Chapter 2

Concept of Discrete Multi-Tone

2.1 Characteristics of the Transmission Environment

In the wired-line transmission environment, many imperfect factors must be considered, including the distortion and delay by the telephone lines, AWGN, NEXT (Near-End Crosstalk), FEXT (Far-End Crosstalk), radio frequency interferences, and impulse noise [1][2].

2.1.1 Attenuation of the Twisted Pair

Since channel frequency response does not vary with time quickly, it is possible to model the channel with a linear and slowly time-varying filter. The channel frequency response is determined by wire category, wire length and frequency band. Telephone lines typically are 24- or 26- gauge twisted pair. There are seven test loops defined in [3]. Figure 2.1 shows the power spectral densities and impulse responses of some test loops.

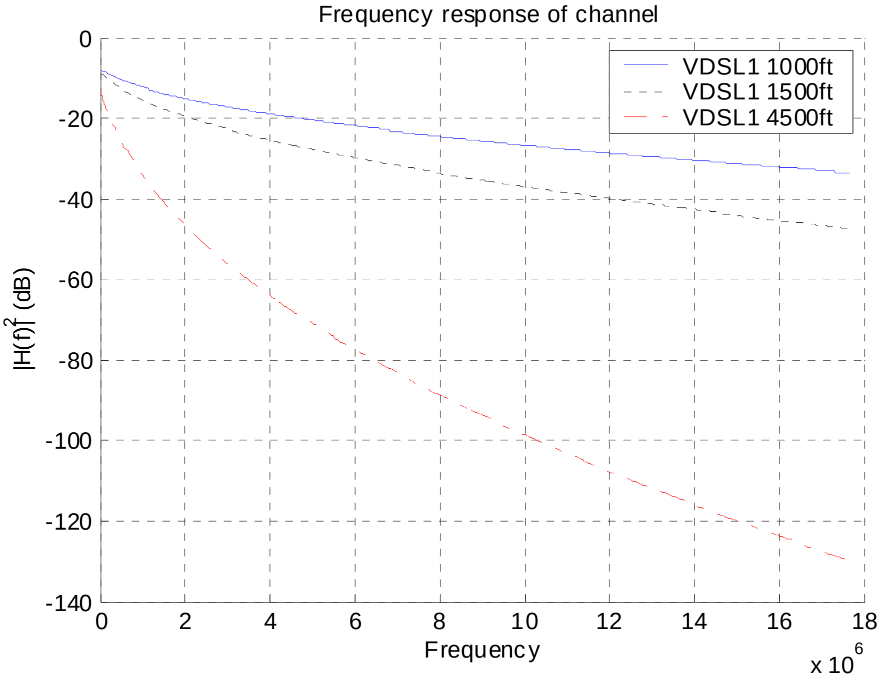

Figure 2.1 (a) Power spectral densities of VDSL#1

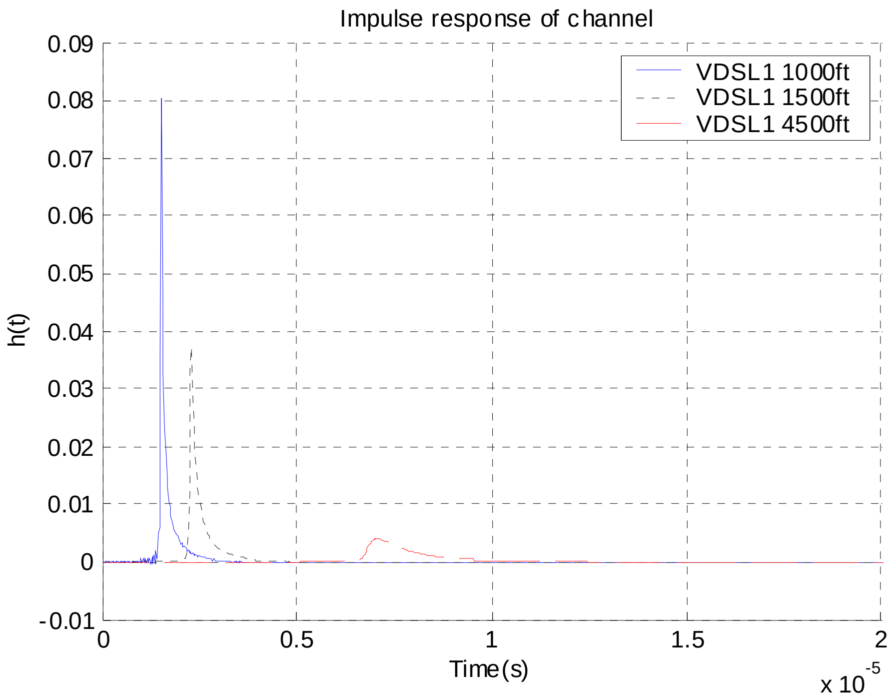

Figure 2.1 (b) Impulse responses of VDSL#1

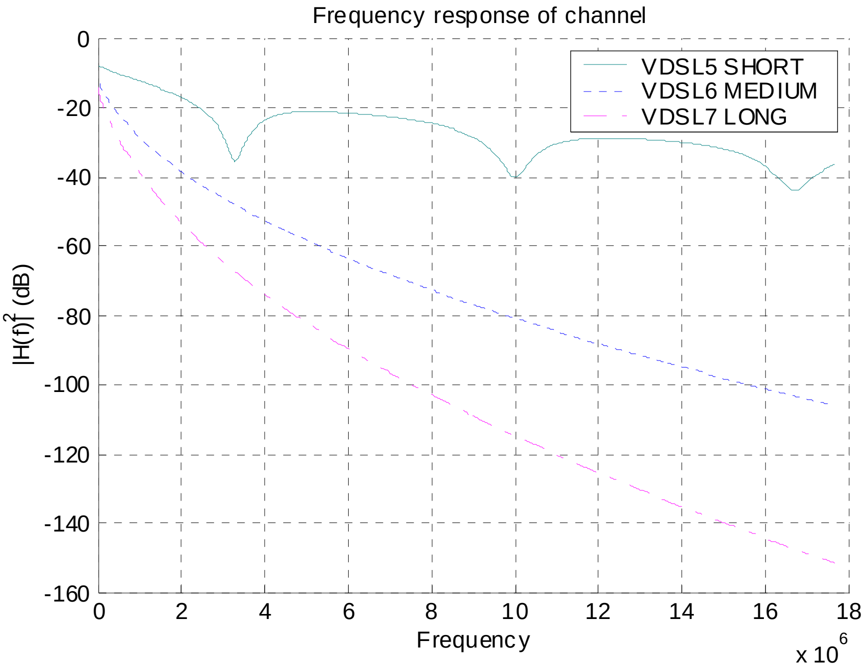

Figure 2.1 (c) Power spectral densities of VDSL#5, VDSL#6 and VDSL#7

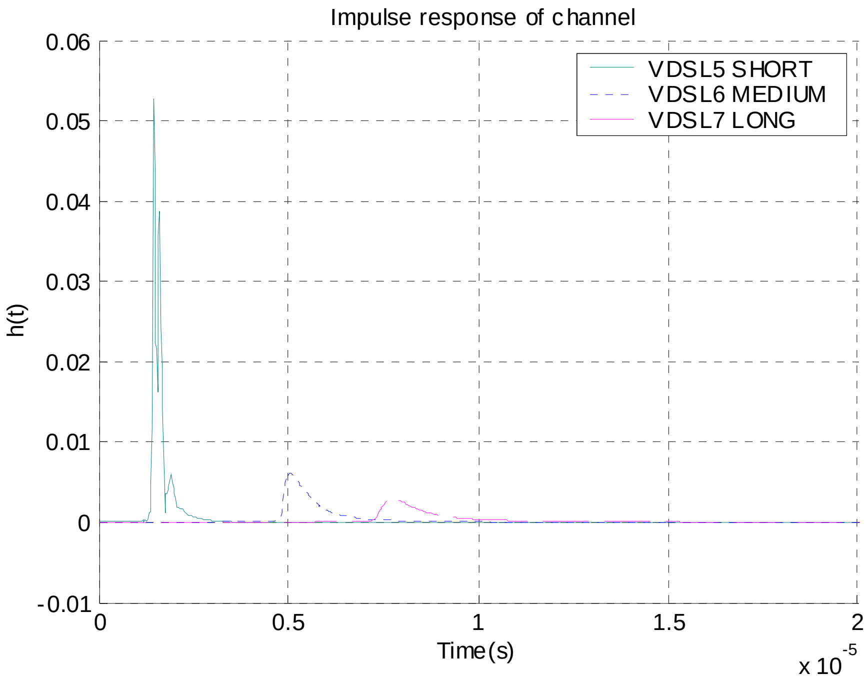

Figure 2.1 (d) Impulse responses of VDSL#5, VDSL#6 and VDSL#7

2.1.2 Additive White Gaussian Noise (AWGN)

The power spectrum desity of AWGN is –140dBm/Hz, while that of the transmitted singal is –60dBm/Hz.

2.1.3 Crosstalk

Corsstalk is a noise on a telephone line that is caused by electromagnetic radiation of other adjacent lines. Such coupling increases with frequency. As shown in Figure 2.2, crosstalk caused by signals traveling in the opposite direction is called Near-End Crosstalk (NEXT), and that caused by signals traveling in the same direction is called Far-End Crosstalk (FEXT). Crosstalk noises are Gaussian signals, and their power spectral densities can be modeled as [3]:

$$PSD_{NEXT} = PSD_{disturber} \cdot (N/49)^{0.6} \cdot (8.818 \times 10^{-14}) \cdot f^{1.5} \qquad (2.1)$$

$$PSD_{FEXT} = PSD_{disturber} \cdot |H(f)|^2 \cdot (N/49)^{0.6} \cdot (7.999 \times 10^{-20}) \cdot d \cdot f^2 \qquad (2.2)$$

where N is the number of cross-talking lines, f is the frequency, d is the length in feet, $|H(f)|^2$ is the channel insertion loss, and $PSD_{disturber}$ is the power spectral density of signals on a cross-talking line. These noises can be very large.

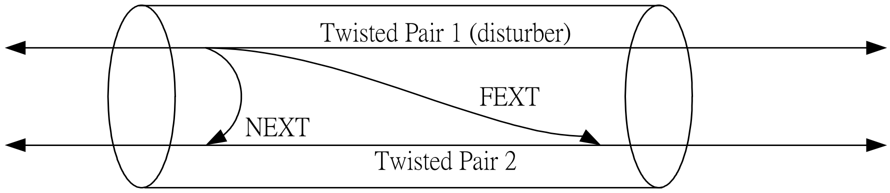

Figure 2.2 Illustration of Near-end and Far-end Crosstalk

2.1.4 Radio Frequency Interference

Radio noise also electromagnetically couples into telephone lines. There are two kinds of radio frequency interferences (RFI), Broadcast Radio Frequency Interference and Amateur Radio Frequency Interference. They must be rejected by the VDSL receivers. Broadcast RFI come from AM broadcast radio signals with fixed central frequencies. Amateur Radio (HAM), with changeable central frequencies, has much lager power than the Broadcast RFI. VDSL must avoid transmission in the known HAM bands. Table 2.1 shows the amateur radio bands recognized by ANSI [4].

Table 2.1 The Amateur Radio Bands Recognized by ANSI

Start frequency (MHz)	End frequency (MHz)
1.810	2.000
3.500	4.000
7.000	7.300
10.100	10.150
14.000	14.350
18.068	18.168
21.000	21.450
24.890	24.990
28.000	29.700

2.1.5 Impulse Noise

Impulse noise is a random pulse waveform with much higher amplitude than AWGN noise. For an unshielded twisted pair, a variety of electronic devices, circuit switching, and even lightening may cause impulse noise. Typically, impulse noise, with 5~20m Volt amplitude, occurs 1~5 times per minute and lasts for 30~150 microseconds [5].

2.2 DMT Basics

DMT (Discrete Multi-tone) [1][2] is a line code technique to transmit high-speed data over copper wires. The transmission channel is partitioned into a number of sub-channels, called *sub-carriers* or *tones*. These tones are orthogonal to each other and overlapped for bandwidth efficiency consideration. Data symbols would be modulated onto these tones, transmitted through the channel, and then demodulated by means of the orthogonality between these tones. Each tone may support a QAM (quadrature amplitude modulation) signal. Data-bit-stream should be split into several segments with variable length of bits. Note that the higher a tone's SNR (signal-to-noise ratio), the more bits are allocated. For further understanding, let's see how DMT modulation works in both continuous-time and discrete-time domains.

2.2.1 Continuous-time DMT Model

As shown in Figure 2.3, data of the nth DMT symbol is modulated onto N_{sc} sub-carriers by QAM as $X_{n,0}, X_{n,1}, \ldots, X_{n,N_{sc}-1}$, and then transmitted simultaneously.

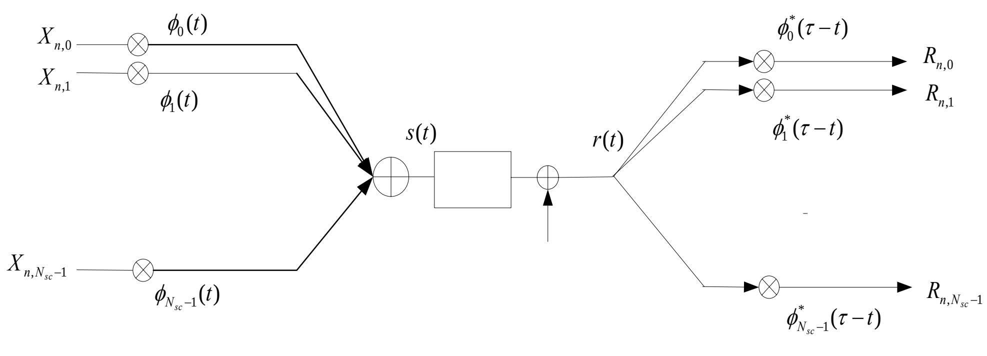

Figure 2.3 Continuous-time DMT model

$\{\phi_k^{(*)}(t) \mid k \in 0,1,\ldots,Nsc-1\}$ is a set of orthogonal basis function, which is usually a set of sinusoidal oscillator with oscillating frequencies $\{0, \Delta f\ ,\ 2\Delta f\ ,\ldots,(N_{sc}-1)\Delta f\ \}$.

Figure 2.4 shows the orthogonality between these sub-carriers in frequency domain. These sub-carriers are overlapped to save bandwidth.

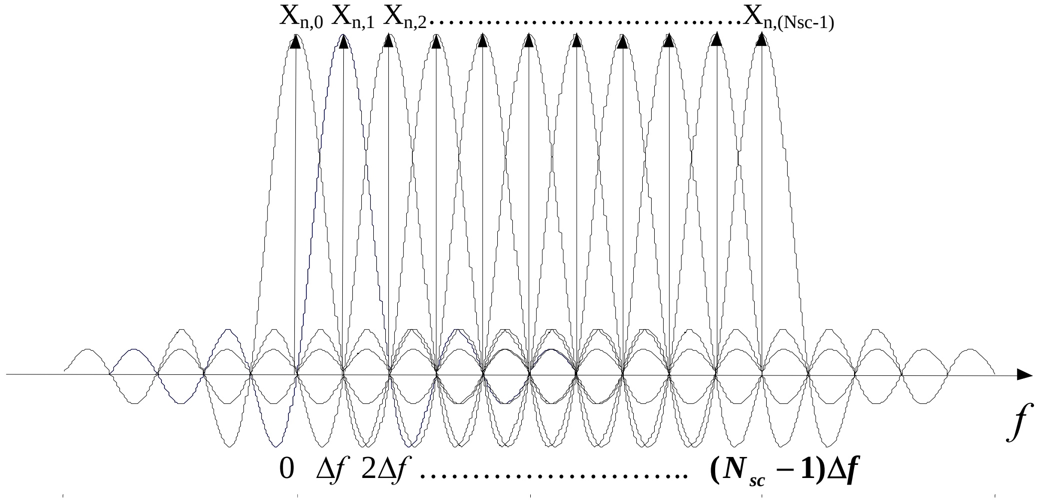

Figure 2.4 Spectral of DMT sub-carriers

As a result, the n[th] DMT symbol can be written as

$$s_n(t) = \sum_{k=0}^{Nsc-1} X_{n,k}\phi_k(t - n(T_g + T_{sym})) \tag{2.3}$$

where
$$\phi_k(t) = \begin{cases} e^{j2\pi k\Delta f t}, & t \in [0,(T_g + T_{sym})) \\ 0, & otherwise \end{cases} \tag{2.4}$$

and then
$$s(t) = \sum_{n=-\infty}^{\infty} s_n(t) = \sum_{n=-\infty}^{\infty}\sum_{k=0}^{Nsc-1} X_{n,k}\phi_k(t - nT) \tag{2.5}$$

where N_{sc} is the number of DMT sub-carriers, T_g is the guard interval duration, T_{sym} is the useful data duration of a DMT symbol and T, which equals to $(T_g + T_{sym})$, is the total duration of a DMT symbol. Note that the guard interval is inserted between two consecutive symbols to avoid inter-symbol interference, which originates from an imperfect channel impulse response. Details are discussed in Section 2.2.3. The received signal $r(t)$ is

$$r(t) = h(t) * s(t) + n(t) \tag{2.6}$$

where $h(t)$ is the channel impulse response and $n(t)$ is AWGN. Applying the orthogonality property between sub-carriers, the demodulated signal from the k[th] tone for the n[th] symbol, $R_{n,k}$, will be:

$$R_{n,k} = \int_{-\infty}^{\infty} r(t - nT)\phi_k^*(t - nT)dt = H_{n,k} X_{n,k} + N_{n,k} \tag{2.7}$$

where $H_k = \int_{-\infty}^{\infty} h(t - nT)\phi_k^*(t - nT)dt$ and $N_k = \int_{-\infty}^{\infty} n(t - nT)\phi_k^*(t - nT)dt$. Clearly, estimation of the k[th] tone's data $\hat{X}_{n,k}$ can be obtained by $\hat{X}_{n,k} = \dfrac{R_{n,k}}{\hat{H}_{n,k}}$, where $\hat{H}_{n,k}$ is an estimate of the channel frequency response at the k[th] tone for the n[th] symbol.

2.2.2 Discrete-time DMT Model

Recall from Eq. (2.3), a sampled version of transmitted signal $s_n(t)$ is

$$s_n(mTs) = \sum_{k=0}^{Nsc-1} X_{n,k} e^{j\frac{2\pi}{N_{sc}Ts}kmTs} = \sum_{k=0}^{Nsc-1} X_{n,k} e^{j\frac{2\pi}{N_{sc}}km}, \quad m = 0,1,\ldots,N_{sc}-1 \tag{2.8}$$

where T_s the sampling period and m is the sample index. Eq. (2.8) is the same as an IDFT (Inverse Discrete Fourier Transform) operation of X_n. Therefore, in the discrete-time domain, IDFT operation is used to modulate data onto sub-carriers. At receiver side, the demodulation process is performed by DFT (Discrete Fourier Transform). Considering hardware complexity and operation speed, IFFT/FFT (Inverse Fast Fourier Transform/Fast Fourier Transform) is frequently used, which is shown in Figure 2.5

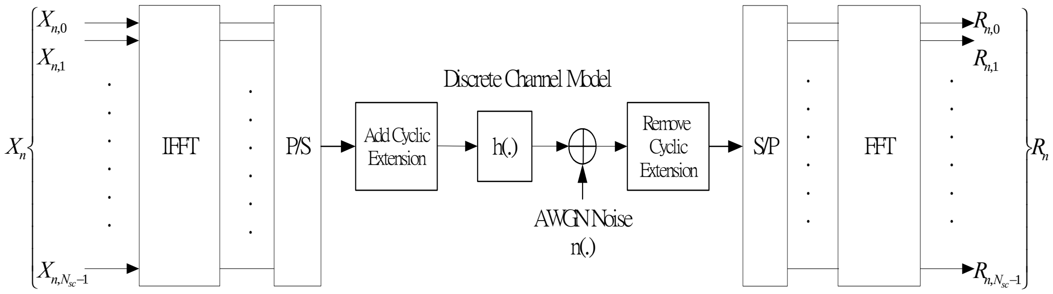

Figure 2.5 Discrete-time DMT model

2.2.3 Guard Interval and Cyclic Extension

Since the impulse response of transmission channel is not perfect, inter-symbol interference (ISI) effect may exist. A solution to this ISI problem is to insert a guard interval (GI) in between every two consecutive symbols. The duration of guard interval is constrained to be larger than that of the channel impulse response. However, another problem arises as an "empty" guard interval is added. As shown in Figure 2.6, a useful symbol window indicates the portion where the received signal is to be demodulated. If the symbol boundary estimation is not precise, which causes the useful symbol window to drift, then the orthogonality between sub-carriers will be destroyed and inter-carrier interferences (ICI) effect will be introduced.

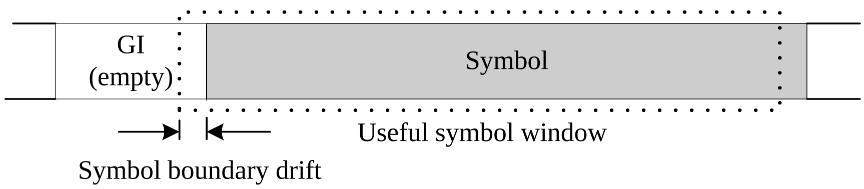

Figure 2.6 Received time-domain DMT signal with empty GI

For a continuous-time DMT model, the empty GI is avoided by simply extending the symbol duration on each subcarrier to (T_g+T_{sym}), as shown in Figure 2.7. Note that the trailing portion of each symbol is exactly the same as GI portion. As long as the symbol boundary estimate locates in GI, a small symbol boundary error only results in a phase shift for the received signal while the orthogonality between sub-carriers is maintained. As for a discrete-time DMT model, it is equivalent to copy the trailing portion of each symbol to the guard interval immediately preceding the symbol, which is called cyclic prefix (CP) extension (as shown in Figure 2.8).

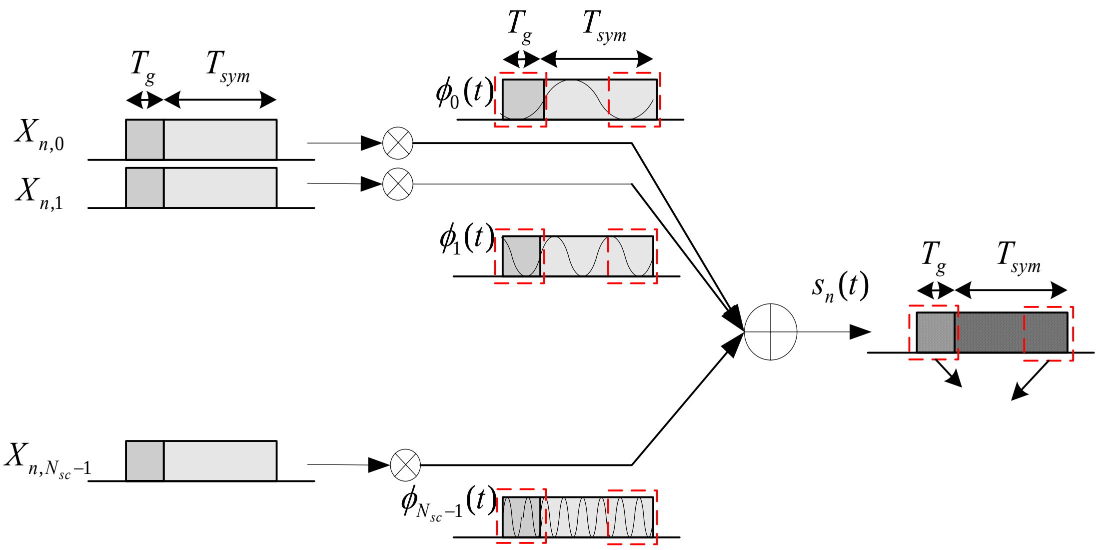

Figure 2.7 A continuous-time domain DMT symbol with non-empty GI

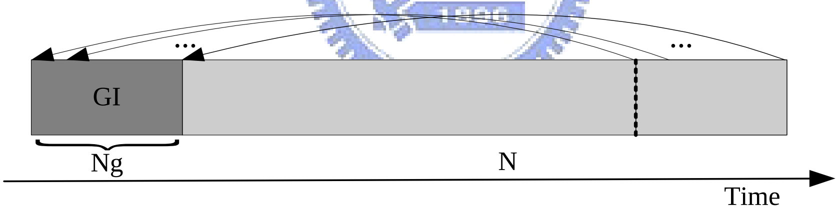

Figure 2.8 A discrete-time domain DMT symbol with cyclic extension in GI

On the other hand, by placing CP in GI, the result of linear convolution between DMT symbols and transmission channel would be the same as that of circular convolution. The n^{th} received symbol in frequency domain can be described as:

$$\begin{aligned} R_n &= FFT(IFFT(X_n) \otimes h + n) \\ &= X_n H_n + N \end{aligned} \tag{2.9}$$

Thus, data carried by the n^{th} DMT symbol can be recovered by

$$\hat{X}_n = \frac{R_n}{\hat{H}_n} \tag{2.10}$$

where $\hat{H}_n$ is an estimate of the channel frequency response while the n^th DMT symbol is received.

2.2.4 Bit Loading

The process of assigning information bits and energy to each tone is called *bit loading.* Each tone has a transmit energy E_k, channel gain H_k, noise power spectral density σ_k^2 and a number of bits b_k, where the subscript *k* indicates the tone index. A gap Γ represents the effective SNR loss with respect to the capacity. For example, QAM achieves a gap of 9.8 dB at a symbol error probability 10^{-7}.

$$b_k = \log_2(1 + \frac{E_k \cdot |H_k|^2}{\Gamma \sigma_k^2}) = \log_2(1 + \frac{SNR_k}{\Gamma}) \qquad (2.11)$$

Thus the total data rate *R* over Nsc tones is

$$R = R_{symbol} \cdot \sum_{k=1}^{Nsc} b_k \ (bps) \qquad (2.12)$$
$$\text{where } R_{symbol} \text{ is the symbol rate}$$

Figure 2.9 shows a possible bit allocation plan versus tone index, which achieves 53M bits per second for the sum of upstream and downstream data rates.

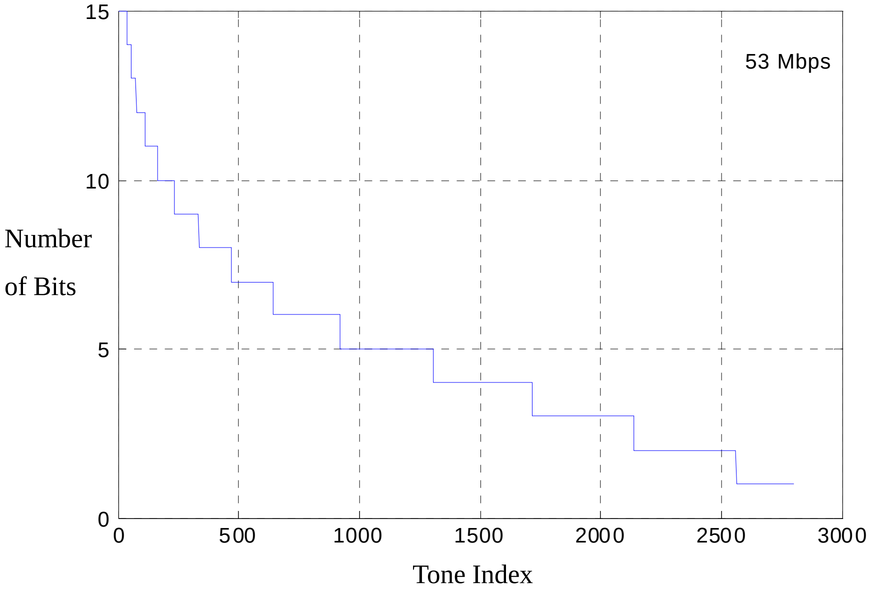

Figure 2.9 An example of bit allocation versus tone index