The main advantage of the pulse modulation technique is that low modulation voltages can be used (several volts). The reason for this is that the number of teeth generated, and hence the spectral width of the comb, is independent of modulation depth. However, to maximise the comb generating efficiency a $\pi/2$ rad modulation depth per Mach–Zehnder arm should be used.

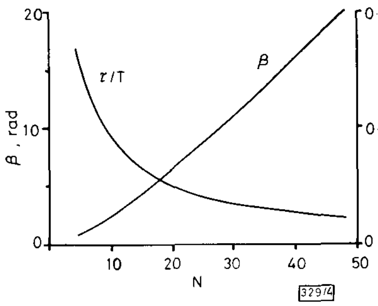

Fig. 4 *β and τ/T values as function of number N of comb teeth generated*

For the sinusoidal technique to generate a similar number of teeth, modulation depths greater than this are needed (Fig. 4), hence the need for higher modulation voltages (tens of volts). It is clear from this that the spectral width of the sinusoidally generated comb will be limited by either the output capabilities of practical power amplifiers, or by the electrical input limitations of optical modulators. The main advantage of the sinusoidal modulation technique is that the number of teeth generated is determined only by the modulation depth. The reason for this is that the number of Bessel sidebands, and hence the spectral width of the comb, is independent of the modulation bandwidth.

Unlike the sinusoidal technique, the pulse approach requires broad bandwidth power amplifiers and optical modulators because the rectangular modulation pulse shape must be preserved. The outcome of this is that the lowest bandwidth device determines the maximum spectral width of the comb, which for a specified number of teeth places an upper limit on the frequency separation that can be used between them. For a reasonably sized comb, the several GHz bandwidth of present-day power amplifiers will probably limit the maximum separation frequency to sub-GHz values, whereas for the sinusoidal case, values exceeding an order of magnitude larger are in principle feasible.

Conclusions: The optical frequency combs generated by a pulse and a sinusoidally driven Mach–Zehnder have been compared. The pulse drive has been shown to have the advantage of using a low drive voltage, and that the limited electrical bandwidths of present power amplifiers will restrict its use to generating combs with sub-GHz tooth spacings. If larger spacings are required, values of up to several tens of GHz should be possible by using a sinusoidal drive, provided the higher voltage requirements can be met.

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MINIMUM EUCLIDEAN DISTANCES OF PARTIAL RESPONSE MULTI-*h* PHASE-CODED MODULATIONS WITH ASYMMETRIC MODULATION INDICES

Indexing terms: Modulation, Information theory

A new concept of asymmetric modulation indices has recently been proposed and applied on full-response multi-*h* phase-coded modulations (MHPM) and improved error probability performance has been found. In this letter, a similar concept is applied to partial response MHPM and the minimum Euclidean distances for linear phase pulse function are evaluated to show further improvements.

Introduction: Multi-*h* phase-coded modulation[1] (MHPM) presents a class of constant envelope and continuous phase modulations with significant improvements in the power/bandwidth design trade-offs relative to conventional techniques. The information-carrying phase function of such schemes varies continuously with time, with modulation indices changed cyclically for successive symbol intervals. These time-varying modulation indices essentially increase the duration of time for which distinct data sequences have distinct phase paths, thereby increasing the minimum Euclidean distances. On the other hand, the technique of partial response yields further improvements in the trade-off between error performance and spectral behaviour as compared with full response systems.[2,3] Partial response multi-*h* phase-coded modulations combine the above two features, for which the upper bounds on minimum Euclidean distance have been well evaluated and discussed.[4] In this letter, the concept of asymmetric indices is applied to partial response multi-*h* schemes to obtain very attractive improvements in the minimum Euclidean distances.

System description: The general form of an MHPM signal is

$$S(t, \alpha) = \sqrt{(2E/T)} \cos(\omega_c t + \phi(t, \alpha) + \phi_0) \quad (1)$$

The information-carrying phase function is

$$\phi(t, \alpha) = 2\pi \sum_{i=-\infty}^{\infty} a_i h_i q(t - (i-1)T) \qquad -\infty \leq t \leq \infty \quad (2)$$

where $\alpha = \{\ldots, a_{-2}, \alpha_{-1}, \alpha_0, \alpha_1, \alpha_2, \ldots\}$ is the transmitted data sequence, h_i is the cyclically varying modulation index corresponding to the ith symbol, and $q(t)$ is a phase pulse function. In practice, the modulation indices are often obtained from a set of rational values of the form $\{l_0/q, l_1/q, \ldots, l_{K-1}/q\}$, where $l_i < q$ for $0 \leq i \leq K-1$, l_i and q are all integers, and K is the number of different modulation indices.

A typical example of the phase pulse function $q(t)$ is the linear phase pulse with length L symbol intervals

$$q(t) = \begin{cases} 0 & t < 0 \\ t/2LT & 0 \leq t \leq LT \\ 1/2 & t \geq LT \end{cases} \quad (3)$$

where $L > 1$ corresponds to a partial response signal. The upper bounds on minimum Euclidean distance for partial response MHPM with such linear phase pulse function and $L = 2$ have been found to be very attractive for $h_i < 1.0$.[4] Such a linear phase function with duration $LT = 2T$ will thus be used in all the following numerical results in this letter.

In conventional or symmetric MHPM schemes, the modulation index h_i for the ith symbol has only one value no matter whether the transmitted datum is $+1$ or -1, and $q(2h_i)$ will always be an even number. However, when the concept of MHPM with asymmetric modulation indices are used in this letter, i.e. the modulation index h_{+i} for the datum $+1$ and h_{-i} for the datum -1 are not necessarily equal, an additional degree of freedom in choosing indices with better performance will be provided.

Minimum Euclidean distances: It is difficult to analyse the error probability performance of a coherent receiver for MHPM schemes. One commonly used approach is to calculate the minimum squared Euclidean distance over several symbol intervals for all possible cyclic shifts of h_i values. The minimum Euclidean distance for minimum shift keying (MSK) is 4.

Table 1 MINIMUM EUCLIDEAN DISTANCES FOR PARTIAL RESPONSE MULTI-h PHASE-CODED MODULATIONS WITH $K = 2$, 3 AND 4

	$K = 2$		$K = 3$		$K = 4$	
q	Conv.	Asym.	Conv.	Asym.	Conv.	Asym.
4	4·36	8·41				
5	6·85	8·83				
6	7·39	8·69				
7	7·90	8·60				
8	8·41	9·03	6·37	9·62		
9	8·85	8·92	6·91	9·79		
10	8·83	8·83	8·76	10·04		
11	8·76	9·02	8·11	10·62		
12	8·69	9·03	8·18	10·32		
13	8·64	8·95	9·15	10·70		
14			9·46	10·53		
15			9·71	10·74		
16			9·62	10·68	9·07	11·22
17			9·49	10·77	9·43	11·21
18					9·03	11·32
19					10·61	11·38
20					10·86	11·56
21					9·90	11·32
22					10·50	11·60
Average	7·87	8·83	8·58	10·38	9·91	11·37

The minimum Euclidean distances for the best combinations of modulation indices with $K = 2$, 3 and 4 have been calculated for partial response MHPM. The results for conventional MHPM are listed in columns 1, 3 and 5 of Table 1, whereas those for MHPM with asymmetric modulation indices are listed in columns 2, 4 and 6. From this Table, we can find that the minimum Euclidean distances for asymmetric MHPM schemes are always larger than those of conventional MHPM for all values of q for which the calculation has been performed. It is also revealed that the minimum Euclidean distances are much less dependent on the choice of q when asymmetric modulation indices are used. Furthermore, it is interesting to find that the average distances of asymmetric MHPM with $K = 2$ and $K = 3$ exceed those of conventional MHPM with $K = 3$ and $K = 4$, respectively. Since the implementation complexity of multi-h systems increases with K, this means asymmetric MHPM can be used to obtain even better error probability performance as compared with conventional MHPM but with less complexity.

In Fig. 1, the minimum Euclidean distances are plotted against q with $K = 2$, 3 and 4 for both full response and partial response MHPMs with asymmetric modulation indices. It is easy to see in the Figure that in asymmetric MHPM schemes partial response provides better performance than full response for K varying from 2 to 4.

Conclusion: In this letter, the concept of asymmetric modulation indices has been applied in partial response multi-h schemes and the minimum Euclidean distances are calculated.

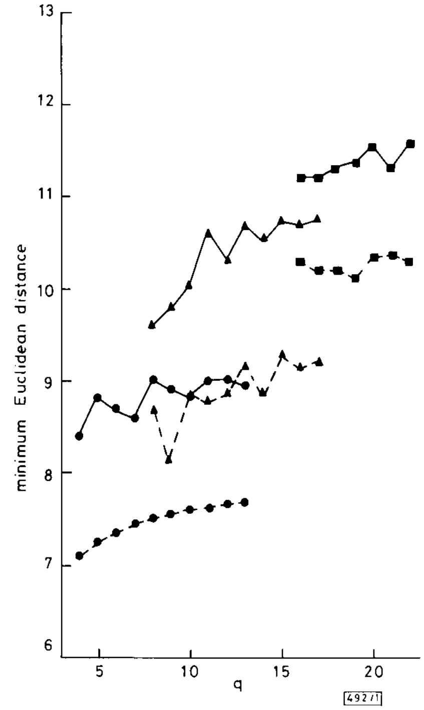

Fig. 1 *Comparison of minimum Euclidean distances for full response and partial response MHPM with asymmetric modulation indices*

——— partial response - - - - full response

● $K = 2$ ▲ $K = 3$ ■ $K = 4$

Reasonable improvements in error probability performance over conventional MHPM with symmetric modulation indices are found, and with asymmetric modulation indices partial response is shown to give better performance than full response.

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