國立交通大學
工業工程與管理學系碩士班

碩士論文

考慮韋伯製程變異數發生變動下之製程能力調整

Process Capability Adjustment for Weibull Processes with Variance Change Consideration

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中華民國九十八年四月

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摘要

製程能力指標被用來衡量製程製造產品符合規格的能力，不僅是提供品質保證的工具，也是在品質改善方面的一個方針。計算製程能力指標需服從製程為穩態的前提假設，也就是在生產過程中平均數和標準差不會改變，但是在實務上製程為動態。當製程之平均數發生微小偏移時，有些管制圖可能無法偵測到，造成製程能力指標高估製程良率，因此必須將製程能力指標進行調整。自從 1980 年代，Motorola 公司提出 6 個標準差的觀念，許多統計學家質疑提倡 6 個標準差的學者，為什麼在衡量製程能力時需要對製程平均數做 1.5 被標準差的調整。Bothe (2002) 提出製程服從常態分配下之製程能力調整方法，他以統計的方法解釋了 1.5 倍標準差的調整之原因。但 Bothe 的研究是在製程服從常態分配的假設之下，而非常態分配製程在業界時常出現，過去的研究也針對了非常態分配的調整方法。事實上，製程標準差也是會改變的，因此本研究在變異數微小變動時，針對製程服從韋伯分配提出製程能力調整方法。在本研究的最後，以實例來說明如何在非常態的製程中，考慮製程變異數發生改變的情況下，調整製程能力指標 C_{pk} 之計算。

關鍵字：非常態、韋伯分配、變異數微小變動、製程能力指標。

Capability Adjustment for Weibull Processes with Variance Change Consideration

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Abstract

Process capability indices (PCIs) have been proposed in the manufacturing industry to provide numerical measures on process reproduction capability, which are effective tools for quality assurance and guidance for process improvement. The assumption that the process is stable (the process mean and variance are not change) must be made before PCIs are calculated. In practice, the process is dynamic. If the process mean has a small shift, and the control chart doesn't detect, then the PCIs will overestimate the true process capability. For this reason, the PCIs have to be adjusted under those cases. Motorola, Inc. introduced its Six Sigma quality initiative to the world in the 1980s. Some quality practitioners questioned why the Six Sigma advocates claim it is necessary to add 1.5σ. Bothe (2002) provided the adjustment method for normality processes. Bothe (2002) provided a statistical reason for including such a shift in the process average that is based on the chart's subgroup size. Data in Bothe' study was assumed to be approximately normally distribution, but the process output is usually not from approximately normally. Some research is about the PCIs adjustment for process output has a non-normal distribution. In fact, the process variance could also change. In this paper, we consider the variance change adjustments to compute reliable estimates for capability index C_{pk} Weibull distribution data. For illustration purpose, an application example is presented.

Keywords: Process capability index, Weibull distribution, Variance Change, Dynamic C_{pk}

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Chapter 1. Introduction

1.1. Research Background and Motivation

Process capability indices (PCIs) which provide numerical measure of production characteristic to reflect the quality of product have been used in the manufacturing industry. Those indices have become popular as unit-less measures on process potential and performance. The most commonly used ones, C_p and C_{pk} discussed in Kane (1986), and more-advanced indices C_{pm} and C_{pmk} developed by Chan *et al.* (1988) and Pearn *et al.* (1992). Many authors have promoted the use of various PCIs for evaluating a supplier's process capability. Based on analyzing the PCIs, a production department can trace and improve a poor process so that the quality level can be enhanced and the requirements of the customers can be satisfied. These PCIs have been defined explicitly as:

$$C_p = \frac{USL - LSL}{6\sigma},\ C_{pk} = \min\left\{\frac{USL-\mu}{3\sigma}, \frac{\mu - LSL}{3\sigma}\right\},\ C_{pm} = \frac{USL - LSL}{6\sqrt{\sigma^2 + (\mu - T)^2}},$$

$$C_{pmk} = \min\left\{\frac{USL-\mu}{3\sqrt{\sigma^2+(\mu-T)^2}}, \frac{\mu-LSL}{3\sqrt{\sigma^2+(\mu-T)^2}}\right\},$$

where USL is the upper specification limit, LSL is the lower specification limit, μ is the process mean, σ is the process standard deviation, and T is the target value.

The first capability index C_p considers the overall process variability relative to the manufacturing tolerance, reflecting product quality consistency. Due to the design is simplicity, C_p can not reflect the tendency of process centering.

The index C_{pk} was created in Japan to offset some of the weaknesses in C_p, primarily because the fact that C_p measures capability in terms of process variation only and does not take process location into consideration. However, C_{pk} considers process variation and the location of process mean. It has been regarded as a yield-based index since it provides lower bounds on process yield, and is always used to measure the quality of the process. For example, when the $C_{pk} = 1$ means that the product's fractions of defectives is not more than 2700 parts per million (ppm) fall outside the specification limits. At $C_{pk} = 1.33$, the defect rate drops to 66 ppm. To achieve that defect rate less than 0.544 ppm, a C_{pk} level of 1.67 is required. At a C_{pk} level of 2.0, the defective rate reduced to 0.002 ppm. The exact number of nonconformities with fixed C_{pk} is very depending upon the location of the process mean and the magnitude of the process variation. Before C_{pk} is calculated, we assumes that the process is stable (the process mean and variation are not change), but in practice, the process is dynamic and the mean and variation always change with small movement for momentary. We use some control charts to monitor our process, but we can not detect this movement obviously. So that the C_{pk} will be underestimated the true number of nonconformities. At the present time, the C_{pk} index is used more than any other index for measuring process capability. It is the reason why we study C_{pk} more than other indices here.

Since Motorola, Inc. introduced its Six Sigma quality initiative, followers of this philosophy notion should add 1.5σ when estimating process capability. When asked the reason for such an adjustment, six-sigma user claim it is necessary, but offer only personal experiences and three dated empirical literature. Bothe (2002) provided a statistical reason to adjust the C_{pk} be overestimated, and he set the adjustment of shift in average that was dependent on the same detection power of the control chart, and the data of Bothe's study was assumed to be approximately normality distribution. However effectively non-normal process occurs frequently in practice. Pyzdek (1995) has mentioned the distributions of certain chemical processes such as zinc plating thickness of a hot-dip galvanizing process are very quite often skewed. Choi (1996) presents an example of a skewed distribution in the 'active area' shaping stage of the wafer's production process. Cygan *et al.* (1989) have mentioned that the lifetimes of polypropylene films under high ac and dc field stresses be shown as a two-parameter Weibull distribution. The Weibull distribution, denoted as Weibull (α, β), with various values of scale parameter α and shape parameter β, covers a wide class of non-normal applications, including product life, product reliability and tensile strength of brittle materials, such as carbon and boron. The abundance of outputs from skewed distribution, the censoring, etc, makes the normality assumption often being illegitimate. Specifically, we assure the product lifetime which be from skewed distribution by statistic test and historical data. It will lead to underrate the probability of nonconformance that using the adjustment for normal case to adjust the non-normal cases.

1.2. Research Purpose

For some non-normal cases, Hsu *et al.* (2007) provided the process capability adjustment for gamma processes, and Li (2007) provided the process capability adjustment for Weibull processes, but they only investigate the change of the process mean shift. In real world, the process is dynamic, the mean and variance could change with small movement for momentary. In this thesis, we focus on the process variance change for non-normal cases.

We investigate Weibull distribution to calculate the ARL(average run length) by simulation. We also show the detection power performance of S^2 chart under variance change. In the cases, we show that the detection power in this control chart is very sensitive. When the data are from Weibull distribution, we provide the statistical derived variance change adjustment based on the chart subgroup size and distribution parameter to calculate the estimate of dynamic C_{pk} when the data is non-normal distribution. It can make sure our process capability do not overestimate.

1.3. Thesis Organization

First, we introduce the research motivation and purpose in Chapter 1. Secondly, a brief introduction of Bothe's study and adjustment reason for mesn shift are included, and adjustment for Weibull process is also in Chapter 2. In Chapter 3, we introduce the characteristic of Weibull distribution, and introduce some properties for S^2 of Weibull process. Then, we compare the difference between normal process and Weibull process on variance distribution. In chapter 4, we use the MATLAB program to create a Monte-Carlo simulation to find upper and lower control limits for detecting variance change. We provide the simulation derived adjustments based on the chart's subgroup size (for Weibull distribution) to calculate the estimator of dynamic C_{pk} when the data is Weibull distribution. For illustrative purpose, application is presented in Chapter 5. Finally, we give some conclusion in Chapter 6.

Chapter 2. Literature Review

The process capability adjustment for mean shift for normal and non-normal distributions had been researched. In this section, we will review those papers about adjustments for normal process and Weibull process.

2.1. Process Capability Adjustment for Normal Process with Mean Shift

Bothe (2002) provided a statistical reason why to add a 1.5σ shift to the average. Assuming the processes approximately normal distribution, control charts can not reliably detect small movement in average. When μ had a small movement (ex: 0.5σ, 1σ) and the detection power of Shewhart $\bar{X}$ control chart is too small to discover. Then, small mean movement affects the PCIs accuracy. However, the probability of nonconformance will increase obviously. For example, when C_{pk} is 1.33, the probability of nonconformance is 64 ppm. If average occur 1σ shift that be difficultly detected by control chart, the probability of nonconformance becomes 1350 ppm. The probability of nonconformance will increase twenty-fold. Bothe considered that adjustments should accord with the same detection standard.

Bothe (2002) considered providing the same detection power in order to define the several adjustments with different subgroup size and called the adjustment S_{50}. By this idea, he set the detecting power to 50 percent and computed the several adjustments for different subgroup size. The reason which Bothe set the power to 50 percent was we want detect the processes out of control immediately if the process mean shifts and the ARL_1(average run length)=1 is the perfect condition. But in fact, the $ARL_1 = 1$ is impossible. For this reason we can just only set the $ARL_1 = 2$, and the detection power is $1/ARL_1$, so we can know if $ARL_1 = 2$ the detecting power is 0.5. The results showed in Table 1. Table 1 displays shift sizes that have 50 percent chance of remaining undetected for subgroup sizes 1 through 6. Because shifts ranging in size from 0 up to $S_{50}\sigma$ are the ones likely to remain undetected, a conservative approach is to assume that every missed shift is as large as $S_{50}\sigma$. And Bothe invented dynamic C_{pk} be defined as

$$dynamic\ C_{pk} = \min\left[\frac{USL-(\mu+S_{50}\sigma)}{3\sigma}, \frac{(\mu-S_{50}\sigma)-LSL}{3\sigma}\right].$$

Bothe (2002) suggested that the adjustment value for normal distribution should be determined by the subgroup size n.

Table 1. Adjustment values for normal distribution with several subgroup size.

Subgroup Size	S_{50}
1	3
2	2.12
3	1.73
4	1.5
5	1.34
6	1.22

2.2. Process Capability Adjustment for Weibull Process with Mean Shift

Li (2007) provided the process capability adjustment for non-normal processes. Weibull distribution does not have reproductive property, and the distribution of the $\bar{X}$ distribution is analytically intractable. Lu (2003) provided to approximate the cumulative density function of $\bar{X}_n$ for Weibull processes. The UCL and LCL was set to 99.865^{th} and 0.135^{th} percentile of $\bar{X}_n$ distribution. We call the control chart they used as percentile Weibull control chart. Then, these two papers used the control limits to calculate the detection power for Weibull processes under various subgroup sizes n and shape parameter β.

Since the shape of the Weibull distribution changing from positive skewness to negative skewness with increasing the shape parameter, Li (2007) discussed two different cases. Process mean had right and left shifts. He used this cumulative density function to compute the relationship between the mean shift and Type II error and calculate the mean shift adjustment AS_{50} which means that the processes mean shift $AS_{50}\,\sigma$ when the detection power of control chart is 0.5. The result is as below:

Table 2. AS_{50} values for several n and various β values with mean right shift.

AS_{50}	Weibull distribution(1, β) for right shift									
n	1	2	3	4	5	6	7	8	9	10
2	3.611	2.492	2.009	1.767	1.632	1.536	1.470	1.424	1.387	1.359
3	2.735	1.967	1.642	1.482	1.373	1.307	1.261	1.228	1.197	1.182
4	2.250	1.663	1.448	1.309	1.232	1.175	1.138	1.103	1.087	1.071
5	1.944	1.484	1.301	1.196	1.127	1.084	1.047	1.025	1.006	0.988
6	1.716	1.343	1.201	1.104	1.043	1.009	0.981	0.960	0.942	0.932
7	1.569	1.239	1.119	1.037	0.990	0.954	0.928	0.907	0.892	0.881
8	1.440	1.159	1.051	0.984	0.939	0.905	0.883	0.864	0.852	0.839
9	1.340	1.086	0.991	0.930	0.891	0.865	0.845	0.828	0.814	0.805
10	1.251	1.031	0.943	0.889	0.853	0.828	0.811	0.797	0.784	0.773

Table 3. AS_{50} values for several n and various β values with mean left shift.

AS_{50}	Weibull distribution(1, β) for left shift									
n	1	2	3	4	5	6	7	8	9	10
2	0.820	1.532	1.888	2.098	2.236	2.333	2.405	2.461	2.504	2.540
3	0.813	1.356	1.591	1.723	1.808	1.866	1.909	1.941	1.967	1.987
4	0.802	1.225	1.399	1.494	1.554	1.596	1.626	1.649	1.667	1.681
5	0.776	1.125	1.263	1.337	1.384	1.416	1.439	1.456	1.470	1.481
6	0.749	1.047	1.160	1.221	1.259	1.285	1.304	1.318	1.329	1.338
7	0.724	0.983	1.079	1.131	1.163	1.185	1.201	1.213	1.222	1.230
8	0.700	0.929	1.013	1.058	1.086	1.105	1.118	1.129	1.137	1.144
9	0.678	0.884	0.958	0.998	1.022	1.039	1.051	1.060	1.067	1.073
10	0.658	0.844	0.911	0.947	0.969	0.984	0.994	1.003	1.009	1.014

Table 2 and Table 3 display the magnitude of mean shift adjustments AS_{50} based on the detection power set to 0.5 and data from Weibull (1, β) distribution for various value of $\beta = 1(1)10$ and n=2(1)10 with right shift ($k > 0$) and left shift ($k < 0$). They also used the most common method for modifying PCIs in the non-normal case is the technique of quantile estimation, and the dynamic C_{pk} was as the same as gamma processes which Hsu *et al.* (2007) provided.

Noting that a process will experience shifts in $X_{0.50}$ (median) of various magnitudes and knowing that not all of these will be discovered, some allowance for them must be made when estimating outgoing quality so customers are not disappointed. Because shifts ranging in size from 0 up to $AS_{50}\sigma$ are likely to remain undetected, a conservative approach it to assume that every missed shift it as large as AS_{50}. When estimating capability, $X_{0.5}$ minus $AS_{50}\sigma$ is used to evaluate how well the process output meets the LSL and $X_{0.5}$ plus $AS_{50}\sigma$ is used for determining conformance to the USL. Both of these adjustments are incorporated into the C_{pk} formula, now called the "dynamic" C_{pk} index, by making the following modifications:

$$
\begin{aligned}
dynamic\ C_{pk} &= \min\{\frac{(X_{0.5} - AS_{50}\sigma) - \mathrm{LSL}}{X_{0.5} - X_{0.135}}, \frac{\mathrm{USL} - (X_{0.5} + AS_{50}\sigma)}{X_{99.865} - X_{0.5}}\} \\
&= \min\{\frac{X_{0.5} - AS_{50}\sigma - \mathrm{LSL}}{X_{0.5} - X_{0.135}}, \frac{\mathrm{USL} - X_{0.5} - AS_{50}\sigma}{X_{99.865} - X_{0.5}}\}
\end{aligned}
$$

The capabability adjustments for Weibull processes are related to which control chart chosen to control the process. The Shwehart $\bar{X}$ control chart assumes that the process data come from a normal or near-normal distribution. When the data comes from Weibull distribution, we should choose control charts for non-normal processes or for Weibull processes to control production process. Padgett and Supurrier (1990) used Monte Carlo simulation to construct Shewhart-type control charts for percentiles of strength distributions. Chan and Cui (2003) provided a skewness correction $\bar{X}$ and R charts for skewed distributions. This control chart proposed a skewness correction method for constructing the $\bar{X}$ and R control charts for skewed process distributions. Their asymmetric control limits are based on the degree of skewness estimated from the subgroups. Nichols and Padgett (2006) provided a bootstrap Weibull control chart. This control chart use bootstrap method to simulate the UCL and LCL for monitoring Weibull percentiles. Erto (2007) provided a Weibull control chart which used Bayes theorem to calculate the sampling distribution of Weibull percentile.

Lu (2008) considered the problem of how to determine the adjustments for process capability with mean shift when data follows the Weibull distribution. Lu (2008) compared the detection powers of the percentile Weibull control chart, bootstrap Weibull control chart and the Erto's Weibull control chart under the Bothe's adjustments, and showed the Bothe's adjustments are inadequate when data come from Weibull processes. He finds the Erto's Weibull control chart is the best powerful control chart than the others. For Weibull processes, Lu (2008) calculated the adjustments for various sample sizes (n) and Weibull shape parameter (β) with detection power of the Erto's Weibull control chart fixed to 0.5. Using the adjusted process capability formula, the engineers can determine the actual process capability more accurately.

Chapter 3. Investigation for Weibull Process

In this section, we introduce the characteristic of Weibull distribution first. In the second part, we investigate the Weibull variance distribution to study the effect on the detection power of the S^2 control chart.

3.1. Weibull Process

The Weibull distribution has been widely used in the field of life data analysis due to its flexibility. It has similar behavior of other statistical distributions such as normal and the exponential distributions. The Weibull distributions are also used to model the time until a given technical device fails. In practical, the Weibull has been used include electronic devices such as memory element, mechanical components such as bearings, and structural elements in aircraft and automobiles,...,etc. The Weibull distribution can also be used to model the distribution of wind speeds at a given location on Earth. Moreover, every location is characterized by a particular shape and scale parameter.

If the failure rate of the device decreases over time, one chooses $\beta < 1$ (β is the shape parameter). If the failure rate of the device is constant over time, one chooses $\beta = 1$, again resulting in a decreasing function *f*. If the failure rate of the device increases over time, one chooses $\beta > 1$ and obtains a density f which increases towards a maximum and then always decreases.

In this thesis, we consider Weibull distribution as the process population to study the effect on the capability estimates when the process output has a non-normal distribution with process variance change. Observations from the Weibull distribution is non-negative. The Weibull distribution can be denoted as Weibull (α, β) with cumulative density function and the probability density function given by

$$F_X(x) = 1 - \mathrm{e}^{-(x/\alpha)^\beta}, x \geq 0,$$

and

$$f(x) = \beta\alpha^{-\beta}x^{\beta-1}e^{-(\frac{x}{\alpha})^\beta}, x \geq 0,$$

where α (>0) is the scale parameter, and β (>0) is the shape parameter. The mean and variance of Weibull distribution are

$$\mu = \alpha[\Gamma(1+\beta^{-1})],$$

and

$$\sigma^2 = \alpha^2[\Gamma(1+2\beta^{-1}) - \Gamma^2(1+\beta^{-1})].$$

The Weibull distribution is very flexible, and by appropriate selection of the parameters α and β, we can obtain a various shapes. Since the Weibull distribution is skewed, we utilize the coefficient of skewness and kurtosis to explain how this distribution is different from normal distribution, the coefficient of skewness and kurtosis of Weibull distribution can be expressed as follows:

$$\gamma_1 = \frac{2\Gamma^3(1+\beta^{-1}) - 3\Gamma(1+\beta^{-1})\Gamma(1+2\beta^{-1}) + \Gamma(1+3\beta^{-1})}{[\Gamma(1+2\beta^{-1}) - \Gamma^2(1+\beta^{-1})]^{3/2}},$$

and

$$\gamma_2 = \frac{f(\beta)}{[\Gamma(1+2\beta^{-1}) - \Gamma^2(1+\beta^{-1})]^2},$$

where $\Gamma(x)$ is the gamma function and

$$f(\beta) \equiv -6\Gamma^4(1+\beta^{-1}) + 12\Gamma^2(1+\beta^{-1})\Gamma(1+2\beta^{-1}) - 3\Gamma^2(1+2\beta^{-1}) - 4\Gamma(1+\beta^{-1})\Gamma(1+3\beta^{-1}) + \Gamma(1+4\beta^{-1}).$$

Table 4 presents the coefficient of skewness and the coefficient of kurtosis of the Weibull distribution under study.

Table 4. Values of skewness and kurtosis of various Weibull distributions.

Weibull(α, β)	skewness	Kurtosis
Normal(0,1)	0	0
Weibull(1,1)	2	6
Weibull(1,2)	0.631111	0.245089
Weibull(1,3)	0.168103	-0.27054
Weibull(1,3.6)	0	-0.283255
Weibull(1,4)	-0.087237	-0.25217
Weibull(1,5)	-0.25411	-0.11971
Weibull(1,6)	-0.373262	0.035455
Weibull(1,7)	-0.46319	0.187183
Weibull(1,8)	-0.533726	0.327676
Weibull(1,9)	-0.590657	0.455204
Weibull(1,10)	-0.637637	0.570166

Figure 1 shows the plot of probability density function of Weibull distribution with various values of α, while β is fixed. From Figure 1, we can find the scale parameter only control the mean and the variance to adjust the shape of the distribution. Figure 2 presents Weibull distribution with various β, while α is fixed. In Figure 3, we observed that the Weibull distribution is more similar to normal distribution while the shape parameter β exceeds 2. From Table 1, we also observe that when β is 3.6 with the coefficient of skewness is 0, but the coefficient of kurtosis is -0.28, so it has a more acute “peak” leptokurtic. The coefficient of kurtosis is most similar to normal when β fall into the interval [5, 6].

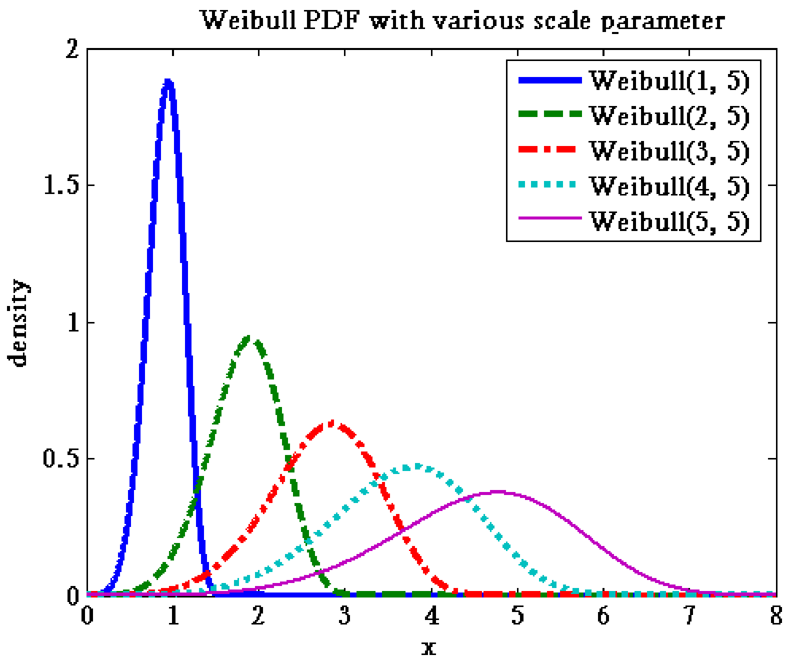

Figure 1. Weibull distribution with various α .

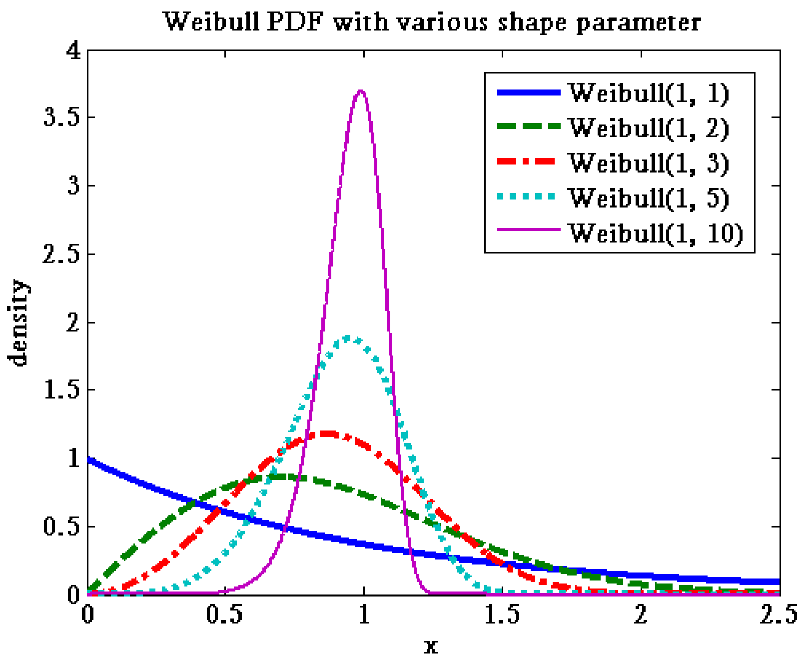

Figure 2. Weibull distribution with various β

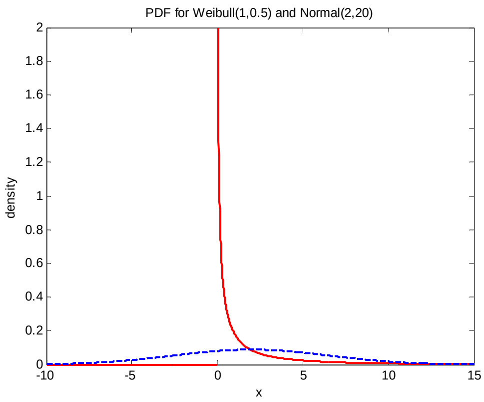

Figure 3(a). Probability density functions for Weibull(1,0.5) along with a Normal distribution.

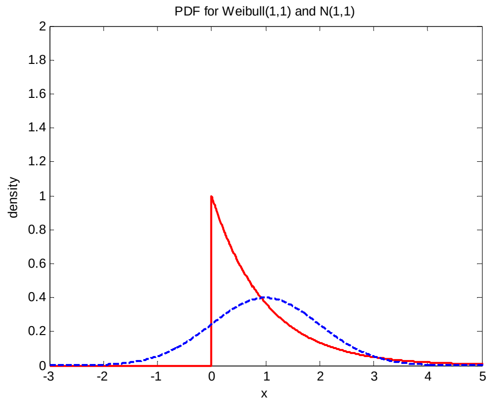

Figure 3(b). Probability density functions for Weibull(1,1) along with a Normal distribution.

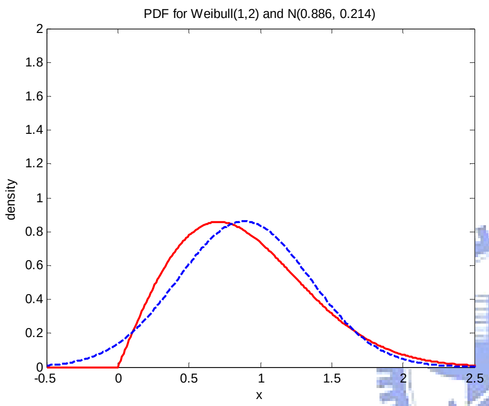

Figure 3(c). Probability density functions for Weibull(1,2) along with a Normal distribution.

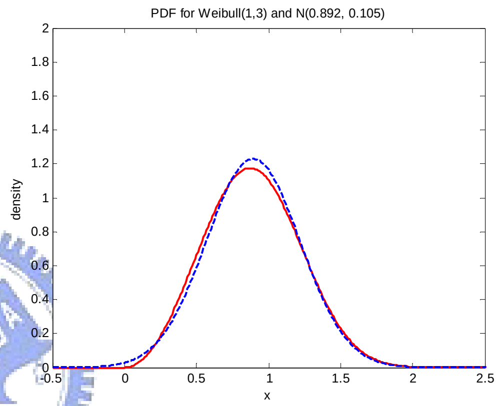

Figure 3(d). Probability density functions for Weibull(1,3) along with a Normal distribution.

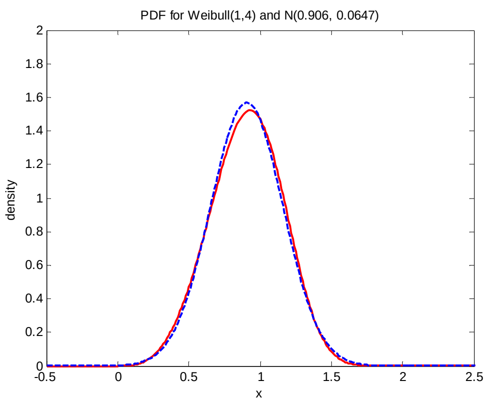

Figure 3(e). Probability density functions for Weibull(1,4) along with a Normal distribution.

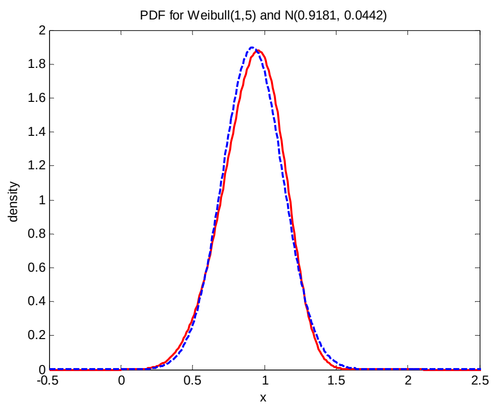

Figure 3(f). Probability density functions for Weibull(1,5) along with a Normal distribution.

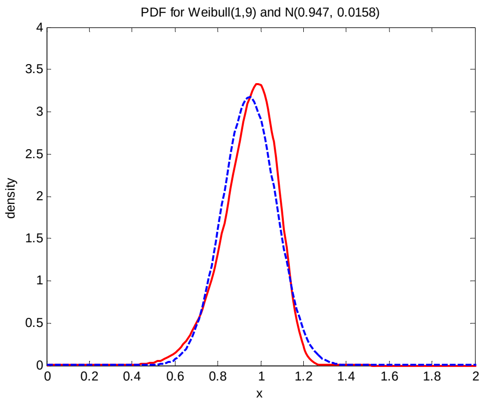

Figure 3(g). Probability density functions for Weibull(1,9) along with a Normal distribution.

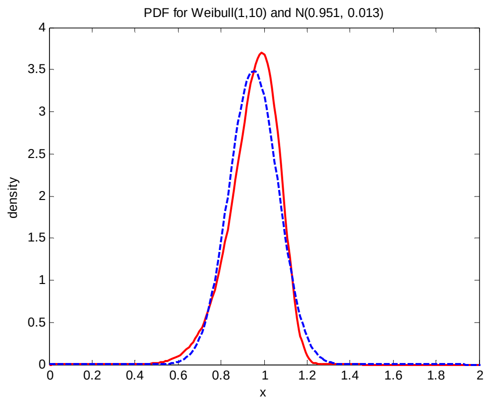

Figure 3(h). Probability density functions for Weibull(1,10) along with a Normal distribution.

Figure 3(a)-3(h). Probability density functions for Weibull distributions along with a Normal distribution for the same mean and variance. Let β = 0.5, 1, 2, 3, 4, 5, 9, and 10.

3.2. Sampling Distribution of Sample Variance for Weibull Process

Because the sampling distribution of variance for Weibull process is difficult to find, so we use the Matlab program to generate 1,000,000 preliminary samples from Weibull(α,β), each of size k, and let S_i be the variance of the *i*th sample to simulate the empirical distribution. We want to know about the distribution when shape parameter changed for Weibull distribution. In Figure 4, we draw several empirical probability density functions of the sample variance when data come from Weibull and normal populations having the same mean and variance, we let β = 0.5, 1, 2, 3, 4, 5, 6, 7, 8, and 9. The sample size is set to be 11. We can find the empirical sampling distribution of variance as below:

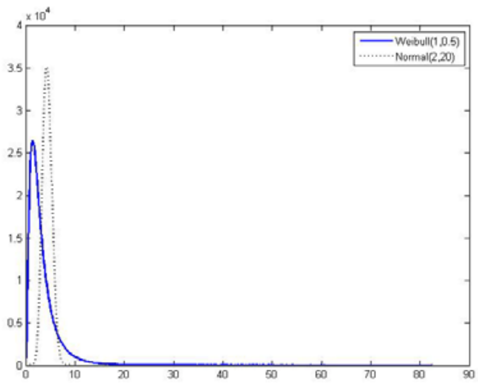

Figure 4(a). Empirical distribution of sample variance for Weibull(1,0.5) and Normal(2,20).

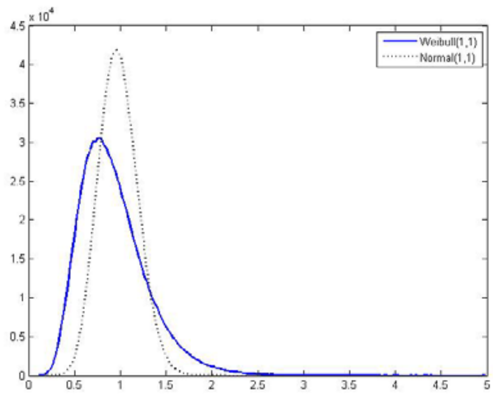

Figure 4(b). Empirical distribution of sample variance for Weibull(1,1) and Normal(1,1).

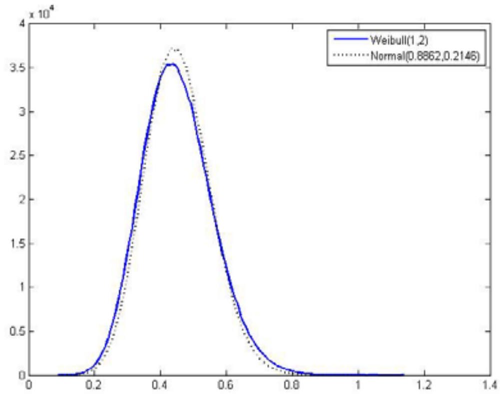

Figure 4(c). Empirical distribution of sample variance for Weibull(1,2) and Normal(0.8862,0.2146).

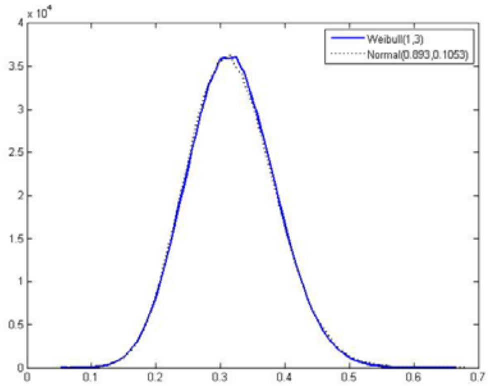

Figure 4(d). Empirical distribution of sample variance for Weibull(1,3) and Normal(0.893,0.1053).

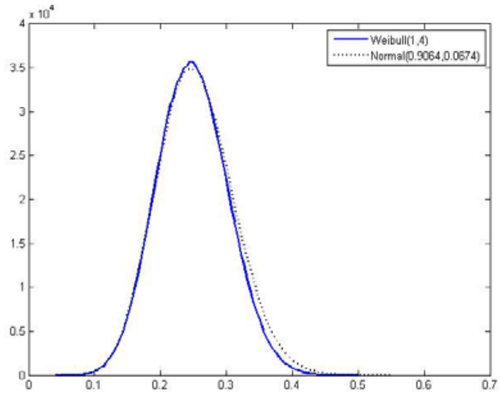

Figure 4(e). Empirical distribution of sample variance for Weibull(1,4) and Normal(0.9064,0.0674).

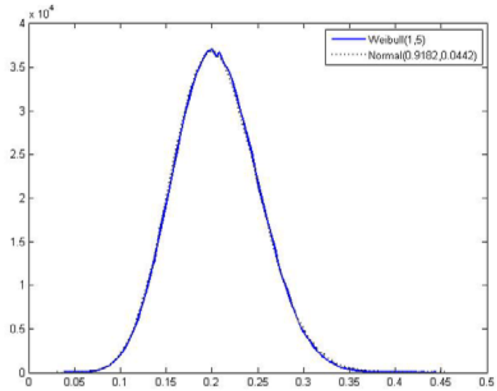

Figure 4(f). Empirical distribution of sample variance for Weibull(1,5) and Normal(0.9182,0.0442).

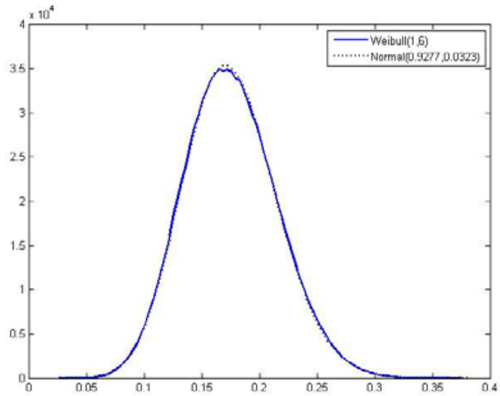

Figure 4(g). Empirical distribution of sample variance for Weibull(1,6) and Normal(0.9277,0.0323).

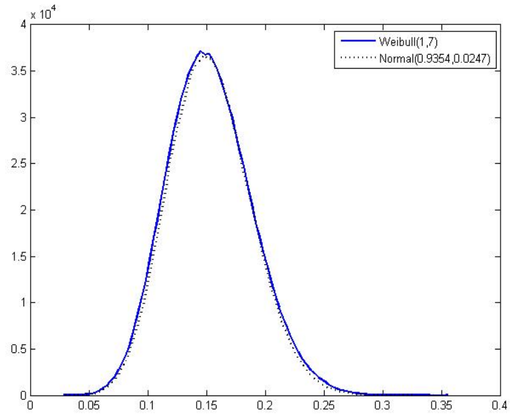

Figure 4(h). Empirical distribution of sample variance for Weibull(1,7) and Normal(0.9354,0.0247).

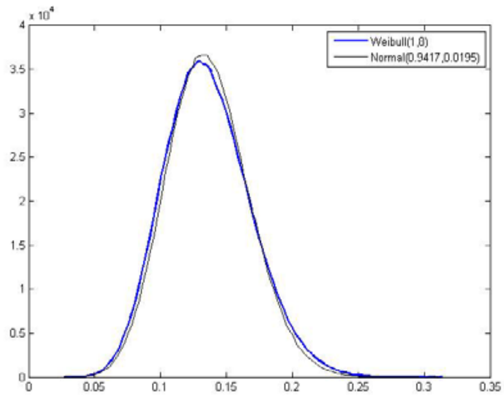

Figure 4(i). Empirical distribution of sample variance for eibull(1,8) and Normal(0.9417,0.0195).

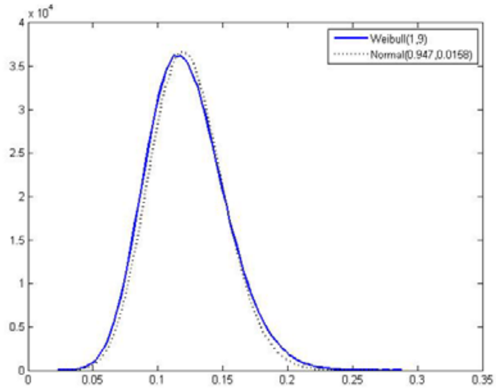

Figure 4(j). Empirical distribution of sample variance for Weibull(1,9) and Normal(0.947,0.0158).

Figure 4(a)-4(j). Empirical distribution of sample variance for Weibull distribution along with Normal distribution on the same mean and variance.

It follows from Figure 3 that when the shape parameter is larger than 2, the Weibull distribution appears to be near normal distribution. So we can infer that the sampling distribution of variance from Weibull distribution is close to the sampling distribution of variance from normal population when β is larger than 2. This phenomenon can be verified from the simulation result shown in Figure4. Moreover, From Figure 4 we also observe that as β is smaller than 2, the tail will be more elongate (distribution is strongly skewed). In Figure 5, we draw several empirical sampling distribution of variance for Weibull with β = 0.5, 0.7, 0.9, 1.1, 1.3, 1.5, 1.7, and 1.9 , and sample size = 11.

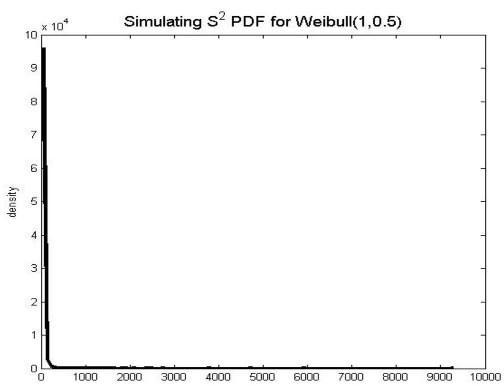

Figure 5(a). Empirical distribution of sample variance for Weibull(1,0.5).

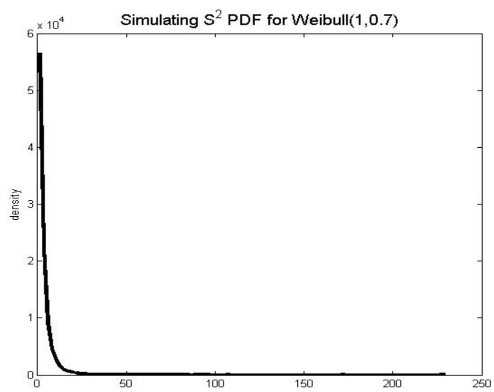

Figure 5(b). Empirical distribution of sample variance for Weibull(1,0.7).

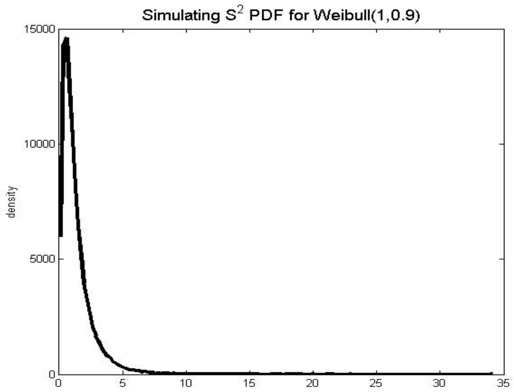

Figure 5(a). Empirical distribution of sample variance for Weibull(1,0.9).

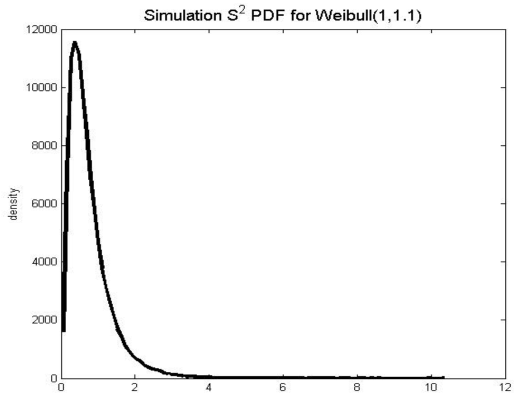

Figure 5(b). Empirical distribution of sample variance for Weibull(1,1.1).

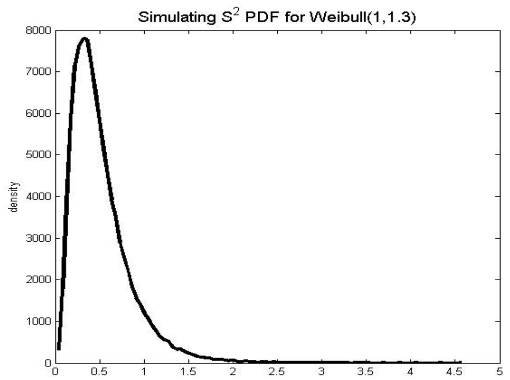

Figure 5(b). Empirical distribution of sample variance for Weibull(1,1.3).

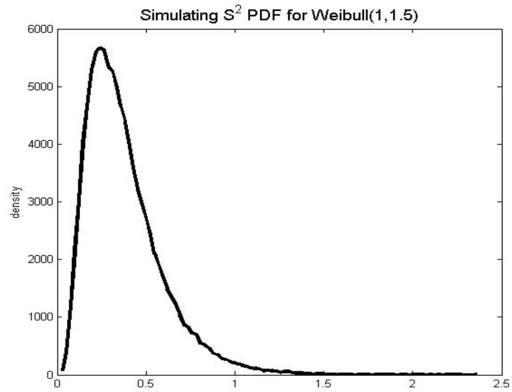

Figure 5(b). Empirical distribution of sample variance for Weibull(1,1.5).

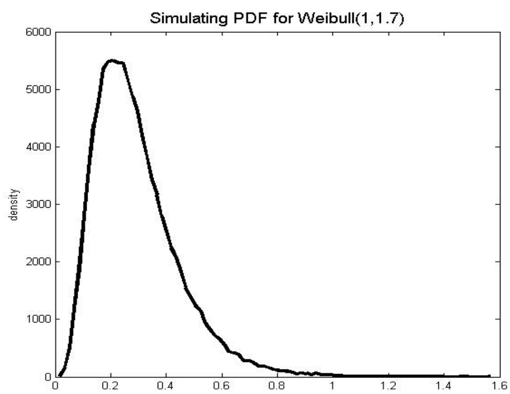

Figure 5(b). Empirical distribution of sample variance for Weibull(1,1.7).

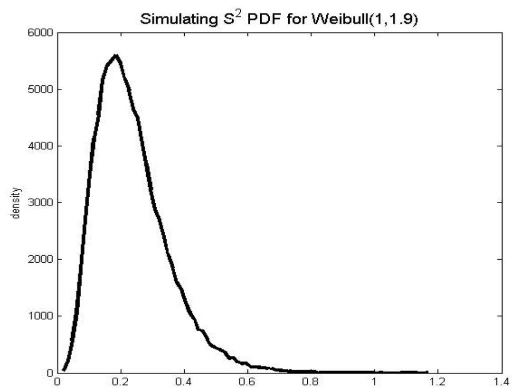

Figure 5(b). Empirical distribution of sample variance for Weibull(1,1.9).

Figure 5(a)-5(h). Empirical distribution of sample variance for Weibull distributions with β = 0.5, 0.7, 0.9, 1.1, 1.3, 1.5, 1.7, and 1.9.

Chapter 4. Process Variance Change Investigation for Weibull Process

In this chapter, we introduce a percentile control chart to investigate the detection power for the process variance change. Then we use it to find the modified standard deviation adjustment for Weibull process.

4.1. Average Run Length

ARL of the control chart means the average number of points that must be poltted before a point indicates an out-of-control condition. If the process observations have some problems, the ARL can be calculated as below:

$$\mathrm{ARL}=\frac{1}{\mathrm{p}},$$

where p means the probability of any point exceeds the control limits.

There are two kinds of ARL for any Shewhart control chart. The first one we can be expressed as ARL_0 for the in-control ARL. It means that the process observations are corrected, but there will be an out of control single generated every ARL_0 samples, on average. We can easily calculate as below:

$$\mathrm{ARL}_0=\frac{1}{\alpha},$$

where α is the probability of indicating a shift when none has occurred.

The second one for the out-of-control ARL can be also be expressed as ARL_1. It means the process observations are uncorrected, so we will find out the out of control single generated every ARL_1 samples, on average. It can be calculated as below:

$$\mathrm{ARL}_1=\frac{1}{1-\beta},$$

where β is the probability of failing to indicate a real shift in process level.

In this paper, we will use the ARL_1 to find the modified standard deviation adjustment for Weibull process, then we can use it to modify our process capability index C_{pk}.

4.2. Monte-Carlo Simulation for Determining UCL and LCL

The main purpose of individuals control chart is assisting on identifying shifts or drifts in processes and it's easily to be implemented. In this paper we study the effects on the capability estimates when the process output obeys gamma distribution with process variance change is remained unknown, so the S^2 control chart is a convention tool to monitor process variability and can help us quickly determine whether the process is stable or not. But, when we adopt the control chart, some assumptions should be satisfied, such as the process characteristics must follow normal distribution. However, since our study is based on the gamma processes, violating the assumption, we will need to replace the traditional upper and lower control limits, $\left(\bar{S}^2/n-1\right)\chi^2_{\alpha/2,n-1}$ and $\left(\bar{S}^2/n-1\right)\chi^2_{1-(\alpha/2),n-1}$, as quantiles of the cumulative distribution function from different parameters of Weibull(α,β), where $\bar{S}^2$ is an unbiased estimator of σ^2.

In order to calculate the probability of misjudgment, one will first need to know the upper and lower control limits (UCL and LCL, respectively) of the process run. It is extremely difficult, if not impossible, to obtain the explicit formula of the distribution of sample variance when data follow gamma distribution. In this paper, Monte-Carlo simulation method was performed to investigate the behavior of sampling distribution of variance for gamma data and determine the estimated upper and lower control limits. So, in our study, the UCL and LCL are estimated through Monte-Carlo simulation method. The steps of Monte-Carlo algorithm to determine the control limits of S^2 control chart are summarized as follows:

Step1: We generate N preliminary samples from Weibull(α, β), each of size k, and let S_i be the variance of the *i*th sample.

Step2: To sort $\mathrm{S_i}$, we obtain $\mathrm{S_{(1)}<S_{(2)}<\ldots<S_{(N)}}$, let $\hat{\mathrm{t}}_\mathrm{p}$ be the percentile for $\mathrm{S_i}$. For example $\mathrm{N}=10^6$, then $\hat{\mathrm{t}}_{0.1}=\mathrm{S}_{(10^6)}$, so $\hat{\mathrm{t}}_\mathrm{p}=\mathrm{S_{(N*p)}}$.

Step3: The upper and lower control limits for Weibull(α, β) can be estimated by $\hat{\mathrm{t}}_{0.99865}$ and $\hat{\mathrm{t}}_{0.00135}$.

4.3. Detection Power of S^2 for Weibull Data

Utilizing the UCL and LCL obtained by Monte-Carlo approach, we derived the power of S^2 for Weibull process data. Since the Type II error β is

$$\begin{aligned}\beta &= P\left(LCL \le S^2 \le UCL \mid \sigma_1 = \mathrm{k}\sigma_0\right)\\ &= P\left(F_{0.00135} \le S^2 \le F_{0.99865} \mid \sigma_1 = \mathrm{k}\sigma_0\right)\\ &= G_{S^2}\left(F_{0.99865}\right) - G_{S^2}\left(F_{0.00135}\right),\end{aligned}$$

where $1-\beta$ is the detection power of the process, $G_{S^2}(\cdot)$ represents the empirical cumulative distribution function of sample variance from Weibull distribution with that variance has changed and σ_1 is the standard deviation after process change (σ_0 is the standard deviation of the original process). The control limits *LCL* and *UCL* are calculated as $F_{0.00135}$ and $F_{0.99865}$ respectively.

We develop a Matlab program to compute the probability of process variance out of control limits. When process variance changes from σ^2 to $\mathrm{k}^2 \times \sigma^2$, and mean is fixed, the parameters α and β will change to new parameters α' and β', then we can obtain the detection power under the situation that the process variance changes. The parameters α' and β' can be found by untilizing the following steps:

Step1: Assume the new standard deviation $\sigma_1 = \mathrm{k} \times \sigma$, and k, μ, and σ are all known.

Step2: The mean and variance of Weibull distribution are $\mu = \alpha[\Gamma(1+\beta^{-1})]$ and $\sigma^2 = \alpha^2[\Gamma(1+2\beta^{-1}) - \Gamma^2(1+\beta^{-1})]$. Then, we compute σ_1 divided by μ as below:

$$\frac{\sigma_1}{\mu} = \frac{\sqrt{\alpha^2[\Gamma(1+2\beta^{-1}) - \Gamma^2(1+\beta^{-1})]}}{\alpha[\Gamma(1+\beta^{-1})]}$$

$$= \frac{\sqrt{[\Gamma(1+2\beta^{-1}) - \Gamma^2(1+\beta^{-1})]}}{[\Gamma(1+\beta^{-1})]}.$$

Step3: We can find the new scale and shape parameters α' and β'.

Table 5 displays the detection power when data come from Weibull distribution with $\alpha = 1$ and $\beta = 3, 4,$ and 5. The change magnitude is 1.0(0.5)3.5 adjustments. From Table 5, we observe the detection power gets larger as sample size (n) increasing.

Table 5. Detection power for various weibull distributions.

Weibull(1,3)					
Magnitude of change in σ	Subgroup Size n				
	9	10	11	12	13
1	0.00272	0.00286	0.00268	0.00266	0.00276
1.5	0.23732	0.26349	0.28855	0.31660	0.34155
2	0.58041	0.62368	0.66314	0.70087	0.73517
2.5	0.72655	0.76466	0.80016	0.83068	0.85645

3	0.78041	0.81628	0.84627	0.87217	0.89399
3.5	0.80037	0.83348	0.86098	0.88494	0.90451

Weibull(1,4)					
Magnitude of change in σ	Subgroup Size n				
	9	10	11	12	13
1	0.00264	0.00277	0.00259	0.00273	0.00275
1.5	0.23987	0.27072	0.29748	0.32572	0.35763
2	0.64447	0.69275	0.73354	0.77036	0.80625
2.5	0.81486	0.85297	0.88250	0.90716	0.92722
3	0.87588	0.90519	0.92758	0.94490	0.95854
3.5	0.89843	0.92341	0.94229	0.95637	0.96773

Weibull(1,5)					
Magnitude of change in σ	Subgroup Size n				
	9	10	11	12	13
1	0.00262	0.00261	0.00267	0.00271	0.00275
1.5	0.21636	0.24175	0.27207	0.29781	0.32945
2	0.65080	0.69954	0.74586	0.78218	0.81768
2.5	0.84854	0.88345	0.91227	0.93255	0.95023
3	0.91675	0.93968	0.95792	0.97026	0.97916
3.5	0.94173	0.95917	0.97233	0.98074	0.98706

4.4. Modified Standard Deviation Adjustment for Weibull Process

We set a given sample size (n) $\alpha = 1$ and given β, then sampling large data (10^7) which are from Weibull distribution to estimate the control limits and compute the detection power of S^2 for Weibull data with the given change magnitude and n.

From the mentioned above, we fix power $= P\left(LCL \le S^2 \le UCL \mid \sigma_1 = k\sigma_0\right)$ = 0.5 to find k. We develop a Matlab program to compute the standard deviation change adjustment AS_{50}. The standard deviation adjustment AS_{50} means that the detection power is fifty percent when process variance change from σ^2 to $AS_{50}^2 \cdot \sigma^2$. The AS_{50} is adjustment in C_{pk}, and it was calculated by how distance change the detection power reach 50 percent. Therefore, Table 6 display the magnitude of standard deviation adjustments AS_{50} based on the detection power is 50 percent and data from Weibull($1, \beta$) distribution for various value of $\beta = 1(1)11$ and n=8(1)35. For example, if β is 3 with n=10, the standard deviation change adjustment AS_{50} is 1.785. When β=1, AS_{50} are all greater than 5. From Figure 4, we find the shape is extraordinarily unlike the normal distribution when β=1. When shape parameter is smaller than 1.5 (see Figure 5), we note that the distribution is a long-tail right skewed distribution.

Why not discuss the relationship between AS_{50} and scale parameter α? By Lu (2003), we can compute the probability of $\bar{X}_n$ when $X_1,\ldots,X_n$ is a random sample from Weibull(α,β), and if we let $Y_i = X_i/\alpha$ then we have

$$Y_i = X_i/\alpha \sim \text{Weibull}(1,\beta) \text{ and } \bar{Y} = \frac{\sum_{i=1}^{n} Y_i}{n} = \frac{\sum_{i=1}^{n} X_i/\alpha}{n} \sim \text{Weibull}(\frac{1}{n},\beta). \qquad (1)$$

From (1) we get

$$\begin{aligned} &P\left\{LCL \le \bar{X} \le UCL\right\} \\ &= P\left\{\frac{LCL}{\alpha} \le \bar{Y} \le \frac{UCL}{\alpha}\right\}. \end{aligned} \qquad (2)$$

So from (2), without loss of generality, we can set $\alpha = 1$ to find the AS_{50}. Because of that, we may infer the standard deviation adjustments AS_{50} would not be affected by the scale parameter α. But, we can not prove this result theoretically. Figure 6 depicts the AS_{50} curves of the Weibull process with scale parameter α=1 and α=3 for subgroup sizes n=10, 15, and 20. It can be seen that the magnitude of standard deviation change would not change for α values

Table 6. AS_{50} values for several subgroup size n and various β values.

AS_{50}	Weibull distribution(1, β)										
n	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)	(1,7)	(1,8)	(1,9)	(1,10)	(1,11)
8	11.865	2.672	1.934	1.854	1.861	1.906	1.943	1.992	2.031	2.072	2.109
9	11.797	2.383	1.848	1.787	1.797	1.836	1.873	1.914	1.957	2.000	2.031
10	11.563	2.195	1.785	1.729	1.748	1.779	1.824	1.861	1.896	1.934	1.967
11	11.250	2.078	1.727	1.688	1.699	1.740	1.771	1.814	1.847	1.884	1.904
12	11.094	1.984	1.689	1.646	1.670	1.695	1.736	1.770	1.799	1.834	1.861
13	10.660	1.914	1.656	1.618	1.635	1.665	1.699	1.731	1.764	1.795	1.824
14	10.195	1.857	1.617	1.592	1.605	1.637	1.670	1.705	1.734	1.759	1.785
15	9.758	1.809	1.592	1.566	1.584	1.613	1.643	1.677	1.706	1.732	1.756
16	9.854	1.768	1.568	1.543	1.559	1.590	1.620	1.649	1.679	1.705	1.729
17	9.703	1.734	1.549	1.527	1.543	1.570	1.600	1.627	1.654	1.682	1.705
18	9.047	1.703	1.523	1.508	1.525	1.550	1.580	1.604	1.633	1.657	1.680
19	8.500	1.670	1.508	1.494	1.509	1.536	1.563	1.589	1.611	1.638	1.659
20	8.227	1.648	1.492	1.478	1.497	1.518	1.544	1.574	1.598	1.618	1.643
21	8.063	1.627	1.479	1.465	1.484	1.503	1.531	1.557	1.580	1.601	1.620
22	7.297	1.607	1.467	1.456	1.467	1.495	1.518	1.543	1.563	1.586	1.606
23	6.422	1.586	1.453	1.442	1.457	1.480	1.506	1.528	1.552	1.574	1.590
24	6.094	1.575	1.446	1.432	1.449	1.471	1.491	1.515	1.541	1.558	1.573
25	5.656	1.552	1.434	1.426	1.438	1.459	1.482	1.503	1.527	1.548	1.563
26	5.109	1.540	1.422	1.413	1.428	1.449	1.473	1.494	1.514	1.535	1.550
27	4.445	1.529	1.412	1.405	1.420	1.438	1.463	1.483	1.504	1.522	1.540
28	3.953	1.516	1.406	1.397	1.411	1.432	1.453	1.476	1.496	1.515	1.532
29	3.748	1.504	1.398	1.391	1.402	1.425	1.444	1.465	1.484	1.503	1.521
30	3.516	1.494	1.392	1.382	1.397	1.414	1.437	1.457	1.476	1.493	1.508
31	3.270	1.480	1.384	1.377	1.389	1.408	1.427	1.446	1.47	1.484	1.500
32	3.254	1.479	1.380	1.368	1.377	1.401	1.428	1.447	1.469	1.467	1.490
33	3.133	1.464	1.375	1.365	1.381	1.395	1.412	1.438	1.454	1.473	1.484
34	2.907	1.455	1.366	1.362	1.371	1.395	1.410	1.429	1.442	1.462	1.479
35	2.859	1.451	1.360	1.347	1.366	1.387	1.406	1.411	1.432	1.453	1.465

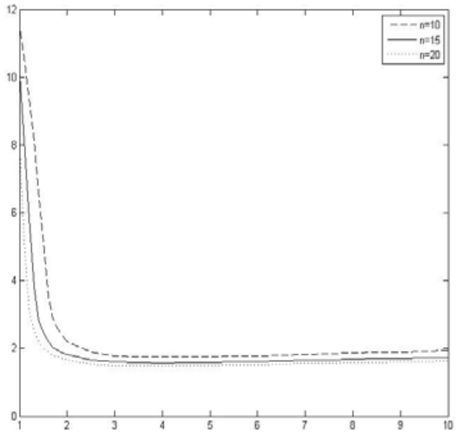

Figure 6(a). The AS_{50} curves of the Weibull process with $\alpha = 1$ for different n values on the horizontal.

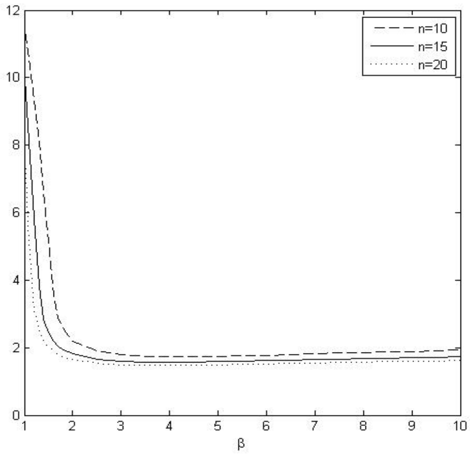

Figure 6(b). The AS_{50} curves of the Weibull process with $\alpha = 3$ for different n values on the horizontal.

Figure 6. AS_{50} values with different α values.

Figure 7 presents the power curves of estimated S^2 control chart for various sample size. The mean of power curve is detection power with various vary magnitude units for standard deviation. For small change in S^2 all curves are close to zero. As the magnitude of change creasing, so does the power of chart to detect it. The horizontal line drawn on this graph shows that is a 50% chance of missing a 1.777 times the size of standard deviation when n is 11, where as σ must change 1.873 times to have this same probability when n is 9.

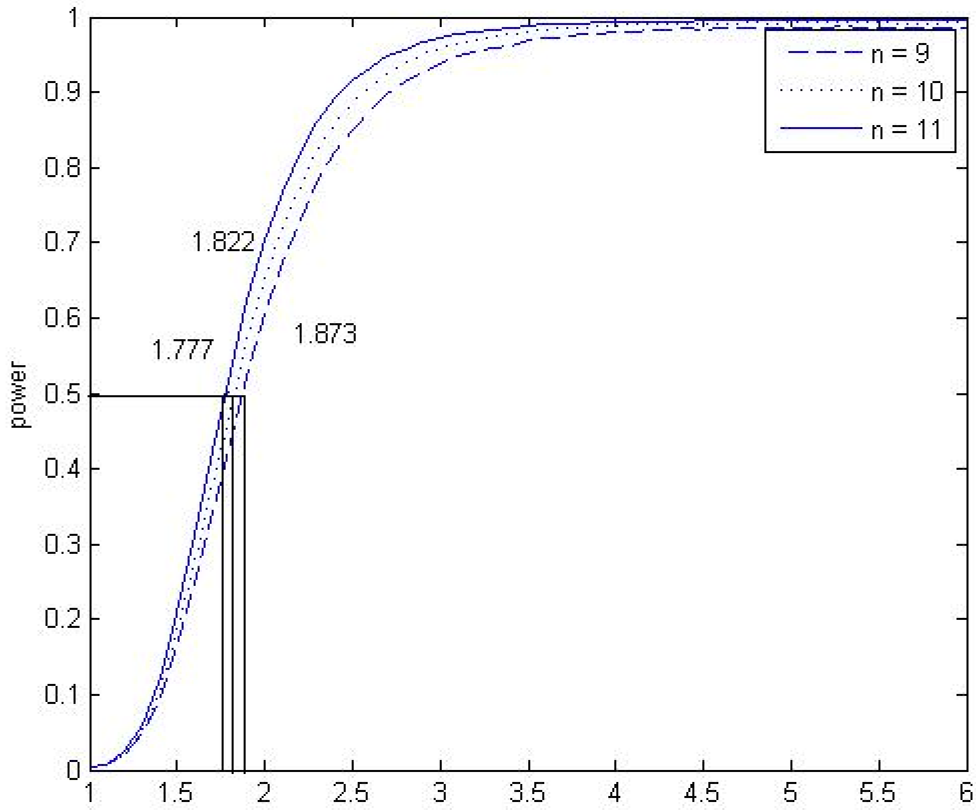

Figure 7. Power curves of estimated S^2 control chart for subgroup size 9, 10, and 11 when $(\alpha,\beta) = (1,7)$.

4.5. Capability Adjustment for Weibull Process

The index C_{pk} has been referred to as a yield-based index since it provides bound on the process on the process yield for a normality distribution process with a fixed value of C_{pk} .This index C_{pk} is defined in chapter 1. The proper use of process capability, is based on several assumptions. One of the most important assumption is that the process monitored is supposed to be stable and the output is approximately normal distribution.

When the distribution of a process characteristic is non-normal, PCIs calculated using existing method often lead to erroneous and misleading interpretation of the process capability. Several approaches to the problems of PCIs for the non-normal populations have been suggested. Chen and Pearn (1997) consider come generalizations of these basic capability indices to cover non-normal distribution. In the non-normal case, if we are able to find a better distribution from the data, which provides a vary satisfactory fit (this can be tested by means of goodness-of-fit tests), we can obtain more accurate measures of the three quantiles ($X_{0.00135}$, $X_{0.05}$, and $X_{0.99865}$) under consideration, the corresponding C_{pu} and C_{pl} are defined as:

$$C_{pu} = \frac{USL - X_{0.5}}{X_{0.99865} - X_{0.5}}, \text{ and } C_{pl} = \frac{X_{0.5} - LSL}{X_{0.5} - X_{0.00135}}$$

The index C_{pk} will then be calculated as the minimum of C_{pu} and C_{pl}, namely:

$$C_{pk} = \min\left\{C_{pu}, C_{pl}\right\}$$

$$= \min\left\{\frac{USL - X_{0.5}}{X_{0.99865} - X_{0.5}}, \frac{X_{0.5} - LSL}{X_{0.5} - X_{0.00135}}\right\}$$

Acknowledging that a process will experience change in $X_{0.99865} - X_{0.5}$ or $X_{0.5} - X_{0.00135}$ of various magnitudes and knowing that not all of these will be discovered, some allowance for them must be made when estimating outgoing quality so customers are not disappointed. Because change ranging in times from 0 up to AS_{50} are the likely to remain undetected, a conservative approach it to assume that every missed change it as large as AS_{50}.

When utilizing dynamic C_{pk} to estimate process capability, we replace $X_{0.99865} - X_{0.5}$ and $X_{0.5} - X_{0.00135}$ with $AS_{50}(X_{0.99865} - X_{0.5})$ and $AS(X_{0.5} - X_{0.00135})$ in the C_{pk} formula just mentioned above, respective. Both of these adjustments are incorporated into the C_{pk} formula, now called the "dynamic" C_{pk} index, by making the following modifications:

$$\text{Dynamic } C_{pk} = \min\{\frac{X_{0.5} - \text{LSL}}{AS_{50}(X_{0.5} - X_{0.00135})}, \frac{\text{USL} - X_{0.5}}{AS_{50}(X_{0.99865} - X_{0.5})}\}$$

$$= \min\{\frac{X_{0.5} - \text{LSL}}{AS_{50} \cdot X_{0.5} - AS_{50} \cdot X_{0.00135}}, \frac{\text{USL} - X_{0.5}}{AS_{50} \cdot X_{0.99865} - AS_{50} \cdot X_{0.5}}\}$$

By including the adjustment in this assessment for undetected variance change, the estimate of capability decreases and the number nonconforming parts measured (calculated) will increase.

Chapter 5. Application

Surface-mount technology (SMT) is a method for constructing electronic circuits in which the components (SMC, or Surface Mounted Components) are mounted directly onto the surface of printed circuit boards (PCBs). Electronic devices so made are called surface-mount devices or SMDs. In the industry it has largely replaced the through-hole technology construction method of fitting components with wire leads into holes in the circuit board.

An SMT component is usually smaller than its through-hole counterpart because it has either smaller leads or no leads at all. It may have short pins or leads of various styles, flat contacts, a matrix of solder balls (BGAs), or terminations on the body of the component.

The SMD resistors come into several possible case sizes. Each size is described as a 4 digits number. The first 2 digits indicate the length; the last 2 indicate the width (in 0.01", or 10 mils units). Figure 9 display a view on common SMDs. For example, the three most popular sizes are “0603”, “0805”, and “1206”. That mean 1.6×0.3mm, 1.8×0.5mm, and 11.2×0.6mm.

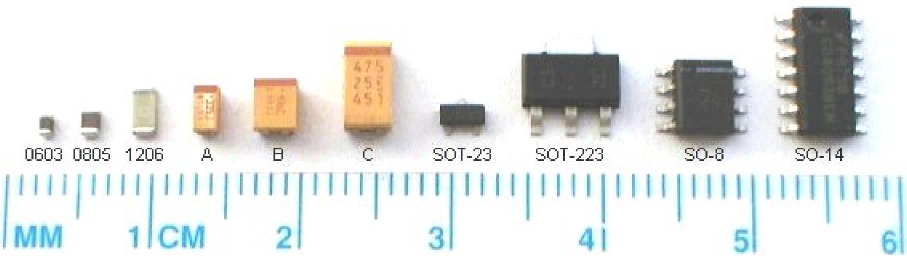

Figure 8. A view on common SMDs.

At SMT process, one of the most important factors is the size of the SMD. The SMD resistor “0603” as shown in figure 9, we let the LSL and USL of the length for line segment “H” are 0.1mm and 0.5mm. This company utilize S^2 control chart to monitor the process. Generally, S^2 charts are preferable to their more familiar counterparts, $\bar{x}-R$ charts, when either

1. The sample size n is moderately large-say, $n>10$ or12.
2. The sample size n is variable.

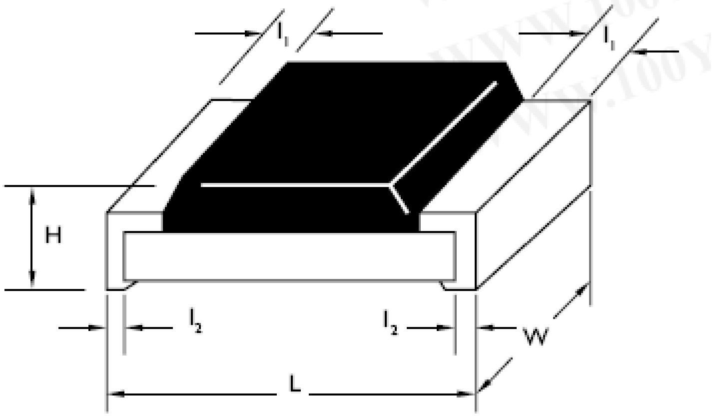

Figure 9. Dimension of SMD.

This company use $n = 10$ to monitor the process. The collected sample data (a part of historical data) are shown in Table 7. From Figure 10, we use Minitab program to conclude the data collected from the factory are not normal distributed. The data analysis results justify that the process is significantly away from the normal distribution. By the goodness-of-fit tests as shown in figure 11, the historical data indicates that the process pretty approximates to be distributed as Weibull distribution. The parameters α and β of this Weibull process could be estimated from the historical data, giving $\hat{\alpha} = 0.304$ and $\hat{\beta} = 6.299$ by MLE (maximum likelihood estimate)

$$\hat{\alpha} = \left[\frac{1}{n} \sum_{i=1}^{n} X_i^{\hat{\beta}} \right]^{1/\hat{\beta}}, \tag{2}$$

and

$$\frac{1}{\hat{\beta}} = \frac{\sum_{i=1}^{n} X_i^{\hat{\beta}} \ln X_i}{\sum_{i=1}^{n} X_i^{\hat{\beta}}} - \frac{1}{n} \sum_{i=1}^{n} \ln X_i \,. \tag{3}$$

Using the maximum likelihood Equations (2) and (3), we can estimate β and α parameter when data are from Weibull distribution with the samples. Then we use the bootstrap to compute confidence intervals for shape parameter.

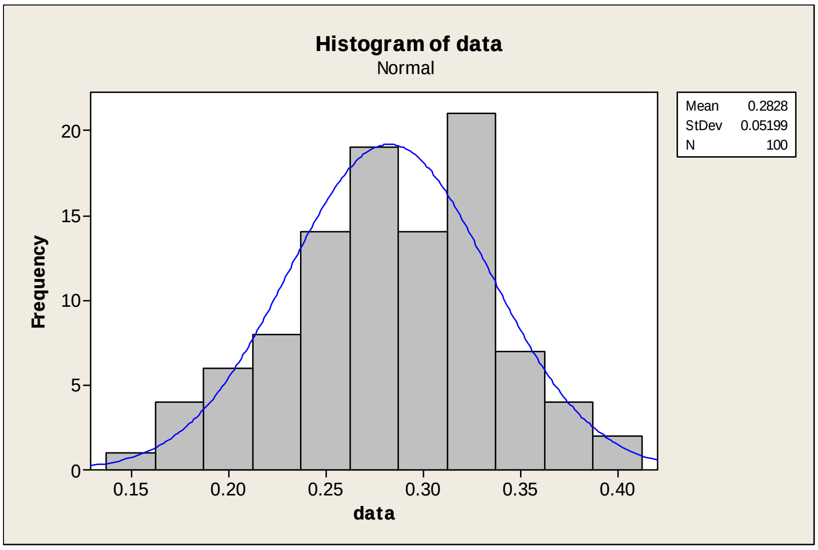

Figure 10. Histogram plot of the historical data.

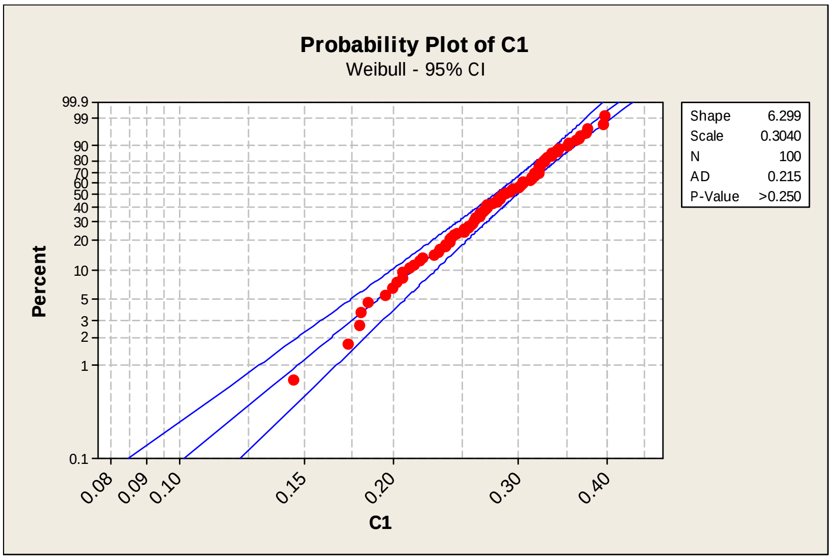

Figure 11. Weibull probability plot of the historical data.

Table 7. The 100 observations are collected from the historical data.

0.1795	0.2641	0.2689	0.3114	0.3333	0.2827	0.3735	0.2584	0.3206	0.2433
0.2018	0.3194	0.259	0.3329	0.2876	0.2795	0.2866	0.1837	0.3523	0.3727
0.3154	0.2916	0.3195	0.2989	0.2545	0.3281	0.2697	0.2405	0.3196	0.3498
0.3191	0.2816	0.2758	0.2636	0.3037	0.2802	0.3008	0.3152	0.2396	0.2844
0.3018	0.2514	0.3949	0.2572	0.3235	0.3631	0.3398	0.2659	0.2357	0.2052
0.3122	0.3035	0.2447	0.3932	0.3259	0.31	0.3268	0.2792	0.3152	0.2646
0.231	0.2544	0.2057	0.3325	0.3407	0.3198	0.2508	0.2998	0.2099	0.268
0.2605	0.322	0.1443	0.3601	0.3039	0.2706	0.3125	0.3209	0.2177	0.1992
0.2606	0.293	0.3395	0.2319	0.2274	0.2817	0.219	0.1726	0.3661	0.2686
0.1785	0.2933	0.2706	0.1941	0.2392	0.2366	0.3512	0.3196	0.3254	0.2136

We utilize this control chart to monitor the process, and collect another new dara are shown in Table 8. By the goodness-of-fit tests as shown in figure 12, the new data indicates that the process pretty approximates to be distributed as Weibull distribution. The parameters α and β of this Weibull process could be estimated from the historical data, giving $\hat{\alpha} = 0.3041$ and $\hat{\beta} = 0.5677$ by MLE.

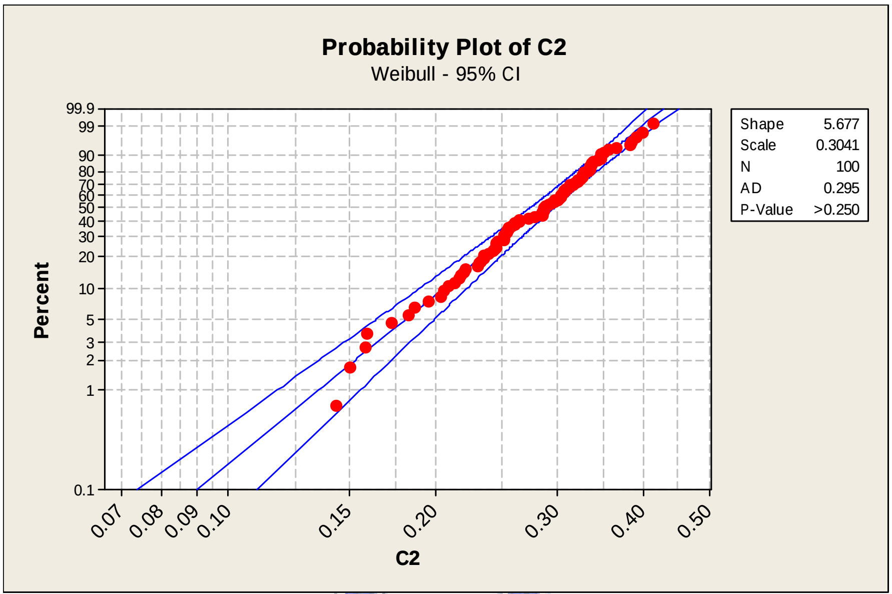

Figure 12. Weibull probability plot of the new data.

Table 8. The 100 observations of the new data.

0.3114	0.2577	0.3851	0.2976	0.2509	0.2785	0.2032	0.2085	0.3123	0.2167
0.2172	0.2125	0.3085	0.2436	0.3209	0.2847	0.297	0.159	0.3274	0.2532
0.2908	0.2597	0.2209	0.2545	0.3907	0.272	0.2348	0.3345	0.2425	0.2379
0.2448	0.3157	0.3358	0.1581	0.3013	0.2311	0.2884	0.3055	0.1951	0.3037
0.1431	0.3473	0.2516	0.3034	0.2848	0.2636	0.3981	0.307	0.4135	0.2855
0.3433	0.3175	0.2944	0.3351	0.1861	0.3503	0.2198	0.2506	0.3348	0.2551
0.2448	0.3551	0.308	0.2301	0.1826	0.3463	0.2598	0.3072	0.3279	0.2644
0.1497	0.2452	0.383	0.2449	0.3383	0.3208	0.3235	0.2054	0.3257	0.2866
0.3644	0.3269	0.286	0.2341	0.2872	0.2883	0.2513	0.3035	0.347	0.3135
0.2634	0.2871	0.33	0.3247	0.2325	0.3333	0.3359	0.1721	0.3007	0.2539

Therefore, it is appropriate to use this approach and we can obtain more accurate measures of three quantile: $X_{0.00135} = 0.102105$, $X_{0.5} = 0.282017$, and $X_{0.99865} = 0.411047$ under consideration. Then "dynamic" C_{pk} index can be calculate as follows:

$$\text{dynamic } C_{pk} = \min\left\{\frac{\mathrm{USL}-X_{0.5}}{AS_{50}(X_{0.99865}-X_{0.5})}, \frac{X_{0.5}-\mathrm{LSL}}{AS_{50}(X_{0.5}-X_{0.00135})}\right\}$$
$$= \min\left\{\frac{0.5-281962}{1.779\times(0.411047-0.281962)}, \frac{0.281962-0.1}{1.779\times(0.281962-0.102105)}\right\}$$
$$= \min\{0.949468, 0.5687\}$$
$$= 0.5687,$$

with AS_{50} =1.779 for n=10 from Table 6. Compared it to the value of the following index:

$$C_{pk} = \min\left\{\frac{\mathrm{USL}-X_{0.5}}{X_{0.99865}-X_{0.5}}, \frac{X_{0.5}-\mathrm{LSL}}{X_{0.5}-X_{0.00135}}\right\}$$
$$= \min\left\{\frac{0.5-0.281962}{0.411047-0.281962}, \frac{0.281962-0.1}{0.281962-0.102105}\right\}$$
$$= \min\{1.6891, 1.0117\}$$
$$= 1.0117,$$

That we do not consider the change in σ, we can find that the value of dynamic C_{pk} much smaller. By increasing n, change in σ have a higher probability to be detected. For example, if n=15, the AS_{50} would be 1.613 for Weibull distribution ($\alpha = 0.3$ and $\beta = 6$) then

$$\text{dynamic } C_{pk} = \min\left\{\frac{\mathrm{USL}-X_{0.5}}{AS_{50}(X_{0.99865}-X_{0.5})}, \frac{X_{0.5}-\mathrm{LSL}}{AS_{50}(X_{0.5}-X_{0.00135})}\right\}$$
$$= \min\left\{\frac{0.5-0.281962}{1.613\times(0.411047-0.281962)}, \frac{0.281962-0.1}{1.613\times(0.281962-0.102105)}\right\}$$
$$= \min\{1.04718, 0.6272\}$$
$$= 0.6272,$$

Increasing n from 12 to 15 increases the dynamic C_{pk} index from 0.5687 to 0.6272.

Chapter 6. Conclusion

This paper has considered the problem for adjusting estimates of process capability by including a variance change when data is from non-normal distribution. In the Bothe' studies, statistically derived adjustments are proposed under the data assumed to be approximately normally distributed. But the case of non-normal processes occurs frequently in practice. We also provide tables for the engineers to use for their in-plant applications. However, this "Dynamic" C_{pk} index assume μ remain stable when σ change. If μ and σ subjected to undetected increases and decreases? Further studies are need to determine how those change would affect estimates of outgoing quality.

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Appendix A. AS_{50} values for several subgroup sizes and various shape parameter.

Table 9. AS_{50} values for n=8(1)35 and β=12(1)21 values.

AS_{50}	Weibull distribution(1, β)										
n	(1,12)	(1,13)	(1,14)	(1,15)	(1,16)	(1,17)	(1,18)	(1,19)	(1,20)	(1,21)	N(0,1)
8	2.154	2.167	2.198	2.231	2.255	2.267	2.285	2.309	2.319	2.334	1.928
9	2.062	2.093	2.110	2.140	2.156	2.176	2.206	2.219	2.229	2.243	1.858
10	1.998	2.021	2.051	2.063	2.082	2.104	2.122	2.137	2.152	2.171	1.802
11	1.941	1.957	1.988	2.008	2.029	2.045	2.057	2.075	2.090	2.096	1.755
12	1.887	1.914	1.936	1.951	1.973	1.983	2.004	2.017	2.032	2.052	1.716
13	1.848	1.869	1.887	1.906	1.926	1.941	1.954	1.970	1.983	1.995	1.682
14	1.807	1.826	1.852	1.873	1.885	1.899	1.916	1.933	1.937	1.959	1.652
15	1.777	1.801	1.820	1.834	1.852	1.863	1.882	1.893	1.904	1.910	1.626
16	1.752	1.771	1.791	1.805	1.822	1.832	1.842	1.855	1.867	1.880	1.602
17	1.728	1.741	1.762	1.777	1.791	1.801	1.812	1.826	1.839	1.849	1.581
18	1.699	1.720	1.736	1.754	1.770	1.773	1.789	1.803	1.810	1.816	1.562
19	1.677	1.699	1.711	1.726	1.741	1.754	1.765	1.769	1.784	1.790	1.545
20	1.656	1.678	1.688	1.705	1.719	1.729	1.744	1.756	1.763	1.769	1.529
21	1.641	1.656	1.671	1.688	1.695	1.715	1.722	1.738	1.744	1.749	1.514
22	1.625	1.639	1.652	1.670	1.682	1.693	1.698	1.715	1.721	1.728	1.501
23	1.610	1.622	1.639	1.648	1.659	1.678	1.681	1.692	1.705	1.712	1.488
24	1.592	1.610	1.623	1.637	1.644	1.661	1.669	1.680	1.689	1.693	1.477
25	1.578	1.595	1.608	1.622	1.631	1.644	1.648	1.660	1.672	1.680	1.466
26	1.564	1.582	1.592	1.605	1.619	1.625	1.634	1.643	1.656	1.663	1.456
27	1.557	1.567	1.580	1.595	1.607	1.617	1.626	1.631	1.638	1.650	1.446
28	1.547	1.555	1.572	1.580	1.594	1.604	1.611	1.619	1.625	1.633	1.438
29	1.535	1.549	1.558	1.570	1.582	1.592	1.598	1.607	1.612	1.623	1.429
30	1.521	1.534	1.551	1.561	1.571	1.578	1.585	1.594	1.601	1.608	1.421
31	1.521	1.526	1.545	1.544	1.556	1.571	1.573	1.583	1.590	1.600	1.414
32	1.510	1.520	1.528	1.542	1.545	1.560	1.568	1.566	1.571	1.599	1.406
33	1.496	1.507	1.521	1.524	1.539	1.543	1.546	1.564	1.575	1.566	1.400
34	1.488	1.504	1.506	1.522	1.532	1.540	1.548	1.551	1.557	1.572	1.393
35	1.481	1.492	1.507	1.513	1.537	1.528	1.523	1.537	1.552	1.558	1.387

Table 10. AS_{50} values for n=8(1)35 and β=22(1)31 values.

AS_{50}	Weibull distribution(1, β)										
n	(1,22)	(1,23)	(1,24)	(1,25)	(1,26)	(1,27)	(1,28)	(1,29)	(1,30)	(1,31)	N(0,1)
8	2.346	2.373	2.375	2.390	2.388	2.413	2.423	2.436	2.435	2.438	1.928
9	2.253	2.277	2.277	2.284	2.298	2.313	2.313	2.331	2.343	2.345	1.858
10	1.998	2.021	2.051	2.063	2.082	2.104	2.122	2.137	2.152	2.171	1.802
11	1.941	1.957	1.988	2.008	2.029	2.045	2.057	2.075	2.090	2.096	1.755
12	1.887	1.914	1.936	1.951	1.973	1.983	2.004	2.017	2.032	2.052	1.716
13	1.848	1.869	1.887	1.906	1.926	1.941	1.954	1.970	1.983	1.995	1.682
14	1.807	1.826	1.852	1.873	1.885	1.899	1.916	1.933	1.937	1.959	1.652
15	1.777	1.801	1.820	1.834	1.852	1.863	1.882	1.893	1.904	1.910	1.626
16	1.752	1.771	1.791	1.805	1.822	1.832	1.842	1.855	1.867	1.880	1.602
17	1.728	1.741	1.762	1.777	1.791	1.801	1.812	1.826	1.839	1.849	1.581
18	1.699	1.720	1.736	1.754	1.770	1.773	1.789	1.803	1.810	1.816	1.562
19	1.677	1.699	1.711	1.726	1.741	1.754	1.765	1.769	1.784	1.790	1.545
20	1.656	1.678	1.688	1.705	1.719	1.729	1.744	1.756	1.763	1.769	1.529
21	1.641	1.656	1.671	1.688	1.695	1.715	1.722	1.738	1.744	1.749	1.514
22	1.625	1.639	1.652	1.670	1.682	1.693	1.698	1.715	1.721	1.728	1.501
23	1.610	1.622	1.639	1.648	1.659	1.678	1.681	1.692	1.705	1.712	1.488
24	1.592	1.610	1.623	1.637	1.644	1.661	1.669	1.680	1.689	1.693	1.477
25	1.578	1.595	1.608	1.622	1.631	1.644	1.648	1.660	1.672	1.680	1.466
26	1.564	1.582	1.592	1.605	1.619	1.625	1.634	1.643	1.656	1.663	1.456
27	1.557	1.567	1.580	1.595	1.607	1.617	1.626	1.631	1.638	1.650	1.446
28	1.547	1.555	1.572	1.580	1.594	1.604	1.611	1.619	1.625	1.633	1.438
29	1.535	1.549	1.558	1.570	1.582	1.592	1.598	1.607	1.612	1.623	1.429
30	1.521	1.534	1.551	1.561	1.571	1.578	1.585	1.594	1.601	1.608	1.421
31	1.615	1.605	1.618	1.612	1.619	1.641	1.625	1.612	1.661	1.651	1.414
32	1.583	1.593	1.602	1.611	1.617	1.624	1.620	1.611	1.638	1.624	1.406
33	1.571	1.595	1.602	1.592	1.594	1.604	1.613	1.592	1.620	1.612	1.400
34	1.569	1.571	1.590	1.584	1.589	1.595	1.611	1.584	1.610	1.611	1.393
35	1.561	1.559	1.579	1.575	1.582	1.583	1.598	1.575	1.601	1.610	1.387

Appendix B. The Average Run Length of Weibull Distributions.

Table 11. Average run length of Weibull with 1.5 times standard deviation change.

ARL	Weibull(1, β)										Normal
n	1	2	3	3.6	4	5	6	7	8	9	10	N(0,1)
7	20.964	7.457	5.467	5.147	5.671	6.186	7.251	8.622	9.547	11.531	12.376	7.222
8	19.033	6.660	4.664	4.616	4.757	5.211	6.068	7.179	8.497	9.633	10.723	6.175
9	17.762	6.044	4.158	4.033	4.200	4.612	5.450	6.115	7.350	7.700	8.971	5.402
10	15.941	5.191	3.905	3.582	3.694	4.084	4.446	5.132	5.959	6.854	7.792	4.768
11	15.340	4.683	3.457	3.240	3.359	3.707	4.209	5.050	5.393	6.096	6.671	4.272
12	14.51	4.475	3.179	3.002	3.06	3.392	3.775	4.381	4.592	5.414	5.849	3.840
13	14.055	4.166	2.912	2.826	2.827	3.025	3.436	3.937	4.285	4.78	5.552	3.542
14	13.300	3.826	2.774	2.627	2.648	2.789	3.212	3.674	3.866	4.557	4.854	3.222
5	12.781	3.618	2.527	2.482	2.448	2.720	2.847	3.287	3.712	4.136	4.732	2.992
16	12.021	3.421	2.421	2.296	2.288	2.514	2.801	2.914	3.450	3.835	4.113	2.781
17	11.639	3.283	2.278	2.113	2.134	2.193	2.524	2.908	3.062	3.273	3.835	2.594
18	11.312	3.033	2.154	1.988	2.070	2.178	2.388	2.695	2.935	3.270	3.540	2.450
19	11.018	2.897	2.053	1.983	1.927	2.020	2.234	2.442	2.882	3.003	3.497	2.328
20	10.416	2.749	1.983	1.851	1.848	1.986	2.135	2.313	2.576	2.808	3.173	2.203
21	9.966	2.651	1.875	1.782	1.776	1.895	1.981	2.284	2.365	2.813	2.964	2.094
22	9.838	2.545	1.784	1.731	1.714	1.815	1.929	2.127	2.314	2.618	2.802	2.000
23	9.532	2.340	1.738	1.69	1.658	1.746	1.859	2.028	2.299	2.456	2.705	1.924
24	9.231	2.346	1.679	1.611	1.612	1.647	1.772	1.986	2.209	2.194	2.500	1.842
25	8.797	2.348	1.645	1.564	1.568	1.606	1.658	1.876	2.066	2.156	2.380	1.777
26	8.365	2.184	1.57	1.498	1.528	1.538	1.646	1.860	1.962	2.064	2.255	1.711
27	8.103	2.152	1.528	1.478	1.480	1.496	1.624	1.719	1.923	1.998	2.174	1.668
28	7.983	2.089	1.514	1.448	1.446	1.445	1.575	1.664	1.759	1.942	2.039	1.619
29	7.508	2.038	1.480	1.411	1.387	1.430	1.504	1.632	1.746	1.827	1.980	1.560
30	7.432	2.007	1.449	1.359	1.399	1.379	1.464	1.599	1.686	1.812	1.908	1.526
31	7.268	1.898	1.406	1.349	1.350	1.368	1.418	1.524	1.651	1.782	1.907	1.494
32	7.142	1.886	1.401	1.317	1.301	1.346	1.408	1.490	1.628	1.710	1.846	1.451
33	6.960	1.819	1.363	1.308	1.275	1.323	1.390	1.487	1.556	1.653	1.797	1.419
34	6.765	1.802	1.335	1.272	1.267	1.305	1.35	1.448	1.510	1.614	1.683	1.390
35	6.507	1.744	1.316	1.254	1.249	1.262	1.349	1.422	1.470	1.555	1.639	1.366

Table 12. Average run length of Weibull with 2 times standard deviation change.

ARL	**Weibull(1, β)**										Normal
n	1	2	3	3.6	4	5	6	7	8	9	10	N(0,1)
7	8.388	3.027	2.101	1.952	1.891	1.883	1.948	2.075	2.246	2.36	2.564	2.046
8	7.722	2.65	1.884	1.766	1.711	1.688	1.739	1.868	1.949	2.114	2.184	1.814
9	7.179	2.466	1.736	1.607	1.545	1.523	1.578	1.652	1.704	1.814	1.947	1.635
10	6.722	2.299	1.614	1.497	1.433	1.434	1.481	1.543	1.589	1.705	1.800	1.510
11	6.629	2.103	1.512	1.394	1.380	1.361	1.375	1.427	1.473	1.567	1.638	1.416
12	6.360	1.993	1.422	1.323	1.297	1.277	1.297	1.341	1.380	1.455	1.514	1.335
13	5.543	1.889	1.377	1.282	1.246	1.216	1.244	1.262	1.339	1.398	1.445	1.276
14	5.423	1.780	1.304	1.230	1.205	1.188	1.195	1.219	1.264	1.321	1.349	1.142
5	5.333	1.685	1.265	1.184	1.164	1.146	1.161	1.196	1.223	1.256	1.286	1.187
16	5.128	1.643	1.221	1.155	1.132	1.119	1.136	1.157	1.181	1.213	1.245	1.155
17	4.853	1.569	1.199	1.139	1.110	1.098	1.107	1.121	1.181	1.165	1.208	1.126
18	4.638	1.542	1.170	1.113	1.090	1.078	1.088	1.106	1.126	1.139	1.177	1.105
19	4.543	1.449	1.146	1.093	1.079	1.067	1.073	1.085	1.094	1.12	1.140	1.087
20	4.480	1.417	1.125	1.074	1.070	1.053	1.059	1.069	1.086	1.099	1.118	1.072
21	4.236	1.397	1.112	1.069	1.053	1.044	1.049	1.056	1.066	1.083	1.096	1.059
22	4.024	1.360	1.096	1.058	1.044	1.037	1.037	1.048	1.055	1.066	1.083	1.049
23	3.919	1.319	1.078	1.047	1.039	1.032	1.031	1.041	1.045	1.060	1.070	1.041
24	3.865	1.300	1.072	1.041	1.031	1.023	1.027	1.030	1.037	1.046	1.06	1.034
25	3.576	1.274	1.061	1.033	1.027	1.02	1.021	1.024	1.03	1.037	1.048	1.028
26	3.703	1.255	1.056	1.029	1.021	1.017	1.017	1.021	1.026	1.032	1.042	1.022
27	3.528	1.232	1.045	1.024	1.019	1.013	1.015	1.016	1.021	1.028	1.036	1.018
28	3.403	1.208	1.041	1.021	1.014	1.011	1.012	1.014	1.018	1.024	1.031	1.015
29	3.277	1.199	1.034	1.017	1.012	1.008	1.008	1.011	1.014	1.019	1.025	1.013
30	3.111	1.181	1.028	1.015	1.010	1.006	1.006	1.009	1.012	1.015	1.021	1.011
31	3.097	1.171	1.027	1.012	1.009	1.005	1.005	1.007	1.010	1.014	1.016	1.008
32	3.050	1.156	1.022	1.010	1.007	1.004	1.004	1.006	1.008	1.011	1.013	1.007
33	2.963	1.145	1.021	1.009	1.005	1.003	1.004	1.005	1.006	1.008	1.011	1.006
34	2.854	1.132	1.017	1.007	1.004	1.003	1.003	1.003	1.005	1.006	1.009	1.004
35	2.781	1.129	1.015	1.006	1.004	1.002	1.002	1.003	1.004	1.006	1.007	1.004

Table 13. Average run length of Weibull with 2.5 times standard deviation change.

ARL	Weibull(1, β)											Normal
n	1	2	3	3.6	4	5	6	7	8	9	10	N(0,1)
7	6.064	2.286	1.597	1.458	1.403	1.339	1.327	1.34	1.384	1.44	1.485	1.338
8	5.551	2.085	1.466	1.35	1.313	1.247	1.231	1.253	1.264	1.299	1.333	1.239
9	5.067	1.926	1.384	1.273	1.224	1.184	1.162	1.174	1.194	1.228	1.24	1.173
10	4.786	1.798	1.307	1.211	1.172	1.133	1.125	1.126	1.145	1.157	1.175	1.126
11	4.56	1.674	1.259	1.169	1.127	1.094	1.088	1.098	1.105	1.115	1.134	1.092
12	4.343	1.618	1.201	1.129	1.101	1.072	1.062	1.069	1.073	1.081	1.103	1.066
13	4.171	1.525	1.173	1.103	1.08	1.051	1.047	1.048	1.054	1.064	1.077	1.047
14	3.896	1.467	1.137	1.083	1.065	1.039	1.033	1.035	1.038	1.047	1.052	1.035
5	3.895	1.413	1.115	1.069	1.049	1.03	1.025	1.027	1.027	1.034	1.039	1.026
16	3.616	1.373	1.093	1.053	1.036	1.022	1.017	1.019	1.02	1.025	1.03	1.019
17	3.545	1.318	1.08	1.042	1.03	1.018	1.012	1.012	1.015	1.016	1.019	1.013
18	3.434	1.302	1.068	1.033	1.022	1.012	1.01	1.01	1.01	1.013	1.015	1.01
19	3.319	1.267	1.057	1.025	1.017	1.008	1.007	1.008	1.007	1.008	1.012	1.007
20	3.175	1.24	1.045	1.02	1.013	1.007	1.004	1.005	1.006	1.007	1.008	1.005
21	3.044	1.215	1.038	1.016	1.01	1.004	1.003	1.003	1.004	1.005	1.006	1.003
22	3.041	1.196	1.033	1.013	1.007	1.003	1.003	1.002	1.002	1.003	1.004	1.003
23	2.921	1.174	1.026	1.01	1.006	1.002	1.002	1.002	1.002	1.002	1.003	1.002
24	2.765	1.16	1.021	1.009	1.005	1.002	1.001	1.001	1.001	1.002	1.002	1.001
25	2.72	1.146	1.019	1.006	1.003	1.002	1.001	1.001	1.001	1.001	1.001	1.001
26	2.662	1.135	1.016	1.005	1.003	1.001	1.001	1.001	1.001	1.001	1.001	1.001
27	2.6	1.119	1.012	1.004	1.002	1.001	1	1	1	1.001	1.001	1
28	2.521	1.108	1.011	1.003	1.001	1	1	1	1	1	1	1
29	2.446	1.103	1.008	1.002	1.001	1	1	1	1	1	1	1
30	2.4	1.087	1.008	1.002	1.001	1	1	1	1	1	1	1
31	2.393	1.08	1.006	1.001	1.001	1	1	1	1	1	1	1
32	2.289	1.072	1.005	1.001	1	1	1	1	1	1	1	1
33	2.279	1.067	1.004	1.001	1	1	1	1	1	1	1	1
34	2.223	1.059	1.004	1.001	1	1	1	1	1	1	1	1
35	2.191	1.055	1.003	1.001	1	1	1	1	1	1	1	1

Table 14. Average run length of Weibull with 3 times standard deviation change.

ARL	**Weibull(1, β)**											Normal
n	1	2	3	3.6	4	5	6	7	8	9	10	N(0,1)
7	4.773	2.081	1.449	1.315	1.267	1.192	1.155	1.157	1.161	1.168	1.178	1.139
8	4.494	1.896	1.352	1.24	1.193	1.131	1.109	1.105	1.1	1.115	1.122	1.09
9	4.303	1.747	1.278	1.18	1.143	1.091	1.072	1.066	1.07	1.073	1.079	1.059
10	3.993	1.642	1.224	1.137	1.107	1.062	1.047	1.042	1.044	1.045	1.051	1.037
11	3.825	1.563	1.18	1.105	1.078	1.041	1.032	1.029	1.029	1.03	1.033	1.024
12	3.739	1.495	1.145	1.081	1.058	1.031	1.022	1.019	1.019	1.02	1.021	1.016
13	3.515	1.422	1.121	1.064	1.043	1.022	1.014	1.013	1.012	1.013	1.014	1.009
14	3.411	1.377	1.097	1.049	1.034	1.014	1.01	1.007	1.008	1.008	1.009	1.007
5	3.334	1.334	1.084	1.038	1.025	1.012	1.006	1.005	1.005	1.005	1.006	1.004
16	3.125	1.301	1.065	1.029	1.018	1.008	1.004	1.003	1.003	1.003	1.004	1.002
17	2.996	1.265	1.054	1.023	1.014	1.005	1.003	1.002	1.002	1.002	1.002	1.002
18	2.933	1.239	1.046	1.017	1.01	1.003	1.002	1.001	1.001	1.001	1.002	1.001
19	2.836	1.216	1.035	1.014	1.007	1.002	1.001	1.001	1.001	1.001	1.001	1.001
20	2.735	1.192	1.03	1.011	1.006	1.002	1.001	1.001	1.001	1	1.001	1
21	2.661	1.173	1.025	1.008	1.004	1.001	1	1	1	1	1	1
22	2.578	1.153	1.02	1.006	1.003	1.001	1	1	1	1	1	1
23	2.507	1.138	1.016	1.005	1.002	1	1	1	1	1	1	1
24	2.479	1.124	1.014	1.004	1.002	1	1	1	1	1	1	1
25	2.419	1.115	1.011	1.003	1.001	1	1	1	1	1	1	1
26	2.307	1.105	1.009	1.002	1.001	1	1	1	1	1	1	1
27	2.344	1.09	1.007	1.002	1.001	1	1	1	1	1	1	1
28	2.231	1.08	1.006	1.001	1.001	1	1	1	1	1	1	1
29	2.181	1.073	1.005	1.001	1.001	1	1	1	1	1	1	1
30	2.164	1.07	1.004	1.001	1	1	1	1	1	1	1	1
31	2.104	1.061	1.003	1.001	1	1	1	1	1	1	1	1
32	2.078	1.054	1.003	1	1	1	1	1	1	1	1	1
33	2.025	1.05	1.002	1	1	1	1	1	1	1	1	1
34	2.016	1.046	1.002	1	1	1	1	1	1	1	1	1
35	1.971	1.04	1.001	1	1	1	1	1	1	1	1	1

Table 15. Average run length of Weibull with 3.5 times standard deviation change.

ARL	Weibull(1, β)										Normal
n	1	2	3	3.6	4	5	6	7	8	9	10	N(0,1)
7	4.198	1.98	1.406	1.274	1.216	1.141	1.109	1.091	1.085	1.082	1.083	1.064
8	3.941	1.813	1.318	1.199	1.153	1.094	1.067	1.053	1.05	1.053	1.051	1.037
9	3.722	1.679	1.255	1.15	1.113	1.061	1.041	1.034	1.029	1.029	1.031	1.022
10	3.605	1.589	1.202	1.116	1.082	1.042	1.026	1.019	1.018	1.017	1.017	1.013
11	3.442	1.52	1.159	1.087	1.064	1.028	1.017	1.013	1.011	1.01	1.011	1.007
12	3.263	1.452	1.131	1.067	1.044	1.02	1.01	1.007	1.006	1.006	1.006	1.004
13	3.153	1.398	1.102	1.053	1.035	1.013	1.007	1.004	1.004	1.003	1.003	1.002
14	3.040	1.349	1.088	1.039	1.026	1.009	1.004	1.003	1.002	1.002	1.002	1.001
5	2.962	1.313	1.068	1.031	1.019	1.006	1.003	1.002	1.001	1.001	1.001	1.001
16	2.855	1.275	1.056	1.024	1.014	1.004	1.002	1.001	1.001	1.001	1.001	1.001
17	2.791	1.248	1.048	1.019	1.01	1.003	1.001	1	1	1	1	1
18	2.674	1.214	1.04	1.015	1.008	1.002	1.001	1	1	1	1	1
19	2.578	1.194	1.032	1.011	1.005	1.001	1	1	1	1	1	1
20	2.508	1.176	1.025	1.009	1.004	1.001	1	1	1	1	1	1
21	2.459	1.156	1.021	1.006	1.003	1.001	1	1	1	1	1	1
22	2.39	1.147	1.019	1.005	1.002	1	1	1	1	1	1	1
23	2.37	1.127	1.015	1.004	1.002	1	1	1	1	1	1	1
24	2.247	1.114	1.012	1.003	1.001	1	1	1	1	1	1	1
25	2.249	1.105	1.01	1.003	1.001	1	1	1	1	1	1	1
26	2.175	1.092	1.008	1.002	1.001	1	1	1	1	1	1	1
27	2.129	1.086	1.007	1.001	1	1	1	1	1	1	1	1
28	2.089	1.075	1.006	1.001	1	1	1	1	1	1	1	1
29	2.035	1.07	1.005	1.001	1	1	1	1	1	1	1	1
30	2.012	1.062	1.003	1.001	1	1	1	1	1	1	1	1
31	1.994	1.058	1.003	1.001	1	1	1	1	1	1	1	1
32	1.938	1.052	1.003	1	1	1	1	1	1	1	1	1
33	1.911	1.045	1.002	1	1	1	1	1	1	1	1	1
34	1.868	1.041	1.002	1	1	1	1	1	1	1	1	1
35	1.874	1.038	1.001	1	1	1	1	1	1	1	1	1