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Two-quasiparticle states in the interacting boson approximation and the structure of even Pt isotopes

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The structures of even Pt isotopes are studied with the traditional interacting boson approximation plus one-fermion-pair model. The energy spectra can be reproduced reasonably well if mixing of two bands of different deformation at low spins in these isotopes is considered. An apparent change of structure transiting from ^{188}Pt to ^{186}Pt was reflected in the different interactions needed to reproduce the energy spectra for heavier Pt isotopes ($188 \leq A \leq 192$) and lighter Pt isotopes ($182 \leq A \leq 186$), respectively. The compression of level spacing occurring between 10^+ and 12^+ states in heavier Pt isotopes is also obtained. The $B(E2)$ values for ^{184}Pt yrast band versus the spins of the depopulating states are calculated and compared with the observed data.

I. INTRODUCTION

The recent studies[1–8] of high-spin states of the even-mass Pt isotopes reveal an interesting anomaly along the yrast line around angular momentum 10^+ to 12^+ which could be called a "backbending." Garg *et al.*[7] measured $B(E2)$ values in ^{184}Pt and found a monotonic increase of $B(E2)$ values with spin I which goes from 2^+ to 10^+ and then decreases quite rapidly with spin between $I=10^+$ and 14^+. In order to interpret these behaviors, many efforts[2–5,9–16] have suggested a band crossing within a model where two quasiparticles may be excited and coupled to the rotation of the core. Yoshida *et al.*[17] proposed a model to describe the high-spin states[18,19] by breaking one of the bosons of proton-neutron interacting boson approximation (IBA-2) model into a pair of nucleons. Alonso *et al.*[20] applied the model of Yoshida *et al.* to calculate the high-spin states in the even $^{154-158}$Dy isotopes by allowing one neutron boson or one proton boson to break into two $1\nu\, i_{13/2}$ and $1\nu\, h_{9/2}$ neutron of $1\pi\, h_{11/2}$ protons. Using the weak-coupling technique and making use of the previous results of the IBA-2 calculation,[21] they can reproduce the backbending of the Dy isotopes very successfully. The model of Yoshida *et al.* can successfully describe the high-spin anomaly. However, as the proton number goes away from the closed shells, the number of basic states of IBA-2 increases rapidly, thus the IBA-2-plus-one-fermion-pair model becomes too tedious to be feasible. It is known[22] that the phenomenological interacting boson approximation (IBA-1) is a valid approximation when the value of $N_\pi+N_\nu$ is quite large as compared with the value of $|N_\pi-N_\nu|$. Therefore, it is hopeful to expect that the high-spin anomalies of some nuclei in the region far away from the closed shell could be described in the IBA-1 (Ref. 23) -plus-one-fermion-pair model. Morrison *et al.*[24] used a semimicroscopic model for the inclusion of two quasiparticle states[17,22] in the interacting-boson-fermion model.[25] Their model was successful in reproducing the main features of the energy spectra of the $^{194-198}$Hg isotopes. This, to a certain extent, justified that the IBA-1-plus-one-fermion-pair model is appropriate to describe the high-spin anomalies.

The purpose of this work is twofold. First, we want to present a systematic study of the even-mass Pt isotopes. Second, and most important, we desire to investigate to what extent the observed irregularities, e.g., the backbending of the moment of inertia and the reduction in the $B(E2)$ values, can be understood in terms of the IBA-1-plus-one-fermion-pair model. In the present work, it is assumed that one of the bosons can be broken to form a fermion pair which may be excited to the $1i_{13/2}$ or $1h_{9/2}$ orbitals with total spin $J=4,6,\ldots,12$ and coupled to the core. The couplings to $J=0,2$ are excluded in order to prevent double counting.

II. THE MODEL

The Pt isotopes with $Z=78$ and $104 \leq N \leq 114$ will be taken as testing examples. Thus, valence protons and valence neutrons are in the 50–82 and 82–126 major shell, respectively. Taking $Z=82$ and $N=126$ as the core, the IBA-1 assumes valence boson numbers 8, 9, 10, 11, 12, and 13 for $^{192-182}$Pt, respectively. To describe the high-spin states, one of the bosons is allowed to break into two fermions which can be excited to the $1i_{13/2}$ and $1h_{9/2}$ single-particle orbitals. The reasons to include $i_{13/2}$ and $h_{9/2}$ orbit are as follows: (1) To reproduce the shrinking of the energy level spacing between $I=10$ and $I=12$ for ^{192}Pt, one has to include two-body matrix elements of $J=12$ and $J=10$ states of the fermion configuration which can only be produced by the $(i_{13/2})^2$ configuration. (2) A measurement of lifetimes in ^{172}W manifests a reduction of transition quadrupole moments, which indicates that the rotation-induced structure change is due not only to $i_{13/2}$ but also to $h_{9/2}$ proton.[26]

Our model space includes the IBA-1 space with N bosons and space with $N-1$ bosons plus two nucleons. The model Hamiltonian is[17]

$$H=H_B+H_F+V_{BF} \ ,$$

where H_B is the IBA boson Hamiltonian

$$H_B=a_0\varepsilon_d+a_1p^\dagger\cdot p+a_2L\cdot L+a_3Q\cdot Q \ .$$

The octopole term $T_3\cdot T_3$ and the hexadecapole term $T_4\cdot T_4$ have been omitted in H_B since they are generally believed to be less important. The fermion Hamiltonian H_F is

$$H_F=\sum_j \varepsilon_j\sqrt{2j+1}[a_j^\dagger\times\tilde{a}_j]^{(0)} +\frac{1}{2}\sum_J {}_j V^J\sqrt{2J+1}[(a_j^\dagger\times a_j^\dagger)\times(\tilde{a}_j\times\tilde{a}_j)^J]^{(0)} \ ,$$

with $a_j^\dagger$ being the nucleon creation operator. The mixing Hamiltonian V_{BF} is assumed

$$V_{BF}=Q^B\cdot Q-Q^B\cdot Q^B \ ,$$

where

$$Q^B=(d^\dagger\times\tilde{s}+s^\dagger\times\tilde{d})^{(2)}-\frac{\sqrt{7}}{2}(d^\dagger\times\tilde{d})^{(2)} \ ,$$

and

$$Q=Q^B+\alpha(a_j^\dagger\times\tilde{a}_j)^{(2)}+\beta[(a_j^\dagger\times a_j^\dagger)^{(4)}\times\tilde{d} -d^\dagger\times(\tilde{a}_j\times\tilde{a}_j)^{(4)}]^{(2)} \ .$$

Two types of fermion interaction potential, i.e., the Yukawa potential with Rosenfeld mixture and the surface delta interaction potential, are used. Both yield almost the same result. In the final calculation, the Yukawa potential is used and the oscillator constant $\nu=0.96A^{-1/3}$ fm^{-2} with $A=160$ assumed. The interaction strength is adjusted so that the $J=0$ state is lower than the $J=2$ state by 2 MeV. The single-particle energies are obtained as a result of fitting. The other parameters contained in the boson Hamiltonian H_B and V_{BF} were chosen to reproduce the energy-level spectra of even Pt isotopes with mass number between 182 and 192. In our calculation, the interaction parameters contained in the boson part for each nucleus are unified for both the N-boson configuration and the $N-1$-boson-plus-one-fermion-pair configuration. On the contrary, the previous works[18–20] adopted different boson interactions[21] for different configurations in each isotope. In our calculation, the two energy-level bands with the former two kinds of configurations are mixed through the diagonalization of the energy matrix in the whole model space which consists of spaces of N bosons and of $N-1$ bosons plus two nucleons. This is in contrast to the weak-coupling treatment of the previous work of Alonso *et al.*

III. RESULTS

The interaction strengths and the single-particle energies for each isotope are allowed to be mass-number dependent. Table I lists the best-fitted interaction strengths and single-particle energies for all isotopes. One can see that the interaction strengths for all isotopes can be categorized into two groups. The parameters contained in lighter-mass isotopes with $182\le A\le 186$ are rather different from those of heavier-mass isotopes with $188\le A\le 192$. It can be seen from Table I that the strengths of the terms ε_d, a_2, α, β, $\varepsilon(i_{13/2})$, and $\varepsilon(h_{9/2})$ can be almost unified for each group. There is a steep change of the strengths when transiting from ^{188}Pt to ^{186}Pt, while the variations of the parameters contained in each group are rather mild. The increasing of the absolute magnitude of quadrupole term $Q\cdot Q$ and the decreasing of the magnitude of pairing term $P^\dagger\cdot P$ as mass number A decreases are consistent with the tendency of deviating away from O(6) symmetry to become the SU(3) symmetry. This is consistent with the results of Casten and Cizewski.[27] The calculated and observed energy spectra for the string of Pt isotopes are shown in Figs. 1–3. Figure 1 shows the ground-state bands. One can note from the figure that the agreements between the calculated and the observed energy levels are in general quite good. The high-spin states can be well reproduced. There is a steep drop of energy levels transiting from ^{188}Pt to ^{186}Pt. This apparent change of structure was already reflected in the different interactions needed to obtain the energy spectra. Figures 2 and 3 show the energy levels of quasi-β and quasi-γ bands. A primitive calculation on Pt isotopes including only the $1i_{13/2}$ single-particle orbit was performed.[28] A comparison between these two works exhibits that the agreements between the calculated and the observed energy spectra of the present work are better than that of the previous work. This manifests the importance of the $1h_{9/2}$ orbital. The effects

TABLE I. The interaction parameters (in MeV) adopted in this work.

Parameter (MeV) nucleus	a_0	a_1	a_2	a_3	α	β	$\varepsilon_{9/2}$	$\varepsilon_{13/2}$
^{192}Pt	0.579	0.104	0.0080	0.0008	0.04	0.03	1.68	1.36
^{190}Pt	0.552	0.100	0.0110	−0.0019	0.04	0.03	1.68	1.36
^{188}Pt	0.552	0.076	0.0098	−0.0028	0.04	0.03	1.68	1.36
^{186}Pt	0.468	0.038	0.0090	−0.0057	0.04	0.03	1.65	1.13
^{184}Pt	0.468	0.034	0.0090	−0.0061	0.04	0.03	1.65	1.13
^{182}Pt	0.478	0.028	0.0090	−0.0068	0.04	0.03	1.65	1.13

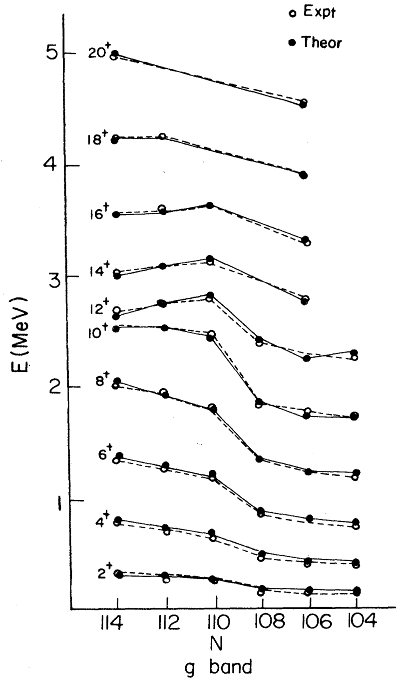

FIG. 1. The calculated and observed yrast energy spectra for Pt isotopes. The experimental data are taken from Ref. 29.

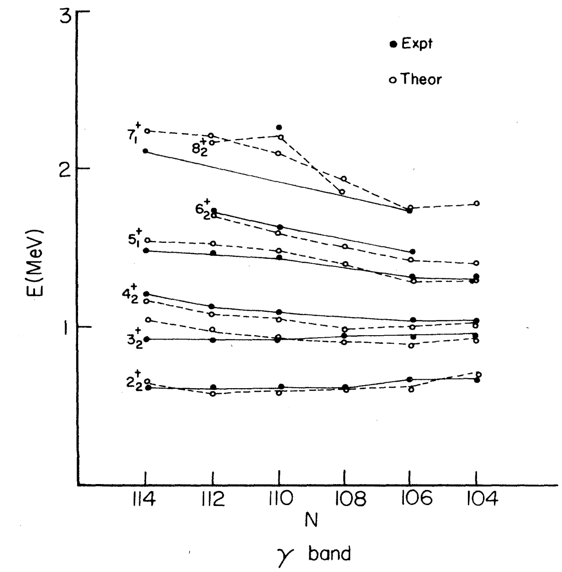

FIG. 3. The calculated and observed quasi-γ band levels for Pt isotopes. The experimental data are taken from Ref. 29.

of the $1h_{9/2}$ orbital is especially important to the energy levels of the quasi-β and quasi-γ bands. This is consistent with the conclusion of Ref. 28. The deviations of our calculated energy levels from the experimental ones increase as the number of active holes decreases.

The relative intensities of wave functions corresponding to N boson and $N-1$-boson-plus-two-$h_{9/2}$-or-$i_{13/2}$-fermions configurations for each state are listed in Tables II and III. The total intensity of N boson, $N-1$-boson-plus-two-$h_{9/2}$-fermions, and $N-1$-boson-plus-two-$i_{13/2}$-fermions configurations for each state is normalized to 1000. One can see, in general, the β band, the lower-lying levels of the γ band, and the yrast levels with angular momentum up to 8 in heavier Pt isotopes and up to 10 in lighter Pt isotopes are dominated by the pure boson configurations. The importance of the $N-1$-boson-plus-two-$h_{9/2}$-fermions configuration is less than that of the $N-1$-boson-plus-two-$i_{13/2}$-fermions configuration. However, it still cannot be neglected, especially in the states of 4_3, 7_1, and 8_2 of ^{192}Pt; 8_2 of ^{190}Pt and ^{188}Pt; 6_2 and 8_2 of ^{186}Pt; and 6_2 of ^{184}Pt. Therefore, if we increase the $h_{9/2}$ single-particle energy so that this orbit becomes effectively irrelevant, then the agreements between the calculated and the observed high-spin levels will become worse. Especially, the good agreement of the theoretical and experimental energy levels around the first backbends of the moments of inertia will deteriorate. One can also find that the mixing between different configurations is small in the general case. However, the mixing between different configurations becomes more and more apparent if one passes from heavier isotopes to lighter isotopes. The configuration with $N-1$ bosons and two $i_{13/2}$ fermions are dominant in the states with spin $I \geq 10$ for $^{192-190}$Pt and the states with $I \geq 12$ for the other Pt isotopes. The different structures between lighter and heavier isotope strings are also revealed in the apparent change of wave function intensities as one passes from ^{188}Pt to ^{186}Pt. It is found that the mixing between different configurations and the fermion-pair excitation becomes more and more important as the mass number decreases. Furthermore, the higher the spin I of the state, in general the more the intensity of the fermion-pair-excitation configuration is contained. Such a variation of intensities of different configurations with spins suggests that two bands of different deformations are

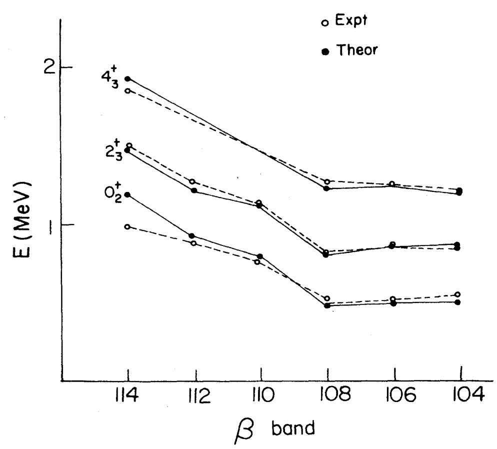

FIG. 2. The calculated and observed quasi-β band levels for Pt isotopes. The experimental data are taken from Ref. 29.

TABLE II. The relative intensities of the configuration with fermion-pair excitation for wave functions of energy spectra in heavier Pt isotopes. The total intensity of configurations with and without fermion-pair excitation for each state is normalized to 1.

Nucleus		^{192}Pt			^{190}Pt			^{188}Pt	
States	0	$h_{9/2}^2$	$i_{13/2}^2$	0	$h_{9/2}^2$	$i_{13/2}^2$	0	$h_{9/2}^2$	$i_{13/2}^2$
0_1	0.997	0.001	0.002	0.993	0.002	0.005	0.991	0.003	0.006
2_1	0.993	0.002	0.005	0.987	0.004	0.009	0.983	0.005	0.012
4_1	0.982	0.003	0.015	0.973	0.005	0.021	0.967	0.007	0.025
6_1	0.961	0.003	0.036	0.946	0.006	0.048	0.941	0.009	0.050
8_1	0.912	0.003	0.086	0.879	0.004	0.117	0.893	0.007	0.100
10_1	0.000	0.011	0.989	0.003	0.034	0.963	0.745	0.012	0.243
12_1	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	1.000
14_1	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	1.000
16_1	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	1.000
18_1	0.000	0.000	1.000	0.000	0.000	1.000			
20_1	0.000	0.000	1.000						
0_2	0.988	0.004	0.008	0.979	0.006	0.015	0.972	0.008	0.020
2_3	0.982	0.004	0.014	0.973	0.007	0.021	0.962	0.009	0.028
4_3	0.005	0.322	0.673	0.974	0.006	0.020	0.970	0.007	0.023
2_2	0.990	0.003	0.007	0.986	0.004	0.009	0.984	0.005	0.011
3_1	0.984	0.004	0.012	0.980	0.005	0.015	0.975	0.007	0.018
4_2	0.978	0.004	0.018	0.975	0.005	0.020	0.971	0.007	0.022
5_1	0.967	0.004	0.029	0.962	0.006	0.032	0.956	0.008	0.036
6_2	0.927	0.011	0.061	0.952	0.006	0.042	0.950	0.008	0.042
7_1	0.072	0.263	0.665	0.895	0.010	0.095	0.921	0.007	0.072
8_2	0.033	0.242	0.725	0.052	0.276	0.672	0.032	0.292	0.674

mixed at some low-spin region. In order to investigate the effect of the mixing between two energy bands produced by configurations of pure N bosons and of $N-1$-bosons-plus-two-fermions, a calculation of energy spectra with and without mixing parameter β was performed. Figure 4 presents the energy spectra of ^{190}Pt as an illustration. The first and second columns are the ground-state band for $\beta=0$, while the third column is the ground band for $\beta\neq 0$. The energy-level band produced by the pure N boson configuration is shown in the first column (called configuration I). One can see this pure boson configuration is only able to produce a rotationlike band,

TABLE III. The relative intensities of the configuration with fermion-pair excitation for wave functions of energy spectra in ligher Pt isotopes. The total intensity of configurations with and without fermion-pair excitation for each state is normalized to 1.

Nucleus		^{186}Pt			^{184}Pt			^{182}Pt	
States	0	$h_{9/2}^2$	$i_{13/2}^2$	0	$h_{9/2}^2$	$i_{13/2}^2$	0	$h_{9/2}^2$	$i_{13/2}^2$
0_1	0.976	0.005	0.019	0.959	0.009	0.032	0.937	0.014	0.049
2_1	0.959	0.007	0.034	0.942	0.011	0.048	0.921	0.015	0.064
4_1	0.927	0.008	0.065	0.910	0.012	0.078	0.892	0.016	0.091
6_1	0.852	0.006	0.142	0.844	0.010	0.147	0.834	0.014	0.152
8_1	0.620	0.015	0.365	0.670	0.010	0.320	0.702	0.011	0.288
10_1	0.195	0.071	0.735	0.306	0.046	0.649	0.418	0.026	0.556
12_1	0.022	0.061	0.917	0.060	0.069	0.870	0.119	0.060	0.821
14_1				0.001	0.006	0.993			
16_1				0.000	0.003	0.997			
18_1				0.000	0.001	0.999			
20_1				0.000	0.000	1.000			
0_2	0.953	0.007	0.040	0.948	0.009	0.043	0.945	0.010	0.045
2_3	0.934	0.008	0.057	0.928	0.011	0.061	0.923	0.013	0.063
4_3	0.025	0.251	0.725	0.895	0.011	0.094	0.889	0.015	0.097
2_2	0.973	0.005	0.022	0.964	0.007	0.029	0.952	0.009	0.039
3_1	0.954	0.006	0.039	0.943	0.009	0.049	0.929	0.012	0.059
4_2	0.947	0.006	0.047	0.936	0.008	0.056	0.921	0.011	0.068
5_1	0.913	0.006	0.081	0.901	0.009	0.090	0.888	0.012	0.099
6_2	0.080	0.243	0.677	0.344	0.158	0.498	0.640	0.067	0.292
7_1	0.740	0.011	0.249	0.767	0.008	0.225	0.776	0.010	0.214
8_2	0.257	0.202	0.541				0.418	0.026	0.556

and thus the shrinking of energy spacing between $I=10$ to 12 cannot be described by IBA-1. The second column of Fig. 4 presents the calculated energy levels obtained with configuration of $N-1$-bosons-plus-a-fermion-pair with the pairing parameters α and β being set to be zero (called configuration II). In this column the boson number and fermion number are all kept constant. The states with spin 0 and 2 are not included in the second column to avoid double counting of these states which are included already through s and d bosons, respectively. It can be seen that this configuration is able to yield the compression of energy spacing between spin 4 and 12. The third column in Fig. 4 presents yrast energy band obtained by whole model Hamiltonian. From the first three columns, it reveals that the states with low spin ($I\le 8$) are mainly dominated by configuration I, and states with high spin ($I\ge 10$) are mainly dominated by configuration II. The experimental energy spectra are also listed in the final column for comparison. It was found that the agreements between the calculated and observed energy levels are very good.

The high-spin states in $^{192-188}$Pt exhibit an anomaly, i.e., shrink of 10^+ to 12^+ level spacing, while the other energy spacings show a nearly monotonic increase with spin. This can also be revealed in the most sensitive backbending plot of the conventional $2\mathcal{J}/\hbar^2$ versus $(\hbar\omega)^2$ curve with

$$2\mathcal{J}/\hbar^2=\frac{4I-2}{E_{I+2}-E_I},$$

and

$$(\hbar\omega)^2=\left\{\frac{E_{I+2}-E_I}{[I(I+1)]^{1/2}-[(I-2)(I-1)]^{1/2}}\right\}^2.$$

Figure 5 shows the calculated and the observed $2\mathcal{J}/\hbar^2$ versus $(\hbar\omega)^2$ curve for whole string Pt isotopes. It is clear from the figure that backbendings happened in isotopes $^{192-188}$Pt but not for isotopes $^{186-182}$Pt. This also manifests a structure change that happened as transiting from ^{188}Pt and ^{186}Pt. It is expected that one-fermion-pair excitation will produce some effects on $B(E2)$ values. The study of these values will give us a good test of the model wave functions. Unfortunately, there is insufficient observed $B(E2)$ data for Pt isotopes. Recently, Garg *et al.*[7] measured $B(E2)$ values of ^{184}Pt and interpreted them by a rigid rotor model and a simple band-mixing calculation.[6] Although both treatments can reproduce the qualitative feature of the increase of $B(E2)$ values between spin $I=2$ and 10, the important decline features beyond $I=10$ are completely unable to obtain. In our calculation, the electric quadrupole operator is written as

$$T(E2)=e^BQ+e^F\alpha\sum_j(a_j^\dagger\times\tilde{a}_j)^{(2)}+\beta e^B\sum_j[(a_j^\dagger\times a_j^\dagger)^{(4)}\times\tilde{d}-d^\dagger\times(\tilde{a}_j\times\tilde{a}_j)^{(4)}]^{(2)},$$

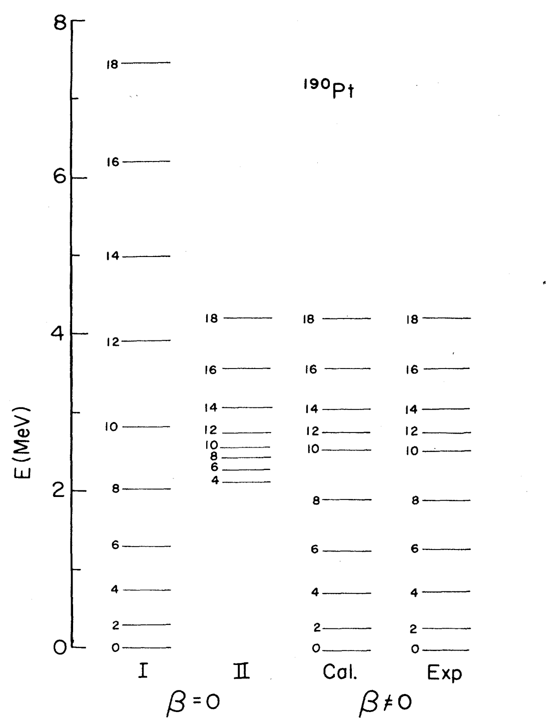

FIG. 4. The comparison of the calculated (3rd column) and observed yrast state energy spectra (4th column) of ^{190}Pt isotope. The first and the second columns are for the case without and with fermion-pair configuration. Both are obtained by setting $\beta=0$ in H_{BF}. The 3rd column is obtained by the whole model Hamiltonian.

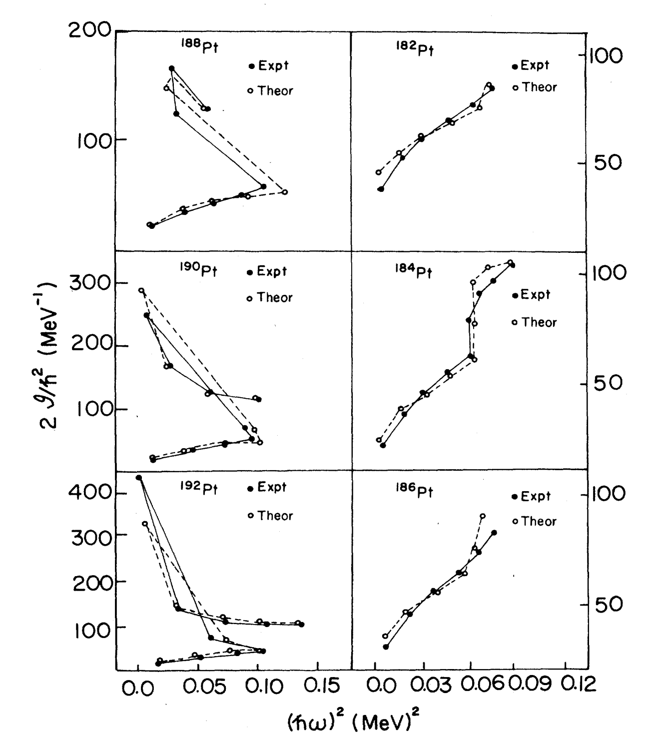

FIG. 5. The calculated and observed moment of inertia $2\vartheta/\hbar^2$ vs $(\hbar\omega)^2$ for yrast levels of $^{184-192}$Pt. Open circles represent the calculated values and solid circles are observed ones.

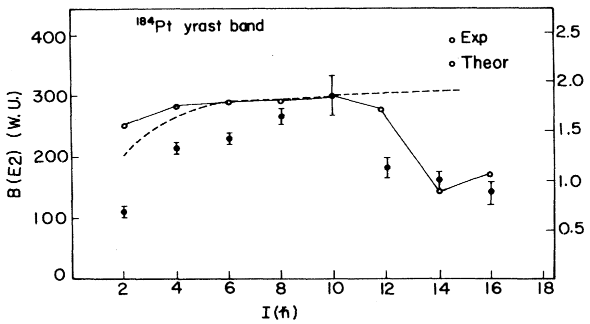

FIG. 6. The calculated (open circles) and observed (solid circles) $B(E2)$ values for ^{184}Pt yrast band vs the spins of the depopulationg states. The dashed line represents the calculated results with the rigid rotor model proposed in Ref. 7.

where Q is taken as

$$Q=(d^\dagger \times \tilde{s}+s^\dagger \times \tilde{d})^{(2)}+\kappa(d^\dagger \times \tilde{d})^{(2)} .$$

For the fermion effective charge e^F, an average value 0.37 of the proton and neutron obtained by Alonso *et al.* is adopted. The boson effective charge in the $T(E2)$ operator is determined by normalizing the largest calculated $B(E2)$ value to the corresponding observed datum. The parameters α and β are assumed the same values as used in the mixing Hamiltonian. Since the structure of ^{184}Pt is close to the SU(3) symmetry, the value of κ is chosen to be $-\sqrt{7/2}$ which is the value for the generator of the SU(3) group. Figure 6 shows the calculated and the observed $B(E2)$ values versus the spins of the depopulating states. From the figure, it can be seen that the increasing feature from spin 2 to 10 is reproduced qualitatively. The decreasing feature beyond spin 10 is also obtained reasonably well, compared to the results of Garg *et al.*

IV. SUMMARY AND DISCUSSION

In summary, we have investigated the structure of the energy spectra and the backbending phenomena of the isotope string of Pt with mass number between 184 and 192. We extend the IBA-1 model to allow a boson to be decoupled to form a fermion pair which can occupy the $1h_{9/2}$ and $1i_{13/2}$ orbitals. The calculated energy levels including the ground-state, β-, and γ-bands are in satisfactory agreement with the observed values for the whole string of Pt isotopes. Backbendings of the moment of inertia of the yrast states can be reproduced reasonably well. We also calculated $B(E2)$ values for ^{184}Pt and yielded satisfactory agreement with the observed data.

The energy spectra reveal a transition from O(6) symmetry to SU(3) symmetry as mass number decreases from ^{188}Pt to ^{186}Pt. This structure change is apparently manifested in the steep change of the Hamiltonian between these two nuclei. The effects of two-fermion configuration are manifested in the improvement of the shrinking of energy level spacing between $I=12$ and 10 of $^{192-188}$Pt and the decline nature of $B(E2)$ values beyond $I=10$ of ^{184}Pt. The couplings to angular momenta $J=4,6,8,\ldots$ for the quasiparticles in $h_{11/2}$ and $i_{13/2}$ orbitals might be considered as implicitly including the higher angular momentum bosons, such as g boson and the I boson, etc., and therefore could make the IBA-1 model space more complete. The effects are also manifested in the analysis of the wave functions. The high-spin states are usually dominated by the $N-1$-boson-plus-two-fermions configurations and thus cannot be reproduced by pure IBA-1. The relative intensities for configurations of pure N bosons and of $N-1$-bosons-plus-two-fermions excitation show a two-band crossing at the low-spin region. In general, the mixings between these two configurations are small and the $N-1$-boson-plus-two-$i_{13/2}$-fermions configurations are more important than $N-1$-boson-plus-two-$h_{9/2}$-fermions configurations for all isotopes. The main feature of $B(E2)$ values versus the spins of the depopulating states can be obtained reasonably well. The model will be particularly useful when a similar calculation with IBA-2 is not feasible.

Recently, very high spin states up to $I \simeq 40$ and a double backbending have been observed in some nuclei in the rare-earth region.[26,30,31] These phenomena might be hopefully interpreted as that two or more bosons are decoupled to form more fermion pairs and make more band crossings to form the double backbending. In conclusion, our calculation suggests a feasible model which, hopefully, can be extended to handle the recently observed very high spin states and the double backbending phenomena in some rare-earth nuclei.

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