國　立　交　通　大　學

財 務 金 融 研 究 所

碩 士 論 文

雙佔代理市場下獨家代理權之最適轉換策略：實質選擇權賽局之應用

Optimal Switching Strategy of an Exclusive Agency in a Duopoly Agent Market: An Application of Real Options Game

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摘　要：

本研究針對雙佔代理市場來分析最適轉換獨家代理商的決策並根據 Shackleton et al.（2004）所提出的模型加以修改更符合雙佔代理市場的一般假設。本研究考慮一個新的變數：佣金比率。透過實質選擇權賽局的方法得到最適轉換策略，此最適轉換策略隱含市場均衡。根據本研究結果顯示，發現高淨成長率以及低獲利波動度的代理商最容易成為獨家代理商，並且在佣金比率越高以及獲利波動相關係數越高和轉換成本越高的環境下，轉換越不容易發生。最後，本研究發現當遲滯效果變大不能代表轉換機率變小。

關鍵字：實質選擇權賽局、最適轉換策略、獨家代理

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Abstract:

This paper modifies from Shackleton et al. (2004) to analyze the optimal switching strategy decision in a duopoly agent market. We introduce a commission rate. By the real options game approach, we derive the optimal switching strategy showing the equilibrium in the market. The results demonstrate that the agent with higher net growth rate and lower volatility is more likely to be the exclusive agent. In addition, under the conditions of high commission rate, high correlation coefficient of the net profitability volatility and high total switching costs, switching is less likely to appear. Moreover, we find out the hysteresis is not the factor to affect the switching probability.

Keywords: real options game, optimal switching strategy, exclusive agent

Graduate Institute of Finance

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1. Introduction

1.1 Research Motivation

GDP in Taiwan rapidly increases from NTD 10.8 trillion in 2003 to NTD 12.1 trillion in 2009.[1] After Taiwan joined WTO in 2002, the import duties are decreased from average 30% to 17.5%. Therefore, there are more choices for consumers buying imported products. On demand side, imported products are not only a necessity but also an emblem to present personal status which demonstrates how prestigious the person is. On supply side, each foreign brand wants to promote its products in Taiwan. However, the brand holder does not know the market in Taiwan. Most brands choose an agent in Taiwan to operate the brand and to sell the products. Finally, there are more and more agents selling imported products in Taiwan.

Recently, when the economic condition becomes unstable and the financial tsunami hits to the world, every business project should be surveyed. We want to understand how the brand holder selects the exclusive agent under uncertainty situation.[2] The Figure 1.1 explains the brand holder's decision in a duopoly agent market.

[1] The data comes from Ministry of Economic Affairs in Taiwan.
[2] The exclusive contract is about only an exclusive agent is allowed to purchase products from the producer of brand.

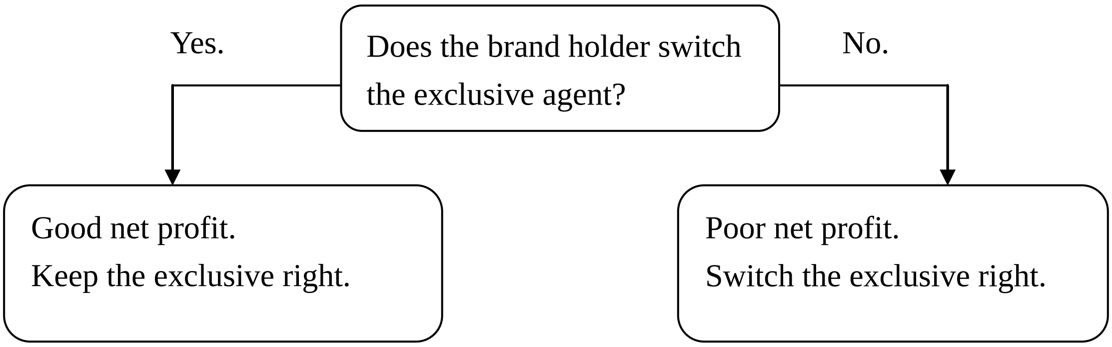

Figure 1.1

The Process of the Brand Hold's Decision

1.2 Research Purposes

The purposes of this paper are as follows:

1. Derive the optimal switching strategy by using the real options game method.
2. Examine what characteristics of an exclusive agent are ideal for foreign brand holders.
3. Find out under what kind of circumstances, switching is more likely to appear.

1.3 Structure

The rest of this paper is organized as follows: Section 2 reviews literatures of the switch options, optimal exercise policies and model's framework. Section 3 demonstrates the model in this paper. Section 4 derives the optimal solutions of switching and calculates the switching probability. Section 5 describes the sensitive analysis. Section 6 presents our conclusion and suggestion. The Figure 1.2 explains the process of this paper.

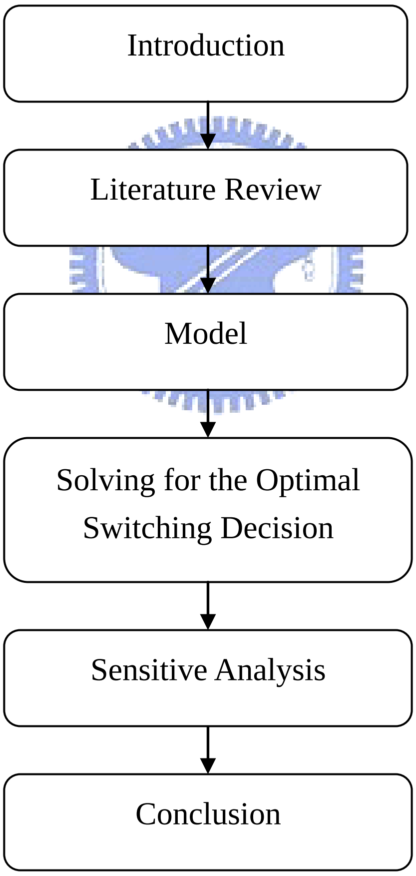

Figure 1.2

The Structure of this Paper

2. Literature Review

When we execute the investment project, we always use the net present value (NPV) approach to evaluate the project. Hayes and Abernathy (1980) and Hayes and Garvin (1982) mentioned some disadvantages on the NPV method. Afterwards, Myers (1987) suggested evaluating the project by using the structure of option. Later, Dixit and Pindyck (1994) provided a survey of the real options literatures. There are three important characteristics using the real options method. Firstly, the investment is irreversible totally. Secondly, the future returns from investment are uncertainty. Third, there are options to decide the timing for investing. Real options, including the value of flexibility apply the thoughts of traditional financial options to evaluate a project. Using real options approach, the manager adapts optimal strategy to maximize the net profit. There are some real options such as switch option, option to defer, growth option and other options. This paper focuses on switch option.

2.1 Switch Options

Switch options mean that the manager has an option to change the composition of products when price or demand changes. In addition, he can have an option to choose the production procedure to produce the same output. There are some typical industries which previous literatures applied for. The first part of the industry is product switches, e.g., consumer electronics, toys, machine parts and automobiles. The second part is input switches, e.g., oil, electric power (oil/gas) and crop switching. There are some literatures about switch option as follows:

Margrabe (1978) developed an equation for the value of the option to exchange one risky asset for another within a stated period. The formula applied to American options and European options. Thus, Margrabe (1978) found a closed-form expression for American options and a put-call parity.

Kensinger (1987) assumed the binominal distribution with raw material (input) and product (output) and discussed the situations in one switch and multiple switches. The results demonstrated that with more flexibility, the switch option is more valuable.

Kulatilaka and Trigeorgis (1994) focused the executing process of investment project. On one hand, we can input different resources to produce specific products. On the other hand, we can also input the same resources to produce different products. Therefore, they presented the value of flexibility into switch operating modes and then developed the general model.

2.2 Optimal exercise policies under duopolistic strategic competition

Under a duopolistic strategic competition, optimal exercise policies were the focus of Smets (1991), Grenadier (1996) and Lambrecht and Perraudin (2003).

Smets (1991) introduced the symmetrical duopoly model under uncertainty and examines entry strategy in a duopoly market facing the stochastic demand. He then found the equilibrium of asymmetric leader and follower.

Grenadier (1996) used game theory method to analyze strategic options. He assumed two decision makers, one is the leader and the other is the follower. He then used sub-game perfect Nash equilibrium to obtain the optimal investment thresholds. Finally, he emphasized the timing of real estate development.

Lambrecht and Perraudin (2003) used real options approach to discuss the perfect competition firms' optimal investment strategy under incomplete information and advantages of first mover. They find out not only the growth opportunity in aircraft industry but also the timing of optimal investment.

2.3 Model

Dixit (1989) mentioned that the hysteresis is produced by entry costs.[3] Shackleton et al. (2004) modified entry/exit model from Dixit (1989). Their model focused on a two player game, and each firm can be "monopolist" for a period. There were some important points in the paper. First of all, only one firm existed in a duopoly market. Secondly, only two firms competed to survive in the market, and each idle firm has an option to claim the market by sinking the investment costs. Thirdly, in order to present the equilibrium in the market, there was a fictitious central planner who can decide the active firm and maximize total market value.[4] Fourthly, they replaced the absolute magnitudes of two firms' net profit with relative magnitudes of two firms' net profit. Fifthly, they used the dynamic programming method to evaluate the optimal thresholds and calculate the switching probability. Finally, they applied their model to the aircraft industry.

[3] Hysteresis means the interval between two thresholds. In the hysteresis, we wait and see. Besides, there are more details in Dixit (1989).

[4] Shackleton et al. (2004) mentioned that the concept of fictitious central planner followed the result of Slade (1994). Besides, Baldursson (1998) also brought up the notion in application.

3. Model

This paper's model follows Shackleton et al. (2004) and focuses on a duopoly agent market. In this paper, the brand holder has switch options and each agent can be an exclusive agent for a period. Our model modifies some points from their model as Table 3.1.

Table 3.1

Difference between the Model of Shackleton et al. (2004) and this Paper

	Shackleton et al.	In this paper
Applied industry	Duopoly aircraft market	Duopoly agent market
Central planner	Fictitious central planner	The brand holder
Commission rate	No	Yes

3.1 The Basic Setting

In the market, there are two agents, i and j, are competing for an exclusive right. Each agent has its own marketing skills and dealer-operated locations to promote the product. Therefore, each agent's net profit is different from its rival.

We define $S_i(t)$ and $S_j(t)$ as net profit of the product that each agent earn at time t, if the agent acquires exclusive right. $S_i(t)$ and $S_j(t)$ abide by the Geometric Brownian Motion (GBM), and the equations are given:

$$\frac{dS_i(t)}{S_i(t)} = (u_i - \delta_i)\, dt + \sigma_i\, dZ_i(t), \text{ and} \tag{3.1}$$

$$\frac{dS_j(t)}{S_j(t)} = (u_j - \delta_j)\, dt + \sigma_j\, dZ_j(t). \tag{3.2}$$

In the equations, u_i and u_j are the each agent's net profit growth rates considering the operating costs, inventory costs and human resources costs and reflect the final earnings in operation. σ_i and σ_j are the standard deviations of $S_i(t)$ and $S_j(t)$ and reflect the final earning's volatility. δ_i and δ_j are the delay costs of each agent which mean the time value of delaying the option such like the interest rate. These parameters are constants and greater than zero.[5]

The future net profit of each agent is uncertain and can be fluctuated from exogenous shocks, specified as the increments of standard Wiener process, $dZ_i(t)$ and $dZ_j(t)$. These shocks can either be agent-specific (e.g., an entrepreneurial

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[5] McDonald and Siegel (1986) mentioned V(t), the stochastic present value of revenues from operating a fixed scale project. The project earns a random cash flow S(t) :

$$\frac{dS(t)}{S(t)} = \alpha dt + \sigma dZ(t)$$

Then the expected present value is

$$V(t) = E\left[\int_t^{+\infty} S(k)e^{-uk}dk\right] = \frac{S(t)}{u-\alpha}$$

E denotes the real-world expectations operator and u is the risk-adjusted rate at which future cash flows are discounted. Because there is a bound for the project value, we require $\delta = u - \alpha > 0$. Expectations can be taken under the risk-neutral measure, in which case the discount rate would be risk-free rate r. According to these reasons, we directly replace α with $u - \delta$ in this paper.

characteristic, a marketing skill in product sales and location of store) or consumer behavior changing (e.g., an unexpected shift in market demand or changing in customers' tastes). Each agent can be the exclusive agent in a duopoly agent market. We allow $S_i(t)$ and $S_j(t)$ to be correlated Brownian motions in order to reflect the fluctuation with the common economic factors. The relation can be expressed as $dZ_i(t)dZ_j(t)=\rho dt$, where ρ is the correlation coefficient of the two agents' net profit and assumed constant.

This paper considers the total switching costs, K, including the switching costs (such as the costs to change company and product names and the costs to move the inventory) and the penalty costs (such as the costs to break the contract). In addition, we force the exclusive agent to be the most "efficient" agent. Net profit of each agent can express operating the market efficiently. For example, when agent j is currently active and $S_i(t)$ is higher than $S_j(t)$, we would expect the brand holder will switch the exclusive right from agent j to agent i. In the market, exclusive right can shift instantaneously.

3.2 Solution Method

Costs have been stressed in the real options literature reviews. In this paper, there are not only switching costs but also penalty costs. The total of switching costs which includes the switching costs and the penalty costs are the reason why there is an option value of delaying. When each agent is idle, it has an option to claim the market from its rival by sinking the total switching costs. The exercise strategy of each agent would specify the optimal stopping time for sinking the total switching costs. The problem is complicated that each agent's exercise strategy should take its rival's strategy into account. In a duopoly agent market, the exercise strategy of each agent has to be simultaneously determined. Therefore, these exercise strategies will be an optimal equilibrium behavior.

However, our problem of finding the equilibrium in the market can be converted to one dimension, thanks to the result in Slade (1994). She mentioned a general N-player game where each firm acts strategically is identical to a fictitious central planner's optimization problem.

We assume that the market is perfect and frictionless. We then use the equilibrium to present the brand holder's optimization problem. The brand holder chooses only one agent to be the exclusive agent of the product. He chooses agent i (j) to be an exclusive agent when $S_i(t)$ is higher (less) than $S_j(t)$. Besides, we assume the brand holder is risk neutral and wants to maximize the expected present value of net profit from the market, net of switching costs and penalty costs. We can use dynamic programming to solve the brand holder's optimization problem.

We assume agent j is currently active and define $F_j(S_i(t),S_j(t))$ as a value of switch option from agent j to agent i. The return of holding this option for the brand holder is composed of two parts. One is the expected capital gain, $E(dF_j(S_i(t),S_j(t)))/dt$, and the other is the dividend, $(1-q)S_j(t)$, where q is the commission rate. The expected capital gain plus the dividend equal the normal return. The normal return is assumed to be risk-free rate. We can write the equations as follows:

$$E(dF_j(S_i(t),S_j(t)))+(1-q)S_j(t)dt = r\,F_j(S_i(t),S_j(t))dt, \text{ and} \tag{3.3}$$

$$E(dF_i(S_i(t),S_j(t)))+(1-q)S_i(t)dt = r\,F_i(S_i(t),S_j(t))dt, \tag{3.4}$$

where r is risk-free rate and assumed to be constant.

We will use Ito's lemma to calculate the expected capital gain and transfer to a second-order partial differential equation (PDE) from the equation (3.3). [6] Then, we will obtain the PDE of $F_j(S_i,S_j)$ as follows:

$$\begin{aligned}&\frac{1}{2}\left(\frac{\partial^2 F_j(S_i,S_j)}{\partial S_i^{\,2}}(S_i\sigma_i)^2+\frac{\partial^2 F_j(S_i,S_j)}{\partial S_j^{\,2}}(S_j\sigma_j)^2+2\times\frac{\partial^2 F_j(S_i,S_j)}{\partial S_i\partial S_j}\rho\times(S_i\sigma_i)\times(S_j\sigma_j)\right)\\&+\frac{\partial F_j(S_i,S_j)}{\partial S_i}S_i(r-\delta_i)+\frac{\partial F_j(S_i,S_j)}{\partial S_j}S_j(r-\delta_j)+(1-q)S_j-r\,F_j(S_i,S_j)\;=0.\end{aligned} \tag{3.5}$$

Following the same steps from the equation (3.4), we can acquire PDE of $F_i(S_i,S_j)$ as follows:

[6] Dixit and Pindyck (1994) mentioned Ito's lemma in their book at p.79.

$$\frac{1}{2}\left(\frac{\partial^2 F_i(S_i,S_j)}{\partial S_i^{\,2}}(S_i\sigma_i)^2+\frac{\partial^2 F_i(S_i,S_j)}{\partial S_j^{\,2}}(S_j\sigma_j)^2+2\times\frac{\partial^2 F_i(S_i,S_j)}{\partial S_i\partial S_j}\rho\times(S_i\sigma_i)\times(S_j\sigma_j)\right)$$
$$+\frac{\partial F_i(S_i,S_j)}{\partial S_i}S_i(r-\delta_i)+\frac{\partial F_i(S_i,S_j)}{\partial S_j}S_j(r-\delta_j)+(1-q)S_i-rF_i(S_i,S_j)=0. \qquad (3.6)$$

3.3 Reducing the Problem's Dimensionality

In equations (3.5) and (3.6), solving for two stochastic variables is difficult. By using natural homogeneity, we can reduce to one dimension. This concept comes from McDonald and Siegel (1986). Generally, the brand holder does not focus on the absolute net profit of the product. In order to execute the optimal switching policy, the brand holder needs to focus on the relative net profit of the two agents. Hence, we define $P=\frac{S_i}{S_j}$ as the relative net profit of two agents and can acquire the relations between $F(S_i,S_j)$ and $f(P)$ as follows:

$$F_j(S_i,S_j)=S_j\times f_j\left(\frac{S_i}{S_j}\right)=S_j\times f_j(P),\ \text{and} \qquad (3.7)$$

$$F_i(S_i,S_j)=S_j\times f_i\left(\frac{S_i}{S_j}\right)=S_j\times f_i(P), \qquad (3.8)$$

where $f_j(P)$ and $f_i(P)$ is the homogeneous degree one function.

Substituting the equation (3.6) into the equation (3.4), we can obtain the equation:

$$\frac{1}{2}f_j''(P)\times P^2\times v^2+f_j'(P)\times P\times(\delta_j-\delta_j)-f_j(P)\times(\delta_j)+(1-q)=0, \qquad (3.9)$$

where $v^2=\sigma_i^{\,2}-2\rho\sigma_i\sigma_j+\sigma_i^{\,2}$. Following same steps, we can thus acquire the equation as follows:

$$\frac{1}{2} f_i''(P) \times P^2 \times v^2 + f_i'(P) \times P \times (\delta_j - \delta_i) - f_i(P) \times (\delta_j) + (1-q)P = 0. \qquad (3.10)$$

The equation (3.9) is an ordinary differential equation of unknown function $f_j(P)$. We calculate the general solution and the particular solution to acquire the $f_j(P)$ function, written: $f_j(P) = AP^a + BP^b + \frac{1-q}{\delta_j}$, where constant a>1 and b<0.[7]

In addition, we need to consider the boundaries. When agent j is active and P approaches zero, the switch option from agent j to agent i will be worthless. According to this condition, B must to be zero. Hence, we can write $f_j(P)$ as follows:

$$f_j(P) = AP^a + \frac{1-q}{\delta_j}, \qquad (3.11)$$

where a > 1, A is constant and to be determined.

Following the same steps, we can obtain $f_i(P)$ as follows:

$$f_i(P) = BP^b + \frac{1-q}{\delta_i} P, \qquad (3.12)$$

where b < 0, B is constant and to be determined.

---

[7] Define $f_j(P) = P^x$. The characteristic quadratic function, $\frac{1}{2} v^2 x(x-1) + (\delta_j - \delta_i)x - \delta_j = 0$, has roots a>1 and b<0 given by

$$a, b = 0.5 - \frac{\delta_j - \delta_i}{v^2} \pm \sqrt{\left(\frac{\delta_j - \delta_i}{v^2} - 0.5\right)^2 + \frac{2\delta_j}{v^2}}.$$

4. Solving for the Optimal Switching Decision

This paper reduces the complicate problem which is two dimensions to one dimension in order to force switch policy to be determined easily. The boundaries are $\overline{P}$ and $\underline{P}$, where $\overline{P} > 1$ and $1 > \underline{P} > 0$. While agent j is currently active and switch option value is deeply "in-the-money" ($P = \overline{P} > 1$), the brand holder switches the exclusive agent from agent j to agent i by paying total switching costs, K. Using the dynamic programming method, we can obtain the boundaries of optimal switching and acquire hysteresis which is the interval between two thresholds.

4.1 Thresholds Solution by Dynamic Programming Method

We can use the dynamic programming method to obtain the optimal boundaries. The optimal switch policy is determined by value-matching conditions as follows:[8]

$$f_j\left(\overline{P}\right) = f_i\left(\overline{P}\right) - K, \text{ and} \tag{4.1}$$

$$f_j\left(\underline{P}\right) - K = f_i\left(\underline{P}\right); \tag{4.2}$$

and smooth-pasting conditions as follows:[9]

$$f_j'\left(\overline{P}\right) = f_i'\left(\overline{P}\right), \text{ and} \tag{4.3}$$

$$f_j'\left(\underline{P}\right) = f_i'\left(\underline{P}\right). \tag{4.4}$$

[8] Value-matching condition means that the two situations are not different. The two situations are holding the option to switch from agent j to agent i and holding the option to switch from agent i to agent j.

[9] Smooth-pasting condition ensures the optimal point on P. Besides, there are more details in Dixit and Pindyck (1994).

The optimal solutions, A, B, $\bar{P}$ and $\underline{P}$, can be determined by solving the equations (4.1) to (4.4). Because the equations are non-linear in $\bar{P}$ and $\underline{P}$, we have to use the numerical method to evaluate the thresholds. We can establish the whole system to be a matrix. Substituting equations (3.11) and (3.12) into the equations (4.1) to (4.4), we can acquire the following equations:

$$A\bar{P}^{a} + \frac{1-q}{\delta_j} = B\bar{P}^{b} + \frac{1-q}{\delta_i}\bar{P} - K, \tag{4.5}$$

$$A\underline{P}^{a} + \frac{1-q}{\delta_j} - K = B\underline{P}^{b} + \frac{1-q}{\delta_i}\underline{P}, \tag{4.6}$$

$$Aa\bar{P}^{a} = Bb\bar{P}^{b} + \frac{1-q}{\delta_i}\bar{P}, \text{ and} \tag{4.7}$$

$$Aa\underline{P}^{a} = Bb\underline{P}^{b} + \frac{1-q}{\delta_i}\underline{P}. \tag{4.8}$$

We use the numerical method to evaluate A, B, $\bar{P}$ and $\underline{P}$. In order to simplify the equations, we define $G_j = \frac{1-q}{\delta_j}$, and $G_i = \frac{1-q}{\delta_i}$.

Setting all equations into a matrix, we can obtain the solutions as follows:

$$\begin{bmatrix} \bar{P} \\ \underline{P} \\ A \\ B \end{bmatrix} = \begin{bmatrix} \frac{1}{a} \times \frac{\underline{P}\bar{P}^{b+a} - \underline{P}^{b}\bar{P}^{1+a}}{\bar{P}^{b}\underline{P}^{a} - \underline{P}^{b}\bar{P}^{a}} + \frac{G_j}{G_i} - \frac{1}{b}\frac{\underline{P}\bar{P}^{a+b} - \bar{P}^{1+b}\underline{P}^{a}}{\bar{P}^{b}\underline{P}^{a} - \underline{P}^{b}\bar{P}^{a}} + \frac{K}{G_i} \\ \frac{1}{a} \times \frac{\underline{P}^{1+a}\bar{P}^{b} - \underline{P}^{b+a}\bar{P}}{\bar{P}^{b}\underline{P}^{a} - \underline{P}^{b}\bar{P}^{a}} + \frac{G_j}{G_i} - \frac{1}{b}\frac{\underline{P}^{1+b}\bar{P}^{a} - \bar{P}\underline{P}^{a+b}}{\bar{P}^{b}\underline{P}^{a} - \underline{P}^{b}\bar{P}^{a}} - \frac{K}{G_i} \\ \frac{G_i}{a} \times \frac{\underline{P}\bar{P}^{b} - \underline{P}^{b}\bar{P}}{\bar{P}^{b}\underline{P}^{a} - \underline{P}^{b}\bar{P}^{a}} \\ \frac{G_i}{b} \times \frac{\underline{P}\bar{P}^{a} - \bar{P}\underline{P}^{a}}{\bar{P}^{b}\underline{P}^{a} - \underline{P}^{b}\bar{P}^{a}} \end{bmatrix}. \tag{4.9}$$

Finally, these two switch thresholds, $\bar{P}$ and $\underline{P}$, are determined by dynamic programming method.

4.2 Calculating the Switching Probability

This section calculates the switching probability. The exclusive agent operates the market for a period until the brand holder executes the switch option. We define $P=\frac{S_i}{S_j}$ and obtain the equation as follows:[10]

$$\frac{dP(t)}{P(t)}=\left(\xi+\frac{1}{2}v^2\right)dt+vdW(t), \tag{4.10}$$

where
$$\xi=u_i-u_j-\left(\delta_i-\delta_j\right)\ \frac{1}{2}\left(\sigma_i^2-\sigma_j^2\right),$$

$$v^2=\sigma_i^{\,2}-2\rho\sigma_i\sigma_j+\sigma_j^{\,2}, \text{ and}$$

$$dW=\frac{1}{v}\left(\sigma_i dZ_i-\sigma_j dZ_j\right).$$

We then define the stopping time, $\tau_j=\inf\left\{t\geq 0:P(t)\geq \overline{P}\right\}$. The stopping time is the first time when $P=\overline{P}$ and P starting from $P(0)\in\left(\overline{P},\underline{P}\right)$. We follow Shackleton et al. (2004) and define $P(0)=\frac{\overline{P}+\underline{P}}{2}$. Time τ_j is a random variable measuring the time interval between now and the time when switching appears. We use the Corollary 7.2.2 in Shreve (2004) and change the variable to obtain the equation, written as:

$$\Pr\left\{\tau_j\leq T\right\}=N\left(\frac{-\ln\left(\frac{\overline{P}}{P(0)}\right)+\xi T}{v\sqrt{T}}\right)+\left(\frac{\overline{P}}{P(0)}\right)^{2\xi/v^2}N\left(\frac{-\ln\left(\frac{\overline{P}}{P(0)}\right)-\xi T}{v\sqrt{T}}\right), \tag{4.11}$$

where ξ and v are defined above and N(·) is the standard normal cumulative distribution function. Equation (4.11) measures the switching probability in time T.

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[10] We obtain the equation (4.10) by using Eq.(1.11) of Harrison (1985)

5. Sensitive Analysis

This section discusses the sensitive analysis based on relevant parameters and is divided into two parts; one is for the thresholds under the changes of environmental parameters in section 5.1, and the other is for switching probability under the changes of the parameters in section 5.2. We observe the influence on the optimal switching decision in $\overline{P}$ and $\underline{P}$ under different conditions. All of the conditions are assumed to be fixed and focus on one fluctuated parameter, and then we acquire the tendency of thresholds. Furthermore, we analyze the results and illustrate the meaning of the figures. In section 5.2, we use the equation (4.11) to present the probability of switching the exclusive agent in time T. Finally, we demonstrate all of the relations in tables to summarize the results.

5.1 Thresholds under the Changes of the Environmental parameters

We analyze the influences of the environmental parameters on the switching thresholds. Under different conditions, we observe the tendency of boundaries. Because we assume that two agents have the same profitability to sell the product, two agents' parameters are the same. Table 5.1 shows all of the parameters in this paper.

Table 5.1

Parameters for this Paper

Parameters	**Idle agent (agent i)**	**Active agent (agent j)**
Growth rate	u_i	u_j
Standard deviation of net profit	σ_i	σ_j
Cost of delay	δ_i	δ_j
Total switching costs	K	
Correlation coefficient of the two agents	ρ	
Commission rate	q	
Time from now	T	

We hypothesize all of the parameters are fixed; except for that one parameter is fluctuated. We assume the value of parameter as following: K =2, $u_i = u_j$ =0.11, $\delta_i = \delta_j$ =0.03, $\sigma_i = \sigma_j$ =0.03, ρ =0.5, q=0.3 and T=5. In this section, the changes of environmental parameters mean that each agent can not be avoided because of the industry changes. When the economy becomes unstable, the volatilities of both agents change in the same way and at the same time. We then define the total volatility as $\sigma = \sigma_i = \sigma_j$. We divide this section into four parts as follows: change of total switching costs, changes of total volatility, changes of correlation coefficient and change of commission rate.

1. The influence of total switching costs on the most optimal switching thresholds is demonstrated in Figure 5.1

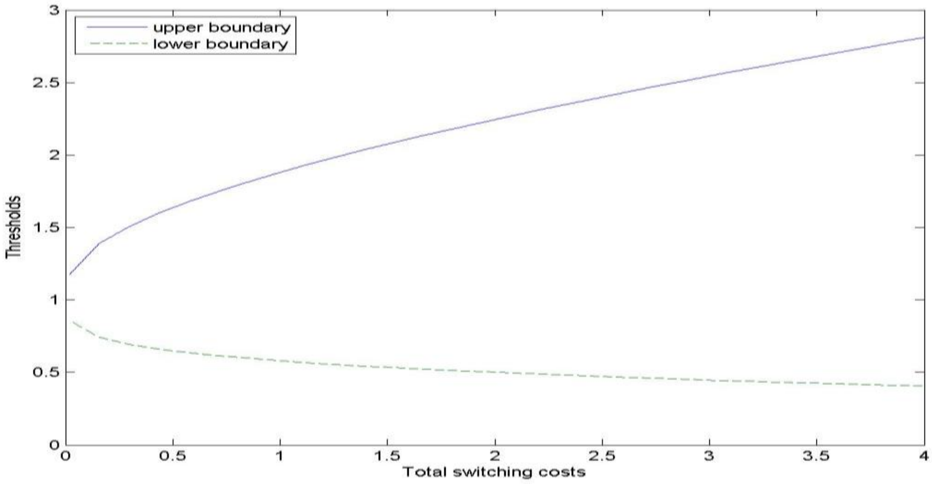

Figure 5.1

Relation between Total Switching Costs and Thresholds

Figure 5.1 explains when the total switching costs, K, including the switching costs and penalty costs increase from 0 to 3, the switching threshold j→i increases, and the switching threshold i→j decreases; therefore, the hysteresis increases. In a duopoly agent market, when the total switching costs increase, the brand holder may not be willing to change the exclusive right by paying more expensive switching costs than before and needs more net profit difference of two agents to cover the increasing switching costs. Therefore, the interval of thresholds becomes bigger. In other words, when the exercise cost increases, the brand holder may not be willing to exercise option until the switch option value is sufficiently in the money. Finally, the gap of two thresholds increases.

2. The influence of the total volatility of net profit on the most optimal switching thresholds is demonstrated in Figure 5.2

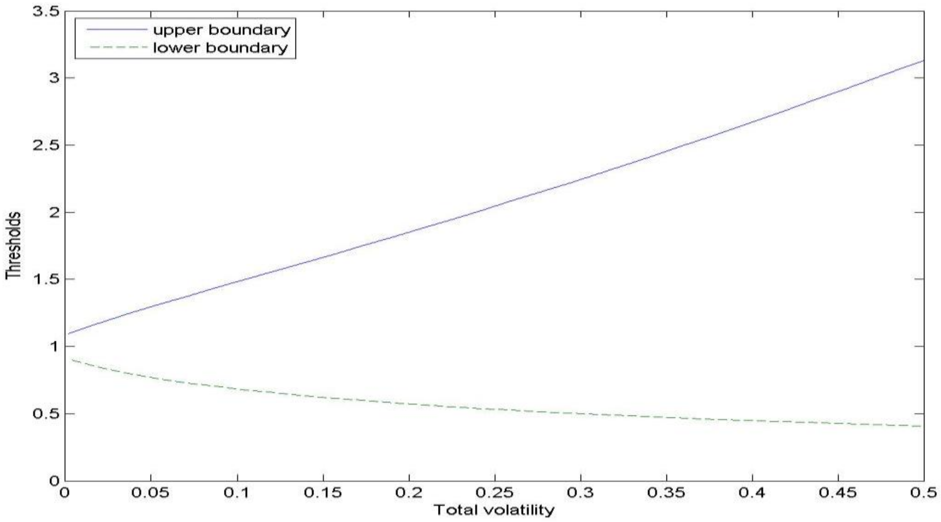

Figure 5.2

Relation between Total Volatility and Thresholds

Figure 5.2 explains when the total volatility, σ, increases from 0 to 0.6, the switching threshold j→i increases, and the switching threshold i→j decreases; therefore, the hysteresis increases. When the situation of industry is more fluctuated than before, there is a bigger possibility for active agent to make the net profit well. Then the brand holder may not be willing to change the exclusive agent until net profit difference of two agents becomes bigger. In other words, when the total volatility increases, the risk which the brand holder faces increases. Since needing more risk premium to face a more uncertain environment, the brand holder may not be willing to exercise the option until the switch option value is sufficiently in the money. Finally, the gap of two thresholds increases.

3. The influence of the correlation coefficient between two agent's net profit on the most optimal switching thresholds is demonstrated in Figure 5.3

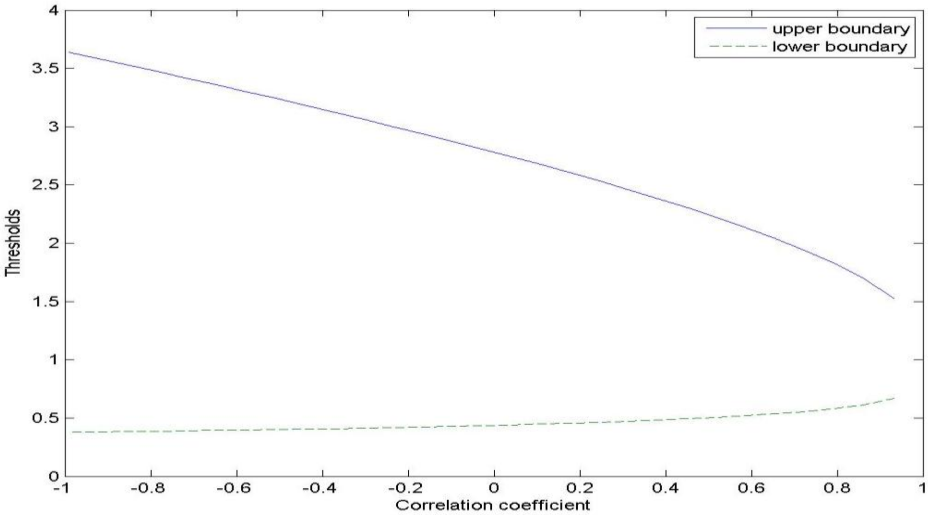

Figure 5.3

Relation between Correlation Coefficient and Thresholds

Figure 5.3 explains when the correlation coefficient, ρ, increases from -1 to 1, the switching threshold j→i decreases, and the switching threshold i→j increases; therefore, the hysteresis decreases. When the correlation coefficient increases, the the risk which the brand holder faces decreases. Since not needing more risk premium, the brand holder may be willing to switch the exclusive right under small net profit difference of two agents. The result shows that when the correlation coefficient increases, the gap of two thresholds decreases.

4. The influence of the commission rate on the most optimal switching thresholds is demonstrated in Figure 5.4

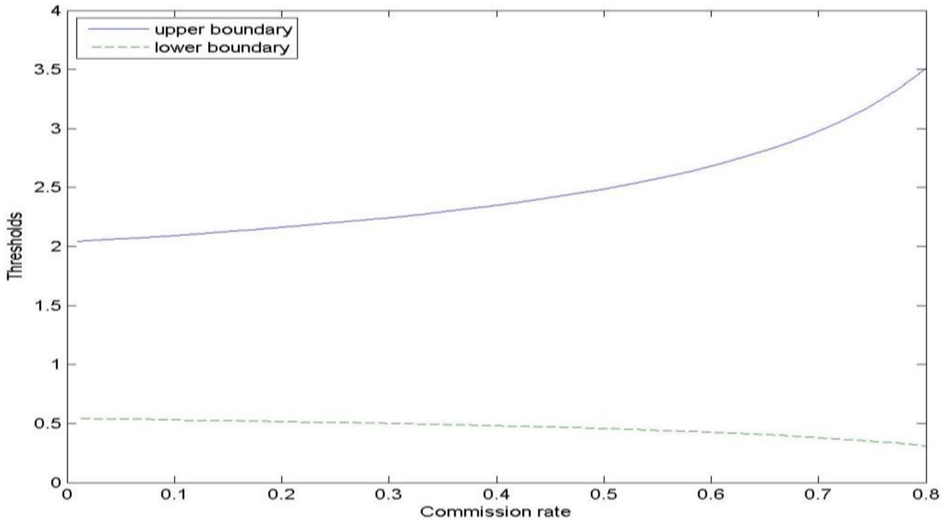

Figure 5.4

Relation between Commission Rate and Thresholds

Figure 5.4 explains when the commission rate, q, increases from 0.1 to 0.9, the switching threshold j→i increases, and the switching threshold i→j decreases; therefore, the hysteresis increases. The brand holder obtains the cash flow from active agent's net profit. Due to the increasing commission rate, the cash flow and the net profit difference of two agents decreases. The brand holder may not be willing to switch the exclusive agent until the net profit difference of two agents is sufficiently bigger. In other words, we can think the commission rate as a variable cost. When the variable cost increases, the total net profit of each agent decreases. The brand holder needs more the net profit difference to cover the increasing variable cost. Finally, the interval of two thresholds increases.

We acquire all of the relations between environmental parameters and thresholds. Table 5.2 shows all of the environmental parameters influence on thresholds and hysteresis.

Table 5.2

Relations between Environmental Parameters and Thresholds

Parameters / Thresholds	Total switching costs K	Total Volatility σ	Correlation coefficient ρ	Commission rate q
$\overline{P}$	(+)	(+)	(-)	(+)
$\underline{P}$	(-)	(-)	(+)	(-)
Hysteresis	(+)	(+)	(-)	(+)

To summarize, we can distinguish the results into three factors. One is the total switching costs, another is risk which the brand holder faces, and the other is cash flow for the brand holder. Hysteresis is increased by increasing the total switching costs and the risk which the brand hold faces and by decreasing the cash flow for the brand holder. Most of results are similar with Dixit (1994).

5.2 Switching Probability under the Changes of the Parameters

Section 5.2 discusses the sensitive analysis under one parameter changed. We obtain the separate relations between the switching probability (4.11) and each parameter. Furthermore, we analyze the results and explain financial implications. Finally, we tabulate the relations.

We set one parameter fluctuated and others fixed at one time. We assume agent j is currently active and set the value of parameter as follows: K=2, $u_i = u_j$=0.11, $\delta_i = \delta_j$=0.03, $\sigma_i = \sigma_j$=0.03, ρ=0.5, q=0.3 and T=5. In this section, each parameter can change independently. Besides, each agent has own strategy to make its parameter different from its rival. We analyze the switching probability with each parameter changing at one time.

1. The influence of the correlation coefficient between two agents' net operating profit on the probability of switching the exclusive right in time T is demonstrated in Figure 5.5

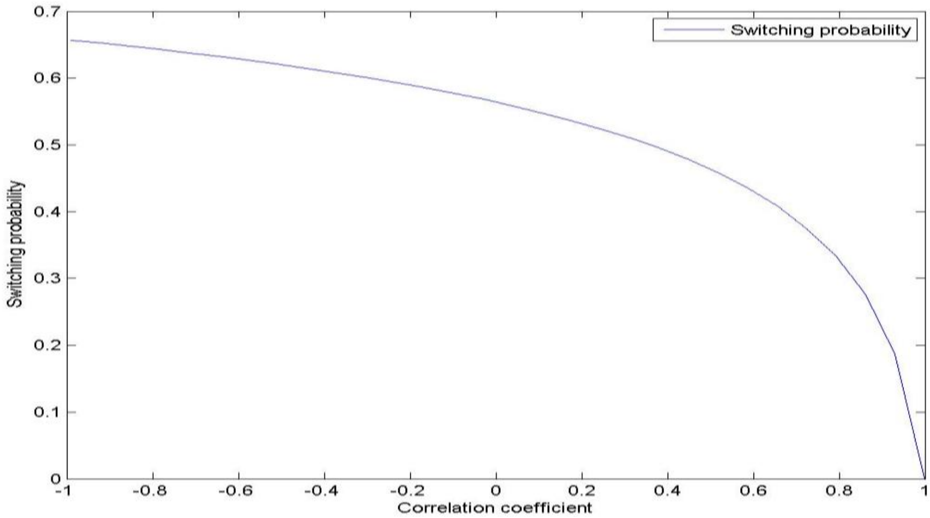

Figure 5.5

Relation between Correlation Coefficient and Switching Probability

Figure 5.5 explains that when the correlation coefficient, ρ, increases from -1 to 1, the switching probability decreases. We know that the brand holder exercises the switch option depend on the net profit difference of two agents. When the correlation coefficient increases, the net profit difference decreases. With the same total switching costs, the brand holder may not be willing to switch the exclusive right because of decreasing the net profit difference. Finally, there is a negative relation between the correlation coefficient and the switching probability.

2. The influence of the commission rate on the probability of switching the exclusive right in time T is demonstrated in Figure 5.6

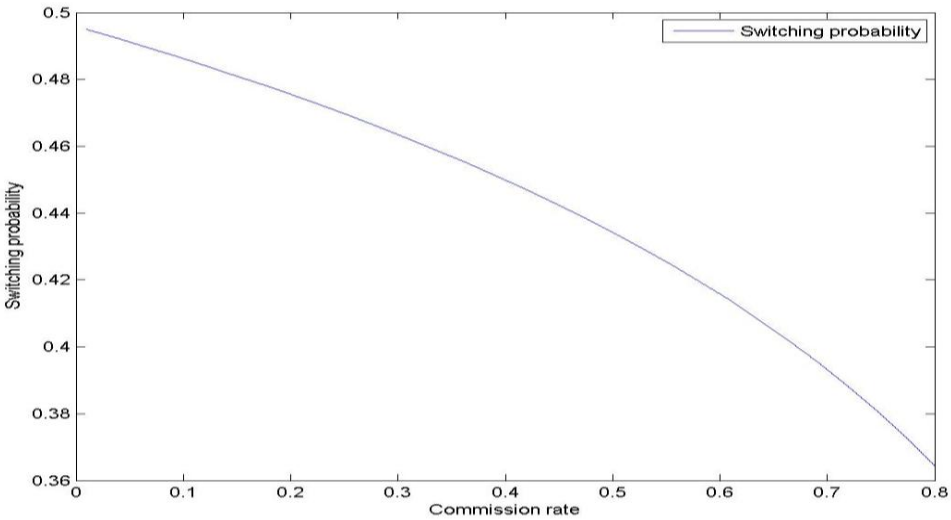

Figure 5.6

Relation between Commission Rate and Switching Probability

Figure 5.6 explains when the commission rate, q, increases from 0 to 0.8, the probability of switching the exclusive right in time T decreases. The brand holder obtains the cash flow from active agent's net profit. Due to the increasing commission rate, the cash flow and the net profit difference decrease. The brand holder may not be willing to switch the exclusive right under lower net profit difference. Finally, there is a negative relation between the commission rate and the switching probability.

3. The influence of the Total switching costs on the probability of switching the exclusive right in time T is demonstrated in Figure 5.7

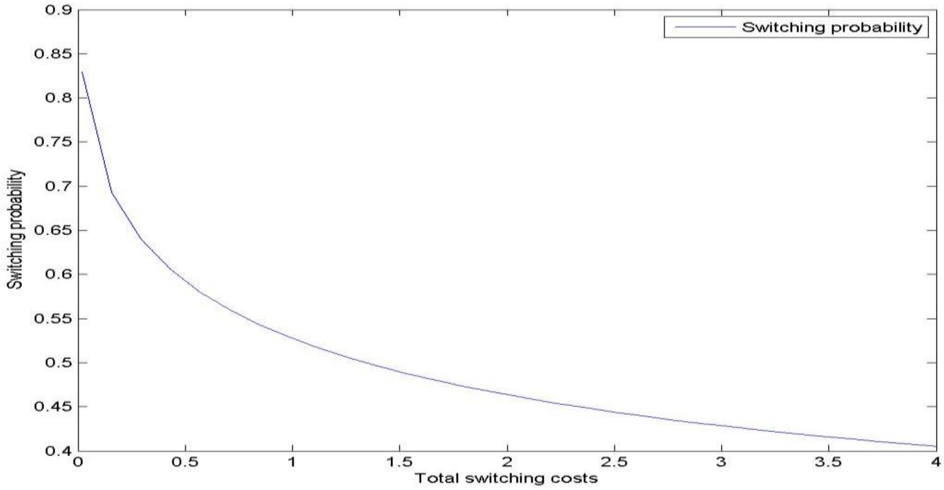

Figure 5.7

Relation between Total Switching Costs and Switching Probability

Figure 5.7 explains when the total switching costs, K, increase from 0 to 4, the probability of switching the exclusive right in time T decreases. The total switching costs are obstacles to entrance. When the obstacles to entrance increase, the brand holder may not be willing to switch the exclusive agent and the switching probability decreases. Finally, there is a negative relation between the total switching costs and the switching probability.

4. The influence of the delay cost for idle agent on the probability of switching the exclusive right in time T is demonstrated in Figure 5.8

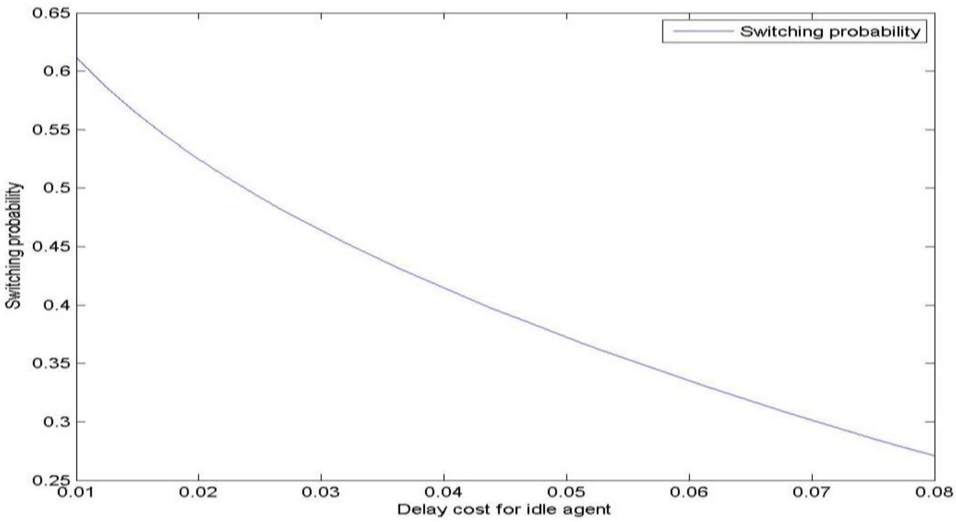

Figure 5.8

Relation between Idle Agent's Delay Cost and Switching Probability

Figure 5.8 explains when the idle agent's delay cost, δ_i, increases from 0.01 to 0.08, the probability of switching the exclusive right in time T decreases. The idle agent's delay cost increases, and the idle agent's net growth rate decreases. The probability of switching the exclusive right decreases because the agent with higher growth rate is easier to be the optimal exclusive agent. Finally, we obtain the negative relation between the idle agent's delay cost and the switching probability.

5. The influence of the idle agent's volatility on the probability of switching the exclusive right in time T is demonstrated in Figure 5.9

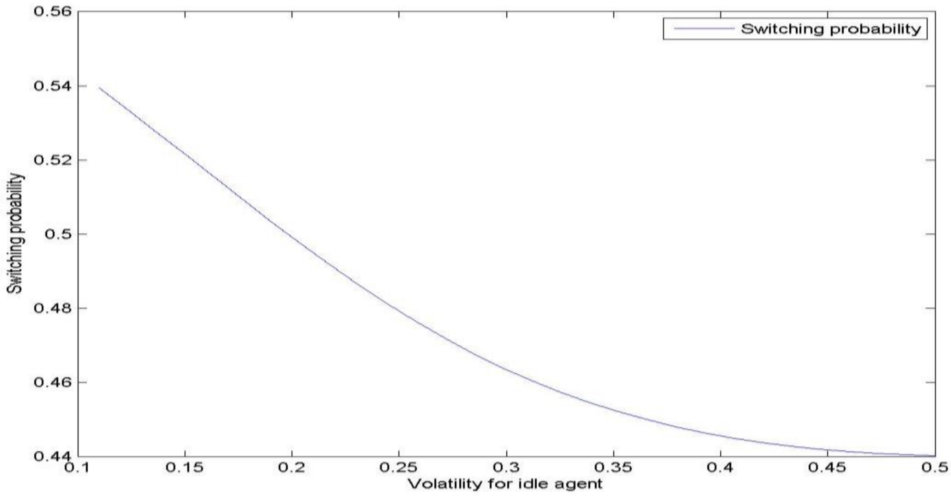

Figure 5.9

Relation between Idle Agent's Volatility and Switching Probability

Figure 5.9 explains when the idle agent's volatility, σ_i, increases from 0.1 to 0.5, the probability of switching the exclusive right in time T decreases. Because the volatility means the risk, the brand holder may not switch the exclusive right to the agent with high volatility. Therefore, when idle agent's volatility increases, switching probability decreases. Finally, we obtain the negative relation between the idle agent's volatility and the switching probability.

6. The influence of the idle agent's growth rate of net profit on the probability of switching the exclusive right in time T is demonstrated in Figure 5.10

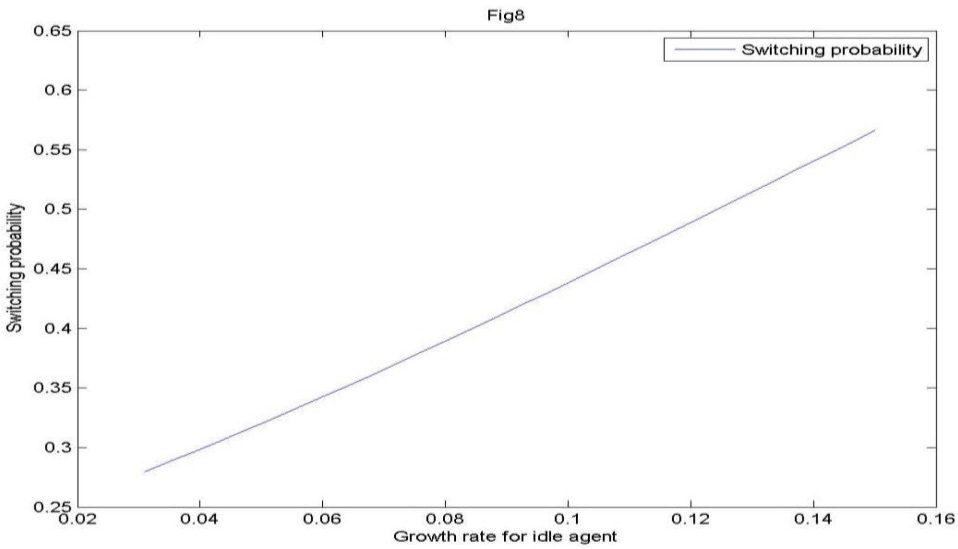

Figure 5.10

Relation between Idle Agent's Growth Rate and Switching Probability

Figure 5.10 explains when the idle agent's growth rate, U_i, increases from 0.03 to 0.15, the probability of switching the exclusive right in time T increases. The brand holder expects that the agent with high growth rate is easier to be the optimal exclusive agent. Therefore, when the idle agent's growth rate increases, the switching probability increases. Finally, we obtain the positive relation between the idle agent's growth rate and the switching probability.

7. The influence of the active agent’s delay cost on the probability of switching the exclusive right in time T is demonstrated in Figure 5.11

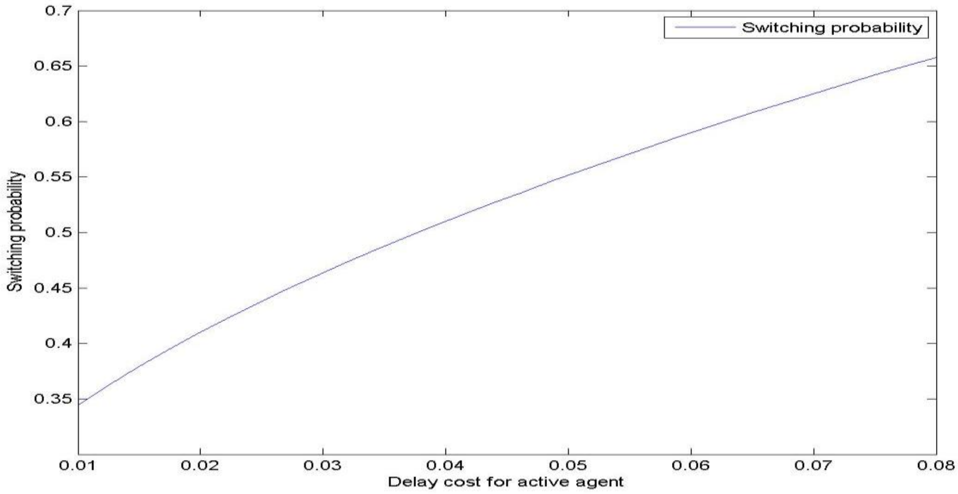

Figure 5.11

Relation between Active Agent’s Delay Cost and Switching Probability

Figure 5.11 explains when the active agent’s delay cost, δ_j, increases from 0.01 to 0.08, the probability of switching the exclusive right in time T increases. The active agent’s delay cost increases, and the active agent’s net growth rate decreases. The brand holder expects that the agent with high growth rate is easier to be the optimal exclusive agent. The switching probability increases because the active agent’s growth rate decreases. Finally, we acquire the positive relation between the active agent’s delay cost and the switching probability.

8. The influence of the active agent's volatility on the probability of switching the exclusive right in time T is demonstrated in Figure 5.12

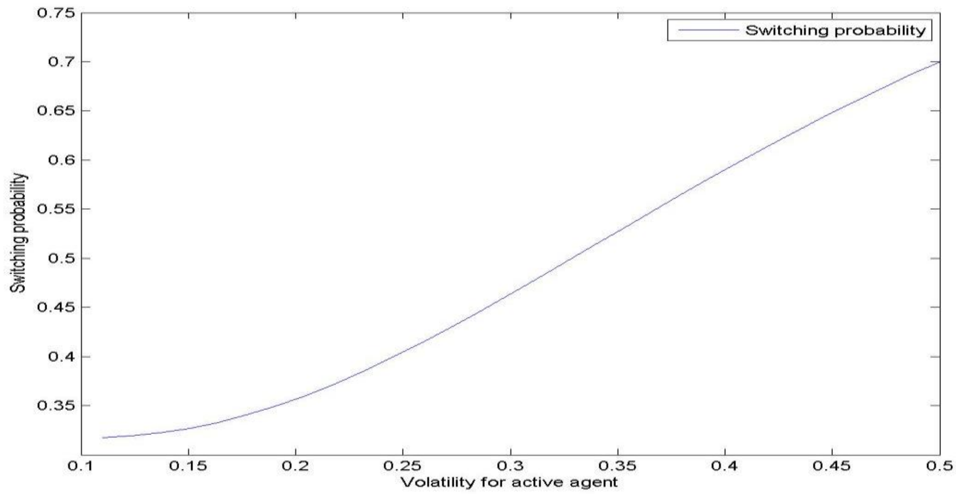

Figure 5.12

Relation between Active Agent's Volatility and Switching Probability

Figure 5.12 explains when the active agent's volatility, σ_j, increases from 0.1 to 0.5, the probability of switching the exclusive right in time T increases. Because the volatility means the risk for the brand holder, he may not switch the exclusive right to the agent with high volatility. Therefore, when active agent's volatility increases, switching probability increases. Finally, we obtain the positive relation between the active agent's volatility and the switching probability.

9. The influence of the active agent's growth rate of net profit on the probability of switching the exclusive right in time T is demonstrated in Figure 5.13

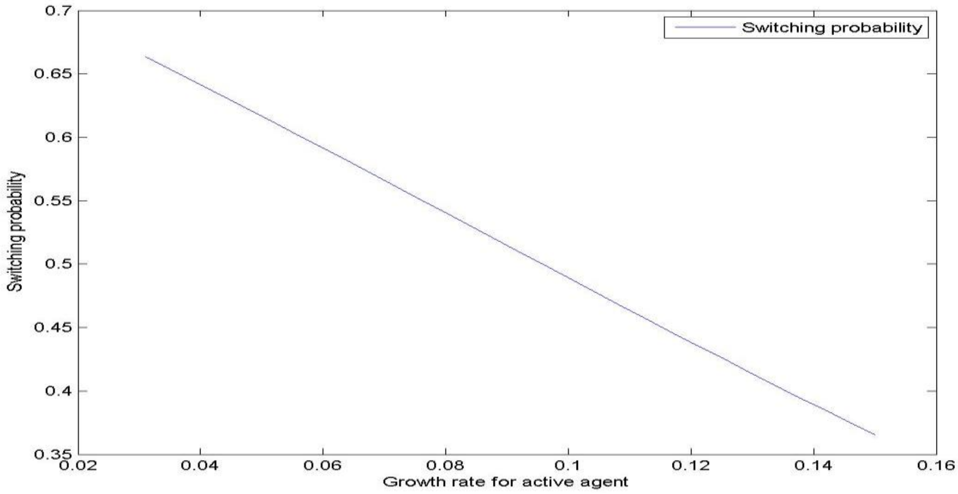

Figure 5.13

Relation between Active Agent's Growth Rate and Switching Probability

Figure 5.13 explains when the active agent's growth rate, U_j, increases from 0.03 to 0.15, the probability of switching the exclusive right in time T decreases. The brand holder expects that the agent with high growth rate is easier to be the optimal exclusive agent. Therefore, when the active agent's growth rate increases, the switching probability is decreased. Finally, we acquire the negative relation between the active agent's growth rate and the probability of switching.

We acquire all of the relations between each parameter and switching probability. Table 5.3 shows that how each parameter influences on switching probability.

Table 5.3

Relations between Parameters and Switching Probability

Environmental parameters	ρ	Κ	q
Switching probability	(-)	(-)	(-)
Parameters of idle agent	σ_i	δ_i	u_i
Switching probability	(-)	(-)	(+)
Parameters of active agent	σ_j	δ_j	u_j
Switching probability	(+)	(+)	(-)

To summarize, we can distinguish the results into five factors, which are the net profit difference, the total switching costs, the cash flow for the brand holder, risk of each agent and the growth rate of each agent. Switching probability increases when the net profit difference, the total switching costs and the cash flow for the brand holder decrease. The brand holder expects that the agent with high growth rate and low volatility is easier to be the optimal exclusive agent. In addition, we can find that increasing the hysteresis does not mean that the switching probability decreases.

6. Conclusion and Suggestion

When two agents are competing for the exclusive right, they can earn the different and uncertain net profits in future. Therefore, claim timing is an important strategy decision variable for each agent, and it can be optimized with conjecturing the rival's responses. The idle agent has an option to claim the market by sinking the total switching costs. This paper solves a stochastic real options game in a duopoly agent market. In order to reduce one dimension, we assume that the optimal strategy problem is natural homogeneity. In our model, there are some restrictions. First of all, only one agent can be active in the market. Secondly, we only consider that there are only two agents competing for the market. Finally, the optimal strategy decisions of two agents can be converted into optimal switching decisions of the brand holder. Under these conditions, we can obtain the solutions by using the real options game method. Using the results, we expect that the agent with high growth rate and low volatility is a better choice for the brand holder.

This paper analyzes the optimal switching decision under the varied condition and introduces commission rate in the market. The commission rate does not affect the results of original model. The results demonstrate how the factors affect hysteresis and switching probability. The hysteresis increases when the total switching costs, the total volatility and the commission rate increase and the correlation coefficient decreases. Under the conditions of high commission rate, high correlation coefficient and high total switching costs, switching is less likely to appear. However, I find out if the hysteresis increases, it does not mean that the switching probability is less likely to appear. Switching probability changes depend on each agent's parameters and industry factors.

There are three recommendations for the future research. Firstly, this study focuses on a duopoly agent market. In real world, there are many agents in an agent market. We should extend the model to a multiple agents market in future research. Secondly, in our model, monopolist exists by the exclusive contract. If applying the model in the other competing market, the patent can also be considered in order to represent the technical monopolist in the market. Finally, we can apply the model to the supply chain analysis. For example, Taiwan Semiconductor Manufacturing Company (TSMC) selects the Original Equipment Manufactures (OEMs) and the process is one item of the supply chain. Therefore, our model applying to the selection process is contributable for future research.

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