A Computationally Efficient Method of Evaluating Green's Functions for 1-, 2-, and 3-D Enclosures

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Abstract

When modeling objects in presence of scatterers or enclosures with the method of moments (MOM), the use of specialized (as an alternative to free space) Green's functions reduces the number of problem unknowns significantly. This advantage, however, is often lost with the increased complexity in the evaluation of these Green's functions.

This paper discusses using a Model-Based Parameter Estimation (MBPE) technique to efficiently evaluate the Green's functions associated with parallel plates, rectangular waveguides and cavities. The numerical model uses a novel approach to break the dyadic Green's function into simpler functions such that a set of regressed polynomials would reconstruct the function. Examples of method of moments calculations incorporating MBPE are presented that show the computational advantage as compared to the direct approach.

Introduction

When the fields of known or impressed currents on an object in the presence of an enclosure (or any other scatterer) are sought, the use of specialized Green's functions as an alternative to the free space Green's function have been shown to reduce the volume of computations for many scatterers and enclosures [1]. The tradeoff for this alternative is replacing a high number of unknowns with a much more complex kernel (as opposed to its free space counterpart in the integral equation).

In spite of the complexity of the closed-form expressions of many specialized Green's functions, in many instances an approximate and simpler model, based on a set of adjustable constant parameters computed from numerical samples of the rigorous and complex function, can be developed to estimate the true function closely. This process has been termed Model-Based Parameter Estimation (MBPE) [2]. MBPE can yield significant computational advantages since it strives to model observed interactions rather than the underlying physics. This paper presents an application of MBPE for the evaluation of the specialized Green's functions associated with scatterers inside rectangular guided-wave structures and cavities.

In the following sections, the evaluation of Green's function for rectangular guided structures based on image theory is developed. This development includes a novel approach to reduce the number of independent variables of the problem, thereby making it best suited for MBPE. The development starts with the one-dimensional case and is then generalized to two and three dimensions. Second, the MBPE model is formulated and the techniques used in its numerical implementation are described. Finally, the results section shows the accuracy and efficiency of the model as compared to the direct method when used with a method of moments (MOM) code.

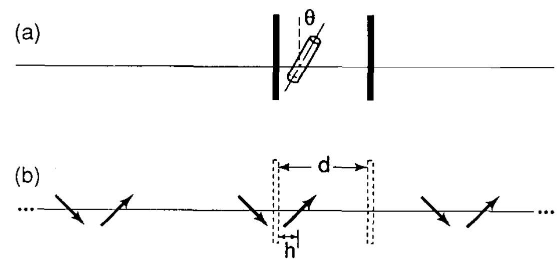

Figure 1: Parallel plate waveguide with wire scatterer (a) and images (b).

Green's Function

The electric field of a current distribution, $\bar{J}(\bar{r})$, inside an enclosure can always be represented as

$$\bar{E}(\bar{r}) = \int_{volume} \overline{\overline{G}}_e(\bar{r},\bar{r}')\cdot\bar{J}(\bar{r}')dv', \tag{1}$$

where $\overline{\overline{G}}_e(\bar{r},\bar{r}')$ is the dyadic Green's function of the electric type. Equation(1) is valid when $\overline{\overline{G}}_e(\bar{r},\bar{r}')$ takes into account all the scattering characteristics of the enclosure and the radiation of the source current distribution itself.

Figure 1(a) shows a one-dimensional rectangular guided wave structure with a wire scatterer. This structure can be used as a simple scatterering range, sometimes called a rail-line range. In order to calculate the scattering characteristics of any scatterer inside the rail-line, the Green's function of the parallel plate stucture must be known. Expressions for the various components of the dyadic Green's function associated with the parallel plate can be developed using successive applications of image theory [3]. Starting with the single source current in the presence of the infinite conducting sheets, the plates can be removed and replaced by an infinite number of images as shown in Figure 1(b). The fields at any point within the waveguide can easily be expressed as an infinite sum of all the image fields produced by the induced current distribution on the wire:

$$\bar{E}_s(\bar{r}) = \sum_{i=-\infty}^{\infty} \int \overline{\overline{G}}_0(\bar{r},\bar{r}'_i)\cdot\bar{J}_i(\bar{r}'_i)dv', \tag{2}$$

where $\overline{\overline{G}}_0(\bar{r},\bar{r}'_i)$ is the free space dyadic Green's function and $\bar{J}_i(\bar{r}'_i)$ is the i^{th} current image inside the guide.

In order to identify simplifications of this analysis, the infinite sequence of images of Figure 1(b) can be split into two even and odd subarrays for each x and z component, resulting in four separate subarrays. Here, the even and odd labels refer to the number of times that the source current has been imaged into the ground planes to produce each image. The normal (x) components of these even and odd subarrays are shown in Figure 2(a-c).

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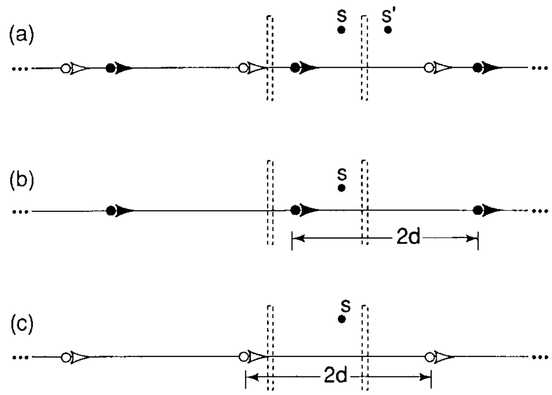

Figure 2: Total x-directed images (a) and even (b) and odd (c) subarrays.

Model-Based Parameter Estimation

The analysis of the preceeding section has shown that the Green's function for a rectangular guided wave enclosure can be expressed in terms of the fields of two similar infinite arrays of equally spaced source elements. A most important part of this analysis is that each component of the dyadic Green's function is calculated by twice evaluating a single function of one argument.

The reason for splitting the infinite images into even and odd subarrays is that each subarray is made up of an infinite number of sources spaced by the same distance, 2d. Thus, the only thing different about each of these subarrays is its relative position (along the x dimension) with respect to the field point s. Thus, if the functional variations of the fields due to either the even or odd subarrays are known, the appropriate component of the dyad can be reconstructed.

Using this framework, any of the components of the dyadic Green's function can be written in terms of the fields of a single infinite subarray of equally spaced source elements. As an example, consider the component G_{xx}. Using the perspective of Figure 2, G_{xx} at any arbitary point s can be written as

$$G_{xx}(s) = F(s) + F(s'), \tag{3}$$

where $F(s)$ is a function that represents the field of an infinite even array of x-directed elements, equally spaced by distance 2d, and s' is the equivalent point for the even subarray that gives the field of the odd subarray at point s.

Notice in Equation (3) that since the even and odd subarrays differ only in polarity and orientation relative to the observer, the function $F(s)$ is used twice with different arguments to obtain the desired dyad component. Thus, although the Green's function itself is a function of the two arguments (the positions of the source and observer), the use of even and odd arrary analysis results in twice evaluating a vector function of a single variable.

The extension of this method of expressing the Green's function to two- and three-dimensional rectangular guided structures is straightforward. In each case, the Green's function can still be expressed as an infinite array of images. This, in turn, can be divided into even and odd order arrays that consist of elements spaced by the distances dictated by opposite condutor walls.

In spite of the attractiveness of writing each component of the Green's function as in Equation (2), it must be remembered that the function $F(s)$ is, at this point, still a slowely converging infinite sum (single, double, or triple for one-, two- and three-dimensional problems, respectively) associated with the fields of each of the images in the even subarray. However, it can be noticed that $F(s)$ itself is not a particularly ill-behaved function when the field of the source element itself is subtracted from $F(s)$. This suggests better methods of calculating $F(s)$ than merely summing the infinite series.

One efficient method of evaluating $F(s)$ is to use polynomial regression. Here, $F(s)$ is evaluated by brute force (i.e., using the actual infinite series expression) over a rectangular grid of points large enough to adequately describe $F(s)$ so that G_{ij} can be approximated over a sufficiently large volume of the waveguide. In order to determine this polynomial, the region of interest where the scatterer is likely to fall is gridded uniformely and the actual value of $F(s)$ (minus the source contribution) for a fixed source location is calculated at all the grid (observation) points to the highest degree of precision possible via brute forcing the infinite sum. The values of G_{ij} thus obtained are regressed in a two- or three-dimensional sense with the lowest order polynomial that best fits the data. The result is a polynomial of degree m in x:

$$F(s) \approx P_m(x, y, z) = a_0 + a_1 x + a_2 x^2 + \ldots\ldots + a_m x^m, \tag{4}$$

where each coefficient a_i is itself a polynomial of degree m' in z:

$$a_i(z, y) = b_0 + b_1 z + b_2 z^2 + \ldots\ldots + b_{m'} z^{m'}. \tag{5}$$

Again b_i coefficints are polynomials of degree m'' in y:

$$b_i(y) = c_0 + c_1 y + c_2 y^2 + \ldots\ldots + c_{m''} y^{m''}. \tag{6}$$

Therefore, by storing the a_i's, b_i's and c_i's in proper manner and supplying the coordinates of the source observer, the value of $F(s)$ is readily computed, from which the entire Green's function can be determined.

Several points need to be mentioned regarding the above model:

1. A polynomial regression of data points is used instead of a polynomial interpolation. The reason interpolation is not a good choice is twofold. First, a polynomial passing through all the data points exactly is not called for since the data maybe contaminated in the first place. Second, polynomials of high degrees resulting from interpolating at all the grid points have inherent oscillatory behavior between data points not compatible with the true nature of the fields inside the guided structure. In this context, our purpose is better served by lower degree polynomials that fit the data approximately in a least square sense (curvilinear regression).

2. Since the fields are predominately due to source and "near-neighbor" images, the field of these sources can be removed from the regression and added back later in order to minimize the modeling error.

3. Near the edge points of the grided region of interest, the polynomials will approximate the function poorly due to lack of information on how the function behaves past those points. Therefore, the mesh of grids should be extended a few points farther than the region of interest to compensate for this.

4. The regression analysis should be repeated for both real and imaginary parts of $F(s)$ rather than magnitude and phase, because phase variations are very abrupt in some instances causing considerable error in regressing polynomials, whereas real and imaginary parts of $F(s)$ are smooth functions of position.

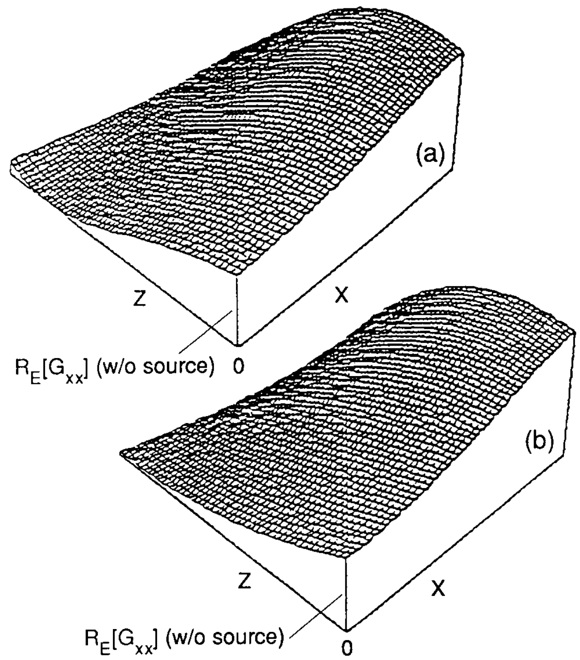

Figure 3: Real part of G_{xx} (source effect excluded) using brute force calculation (a) and MBPE (b).

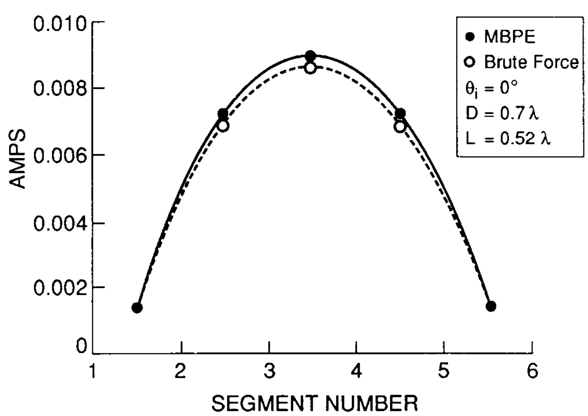

Figure 4: Current distribution on a $\lambda/2$ dipole using brute force and MBPE.

Results

This MBPE analysis has been done to evaluate the Green's function associated with a parallel plate waveguide for both the source and observer in a plane $(x-z)$ perpendicular to the plates. A mesh of 11x11=121 points extending from $z = 0$ to $z = 0.7\lambda$ inside a rail-line range with a plate separation of $d = 0.7\lambda$ was examined. The regression was done on individual rows resulting in 11 polynomials of order 3 in x. The regression was then repeated on coefficients of x^0, x^1, x^2, x^3 with z as the variable. Here again, a third order polynomial fits the data points in a least square sense. The procedure was repeated for real and imaginary components of G_{ij}, resulting in a polynomial in x and z with a total of 16 coefficients to be stored. Single precision calculation was used throughout, although added precision for evaluation of polynomial coefficients could be justified. Using these coefficients, the field values due to arbitary source locations were computed over many observation points inside the waveguide and were compared with brute force computations.

The results turned out to be close for most source-obsevation pairs. A worst case comparison is shown in Figure 3 (a-b) for the real part of G_{xx} (source effect excluded). The results for other components of the Green's function are similar or better.

To examine the propagation of errors once the integral equation operator is used, the model described was used in a MOM code to compute the current distribution on a wire scatterer placed inside the rail-line range with different orientations. Figure 4 compares the current on this wire when brute force and MBPE Green's functions are used. The results obtained would be improved if double precision or a finer grid were used in computing the polynomial coefficients.

As far as computation time is concerned, the MBPE of Green' function is most efficient, especially for larger number of segments on a given scatterer. According to our timings on a VAX 8650, the computation of the current on a $\lambda/2$ wire with 6 segments was 20 times faster with our MOM code incorporating MBPE as compared to the brute force Green's function. Significantly greater time savings would be expected for 2- and 3-D enclosures since the MBPE numerical grid would be relatively unchanged, whereas the "brute force" Green's functions contain double, and triple infinte sums, respectively.

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