國立交通大學
工業工程與管理學系碩士班

碩士論文

考慮多品質特性製程之計量抽樣計劃

A Variables Sampling Plan for Processes with Multiple Characteristics

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摘要

允收抽樣計劃在品質管制的應用中是相當重要的工具之一，它是在產品訂購的品質契約上，品質保證應用的實務工具。抽樣計劃提供了生產者和消費者對於產品是否接受或拒絕的決策準則，同時也可降低生產者所提供之產品品質、數量與訂單數量之間的差異。根據現今的品質改善理論，產品製程良率涉及一種以上之品質特性是相當普遍的，因此本研究提出了製程能力指標 S_{pk}^{T} 的計量抽樣計劃來處理多品質特性製程之產品允收決策準則，而工程師或實務界可以利用本研究所提出之方法來決定所需檢驗之樣本數及產品允收之臨界值，並做出可靠有效之決策。在本研究的最後，我們利用一個實例來說明如何建構允收抽樣計劃之操作程序及應用。

關鍵字：允收抽樣計劃、製程能力指標、製程良率、多品質特性、允收臨界值、決策

A variables sampling plan for processes with multiple characteristics

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Abstract

Acceptance sampling plans have been one of the most practical tools used in classical quality control applications. It is a practical tool for quality assurance applications involving quality contract on product orders. The sampling plans provide the vendor and buyer decision rules for product acceptance to meet the present product quality requirement. A well-designed sampling plan can effectively reduce the difference between the actual supply quantity and order quantity. According to today's modern quality improvement theory, the manufactured product involving more than one quality characteristic is quite common. Therefore, in this paper, we introduce a new variables sampling plan based on process capability index S_{pk}^{T} to deal with product acceptance determination for multiple characteristics. Practitioners can use the proposed method to determine the number of required in section units, the critical acceptance value, and make reliable decisions in product acceptance. At the end, a case study is also presented to illustrate how the proposed procedure can be constructed and applied to the real applications.

Keywords: Acceptance sampling plans, Process capability index, Process yield, Multiple characteristics, Critical acceptance values, Decision making.

誌謝

從來沒有想過會在交大就讀的我，如今要從這裡畢業了，雖然才短短的兩年，但這一切的奇蹟，真的要感謝很多人，首先要感謝我的指導老師彭文理老師，謝謝您平常除了在課業上對我們的指導外，在生活上及未來的旅途上也都能給予我們建議及關心，真的是受益良多，也很感謝我的口試委員，鐘淑馨老師、洪志真老師、洪一薰老師，對我的論文所做出的指正及建議，這才使得我的論文能夠更完善。當然，這兩年也要感謝我的父母，謝謝你們不讓我為家裡經濟操心，並願意支持我完成學業，以及我的弟弟們，願意在我遇到挫折時聽我哭訴，還有台中技術學院統計系的老師們，感謝您們當初的鼓勵及支持，也很感謝您們，即使在我畢業後也還是對我如此的關心及鼓勵。

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Chapter 1. Introduction

1.1. Research Background and Motivation

Inspection of raw materials, semiconductor finished products, or finished products are one aspect of quality assurance. Acceptance sampling plans are practical tools for quality assurance applications, which involves quality contract on production orders between the factories and customers. Acceptance sampling plans provide the producer and the consumer general decision rules for product acceptance while meeting their needs for product quality. A well-designed sampling plan can effectively reduce the difference between the actual supply quantity and order quantity. Acceptance sampling plan, however, cannot avoid the risk of accepting bad product lots or rejecting good product lots even when 100% inspection is implemented, because of human error and fatigue, we are never ensured that the decision will be the right one. Acceptance sampling plans set the required sample size for product inspection and the associated acceptance or rejection criteria for sentencing each individual product lot. However, the criteria used for measuring the performance of an acceptance sampling plan is usually based on the operating characteristic (OC) curve quantifying the risk for producer and consumer.

The operating characteristic (OC) curve is often viewed in the sense of an adversary relationship between the producer and the consumer. Figure 1 displays the OC curve of the sampling plan. The OC curve plots the probability of accepting the lot against actual lot fraction defective, which displays the discriminatory power of the sampling plan. That is, the OC curve shows the probability of accepting a lot submitted with a certain fraction of defectives which results in the producer and the consumer having a common base for judging whether the sampling plan is appropriate. The producer is primarily interested in insuring that good lots are accepted while the consumer wants to be reasonably sure that bad lots will be rejected. Therefore, for product quality protection and company's profit, the producer or suppliers usually look at a specific level of product quality on OC curve, traditionally referred to as average quality level (AQL), which would yield a high probability of acceptance. The AQL presents the poorest level of quality for the producer's process that the consumer would consider acceptable as a process average. The consumer would seek a sampling procedure with OC curve providing a high probability of acceptance at the AQL. The consumer would also look at the other end of the OC curve, called lot tolerance percent defective (LTPD). The LTPD is the poorest quality level that the consumer is willing to accept.

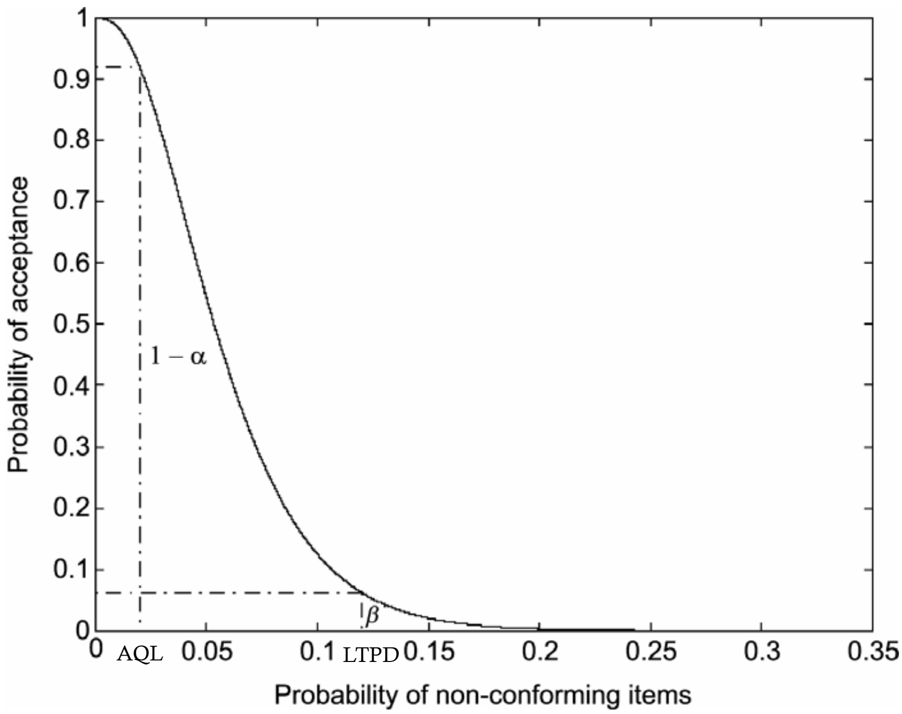

Figure 1. OC curve of the sampling plan.

Acceptance sampling plan basically consists of a sample size for inspection and an acceptance criterion. Therefore, sampling involves risks that the sample will not adequately reflect the totality of quality conditions of the product. α is the probability of the Type I error, for a given sampling plan, of rejecting the product that has a defect level equal to the AQL. The producer suffers when this occurs because a product with acceptable quality is rejected. Type II error (β) is the probability, for a given sampling plan, of accepting the product with defect level equal to the LTPD. The consumer suffers when this occurs, because product with unacceptable quality is accepted. A well-designed sampling plan must provide the probability of at least $1-\alpha$ of accepting a lot if the lot fraction of defectives is at the contracted value AQL. Analogously, the sampling plan must also provide the probability of acceptance no more than β if the lot fraction of defectives is at the LTPD level which is an undesired level designated by the consumer. That is, the acceptance sampling plan must have its OC curve passing through those two designated points (AQL, $1-\alpha$) and (LTPD, β).

1.2. Research Purpose and Objectives

There are a number of different ways to classify acceptance sampling plans. One major classification is by attributes and variables. When a quality characteristic is measurable on a continuous scale and is known to have a distribution of a specified type, it may be appropriate to use variables sampling plans rather than attributes sampling plans for product acceptance applications. The variables sampling plan has the primary advantage that the same operating characteristic curve can be obtained with a smaller sample size than would be required using an attributes sampling plan. That is, a variables sampling plan that has the same protection as an attributes acceptance sampling plan would require less sampling. The precise measurements required by a variables plan would probably cost more than the simple classification of items required by an attributes plan, but the reduction in sample size may more than offset this increased cost. Such savings may be especially marked if the inspection is destructive and item is expensive (see, e.g., Schilling (1982), Duncan (1986), Montgomery (2001)).

For the attributes and variables sampling plans, there have been many researchers who have investigated the sampling plans problems. In the attributes sampling plans, Guenther (1969) developed a systematic search procedure, which can be used with published tables of binomial, hyper-geometric, and Poisson distributions to obtain the desired acceptance sampling plans. Stephens (1978) provided a closed form solution for single sample acceptance sampling plans using a normal approximation to the binomial distribution. Hailey (1980) presented a computer program to obtain single sampling plans with a minimum sample size based on either the Poisson or binomial distribution. Hald (1981) gave a systematic exposition of the existing statistical theory of lot-by-lot sampling inspection by attributes and provided some tables for the sampling plans. Comparisons between variables sampling plans and attributes sampling plans were investigated by Kao (1971) and Hamaker (1979), who concluded that the expected sample size required by variables sampling is smaller than those for comparable attributes sampling plans.

As for the variables sampling plans, the basic concepts and models of statistically based on variables sampling plans were introduced by Jennett and Welch (1939). Lieberman and Resnikoff (1955) developed extensive tables and OC curves for various AQLs for MIL-STD-414 sampling plan. Owen (1967) considered variables sampling plans based on the normal distribution, and developed sampling plans for various levels of probabilities of type I error when the standard deviation is unknown. Das and Mitra (1964) have investigated the effect of non-normality on the performance of the sampling plans. Bender (1975) considered sampling plans for assuring the percent defective in the case of the product quality characteristics obeying a normal distribution with unknown standard deviation, and presented a procedure using iterative computer program calculating the non-central t-distribution. Govindaraju and Soundararajan (1986) developed variables sampling plans that match the OC curves of MIL-STD-105D. Suresh and Ramanathan (1997) developed a sampling plan based on a more general symmetric family of distributions. In addition to the graphical procedure for designing sampling plans with specified OC curves, tabular procedures are also available for the same purpose. Duncan (1986) gave a good description of these techniques.

As the rapid advancement of manufacturing technology, suppliers require their products to be of high quality with a very low fraction of defectives. Due to the sampling cannot guarantee that every defective item in a lot will be inspected, the sampling involves risks of not adequately reflecting the quality conditions of the lot. Particularly, when the fraction of nonconforming products is required very low, such as the required fraction of defectives is often lower than 0.01%, and is measured in parts per million (PPM). Unfortunately, traditional methods for calculating the fraction nonconforming no longer work since any sample of reasonable size will probably contain no defective product items. An alternative method of measuring the fraction of defectives is to use process capability indices. However, the manufactured product has multiple correlated characteristics is quite common. Therefore, in this paper we introduce an effective acceptance sampling plan for lot sentencing based on the index S_{pk}^{T} as a quality benchmark for product acceptance, specifically for normally distributed processes with low fraction of defectives.

1.3. Thesis Organization

This thesis is organized as follows. First, we introduce the research motivation and purpose in Chapter 1. Secondly, a brief introduction of variables acceptance sampling plans for index C_{pk}, C_{pm} and C_{pmk} in Chapter 2. In Chapter 3, we introduce the process capability indices and introduce estimation of S_{pk}^{T} and it sampling distribution. In Chapter 4, we introduce the calculation of S_{pk}^{T} variables sampling plans, and show the sampling procedure and decision making. For illustrative purpose, an example to demonstrate the model used for S_{pk}^{T} sampling plans is presented in Chapter 5. Finally, we give some conclusions in Chapter 6.

Chapter 2. Literature Review

The acceptance sampling plans had been many researched. But as today's modern quality improvement philosophy, suppliers require their products to be of high quality with a very low fraction of defectives. Acceptance sampling plans of traditional methods for calculating the fraction nonconforming no longer work. An alternative method of measuring the fraction of defectives is to use process capability indices. Therefore, in this section, we will review these papers about acceptance sampling plans for index C_{pk}, C_{pm} and C_{pmk}.

2.1. Acceptance Sampling Plans Based on C_{pk}

Process capability analysis has become an important and integrated part in the applications of statistical process control for continuous improvement in productivity and quality assurance. Process capability indices (PCIs), establishing the relationship between the actual process performance and the manufacturing specifications, have been the focus of recent research in quality assurance and statistical literature. The most commonly used indices include C_p, C_{pk}, C_{pm} and C_{pmk}. The C_p and C_{pk} have been proposed for a long time and widely discussed in the paper of Kane (1986). The indices C_{pm} and C_{pmk} were originally developed by Chan *et al.* (1988) and Pearn *et al.* (1992), respectively. Based on analyzing the PCIs, a production department can trace and improve a poor process so that the quality level can be enhanced and the requirements of the customers can be satisfied. Index C_p has been defined as $C_p = (USL - LSL)/6\sigma$, where *USL* and *LSL* are the upper and lower specification limit, respectively, σ is the process standard deviation. In process capability analysis, C_{pk} is the most popular index. It has been defined as:

$$C_{pk} = \min\left\{\frac{USL-\mu}{3\sigma}, \frac{\mu - LSL}{3\sigma}\right\},$$

where *USL* is the upper specification limit, *LSL* is the lower specification limit, μ is the process mean, and σ is the process standard deviation.

The C_{pk} index is an appropriate measure of progress for quality improvement paradigms in which reduction of variability is the guiding principle and process yield is the primary measure of success. Pearn and Wu (2007) provided an acceptance sampling plan for C_{pk} index as a quality benchmark for product acceptance. Since the quality characteristic is variable, the lower specification limit and the upper specification limit can be used to define the acceptable values of this parameter. It is easy to design a sampling plan with a specified OC curve. Let (AQL, $1-\alpha$) and (LTPD, β) be the two points on the OC curve of interest.

For processes with target value set to the mid-point of the specification limits (i.e. $T = M$), the index may be rewritten as $C_{pk} = (d/\sigma - |\xi|)/3$, where $\xi = (\mu - M)/\sigma$, T is the target value, $d = (USL - LSL)/2$ is the half length of the specification interval, $m = (USL + LSL)/2$ is the midpoint of the specification limits. It's noted, the sampling distribution of $\hat{C}_{pk} = (d - |\bar{X} - M|)/3S$ is expressed in terms of mixture of the chi-square and the normal distributions. Given $C_{pk} = C$, $b = d/\sigma$ can be expressed as $b = 3C + |\xi|$. Thus, the probability of accepting the product can be expressed as

$$\pi_A(c) = P(\hat{C}_{pk} \geq c_0 | C_{pk} = c)$$
$$= \int_0^{b\sqrt{n}} G\left(\frac{(n-1)(b\sqrt{n}-t)^2}{9nc_0^2}\right) \times (\phi(t+\xi\sqrt{n}) + \phi(t-\xi\sqrt{n}))dt$$

where $G(\cdot)$ is the cumulative distribution function of the chi-square distribution with degree of freedom $n - 1$, χ^2_{n-1}, and $\phi(\cdot)$ is the probability density function of the standard normal distribution $N(0, 1)$.

Therefore, the required inspection sample size n and critical acceptance value c_0 for the sampling plan are the solutions to the following two nonlinear simultaneous equations:

$$S_1(n, c_0) = \int_0^{b_1\sqrt{n}} G\left(\frac{(n-1)(b_1\sqrt{n}-t)^2}{9nc_0^2}\right) \times (\phi(t+\xi\sqrt{n}) + \phi(t-\xi\sqrt{n}))dt - (1-\alpha),$$

$$S_2(n, c_0) = \int_0^{b_2\sqrt{n}} G\left(\frac{(n-1)(b_2\sqrt{n}-t)^2}{9nc_0^2}\right) \times (\phi(t+\xi\sqrt{n}) + \phi(t-\xi\sqrt{n}))dt - \beta,$$

where $b_1 = 3C_{AQL} + |\xi|$ and $b_2 = 3C_{LTPD} + |\xi|$, $C_{AQL} > C_{LTPD}$. Here C_{AQL} and C_{LTPD} represent the capability requirements corresponding to the AQL and the LTPD based on C_{pk} index, respectively.

For practical application purposes, we calculate and tabulate the critical acceptance values (c_0) and required sample sizes (n) for the sampling plans, with commonly used α, β, C_{AQL} and C_{LTPD}. The results obtained are useful to the practitioners in making reliable decisions.

2.2. Acceptance Sampling Plans Based on C_{pm}

Pearn and Wu (2006) developed the acceptance sampling plan for C_{pm} index. Hsiang and Taguchi (1985) introduced the index C_{pm}, which was also proposed independently by Chan *et al.* (1988). The index is related to the idea of squared error loss $loss(X) = (X-T)^2$. This loss based process capability index C_{pm} has also been called the Taguchi capability index. The index emphasizes on measuring the ability of the process to cluster around the target, which reflects the degrees of process targeting (centering). The index C_{pm} incorporates with the product variation with respect to the target value and the manufacturing specifications preset in the factory. The index C_{pm} is defined as

$$C_{pm} = \frac{USL - LSL}{6\sqrt{\sigma^2 + (\mu - T)^2}},$$

where μ is the process mean, σ is the process standard deviation, and T is the target value.

According to today's modern quality improvement philosophy, customers do notice unit-to-unit differences in these characteristics, especially if the variance is large and /or the mean is offset from the target. With the increasing importance of clustering around the target rather than conforming to specification limits, the understanding of loss functions is the guiding principle to assess the process capability. Therefore, for this reason the C_{pm} index can be used as a quality benchmark for acceptance of a production lot.

The probability of accepting the lot can be expressed as:

$$\begin{aligned}\pi_A(c) &= P(\hat{C}_{pm} \geq c_0 \,|\, C_{pm} = c) \\ &= \int_0^{b\sqrt{n}/3(c_0)} G\left(\frac{b^2 n}{9c_0^2} - t^2\right) \times (\phi(t + \xi\sqrt{n}) + \phi(t - \xi\sqrt{n}))dt\end{aligned},$$

where $b = d/\sigma$, $\xi = (\mu - T)/\sigma$, $G(\cdot)$ is the cumulative distribution function of the χ^2 distribution with degree of freedom n – 1, χ^2_{n-1}, and $\phi(\cdot)$ is the probability density function of the standard normal distribution $N(0,1)$. Therefore, the required inspection sample size n and critical acceptance value c_0 of $\hat{C}_{pm}$ for the sampling plans can be obtained by solving the following two nonlinear simultaneous equations:

$$S_1(n, c_0) = \int_0^{b_1\sqrt{n}/3(c_0)} G\left(\frac{b_1^2 n}{9c_0^2} - t^2\right) \times (\phi(t + \xi\sqrt{n}) + \phi(t - \xi\sqrt{n}))dt - (1 - \alpha),$$

$$S_2(n, c_0) = \int_0^{b_2\sqrt{n}/3(c_0)} G\left(\frac{b_2^2 n}{9c_0^2} - t^2\right) \times (\phi(t + \xi\sqrt{n}) + \phi(t - \xi\sqrt{n}))dt - \beta,$$

where $b_1 = 3C_{AQL}(1 + \xi^2)^{1/2}$ and $b_2 = 3C_{LTPD}(1 + \xi^2)^{1/2}$, $C_{AQL} > C_{LTPD}$. Here C_{AQL} and C_{LTPD} represent the capability requirements corresponding to the AQL and the LTPD based on C_{pm} index, respectively.

Pearn and Wu (2006) also tabulated the required sample size n and the critical acceptance value c_0 for various α-risks, β-risks, and the fraction of defectives of process that correspond to acceptable quality levels. Practitioners can determine the number of required inspection units and the critical acceptance value, and make reliable decisions.

2.3. Acceptance Sampling Plans Based on C_{pmk}

The index C_{pmk} is constructed by combining the yield-based index C_{pk} and the loss-based index C_{pm}, taking into account the process yield (meeting the manufacturing specifications) as well as the process loss (variation from the target). So, the C_{pmk} index is defined as

$$C_{pmk} = \min\left\{\frac{USL - \mu}{3\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{\mu - LSL}{3\sqrt{\sigma^2 + (\mu - T)^2}}\right\},$$

where USL is the upper specification limit, LSL is the lower specification limit, μ is the process mean, σ is the process standard deviation, and T is the target value.

When the process mean μ departs from the target value T, the reduced value of C_{pmk} is more significant than those of C_p, C_{pk}, and C_{pm}. Hence, the index C_{pmk} responds to the departure of the process mean μ from the target value T faster than the other three basic indices C_p, C_{pk}, and C_{pm}, while it remains sensitive to the changes of process variation (see Pearn and Kotz, 1994-1995). Thus, the index C_{pmk} indeed provides more quality assurance with respective to process yield and process loss to the customers than the two indices C_{pk} and C_{pm}.

According to today's modern quality improvement theory, reduction of the process loss is as important as the process yield, C_{pmk} can be used as a quality benchmark for acceptance of a product lot. Therefore, Wu and Pearn (2008) provided the acceptance sampling plan for C_{pmk} index.

The probability of accepting the lot can be expressed as:

$$\pi_A(c) = P(\hat{C}_{pmk} \geq c_0 \,|\, C_{pmk} = c)$$
$$= \int_0^{b\sqrt{n}/(1+3c_0)} G\left(\frac{(b\sqrt{n}-t)^2}{9c_0^2} - t^2\right) \times (\phi(t+\xi\sqrt{n}) + \phi(t-\xi\sqrt{n}))dt,$$

where $b = d/\sigma$, $\xi = (\mu - T)/\sigma$, $G(\cdot)$ is the cumulative distribution function of the chi-square distribution with degree of freedom n – 1, χ^2_{n-1}, and $\phi(\cdot)$ is the probability density function of the standard normal distribution $N(0,1)$. Therefore, the required inspection sample size n and critical acceptance value c_0 of $\hat{C}_{pmk}$ for the sampling plans can be obtained by solving the following two nonlinear simultaneous equations:

$$S_1(n, c_0) = \int_0^{b_1\sqrt{n}/(1+3c_0)} G\left(\frac{(b_1\sqrt{n}-t)^2}{9c_0^2} - t^2\right) \times (\phi(t+\xi\sqrt{n}) + \phi(t-\xi\sqrt{n}))dt - (1-\alpha),$$

$$S_2(n, c_0) = \int_0^{b_2\sqrt{n}/(1+3c_0)} G\left(\frac{(b_2\sqrt{n}-t)^2}{9c_0^2} - t^2\right) \times (\phi(t+\xi\sqrt{n}) + \phi(t-\xi\sqrt{n}))dt - \beta,$$

where $b_1 = 3C_{AQL}(1+\xi^2)^{1/2} + |\xi|$ and $b_2 = 3C_{LTPD}(1+\xi^2)^{1/2} + |\xi|$, $C_{AQL} > C_{LTPD}$. Here C_{AQL} and C_{LTPD} represent the capability requirements corresponding to the AQL and the LTPD based on C_{pmk} index, respectively.

Wu and Pearn (2008) developed a method of acceptance sampling plan for obtaining the required sample size for inspection and the corresponding critical acceptance values based on the exact sampling distribution, which provides the desired levels of protection for both producers and consumers.

Chapter 3. Process Capability Indices

3.1. Process Capability Indices and Product Quality

Process yield is the most common and standard criteria used in the manufacturing industries for judging process performance. Process yield is currently defined as the percentage of processed product unit passing inspection. That is, the product characteristic must fall within the manufacturing tolerance. For processes with high yield, it produces few percentages of non-conforming products. That is, most of the products produced in this process satisfy the requirement of specifications. Enterprises get more profit and cost down with high process yield, hence companies make their efforts to increase the process yield. Thus, the connections between the capability indices and the process yield are important. However, none of the above indices can provide good enough measure on the production yield. To overcome this shortage, Boyles (1994) proposed a yield index referred to as S_{pk} for normally distributed processes. The index S_{pk} is defined as below, where $\Phi(\cdot)$ is the cumulative distribution function (CDF) of the standard normal distribution, and $\Phi^{-1}(\cdot)$ is the inverse function of $\Phi(\cdot)$:

$$S_{pk} = \frac{1}{3}\Phi^{-1}\left\{\frac{1}{2}\Phi\left(\frac{USL-\mu}{\sigma}\right)+\frac{1}{2}\Phi\left(\frac{\mu-LSL}{\sigma}\right)\right\}.$$

To estimate the yield measure S_{pk}, we consider the following natural estimator $\hat{S}_{pk}$, involving the statistics $\bar{x}=\sum_{i=1}^{n} x_i/n$, and $s=[\sum_{i=1}^{n}(x_i-\bar{x})^2/(n-1)]^{1/2}$ are the sample mean and the sample standard deviation being the conventional estimators of μ and σ, respectively, obtained from a well-controlled process. The estimator is evidently

$$\hat{S}_{pk} = \frac{1}{3}\Phi^{-1}\left\{\frac{1}{2}\Phi\left(\frac{USL-\bar{x}}{s}\right)+\frac{1}{2}\Phi\left(\frac{\bar{x}-LSL}{s}\right)\right\}.$$

The index C_p measures the overall process variation (process potential or process precision) relative to the specification tolerance, therefore it only reflects the consistency of the product quality characteristic. The index C_{pk} takes into account the magnitude of process variation as well as the departures of process mean from the mid-point of specification limits. However, it can only provide approximate measure on the production yield. The index C_{pm} emphasizes measuring the ability of process to cluster around the target, which reflects the degrees of process loss. The index C_{pmk} is constructed by combining the yield-based index C_{pk} and the loss-based index C_{pm}, taking into account the process yield (meeting the manufacturing specifications) as well as the process loss (variation from the target). Thus, the index C_{pmk} indeed provides more quality assurance with respective to process yield and process loss to the customers than the two indices C_{pk} and C_{pm}. Unfortunately, the C_{pmk} index still can not provide an adequate measure on production yield. Only the index S_{pk} establishes a one-to-one relationship between the index value and the production yield. For normally distributed processes, the number of non-conformities corresponding to a capable process with S_{pk} =1.00 is 2700 ppm, a satisfactory process with S_{pk} =1.33 is 63 ppm, an excellent process with S_{pk} =1.67 is 0.6 ppm, and a super process with S_{pk} =2.00 is 0.002 ppm, as summarized in Table 1. Therefore, for a process with S_{pk} = *c*, the process yield can be expressed as $Yield = 2\Phi(3S_{pk}) - 1$. Obviously, there is a one-to-one relationship between S_{pk} and the process yield. Thus, the yield index S_{pk} provides an exact measure of the process yield.

Table 1. Some minimum capability requirements of S_{pk} .

S_{pk}	Process yield	Production process types
1.00	0.997300204	Capable process
1.33	0.999933927	Satisfactory process
1.67	0.999999456	Excellent process
2.00	0.999999998	Super process

3.2. Processes with Multiple Characteristics

The mentioned indices C_p , C_{pk} , C_{pm} , C_{pmk} and S_{pk} are appropriate to be used for processes with a single characteristic. Often, a manufactured product is described in multiple characteristics. That is, manufactured items require values of several different characteristics for adequate description of their quality. However, capability measurement for processes with multiple characteristics is comparatively neglected. For processes with multiple characteristics, Chen *et al.* (2003) propose the following capability index, which is referred to as S_{pk}^T :

$$S_{pk}^T = \frac{1}{3}\Phi^{-1}\left\{\left[\prod_{j=1}^{v}(2\Phi(3S_{pkj}) - 1) + 1\right] \Big/ 2\right\},$$

where S_{pkj} denotes the S_{pk} value of the *j*th characteristic for *j*=1,2,…,*v*, and *v* is the number of characteristics. The index S_{pk}^T can be viewed as a generalization of the single characteristic yield index S_{pk} , proposed by Boyles (1994). The index provides an exact measure of the overall process yield while the characteristics are mutually independent and normally distributed. A one-to-one correspondence relationship between the index S_{pk}^T and the overall production yield can be expressed as follows:

$$Yield = \prod_{j=1}^{v}\left[2\Phi(3S_{pkj}) - 1\right] = 2\Phi(3S_{pk}^T) - 1 .$$

For example, if the capability value of a process is S_{pk}^T = 1.00, then the entire process yield is exactly 0.997300204. Table 2 displays the corresponding production yields as well as non-conformities in parts per million (PPM) for $S_{pk}^T = 1.0(0.05)2.0$, including the some commonly used performance requirements: 1.00, 1.33, 1.50, 1.67 and 2.00.

Table 2. Various S_{pk}^T values and the corresponding process yield.

S_{pk}^T	*Yield*	PPM	S_{pk}^T	*Yield*	PPM	S_{pk}^T	*Yield*	PPM
1.00	0.9973002	2699.79	1.35	0.99994878	51.21	1.70	0.99999966	0.340
1.05	0.9983672	1632.70	1.40	0.99997330	26.69	1.75	0.99999984	0.152
1.10	0.9990331	966.848	1.45	0.99998638	13.61	1.80	0.99999993	0.067
1.15	0.9994394	560.587	1.50	0.99999320	6.795	1.85	0.99999997	0.029
1.20	0.9996817	318.217	1.55	0.99999668	3.319	1.90	0.99999998	0.012
1.25	0.9998231	176.835	1.60	0.99999841	1.587	1.95	0.99999999	0.005
1.30	0.9999038	96.193	1.65	0.99999925	0.742	2.00	0.99999999	0.002
1.33	0.9999339	66.073	1.67	0.99999945	0.544			

3.3. Estimation of S_{pk}^T and Its Sampling Distribution

In order to handle the issue for cases with multiple quality characteristics, Pearn *et al.* (2006) derived the asymptotic distribution for an estimator of S_{pk}^T. The natural estimator $\hat{S}_{pk}^T$ is defined as

$$\hat{S}_{pk}^T = \frac{1}{3}\Phi^{-1}\left\{\left[\prod_{j=1}^{v}(2\Phi(3\hat{S}_{pkj})-1)+1\right]\Big/2\right\},$$

where $\hat{S}_{pkj}$ denotes the estimator of S_{pkj}, and all $\hat{S}_{pkj}$ s are mutually independent. Consequently, the distribution of $\hat{S}_{pk}^T$ can be shown as an asymptotic normal distribution as

$$\hat{S}_{pk}^T \dot{\sim} N\left(S_{pk}^T, \frac{1}{36n(\phi(3S_{pk}^T))^2}\left(\sum_{j=1}^{v}\left\{(a_j^2+b_j^2)\left[\frac{\prod_{i=1}^{v}(2\Phi(3S_{pki})-1)^2}{(2\Phi(3S_{pkj})-1)^2}\right]\right\}\right)\right),$$

where

$$a_j = \frac{1}{\sqrt{2}}\left\{\frac{1-C_{drj}}{C_{dpj}}\phi\left(\frac{1-C_{drj}}{C_{dpj}}\right)+\frac{1+C_{drj}}{C_{dpj}}\phi\left(\frac{1+C_{drj}}{C_{dpj}}\right)\right\},\quad b_j = \phi\left(\frac{1-C_{drj}}{C_{dpj}}\right)-\phi\left(\frac{1+C_{drj}}{C_{dpj}}\right),$$

$$C_{drj} = \frac{(\mu_j - m_j)}{d_j},\quad C_{dpj} = \frac{\sigma_j}{d_j},$$

$m_j = (USL_j + LSL_j)/2$ is the midpoint of the specification limits of the *j*th characteristic, $d_j = (USL_j - LSL_j)/2$ is the half length of the specification interval the *j*th characteristic, and $\phi(\cdot)$ is the probability density function (PDF) of the standard normal distribution.

It should be noted that the asymptotic variance of $\hat{S}_{pk}^T$ is difference when we have the same S_{pk}^T. Table 3 shows a few examples for processes with three measured characteristics for S_{pk}^T =1. This is an undesirable consequence. To overcome this, we perform extensive calculation to fine out the largest variance for a fixed S_{pk}^T. The results of our calculation shows that (i) for a fixed S_{pk}^T, variance of $\hat{S}_{pk}^T$ is maximal at S_{pki} = S_{pk}^T and $S_{pkj} = \infty$, where $j \neq i$, (also variance of $\hat{S}_{pk}^T$ is minimal while all v S_{pkj} are equal); (ii) for fixed S_{pkj}, where j = 1, …, v, the variance of $\hat{S}_{pk}^T$ reaches its maximum at b_j = 0, i.e. the mean vector is on-center. Hence, in the calculation of critical value or lower confidence bound of S_{pk}^T, we will set $S_{pki} = S_{pk}^T$ and $S_{pkj} = \infty$, for all $j \neq i$, $a_j = \sqrt{2}(3S_{pkj})\phi(3S_{pkj})$, and $b_j = 0$ for all $j = 1, \ldots, v$.

We note that with the above parameter settings, the sampling distribution of S_{pk}^T can be rewritten in a shorter and simpler form:

$$\hat{S}_{pk}^T \dot{\sim} N\left(S_{pk}^T, \frac{\left(S_{pk}^T\right)^2}{2n}\right).$$

Table 3. Combinations of the parameters and the corresponding $nVar(\hat{S}_{pk}^T)$ for $S_{pk}^T = 1$.

S_{pk1}	S_{pk2}	S_{pk3}	a_1	a_2	a_3	b_1	b_2	b_3	$nVar(\hat{S}_{pk}^T)$
1.10661	1.10661	1.10661	0.00757	0.00757	0.00757	0.00000	0.00000	0.00000	0.242496
1.10661	1.10661	1.10661	0.00754	0.00754	0.00754	0.00071	0.00071	0.00071	0.242483
1.10661	1.10661	1.10661	0.00749	0.00749	0.00749	0.00112	0.00112	0.00112	0.242413
2.10661	1.10661	1.04043	0.00000	0.00757	0.01350	0.00000	0.00000	0.00000	0.337989
2.10661	1.10661	1.04043	0.00000	0.00754	0.01344	0.00000	0.00071	0.00126	0.337973
2.10661	1.10661	1.04043	0.00000	0.00749	0.01335	0.00000	0.00112	0.00198	0.337890
2.10661	2.10661	1.00000	0.00000	0.00000	0.01880	0.00000	0.00000	0.00000	0.500000
2.10661	2.10661	1.00000	0.00000	0.00000	0.01872	0.00000	0.00000	0.00175	0.499981
2.10661	2.10661	1.00000	0.00000	0.00000	0.01860	0.00000	0.00000	0.00275	0.499880

Thus, in the following sections we will use the simpler form of the distribution of $\hat{S}_{pk}^T$ to propose an acceptance sampling plan based on S_{pk}^T for processes with multiple characteristics. In this way, the level of confidence can be ensured, and the decisions made based on such an approach are indeed more reliable.

Chapter 4. Variables Sampling Plans for S_{pk}^T

4.1. Designing S_{pk}^T Variables Sampling Plans

Consider an acceptance sampling plan by variables to control the lot or process fraction of nonconformities. Since the quality characteristic is variable, the lower specification limit (LSL) and the upper specification limit (USL) can be used to define the acceptable values of this parameter. According to today's modern quality improvement theory, the manufactured product involves more than one quality characteristic is quite common. That is, manufactured items require values of several different characteristics for adequate description of their quality. Therefore, the S_{pk}^T index can be used as a quality benchmark for acceptance of a product lot. Thus, design a variables sampling plan with a specified OC curve is easy. Let (AQL, $1-\alpha$) and (LTPD, β) be the two points on the OC curve of interest.

As indicated earlier, the index S_{pk} establishes a relationship between the manufacturing specifications and the actual process performance, which provides an exact measure of the process yield. Considering processes with multiple characteristics, the S_{pk}^T index is one-to-one correspondence relationship between the index and overall process yield. So, the S_{pk}^T index provides an exact measure of the overall process yield when the characteristics are mutually independent. Therefore, the S_{pk}^T index can be used as a quality benchmark for product acceptance. The concept of the new variables sampling plan may be constructed as

If $S_{pk}^T \geq S_{AQL}$, then the lot should be accepted with producer's risk α, and

If $S_{pk}^T \leq S_{LTPD}$, then the lot should be rejected with consumer's risk β,

where S_{AQL} and S_{LTPD} represent the capability requirements corresponding to the AQL and the LTPD based on S_{pk}^T index, respectively.

A well-designed sampling plan must provide a probability of at least $1-\alpha$ of accepting a lot if the lot fraction of defectives is at the contracted AQL. The sampling plan must also provide a probability of acceptance no more than β if the lot fraction of defectives is at the LTPD level, the designated undesired level preset by the consumer. Therefore, the required inspection sample size n and critical acceptance value c_0 for the sampling plans are the solution to the following two nonlinear simultaneous equations.

$$\Pr\{\text{Accepting the lot} \mid p = \text{AQL}\} \geq 1-\alpha, \tag{1}$$

$$\Pr\{\text{Accepting the lot} \mid p = \text{LTPD}\} \leq \beta. \tag{2}$$

As described earlier, the sampling asymptotic distribution of $\hat{S}_{pk}^T$ is normally distributed with mean S_{pk}^T and variance $(S_{pk}^T)^2/2n$. The approximate probability of accepting the lot can be expressed as:

$$\pi_A(c) = P(\hat{S}_{pk}^T \geq c_0 \mid S_{pk}^T = c)$$
$$= \int_{c_0}^{\infty} \frac{\sqrt{n}}{c\sqrt{\pi}} \exp\left[-\frac{n(x-c)^2}{c^2}\right] dx \tag{3}$$

Therefore, the required inspection sample size n and critical acceptance value c_0 of $\hat{S}_{pk}^T$ for the sampling plans can be obtained by solving the following two nonlinear simultaneous equations (4) and (5).

$$1-\alpha \leq \int_{c_0}^{\infty} \frac{\sqrt{n}}{S_{AQL}\sqrt{\pi}} \exp\left[-\frac{n(x-S_{AQL})^2}{S_{AQL}^2}\right] dx \tag{4}$$

$$\beta \geq \int_{c_0}^{\infty} \frac{\sqrt{n}}{S_{LTPD}\sqrt{\pi}} \exp\left[-\frac{n(x-S_{LTPD})^2}{S_{LTPD}^2}\right] dx \tag{5}$$

where $S_{AQL} > S_{LTPD}$. We note that the required sample size n is the smallest possible value of n satisfying equations (4) and (5), and determining the $[n]$ as sample size, where $[n]$ is the least integer greater than or equal to n.

4.2. Solving Nonlinear Simultaneous Equations

In order to solve the above two nonlinear simultaneous Equations (4) and (5), we let

$$S_1(n,c_0) = \int_{c_0}^{\infty} \frac{\sqrt{n}}{S_{AQL}\sqrt{\pi}} \exp\left[-\frac{n(x-S_{AQL})^2}{S_{AQL}^2}\right] dx - (1-\alpha)\,, \tag{6}$$

$$S_2(n,c_0) = \int_{c_0}^{\infty} \frac{\sqrt{n}}{S_{LTPD}\sqrt{\pi}} \exp\left[-\frac{n(x-S_{LTPD})^2}{S_{LTPD}^2}\right] dx - \beta\,. \tag{7}$$

For S_{AQL}=1.33 and S_{LTPD}=1.00, Figures 2(a)-(b) and Figures 3(a)-(b) display the surface and contour plots of Equations (6) and (7), respectively, with α-risk=0.05 and β-risk=0.10. Figures 4(a)-(b) display the surface and contour plots of Equations (6) and (7) simultaneously with α-risk =0.05 and β-risk =0.10 under S_{AQL}=1.33 and S_{LTPD}=1.00, respectively.

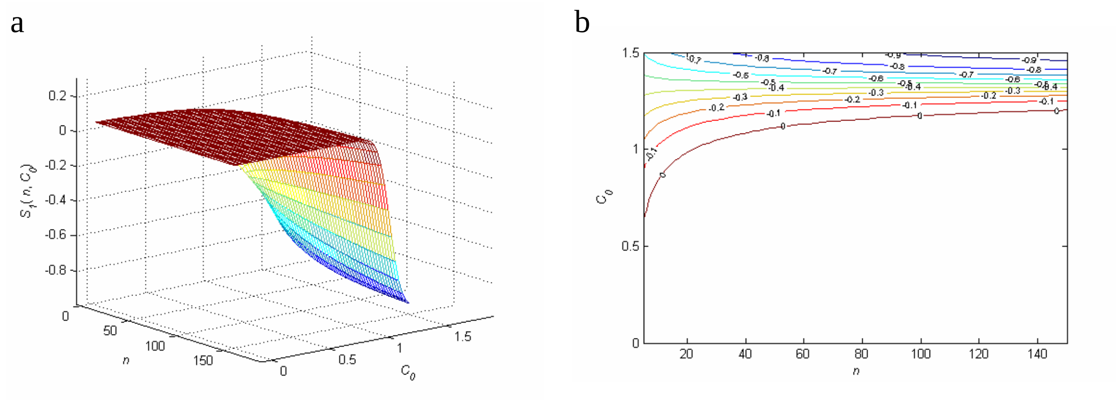

Figure 2. (a) Surface plot of $S_1(n, c_0)$. (b) Contour plot of $S_1(n, c_0)$.

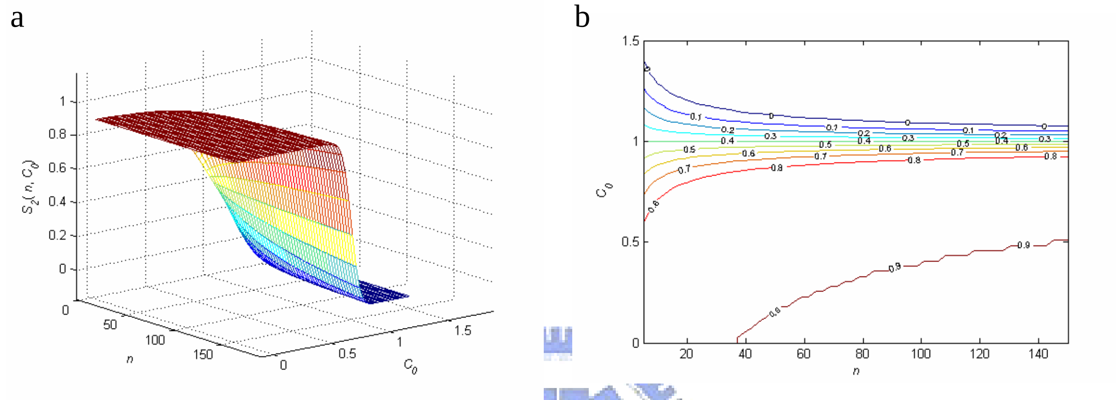

Figure 3. (a) Surface plot of $S_2(n, c_0)$. (b) Contour plot of $S_2(n, c_0)$.

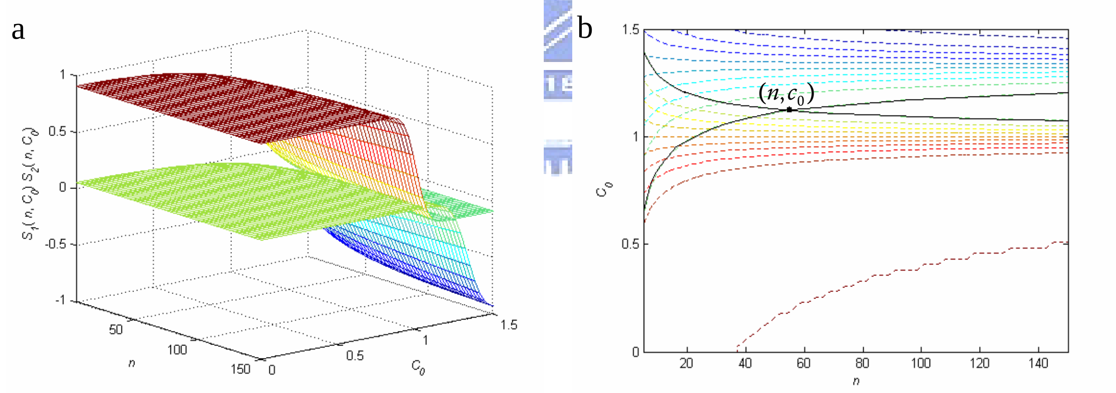

Figure 4. (a) Surface plot of S_1 and S_2. (b) Contour plot of S_1 and S_2.

From Figure 4(b), we can see that the intersection of $S_1(n, c_0)$ and $S_2(n, c_0)$ contour curves at level 0 is $(n, c_0) =$ (56, 1.1219), which is uses MATLAB software to solution the nonlinear simultaneous equations (4) and (5). That is, in this case, the minimum required sample size n=56 and critical acceptance value c_0=1.1219 of the sampling plan based on the capability index S_{pk}^T.

To investigate the behavior of the critical acceptance values, required sample sizes with various parameters and practical application purposes, we perform extensive calculations to obtain the solution of (4) and (5) and tabulate the critical acceptance values (c_0) and required sample sizes (n) for the sampling plans, with commonly used α, β, S_{AQL} and S_{LTPD}. Table 4 displays (n, c_0) values for producer's α-risk=0.01, 0.025, 0.05, 0.075, 0.10 and consumer's β-risk=0.01, 0.025, 0.05, 0.075, 0.10, with various benchmarking quality levels, (S_{AQL}, S_{LTPD})=(1.33, 1.00), (1.50, 1.00), (1.50, 1.33), (1.67, 1.33), (1.67, 1.50), (2.00, 1.67). For example, if the benchmarking quality level (S_{AQL}, S_{LTPD}) is set to (1.33, 1.00) with producer's α-risk =0.01 and consumer's β-risk =0.05, then the corresponding sample size and critical acceptance value can be obtained as (n, c_0)=(104, 1.1145). That is, the lot will be accepted if the 104 inspected product items yield measurements with $\hat{S}_{pk}^T \geq 1.1145$.

From the table results, we observe that the greater of the risk (α and/or β) which producer or customer could suffer, the smaller is the required sample size n. This phenomenon can be interpreted intuitively, as if we expect that the chance of wrongly concluding a bad process as good or good lots as bad ones is smaller, the more sample information need to judge the lots. Further, for fixed α-risk, β-risk and S_{LTPD}, the required sample sizes become smaller when the S_{AQL} becomes larger. This can also be explained by the same reasoning, as the judgment will be more correct with a larger value of difference between the S_{AQL} and S_{LTPD}.

4.3. Sampling Procedure and Decision Making

Both producer and consumer will lay down their requirements in the contract: the producer demands that not too many 'good' lots shall be rejected by the sampling inspection, and consumer demands that not too many 'bad' lots shall be accepted. Therefore, selection of a meaningful critical value for capability test requires specification of an acceptable quality level (AQL) and a lot tolerance percent defective (LTPD) for the S_{pk}^T value. The AQL is simply a standard against which to judge the lots. It is hoped that the producer's process will operate at a fallout level that is considerably better than the AQL. In choosing a sampling plan attempts will be made to meet these somewhat opposing requirements. Thus, both producers and consumers may set their own safeguard line to protect their benefits.

In order to judge whether a given process meets the capability requirement, the first step is determine the specified value of the capability requirement S_{AQL} and S_{LTPD} (or fraction of defectives AQL and LTPD), and the α-risk, β-risk. Two kinds of risks are balanced using a well-designed sampling plan. That is, if production process capability with $S_{pk}^T = S_{AQL}$ (in high quality), the probability of acceptance must be larger than $1-\alpha$. And if the producer's capability is only with $S_{pk}^T = S_{LTPD}$ (in low quality), consumer would accept no more than β. Then, by checking Table 4, we would obtain the sample size n and the critical value c_0 based on given values of α-risk, β-risk, S_{AQL} and S_{LTPD}. If the estimated S_{pk}^T value is greater than the critical value c_0, then the consumer will accept the entire product. Otherwise, we do not have sufficient information to conclude that the process meets the present capability requirement. In this situation, the consumer will reject the product.

For the proposed sampling plan to be practical and convenient to use, a step-by-step procedure is provided as below.

Step 1: Decide the process capability requirements (i.e. set the values of S_{AQL} and S_{LTPD}), and choose the α-risk, the chance of wrongly concluding a capable process as incapable, and the β-risk, the chance of wrongly concluding a bad lot as good one.

Step 2: Check Table 4 to find the critical value (or acceptance criterion) and the required number of product units for inspection, (n, c_0), based on given values of α-risk, β-risk, S_{AQL} and S_{LTPD}.

Step 3: Calculate the value of $\hat{S}_{pk}^{T}$ (sample estimator) from these n inspected samples.

Step 4: Make decisions to accept the entire lot if the estimated $\hat{S}_{pk}^{T}$ value is greater than the critical value c_0 ($\hat{S}_{pk}^{T} > c_0$). Otherwise, we reject the entire products.

Table 4. Required sample sizes (n) and critical acceptance values (c_0) for various α-risk and β-risk with selected (S_{AQL}, S_{LTPD}).

α	β	$S_{AQL}=1.33$ $S_{LTPD}=1.00$		$S_{AQL}=1.50$ $S_{LTPD}=1.00$		$S_{AQL}=1.50$ $S_{LTPD}=1.33$		$S_{AQL}=1.67$ $S_{LTPD}=1.33$		$S_{AQL}=1.67$ $S_{LTPD}=1.50$		$S_{AQL}=2.00$ $S_{LTPD}=1.67$	
		n	c_0	n	c_0	n	c_0	n	c_0	n	c_0	n	c_0
	0.010	135	1.1416	68	1.2000	750	1.4099	211	1.4807	941	1.5804	335	1.8202
	0.025	118	1.1280	60	1.1798	643	1.4027	183	1.4665	806	1.5732	289	1.8063
0.010	0.050	104	1.1145	53	1.1602	558	1.3955	160	1.4525	699	1.5660	252	1.7925
	0.075	95	1.1048	49	1.1460	506	1.3902	146	1.4422	633	1.5607	229	1.7824
	0.100	88	1.0967	46	1.1343	467	1.3858	136	1.4337	584	1.5563	212	1.7740
	0.010	112	1.1556	56	1.2209	630	1.4172	176	1.4952	792	1.5877	280	1.8343
	0.025	96	1.1416	49	1.2000	533	1.4099	150	1.4807	668	1.5804	238	1.8202
0.025	0.050	83	1.1277	43	1.1794	455	1.4025	129	1.4662	571	1.5731	205	1.8060
	0.075	76	1.1174	39	1.1643	408	1.3970	117	1.4555	511	1.5676	184	1.7954
	0.100	70	1.1088	36	1.1518	374	1.3924	108	1.4464	468	1.5629	169	1.7865
	0.010	94	1.1701	46	1.2427	536	1.4246	148	1.5101	673	1.5951	237	1.8487
	0.025	79	1.1559	40	1.2214	446	1.4173	124	1.4955	560	1.5879	198	1.8346
0.050	0.050	68	1.1416	34	1.2000	375	1.4099	106	1.4807	471	1.5804	168	1.8202
	0.075	61	1.1310	31	1.1842	333	1.4043	94	1.4696	417	1.5748	149	1.8093
	0.100	56	1.1219	29	1.1709	302	1.3995	86	1.4602	378	1.5700	136	1.8001
	0.010	83	1.1810	41	1.2593	478	1.4301	131	1.5213	601	1.6007	211	1.8595
	0.025	69	1.1669	34	1.2379	393	1.4230	109	1.5069	495	1.5935	174	1.8456
0.075	0.050	59	1.1525	29	1.2162	327	1.4156	92	1.4920	411	1.5861	146	1.8311
	0.075	52	1.1416	26	1.2000	288	1.4099	81	1.4807	361	1.5804	129	1.8202
	0.100	47	1.1323	24	1.1862	259	1.4050	74	1.4711	324	1.5755	116	1.8107
	0.010	75	1.1905	37	1.2738	436	1.4349	119	1.5310	549	1.6054	191	1.8688
	0.025	62	1.1765	31	1.2524	355	1.4278	98	1.5167	447	1.5984	157	1.8551
0.100	0.050	52	1.1621	26	1.2306	293	1.4205	82	1.5019	368	1.5910	130	1.8407
	0.075	46	1.1511	23	1.2141	255	1.4148	72	1.4905	320	1.5854	114	1.8297
	0.100	41	1.1416	21	1.2000	228	1.4099	64	1.4807	286	1.5804	102	1.8202

Chapter 5. An Application

Silicon photodiodes are semiconductor devices used for the detection of light in ultraviolet, visible and infrared spectral regions. Because of their small size, low noise, high speed and good spectral response, silicon photodiodes are being used for both civilian and defense related applications. Depending on the requirement of any particular application, photodiodes can be made in any desired geometry, and provided in a special package with a filter for any special application such as Mouse, Remote control, Receiver module, Wireless communication, etc. Figure 5 shows a particular chip of silicon photodiodes.

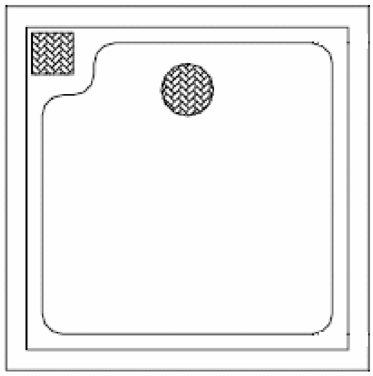

Figure 5. A silicon photodiode chip.

Silicon photodiodes are constructed from single crystal silicon wafers similar to those used in the manufacture of integrated circuits. The major difference is that photodiodes require higher purity silicon. A cross section of a typical silicon photodiode chip is shown in Figure 6. The bulk N-type silicon is the starting material. A thin "P" layer is formed on the front surface of the device by thermal diffusion or ion implantation of the appropriate doping material. The interface between the "P" layer and the "N" silicon is known as a P-N junction. Small metal contacts are applied to the front surface of the device and the entire back is coated with a contact metal. The back contact is the cathode, and the front contact is the anode. The active area is coated with either silicon nitride, silicon monoxide or silicon dioxide for protection and to serve as an anti-reflection coating. The thickness of this coating is optimized for particular irradiation wavelengths.

The following case is taken from a manufacturing factory located in a science-based industrial park at Hsinchu, Taiwan, making various types of silicon photodiode chips. The particular silicon photodiode chip we investigate has multiple concerned characteristics including the chip length (L), chip width (W), chip thickness (T) and P bonding pad (P). The product specification limits for the L, W, T and P characteristics of the silicon photodiode chip are set at (USL, LSL) = (35.984 mil, 34.016 mil), (35.984 mil, 34.016 mil), (12.784 mil, 10.816 mil) and (5.393 mil, 4.607 mil), respectively.

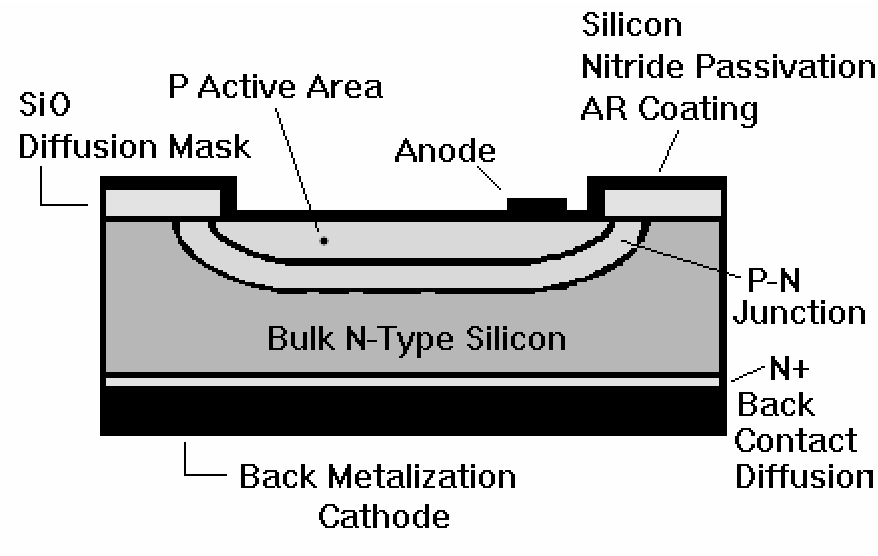

Figure 6. A cross section of a typical silicon photodiode chip.

Table 5. Sample data of the silicon photodiode chip characteristics.

chip length (*L*) (unit: mil)									
35.022	35.410	34.914	35.207	34.796	34.672	35.173	34.638	34.662	34.843
34.613	35.145	34.928	34.664	34.769	34.744	34.981	34.372	34.819	34.638
35.086	35.156	35.038	34.974	34.315	35.007	34.772	34.931	34.915	34.699
35.151	35.062	34.909	35.232	35.410	34.712	34.993	34.711	35.195	35.130
34.866	34.855	34.804	35.241	34.935	34.961	34.673	35.081	35.699	35.075
34.630	35.064	34.627	35.039	34.560	35.187	35.297	35.404	35.360	34.803
35.115	35.237	34.721	35.052	35.198	34.665	34.752	35.202		
chip width (*W*) (unit: mil)									
34.998	35.168	35.437	35.154	34.748	35.241	34.947	34.739	35.670	35.026
34.700	35.203	35.538	35.190	35.253	35.034	34.949	35.279	34.938	35.024
34.831	34.781	34.914	34.620	35.080	35.033	34.987	34.281	34.837	35.080
35.118	35.057	34.988	34.487	35.109	34.958	35.155	35.192	34.972	35.104
34.893	34.897	34.878	34.695	35.075	35.395	34.815	34.840	35.231	35.013
35.493	34.456	34.669	34.756	35.696	35.138	34.858	35.010	34.899	35.313
35.006	35.262	35.225	34.926	34.964	35.041	34.744	34.924		
chip thickness (*T*) (unit: mil)									
11.783	11.606	11.978	11.585	11.665	11.578	12.002	11.671	11.472	11.955
12.074	11.928	11.826	11.637	11.657	12.034	12.019	11.684	11.953	11.841
11.729	11.642	11.855	11.454	11.501	12.183	11.786	12.299	11.754	11.833
11.975	11.766	11.701	12.072	11.635	11.953	11.703	11.941	11.961	11.642
11.901	11.996	11.600	11.869	11.793	11.567	11.748	11.770	11.594	11.429
11.627	11.799	11.473	12.040	11.686	11.408	11.821	12.083	11.810	11.831
11.795	11.833	11.968	11.835	11.944	11.897	11.825	11.853		
P bonding pad (*P*) (unit: mil)									
4.882	4.887	4.998	5.043	4.893	5.097	5.013	5.328	5.049	4.843
5.046	5.060	4.939	5.028	4.928	5.002	5.050	5.143	5.093	4.683
5.034	5.099	4.999	5.103	5.093	5.157	5.148	5.115	4.801	4.881
5.082	4.883	4.927	5.082	5.002	5.128	4.839	5.058	4.804	4.827
5.035	4.983	4.864	4.965	4.930	4.892	5.030	4.821	5.038	4.757
5.063	5.041	5.091	4.917	5.082	4.871	5.108	4.745	5.034	4.912
4.907	5.018	4.960	5.116	4.987	5.122	4.943	5.008		

Once the characteristic data do not fall within the specification limits, the lifetime or reliability of the silicon photodiode will be discounted. In the contract, the performance requirement S_{AQL} and S_{LTPD} are set to be 1.33 and 1.00 with α-risk and β-risk both set to be 0.05. Then, the problem for the inspection practitioners is to determine the critical acceptance value and the required sample size for the sampling plan that provides desired levels of protection for both the producer and the consumer. Based on the proposed procedure and Table 4, the practitioners can acquire the critical acceptance value and inspected sample size as (n, c_0) = (68, 1.1416). The required samples for inspection are randomly taken, and the observations are displayed in Table 5.

Based on the observations, we calculate the sample estimate $\hat{S}_{pk}^{T}$ of S_{pk}^{T} as follows. Table 6 presents the sample average ($\bar{X}_j$), sample standard deviation (S_j) and $\hat{S}_{pkj}$ for each characteristic. Thus, we can obtain that

$$\hat{S}_{pk}^{T} = \frac{1}{3}\Phi^{-1}\left\{\left[\prod_{j=1}^{v}(2\Phi(3\hat{S}_{pkj})-1)+1\right]\Big/2\right\} = 1.0763.$$

Table 6. The average ($\bar{X}$), standard deviation (S) and $\hat{S}_{pk}$ of each silicon photodiode chip characteristics.

characteristics	$\bar{X}_j$	S_j	$\hat{S}_{pkj}$
chip length (L)	34.9487	0.2642	1.2202
chip width (W)	35.0136	0.2614	1.2531
chip thickness (T)	11.7960	0.1884	1.7405
P bonding pad (P)	4.9899	0.1170	1.1152

Therefore, in this case, the consumer would reject the entire lot, since the sample estimate $\hat{S}_{pk}^{T}$ = 1.0763 is smaller than the critical acceptance value 1.1416 of the sampling plan. The process yield is exactly 0.9988 for index $\hat{S}_{pk}^{T}$ = 1.0763. Note that for existing sampling plans, it is almost certain that any samples of 68 silicon photodiode chips will contain zero defective items. All the products therefore will be accepted, which obviously provide no protection to the consumer at all.

Chapter 6. Conclusion

Acceptance sampling plans are practical tools for quality assurance applications. It provides the producer and the consumer a decision rule for product sentencing to meet their needs. However, as the rapid advancement of the manufacturing technology and stringent customers demand is enforced, the manufactured items require values of several different characteristics for adequate description of their quality. The S_{pk}^{T} index measures the overall process yield when the processes with multiple characteristics. Therefore, in this paper, we developed a variables sampling plan based on the process capability index S_{pk}^{T} to deal with lot sentencing problem even when the lots or processes with multiple characteristics. We developed a method to determine the sample size required for inspection and the corresponding acceptance criterion, to provide the desired levels of protection to both producers and consumers. We tabulated the required sample size and the corresponding critical acceptance value for various producer's risks, the consumer's risks with the capability requirement AQL and the LTPD. The results obtained in this paper are useful to the practitioners in making reliable decisions.

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