國立交通大學

經營管理研究所

碩 士 論 文

風險值衡量：實現變幅的應用
Estimating Value at Risk with Realized Range

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經營管理研究所
碩士論文

A Thesis
Submitted to Institute of Business and Management
College of Management
National Chiao Tung University
in Partial Fulfillment of the Requirements
for the Degree of
Master of Business Administration

June 2009
Taipei, Taiwan, Republic of China

中華民國 九十八 年 六 月

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中文摘要

本篇論文將實現變幅（realized range）概念應用在風險值模型中，利用Martens and van Dijk (2007) 所提出的修正誤差方法，並使用MEM（Multiplicative Error Model）來預測下一期的波動性，得到實現變幅基礎下的風險值模型。此外，本研究也利用常態分配假設下的變異數-共變異數法（variance-covariance method），以及厚尾性質的極值理論（extreme value theory）兩種不同假設的風險值模型來一起做比較。在實證上，以標準普爾500（S&P 500）指數與那斯達克（Nasdaq）指數的高頻率資料作為研究對象，進行實現變幅、報酬與變幅基礎下的風險值模型在風險值的預測能力比較。實證結果顯示，以實現變幅為基礎下的風險值模型表現優於其他的風險值模型。

關鍵字：實現變幅、日內資料、風險值、極值理論、變幅、波動性

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ABSTRACT

This paper investigates the concept of realized range into the Value-at-Risk estimation. We follow the bias-correction method of Martens and van Dijk (2007) and use MEM model（Multiplicative Error Model）to forecast volatility and VaR estimation. In addition, we apply two different VaR methods to make the comparison: Variance-covariance method and Extreme value theory. In empirical research, we use the intra-day data of S&P 500 and Nasdaq Index to compare the forecast ability of VaR with realized range, daily return and daily range data. The comparing result shows that realized-range-based VaR model performs better than other models.

Keywords: Realized range, Intra-day data, Value at Risk, Extreme value theory, Range, Volatility

謝辭

兩年的研究所生涯過得很快，也即將邁向另一個階段。在這段期間，首先要感謝的是我的指導教授 周雨田老師，謝謝他對我的耐心教導，以及開啟我對風險管理的興趣；另外，老師也總是不厭其煩地聽我的論文報告，並總是很親切地表達他的關心。也感謝口試委員 丁承老師、高櫻芬老師及周恆志老師的建議和指教，讓這篇論文能夠更完整，在此，謹表達最深的感謝。

在論文撰寫過程中，要特別感謝炳麟學長，在軟體程式上給予很大的幫助與修正，並提供模型上的建議，讓我的論文過程更加順利。也要感謝研究所的伙伴們，尤其是周家班，以及我的「家人」們：曼慈、悅慈、振儀、泰佑、彥偉、錦宏，有你們的陪伴，讓我的研究所求學之路更加精彩；另外，也特別感激維苡、致宏和唯帆，一直很有義氣地解決我課業上的困難，耐心地回答我各式各樣的問題，讓我擁有了最佳的智囊團來面對不同的挑戰。

最後，謝謝父母親的支持與栽培，讓我無後顧之憂地完成我的學業；也感謝一直陪在我身邊的許多好友，分擔我的喜怒哀樂。謝謝你們！

陳昱如 謹誌

國立交通大學經營管理研究所

民國九十八年六月

TABLE OF CONTENTS

List of Tables

List of Figures

Ⅰ. Introduction

In recent ten years, a number of economies have been highly volatile financial markets, for example, the ‘dot-com’ bubble in 2000 and the subprime mortgage crisis during 2007. What happens to a country may affect the whole financial markets. That causes the increasing of price fluctuation and instability. Most investors care about their expected investment returns and the risk they bear. The financial institutions emphasize not only on the profit, but also on their ability to suffer large losses. Therefore, risk management becomes a popular and important issue for investors, financial institutions and regulators. How to establish a proper system to control the risk is one of the most important goals of academic research and the regulators. A major concern for regulators and owners of financial institutions has had two dimensions: the adequacy of minimum capital requirements as designed by the Basle Committee and the adoption of the Value-at-Risk (called VaR) method calculating market risks. That’s the reason why the VaR method turns into a main risk management technique to avoid the potential damage from bank runs and systematic risks.

VaR is a summary measure of downside risks expressed in percentage. VaR measures market risks by determining how much the value of a portfolio could fall with a given small probability as a result of changes in market prices over a fixed number of days. By Jorion’s (2007) definition, ”VaR is the maximum loss over a target horizon such that there is a low, pre-specified probability that the actual loss will be larger.” The Basle Committee establishes a standard rule of measuring VaR to supervise their members. VaR is simple to explain and is used by financial institutions extensively.

The development of VaR methods to evaluate and forecast the risk of unpredicted loss has moved rapidly. These methods are organized in two main classes: parametric prediction of conditional volatilities such as J. P. Morgan RiskMetrics method, and non–parametric prediction of unconditional volatilities like historical simulation method. There is no absolute answer that which VaR model is the best one. Moreover, some research has shown that financial returns tend to have fat-tailed distribution rather than normal distribution. Traditional VaR methods evaluate risks by normal distribution and that may cause undervaluation of true risks. As a result, the method of estimating tail index has been suggested to capture the true distribution of extreme low returns. Combining tail index with VaR models, the VaR-x model is conducted to measure the downside risks using tail index. In this paper, VaR-normal and VaR-x models are measured and compared by various conditional volatilities forecast. The details are discussed in the next section.

Much research has been devoted to forecasting and measuring volatility of asset returns. Volatility is an important factor in risk management. How to measure ex-post volatility for accurate volatility forecasts is a popular issue in financial research. Recently, there is much research about the use of high-frequency data for measuring volatility, called realized variance, the sum of squared intra-day returns. In theory, the realized volatility is more robust than the volatility measured by the squared daily return. In addition, Parkinson (1980) showed that the daily high-low range is five times more efficient than the squared daily return. In according to previous research, the use of the realized range, the sum of high-low ranges for intra-day intervals, is derived. Moreover, Martens and van Dijk (2007) suggested a bias-correction procedure, scaling the realized range with the average level of the daily range, to eliminate the effects of microstructure frictions. This scaling method can remove both upward biases caused by bid-ask bounce and downward biases as a result of infrequent trading. The realized range significantly improves over realized volatility, especially for the popular sampling frequencies of 5-min and 30-min. Although much work has been done to date, more studies need to be conducted to apply the advantage of realized range to other financial issue, like risk management.

The purpose of this study is to ascertain the excellence of measuring VaR by the scaled realized range as compared with VaR measured by the daily return and daily range. Meanwhile, we consider the fat-tailed character of financial returns and compare VaR models using normal distribution with student-t distribution. Empirical analysis of the S&P500 and the Nasdaq index confirm the advantage of the realized range. This topic is identified as being important to financial risk manager in providing them a more robust method to measure VaR and control the risk.

The reminder of this paper is organized as follows. Chapter 2 describes the reviewing of previous research. Chapter 3 introduces the design and method of the realized range and the competing models. In addition, two VaR models and the comparison methods are discussed in this part. Chapter 4 presents the empirical results for the S&P500 and the Nasdaq index. Finally, chapter 5 makes a conclusion.

Ⅱ. Previous Research

2.1. Volatility Models

Volatility plays an important role in financial research. Traditional econometric models assume that the variance is constant in sample period. In the late 19^{th} century, it has been well established that volatility is both time-varying and predictable. A model named Autoregressive Conditional Heteroscedasticity (called ARCH) is introduced by Engle (1982). These are serially uncorrelated processes with non-constant conditional variance. This model obtained the empirical support by US financial market. Bollerslev (1986) revised ARCH model by adding past conditional variances in the current conditional variance equation and proposed Generalized Autoregressive Conditional Heteroscedasticity (called GARCH) model. GARCH model makes a proper explanation about volatility cluster and has been applied in many financial markets. An overview of the ARCH-type models and a thorough survey of empirical research using financial data are introduced, see Bollerslev, Chou, and Kroner (1992). The advantage of ARCH family is its flexible model of the dynamics of volatilities and its ease of estimation.

In recent ten years, much research has been devoted to using high-frequency data to measure volatility. The sum of squared intra-day returns, named realized volatility, is illustrated by Andersen et al. (2001) and it has become a popular issue for estimating volatility. Realized variance is considered an unbiased and highly efficient estimator. Barndorff-Nielsen and Shephard (2002) presented that when the length of the intra-day intervals are close to zero, realized variance converges to the true integrated variance. However, in practical, there are some market microstructure effects such as bid-ask bounce distorting the accuracy of realized variance. Returns at very high frequencies contaminated by these noises become biased and inconsistent, see Hansen and Lunde (2006). Therefore, it is popular to construct realized variance at a moderate frequency, where the negative effect of noise is small enough to be ignored, but that doesn't lead to loss much of information. Much research has found the proper sampling frequency to strike a balance between the increasing accuracy of high frequencies and the market microstructure noises. Popular frequencies in empirical research are the 5-min and 30-min intervals, see Andersen et al. (2003). Furthermore, Lanne (2006) conducted Multiplicative Error Model (called MEM model) and the realized variance to forecast the realized volatility. In this paper, we use the MEM model to predict the realized range volatility on day $t+1$.

There is an alternative way to measure volatility using the difference between the maximum and minimum prices during a certain period. It has been known for a long time in statistics that range is an unbiased proxy of the volatility. Parkinson (1980) argued the superiority of using range as a volatility estimator as compared with return. The daily range, scaled properly, is an unbiased estimator of volatility and is five times more efficient than the squared daily return. Combining with the range and the time-varying property, Chou (2005) proposed the range-based volatility model: the Conditional Autoregressive Range model (called CARR model). By modeling the dynamics properly, daily range performs better than return-based proxy in forecasting volatility. This model belongs to the family of MEM model and is easy to estimate. For empirical result, CARR model can produce more robust volatility forecast than GARCH model.

Considering the use of intra-day data and high-low range, a new method of estimating volatility has been developed. Several researchers have studied the application of realized range, the sum of high-low ranges for intra-day frequencies. Christensen and Podolskij (2007) first derived the theoretical characters of the realized range. According to Parkinson (1980), the realized range is five times more efficient than the realized variance with the same sampling frequencies in theory. However, as same as realize variance, the realized range is damaged by the effects of market microstructure noise. This paper also presented the solution to the downward bias. Corrado and Truong (2007) showed that the intraday high-low range often provides more significant additional information than the GARCH model. Martens and van Dijk (2007) suggested a bias-correction procedure to the effects of microstructure frictions for both downward and upward biases by scaling the realized range with the average level of the daily range. In addition, from the simulation experiment and empirical research, realized range significantly dominates over realized variance for the popular frequencies of 5-min and 30-min.

2.2. VaR models

VaR models have been developed since the middle of 1990s. Because of the Basle Committee's (1995,1996) internal model approach, the number of VaR methods for such calculations has continued to rise. Popular VaR methods can be classified to four groups: historical simulation method, Monte Carlo simulation method, variance-covariance method and extreme value theory. Jorion (2000) gave a good overview of Value-at-Risk and introduced these four groups more detailed. Moreover, Engle and Manganelli (2004) proposed a new concept, conditional value at risk by quantile regression (called CAViaR), to solve the VaR's statistical problem. CAViaR model focuses on the behavior of quantile instead of the distribution of returns and uses regression quantile estimation to get the parameters of dynamic autoregressive process. CAViaR is a new method in risk management issue.

According to Jorion (2000), historical simulation method is one of the nonparametric methods. It assumes that the variation of future prices can be forecasted by actual past prices. VaR is obtained by sorting returns and picking the given confidence interval. The advantage of this method is that it's easy to measure and it doesn't need to make an assumption on return distribution. This is an improvement over the normal distribution because historical information contains fat tails. Its main disadvantage is that it may produce serious bias in small sample. In addition, if the market structure is different from the past, it may decrease the accuracy.

According to Jorion (2000), Monte Carlo simulation method belongs to parametric method. The movements in risk factors are generated from some prespecified distribution and financial returns can be simulated by this process. Then the returns are sorted to get the VaR. This method is the most flexible and can be used on all financial goods and all risks, including non-linear risks. The main drawback is its enormous computational cost. The users are required to make assumptions on the stochastic process and understand the relationship between risk factors and returns. Therefore, this method is subject to model risk.

The RiskMetrics VaR specification is developed by J.P. Morgan (1994) and is used into practice widely. RiskMetrics VaR model belongs to variance-covariance method and bases assumptions on normality of returns, independence of all observed data and a linear relation between asset prices and market variables. The RiskMetrics method is one of GARCH models and it uses exponentially weighted moving average (called EWMA) to forecast variance. EWMA model assumes that recent price volatility has larger impact on forecast of variance, so the weighted factor is given bigger. EWMA method is the core part of RiskMetrics VaR model. Last, as for extreme value theory, we'll discuss this method in next section.

Some literature has been devoted to applying univariate time series model to VaR estimations. A comparison of VaR specification using ARCH type models and realized volatility is conducted by Giot and Laurent (2004). Hartz, Mittnik, and Paolella (2006) developed a bias-correction method and used bootstrap to forecast precisely VaR estimates with normal-GARCH model.

How to evaluate various VaR models is also a popular issue. Many researchers have mentioned different points of view and proposed various criteria to compare, see Kupiec

(1995), Hendricks (1996), Christoffersen (1998), Lopez (1999), Neftci (2000), Christoffersen, Hahn, and Inoue (2001), Berkowitz and O'brien (2002). Moreover, Engel and Gizycki (1999) classified three dimensions of evaluating VaR models. In this paper, we follow the classified method to compare different VaR models.

2.3. Extreme Value Theory

Numerous articles have investigated the true distribution of financial returns so far. They proposed that the financial returns have the property of extreme value process and fat tails instead of normal distribution. Parkinson (1980) recognized that extreme value contains more useful information than traditional return-based data. Moreover, many researchers have conducted the extreme value behavior of stock market. Jansen and de Vries (1991) applied extreme theory in tail behavior of stock returns instead of considering whole distribution, and investigated the fatness of distribution tails. Longin (1996) showed that the behavior of extreme returns is useful to understand the whole price movements including booms and crashes. The distribution of extreme values is precisely known.

According to Longin (1996), the tail index is helpful to choose a proper model of returns, like normal distribution, student-t distribution, or ARCH process, etc. As the shape of the distributional tail is varied, different value of tail index is obtained. The fatter the distributional tail is, the larger the value of tail index is. In addition, the inverse of tail index estimation is defined as shape parameter, like the degree of freedom in student-t distribution. Some research has shown that Hill's estimator is a better method to estimate the value of tail index, see Longin (1996), Kearns and Pagan (1997). Hill (1975) proposed a simple approach to measure the behavior of a distributional heavy tail. This approach only requires understanding the form of tails instead of the whole distribution.

However, McNeil and Frey (2000) illustrated that Hill's estimator would encounter two problems. One is that Hill's estimator would result in some biases in small samples; the other is that it is difficult to decide proper number of observations to measure the tail index. This means that either long period or high-frequency data is required. In small samples, the overestimation of tail thickness is likely to occur. Because of this shortcoming, Huisman, Koedijk, Kool and Palm (2001) proposed a revised Hill's estimator to obtain tail index. In this method, the tail index is calculated by weighted average of some Hill estimates, which differ in the number of tail observations included. In practical, this research showed that the tail index is obtained reliably even in small samples. Moreover, Huisman et al. (1998) suggested the using of tail index with small samples to estimate risks.

Following the fat-tailed character of financial returns, much research has demonstrated that measuring VaR with assumption of normal distribution would underestimate the true risks. Duffie and Pan (1997) reported that under the fat-tailed character of market factors' probability, the variance-covariance method creates some problems about underestimation of risks. Danielsson and de Vries (2000) compared RiskMetrics model which assumes normal distribution with extreme value model, and found that the latter model forecasts more accurate risks at high confidence level, like 99%. Thus it can be seen that financial returns contain additional downside risks. Combining fat-tailed character with VaR model, Huisman et al. (1998) presented such a measure, VaR-x, to evaluate financial risks under the assumption of student-t distributed returns. First, estimate the value of tail index by using revised Hill's estimator. Then get the inverse of tail index, the degree of freedom of student-t distribution, and measure the value of risks. The accuracy of VaR-x estimates is proofed by the empirical research on US stocks and bonds. The VaR model with extreme value theory has become more and more popular.

Except for the fat-tailed issue, the heteroscedastic property of financial returns is concerned. Some literature has been conducted with the heteroscedasticity and heavy-tailed character of financial data. Pownall and Koedijk (1999) focused on the risks of financial crisis, the periods with additional downside risk to investors, and showed the advantage of using conditional VaR-x method to capture the nature of downside risk in financial tsunami. Moreover, unlike the using of original data in Pownall and Koedijk (1999), McNeil and Frey (2000) proposed the concept of using the residuals of GARCH model to measure the value of tail index. This procedure is better than methods which ignore the fat-tailed property or the stochastic nature of the volatility. In this paper, we integrate the conditional VaR-x model in Pownall and Koedijk (1999) with the concept of residuals of conditional volatility model in McNeil and Frey (2000) to measure the value of risks.

Ⅲ. Model

All models in this paper are discussed in this chapter. The first part describes all conditional volatility models. The second part introduces the extreme value theory and the estimation of tail index. The third part presents the measure of VaR. The final comparison results are shown in the last section.

3.1. Conditional Volatility Models

Two methods are used to measure Value-at-Risk commonly. One is non-parametric method, the other is parametric method. Historical simulation method is a popular procedure of non-parametric model. Past returns are used to forecast future returns instead of making assumptions on the distribution of returns. Parametric method includes variance-covariance method and extreme value method. In this research, we compare five conditional variance-covariance and conditional extreme value models. As a result, the conditional volatility models are discussed first as follows.

3.1.1. Realized range

Let P_t be the security price at time t. To measure one-day realized range, the sum of high-low ranges for intra-day intervals, we normalize the daily interval to unity. Then for the ith interval of length θ on day t, for i = 1,2,…I with $I = 1/\theta$, we define the high price $H_{t,i}$ = maximum price from $(t-1+(i-1)\theta)$ to $(t-1+i\theta)$ and the low price $L_{t,i}$ = minimum price between $(t-1+(i-1)\theta)$ and $(t-1+i\theta)$. An estimator of the so-called realized range is

$$RR_t^{\theta}=\sum_{i=1}^{I}(\ln H_{t,i}-\ln L_{t,i})^2 , \tag{1}$$

the sum of high-low ranges for intra-day intervals. The realized range has two advantages over the previous return or daily range procedures on volatility estimation. First, the realized range observes all data information, like open, close, high and low prices .Second, the high-low range is more efficient than the squared return in any (intra-day) intervals, see Parkinson (1980). However, the realized range is affected more seriously by microstructure noise. That makes the realized range become a biased estimator, like upward bias because of bid-ask bounce and downward bias in presence of infrequent trading. Martens and van Dijk (2007) proposed a bias-correcting method to eliminate the effects of microstructure noise by scaling the realized range with the ratio of the average level of the daily range and the average level of the realized range over the q previous trading days. Therefore, the scaled realized range is defined as below:

$$RR_{S,t}^{\theta} = \left(\frac{\sum_{l=1}^{q} RR_{t-1}}{\sum_{l=1}^{q} RR_{t-1}^{\theta}} \right) RR_{t}^{\theta}, \tag{2}$$

where $RR_t = \left(\ln H_t - \ln L_t\right)^2$, which means the squared daily range. The idea is derived as the daily range is a good estimator of volatility and not influenced by microstructure noise. Moreover, the average level of the daily squared range and the realized range vary over time. In this paper, the previous q=66 trading days are used to compute the scaled realized range, following Martens and van Dijk (2007).

Next, the econometric model for the forecast of the realized range is introduced. Lanne (2006) developed multiplicative error model (called MEM) with time-varying parameters to forecast the realized volatility. The realized volatility, RRV_t, with MEM(p, q) model is evolved as follows,

$$RRV_t = \tau_t \varepsilon_t, \quad t = 1, 2, \ldots, T, \tag{3}$$

where the conditional mean equation

$$\tau_t = \omega^R + \sum_{i=1}^{q} \alpha_i^R RRV_{t-i} + \sum_{j=1}^{p} \beta_j^R \tau_{t-j}, \tag{4}$$

and ε_t is a stochastic positive-valued error term with $E(\varepsilon_t | \pi_{t-1}) = 1$ with $\pi_{t-1} = \{RRV_{t-j}\, j \geq 0\}$. ω^R, α_i^R and β_j^R are the estimated coefficients and are all positive to ensure positivity of τ_t. The parameters ω^R, α_i^R and β_j^R represent the uncertainty in realized volatility, the short-term impact effect, and the long-term effect of shocks to the realized volatility, respectively. For the stationary process, the sum of the impact parameters is restricted smaller than 1 , $\sum_{i=1}^{q} \alpha_i^R + \sum_{j=1}^{p} \beta_j^R < 1$. This model is similar to the CARR model applied to the daily range data, see Chou (2005).

In the previous literature, various distributional assumptions on the error term in MEM model have been conducted. By Engle (2002), the constant quasi-maximum likelihood estimator is obtained under the assumptions on the error term to be exponentially distributed. Instead of assuming exponentially distributed, Lanne (2006) developed a mixture MEM model to forecast the realized volatility. This mixture model includes an error term following a mixture of gamma distributions. In this paper, we use MEM(1,1) model and follow Engle's assumption, the error term with exponentially distributed, to predict the realized range. We define the realized range volatility as the square root of the scaled realized range $\sqrt{RR_{S,t}^{\theta}}$. Let RRV_t be the realized range volatility, and the MEM(1,1) model is shown as follows,

$$RRV_t = \tau_t \varepsilon_t, \quad t = 1, 2, \ldots, T, \tag{5}$$

$$\tau_t = \omega^R + \alpha_1^R RRV_{t-1} + \beta_1^R \tau_{t-1} \quad . \tag{6}$$

The parameters (ω^R, α_i^R and β_j^R) are in the same meaning as discussed before and the realized volatility here stands for the realized range volatility ($\sqrt{RR_{S,t}^{\theta}}$). The sum of impact parameters, $\alpha_1^R + \beta_1^R$, represents the persistence of the square root of the scaled realized range shocks. Also, in order to assure the stationary condition, the restriction, $\alpha_1^R + \beta_1^R$ <1, is imposed. In empirical study, the 5-min and 30-min interval sample length (called RR_5m and RR_30m model, respectively) are observed in this paper.[1]

3.1.2. EWMA model

RiskMetrics approach was established by J.P. Morgan to measure the risk. It is a special case of a normal Integrated GARCH (1,1) model where the variance ξ^2 are forecasted by EWMA model and the sum of the coefficients are set to be 1. In this specification, EWMA model is defined as:

$$\xi_t^2 = (1-\lambda)\varepsilon_{t-1}^2 + \lambda\xi_{t-1}^2 \quad , \tag{7}$$

where λ is the decay factor. Based on the difference between daily data and weekly data, λ is suggested to be a prespecified value of 0.94 and 0.97 respectively. Because of the comparison of the 1-day VaR in this paper, λ is equal to 0.94. Therefore, the EWMA model does not require estimation unknown parameters in the volatility equation. Although it is not a flexible model, it is easy to measure and often gives acceptable forecast value for the short-term volatility. Nowadays, the RiskMetrics model is used widely in practical.

3.1.3. GARCH model

When a series of asset returns are known to be heteroscedastic, it is better to use the conditional model to forecast volatility. The generalized autoregressive conditionally

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[1] Notice that in empirical research, the data with the first order moment is conducted when using the MEM model to evaluate volatility. As a result, the forecast value (τ_t) need to be adjusted. The adjusted forecast value of the realized range volatility is $\tau_{t,adj} = adj \times \tau_t$, with $adj = \frac{\bar{\sigma}}{\bar{\hat{\tau}}}$. $\bar{\sigma}$ is defined as the unconditional standard errors of returns and $\bar{\hat{\tau}}$ is the sample mean of the realized range volatility forecast. We use the adjusted series $(\tau_{1,adj}, ..., \tau_{t,adj})$ to be the proxy of the conditional volatility to measure Value-at-Risk.

heteroscedastic (called GARCH) model developed by Bollerslev (1986) allows the conditional variance to be dependent on previous value of the squared errors and previous own lags, so the conditional variance equation in GARCH(p, q) is now

$$X_t = \mu_t + \varepsilon_t \qquad \varepsilon_t \left| \mathrm{I}_{t-1} \sim N(0, \sigma_t^2) \right. \quad , \tag{8}$$

$$\sigma_t^2 = \omega^G + \sum_{i=1}^{p} \alpha_i{}^G \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j{}^G \sigma_{t-j}^2 \quad , \tag{9}$$

where X_t means the daily return on day t , μ_t is known as the conditional mean and σ_t^2 is the conditional variance. For GARCH(p, q) model, all coefficients would be required to be non-negative. The unconditional variance of μ_t can be defined under $\sum_{i=1}^{q} \alpha_i{}^G + \sum_{j=1}^{p} \beta_j{}^G < 1$, and this is a stationary process. The parameters (ω^G , $\alpha_i{}^G$ and $\beta_j{}^G$) characterize the uncertainty in conditional variance, the short-term impact effect, and the long-term effect of shocks to the conditional variance, respectively. These parameters are estimated by maximum likelihood method. GARCH model is more flexible than the Riskmetrics model. From the precious literature, GARCH(1,1) model can capture the property of the returns and make an accurate estimation. Therefore, GARCH(1,1) is used in this paper to forecast the conditional variance, and measure the Value-at-Risk.

3.1.4. CARR model

Instead of the returns, the Conditional Autoregressive Range Model (called CARR model) is a dynamic model for the high-low range of asset prices within fixed time intervals, see Chou (2005). This model is similar to the GARCH models by using the square root of the range without a constant term in the mean equation, and it belongs to the family of MEM models, used to evaluate the realized range above. The CARR(p, q) model is specified as

$$R_t = \varphi_t \varepsilon_t \quad , \tag{10}$$

$$\varphi_t = \omega^C + \sum_{i=1}^{q} \alpha_i^{\ C} R_{t-i} + \sum_{j=1}^{p} \beta_j^{\ C} \varphi_{t-j} \quad , \tag{11}$$

$$\varepsilon_t \left| \mathrm{I}_{t-1} \sim f(1, \zeta_t) \right. \tag{12}$$

where R_t is defined as the daily high-low range and φ_t is the conditional mean of the range. The restrictions of the parameters (ω^C, $\alpha_i^{\ C}$ and $\beta_j^{\ C}$) are the same as the MEM model discussed on the realized range. These parameters can be obtained by the Quasi-Maximum Likelihood Estimation (called QMLE) method. ω^C, $\alpha_i^{\ C}$ and $\beta_j^{\ C}$ stand for the uncertainty in range, the short-term impact effect, and the long-term effect of shocks to the range, respectively. From Chou (2005), CARR(1,1) model is sufficient to explain the volatility. As a result, we estimate the range with CARR(1,1) model in the empirical study.

3.2. Estimating tail index

The most important thing in VaR is to estimate the probable biggest loss on the worst situation. As a result, the tail distribution of financial returns is considered the most significant issue. Much research has illustrated the fat-tailed character of financial return distribution. The extreme value (called EV) theory is mainly discussed on the property of tail distribution, instead of the whole distribution of returns. Longin (1996) presented that the thickness of tail is measured as the value of tail index. In this paper, we use the revised Hill's estimator to evaluate the tail index.

As mentioned before, users don't need to make assumption of financial returns to measure risks. Instead, true risks are determined by observing the extreme value. The thickness of tail is reflected by the estimation of tail index, γ. The fatter the tail is, the larger the tail index value becomes. Therefore, it is important to estimate accurately the value of tail index for understanding the tail distribution of financial returns.

Hill's estimator is a popular method to measure the tail index, proposed by Hill (1975). It is easy to use for describing the tail behavior, given the values of the extreme order statistics. Suppose that a sample of *n* independent observations is drawn from a population with fat-tailed distribution. Let $x_{(i)}$ be the *i*th-order statistics of the absolute value of observations such that $x_{(i)} \geq x_{(i-1)}$ for *i* = *2*,…, *n*. We choose to contain *k* observations from the left tail to estimate. The Hill's estimator for γ is as follows,

$$\gamma(k) = \frac{1}{k}\sum_{j=1}^{k} \ln(X(n-j+1)) - \ln(X(n-k)) \quad . \tag{13}$$

Huisman, Koedijk, Kool and Palm (2001) pointed out that $\gamma(k)$ is a maximum likelihood estimator for a conditional Pareto distribution. The difficulty of using Hill's estimator is the proper choice of *k*. Dacorogna, Muller, Pictet and de Vries (1995) proposed an asymptotic approximated distribution function of the bias in the Hill's estimator:

$$F(x) = 1 - ax^{-\alpha}(1 + bx^{-\beta}) \quad , \tag{14}$$

where α and β are positive and *a* and *b* are real numbers. Hall (1990) showed that under the given *k*, the asymptotic expected value and variance of the Hill's estimator are as follows,

$$E(\gamma(k)) \approx \frac{1}{\alpha} - \frac{b\beta}{\alpha(\alpha+\beta)} a^{-\frac{\beta}{\alpha}} \left(\frac{k}{n}\right)^{\frac{\beta}{\alpha}} \quad , \tag{15}$$

$$Var(\gamma(k)) \approx \frac{1}{k\alpha^2} \quad . \tag{16}$$

From equation (15) and (16), a small *k* is preferred for the unbiasedness but a large *k* is better from an efficiency viewpoint. It shows the trade-off relationship between accuracy and efficiency. Meanwhile, an important discovery from the equation (15) is that for any *k* exceeding 0, the estimator always encounters the problem of bias. Dacorogna, Muller, Pictet and de Vries (1995) used a simulation method and concluded that there is no significant influence of the estimation of α even with large bias in the assumption of value of β.

According to the analysis above, Huisman, Koedijk, Kool and Palm (2001) proposed a method of revising the Hill's estimator to solve the problem of choosing the value of *k*. They imposed the restriction $\alpha = \beta$ on Hill's estimator to make the asymptotic bias linear in *k*. The equation (13) is transferred as

$$\gamma(k) = \beta_0 + \beta_1 k + \varepsilon(k), \quad k = 1, \ldots, \kappa \ , \tag{17}$$

where β_0 and β_1 are the parameters and $\varepsilon(k)$ is the error term in the regression. Instead of selecting optimal *k* to measure the tail index, Hill's estimates of $\gamma(k)$ for *k* from *1* to κ are computed. This procedure resolves the problem of trade-off relation between bias and variance by using different values of *k* to obtain the estimate of tail index. Evaluation of equation (17) on *k* approaching 0 makes an unbiased estimate of $\gamma(k)$ equal to the intercept β_0. As a result, the unbiased estimate of tail index, $\widehat{\beta_0}$, is obtained by using weighted squares least (WLS) method. From the simulation result in Huisman, Koedijk, Kool and Palm (2001), the choice of value κ has no influence on the estimation of tail index. The using of $\kappa = n/2$ is suggested to get a precise estimate.

As discussed before, the inaccurate VaR may be obtained if the heteroscedastic property of financial returns is ignored. In our research, we follow the concept of McNeil and Frey (2000) that using the standard residual series which fit i.i.d. character to estimate the tail index. The process of producing i.i.d. standard residual series is described as follows. Assume that the dynamics of *X* are given by

$$X_t = \mu_t + \sigma_t Z_t \ , \tag{18}$$

where X_t is the return on day *t*, μ_t is the conditional mean, σ_t^2 is the conditional variance, and the innovations Z_t are a i.i.d. white noise process with zero mean and unit variance. We use autoregressive moving average with one lag period (called ARMA(1,1)) to estimate conditional mean. As for the conditional variance, we use five different conditional volatility models (EWMA, GARCH, CARR, RR_30m and RR_5m) to estimate. These models are set detailed below.

ARMA(1,1)-EWMA(λ =0.94) model is shown as

$$\begin{aligned} &X_t = \mu + \phi_1 X_{t-1} + \theta_1 \varepsilon_{t-1} + \varepsilon_t \quad \varepsilon_t \big| I_{t-1} \sim N(0, \xi_t^2) \\ &\xi_t^2 = (1-\lambda)\varepsilon_{t-1}^2 + \lambda \xi_{t-1}^2 \end{aligned} . \tag{19}$$

Standard residual series of ARMA(1,1)-EWMA model are obtained as follows,

$$(z_{t-n+1}^E, \ldots z_t^E) = \left(\frac{X_{t-n+1} - \hat{\mu}_{t-n+1}}{\hat{\xi}_{t-n+1}}, \ldots, \frac{X_t - \hat{\mu}_t}{\hat{\xi}_t}\right) . \tag{20}$$

ARMA(1,1)-GARCH(1,1) model is defined as

$$\begin{aligned} &X_t = \mu + \phi_1 X_{t-1} + \theta_1 \varepsilon_{t-1} + \varepsilon_t \quad \varepsilon_t \big| I_{t-1} \sim N(0, \sigma_t^2) \\ &\sigma_t^2 = \omega^G + \alpha_1^G \varepsilon_{t-1}^2 + \beta_1^G \sigma_{t-1}^2 \end{aligned} . \tag{21}$$

Standard residual series of ARMA(1,1)-GARCH(1,1) model are as follows,

$$(z_{t-n+1}^G, \ldots z_t^G) = \left(\frac{X_{t-n+1} - \hat{\mu}_{t-n+1}}{\hat{\sigma}_{t-n+1}}, \ldots, \frac{X_t - \hat{\mu}_t}{\hat{\sigma}_t}\right) . \tag{22}$$

ARMA(1,1)-CARR(1,1) model is presented below,

conditional mean equation: $X_t = \mu + \phi_1 X_{t-1} + \theta_1 \varepsilon_{t-1} + \varepsilon_t \quad \varepsilon_t \big| I_{t-1} \sim N(0, \sigma_t^2)$, (23)

conditional volatility equation:
$$\begin{aligned} &R_t = \varphi_t \varepsilon_t \quad \varepsilon_t \big| \mathrm{I}_{t-1} \sim f(1, \zeta_t) \\ &\varphi_t = \omega^C + \alpha_1^C R_{t-1} + \beta_1^C \varphi_{t-1} \end{aligned} . \tag{24}$$

Standard residual series of ARMA(1,1)-CARR(1,1) model are shown as follows,

$$(z_{t-n+1}^C, \ldots z_t^C) = \left(\frac{X_{t-n+1} - \hat{\mu}_{t-n+1}}{\hat{\varphi}_{t-n+1}}, \ldots, \frac{X_t - \hat{\mu}_t}{\hat{\varphi}_t}\right) . \tag{25}$$

ARMA(1,1)-RR_30m(1,1) model is displayed as conditional mean equation: $X_t = \mu + \phi_1 X_{t-1} + \theta_1 \varepsilon_{t-1} + \varepsilon_t \quad \varepsilon_t \left| I_{t-1} \sim N(0, \sigma_t^2) \right.$, (26)

conditional volatility equation: $$\begin{aligned} &RRV\_30m_t = \tau_t^{R30} \varepsilon_t \quad \varepsilon_t \left| \mathrm{I}_{t-1} \sim f(1, \rho_t) \right. \\ &\tau_t^{R30} = \omega^{R30} + \alpha_1^{R30} RRV\_30m_{t-1} + \beta_1^{R30} \tau_{t-1}^{R30} \end{aligned} \tag{27}$$

Standard residual series of ARMA(1,1)- RR_30m(1,1) model are as follows,

$$(z_{t-n+1}^{R30}, \ldots z_t^{R30}) = \left(\frac{X_{t-n+1} - \widehat{\mu}_{t-n+1}}{\widehat{\tau_{t-n+1}^{R30}}}, \ldots, \frac{X_t - \widehat{\mu}_t}{\widehat{\tau_t^{R30}}}\right) \quad . \tag{28}$$

Last, ARMA(1,1)-RR_5m(1,1) model is shown below,

conditional mean equation: $X_t = \mu + \phi_1 X_{t-1} + \theta_1 \varepsilon_{t-1} + \varepsilon_t \quad \varepsilon_t \left| I_{t-1} \sim N(0, \sigma_t^2) \right.$, (29)

conditional volatility equation: $$\begin{aligned} &RRV\_5m_t = \tau_t^{R5} \varepsilon_t \quad \varepsilon_t \left| \mathrm{I}_{t-1} \sim f(1, \pi_t) \right. \\ &\tau_t^{R5} = \omega^{R5} + \alpha_1^{R5} RRV\_5m_{t-1} + \beta_1^{R5} \tau_{t-1}^{R5} \end{aligned} \tag{30}$$

Standard residual series of ARMA(1,1)- RR_5m(1,1) model are as follows,

$$(z_{t-n+1}^{R5}, \ldots z_t^{R5}) = \left(\frac{X_{t-n+1} - \widehat{\mu}_{t-n+1}}{\widehat{\tau_{t-n+1}^{R5}}}, \ldots, \frac{X_t - \widehat{\mu}_t}{\widehat{\tau_t^{R5}}}\right) \quad . \tag{31}$$

All parameters are defined before in the section 3.1. Five standard residual series are obtained by using different conditional volatility models. Then we use the left tail of these standard residual series to estimate the tail index, instead of using the original financial returns.

3.3. Evaluating Value-at-Risk

After discussing the conditional volatility models, we now focus on the application of these volatility models to VaR model. VaR model is the most commonly used technique in risk management to obtain possible losses in financial markets. Within a given confidence interval, VaR measures the market risks by estimating the worst expected loss over a period. Let P_t be the asset price for the time t, and the expected return at time t is shown as $X_t = \ln P_t - \ln P_{t-1}$. Given the initial investment V_0 and a chosen time horizon, the expected value of the investment is:

$$V_t = V_0 \times (1 + X_t) . \tag{32}$$

We are interested to get the lowest investment value at time *t+1* at a particular confidence interval (100(c)%), so the rate of return X_{t+1}^{*} resulting in this lowest investment value V_{t+1}^{*} :

$$V_{t+1}^{*} = V_0 \times (1 + X_{t+1}^{*}) . \tag{33}$$

Assuming that the average return is defined as μ, the estimate for the VaR relative to the mean is developed as:

$$VaR = V_0 \times (1 + X_{t+1}^{*}) - V_0 \times (1 + \mu) . \tag{34}$$

To simplify, it turns to:

$$VaR = V_0 (X_{t+1}^{*} - \mu) . \tag{35}$$

The crux of being able to obtain the accurate VaR is in being able to estimate the expected rate of return X_{t+1}^{*}. Under a particular confidence interval (100(c)%), X_{t+1}^{*} on day t is shown :

$$prob(X_{t+1} \le X_{t+1}^{*} | \mathrm{I}_t) = 1 - c \quad , \tag{36}$$

where I_t represents the information set on day *t*. The cumulative distribution function of X_{t+1}^{*} on day *t* shown by $F(\bullet)$ is written as:

$$F(X_{t+1}^{*}) = 1 - c . \tag{37}$$

As a result, X_{t+1}^{*} can be presented with the inverse function below:

$$X_{t+1}^{*} = F^{-1}(1 - c) . \tag{38}$$

VaR estimation requires knowing the distribution of the returns. However, the true distribution of financial returns is always unknown. That’s the reason that the returns are assumed to a particular distribution before measuring VaR. In this paper, we make assumptions that the distribution of $F(\bullet)$ is normal or student-t distributed, respectively. Two evaluating methods of VaR, variance-covariance with normal distribution and extreme value theory with student-t distribution, are discussed below.

3.3.1. Variance-covariance method

J.P. Morgan first proposed a variance-covariance method, called RiskMetrics, to measure the VaR. There are three important assumptions in this method. First, the distribution of financial returns is assumed to be normal. Second, every observation is considered to be independent. Last, the variation of market factors and the price variation are assumed to be linear. Given the average of the return (μ) and the variance (σ^2), the estimation of the sample mean and variance is $\hat{\mu}$ and $\hat{\sigma}^2$, respectively. X^*_{t+1} can be rewritten as

$$X^*_{t+1} = N^*\hat{\sigma} + \hat{\mu} \quad , \tag{39}$$

where N^* is the critical value in normal distribution under a given confidence level. With substituting X^*_{t+1} in equation (35), the relative VaR is $V_0 N^* \hat{\sigma}$. By using the conditional model, the estimation of volatility is time-varying. Therefore, the relative VaR forecast of day *t+1* on day *t* is shown as follows,

$$VaR_{t+1} = V_0 N^* \hat{\sigma}_{t+1} . \tag{40}$$

The value of V_0 is assumed to be 1 in this paper. We use five different methods to forecast volatility ($\hat{\sigma}_{t+1}$) : EWMA, GARCH, CARR, RR_30m and RR_5m models.

3.3.2. Extreme value theory (VaR-x model)

Instead of assuming the distribution to be normal, Huisman et al. (1998) presented VaR-x model which assumed to be student-t distribution. The degree of freedom must be determined first before estimating N^*. To evaluate VaR-x, we estimate the tail index by revised Hill's estimator first. Then the student-t distribution and the degree of freedom are obtained by inversing the tail index. The following is the detailed steps for evaluating VaR-x. First, the tail index $\widehat{\beta_0}$ is estimated by the revised Hill's estimator for the left tail of standard residual series. Second, the conditional mean μ_{t+1} and the conditional standard error σ_{t+1} are forecasted by five conditional volatility models. Third, let the estimation of degree of freedom $\hat{\nu}$ in student-t distribution equals the inverse of tail index $\widehat{\beta_0}$. That means $\hat{\nu} = \frac{1}{\widehat{\beta_0}}$. Next, find the critical number S^* in the standard student-t distribution with $\hat{\nu}$ degrees of freedom. Because of $S^* \sim t(0, \frac{\hat{\nu}}{\hat{\nu}-2})$, this value S^* needs to be transferred to the real cutoff return $X_{t+1}^* = S^*\Phi + \hat{\mu}$, where Φ is a scale factor given by $\Phi = \frac{\hat{\sigma}}{\sqrt{\hat{\nu}/\hat{\nu}-2}}$. In the last step, with substituting X_{t+1}^* in equation (35), the VaR-x is $V_0 S^* \Phi$.

3.4. Comparison of Value-at-Risk models

The Basel Committee on Banking Supervision enforced a regulation that the financial institutions need to use backtesting to evaluate the accuracy of internal models. Backtesting is a statistical testing framework that checking whether the actual trading losses are in accordance with the VaR. Each exceedence is called a failure. The closer the number of failures and the theoretical value are, the better the model is.

Except the number of failures, some comparing criteria were developed in the previous research. These criteria are classified into three dimensions.

3.4.1. Conservatism

The variation in the size of risk estimates obtained by different models is to evaluate if any model tends to produce high-level risk estimates relative to other models. We characterize those models which make high-risk estimates relatively as conservative ones.

3.4.1.1. Mean Relative Bias (called MRB)

The mean relative bias (*MRB*) method, proposed by Hendricks (1996), captures the degree to which models produce risk estimates of similar average level. Given T days and N VaR models, the *MRB* of model i is measured as:

$$MRB_i = \frac{1}{T}\sum_{t=1}^{T}\frac{VaR_{i,t} - \overline{VaR_t}}{\overline{VaR_t}} \tag{41}$$

where $\overline{VaR_t} = \frac{1}{N}\sum_{i=1}^{N} VaR_{i,t}$ .The bigger MRB_i is, the more conservative the model$_i$ is.

3.4.1.2. Root Mean Squared Relative Bias (called RMSRB)

This criterion evaluates the extent to which model risk estimates tend to vary around the average risk measure of all models for a given day, presented by Hendricks (1996). The *RMSRB* of model i is computed as follows,

$$RMSRB_i = \sqrt{\frac{1}{T}\sum_{t=1}^{T}\left(\frac{VaR_{i,t} - \overline{VaR_t}}{\overline{VaR_t}}\right)^2} \tag{42}$$

where $\overline{VaR_t} = \frac{1}{N}\sum_{i=1}^{N} VaR_{i,t}$, T stands for the number of days and N is the number of VaR models. The bigger $RMSRB_i$ is, the bigger the divergence of model$_i$ compared to others is.

3.4.2. Accuracy

Accuracy takes account of whether the VaR estimates are large enough to cover the true underlying risks. The number of failures and the size of those losses are concerned in this dimension. Under the Banking Supervision's regulations, the failure rate should be equal to or smaller than the significant level of VaR models. Different types of criterion on accuracy are developed below.

3.4.2.1. *The Binary Loss Functions (called BLF)*

The number of failures rather than the size of these failures are concerned in this method. This is a binomial function from the general loss function to measure the number of failures, developed by Lopez (1999). That is,

$$L_{i,t+1} = \begin{cases} 1 \ \ if \ \ \Delta P_{i,t+1} < VaR_{i,t} \\ 0 \ \ if \ \ \Delta P_{i,t+1} \geq VaR_{i,t} \end{cases} \tag{43}$$

This dummy variable means that when actual excess surpass VaR estimates, it is defined as an outlier or failure. The average binary loss function of model *i* is obtained with measuring total number of outlier divided by *T*, where *T* stands for the number of sample days. The average binary loss function is as same as the failure rate. When the failure rate is close to the expected rate, it can be seen as a better model.

3.4.2.2. *Mean Excess*

In this criterion, the size of those failures is used to measure. Mean excess, proposed by Neftci (2000), is obtained by computing the average of exceedence which actual losses surpass VaR estimates. The equation is defined as:

$$mean \ excess \ of \ \ model_i = E(\left|\Delta P_{i,t+1} - VaR_{i,t}\right| \left|\Delta P_{i,t+1} < VaR_{i,t}\right.) \,. \tag{44}$$

The smaller mean excess is, the smaller the unexpected losses are.

3.4.2.3. *LR test of unconditional coverage (called LR_{uc})*

Kupiec (1995) developed a likelihood-ratio test that can test whether the sample estimate is statistically consistent with the given confidence level of these models. If a bank's daily VaR and returns can be assumed to be independent, the number of failures represents a sequence of independent Bernoulli trials. To evaluate the accuracy of VaR models, we conduct a test of the null hypothesis that the probability of failure on each trial ($\widehat{\alpha_c} = \dfrac{L}{T}$) is as same as the model's specified probability (α_c). The Likelihood Ratio test statistic is shown below:

$$LR_{uc} = 2 \cdot \frac{\ln\left[(1-\widehat{\alpha_c})^{T-L}\widehat{\alpha_c}^{\,L}\right]}{\ln\left[(1-\alpha_c)^{T-L}\alpha_c^{\,L}\right]} \sim \chi^2_{1,\alpha_c}, \tag{45}$$

where T stands for the number of days and L is the total number of failures. If the null hypothesis is not rejected, it means that there is not significant difference between failure rate and the theoretical rate.

3.4.2.4. LR test of independence (called LR_{ind})

If a model can capture the conditional distribution and time-varying character of returns, the failures will occur independently and unpredictably in samples. Christoffersen (1998) presented LR_{ind} test to evaluate the independence of failures and the accuracy of these VaR models. The null hypothesis and the test statistic are defined as:

$$\begin{aligned}
&H_0 : \pi_{01} = \pi_{11} = \pi \\
&LR_{ind} = 2 \cdot \frac{\ln\left[(1-\pi_{01})^{n_{00}}\pi_{01}^{\;n_{01}}(1-\pi_{11})^{n_{10}}\pi_{11}^{\;n_{11}}\right]}{\ln\left[(1-\pi)^{(n_{00}+n_{10})}\pi^{(n_{01}+n_{11})}\right]} \sim \chi^2_{1,\alpha_c} \\
&\pi_{01} = \frac{n_{01}}{n_{00}+n_{01}}; \\
&\pi_{11} = \frac{n_{11}}{n_{10}+n_{11}}; \\
&\pi = \frac{n_{01}+n_{11}}{n_{00}+n_{01}+n_{10}+n_{11}}
\end{aligned}, \tag{46}$$

where n_{ij} means the total days for the state j of the difference between actual returns and VaR estimates this period and the state i of that last period. α_c stands for the significant level in VaR models. If the null hypothesis is not rejected, the independence of failures and the accuracy of VaR models are proven.

3.4.2.5. LR test of conditional coverage (called LR_{cc})

According to Christoffersen's (1998) definition, LR_{cc} is the combination with the unconditional coverage LR test and the independence LR test. The test statistic is:

$$LR_{cc} = LR_{uc} + LR_{ind} \sim \chi^2_{2,\alpha_c} \quad . \tag{47}$$

When the null hypothesis is not rejected, it means that the VaR model can evaluate the number of failures precisely and capture the character of time-varying.

3.4.2.6. Multiple to Obtain Coverage (called MOC)

If the failure rate is not equal to the significant level, it is meant that there is a bias for the VaR estimates. To obtain the magnitude of bias, Hendricks (1996) developed the *MOC* method to appraise the accuracy of VaR models. It is presented as follows,

$$F_i = T\alpha_c \text{ , } where\ F_i = \sum_{t=1}^{T} \begin{cases} 1 & if\ \Delta P_{i,t+1} < MOC_i \cdot VaR_{i,t} \\ 0 & if\ \Delta P_{i,t+1} \geq MOC_i \cdot VaR_{i,t} \end{cases} \quad , \tag{48}$$

where T is the number of days and MOC_i means the multiple to obtain coverage of model i. As the *MOC* is greater than 1, the VaR estimates are undervalued; as the *MOC* is smaller than 1, the VaR estimates are overvalued. The closer between *MOC* and 1 are, the more accurate VaR model is.

3.4.3. Efficiency

The efficiency of VaR models means the magnitude of the required minimum capital under the specified accuracy which VaR estimates can cover actual losses. A more efficient VaR model provides more precise resource allocation signals to traders and financial institutions. Two criteria used in this paper are discussed below.

3.4.3.1. Mean Relative Scaled Bias (called MRSB)

This method suggested by Hendricks (1996) combines mean relative bias (*MRB*) and multiple to obtain coverage (*MOC*). The *MRSB* of a VaR model is aimed to determine which approach scaled by *MOC* produces the smallest average risk. The equation of MRSB of model i is set as:

$$MRSB_i = \frac{1}{T}\sum_{t=1}^{T}\frac{MOC_i \cdot VaR_{i,t} - \overline{MOC \cdot VaR_t}}{\overline{MOC \cdot VaR_t}},\qquad (49)$$
$$\overline{MOC \cdot VaR_t} = \frac{1}{N}\sum_{i=1}^{N} MOC_i \cdot VaR_{i,t}$$

where T stands for the number of days and N is the number of VaR models. The most efficient model is the one who has the smallest *MRSB.*

3.4.3.2. Error Efficiency

The VaR estimates must measure the largest losses effectively. However, if the VaR estimates are overvalued excessively, VaR will become meaningless. We define error efficiency as measuring the relative distance of actual returns and VaR estimates, including profits and losses. It is shown as follows,

$$error\ efficiency\ of\ model_i = \frac{1}{T}\sum_{t=1}^{T}\left|\frac{\left|\Delta P_{i,t+1}\right| - VaR_{i,t}}{VaR_{i,t}}\right|,\qquad (50)$$

where T stands for the number of days. Error efficiency considers both the accuracy of unpredicted losses and the cost of predicted losses. The smaller the error efficiency is, the closer between actual returns and all VaR estimates are. In other words, this model can not only make precise estimates but also have more efficiency.

IV. Results

4.1. Data

In this section, we examine daily and intra-day S&P 500 and Nasdaq Index from 1997/1/2 to 2003/12/31. As for S&P 500 Index, 400 prices of 1-min for one day are contained in the database before 2002/10/31; 390 prices of 1-min per day are obtained after 2002/11/1. As for Nasdaq Index, 390 observations of 1-min price are included in our sample period. We acquire daily returns, 5-min ranges, 30-min ranges and daily ranges from original 1-min prices. The data employed in our empirical study comprise 140,294 prices per 5 minutes and 24,361 prices per 30 minutes for S&P 500 Index. For Nasdaq Index, we conduct 137,358 prices per 5 minutes and 22,893 prices per 30 minutes. Daily data reach a total of 1761 observations for both price indices. These data are retrieved from the TickWrite database[2].

4.2. Descriptive Statistics

Figure 1 shows the graphs for close prices, daily returns, daily ranges, realized range volatility with bias-correction procedure for 30-min range and 5-min range (called RRV_30m and RRV_5m, respectively) of S&P 500 and Nasdaq Index over the sample period from January 2, 1997 to December 31, 2003. The data of return and range on S&P500 and Nasdaq Index are defined as follows,

$$return_t = 100 \times \left[\ln(P_t^{close}) - \ln(P_{t-1}^{close}) \right],$$
$$range_t = 100 \times \left[\ln(P_t^{high}) - \ln(P_t^{low}) \right]$$

It is often reported as a percentage (%) by multiplying the above calculation by 100. The descriptive statistics for the daily returns, ranges, RRV_30m and RRV_5m of S&P 500 and

[2] We thank Professor Huimin Chung in Graduate Institute of Finance at National Chiao Tung University for providing the database.

Nasdaq Index are presented in Table 1. It shows the univariate statistics for the time series data over the sample period 1997-2003. The average of return for Nasdaq is larger than S&P 500 and Nasdaq Index shows more volatile than S&P 500. The standard deviations of range-based variables on both indices are smaller than the daily return. In addition, the standard deviations of RRV_5m are the smallest ones. This may imply that the intra-range data can capture the character of volatility better than daily returns and daily ranges. As for the statistic analysis of normality, all variables for S&P 500 and Nasdaq Index exhibit highly significant skewness and kurtosis, especially for the range-based ones. Moreover, the normality test, Jarque-Bera test, shows that the hypotheses of normal distribution for these variables are rejected. These four variables are not normal-distributed.

From the Ljung-Box Q statistics and Q^2 statistics in Table 1, daily returns and the squared daily return of these two indices are significant series autocorrelation. The p-value are all smaller than 0.1. The return-based data may be in the existence of ARMA effect and GARCH effect which stands for volatility-clustered situation. On the other part, the Q statistics of range-based data are all significant. It means that those range-based variables are correlated with their own lag value and not independent. The daily range and realized range volatility may have the character of CARR effect and MEM effect, respectively. In order to estimate accurate tail index, original financial returns are standardized first by ARMA(1,1) and conditional volatility process for obtaining standard residual series[3].

Table 2 shows the estimation of parameters in ARMA(1,1)-GARCH(1,1), CARR(1,1), RR_30m(1,1), RR_5m(1,1) models for the standardizing process. Comparing the sum of coefficients ($\alpha + \beta$), the sequence from big to small ones for S&P 500 Index is GARCH, CARR, RR_30 and RR_5m. For Nasdaq, the order is GARCH, CARR, RR_5m and RR_30m. The bigger the sum is, the stronger the volatility persistence effect is. On the other hand, if the

[3] Much research have demonstrated that ARMA(1,1) is sufficient to capture the property of autocorrelation of financial data.

sum of coefficients is small, the effect of mean reverting is large. As a result, GARCH models for two indices have the strongest effect of volatility persistence. This property is also shown on the biggest standard deviation in daily return. In addition, the coefficient α means the sensitivity to shocks of the short run. RR_5m models of two indices have the largest α value and are the most sensitive models to the short-run fluctuation.

The descriptive statistics for standard residual item of daily return in different models of S&P 500 and Nasdaq Index are presented in Table 3. The Q statistics of residual items become insignificant compared to the original daily returns. It means that ARMA(1,1) model can capture the character of first-order autocorrelation of financial returns. The Q^2 statistics are also insignificant, so the squared residual items are not in existence of autocorrelation. ARMA(1,1)-EWMA with $\lambda=0.94$, ARMA(1,1)-GARCH(1,1), ARMA(1,1)-CARR(1,1), ARMA(1,1)-RR_30m(1,1) and ARMA(1,1)-RR_5m(1,1) are sufficient to catch the property of heteroscedasticity and volatility-clustered. Moreover, the Jarque-Bera value for normality test of residual items is smaller than the daily returns. However, the hypothesis of normal distribution is still rejected, except for ARMA(1,1)-RR_30m(1,1) and ARMA(1,1)-RR_5m(1,1) models of Nasdaq Index.

4.3. Empirical Analysis

4.3.1. Tail index

From precious analysis, the daily returns of S&P 500 and Nasdaq Index are fat-tailed distributed. The thickness of tail distributed can be measured by the tail index. In this section, we use the revised Hill's estimator to estimate the tail index and capture the tail shape of these two indices.

Our purpose in this research is to find a precise method to evaluate downside risks. As a result, the left tail of observations is used to estimate tail index and measure the VaR. We use rolling sample method (Brooks, 2002) with 250 days to estimate the tail index and VaR of day $t+1$. Table 4 shows that the returns of both S&P 500 and Nasdaq Index for the whole sample period (1997~2003) are in existence of fat-tail property. The estimates of β_0 are bigger than 0 and between 0.1033 to 0.3036. According to Koedijk, Schafgans, and de Vries (1990), the distribution of these returns is fatter than normal distribution and belongs to student-t distribution. In addition, the estimated tail index of RR_30m and RR_5m are the smallest two among other models. This may imply that more problems of heteroscedasticity and volatility-clustered situation can be solved by using realized range method to standardize.

4.3.2. Comparison of VaR models

From precious discussion, the downside risks can be captured by tail index. In this section, the result of using variance-covariance and extreme value theory to measure VaR is presented. The average of estimated VaR for all sample period and individual years are listed in Table 5. Moreover, we use backtesting method to compare the ability of forecasting VaR in different models. For detailed analysis, many criteria of comparing VaR are applied and classified to three different dimensions. We focus on the result if it is better to evaluate VaR using realized range models and the comparison of VaR-normal with VaR-x models. In this research, we use rolling sample method and the length of each rolling sample is 250 days. VaR estimate of the next day is based on the precious 250 prices. By this approach, 1512 volatility and VaR estimates are obtained. Figure 2 and 3 show daily returns and VaR-normal estimates of these conditional volatility models for S&P 500 and Nasdaq Index, respectively. Figure 4 and 5 present daily returns and VaR-x estimates of these conditional volatility models for S&P 500 and Nasdaq Index, respectively. The biases of return-based models are obvious in evaluating VaR-x. The detailed analysis of comparing variance-covariance method by normal distribution with extreme value theory of VaR-x model is discussed below.

The models included in our research are described as follows. The first part is the models assumed to be normal distribution of financial returns in different conditional volatility process: EWMA-normal, GARCH-normal, CARR-normal, RR_30m-normal, and RR_5m-normal models. The second part is the models combining VaR-x and extreme value theory in different conditional volatility process: EWMA-VaR-x, GARCH-VaR-x, CARR-VaR-x, RR_30m-VaR-x, and RR_5m-VaR-x models. We use the same order of models in each rolling sample. Table 6 presents the number of failures in these ten conditional VaR models. In 95% confidence level for S&P 500, the failure number of EWMA-normal model and RR_30m-VaR-x model (77 and 75, respectively) are the most closest to the theoretical number (76). The third and fourth better ones are GARCH-normal and RR_5m-VaR-x models in sequence. As for Nasdaq Index, GARCH-normal model is the best one. The second best models are EWMA-normal, RR_5m-normal and CARR-VaR-x models. We conclude that it is better to evaluate risks under normal-distributed assumption in 95% confidence level. In 97.5% confidence level for S&P 500, RR_30m-VaR-x model performs the best. RR_30m-normal, RR_5m-VaR-x and GARCH-VaR-x models are the next. For Nasdaq Index, EWMA-normal is the best one. The second and third better ones are GARCH-normal and GARCH-VaR-x models in sequence. In 99% confidence level for S&P 500, the closest one is CARR-VaR-x model. The next are GARCH-normal, CARR-normal, RR_5m-normal and RR_30m-VaR-x models. As for Nasdaq Index, the five normal ones and GARCH-VaR-x model perform nearly. As a whole, the normal-distributed models and the extreme value ones perform about the same in 97.5% and 99% level. In addition, RR_30m-VaR-x is the most precise model in different confidence level for S&P 500. EWMA-normal and RR_5m-VaR-x are the second better. For Nasdaq Index, EWMA-normal and GARCH-normal are the best. RR_5m-normal and GARCH-VaR-x are the second ones. In empirical research of the failure number, realized range model performs better for S&P 500 than Nasdaq Index. In detail, realized range models with extreme value theory are more proper for S&P 500 and those with normal distribution are more suitable for Nasdaq Index.

Except for the number of failures, there are other testing dimensions to evaluate VaR models. When computing equation of some criteria, like *MRB* in equation (41), *RMSRB* in equation (42), LR_{uc} in equation (45), *MRSB* in equation (49) and error efficiency in equation (50), the number of sample days T is 1512 and the number of VaR models N equals to 10. The backtesting results of VaR models are listed in Table 7 to 12. Considering the conservatism of VaR models of S&P500 in 95% level in Table 7, the average of all models' mean relative bias (MRB) are from -0.0657 to 0.0537. EWMA-VaR-x, GARCH-VaR-x, CARR-normal, and RR_5m-normal models are more conservative because of its large MRB in sequence. According to root mean squared relative bias, CARR-normal, RR_5m-normal, RR_30m-normal model, RR_5m-VaR-x and GARCH-normal are the least divergent method. The range-based models with normal distribution perform more conservative than others in 95% confident interval.

In the accuracy analysis of VaR models, the comparing result of the binary loss functions (BLF) is as same as the number of failures discussed above. RR_30m-VaR-x, EWMA-normal, GARCH-normal and RR_5m-VaR-x are the most accurate in sequence. When discussing the mean excess, RR_5m-VaR-x, GARCH-normal, EWMA-VaR-x and RR_30m-normal model are the smallest ones in sequence in 95% percentile. In regard to the LR test of unconditional coverage, independence, and conditional coverage, all models' assumption are not rejected. In other words, all VaR models pass the statistic test. According to MOC criterion, the result is that EWMA-normal and RR_30m-VaR-x models are the best ones. GARCH-normal and RR_5m-VaR-x are the third and fourth models in 95% percentile. As a whole for accuracy test, GARCH-normal, RR_5m-VaR-x, RR_30m-VaR-x and EWMA-normal models perform well on accuracy test in sequence.

For the efficiency of VaR models, the best two efficient models are RR_30m-VaR-x and

RR_5m-VaR-x models. The next two are RR_5m-normal and RR_30m-normal models. The realized range model performs efficient among others. To summarize, the whole performance of realized range models with normal distritubion is close to EWMA-normal and GARCH-normal. However, realized range with VaR-x models dominate over others. RR_5m-VaR-x model is the best one.

As for Nasdaq in Table 8, the average of all models' mean relative bias (MRB) are from -0.0989 to 0.0288. RR_5m-normal, CARR-normal and RR_30m-normal models are the most conservative ones according to large MRB. For root mean squared relative bias, RR_5m-normal, RR_30m-normal and CARR-VaR-x models are the least divergent method. The range-based models with normal distribution perform more conservative than others. In the accuracy analysis of VaR models, GARCH-normal, EWMA-normal, RR_5m-normal and CARR-VaR-x can produce more accurate value according to BLF. As to mean excess, the four realized range models perform well. As same as S&P500, all LR tests are passed. In regard to MOC, EWMA-normal, GARCH-normal, RR_5m-VaR-x and RR_30m-VaR-x models are closest to 1. Realized-range-based VaR-x models and return-based normal models perform about the same in accuracy. As for efficiency test, CARR-VaR-x and RR_30m-VaR-x models are the most efficient ones. To conclude the result of Table 8, CARR-VaR-x, RR_5m-normal, EWMA-normal and RR_30m-VaR-x are the top four models for Nasdaq Index in 95% percentile.

Considering the conservatism of VaR models of S&P500 in 97.5% level in Table 9, GARCH-VaR-x and EWMA-VaR-x models produce largest number in MRB. As for RMSRB, the value of CARR-normal, RR_5m-VaR-x, RR_5m-normal and RR_30m-normal are the smallest in sequence. Considering the accuracy, RR_30m-VaR-x is the most precise model. RR_30-normal, RR_5m-VaR-x and GARCH-VaR-x models perform well, too. For the efficiency test, CARR-normal, RR_5m-normal and RR_30m-normal models have the smallest value of MRSB and error efficiency. As a whole, realized range models perform much better than return-based and CARR models. RR_30m-normal and RR_5m-VaR-x models are the best two on estimating VaR of S&P 500 in 97.5% confidence interval.

As for Nasdaq Index in 97.5% level in Table 10, CARR-VaR-x model is the most conservative one. In regard to accuracy, EWMA-normal, GARCH-normal, RR_30m-normal and GARCH-VaR-x models perform much better than others. For efficiency, CARR-normal, RR_30m-normal and RR_5m-normal models are more efficient. To summarize the result in Table 10, EWMA-normal and RR_30m-normal are the best models. CARR-normal and RR_5m-normal models are the next best ones.

Table 11 shows the result of S&P 500 in 99% level. For conservatism test, EWMA-VaR-x and GARCH-VaR-x models are the most conservative and divergent ones. As to accuracy, RR_5m-normal is the most precise model. Other range-based models also perform well. According to efficiency, GARCH-normal, EWMA-normal, RR_30m-normal and RR_5m-normal models produce smaller value of MRSB and error efficiency. RR_5m-normal and RR_30m-normal model are the most proper ones to evaluate the risks of S&P 500 in 99% confident interval. In addition, models with normal distribution perform better than the VaR-x models.

Last, the result of Nasdaq in 99% level is presented in Table 12. As for conservatism test, EWMA-VaR-x and GARCH-VaR-x models are the most conservative and divergent ones. For accuracy test, RR_5m-normal and RR_30m-normal models produce more precise estimates. According to efficiency test, EWMA-normal, CARR-normal and RR_30m-normal models perform well. As a whole, the performance of RR_30-normal, RR_5m-normal and CARR-normal is much better than others. Moreover, normal-distributed models are better than extreme value theory.

Various criteria are used to show different importance of VaR dimensions. Meanwhile, accuracy is more popular and important among these three dimensions. Users of VaR models often focus on the difference between failure rate and theoretical rate first. Moreover, financial institutions always don't want to be conservative because they have to spend more costs to reach the restriction of required minimum capital. In practical, conservatism and efficiency seem to be paid less attention. To conclude, realized range models can produce better evaluation of financial risks. In 95% confident interval, realized range models with normal distribution perform as well as other normal models. In regard to VaR-x models, range-based ones are better than return-base method. As the increasing of percentile, realized range models dominate over return-based models and CARR model. Moreover, in 99% confident level, RR_30m-normal and RR_5m-normal models are sufficient to capture the downside risks. It may imply that realized range models with normal distribution can evaluate VaR much better than VaR-x models in high confident level.

V. Conclusion

Considering the additional downside risks is a major part in measuring VaR. Much research has proposed the property of heteroscedasticity and volatility-clustered in financial returns. As a result, finding a proper method that is able to capture the character of financial data is significant. In addition, the realized range is presented to be a proxy of variance in recent years. In this paper, we do empirical research on S&P 500 and Nasdaq Index and compare different conditional VaR models. We use the most popular frequency, 30-min and 5-min range, to measure the realized range. In practical research, realized range models improve its performance compared with others as the confident level increases. It indicates that intra-day range data contain more information than daily return and daily range. Moreover, we find a surprising result that realized range models with normal distribution perform better than the VaR-x models in high confident interval. The possible reason may be that intra-day range is more sensitive to outliers. This discovery may imply that even though the financial returns are existence of fat-tailed property, it can still be captured by normal-distributed realized range model.

Our contribution in this paper is applying realized range method forecasted by MEM model to VaR model. The new method of measuring risks is proved precise by three dimensions of comparing criteria in empirical study. Comparing VaR-normal with VaR-x model, normal-distributed realized range models can produce much better estimates than the other in higher percentile. However, a detailed analysis of this question is left for future research. In addition, the restriction of this paper is that the result is data-oriented. To obtain a more reliable result, Monte Carlo method is an alternative for measuring VaR. With simulating repeated process, the distribution function and the required cutoff values can be estimated. This method can be conducted in future research.

References

Andersen, T., Bollerslev, T., Diebold, F., and Labys, P. (2001), The distribution of realized exchange rate volatility, *Journal of the American Statistical Association*, 96, 42-55.

Andersen, T., Bollerslev, T., Diebold, F., and Labys, P. (2003), Modeling and forecasting realized volatility, *Econometrica*, 71, 579-625.

Barndorff-Nielsen, O., and Shephard, N. (2002), Econometric analysis of realised volatility and its use in estimating stochastic volatility models, *Journal of the Royal Statistical Society Series*, 64, 253-280.

Berkowitz, J., and O'brien, J. (2002), How accurate are Value-at-Risk models at commercial banks, *Journal of Finance*, 57, 1093-1111.

Bollerslev, T. (1986), Generalized autoregressive conditional heteroscedasticity, *Journal of Econometrics*, 31, 307-327.

Bollerslev, T., Chou, R., and Kroner K. (1992), ARCH modeling in finance: A review of the theory and empirical evidence, *Journal of Econometrics*, 52, 5–59.

Brooks, C. (2002), *Introductory Econometrics for Finance*, Cambridge.

Chou, R. (2005), Forecasting financial volatilities with extreme values: The conditional autoregressive range (CARR) model, *Journal of Money, Credit and Banking*, 37, 561-582.

Christensen, K., and Podolskij, M. (2007), Realized range-based estimation of integrated variance, *Journal of Econometrics*, 141, 323-349.

Christoffersen, P. (1998), Evaluating interval forecasts, *International Economic Review*, 39, 841-862.

Christoffersen, P., Hahn, J., and Inoue, A. (2001), Testing and comparing Value-at-Risk measures, *Journal of Empirical Finance*, 8, 325-342.

Corrado, C., and Truong, C. (2007), Forecasting stock index volatility: Comparing implied volatility and the intraday high-low price range, *Journal of Financial Research*, 30, 201-215.

Dacorogna, M., Muller, U., Pictet, O., and de Vries, C. (1995), The distribution of extremal foreign exchange rate returns in extremely large data sets, *Tinbergen Institute Discussion Paper*, 70-95.

Danielsson, J., and de Vries, C. (2000), Value-at-Risk and extreme returns, *Annales* *d'Économie et de Statistique*, 60, 239-270.

Duffie, D., and Pan, J. (1997), An overview of value at risk, *Journal of Derivatives*, 4, 7-49.

Engel, J., and Gizycki, M. (1999), Conservatism, accuracy and efficiency: Comparing Value-at-Risk models, Working paper 2, Australian Prudential Regulation Authority.

Engle, R. (1982), Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation, *Econometrica*, 50, 987-1008.

Engle, R. (2002), New frontiers for ARCH models, *Journal of Applied Econometrics*, 17, 425–446.

Engle, R. and Manganelli, S. (2004), CAViaR: Conditional Autoregressive Value at Risk by Regression Quantiles, *Journal of Business & Economic Statistics*, 22, 367-381.

Giot, P., and Laurent, S. (2004), Modeling daily Value-at-Risk using realized volatility and ARCH type models, *Journal of Empirical Finance*, 11, 379-398.

Hall, P. (1990), Using the bootstrap to estimate mean squared error and select smoothing parameter in nonparametric problems, *Journal of Multivariate Analysis*, 32, 177-203.

Hansen, P., and Lunde, A. (2006), Realized variance and market microstructure noise, *Journal of Business and Economic Statistics*, 24, 127-218.

Hartz, C., Mittnik, S., and Paolella, M. (2006), Accurate value-at-risk forecasting based on the normal-GARCH model, *Computational Statistics & Data Analysis*, 51, 2295-2312.

Hendricks, D. (1996), Evaluation of Value-at-Risk models using historical data, *Federal Reserve Bank of New York Economic Policy Review*, 2, 36-69.

Hill, B. (1975), A simple general approach to inference about the tail of a distribution, *The Annals of Statistics*, 3, 1163-1174.

Huisman, R., Koedijk, K., Kool, C., and Palm, F. (2001), Tail-index estimates in small samples, *Journal of Business and Economic Statistics*, 19, 208-216.

Huisman, R., Koedijk, K., and Pownall, R. (1998), VaR-x: Fat tails in financial risk management, *Journal of Risk*, 1, 47-62.

Jansen, D., and de Vries, C. (1991), On the frequency of large stock returns: Putting booms and busts into perspective, *Review of Economics and Statistics*, 73, 18-24.

Jorion, P. (2000), *Value at Risk*, McGraw Hill, New York.

Jorion, P. (2007), *Financial Risk Manager Handbook*, fourth edition, John Wiley & Sons.

Kearns, P. and Pagan, A. (1997), Estimating the density tail index for financial time series, *Review of Economics and Statistics*, 79, 171-175.

Koedijk, K., Schafgans, M., and de Vries, C. (1990), The tail index of exchange rate returns, *Journal of International Economics*, 29, 93-108.

Kupiec, P. (1995), Techniques for verifying the accuracy of risk measurement models, *Journal of Derivatives*, 3, 73-84.

Lanne, M. (2006), A mixture multiplicative error model for realized volatility, *Journal of Financial Econometrics*, 4, 594-616.

Longin, F. (1996), The asymptotic distribution of extreme stock market returns, *Journal of Business*, 69, 383-408.

Lopez, J. (1999), Methods for evaluating Value-at-Risk estimates, *Federal Reserve Bank of San Francisco Economic Review*, 2, 3-39.

Martens, M., van Dijk, D. (2007), Measuring volatility with the realized range, *Journal of Econometrics*, 138, 181-207.

McNeil, A., and Frey, R. (2000), Estimation of tail-related risk measures for heteroscedastic financial time series: An extreme value approach, *Journal of Empirical Finance*, 7, 271-300.

Neftci, S. (2000), Value at Risk calculations, extreme events, and tail estimation, *Journal of Derivatives*, 7, 23-37.

Parkinson, M. (1980), The extreme value method for estimating the variance of the rate of return, *Journal of Business*, 53, 61-65.

Pownall, R., and Koedijk, K. (1999), Capturing downside risk in financial markets: The case of the Asian Crisis, *Journal of International Money and Finance*, 18, 853-870.

Table 1:

Descriptive Statistics for Daily Returns, Daily Ranges, Realized Range Volatility for 30-min range and 5-min range (called RRV_30m and RRV_5m, respectively) of S&P 500 and Nasdaq Index, 1997/01/02-2003/12/31

$$RR_t^{\theta} = \sum_{i=1}^{I} (\ln H_{t,i} - \ln L_{t,i})^2$$

$$RR_{S,t}^{\theta} = \left(\frac{\sum_{l=1}^{q} RR_{t-1}}{\sum_{l=1}^{q} RR_{t-1}^{\theta}} \right) RR_t^{\theta}$$

$$RRV = \sqrt{RR_{S,t}^{\theta}}$$

where $RR_{S,t}^{\theta}$ is the scaled realized range (after bias-correction) and *RRV* is the realized range volatility. *RRV_30m* and *RRV_5m* stand for the 30-min and 5-min frequency of data used to measure *RRV*. Panel A and B report the descriptive statistics of S&P 500 and Nasdaq Index, respectively. The four variables are in daily percentage units. Std. Dev. denotes standard deviation. Jarque-Bera is the test of normality. Q(12) and $Q^2(12)$ are the Ljung-Box statistic for auto-correlation test with 12 lags. Numbers in parentheses are *p*-values.

	Panel A - S&P 500			
	RETURN	RANGE	RRV_30m	RRV_5m
Mean(%)	0.0233	1.6285	1.7075	1.7246
Median(%)	0.0355	1.4593	1.5447	1.5699
Maximum(%)	5.3080	8.4792	8.7396	13.5441
Minimum(%)	-7.1127	0.2800	0.3787	0.3499
Std. Dev. (%)	1.3010	0.8604	0.7566	0.7497
Skewness	-0.0675	2.1811	2.2542	3.6054
Kurtosis	5.0744	12.1611	12.6527	41.8617
Jarque-Bera	316.8883	7554.1897	8016.0329	110332.2280
	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Auto-Correlation Test				
Q(12)	21.7515	2187.5000	4681.5000	4940.3000
	(0.0400)	(0.0000)	(0.0000)	(0.0000)
$Q^2(12)$	272.2200			
	(0.0000)			

Table 1:

(continued)

	Panel B - Nasdaq			
	RETURN	RANGE	RRV_30m	RRV_5m
Mean(%)	0.0332	3.0416	3.1548	3.1748
Median(%)	0.1424	2.6787	2.8001	2.8226
Maximum(%)	17.2434	19.2172	17.6370	14.5331
Minimum(%)	-10.4345	0.5445	0.6695	0.5732
Std. Dev. (%)	2.5544	1.6586	1.5263	1.5230
Skewness	0.1486	2.1306	2.0078	1.7255
Kurtosis	5.3436	12.7884	11.7380	8.2205
Jarque-Bera	409.2518	8362.6136	6531.2672	2765.9193
	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Auto-Correlation Test				
Q(12)	27.6100	4194.9000	6646.7000	7664.3000
	(0.0060)	(0.0000)	(0.0000)	(0.0000)
$Q^2(12)$	534.2000			
	(0.0000)			

Table 2:

Estimation of Conditional Models for the Standardizing Process

$$X_t = \mu + \phi_1 X_{t-1} + \theta_1 \varepsilon_{t-1} + \varepsilon_t \quad \varepsilon_t \big| I_{t-1} \sim N(0, \sigma_t^2)$$
$$\sigma_t^2 = \omega^G + \alpha_1^G \varepsilon_{t-1}^2 + \beta_1^G \sigma_{t-1}^2$$
$$R_t = \varphi_t \varepsilon_t \quad \varepsilon_t \big| \mathrm{I}_{t-1} \sim f(1, \zeta_t)$$
$$\varphi_t = \omega^C + \alpha_1^C R_{t-1} + \beta_1^C \varphi_{t-1}$$
$$RRV\_30m_t = \tau_t^{R30} \varepsilon_t \quad \varepsilon_t \big| \mathrm{I}_{t-1} \sim f(1, \rho_t)$$
$$\tau_t^{R30} = \omega^{R30} + \alpha_1^{R30} RRV\_30m_{t-1} + \beta_1^{R30} \tau_{t-1}^{R30}$$
$$RRV\_5m_t = \tau_t^{R5} \varepsilon_t \quad \varepsilon_t \big| \mathrm{I}_{t-1} \sim f(1, \pi_t)$$
$$\tau_t^{R5} = \omega^{R5} + \alpha_1^{R5} RRV\_5m_{t-1} + \beta_1^{R5} \tau_{t-1}^{R5}$$

where X_t is the daily return, R_t is the daily range, RRV_30m_t and RRV_5m_t are the realized range volatility for 30-min and 5-min frequency, respectively. Panel A, B, C and D report the estimation of parameters in ARMA(1,1)-GARCH(1,1), CARR(1,1), RR_30m(1,1) and RR_5m(1,1) models. EWMA model is not required to estimate unknown parameters, so it is not shown below. Numbers in parentheses are *p*-values.

Panel A	ARMA(1,1)-GARCH(1,1)	
	S&P 500	Nasdaq
μ	0.0614	0.1220
	(0.0301)	(0.0022)
ψ_1	-0.9076	0.7381
	(0.0000)	(0.0000)
θ_1	0.8858	-0.7843
	(0.0000)	(0.0000)
ω^G	0.0615	0.0649
	(0.0000)	(0.0020)
α_1^G	0.0925	0.0765
	(0.0000)	(0.0000)
β_1^G	0.8726	0.9142
	(0.0000)	(0.0000)

Table 2:

(continued)

Panel B	CARR(1,1)	
	S&P 500	Nasdaq
ω^C	0.0571	0.0799
	(0.0001)	(0.0003)
$\alpha_1{}^C$	0.1695	0.2145
	(0.0000)	(0.0000)
$\beta_1{}^C$	0.7952	0.7590
	(0.0000)	(0.0000)
Panel C	**RR_30m(1,1)**	
	S&P 500	Nasdaq
ω^{R30}	0.0662	0.1261
	(0.0000)	(0.0000)
$\alpha_1{}^{R30}$	0.3098	0.4091
	(0.0000)	(0.0000)
$\beta_1{}^{R30}$	0.6516	0.5513
	(0.0000)	(0.0000)
Panel D	**RR_5m(1,1)**	
	S&P 500	Nasdaq
ω^{R5}	0.0793	0.1217
	(0.0000)	(0.0000)
$\alpha_1{}^{R5}$	0.3898	0.4342
	(0.0000)	(0.0000)
$\beta_1{}^{R5}$	0.5645	0.5279
	(0.0000)	(0.0000)

Table 3:

Descriptive Statistics for Standard Residual Item of Daily Returns in different models of S&P 500 and Nasdaq Index, 1997/01/02-2003/12/31

ARMA(1,1)-EWMA: $(z^E_{t-n+1},\ldots z^E_t) = (\dfrac{X_{t-n+1}-\widehat{\mu}_{t-n+1}}{\hat{\xi}_{t-n+1}},\ldots,\dfrac{X_t-\widehat{\mu}_t}{\hat{\xi}_t})$

ARMA(1,1)-GARCH(1,1): $(z^G_{t-n+1},\ldots z^G_t) = (\dfrac{X_{t-n+1}-\widehat{\mu}_{t-n+1}}{\widehat{\sigma}_{t-n+1}},\ldots,\dfrac{X_t-\widehat{\mu}_t}{\widehat{\sigma}_t})$

ARMA(1,1)-CARR(1,1): $(z^C_{t-n+1},\ldots z^C_t) = (\dfrac{X_{t-n+1}-\widehat{\mu}_{t-n+1}}{\widehat{\varphi}_{t-n+1}},\ldots,\dfrac{X_t-\widehat{\mu}_t}{\widehat{\varphi}_t})$

ARMA(1,1)-RR_30m(1,1): $(z^{R30}_{t-n+1},\ldots z^{R30}_t) = (\dfrac{X_{t-n+1}-\widehat{\mu}_{t-n+1}}{\widehat{\tau^{R30}_{t-n+1}}},\ldots,\dfrac{X_t-\widehat{\mu}_t}{\widehat{\tau^{R30}_t}})$

ARMA(1,1)-RR_30m(1,1): $(z^{R5}_{t-n+1},\ldots z^{R5}_t) = (\dfrac{X_{t-n+1}-\widehat{\mu}_{t-n+1}}{\widehat{\tau^{R5}_{t-n+1}}},\ldots,\dfrac{X_t-\widehat{\mu}_t}{\widehat{\tau^{R5}_t}})$

where $\widehat{\mu}_t$, $\hat{\xi}_t$, $\widehat{\sigma}_t$, $\widehat{\varphi}_t$, $\widehat{\tau^{R30}_t}$ and $\widehat{\tau^{R5}_t}$ are estimated by those models above, X_t is the daily return, and z_t is the standard residual item. The decay factor λ in EWMA model is set to be 0.94. Panel A and B report the descriptive statistics of S&P 500 and Nasdaq Index, respectively. Std. Dev. denotes standard deviation. Jarque-Bera is the test of normality. Q(12) and $Q^2(12)$ are the Ljung-Box statistic for auto-correlation test with 12 lags. Numbers in parentheses are *p*-values.

	Panel A - S&P 500				
	ARMA -EWMA	ARMA -GARCH	ARMA -CARR	ARMA -RR_30m	ARMA -RR_5m
Mean	-0.0478	-0.0440	-0.0440	-0.0452	-0.0448
Std. Dev.	1.0384	0.9990	0.9579	0.9647	0.9660
Skewness	-0.4081	-0.3192	-0.2498	-0.1957	-0.1822
Kurtosis	4.9656	4.3351	3.7251	3.5380	3.5420
Jarque-Bera	331.9879	160.5067	56.8348	31.2622	30.1309
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Auto-Correlation Test					
Q(12)	14.3890	14.6150	15.1220	13.9750	14.8030
	(0.2770)	(0.2630)	(0.2350)	(0.3020)	(0.2520)
$Q^2(12)$	12.0960	10.2340	10.6650	12.4420	11.2760
	(0.4380)	(0.5950)	(0.5580)	(0.4110)	(0.5050)

Table 3:

(continued)

	Panel B - Nasdaq				
	ARMA -EWMA	ARMA -GARCH	ARMA -CARR	ARMA -RR_30m	ARMA -RR_5m
Mean	-0.0399	-0.0424	-0.0396	-0.0329	-0.0312
Std. Dev.	1.0268	0.9995	0.9305	0.9393	0.9425
Skewness	-0.1734	-0.1507	-0.1401	-0.1066	-0.0939
Kurtosis	3.8944	3.4617	3.0817	2.9131	2.9365
Jarque-Bera	67.4501	22.2819	6.2434	3.7458	2.7750
	(0.0000)	(0.0000)	(0.0441)	(0.1537)	(0.2497)
Auto-Correlation Test					
Q(12)	7.4781	8.0460	9.0003	8.9905	8.9606
	(0.8240)	(0.7820)	(0.7030)	(0.7040)	(0.7060)
$Q^2(12)$	19.1900	13.6340	13.7970	15.2810	15.0560
	(0.0840)	(0.3250)	(0.3140)	(0.2260)	(0.2380)

Table 4:

Estimation of Tail Index for S&P 500 and Nasdaq Index, 1997/01/02-2003/12/31

$$\gamma(k) = \frac{1}{k}\sum_{j=1}^{k} \ln(X(n-j+1)) - \ln(X(n-k))$$

$$\gamma(k) = \beta_0 + \beta_1 k + \varepsilon(k), \quad k = 1, \ldots, \kappa$$

β_0 is the estimation of tail index by using the standard residual series in ARMA(1,1)-EWMA, ARMA(1,1)-GARCH(1,1), ARMA(1,1)-CARR(1,1), ARMA(1,1)-RR_30m(1,1) and ARMA(1,1)-RR_5m(1,1) models. v is the inverse value of β_0 and the degree of freedom in student-t distribution. The estimates below are using data from 1997/01/02 to 2003/12/31.

	S&P 500				
	EWMA	GARCH	CARR	RR_30m	RR_5m
β_0	0.3036	0.2653	0.2447	0.2293	0.2412
v	3.2935	3.7690	4.0858	4.3608	4.1467

	Nasdaq				
	EWMA	GARCH	CARR	RR_30m	RR_5m
β_0	0.2006	0.1346	0.1322	0.1033	0.1196
v	4.9855	7.4280	7.5623	9.6794	8.3618

Table 5:

The Average of Estimated VaR of S&P 500 and Nasdaq Index

The average VaR of total sample period and individual year are presented. In this table, the absolute value of VaR (positive-valued) is shown below. Panel A and B report the average VaR of S&P 500 and Nasdaq Index, respectively.

Panel A – S&P 500

	average VaR - 95%						
	Total	1998	1999	2000	2001	2002	2003
EWMA-normal	2.0905	1.9646	1.8962	2.1636	2.2363	2.5226	1.7634
GARCH-normal	2.1351	2.1024	1.9989	2.1069	2.2489	2.4872	1.8684
CARR-normal	2.1826	2.1381	2.0285	2.2014	2.2485	2.6122	1.8683
RR_30m-normal	2.1777	2.1336	2.0119	2.2137	2.2320	2.6316	1.8451
RR_5m-normal	2.1780	2.1476	2.0138	2.2209	2.2305	2.6278	1.8286
EWMA-VaR-x	2.2182	3.1776	1.8431	1.9984	2.0805	2.4403	1.7558
GARCH-VaR-x	2.1909	3.0158	1.9537	1.8840	2.0048	2.4182	1.8559
CARR-VaR-x	2.0149	1.3708	1.9346	2.1504	2.1828	2.5977	1.8633
RR_30m-VaR-x	2.0960	1.8351	1.9947	2.1638	2.1498	2.5997	1.8369
RR_5m-VaR-x	2.1031	1.9823	1.9846	2.1808	2.1706	2.4941	1.8088

	average VaR - 97.5%						
	Total	1998	1999	2000	2001	2002	2003
EWMA-normal	2.4908	2.3408	2.2593	2.5780	2.6645	3.0056	2.1011
GARCH-normal	2.5439	2.5050	2.3816	2.5104	2.6796	2.9635	2.2262
CARR-normal	2.6005	2.5475	2.4170	2.6229	2.6791	3.1124	2.2261
RR_30m-normal	2.5948	2.5422	2.3971	2.6376	2.6594	3.1355	2.1984
RR_5m-normal	2.5951	2.5589	2.3994	2.6462	2.6576	3.1310	2.1788
EWMA-VaR-x	3.2501	5.8983	2.4002	2.9918	2.8126	3.2095	2.1494
GARCH-VaR-x	3.1016	5.5330	2.4804	2.4269	2.7766	3.0959	2.2625
CARR-VaR-x	2.6099	2.0150	2.4718	2.7376	2.9160	3.2757	2.2554
RR_30m-VaR-x	2.6854	2.5607	2.4509	2.7529	2.7529	3.3566	2.2410
RR_5m-VaR-x	2.6955	2.7657	2.4622	2.7588	2.7456	3.2488	2.1917

Table 5:
(continued)

	average VaR - 99%						
	Total	1998	1999	2000	2001	2002	2003
EWMA-normal	2.9559	2.7779	2.6812	3.0593	3.1621	3.5669	2.4934
GARCH-normal	3.0190	2.9728	2.8264	2.9792	3.1799	3.5169	2.6419
CARR-normal	3.0861	3.0232	2.8683	3.1127	3.1794	3.6936	2.6418
RR_30m-normal	3.0793	3.0169	2.8448	3.1301	3.1559	3.7210	2.6089
RR_5m-normal	3.0797	3.0367	2.8474	3.1404	3.1538	3.7156	2.5857
EWMA-VaR-x	5.5669	13.6267	3.2605	5.3126	4.0252	4.3979	2.6578
GARCH-VaR-x	5.0499	12.6356	3.2346	3.2503	4.2254	4.0765	2.7737
CARR-VaR-x	3.5504	3.3920	3.2485	3.5835	4.1283	4.2215	2.7398
RR_30m-VaR-x	3.5595	3.8553	3.0383	3.5898	3.6320	4.4837	2.7556
RR_5m-VaR-x	3.5750	4.1645	3.0956	3.5681	3.5516	4.3933	2.6694

Panel B – Nasdaq

	average VaR - 95%						
	Total	1998	1999	2000	2001	2002	2003
EWMA-normal	4.1276	3.1161	3.4272	5.4083	5.8034	4.3808	2.6682
GARCH-normal	4.1884	3.0189	3.5761	5.3938	5.9496	4.3211	2.9125
CARR-normal	4.2096	3.0749	3.6114	5.5295	5.8093	4.4097	2.8615
RR_30m-normal	4.2185	3.1635	3.6191	5.5623	5.7974	4.3618	2.8448
RR_5m-normal	4.2294	3.2158	3.6016	5.6160	5.7992	4.3554	2.8252
EWMA-VaR-x	3.7958	2.8313	2.3963	5.2670	5.6273	4.0353	2.6580
GARCH-VaR-x	4.0191	2.7196	3.0801	5.3334	5.9062	4.2183	2.9023
CARR-VaR-x	4.0994	2.9378	3.2937	5.4785	5.7579	4.3254	2.8432
RR_30m-VaR-x	4.1542	2.9677	3.5619	5.5165	5.7708	4.3259	2.8221
RR_5m-VaR-x	4.1464	2.9231	3.5019	5.5864	5.7815	4.3225	2.8038

Table 5:
(continued)

	average VaR - 97.5%						
	Total	1998	1999	2000	2001	2002	2003
EWMA-normal	4.9179	3.7128	4.0835	6.4439	6.9147	5.2197	3.1792
GARCH-normal	4.9904	3.5970	4.2609	6.4267	7.0889	5.1485	3.4702
CARR-normal	5.0157	3.6638	4.3030	6.5884	6.9217	5.2541	3.4095
RR_30m-normal	5.0263	3.7692	4.3122	6.6274	6.9076	5.1970	3.3895
RR_5m-normal	5.0393	3.8316	4.2913	6.6914	6.9097	5.1894	3.3662
EWMA-VaR-x	5.1142	4.6938	3.3161	6.7204	7.1913	5.5914	3.2105
GARCH-VaR-x	5.3332	4.1710	4.2156	6.7848	7.5117	5.8608	3.5040
CARR-VaR-x	5.1457	3.8193	4.4451	6.6984	7.0589	5.4268	3.4721
RR_30m-VaR-x	5.1429	3.8587	4.5336	6.7599	7.0045	5.2908	3.4545
RR_5m-VaR-x	5.1518	3.8940	4.5788	6.7872	6.9599	5.3008	3.4341

	average VaR - 99%						
	Total	1998	1999	2000	2001	2002	2003
EWMA-normal	5.8363	4.4061	4.8460	7.6472	8.2059	6.1944	3.7728
GARCH-normal	5.9223	4.2687	5.0566	7.6268	8.4127	6.1099	4.1182
CARR-normal	5.9523	4.3479	5.1065	7.8187	8.2142	6.2352	4.0461
RR_30m-normal	5.9649	4.4731	5.1174	7.8650	8.1975	6.1675	4.0225
RR_5m-normal	5.9803	4.5471	5.0926	7.9409	8.2000	6.1585	3.9948
EWMA-VaR-x	7.5105	9.4854	4.9418	8.8004	9.5675	8.3928	3.8845
GARCH-VaR-x	7.5927	7.5551	6.1836	8.8458	9.9513	8.8219	4.2363
CARR-VaR-x	6.6150	5.1668	6.3858	8.2500	8.7261	6.9443	4.2676
RR_30m-VaR-x	6.4604	5.2217	5.9262	8.3500	8.5320	6.5182	4.2617
RR_5m-VaR-x	6.5199	5.4578	6.2460	8.2798	8.3806	6.5584	4.2390

Table 6:

The Number of Failures of Conditional VaR models under 95%, 97.5% and 99% Confidence Interval for S&P 500 and Nasdaq Index, 1997/01/02-2003/12/31

	95% level		97.5% level		99% level	
	S&P 500	Nasdaq	S&P 500	Nasdaq	S&P 500	Nasdaq
Theoretical Number	76	76	38	38	15	15
Variance-Covariance Method						
EWMA-normal	77	73	48	38	20	12
GARCH-normal	79	75	43	39	21	18
CARR-normal	64	65	33	28	20	12
RR_30m-normal	63	68	36	29	22	12
RR_5m-normal	59	73	34	29	20	12
Extreme Value Theory						
EWMA-VaR-x	85	142	30	79	6	44
GARCH-VaR-x	83	97	40	36	9	12
CARR-VaR-x	97	73	44	26	14	7
RR_30m-VaR-x	75	71	37	25	10	8
RR_5m-VaR-x	72	81	36	27	7	7

Table 7: Results of VaR models for S&P 500 in 95% Confident Interval, 1997/01/02-2003/12/31

	VaR-normal					VaR-x				
	GARCH	EWMA	CARR	RR_30m	RR_5m	GARCH	EWMA	CARR	RR_30m	RR_5m
Theoretical Exceptions	76	76	76	76	76	76	76	76	76	76
Actual Exceptions	79	77	64	63	59	83	85	97	75	72
Conservatism										
MRB	0.0036	-0.0213	0.0182	0.0109	0.0115	0.0385	0.0537	-0.0657	-0.0276	-0.0219
RMSRB	0.1162	0.1347	0.0647	0.0860	0.0965	0.2823	0.3125	0.2184	0.1221	0.1102
Accuracy										
BLF (%)	5.2283	5.0960	4.2356	4.1694	3.9047	5.4931	5.6254	6.4196	4.9636	4.7651
mean excess	0.6467	0.6960	0.6811	0.6572	0.6845	0.7698	0.6494	0.6912	0.6826	0.6426
LR_{uc}	1.9995	1.9999	1.9926	1.9912	1.9836	1.9977	1.9963	1.9837	2.0000	1.9994
LR_{ind}	1.9988	2.0000	1.9992	1.9946	1.9960	1.9879	1.9575	1.9705	1.9999	1.9974
LR_{cc}	3.9983	3.9999	3.9918	3.9858	3.9796	3.9856	3.9538	3.9542	3.9999	3.9967
MOC	1.0100	1.0007	0.9588	0.9627	0.9569	1.0226	1.0392	1.1190	0.9975	0.9869
Efficiency										
MRSB	0.0087	-0.0254	-0.0284	-0.0313	-0.0366	0.0568	0.0897	0.0403	-0.0346	-0.0392
Error Efficiency	0.5915	0.5826	0.5878	0.5844	0.5840	0.6150	0.6013	0.6115	0.5796	0.5761

Table 8: Results of VaR models for Nasdaq in 95% Confident Interval, 1997/01/02-2003/12/31

	VaR-normal					VaR-x				
	GARCH	EWMA	CARR	RR_30m	RR_5m	GARCH	EWMA	CARR	RR_30m	RR_5m
Theoretical Exceptions	76	76	76	76	76	76	76	76	76	76
Actual Exceptions	75	73	65	68	73	97	142	73	71	81
Conservatism										
MRB	0.0256	0.0107	0.0281	0.0274	0.0288	-0.0305	-0.0989	-0.0029	0.0087	0.0030
RMSRB	0.1432	0.1327	0.0956	0.1040	0.1082	0.1765	0.3890	0.0984	0.1152	0.1345
Accuracy										
BLF (%)	4.9636	4.8312	4.3018	4.5003	4.8312	6.4196	9.3977	4.8312	4.6989	5.3607
mean excess	1.1356	1.0446	0.9863	0.8905	0.8402	1.0881	1.1492	0.9706	0.9160	0.8460
LR_{uc}	2.0000	1.9997	1.9940	1.9970	1.9997	1.9837	1.9002	1.9997	1.9990	1.9987
LR_{ind}	1.9994	1.9997	1.9999	1.9984	1.9997	1.9965	1.9620	1.9997	1.9976	1.9980
LR_{cc}	3.9994	3.9994	3.9939	3.9954	3.9994	3.9802	3.8623	3.9994	3.9966	3.9967
MOC	0.9913	0.9918	0.9601	0.9772	0.9876	1.1072	1.3727	0.9785	0.9894	1.0097
Efficiency										
MRSB	-0.0146	-0.0288	-0.0427	-0.0262	-0.0145	0.0383	0.1910	-0.0538	-0.0317	-0.0171
Error Efficiency	0.5750	0.5654	0.5627	0.5610	0.5628	0.5674	0.7851	0.5547	0.5573	0.5614

Table 9: Results of VaR models for S&P 500 in 97.5% Confident Interval, 1997/01/02-2003/12/31

	VaR-normal					VaR-x				
	GARCH	EWMA	CARR	RR_30m	RR_5m	GARCH	EWMA	CARR	RR_30m	RR_5m
Theoretical Exceptions	38	38	38	38	38	38	38	38	38	38
Actual Exceptions	43	48	33	36	34	40	30	44	37	36
Conservatism										
MRB	-0.0524	-0.0747	-0.0391	-0.0460	-0.0455	0.1340	0.1853	-0.0349	-0.0152	-0.0114
RMSRB	0.1498	0.1702	0.1066	0.1212	0.1271	0.4340	0.4970	0.2217	0.1318	0.1175
Accuracy										
BLF (%)	2.8458	3.1767	2.1840	2.3825	2.2502	2.6473	1.9854	2.9120	2.4487	2.3825
mean excess	0.6678	0.6628	0.7648	0.6734	0.7032	0.7694	0.5923	0.7738	0.6677	0.5800
LR_{uc}	1.9964	1.9878	1.9959	1.9995	1.9975	1.9993	1.9881	1.9950	1.9999	1.9995
LR_{ind}	1.9998	1.9990	1.9994	1.9999	1.9996	2.0000	1.9852	1.9797	1.9999	1.9999
LR_{cc}	3.9961	3.9867	3.9953	3.9994	3.9971	3.9993	3.9733	3.9747	3.9998	3.9994
MOC	1.0289	1.0538	0.9648	0.9798	0.9751	1.0075	0.9486	1.0454	0.9944	0.9523
Efficiency										
MRSB	-0.0194	-0.0192	-0.0675	-0.0597	-0.0637	0.1503	0.1323	0.0145	-0.0148	-0.0529
Error Efficiency	0.6321	0.6244	0.6346	0.6335	0.6334	0.6681	0.6697	0.6475	0.6429	0.6436

Table 10: Results of VaR models for Nasdaq in 97.5% Confident Interval, 1997/01/02-2003/12/31

	VaR-normal					VaR-x				
	GARCH	EWMA	CARR	RR_30m	RR_5m	GARCH	EWMA	CARR	RR_30m	RR_5m
Theoretical Exceptions	38	38	38	38	38	38	38	38	38	38
Actual Exceptions	39	38	28	29	29	36	79	26	25	27
Conservatism										
MRB	-0.0127	-0.0275	-0.0104	-0.0111	-0.0098	0.0440	-0.0202	0.0172	0.0139	0.0166
RMSRB	0.1440	0.1340	0.0958	0.1040	0.1073	0.2110	0.4659	0.1071	0.1232	0.1492
Accuracy										
BLF (%)	2.5811	2.5149	1.8531	1.9193	1.9193	2.3825	5.2283	1.7207	1.6545	1.7869
mean excess	1.0566	0.9108	1.0571	0.8904	0.8787	1.0286	1.1042	1.0251	0.9651	0.9449
LR_{uc}	1.9998	2.0000	1.9798	1.9843	1.9843	1.9995	1.8923	1.9684	1.9613	1.9745
LR_{ind}	2.0000	1.9948	1.9974	1.9979	1.9979	1.9897	1.9004	1.9960	1.9951	1.9967
LR_{cc}	3.9998	3.9948	3.9772	3.9822	3.9822	3.9892	3.7927	3.9644	3.9564	3.9712
MOC	1.0174	1.0000	0.9509	0.9538	0.9366	0.9885	1.7597	0.9314	0.9306	0.9218
Efficiency										
MRSB	-0.0289	-0.0610	-0.0894	-0.0871	-0.1024	-0.0077	0.6369	-0.0829	-0.0860	-0.0915
Error Efficiency	0.6186	0.6100	0.6140	0.6131	0.6139	0.6302	0.7288	0.6227	0.6207	0.6229

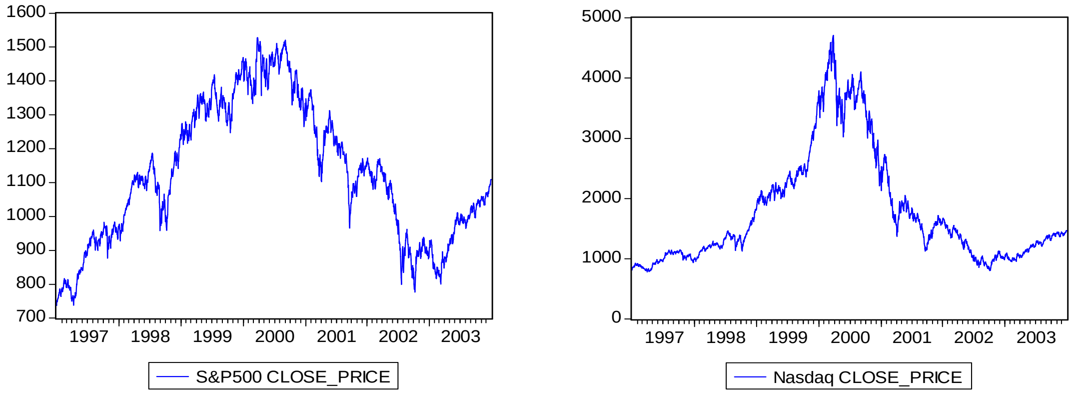
Table 11: Results of VaR models for S&P 500 in 99% Confident Interval, 1997/01/02-2003/12/31

	VaR-normal					VaR-x				
	GARCH	EWMA	CARR	RR_30m	RR_5m	GARCH	EWMA	CARR	RR_30m	RR_5m
Theoretical Exceptions	15	15	15	15	15	15	15	15	15	15
Actual Exceptions	21	20	20	22	20	9	6	14	10	7
Conservatism										
MRB	-0.1306	-0.1499	-0.1194	-0.1259	-0.1255	0.2719	0.3912	0.0038	-0.0079	-0.0078
RMSRB	0.2344	0.2459	0.2050	0.2126	0.2143	0.6821	0.8156	0.2542	0.1694	0.1522
Accuracy										
BLF (%)	1.3898	1.3236	1.3236	1.4560	1.3236	0.5956	0.3971	0.9265	0.6618	0.4633
mean excess	0.7495	0.9248	0.7074	0.5354	0.5824	1.1615	0.8957	1.0965	0.6575	0.6153
LR_{uc}	1.9815	1.9865	1.9865	1.9761	1.9865	1.9484	1.8318	1.9989	1.9676	1.8840
LR_{ind}	1.9901	1.9950	1.9950	1.9943	1.9950	1.9980	1.9988	1.9027	1.9978	1.9985
LR_{cc}	3.9716	3.9814	3.9814	3.9704	3.9814	3.9464	3.8306	3.9016	3.9654	3.8826
MOC	1.0364	1.0409	1.0349	1.0318	1.0343	0.8884	0.8205	0.9849	0.9341	0.9590
Efficiency										
MRSB	-0.0687	-0.0856	-0.0581	-0.0679	-0.0652	0.1833	0.1969	0.0217	-0.0413	-0.0152
Error Efficiency	0.6773	0.6706	0.6836	0.6825	0.6825	0.7373	0.7530	0.7118	0.7171	0.7186

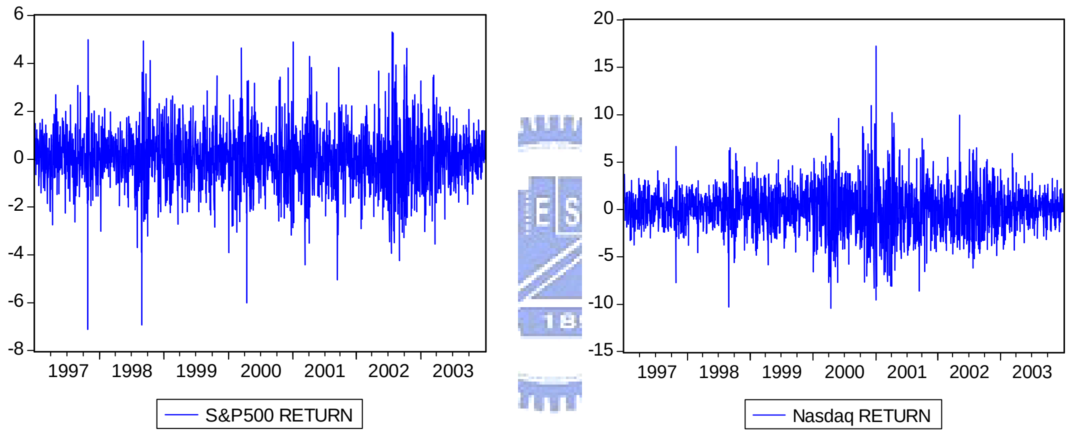
Table 12: Results of VaR models for Nasdaq in 99% Confident Interval, 1997/01/02-2003/12/31

	VaR-normal					VaR-x				
	GARCH	EWMA	CARR	RR_30m	RR_5m	GARCH	EWMA	CARR	RR_30m	RR_5m
Theoretical Exceptions	15	15	15	15	15	15	15	15	15	15
Actual Exceptions	18	12	12	12	12	12	44	7	8	7
Conservatism										
MRB	-0.0688	-0.0833	-0.0669	-0.0677	-0.0665	0.1616	0.1027	0.0418	0.0152	0.0320
RMSRB	0.1745	0.1674	0.1359	0.1421	0.1441	0.3386	0.6233	0.1459	0.1440	0.1859
Accuracy										
BLF (%)	1.1913	0.7942	0.7942	0.7942	0.7942	0.7942	2.9120	0.4633	0.5295	0.4633
mean excess	1.1381	1.3632	0.9610	0.7978	0.7692	0.9455	1.2216	1.0074	0.5326	1.0661
LR_{uc}	1.9946	1.9901	1.9901	1.9901	1.9901	1.9901	1.8305	1.8840	1.9215	1.8840
LR_{ind}	1.9838	1.9973	1.9973	1.9973	1.9973	1.9973	1.7788	1.9985	1.9983	1.9985
LR_{cc}	3.9785	3.9874	3.9874	3.9874	3.9874	3.9874	3.6093	3.8826	3.9198	3.8826
MOC	1.0643	0.9797	0.9804	0.9778	0.9509	0.9286	2.3250	0.9093	0.9137	0.9011
Efficiency										
MRSB	-0.0942	-0.1813	-0.1630	-0.1656	-0.1877	-0.0278	1.2487	-0.1328	-0.1500	-0.1462
Error Efficiency	0.6670	0.6607	0.6671	0.6673	0.6676	0.7174	0.7296	0.6990	0.6922	0.6948

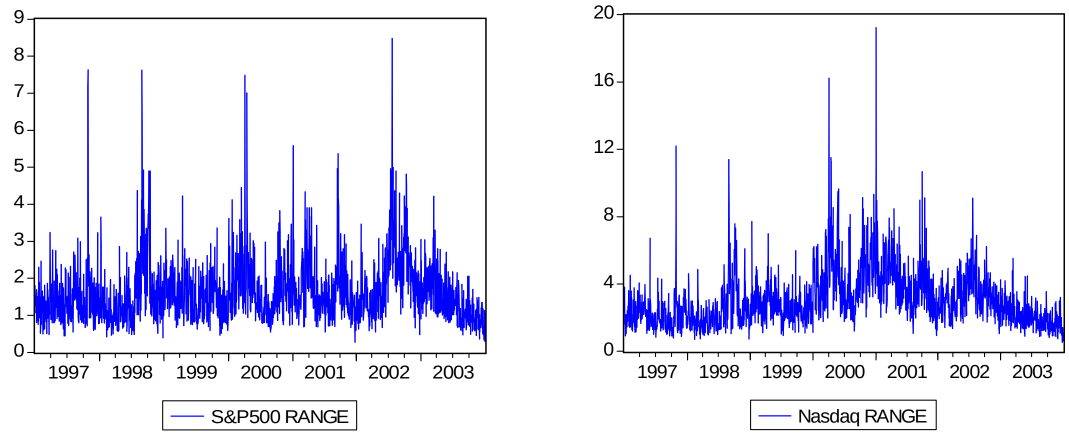
Panel A : Close Prices



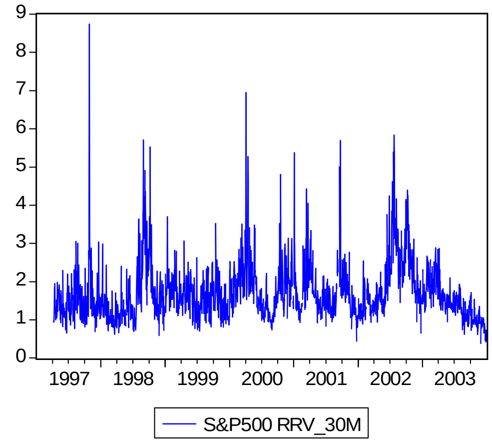
Panel B : Daily Returns

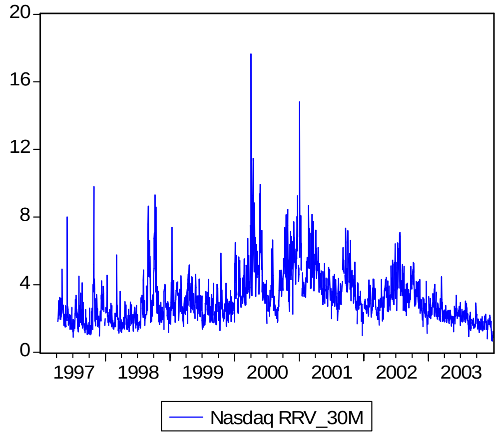


Panel C : Daily Ranges

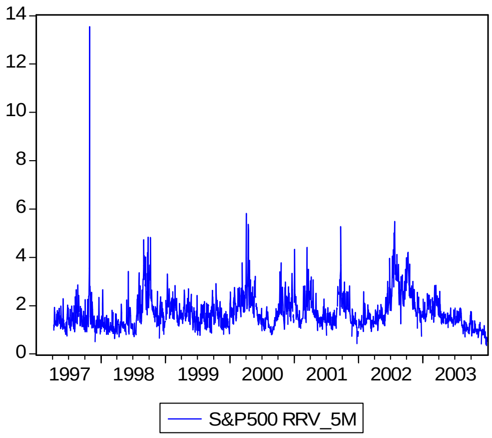


Panel D : RRV_30m





Panel E : RRV_5m



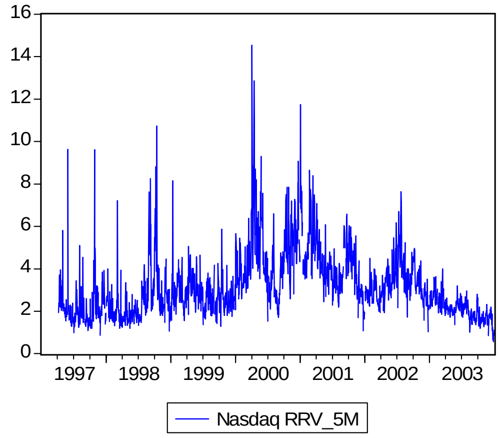


Figure 1: S&P 500 and Nasdaq Index Daily Closing Prices, Returns, Ranges, RRV_30m and RRV_5m, 1997/01/02-2003/12/31

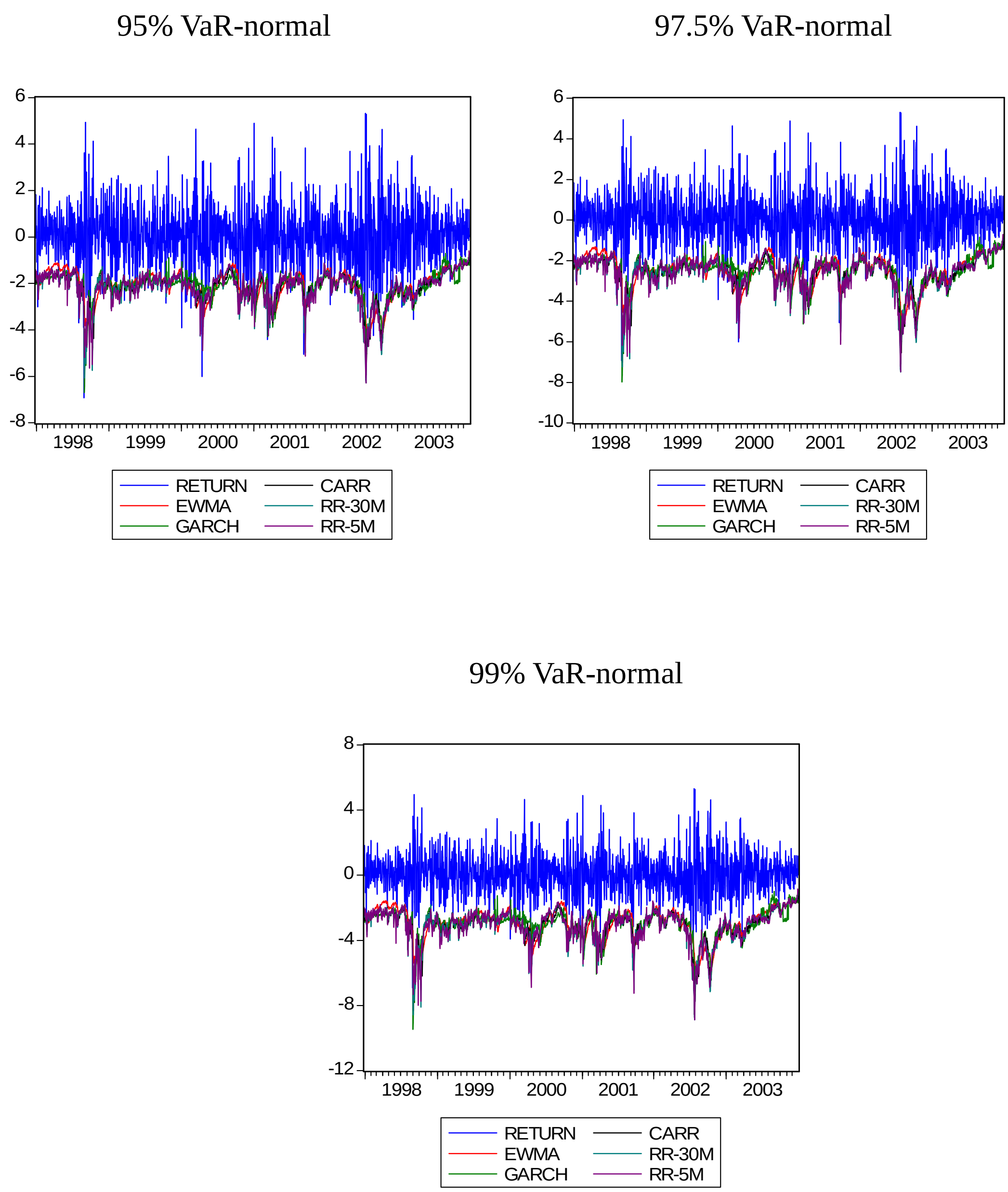


Figure 2: Daily Returns and VaR-normal Estimates for S&P 500 with GARCH, EWMA, CARR model, RR_30m and RR_5m models under 95%, 97.5%, and 99% Confidence Interval

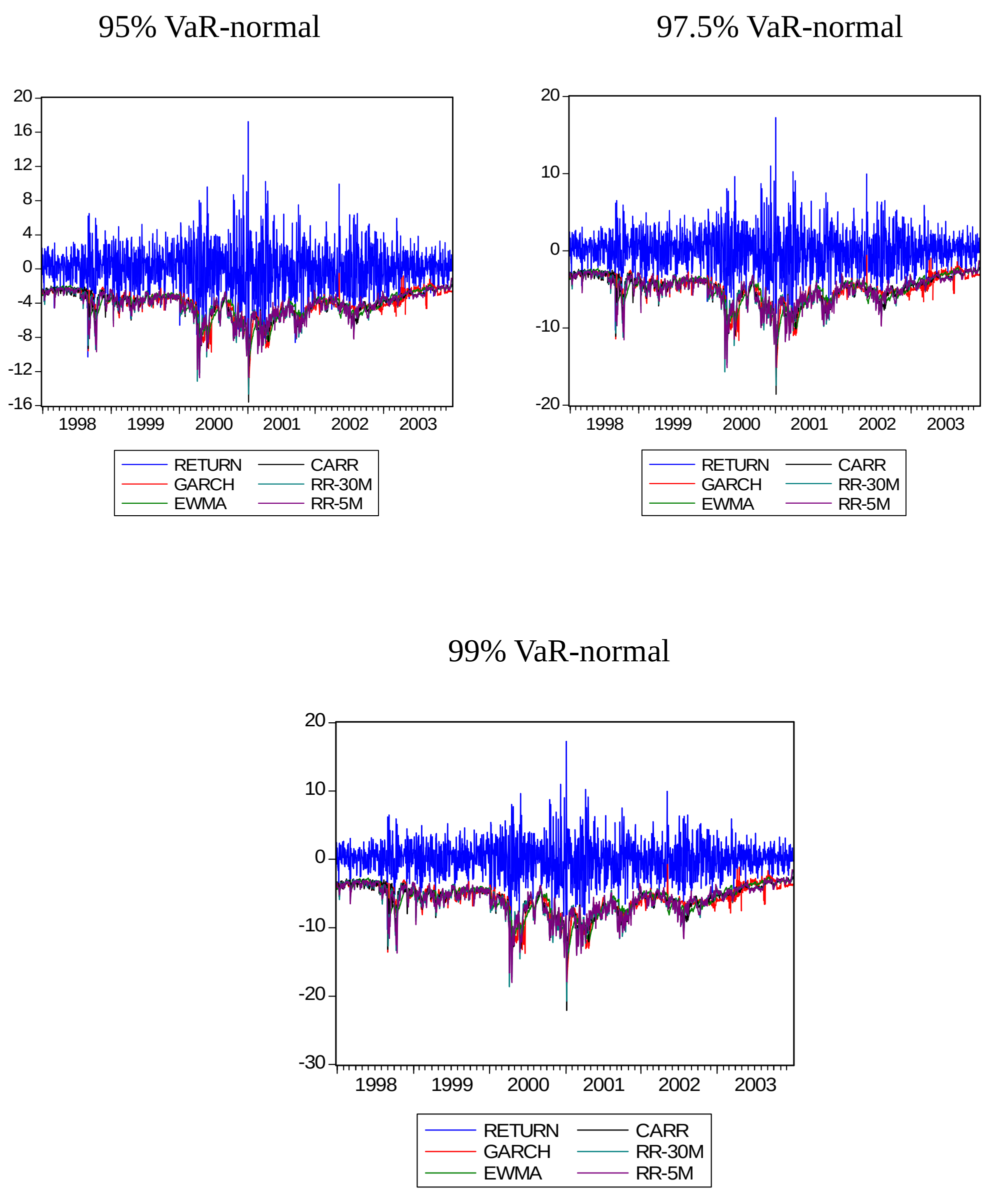


Figure 3: Daily Returns and VaR-normal Estimates for Nasdaq Index with GARCH, EWMA, CARR model, RR_30m and RR_5m models under 95%, 97.5%, and 99% Confidence Interval

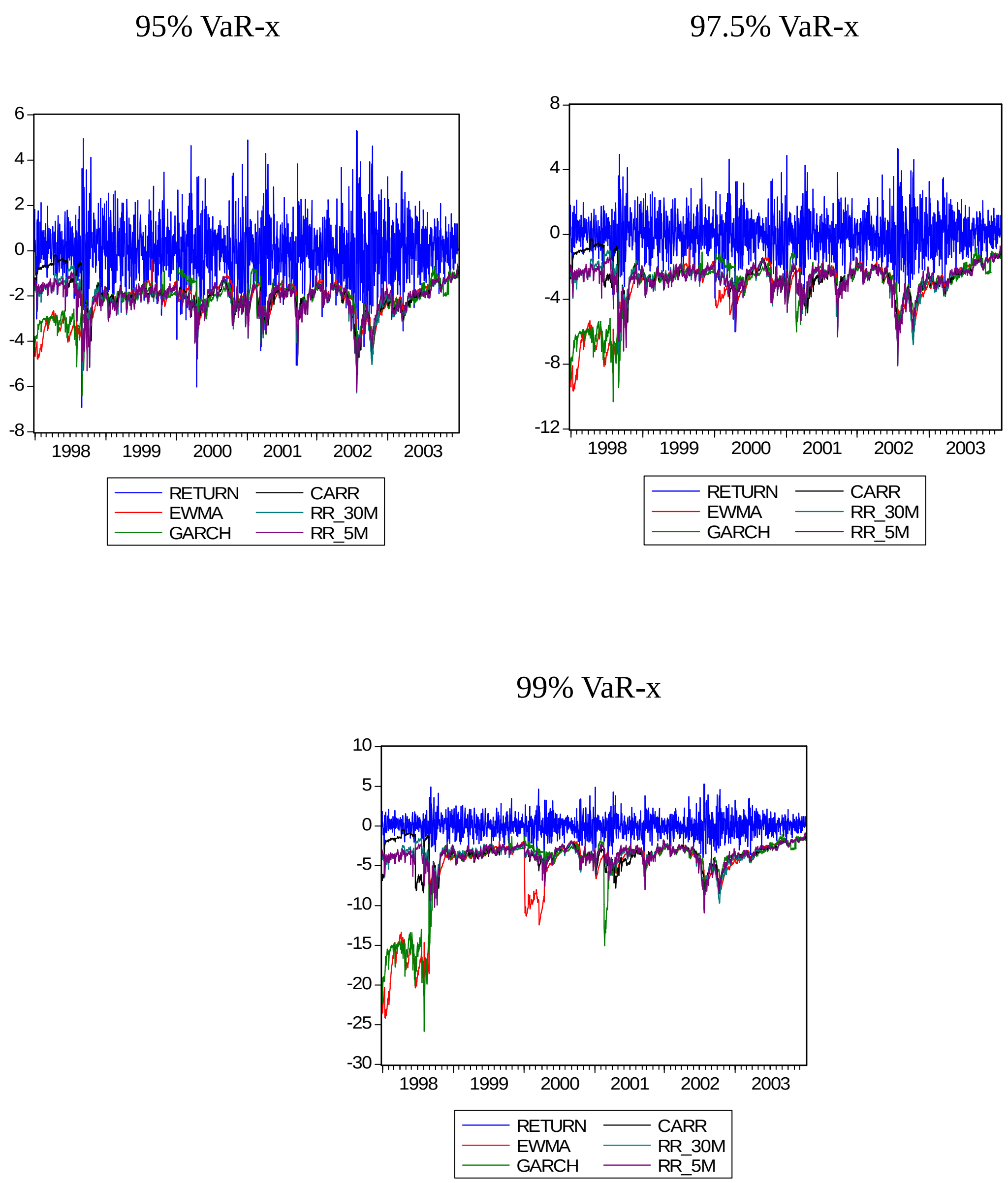


Figure 4: Daily Returns and VaR-x Estimates for S&P 500 Index with GARCH, EWMA, CARR model, RR_30m and RR_5m models under 95%, 97.5%, and 99% Confidence Interval

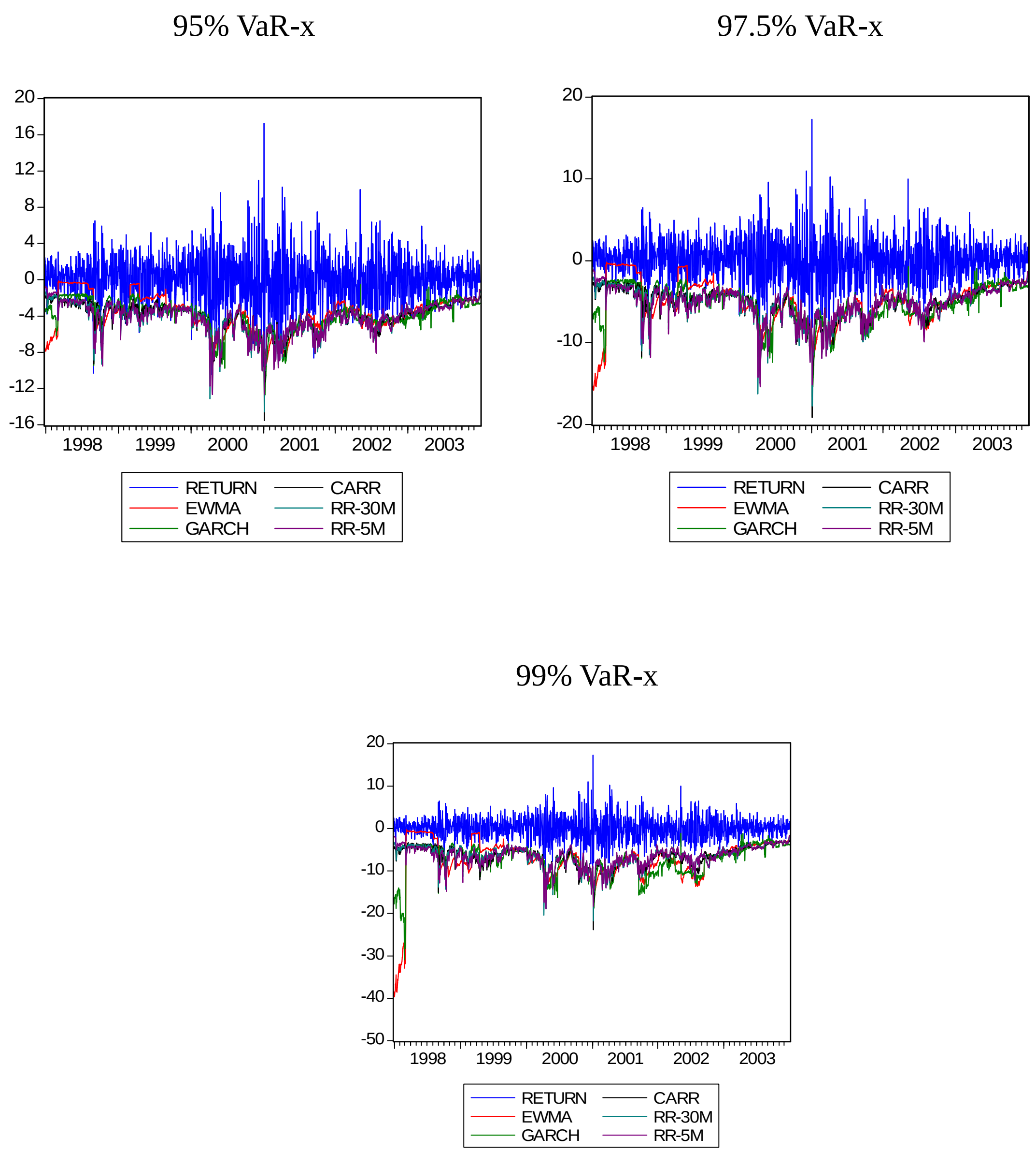


Figure 5: Daily Returns and VaR-x Estimates for Nasdaq Index with GARCH, EWMA, CARR model, RR_30m and RR_5m models under 95%, 97.5%, and 99% Confidence Interval