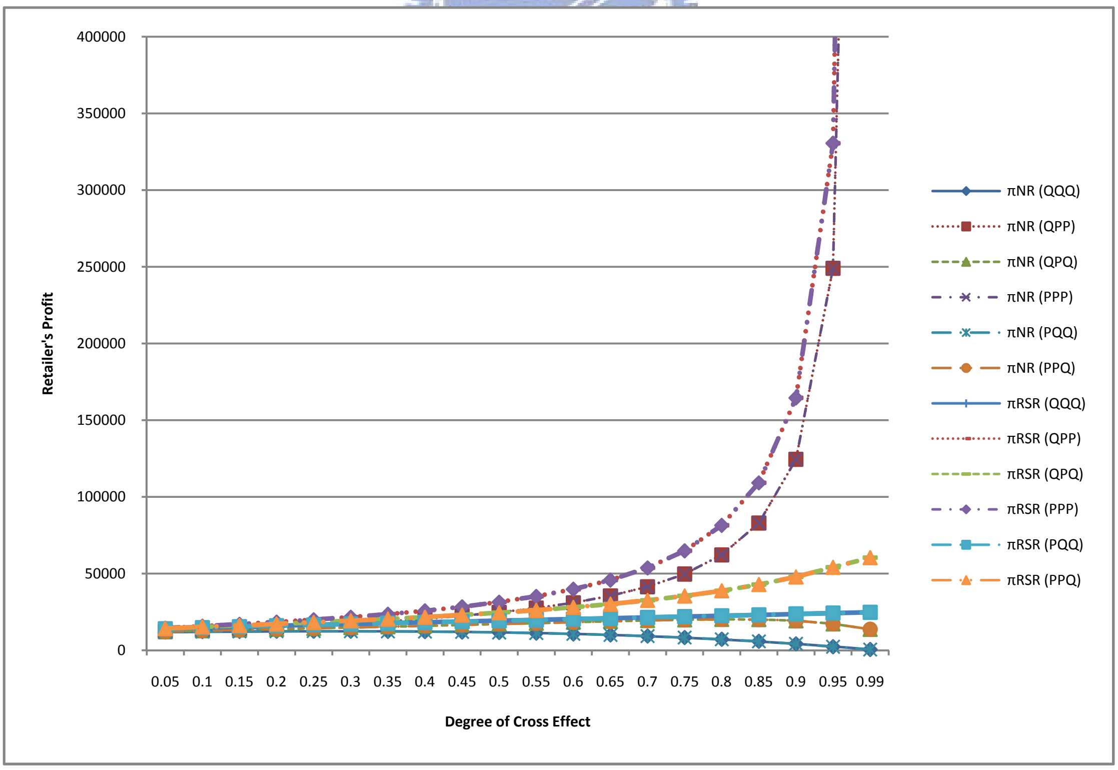



Figure 17. Retailer's profit in different $\gamma^2/\beta_1\beta_2$ (complement products)

## 5.1.3 Substitute Products

In the last section we displayed the equilibrium results correspond to complement products. We now examine the equilibrium results with substitutes. Figure 18 - Figure 20 depict the equilibrium results of Manufacturer1; Manufacturer2's profits are displayed in Figure 21, Figure 22 and Figure 23; Retailer's profit are shown in Figure 24 - Figure 26.

According to our results, we observed the following characteristics:

① $\gamma^2 / \beta_1 \beta_2 = 0.3$

Manufacturer1's profit:

$$\Pi_{M_1}^{N(PQQ)} = \Pi_{M_1}^{N(QQQ)} > \Pi_{M_1}^{N(PPQ)} = \Pi_{M_1}^{N(QPQ)} > \Pi_{M_1}^{N(PPP)} = \Pi_{M_1}^{N(QPP)}$$
$$> \Pi_{M_1}^{RS(PQQ)} = \Pi_{M_1}^{RS(QQQ)} > \Pi_{M_1}^{RS(PPQ)} = \Pi_{M_1}^{RS(QPQ)} > \Pi_{M_1}^{RS(PPP)} = \Pi_{M_1}^{RS(QPP)}$$

Manufacturer2's profit:

$$\Pi_{M_2}^{N(PQQ)} = \Pi_{M_2}^{N(QQQ)} > \Pi_{M_2}^{N(PPQ)} = \Pi_{M_2}^{N(QPQ)} > \Pi_{M_2}^{N(PPP)} = \Pi_{M_2}^{N(QPP)}$$
$$> \Pi_{M_2}^{RS(PQQ)} = \Pi_{M_2}^{RS(QQQ)} > \Pi_{M_2}^{RS(PPP)} = \Pi_{M_2}^{RS(QPP)} > \Pi_{M_2}^{RS(PPQ)} = \Pi_{M_2}^{RS(QPQ)}$$

Retailer's profit:

$$\Pi_{R}^{RS(PPP)} = \Pi_{R}^{RS(QPP)} > \Pi_{R}^{N(PPP)} = \Pi_{R}^{N(QPP)} > \Pi_{R}^{RS(PPQ)} = \Pi_{R}^{RS(QPQ)}$$
$$> \Pi_{R}^{N(PPQ)} = \Pi_{R}^{N(QPQ)} > \Pi_{R}^{RS(PQQ)} = \Pi_{R}^{RS(QQQ)} > \Pi_{R}^{N(PQQ)} = \Pi_{R}^{N(QQQ)}$$

These inequalities interpret that when $\gamma^2 / \beta_1 \beta_2 = 0.3$ and under VN game, whatever manufacturer2 chooses, the best strategy of manufacturer1 is to set quantity, and vice versa. That is, to set quantity is the dominant strategy.

In terms of RS game, although we observe that both manufacturers set quantity still yield the highest profit. However, if manufacturer1 set price as its strategic variable, the best response of manufacturer2 is to set price. That means there is no dominant strategy here when the manufacturers play as Stackelberg follower.

② $\gamma^2 / \beta_1 \beta_2 = 0.6$

Manufacturer1's profit:

$$\Pi_{M_1}^{N(PQQ)} = \Pi_{M_1}^{N(QQQ)} > \Pi_{M_1}^{N(PPQ)} = \Pi_{M_1}^{N(QPQ)} > \Pi_{M_1}^{RS(PQQ)} = \Pi_{M_1}^{RS(QQQ)}$$
$$> \Pi_{M_1}^{RS(PPQ)} = \Pi_{M_1}^{RS(QPQ)} > \Pi_{M_1}^{N(PPP)} = \Pi_{M_1}^{N(QPP)} > \Pi_{M_1}^{RS(PPP)} = \Pi_{M_1}^{RS(QPP)}$$

Manufacturer2's profit:

$$\Pi_{M_2}^{N(PQQ)} = \Pi_{M_2}^{N(QQQ)} > \Pi_{M_2}^{RS(PQQ)} = \Pi_{M_2}^{RS(QQQ)} > \Pi_{M_2}^{N(PPQ)} = \Pi_{M_2}^{N(QPQ)}$$
$$> \Pi_{M_2}^{N(PPP)} = \Pi_{M_2}^{N(QPP)} > \Pi_{M_2}^{RS(PPP)} = \Pi_{M_2}^{RS(QPP)} > \Pi_{M_2}^{RS(PPQ)} = \Pi_{M_2}^{RS(QPQ)}$$

Retailer's profit:

$$\Pi_{R}^{RS(PPP)} = \Pi_{R}^{RS(QPP)} > \Pi_{R}^{N(PPP)} = \Pi_{R}^{N(QPP)} > \Pi_{R}^{RS(PPQ)} = \Pi_{R}^{RS(QPQ)}$$
$$> \Pi_{R}^{N(PPQ)} = \Pi_{R}^{N(QPQ)} > \Pi_{R}^{RS(PQQ)} = \Pi_{R}^{RS(QQQ)} > \Pi_{R}^{N(PQQ)} = \Pi_{R}^{N(QQQ)}$$

These inequalities interpret that, when the degree of cross-effect equals to 0.6, if manufacturer1 sets price, the best response of manufacturer2 is to set price; if manufacturer1 sets quantity, manufacturer2 is better to set quantity. Thus, there is no dominant strategy here and it holds whatever the channel structure is.

③ $\gamma^2 / \beta_1 \beta_2 = 0.9$

Manufacturer1's profit:

$$\Pi_{M_1}^{N(PQQ)} = \Pi_{M_1}^{N(QQQ)} > \Pi_{M_1}^{RS(PQQ)} = \Pi_{M_1}^{RS(QQQ)} > \Pi_{M_1}^{N(PPQ)} = \Pi_{M_1}^{N(QPQ)}$$
$$> \Pi_{M_1}^{RS(PPQ)} = \Pi_{M_1}^{RS(QPQ)} > \Pi_{M_1}^{N(PPP)} = \Pi_{M_1}^{N(QPP)} > \Pi_{M_1}^{RS(PPP)} = \Pi_{M_1}^{RS(QPP)}$$

Manufacturer2's profit:

$$\Pi_{M_2}^{N(PQQ)} = \Pi_{M_2}^{N(QQQ)} > \Pi_{M_2}^{RS(PQQ)} = \Pi_{M_2}^{RS(QQQ)} > \Pi_{M_2}^{N(PPP)} = \Pi_{M_2}^{N(QPP)}$$
$$> \Pi_{M_2}^{RS(PPP)} = \Pi_{M_2}^{RS(QPP)} > \Pi_{M_2}^{N(PPQ)} = \Pi_{M_2}^{N(QPQ)} > \Pi_{M_2}^{RS(PPQ)} = \Pi_{M_2}^{RS(QPQ)}$$

Retailer's profit:

$$\Pi_{R}^{RS(PPP)} = \Pi_{R}^{RS(QPP)} > \Pi_{R}^{N(PPP)} = \Pi_{R}^{N(QPP)} > \Pi_{R}^{RS(PPQ)} = \Pi_{R}^{RS(QPQ)}$$
$$> \Pi_{R}^{N(PPQ)} = \Pi_{R}^{N(QPQ)} > \Pi_{R}^{RS(PQQ)} = \Pi_{R}^{RS(QQQ)} > \Pi_{R}^{N(PQQ)} = \Pi_{R}^{N(QQQ)}$$

These inequalities interpret that, when the degree of cross-effect equals to 0.9, if manufacturer1 sets price, the best response of manufacturer2 is to set price; if manufacturer1 sets quantity, manufacturer2 is better to set quantity. Thus, there is no dominant strategy here and it holds whatever the channel structure is.

As the result performed above, it can be realized that the equilibrium outcome with substitute products is similar to the complements: retailer always benefits by playing the Stackelberg leader and manufacturers gain less profit when they are Stackelberg follower. When the product interaction is considered, choices of strategic variable by manufacturers do have influence on the equilibrium outcome whereas retailer's choice still makes no effect.

Contrary to complement products, when the degree of cross-effect is relatively low, manufacturers' choices of strategic variable play an important role. However, as the cross-effect become larger, channel structure become more critical. As the results we displayed above, the best strategy of manufacturers is depend on the value of $\gamma^2/\beta_1\beta_2$. The relationships between equilibrium profit of each member and $\gamma^2/\beta_1\beta_2$ are depicted in Figure 27-Figure 29.

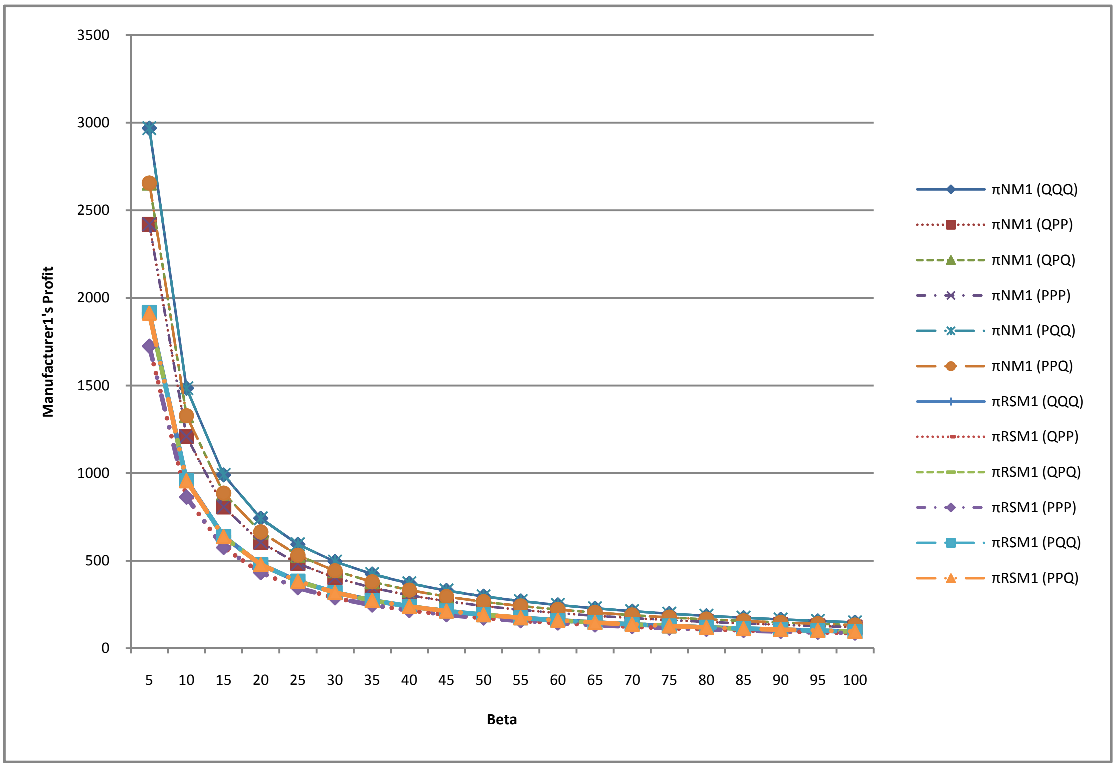



Figure 18. Manufacturer1's profit in different channel structure (substitute products; $\gamma^2/\beta_1\beta_2 = 0.3$)

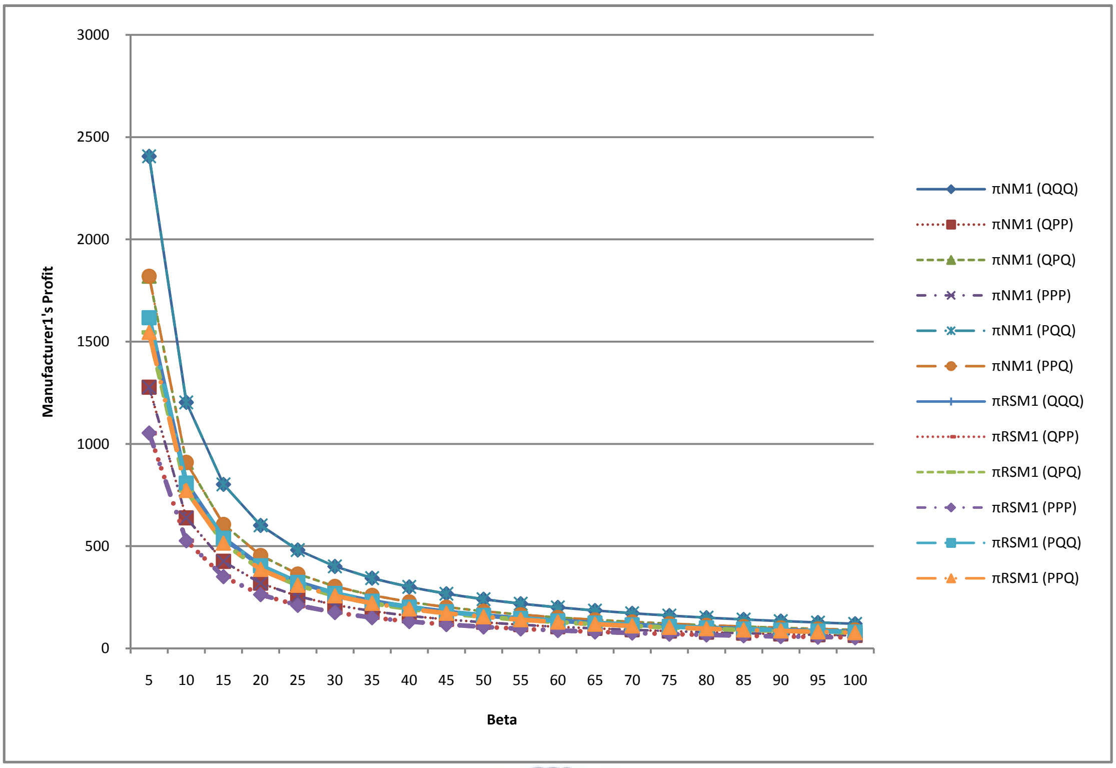



Figure 19. Manufacturer1's profit in different channel structure (substitute products; $\gamma^2/\beta_1\beta_2 = 0.6$)

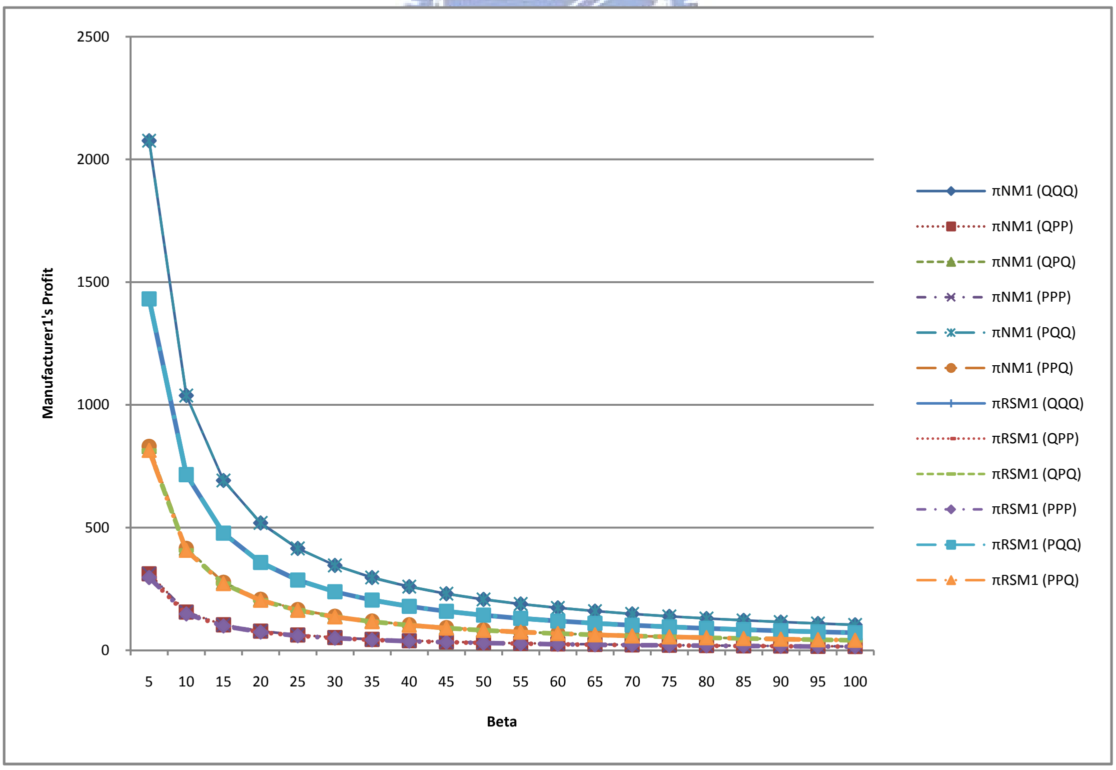



Figure 20. Manufacturer1's profit in different channel structure (substitute products; $\gamma^2/\beta_1\beta_2 = 0.9$)

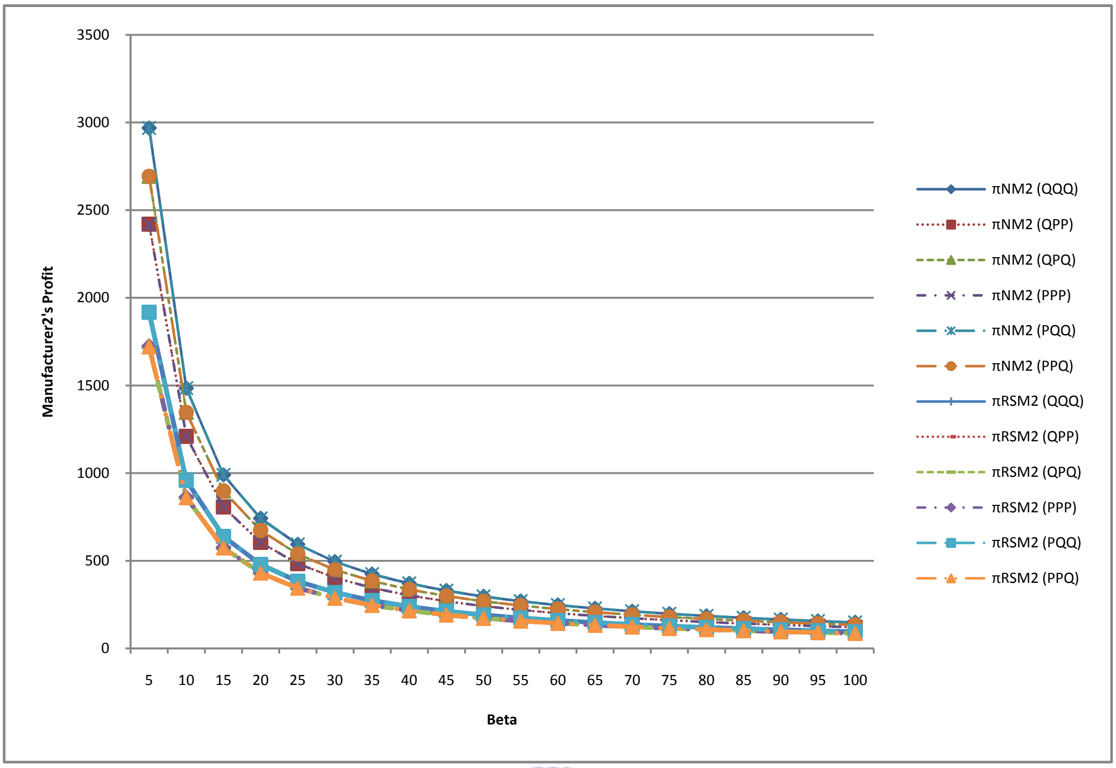



Figure 21. Manufacturer2's profit in different channel structure (substitute products; $\gamma^2/\beta_1\beta_2 = 0.3$)

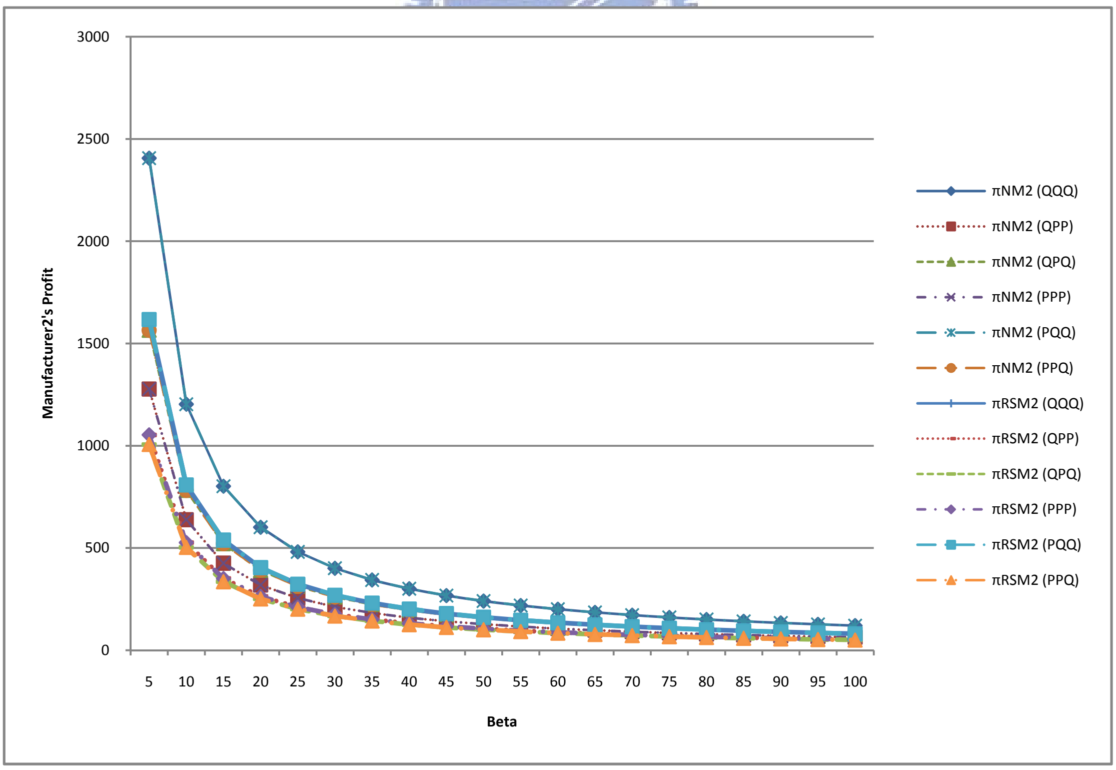



Figure 22. Manufacturer2's profit in different channel structure (substitute products; $\gamma^2/\beta_1\beta_2 = 0.6$)

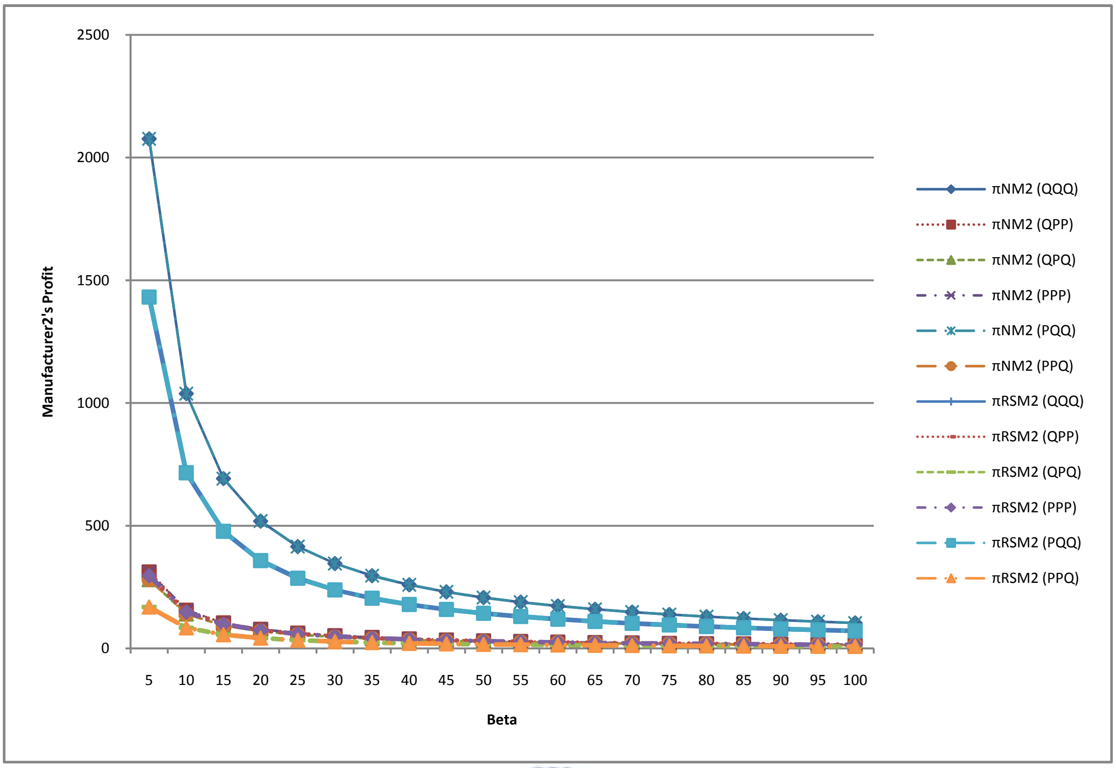



Figure 23. Manufacturer2's profit in different channel structure (substitute products; $\gamma^2 / \beta_1 \beta_2 = 0.9$)

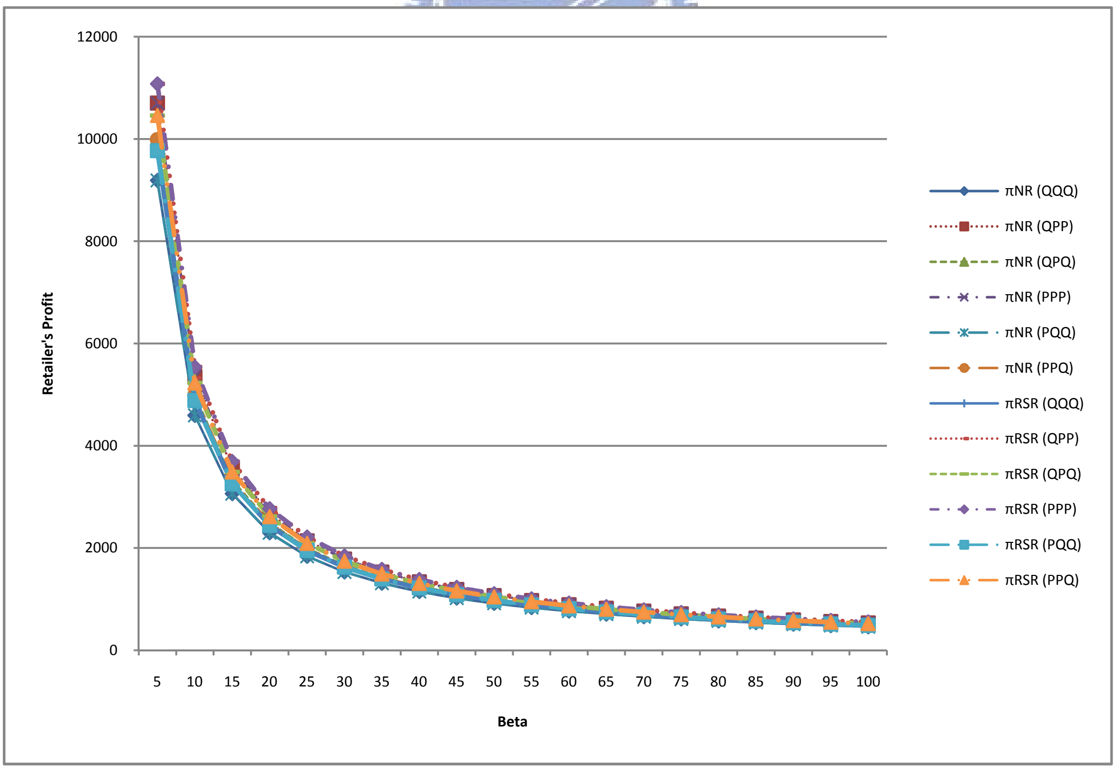



Figure 24. Retailer's profit in different channel structure (substitute product; $\gamma^2 / \beta_1 \beta_2 = 0.3$)

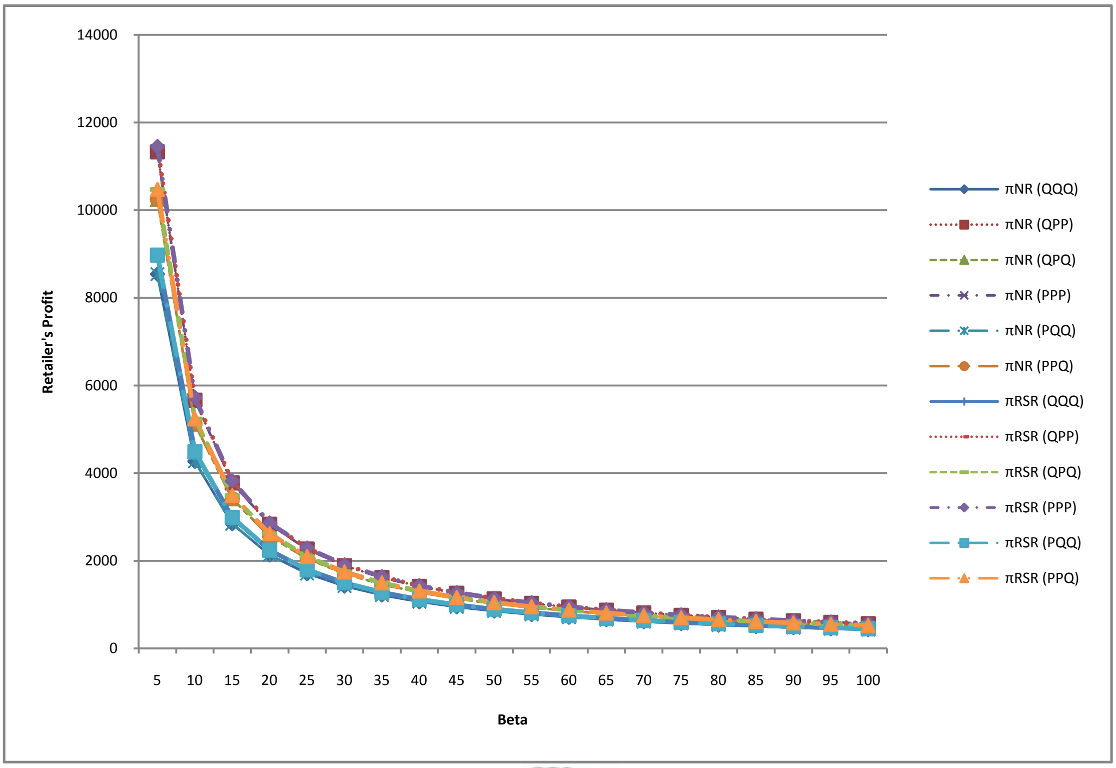



Figure 25. Retailer's profit in different channel structure (substitute products; $\gamma^2/\beta_1\beta_2 = 0.6$)

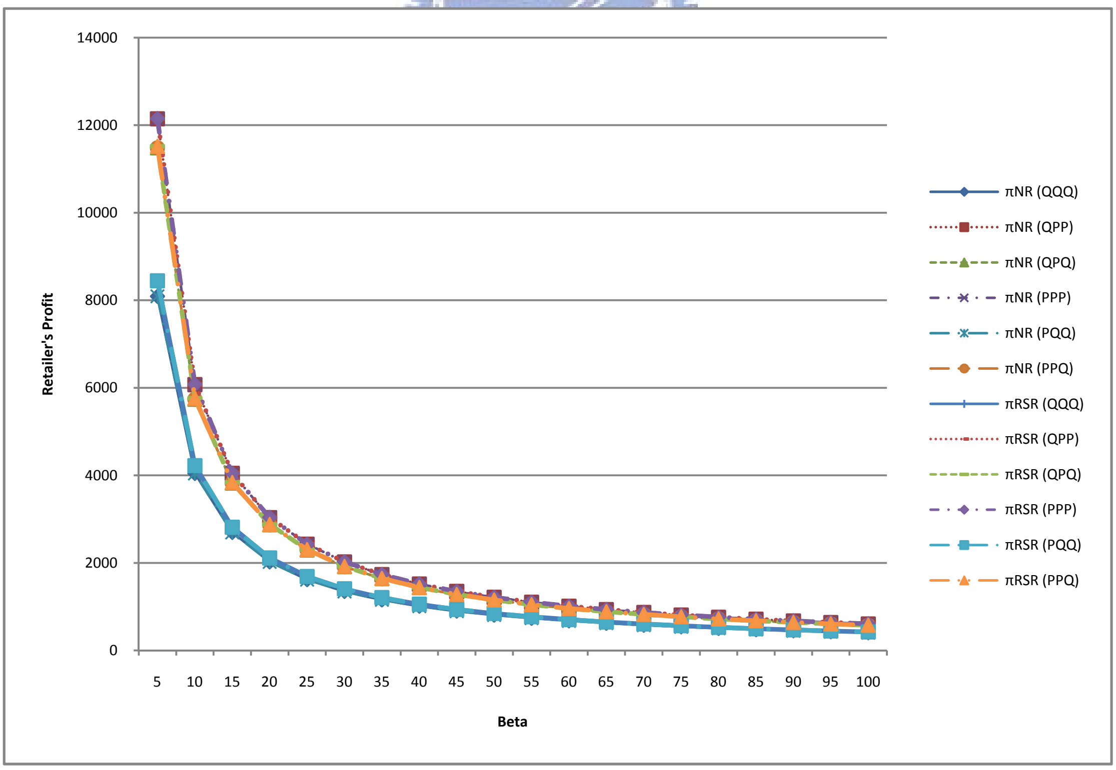



Figure 26. Retailer's profit in different channel structure (substitute products; $\gamma^2/\beta_1\beta_2 = 0.9$)

Table 4. Corresponding parameters applied in Figure 27-29

| $\alpha_1$ | $\alpha_2$ | | $\beta_1$ | | $\beta_2$ | | $\mu_1$ | | $\mu_2$ | |
|---|---|---|---|---|---|---|---|---|---|---|
| 500 | 500 | | 5 | | 5 | | 1 | | 1 | |
| Degree of difference | 0.05 | 0.1 | 0.15 | 0.2 | 0.25 | 0.3 | 0.35 | 0.4 | 0.45 | 0.5 |
| $\gamma$ | 1.118 | 1.581 | 1.936 | 2.236 | 2.500 | 2.739 | 2.958 | 3.162 | 3.354 | 3.536 |
| Degree of difference | 0.55 | 0.6 | 0.65 | 0.7 | 0.75 | 0.8 | 0.85 | 0.9 | 0.95 | 0.99 |
| $\gamma$ | 3.708 | 3.873 | 4.031 | 4.183 | 4.330 | 4.472 | 4.610 | 4.743 | 4.873 | 4.975 |

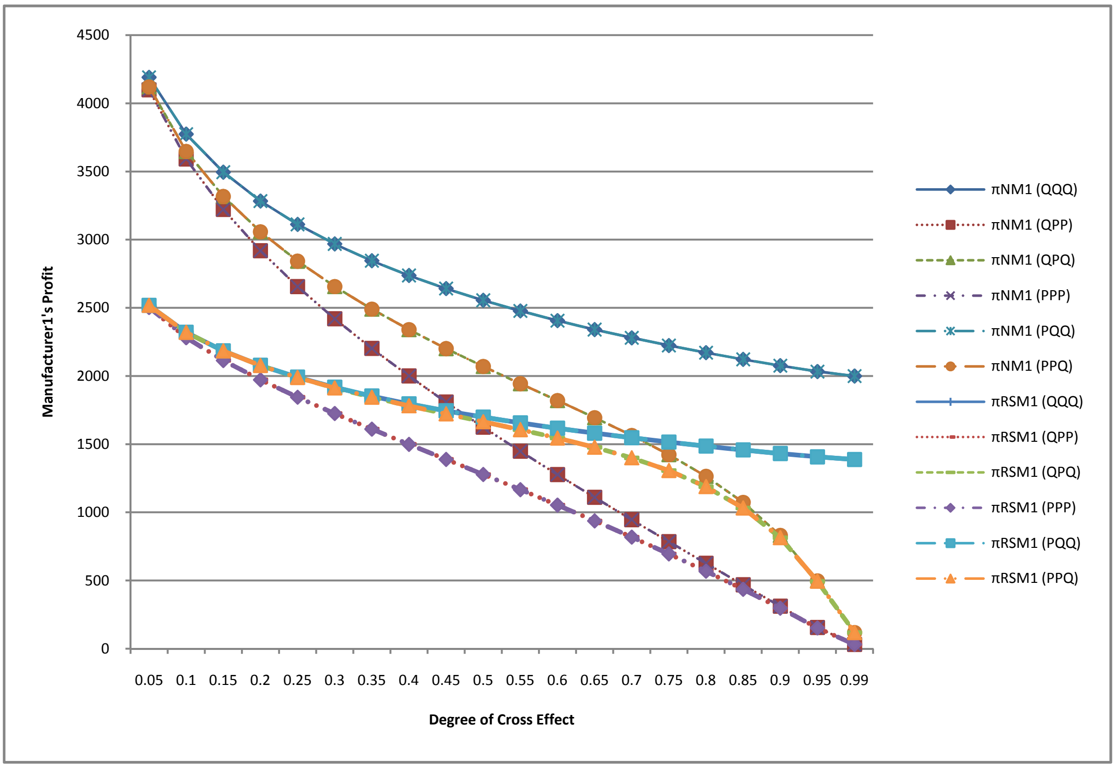



Figure 27. Manufacturer1's profit in different $\gamma^2/\beta_1\beta_2$ (substitute products)

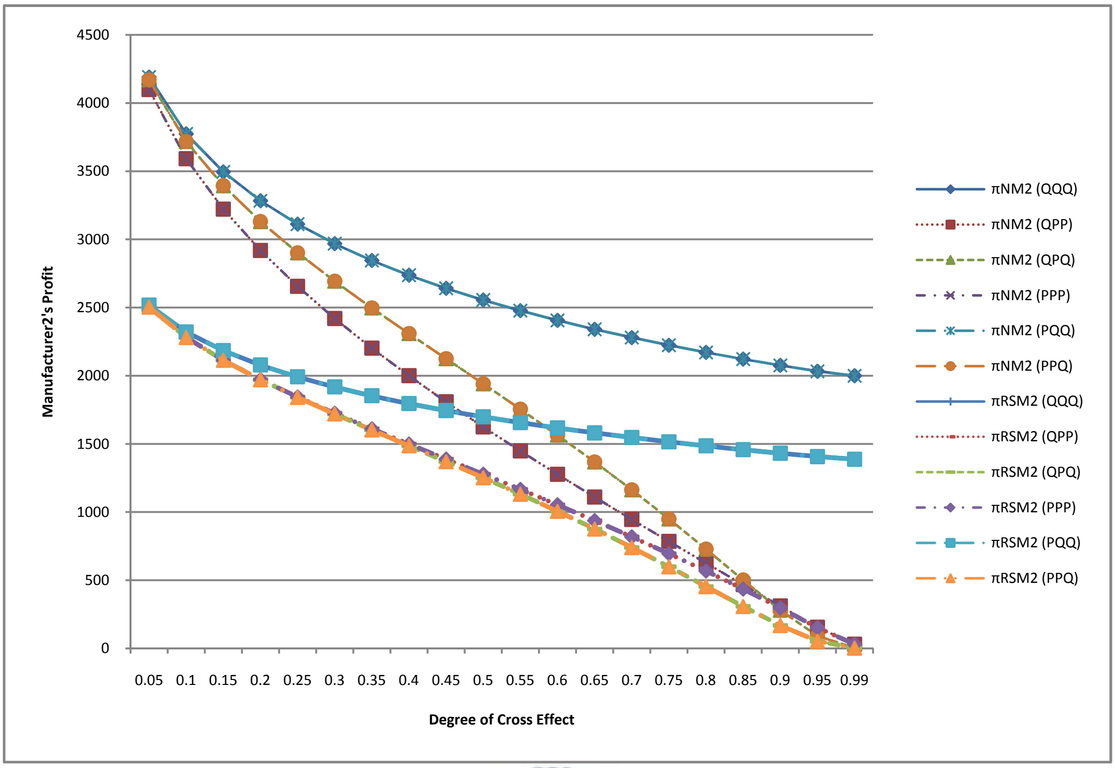



Figure 28. Manufacturer2's profit in different $\gamma^2/\beta_1\beta_2$ (substitute products)

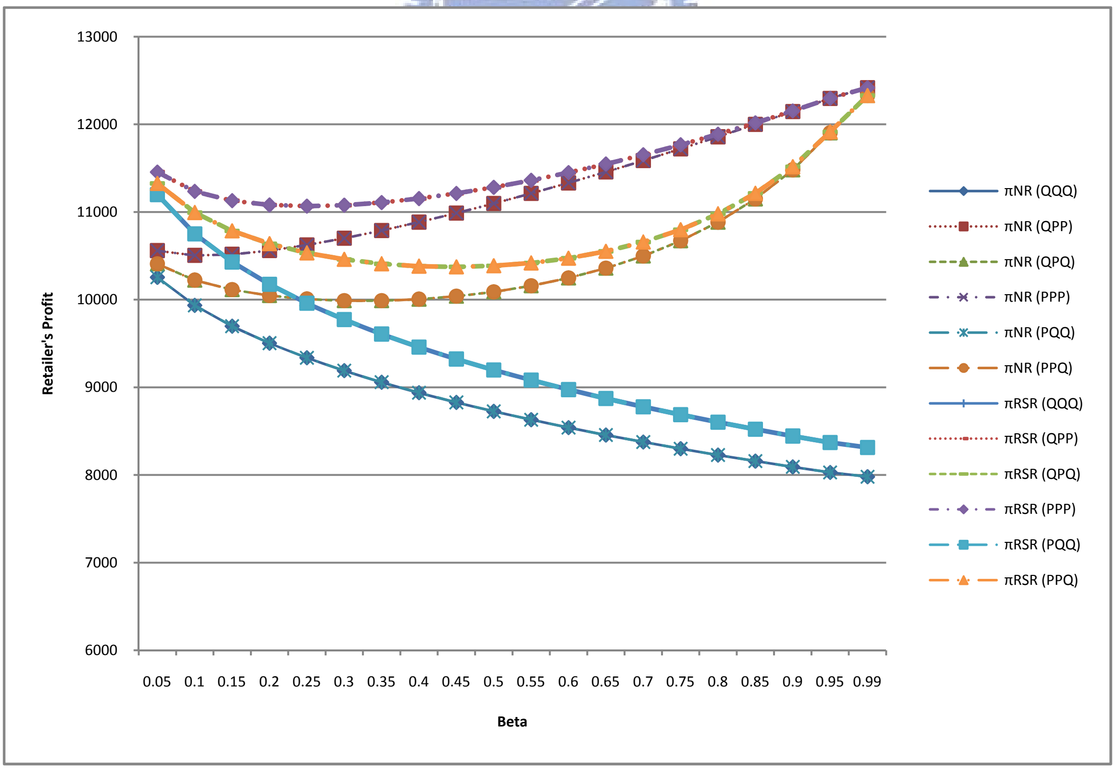



Figure 29. Retailer's profit in different $\gamma^2/\beta_1\beta_2$ (substitute products)

## 5.2 Choice of Strategic Variable with Product Differentiation

In section 4.1 we have learnt about whatever the channel structure is, best strategies of manufacturers are depend on the degree of cross-effect. We now focus on the Retailer-Stackelberg game and make a precise understanding about interactions within the members. Influence by degree of cross-effect is introduced in 4.2.1 and, furthermore, quality differentiation is being considered in 4.2.2.

### 5.2.1 Degree of Cross Effect

According to the results above, we have noticed that best strategy of manufacturer is depended on the degree of cross-effect. However, we have no idea what makes one strategy distinguishing from the others. In this part, equilibrium outcomes we show are not only the profits, but also price, margin and quantity of each member.

#### Complement Products

We examine complement products first. Remaining the same parameters which given in Table 3. In terms of manufacturers, Figure 30 and Figure 31 show that the best outcome is that both manufacturer set price as their strategic variable. As the degree of cross-effect approach to 1, it makes a mighty advantage. The resulting equilibrium wholesale prices, retail prices, manufacturers' margin, retailer's margin, quantities across RS game are depicted in Figure 32-Figure 42.

In accordance to our result, we can observe that no matter how large $\gamma^2/\beta_1\beta_2$ is and what strategies the manufacturers play, retailer margin of the two products remain constant. That is, retail price of each product is depends on the wholesale price. When the manufacturers compete in price, it results in a relatively low wholesale price and stimulates demand. Thanks to the nature of complement products, more of product1 being bought would result in more of product2 also being bought. As a result, the equilibrium quantity of both manufacturers set "quantity" as their strategic variables yields a huge different from others.

With respect to retailer, margin of each product it sells remained as a constant. Total profit of the retailer is base on the quantity it sold. That is, when products are complementary, choosing quantity as strategic variable is the best strategy for manufacturers and the retailer.

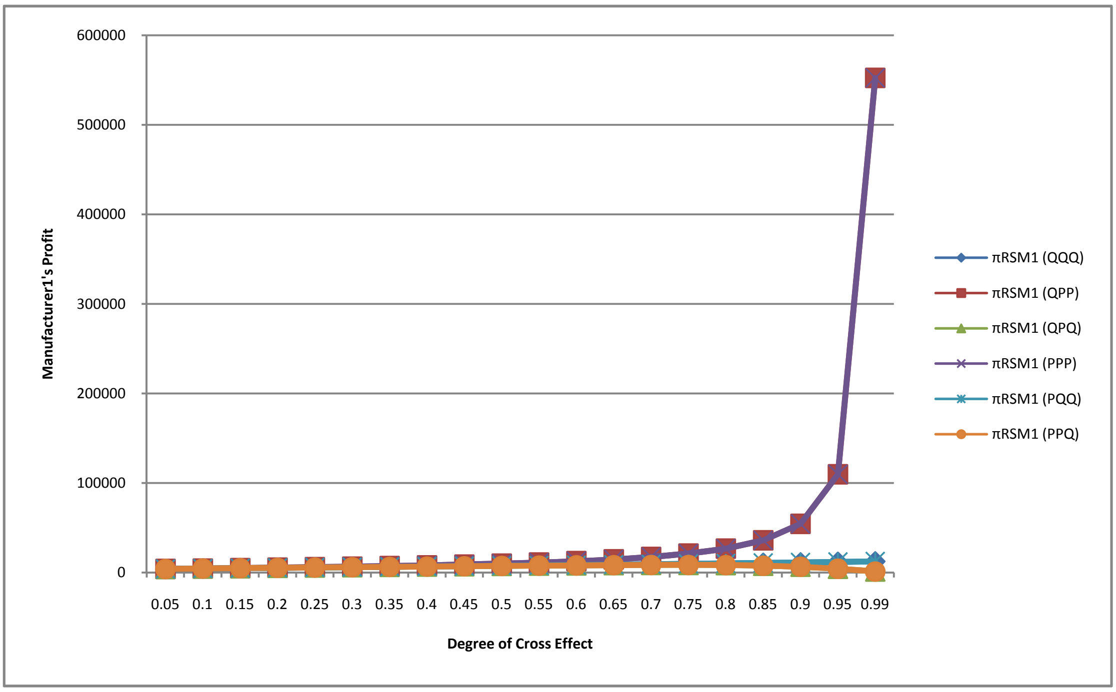



Figure 30 Manufacturer1's profit in RS game (complement products)

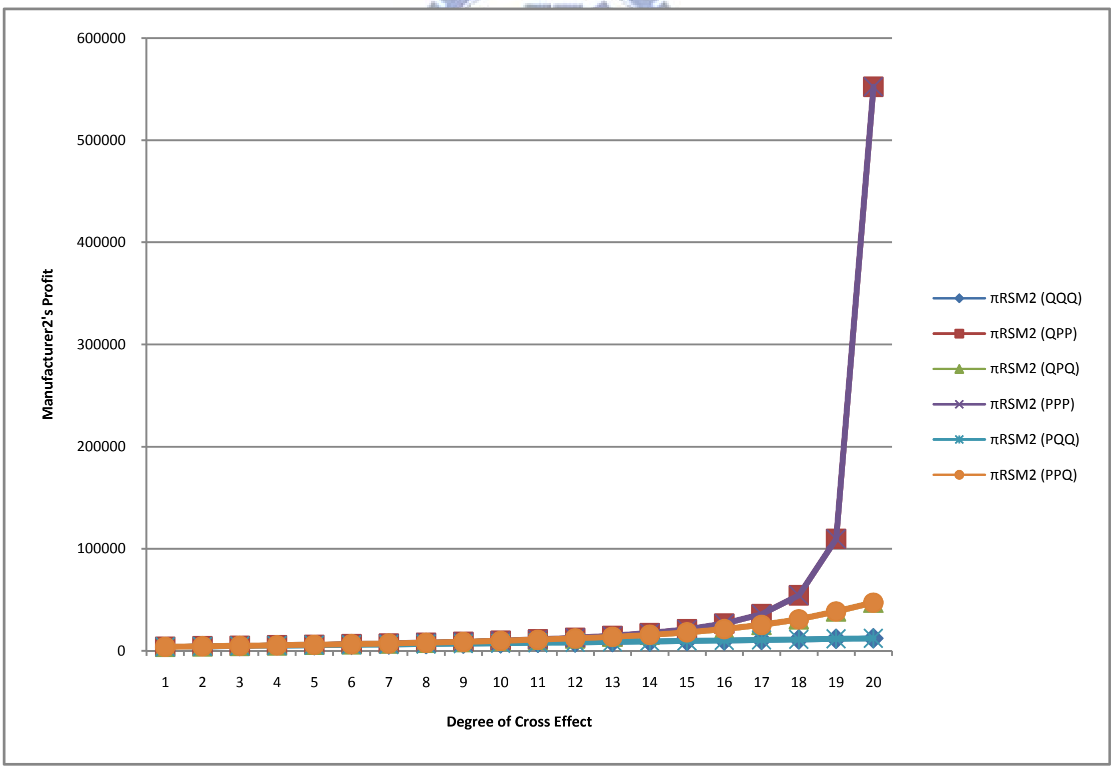



Figure 31 Manufacturer2's profit in RS game (complement products)

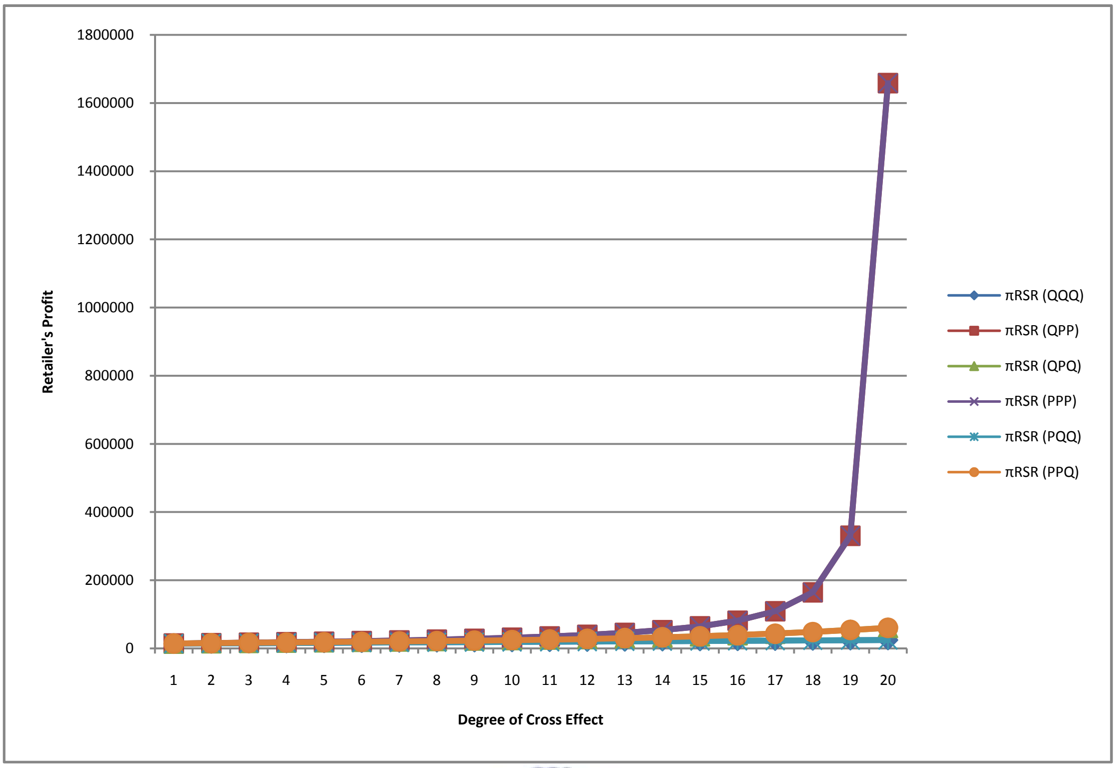



Figure 32 Retailer's profit in RS game (complement products)

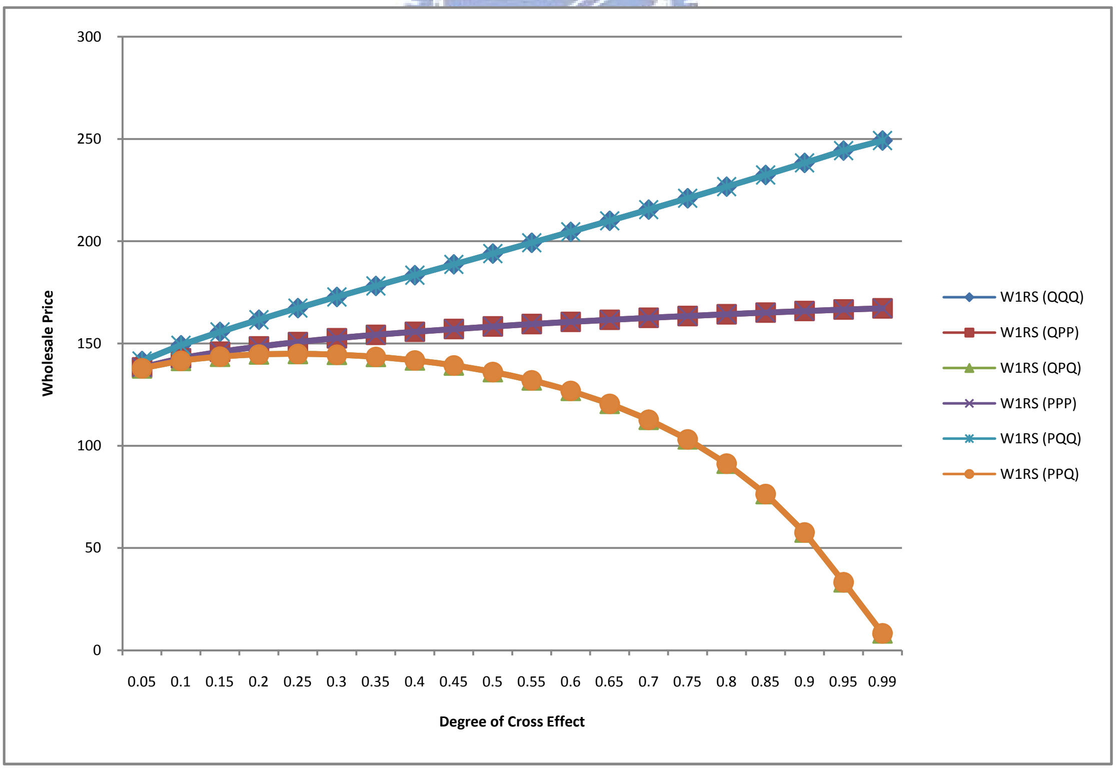



Figure 33 Wholesale price of product1 in RS game (complement products)

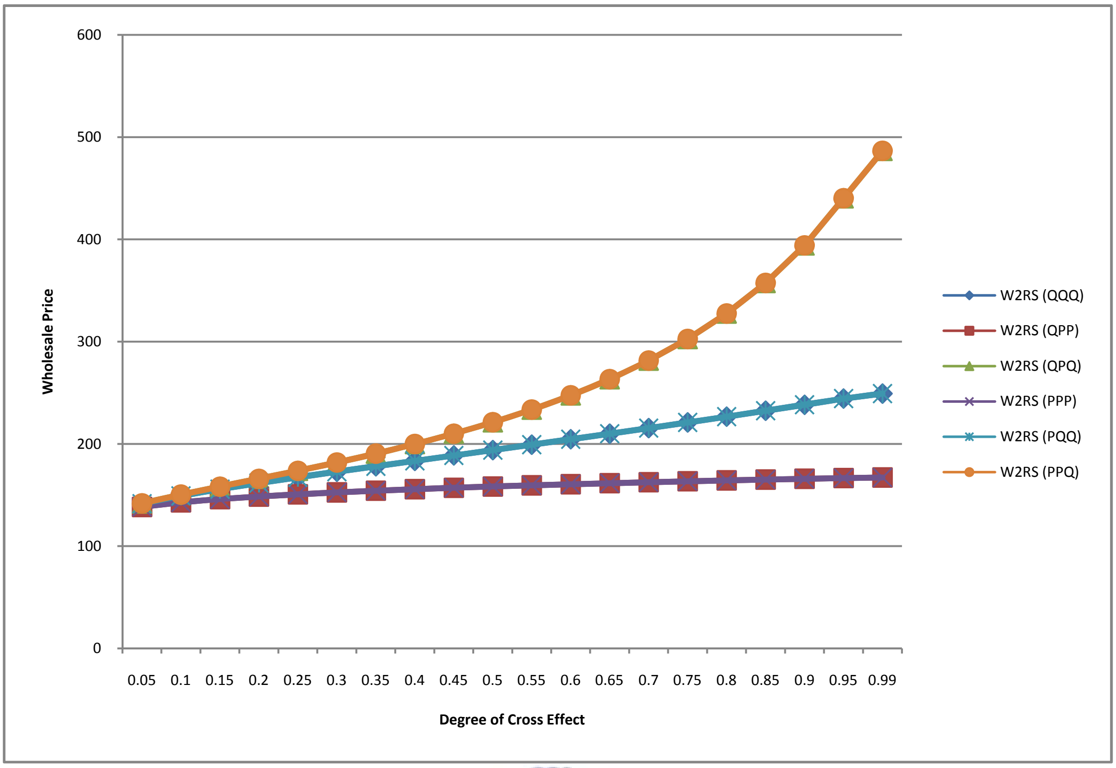



Figure 34 Wholesale price of product2 in RS game (complement products)

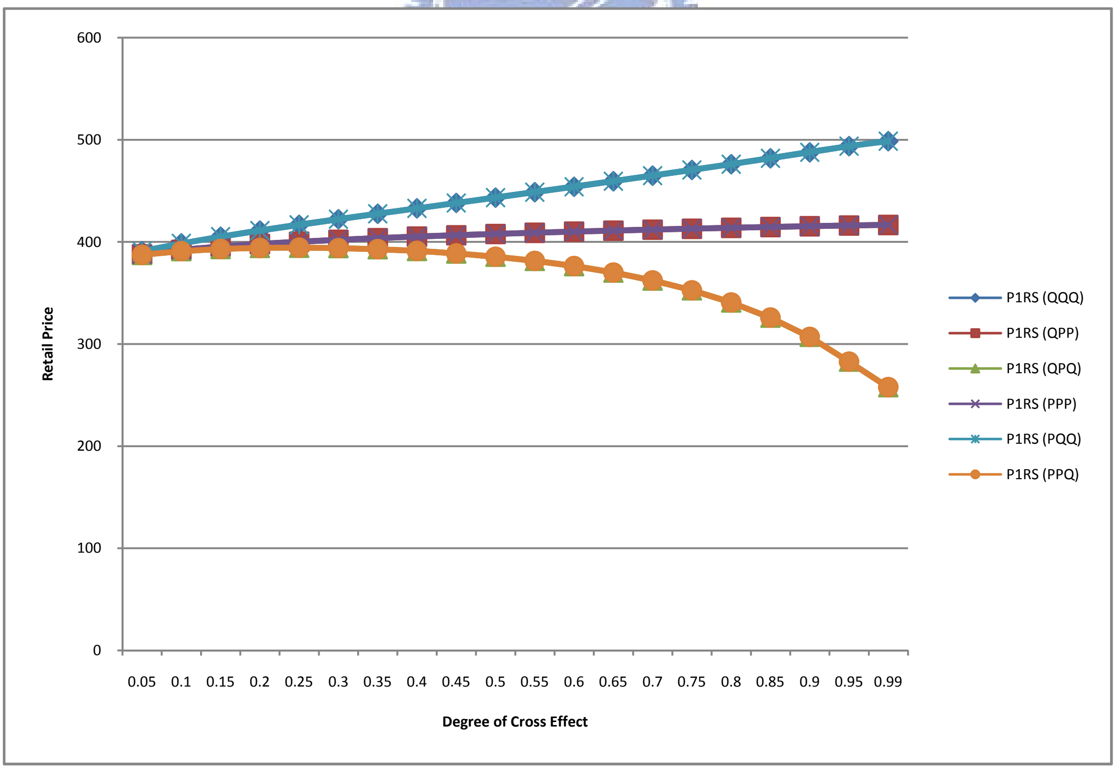



Figure 35 Retail price of product1 in RS game (complement products)

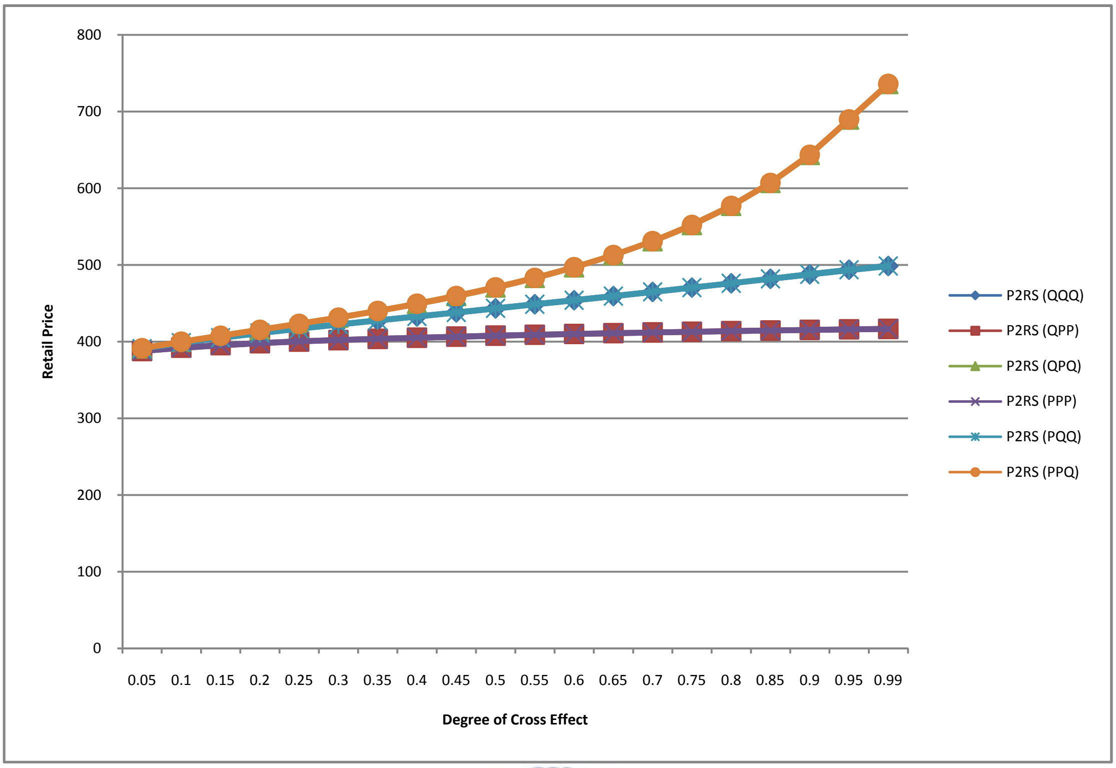



Figure 36 Retail price of product2 in RS game (complement products)

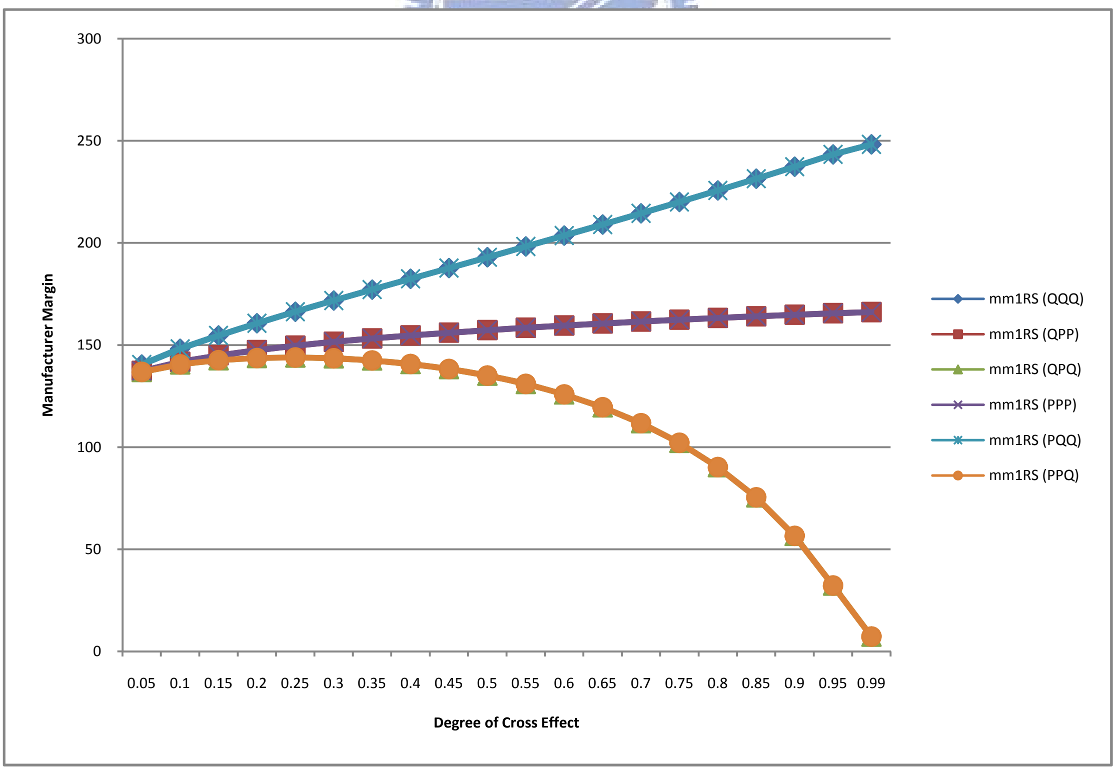



Figure 37 Manufacturer's margin of product1 in RS game (complement products)

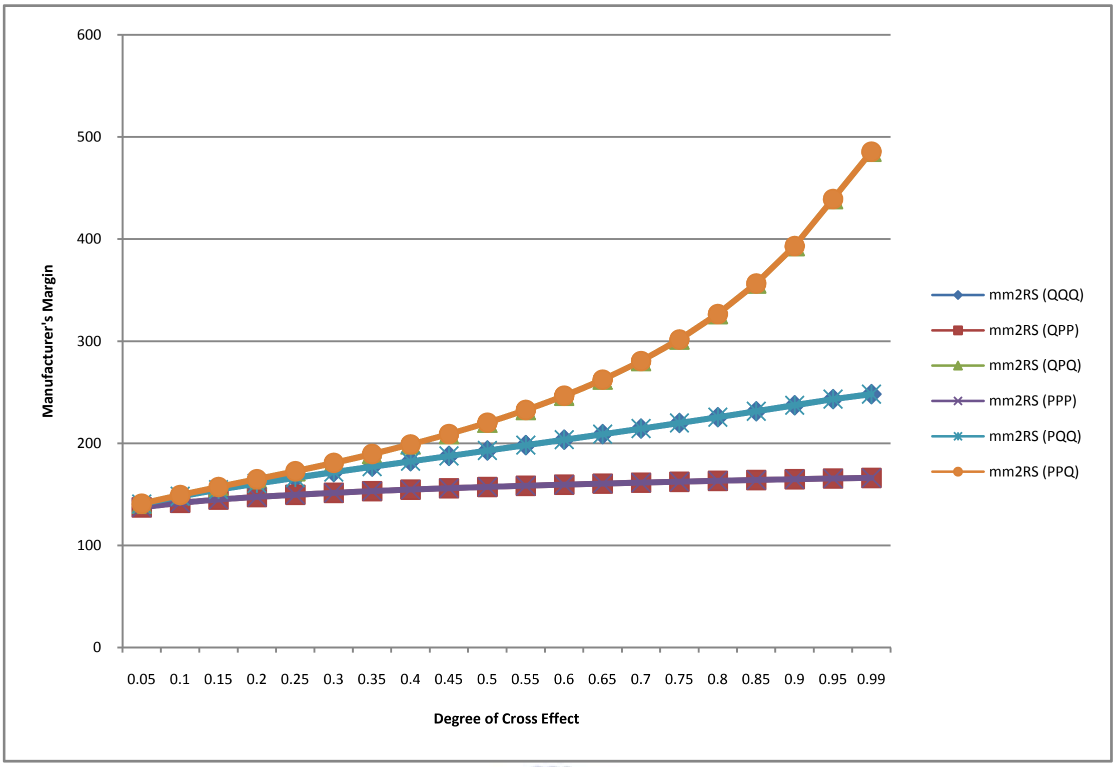



Figure 38 Manufacturer's margin of product2 in RS game (complement products)

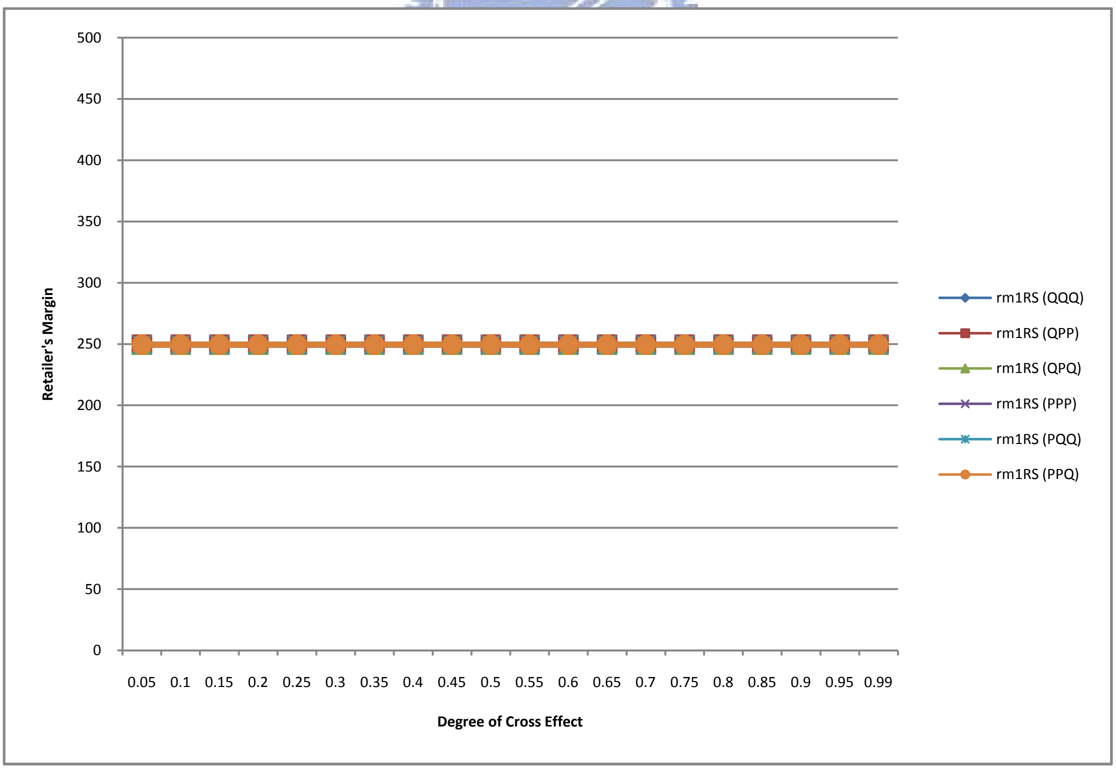



Figure 39 Retailer's margin of product1 in RS game (complement products)

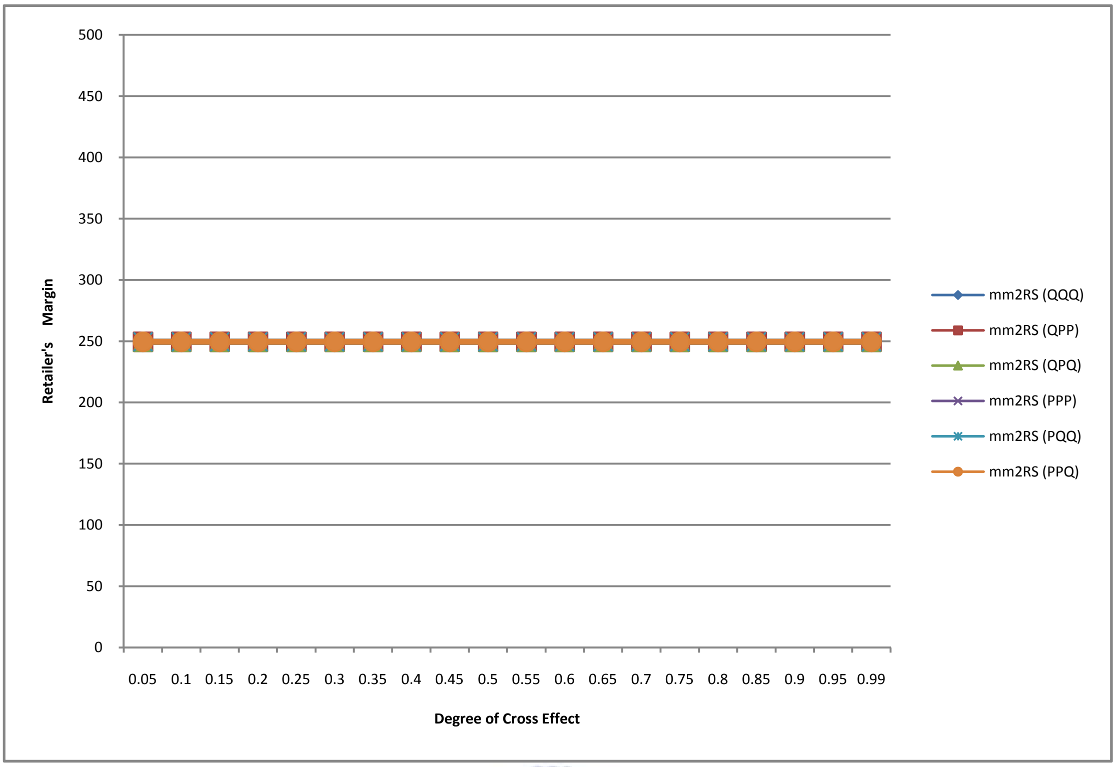



Figure 40 Retailer's margin of product2 in RS game (complement products)

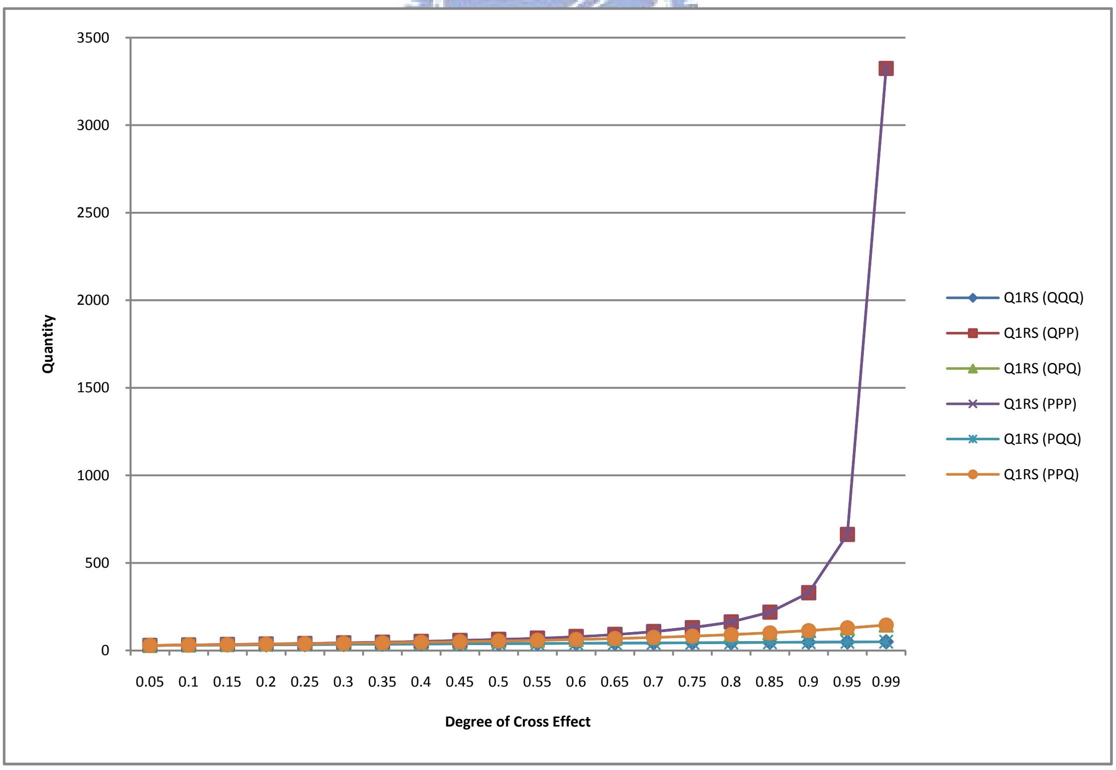



Figure 41 Quantity of product1 in RS game (complement products)

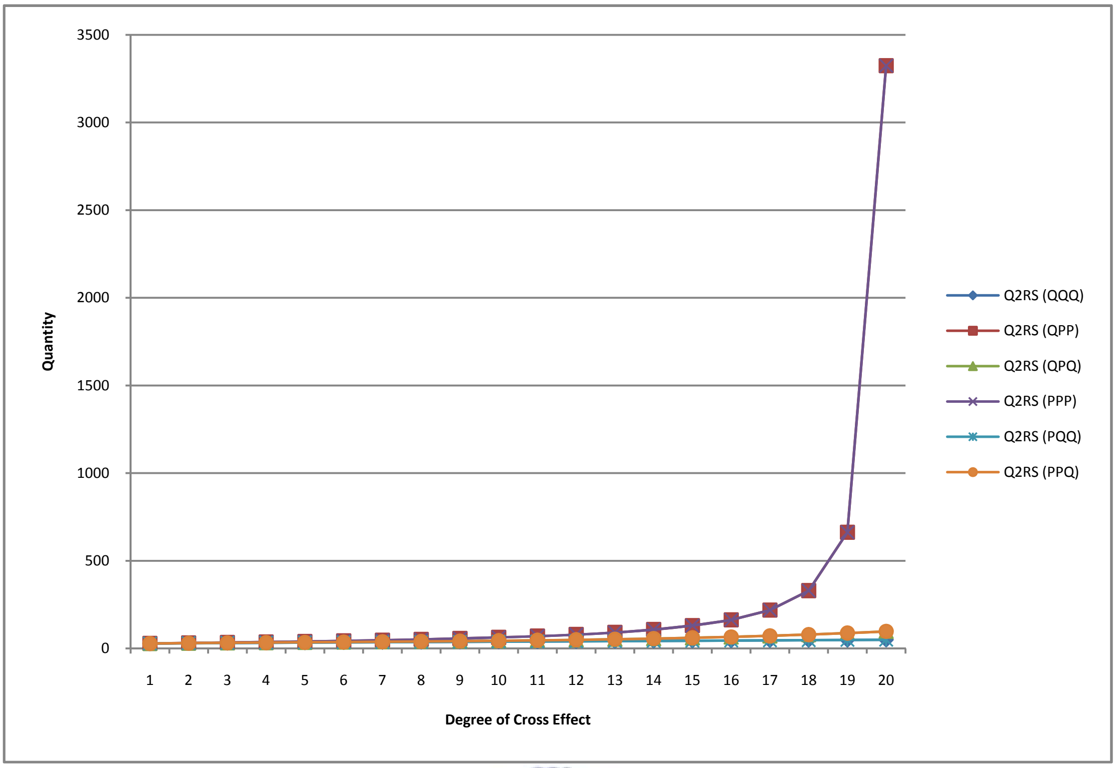



Figure 42 Quantity of product2 in RS game (complement products)

### Substitute Products

We now keep our eyes on the substitute products. Remaining the same parameters which given in Table 4. The resulting equilibrium wholesale prices, retail prices, manufacturers' margin, retailer's margin, quantities across RS game are depicted in Figure 32-Figure 42.

Same as complement products, we can observe that no matter how large $\gamma^2/\beta_1\beta_2$ is and what strategies the manufacturers play, retailer margin of the two products remain constant. Hence, retail price of each product is depends on the wholesale price. When the manufacturers compete in price, it results in a relatively low wholesale price and stimulates demand. In accordance to our results, price competition results in lowest price (both wholesale price and retail price) and a relatively large quantity of demand. However, descending wholesale price squeezes manufacturer's margin. Figure 43 and Figure 44 show that the best outcome is that both manufacturer set quantity as their strategic variable.

With respect to retailer, margin of each product it sells remained as a constant. Total profit of the retailer is base on the quantity it sold. It is the best outcome for retailer when manufacturers fall into price competition.

Therefore, manufacturers under RS game would like to choose quantity as their strategic variable. Nevertheless, if the dominant-retailer is able to influence its suppliers' decision, retailer's profit would be maximized by forcing manufacturers to choose price as strategic variable when products are substitutes.

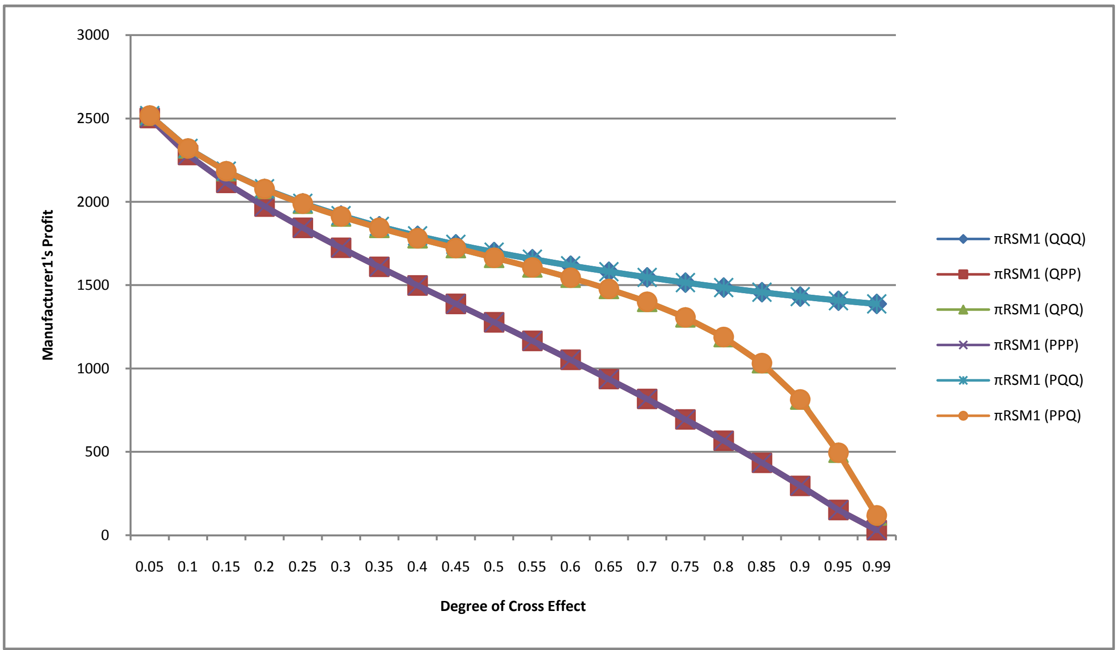



Figure 43 Manufacturer1's profit in RS game (substitute products)

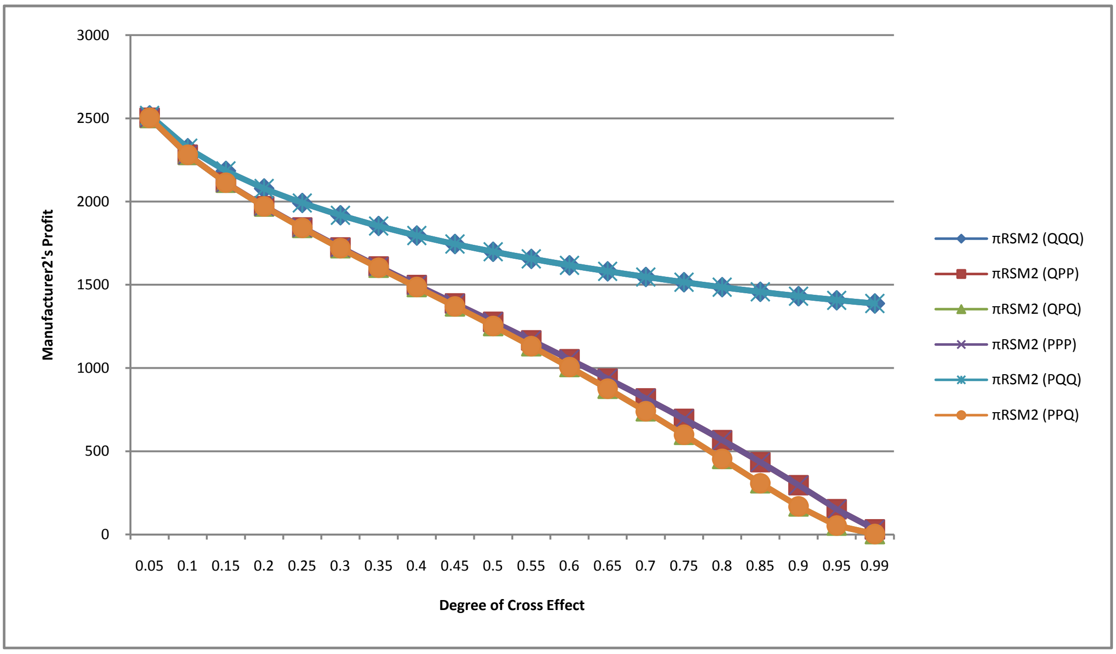



Figure 44 Manufacturer2's profit in RS game (substitute products)

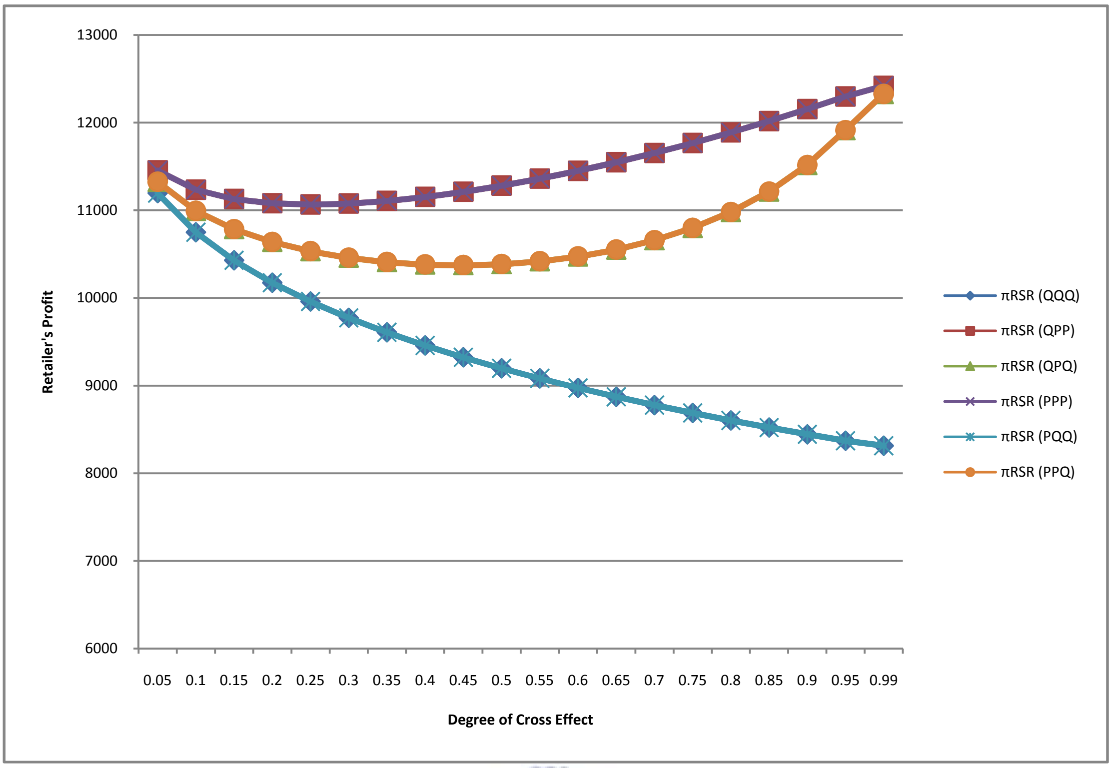



Figure 45 Retailer's profit in RS game (substitute products)

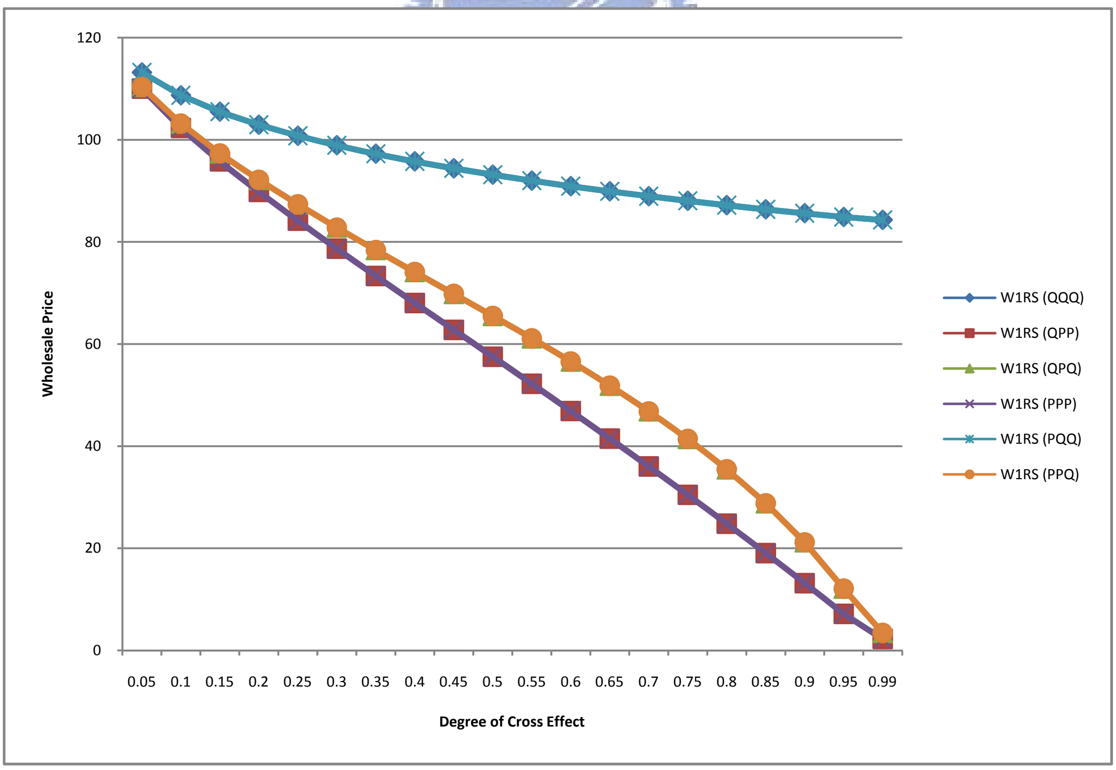



Figure 46 Wholesale price of product1 in RS game (substitute products)

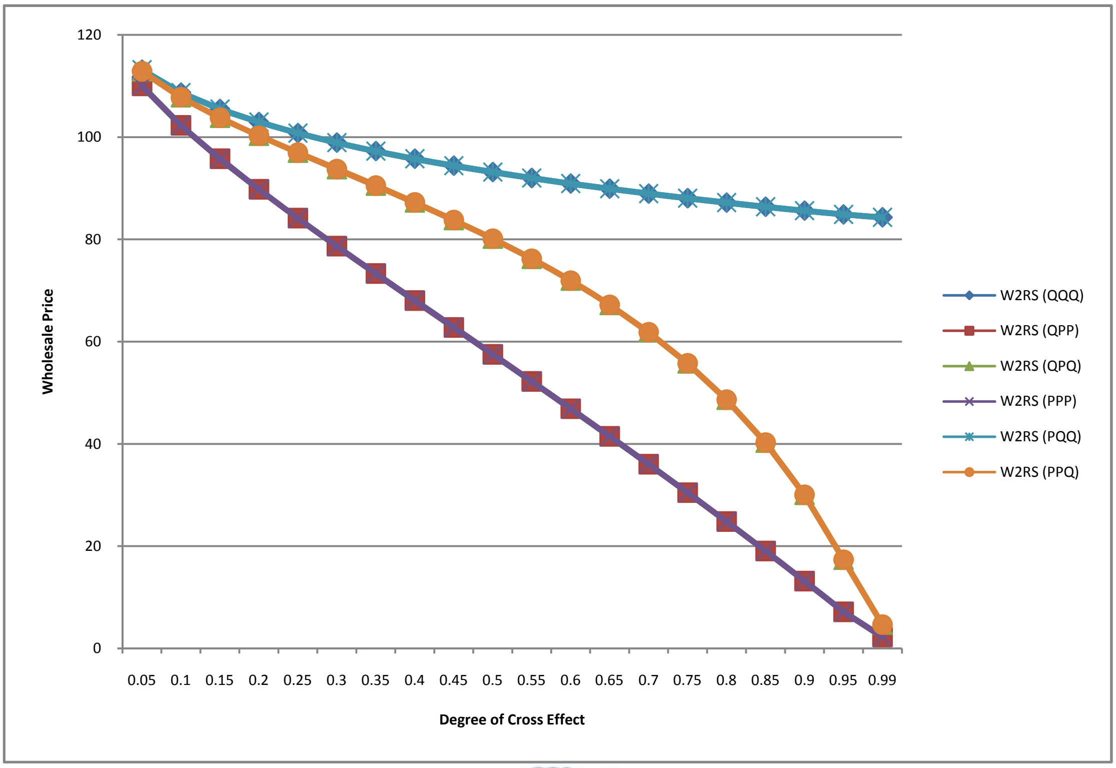



Figure 47 Wholesale price of product2 in RS game (substitute products)

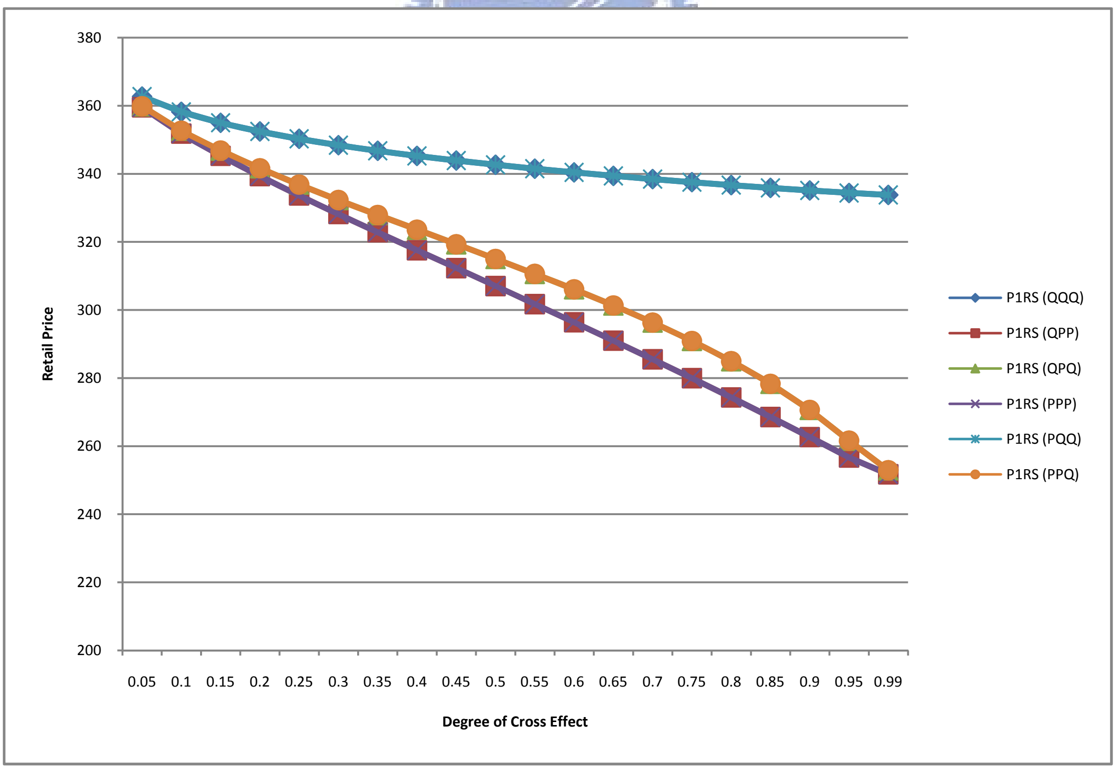



Figure 48 Retail price of product1 in RS game (substitute products)

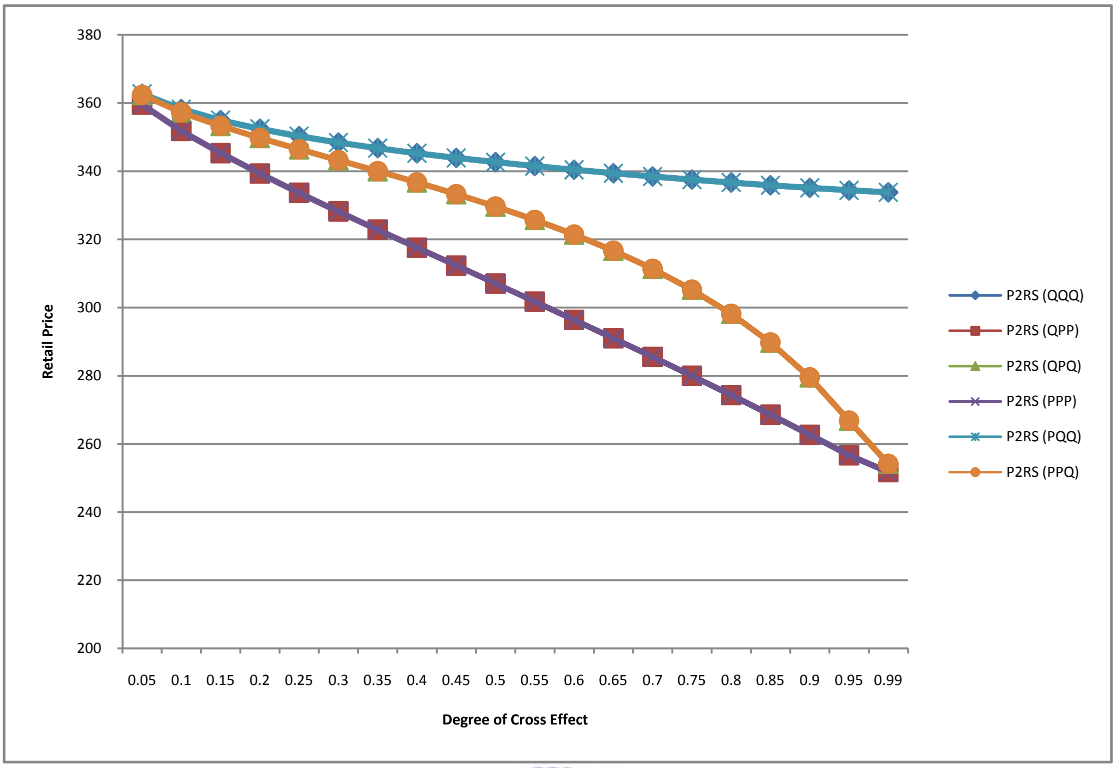



Figure 49 Retail price of product2 in RS game (substitute products)

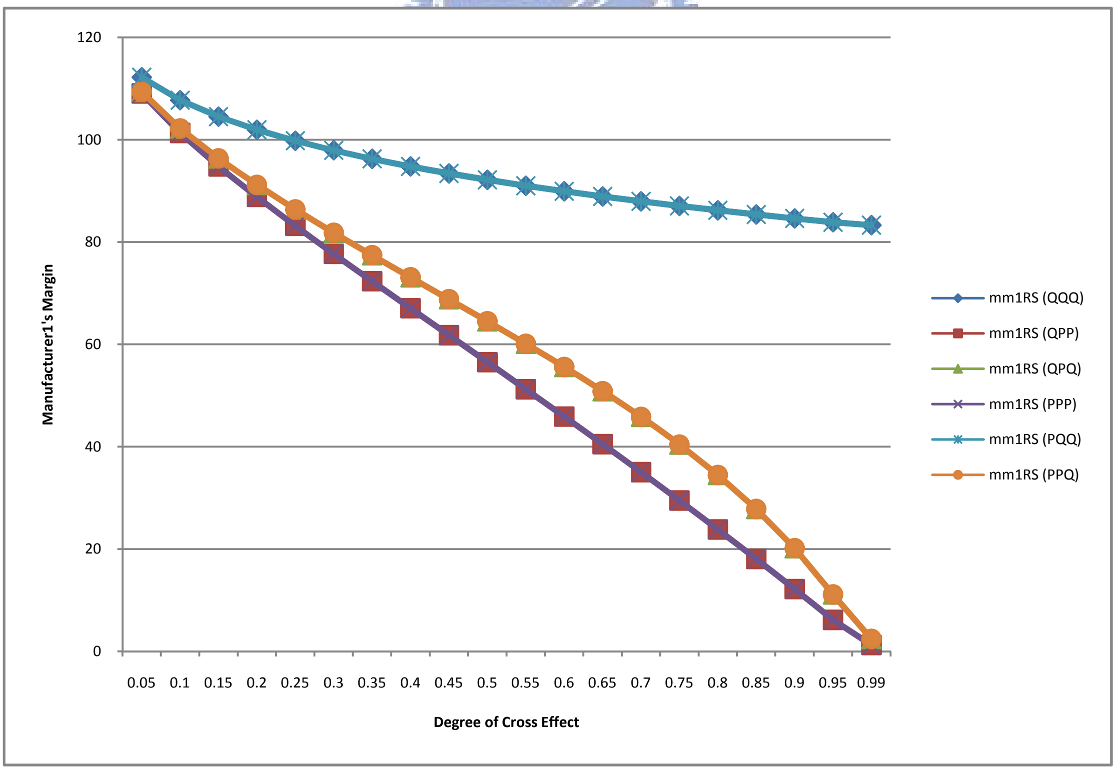



Figure 50 Manufacturer's margin of product1 in RS game (substitute products)

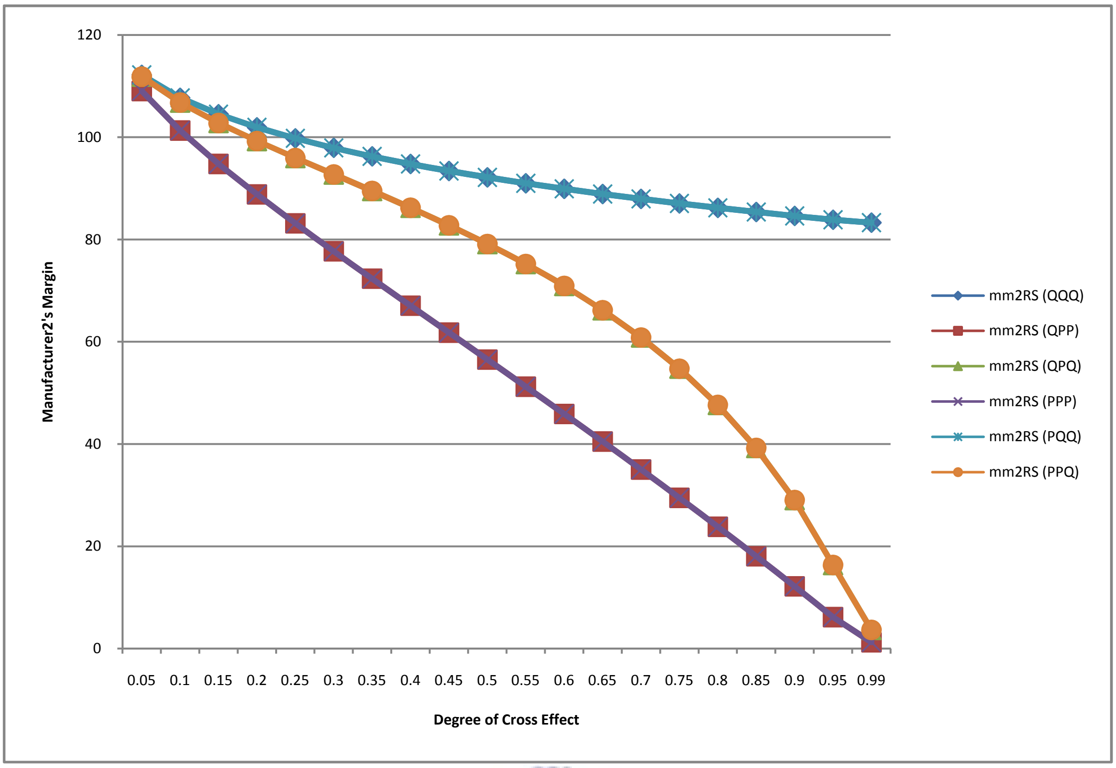



Figure 51 Manufacturer's margin of product2 in RS game (substitute products)

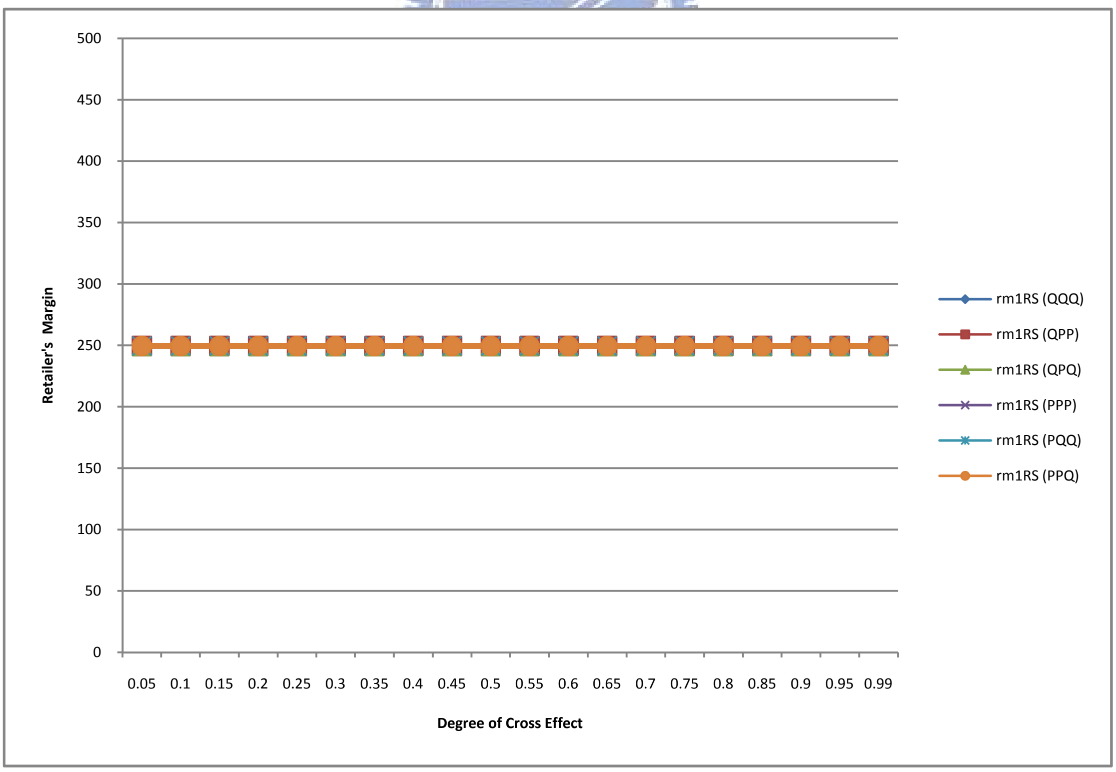



Figure 52 Retailer's margin of product1 in RS game (substitute products)

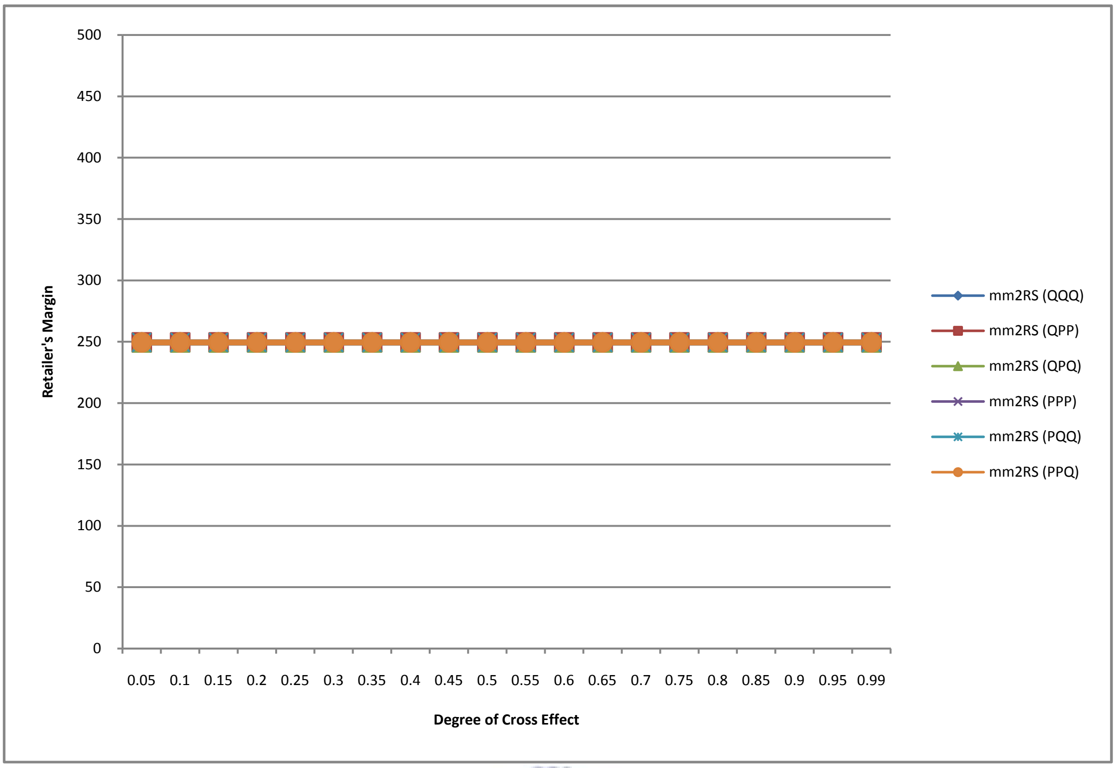



Figure 53 Retailer's margin of product2 in RS game (substitute products)

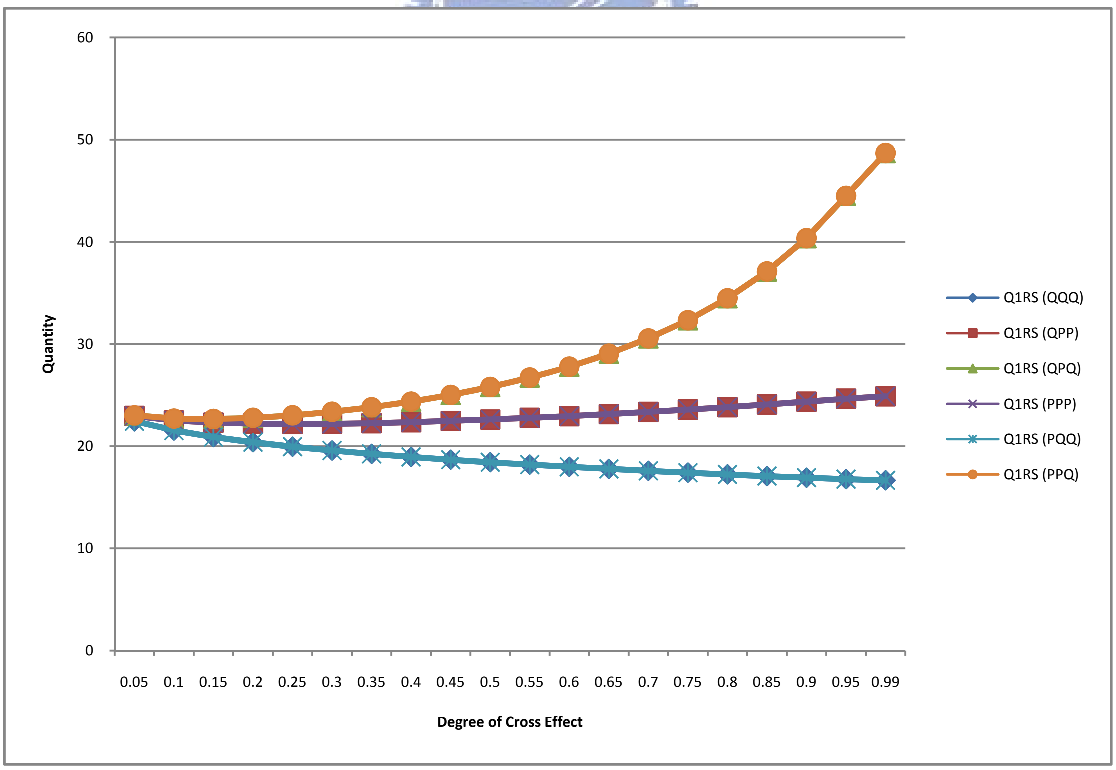



Figure 54 Quantity of product1 in RS game (substitute products)

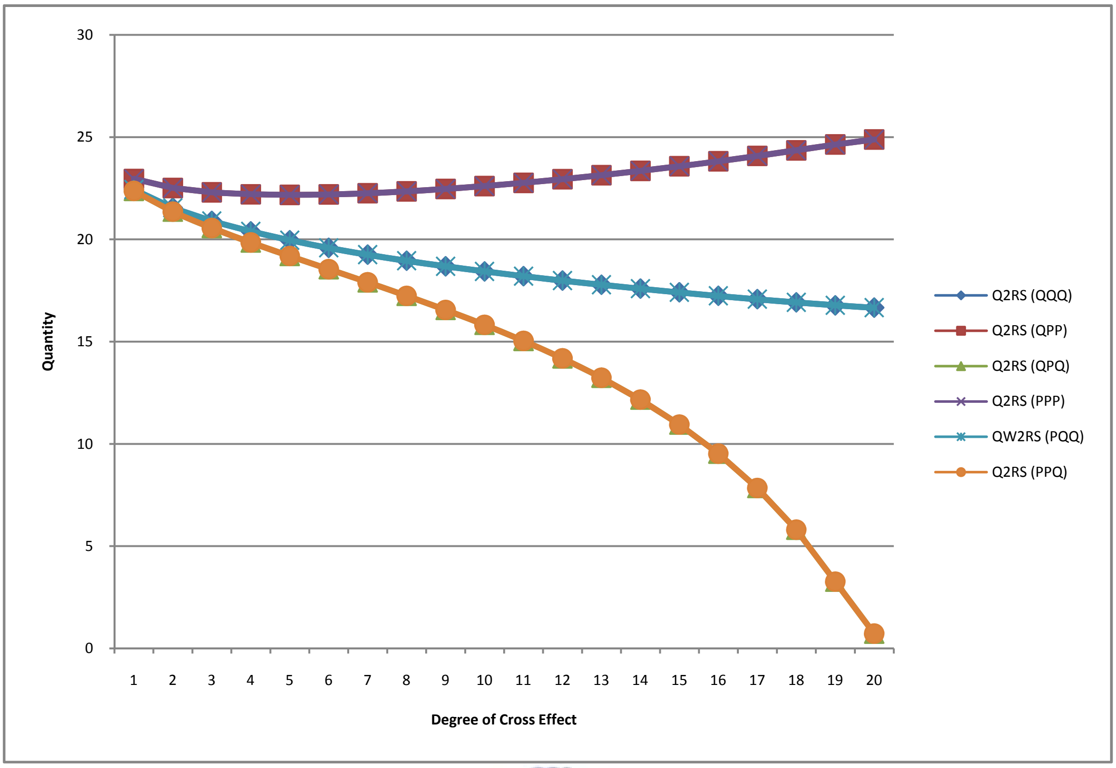



Figure 55 Quantity of product2 in RS game (substitute products)

### 5.2.2 Quality Difference between Two Products

Quality difference may be the result of, for example, asymmetric product R&D investments. We define $\theta = \alpha_1/\alpha_2$ as the degree of quality differentiation. For $\theta > 1$, product 1 has an absolute advantage in demand; for $\theta < 1$, firm 2 has an absolute advantage in demand, because $\alpha_2 > \alpha_1$; and $\theta = 1$ implies no quality difference between the products. Recall that **Assumption 4.2** make restriction on parameters $\alpha_i\beta_j - \alpha_j\gamma > 0$. That is, $\alpha_2\gamma/\beta_2 < \alpha_1 < \alpha_2\beta_1/\gamma$. Effect of quality differential will be examined by complement and substitute products and degree of cross effect is also considered here.

#### Complement Products

As the quality had improved, equilibrium profit of the manufacturer is higher (see Figure 56-Figure 61). This higher profit is attributed to the increase of margin and quantity. Due to the nature of complement products, more of product1 being bought would result in more of product2 also being bought. Hence, all members contained in the channel can be benefited. According to our results, quality differentiation makes no effect on choice of strategic variable.

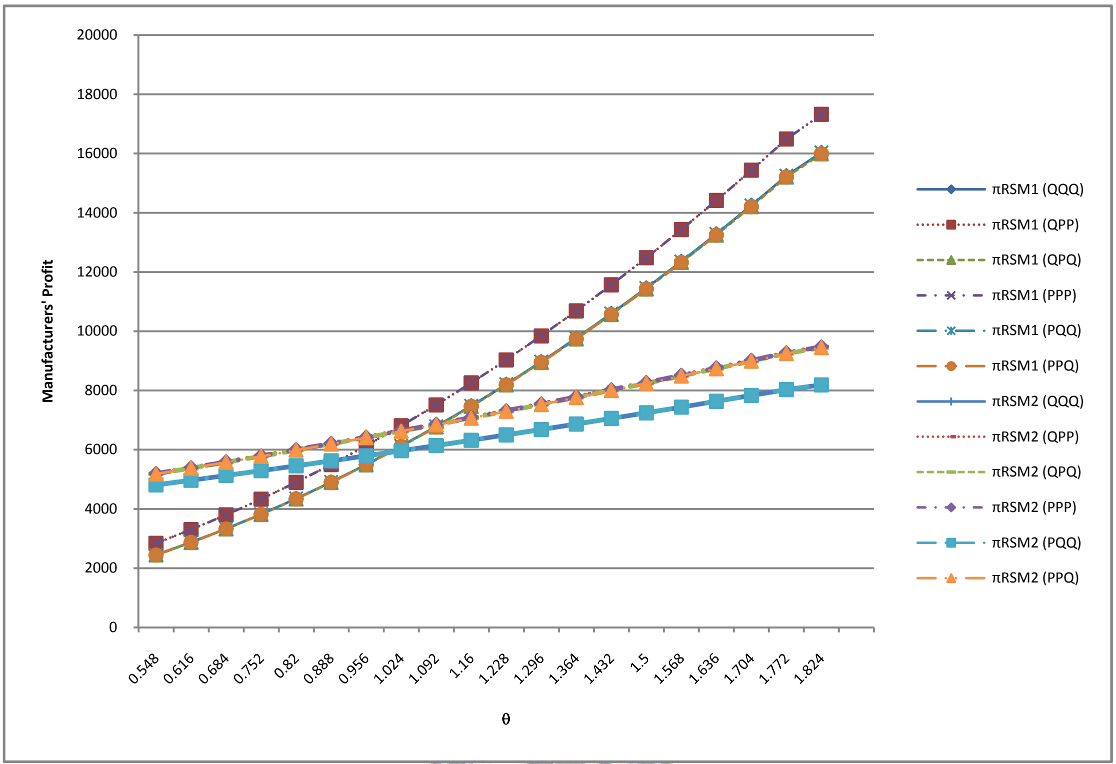



Figure 56 Manufactures' profit with quality differential (complement products; $\gamma^2/\beta_1\beta_2 = 0.3$)

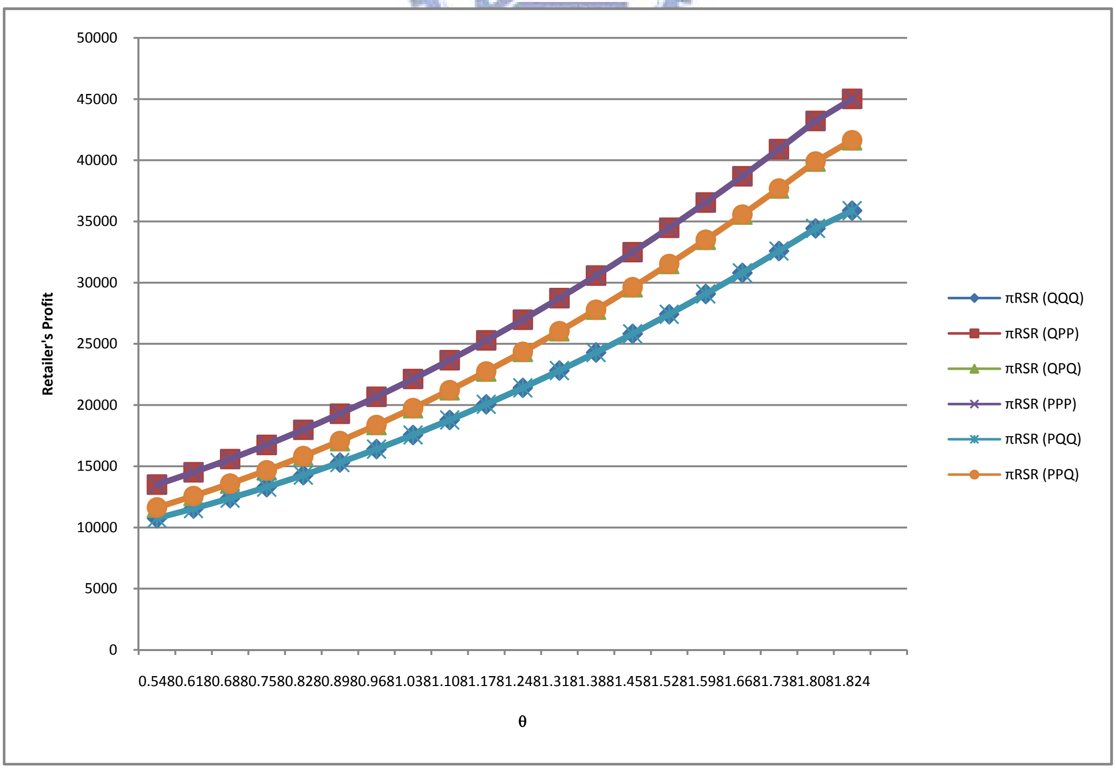



Figure 57 Retailer's profit with quality differential (complement products; $\gamma^2/\beta_1\beta_2 = 0.3$)

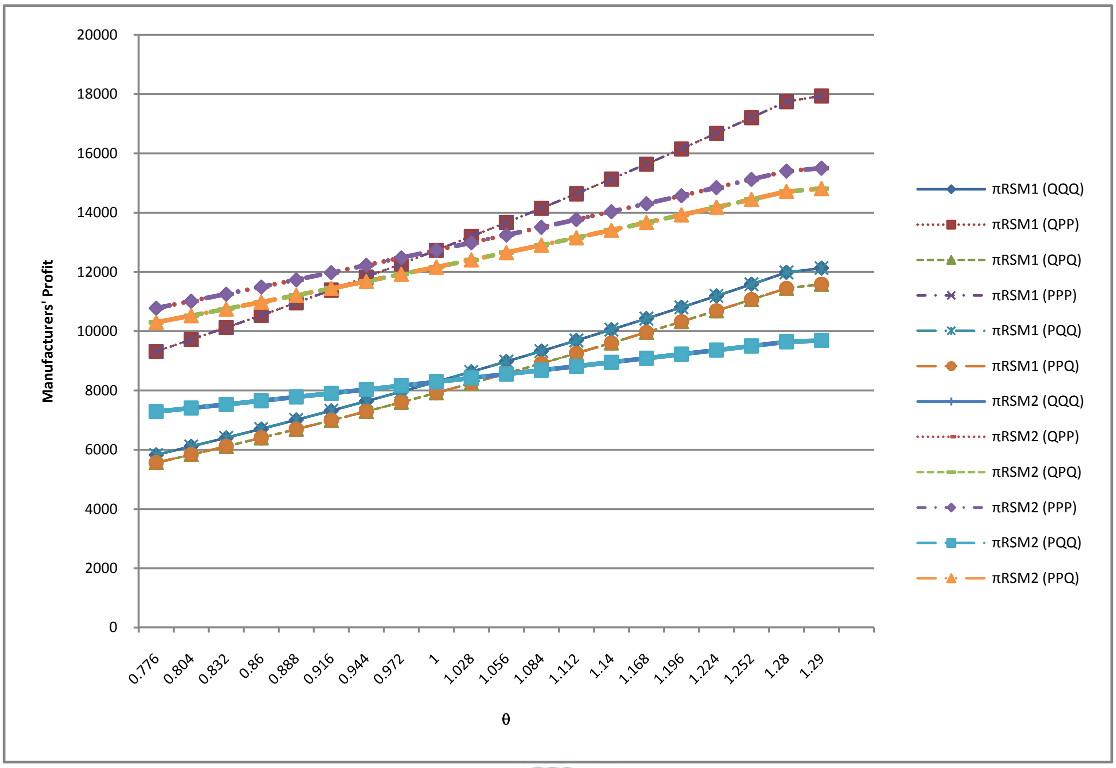



Figure 58 Manufacturers' profit with quality differential (complement products; $\gamma^2/\beta_1\beta_2 = 0.6$)

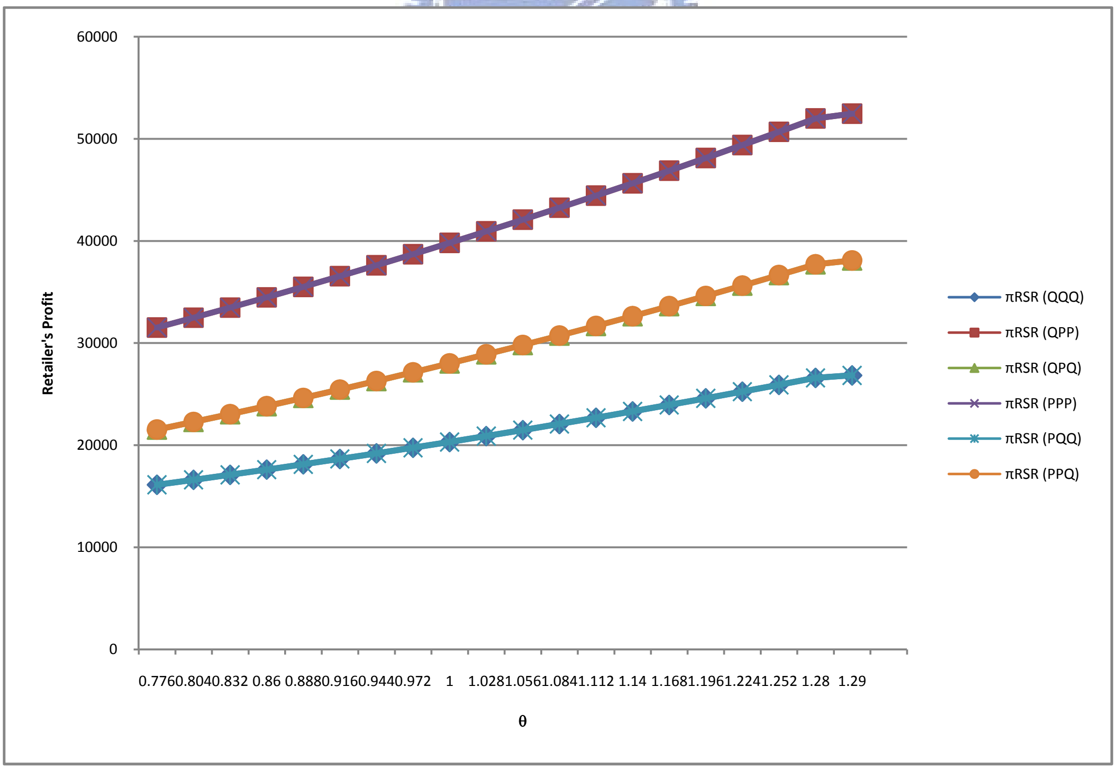



Figure 59 Retailer's profit with quality differential (complement products; $\gamma^2/\beta_1\beta_2 = 0.6$ )

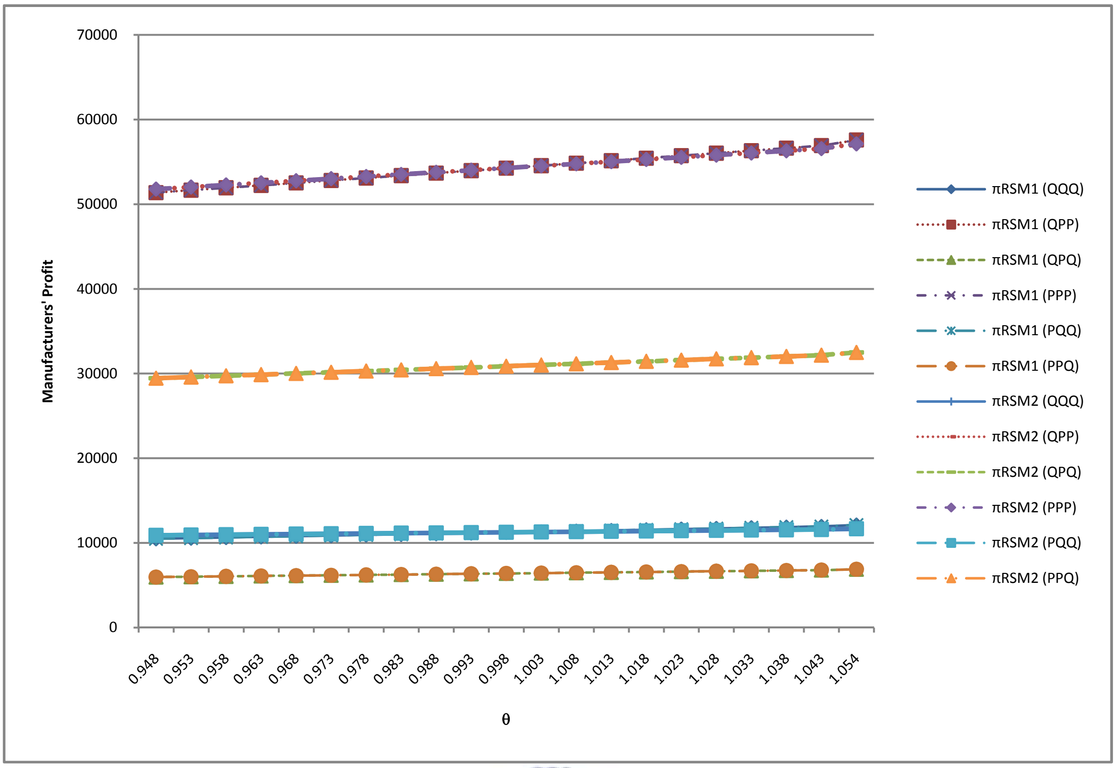



Figure 60 Manufacturers' profit with quality differential (complement products; $\gamma^2/\beta_1\beta_2 = 0.9$)

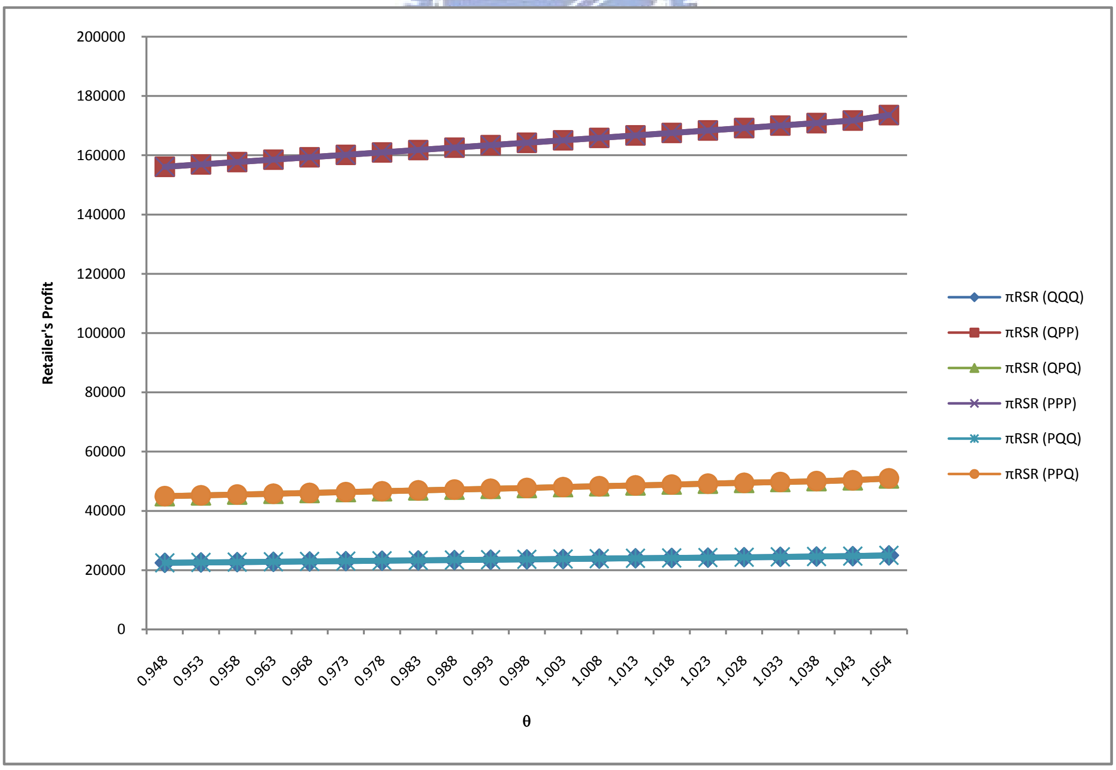



Figure 61 Retailer's profit with quality differential (complement products; $\gamma^2/\beta_1\beta_2 = 0.9$)

## Substitute Products

For the case of substitute products, as the quality had improved, equilibrium profit of the manufacturer and retailer are higher (see Figure 62-Figure 67). This higher profit is attributed to the increase of margin and quantity.

However, for substitute products, more of product1 being bought would result in less of product2 being bought. Where high-quality product possesses a higher profit, it hurts the low-quality manufacturer. According to our results, quality differentiation makes no effect on choice of strategic variable.

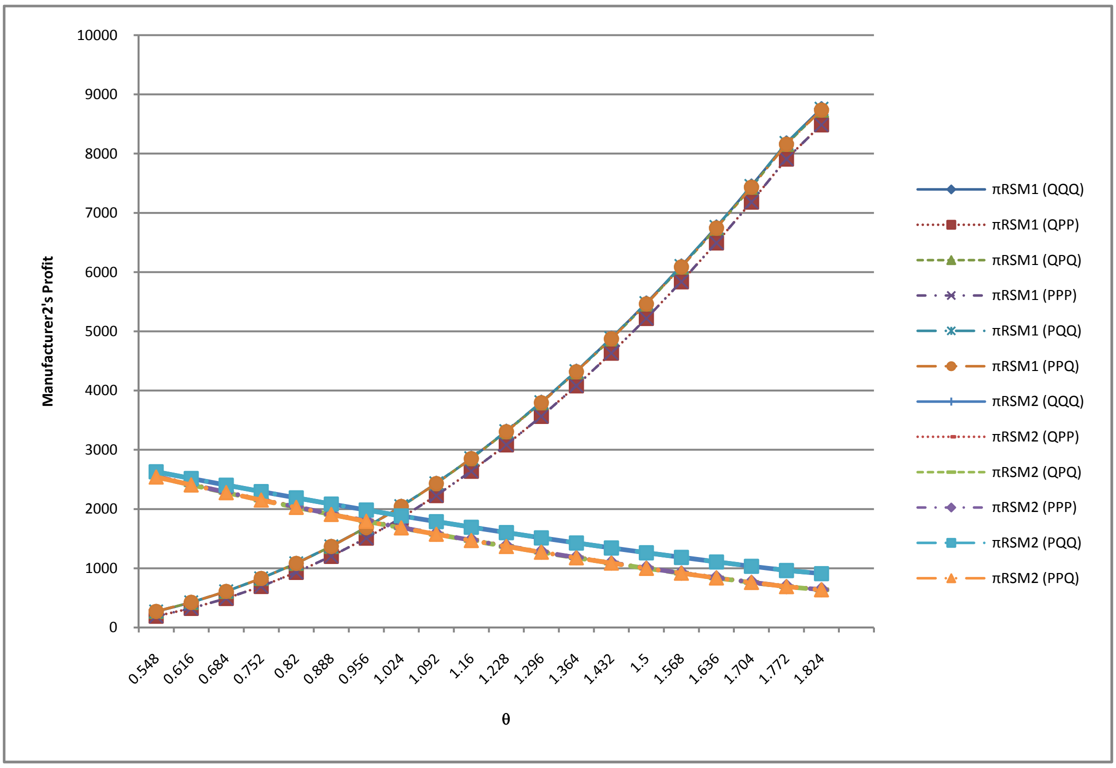



Figure 62 Manufacturers' profit with quality differential (substitute products; $\gamma^2 / \beta_1 \beta_2 = 0.3$ )

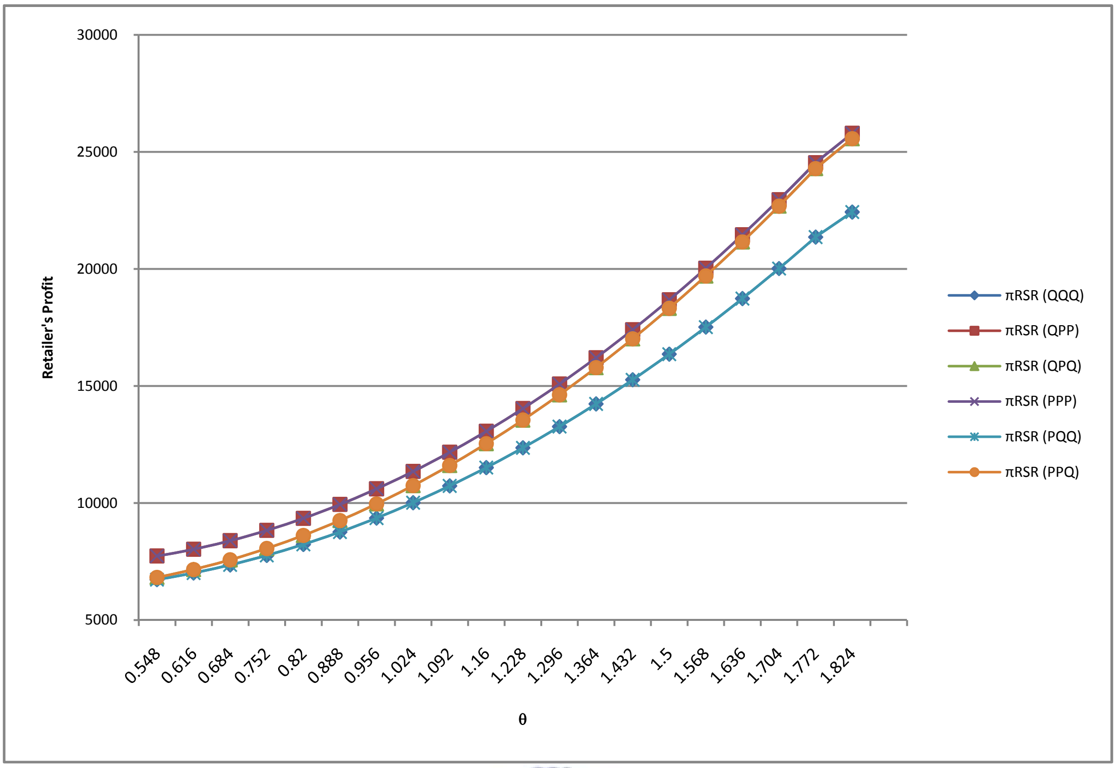



Figure 63 Retailer's profit with quality differential (substitute products; $\gamma^2/\beta_1\beta_2 = 0.3$ )

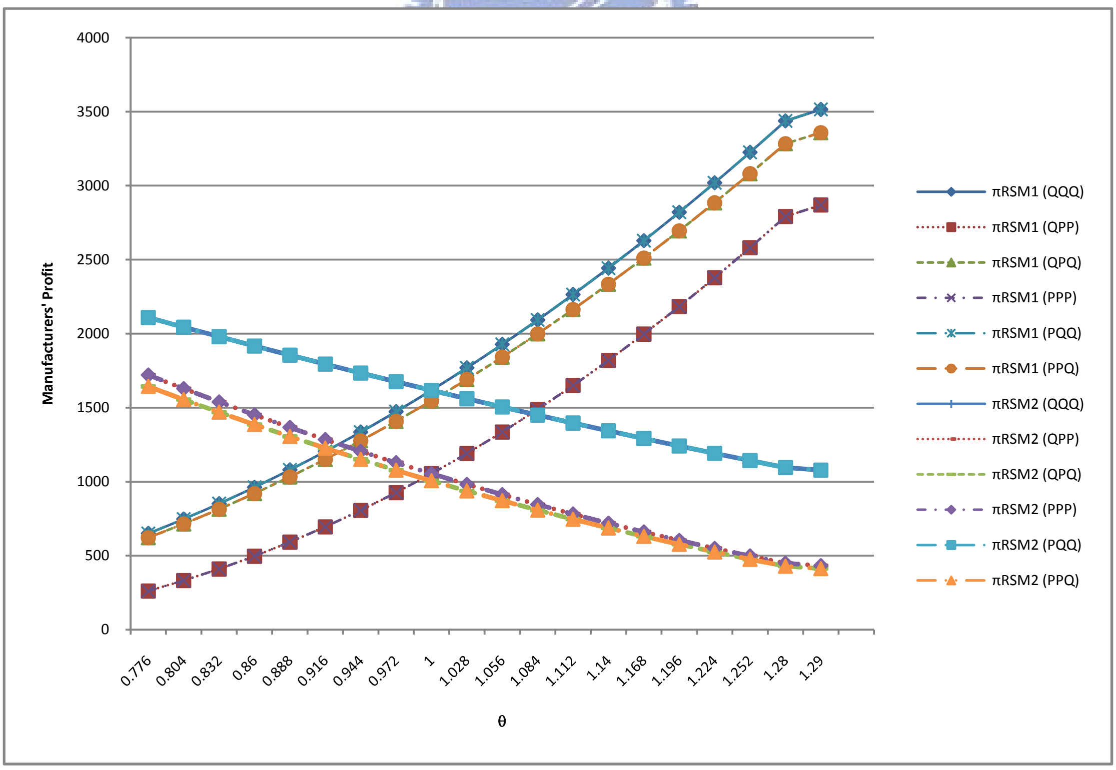



Figure 64 Manufacturers' profit with quality differential (substitute products; $\gamma^2/\beta_1\beta_2 = 0.6$ )

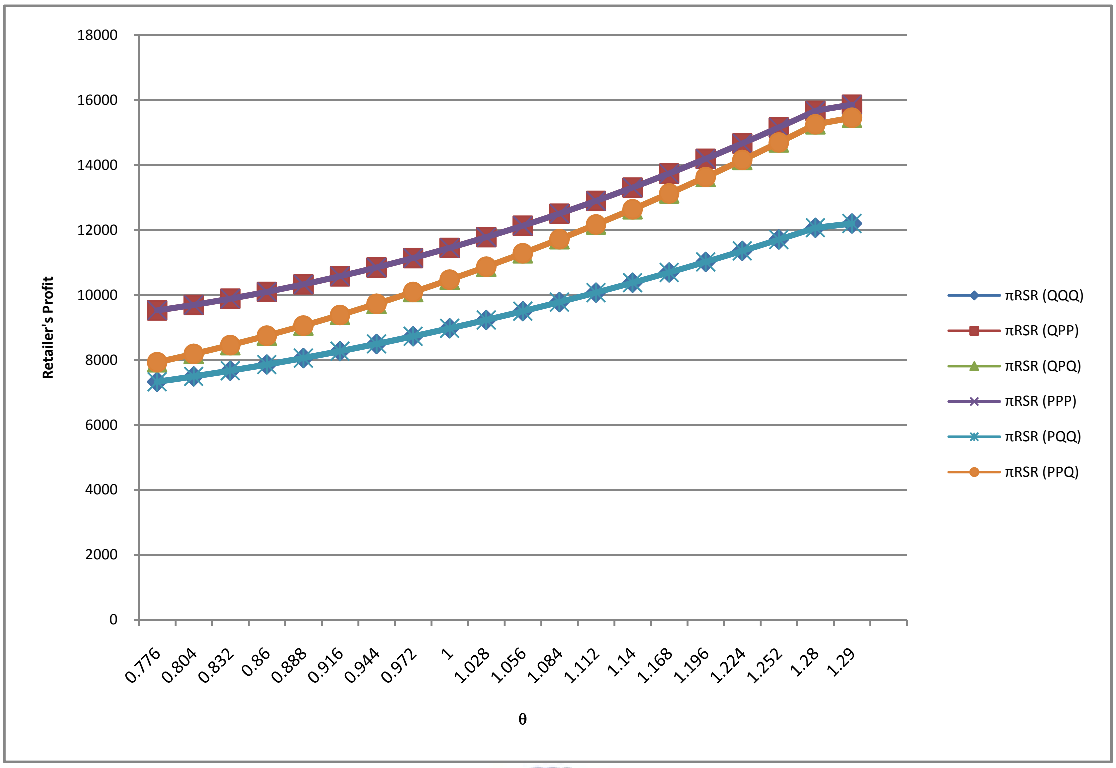



Figure 65 Retailer's profit with quality differential (substitute products; $\gamma^2/\beta_1\beta_2 = 0.6$)

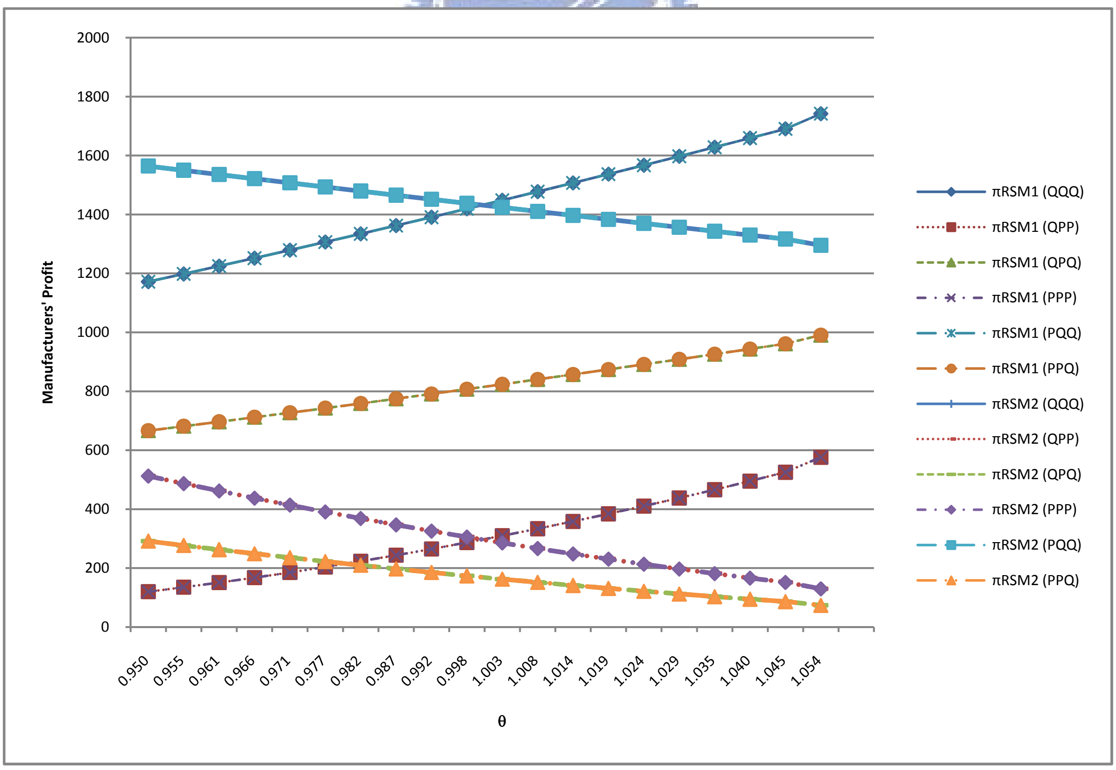



Figure 66 Manufacturers' profit with quality differential (substitute products; $\gamma^2/\beta_1\beta_2 = 0.9$)

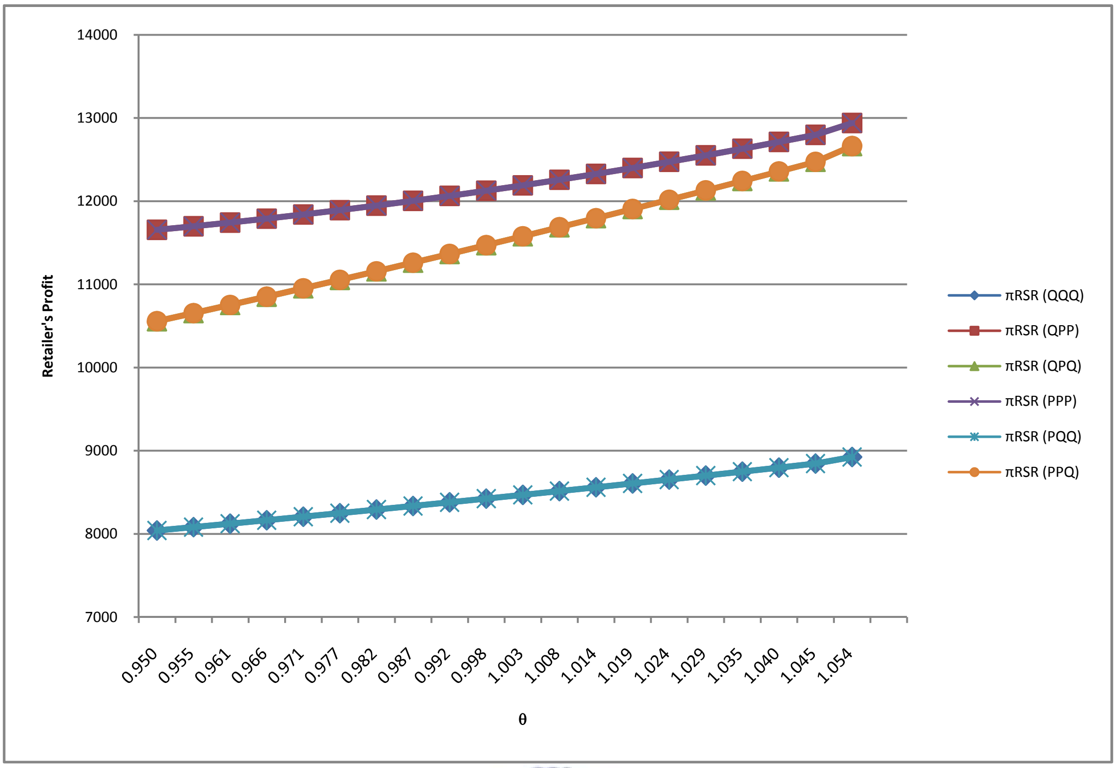



Figure 67 Retailer's profit with quality differential (substitute products; $\gamma^2/\beta_1\beta_2 = 0.9$)

# 6 SUMMARY AND CONCLUSIONS

This research is aimed to provide a precise understanding of price and quantity competition models when differentiated products made by different manufacturer and sold them through a common retailer. We especially focus on the channel structure as Retailer-Stackelberg game mentioned in Choi's paper (1991).

We perform our results in numerical example. In Section 5.1 we analyzed the choices of strategic variable with different channel structure. It was shown that retailer always benefit from being a Stackelberg leader and, whatever the strategic variable retailer chooses, it makes no influence on the equilibrium outcomes. When the degree of cross effect is relatively low, channel structure plays an important role on the equilibrium outcomes. As the degree of cross effect grows, choices of strategic variable by the manufacturers become more critical. With regard to the manufacturers, to set quantity as their strategic variable is a dominant strategy when they produce complement goods. There is no dominant strategy when products are substitutes, but the manufacturers are more likely to set quantity. From retailer point of view, manufacturers fall into price competition would be always preferred.

How the degree of cross effect works upon RS game is analyzed in Section 5.2.1. We found that when retailer possesses a Stackelberg leadership, no matter how large the degree of cross effect is and what strategies the manufacturers play, retailer margin of the two products remain constant. That is, retail price of each product is depends on the wholesale price. Unsurprisingly, manufacturers compete in price result in more quantity demand. Thus retailer always prefers its manufacturer compete in price. For the case of manufacturers, their best strategies are base on the cross-effect of the products. When products are complement and the cross-effect is relatively low, the dominant strategy of manufacturers is to set quantity. As the cross-effect increases, setting quantity is still a best strategy, but the best outcome occurred when both of them set price. That is the so-called "Prisoner's Dilemma". When products are substitute, there is no more dominant strategy, but they are more likely to set quantity.

Quality differential is considered in Section 5.2.2. According to our results, as the quality of product had improved, equilibrium profit of the manufacturer is higher and this higher profit is attributed to the increase of margin and quantity. When products are complements, thanks to the complementary nature, all members contained in the channel can be benefited by product improvement. On the other hand, manufacturer which produces high-quality product and the retailer possess higher profit, but it hurts the low-quality manufacturer. Moreover, quality differentiation makes no effect on choice of strategic variables.

Two manufacturers and only one dominant retailer are considered as members in this research. Hackner (2000) had shown that the results developed in Singh and Vives (1984) are sensitive to the duopoly assumption. Besides, researchers have already probed into the interactions between two retailers and two manufacturers. Furthermore, capacity restriction is not considered in the article. There is space for extension which contains more channel members in the models to help understand a more realistic market.

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