國立交通大學

電機學院　電機與控制學程

碩士論文

具降階估測器之可變結構控制器設計

Reduced-Order Observer-Based Sliding Mode Controller Design

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中華民國九十八年六月

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摘　　　要

本論文主要在設計可變結構控制器用來控制部份狀態變數無法量測之系統，針對無法量測之狀態變數，此控制器結合 Kudva 所使用的 Luenberger 降階估測器，此估測器能在雜訊干擾的情況下準確估測狀態變數，不受雜訊的影響，此外此控制器也使用順滑層來減抑不當的切跳現象並且消除匹配式雜訊。本論文針對三種不同的狀況來設計可變結構控制法則，包括具匹配式雜訊系統之穩定性控制，具非匹配式雜訊系統之穩定性控制，以及具非匹配式雜訊系統之追蹤控制，除了詳細列出控制法則的推導過程外，最後還利用六個不同的範例來進行數值模擬驗證，分別探討雜訊維度對降階估測器的影響，匹配式雜訊與非匹配式雜訊對受控系統穩定性與效能的影響，以及雜訊對輸出追蹤控制的影響，根據模擬結果，此可變結構控制器確實能達到所預設的控制目的。

Reduced-Order Observer-Based Sliding Mode Controller Design

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ABSTRACT

This thesis presents reduced-order observer-based sliding mode controller (ROSMC) design. The reduced-order observer, a kind of Luengerger observer and designed by Kudva, is used to accurately estimate the unmeasurable state variables, even under the influence of undesirable disturbance. Besides, the ROSMC also employs a sliding layer to reduce the chattering phenomenon and eliminate the matched disturbance. There are three cases discussed in this thesis, including stability control of system with matched disturbance, stability control of system with mismatching disturbance, and output tracking control of system with disturbance. To demonstrate the usefulness of the ROSMC, there are six examples given and simulated by the software package MATLAB. The simulation results are mainly used to show the effect on the observer caused by the dimension of unmeasurable state variables, the effect on the system stability and performance caused by matched and mismatching disturbance, the effect on the output tracking control caused by disturbance. Form the simulation results, the developed ROSMC is indeed able to achieve the desired control goal.

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謹以此篇論文獻給所有關心我、照顧我的人

魏吉佑　2009.6

CONTENTS

LIST OF FIGURES

Chapter 1

Introduction

1.1 Motivation

This thesis will focus on the sliding mode controller design for a system with unmeasurable state variables. Unfortunately, a sliding mode control is commonly based on the feedback of full state variables. That means it is difficult to design a sliding mode control when unmeasurable state variables exist. To solve this problem, it is required for the sliding mode control to combine an observer to estimate the unmeasurable state variables. Hence, how to choose an appropriate observer for the sliding mode control becomes an important task in this thesis.

There are several observers proposed to estimate these unmeasurable state variables, including the sliding observer [1] and the reduced order observer [2]. The sliding observer is proposed by Utkin for systems without noise and achieves good performance. However, the sliding observer is only limited to noiseless systems and not extendable to general systems, which often inevitably suffer from noises. Hence, in order to deal with noises existing in general systems, Kudva presented the reduced-order Luenberger observer [2], which has been shown robust to noises. In addition, it has been found that both the sliding mode observer and the reduced-order Luenberger observer are constrained to the same conditions. Clearly, the reduced-order Luenberger observer is indubitably better than the sliding mode observer and more suitable for systems encountering noises. Therefore, this thesis will employ the reduced-order Luenberger observer to estimate the unmeasurable state variables while design the sliding mode control, which is called the reduced-order observer-based sliding mode controller, or ROSMC in short.

1.2 Research background

The feature of sliding mode control system is claimed to result in superb system performance, which includes insensitivity to parameter variations and complete rejection of matched disturbances [3]. The sliding mode control research community has risen to respond to the critical challenge, chattering phenomenon, which is the only obstacle for sliding mode to become one of the most significant discoveries in modern control theory [4-7]. Many analytical design methods were proposed to reduce the affects of chattering [8-12] and the use of sliding layer is the commonest one, which, however, inevitably decreases the control precision [13]. In general, the design of a sliding mode control is composed of two basic steps. For the first step, design a sliding function to guarantee the desired system performance in the sliding mode. For the second step, develop the control law to drive the system trajectories into the sliding layer in a finite time and stay thereafter [1, 13-15].

When unmeasurable state variables exist, the use of observer to estimate is required. The sliding mode observer design method has been proposed by Utkin, where some sufficient conditions of uncertain input need to be satisfied for the asymptotic convergence of the estimated state [16]. By selecting appropriate gains for the sliding mode observer, it can be guaranteed that the convergence of any initial estimated state to its true state in a finite time [17]. Once the estimated state reaches the true state, it will remain on the trajectory of the true state or within a very small region around the true state. However, the sliding mode observer is designed under the assumption that the measurement is not corrupted by noise, so it may not perform well for a system with noise [1].

Recently, many investigators have focused on the reduced-order Luenberger observers for the state estimation of linear systems subject to unknown inputs. Besides the observability conditions, some other conditions in terms of the zeros of the system characteristic polynomial are also needed for the observer to be designable [2, 18-21]. This thesis had tried to develop sliding mode observer to LTI system with disturbance, but found that it required the same restricted conditions as those of the reduced-order Luenberger observer proposed by Kudva. Clearly, the reduced-order Luenberger observer is indubitably better than the sliding mode observer and more suitable for systems encountering disturbance. Hence, this thesis will adopt the observer proposed by Kudva to deal with the unknown-input [2].

1.3 Thesis organization

The thesis is organized as follows. The mathematical model of observer design is introduced in Chapter 2, including the reduced order observer and the sliding mode observer. In Chapter 3, the sliding mode controller is designed based on the reduced-order observer for three cases, which are regulation control for matched disturbance, regulation control for mismatching disturbance, and output tracking control for disturbance system. In chapter 4, the simulation and results will be shown to demonstrate the usefulness of the reduced-order observer-based sliding mode controller (ROSMC). Form the simulation results, the developed ROSMC is indeed able to achieve the desired control goal. Finally, the conclusions and future research will be proposed in Chapter 5.

Chapter 2

The Mathematical Model of Observer Design

In this chapter, two kinds of observers, reduced order observer [2] and sliding mode observer [1], are introduced in Section 2.1 and 2.2 respectively for a class of linear time-invariant systems with matched and mismatched uncertainties and disturbances. Both observers are found restricted to the same conditions within the design process. In fact, the reduced order observer is more convenient when applying to system control problems. Therefore, only the reduced order observer will be adopted in Chapter 3, combined into the sliding mode control design.

2.1 Reduced order observer design

In general, an observer is required when a controller design faces insufficient information of system states. Here, introduce the reduced order observer proposed by Kudva et al. [2] for the linear time-invariant system expressed as

$$\dot{\boldsymbol{x}}(t)=\boldsymbol{A}\boldsymbol{x}(t)+\boldsymbol{B}\boldsymbol{u}(t)+\boldsymbol{E}\boldsymbol{d}(t) \tag{2-1}$$

where $\boldsymbol{x}(t)\in\Re^{n}$ is the system state, $\boldsymbol{u}(t)\in\Re^{m}$ is the control input, and $\boldsymbol{d}(t)\in\Re^{q}$ is the disturbance. Without loss of generality, let the system state be decomposed into two parts, $\boldsymbol{x}_1(t)$ and $\boldsymbol{x}_2(t)$, i.e., $\boldsymbol{x}(t)=\begin{bmatrix}\boldsymbol{x}_1^T(t) & \boldsymbol{x}_2^T(t)\end{bmatrix}^T$, where $\boldsymbol{x}_1(t)\in\Re^{p}$ is measurable and $\boldsymbol{x}_2(t)\in\Re^{n-p}$ is not obtainable. Hence, (2-1) can be rewritten as

$$\begin{bmatrix}\dot{\boldsymbol{x}}_1(t)\\ \dot{\boldsymbol{x}}_2(t)\end{bmatrix}=\begin{bmatrix}\boldsymbol{A}_{11} & \boldsymbol{A}_{12}\\ \boldsymbol{A}_{21} & \boldsymbol{A}_{22}\end{bmatrix}\begin{bmatrix}\boldsymbol{x}_1(t)\\ \boldsymbol{x}_2(t)\end{bmatrix}+\begin{bmatrix}\boldsymbol{B}_1\\ \boldsymbol{B}_2\end{bmatrix}\boldsymbol{u}(t)+\begin{bmatrix}\boldsymbol{E}_1\\ \boldsymbol{E}_2\end{bmatrix}\boldsymbol{d}(t) \tag{2-2}$$

where $\left(\begin{bmatrix}\boldsymbol{A}_{11} & \boldsymbol{A}_{12}\\ \boldsymbol{A}_{21} & \boldsymbol{A}_{22}\end{bmatrix},\begin{bmatrix}\boldsymbol{B}_1\\ \boldsymbol{B}_1\end{bmatrix}\right)$ is controllable since $(\boldsymbol{A},\boldsymbol{B})$ is controllable, and $(\boldsymbol{A}_{12},\boldsymbol{A}_{22})$ is an observable pair. Then, an observer of order of (n-p) to estimate $\boldsymbol{x}_2(t)$ is constructed as below[2]:

$$\dot{z}(t) = Fz(t) + Jx_1(t) + Nu(t) \tag{2-3}$$

where

$$\begin{aligned} F &= A_{22} - LA_{12} \\ J &= FL + A_{21} - LA_{11} \\ N &= B_2 - LB_1 \end{aligned} \tag{2-4}$$

Since (A_{22}, A_{12}) is an observable pair, there exists an $L \in \Re^{(n-p)\times p}$ such that F is of Hurwitz and contain desired stable eigenvalues. Further choose the estimated state of $x_2(t)$ as

$$\hat{x}_2(t) = z(t) + Lx_1(t) \tag{2-5}$$

then find the derivative of $\hat{x}_2(t)$ from (2-2) and (2-3) as

$$\begin{aligned} \dot{\hat{x}}_2(t) &= \dot{z}(t) + L\dot{x}_1(t) \\ &= Fz(t) + Jx_1(t) + Nu(t) + L\left(A_{11}x_1(t) + A_{12}x_2(t) + B_1u(t) + E_1d(t)\right) \end{aligned} \tag{2-6}$$

Define the estimated error as

$$\tilde{x}_2(t) = \hat{x}_2(t) - x_2(t) \tag{2-7}$$

and then achieve the derivative of $\tilde{x}_2(t)$ from (2-2) and (2-6) as

$$\dot{\tilde{x}}_2(t) = \dot{\hat{x}}_2(t) - \dot{x}_2(t) = F\tilde{x}_2(t) + (E_2 - LE_1)d(t) \tag{2-8}$$

where $F = A_{22} - LA_{12}$ has been chosen to contain stable eigenvalues. In order to eliminate the effect caused by the disturbance $d(t)$, the term $E_2 - LE_1$ must vanish to achieve $\tilde{x}_2(t) \to 0$ as $t \to \infty$, i.e., $\hat{x}_2(t) \to x_2(t)$ as $t \to \infty$. Clearly, the reduced order observer should be designed under the following two conditions:

i) $F = A_{22} - LA_{12}$ has stable eigenvalues (2-9)

ii) $E_2 - LE_1 = 0$ (2-10)

In fact, it is not easy to choose L satisfying (2-9) and (2-10) simultaneously. To deal with such problem, an algorithm is given next to suitably design L.

The existence condition of L to satisfy (2-9) and (2-10) has been introduced by Kudva, et. al. as below: [2]

$$rank E_1 = rank \begin{bmatrix} E_1 \\ E_2 \end{bmatrix} = rank\left(\begin{bmatrix} A_{11} & A_{12} \end{bmatrix} E\right) = q \text{, and } \quad p \geq q \tag{2-11}$$

Clearly, E and E_1 are of full rank. Besides, the dimension of the measurable state $x_1(t)$ is not less than that of the disturbance $\boldsymbol{d}(t)$. Once (2-11) is guaranteed, $\boldsymbol{L}$ can be selected as

$$\boldsymbol{L} = \boldsymbol{E}_2\boldsymbol{E}_1^{+} + \boldsymbol{\Gamma}\left(\boldsymbol{I}_p - \boldsymbol{E}_1\boldsymbol{E}_1^{+}\right) \tag{2-12}$$

where $\boldsymbol{E}_1^{+} = \left(\boldsymbol{E}_1^T\boldsymbol{E}_1\right)^{-1}\boldsymbol{E}_1^T$ is the generalized inverse of $\boldsymbol{E}_1$, then $\boldsymbol{E}_1^{+}\boldsymbol{E}_1 = \boldsymbol{I}_q$, and $\boldsymbol{\Gamma} \in \Re^{(n-p)\times p}$ is designed to satisfy (2-9). Consider two conditions of $p = q$ and $p > q$ to choose matrix $\boldsymbol{L}$. For the first condition $p = q$, $\boldsymbol{E}_1$ should be square and invertible, which leads to $\boldsymbol{I}_p - \boldsymbol{E}_1\boldsymbol{E}_1^{+} = 0$. Then, (2-12) becomes

$$\boldsymbol{L} = \boldsymbol{E}_2\boldsymbol{E}_1^{-1} \tag{2-13}$$

and (2-9) can be rewritten as

$$\boldsymbol{F} = \boldsymbol{A}_{22} - \boldsymbol{L}\boldsymbol{A}_{12} = \boldsymbol{A}_{22} - \boldsymbol{E}_2\boldsymbol{E}_1^{-1}\boldsymbol{A}_{12} \tag{2-14}$$

Clearly, the stability of $\boldsymbol{F}$ can not be determined by $\boldsymbol{\Gamma}$. Hence, if $\boldsymbol{F}$ in (2-14) is not stable, then the observer (2-3) is unable to estimate the system state $\boldsymbol{x}$. On the other hand, if $\boldsymbol{F}$ in (2-14) is stable, then the observer (2-3) is available for estimating $\boldsymbol{x}$. For the second condition $p > q$, $\boldsymbol{E}_1$ is no more square and invertible, i.e., $\boldsymbol{I}_p - \boldsymbol{E}_1\boldsymbol{E}_1^{+} \neq 0$. By substituting (2-12) into (2-9), $\boldsymbol{F}$ can be rearranged as

$$\begin{aligned}\boldsymbol{F} &= \boldsymbol{A}_{22} - \left(\boldsymbol{E}_2\boldsymbol{E}_1^{+} + \boldsymbol{\Gamma}\left(\boldsymbol{I}_p - \boldsymbol{E}_1\boldsymbol{E}_1^{+}\right)\right)\boldsymbol{A}_{12} \\ &= \boldsymbol{A}_{22} - \boldsymbol{E}_2\boldsymbol{E}_1^{+}\boldsymbol{A}_{12} + \boldsymbol{\Gamma}\left(\boldsymbol{I}_p - \boldsymbol{E}_1\boldsymbol{E}_1^{+}\right)\boldsymbol{A}_{12} \\ &= \boldsymbol{\Phi}_1 + \boldsymbol{\Gamma}\boldsymbol{\Phi}_2\end{aligned} \tag{2-15}$$

where $\boldsymbol{\Phi}_1 = \boldsymbol{A}_{22} - \boldsymbol{E}_2\boldsymbol{E}_1^{+}\boldsymbol{A}_{12}$ and $\boldsymbol{\Phi}_2 = \left(\boldsymbol{I}_p - \boldsymbol{E}_1\boldsymbol{E}_1^{+}\right)\boldsymbol{A}_{12}$. Clearly, it is required to find a matrix $\boldsymbol{\Gamma}$ which guarantees $\boldsymbol{F}$ is stable. If such a matrix $\boldsymbol{\Gamma}$ does not exist, it is not possible to design the reduced order observer (2-3). In case that $\boldsymbol{\Gamma}$ exists, then $\boldsymbol{L}$ can be chosen as (2-12) accordingly and the design of the observer (2-3) is completed which can successfully estimate $\boldsymbol{x}_2$ as given in (2-5).

2.2 Sliding mode observer design

This section will introduce the sliding mode observer proposed by Utkin et al. [1] for the linear time-invariant system, which is a full order observer and more complicated than the reduced order observer introduced in Section 2.1. In addition, this sliding mode observer and the reduced order observer are restricted to the same conditions given (2-9) and (2-10). Hence, this thesis will only employ the reduced order observer and the sliding mode observer is introduced in this section just for reference.

The sliding mode observer design is applied to the same system (2-2), shown in Section 2.1 and rewritten as

$$\begin{bmatrix} \dot{\boldsymbol{x}}_1(t) \\ \dot{\boldsymbol{x}}_2(t) \end{bmatrix} = \begin{bmatrix} \boldsymbol{A}_{11} & \boldsymbol{A}_{12} \\ \boldsymbol{A}_{21} & \boldsymbol{A}_{22} \end{bmatrix} \begin{bmatrix} \boldsymbol{x}_1(t) \\ \boldsymbol{x}_2(t) \end{bmatrix} + \begin{bmatrix} \boldsymbol{B}_1 \\ \boldsymbol{B}_2 \end{bmatrix} \boldsymbol{u}(t) + \begin{bmatrix} \boldsymbol{E}_1 \\ \boldsymbol{E}_2 \end{bmatrix} \boldsymbol{d}(t) \tag{2-16}$$

where $\boldsymbol{x}_1(t) \in \Re^p$ is obtainable and $\boldsymbol{x}_2(t) \in \Re^{n-p}$ is not measurable. Note that $(\boldsymbol{A}_{12}, \boldsymbol{A}_{22})$ is an observable pair. The sliding mode observer proposed by Utkin is constructed as below[1]:

$$\begin{cases} \dot{\hat{\boldsymbol{x}}}_1(t) = \boldsymbol{A}_{11}\hat{\boldsymbol{x}}_1(t) + \boldsymbol{A}_{12}\hat{\boldsymbol{x}}_2(t) + \boldsymbol{B}_1\boldsymbol{u}(t) - \boldsymbol{v}(t) \\ \dot{\hat{\boldsymbol{x}}}_2(t) = \boldsymbol{A}_{21}\hat{\boldsymbol{x}}_1(t) + \boldsymbol{A}_{22}\hat{\boldsymbol{x}}_2(t) + \boldsymbol{B}_2\boldsymbol{u}(t) + \boldsymbol{L}\boldsymbol{v}(t) \end{cases} \tag{2-17}$$

where $(\hat{\boldsymbol{x}}_1(t), \hat{\boldsymbol{x}}_2(t))$ represents the state estimate for $(\boldsymbol{x}_1(t), \boldsymbol{x}_2(t))$, $\boldsymbol{L} \in \Re^{(n-p)\times p}$ is a constant feedback gain matrix and $\boldsymbol{v} \in \Re^p$ is a discontinuous vector defined component wise by

$$\boldsymbol{v}(t) = M \cdot sgn(\tilde{\boldsymbol{x}}_1(t)) \tag{2-18}$$

where $M \in R_+$ will be determined later. From (2-16) and (2-17), the error dynamics are given by:

$$\begin{cases} \dot{\tilde{\boldsymbol{x}}}_1(t) = \dot{\hat{\boldsymbol{x}}}_1(t) - \dot{\boldsymbol{x}}_1(t) = \boldsymbol{A}_{11}\tilde{\boldsymbol{x}}_1(t) + \boldsymbol{A}_{12}\tilde{\boldsymbol{x}}_2(t) - \boldsymbol{v}(t) - \boldsymbol{E}_1\boldsymbol{d}(t) \\ \dot{\tilde{\boldsymbol{x}}}_2(t) = \dot{\hat{\boldsymbol{x}}}_2(t) - \dot{\boldsymbol{x}}_2(t) = \boldsymbol{A}_{21}\tilde{\boldsymbol{x}}_1(t) + \boldsymbol{A}_{22}\tilde{\boldsymbol{x}}_2(t) + \boldsymbol{L}\boldsymbol{v}(t) - \boldsymbol{E}_2\boldsymbol{d}(t) \end{cases} \tag{2-19}$$

Let

$$\boldsymbol{e}_2(t) = \tilde{\boldsymbol{x}}_2(t) + \boldsymbol{L}\tilde{\boldsymbol{x}}_1(t) \tag{2-20}$$

then the derivative of $\boldsymbol{e}_2(t)$ is

$$\dot{\boldsymbol{e}}_2(t) = \dot{\tilde{\boldsymbol{x}}}_2(t) + \boldsymbol{L}\dot{\tilde{\boldsymbol{x}}}_1(t) = \tilde{\boldsymbol{A}}_{22}\tilde{\boldsymbol{x}}_2(t) + \tilde{\boldsymbol{A}}_{21}\tilde{\boldsymbol{x}}_1(t) - \tilde{\boldsymbol{E}}_2\boldsymbol{d}(t) \tag{2-21}$$

where

$$
\begin{aligned}
\tilde{\boldsymbol{A}}_{22} &= \boldsymbol{A}_{22} + \boldsymbol{L}\boldsymbol{A}_{12} \\
\tilde{\boldsymbol{A}}_{21} &= \boldsymbol{A}_{21} + \boldsymbol{L}\boldsymbol{A}_{11} - \tilde{\boldsymbol{A}}_{22}\boldsymbol{L} \\
\tilde{\boldsymbol{E}}_{2} &= \boldsymbol{E}_{2} + \boldsymbol{L}\boldsymbol{E}_{1}
\end{aligned} \tag{2-22}
$$

Further substituting (2-18) into (2-19) of $\dot{\tilde{\boldsymbol{x}}}_1(t)$ yields

$$
\dot{\tilde{\boldsymbol{x}}}_1(t) = \tilde{\boldsymbol{A}}_{11}\tilde{\boldsymbol{x}}_1(t) + \boldsymbol{A}_{12}\tilde{\boldsymbol{x}}_2(t) - \boldsymbol{v}(t) - \boldsymbol{E}_1\boldsymbol{d}(t) \tag{2-23}
$$

where $\tilde{\boldsymbol{A}}_{11} = \boldsymbol{A}_{11} - \boldsymbol{A}_{12}\boldsymbol{L}$. By premultiplying $\tilde{\boldsymbol{x}}_1^T(t)$ into (2-23), it can be attained that

$$
\begin{aligned}
\tilde{\boldsymbol{x}}_1^T(t)\dot{\tilde{\boldsymbol{x}}}_1(t) &= \tilde{\boldsymbol{x}}_1^T(t)\left(\tilde{\boldsymbol{A}}_{11}\tilde{\boldsymbol{x}}_1(t) + \boldsymbol{A}_{12}\tilde{\boldsymbol{x}}_2(t) - Msgn\left(\tilde{\boldsymbol{x}}_1(t)\right) - \boldsymbol{E}_1\boldsymbol{d}(t)\right) \\
&< \left\|\tilde{\boldsymbol{x}}_1^T(t)\right\|\left(\left\|\tilde{\boldsymbol{A}}_{11}\tilde{\boldsymbol{x}}_1(t) + \boldsymbol{A}_{12}\tilde{\boldsymbol{x}}_2(t)\right\| - M + \left\|\boldsymbol{E}_1\right\|\delta\right)
\end{aligned} \tag{2-24}
$$

Clearly, if M is chosen as

$$
M = max\left(\left\|\tilde{\boldsymbol{A}}_{11}\tilde{\boldsymbol{x}}_1(t) + \boldsymbol{A}_{12}\tilde{\boldsymbol{x}}_2(t)\right\| + \left\|\boldsymbol{E}_1\right\|\delta\right) + \sigma \tag{2-25}
$$

where $\sigma > 0$, then (2-24) becomes

$$
\tilde{\boldsymbol{x}}_1^T(t)\dot{\tilde{\boldsymbol{x}}}_1(t) < -\sigma\left\|\tilde{\boldsymbol{x}}_1(t)\right\| \tag{2-26}
$$

which guarantees $\tilde{\boldsymbol{x}}_1(t)$ reaches zero in a finite time as $t \to \infty$, i.e., $\hat{\boldsymbol{x}}_1(t) \to \boldsymbol{x}_1(t)$ .as $t \to \infty$. Hence (2-21) can be rewritten as

$$
\dot{\boldsymbol{e}}_2(t) = \tilde{\boldsymbol{A}}_{22}\boldsymbol{e}_2(t) - \tilde{\boldsymbol{E}}_2\boldsymbol{d}(t) \tag{2-27}
$$

where the truth of $\boldsymbol{e}_2(t) = \tilde{\boldsymbol{x}}_2(t)$ has been adopted from (2-20). There are two important conditions listed as below:

i) The eigenvalues of $\tilde{\boldsymbol{A}}_{22} = \boldsymbol{A}_{22} + \boldsymbol{L}\boldsymbol{A}_{12}$ are stable. (2-28)

ii) $\tilde{\boldsymbol{E}}_2 = \boldsymbol{E}_2 + \boldsymbol{L}\boldsymbol{E}_1 = 0$ (2-29)

With these two conditions, the estimation error $\boldsymbol{e}_2(t)$ will approach zero as $t \to \infty$, i.e., $\hat{\boldsymbol{x}}_2(t) \to \boldsymbol{x}_2(t)$ as $t \to \infty$. This confirms the success of the sliding mode observer (2-17). However, the conditions shown in (2-28) and (2-29) are the same as (2-9) and (2-10), required for the reduced order observer. Since the sliding mode observer is much more complicated, the reduced order observer will be used in the sliding mode controller design in the next chapter.

Chapter 3

Reduced-Order Observer-Based Sliding Mode Controller Design

In this Chapter, the sliding mode controller is designed based on the reduced-order observer for three cases, including regulation control for matched disturbance in Section 3.1, regulation control for mismatching disturbance in Section 3.2, and tracking control for mismatching disturbance in Section 3.3.

3.1 Reduced-order observer-based sliding mode controller design for matched disturbance

In this section, consider reduced-order observer-based sliding mode controller design for matched disturbance system, expressed as

$$\dot{\boldsymbol{x}}(t)=\boldsymbol{A}\boldsymbol{x}(t)+\boldsymbol{B}\big(\boldsymbol{u}(t)+\boldsymbol{d}(t)\big) \tag{3-1}$$

where $\boldsymbol{x}(t)\in\Re^n$ is the system state, $\boldsymbol{u}(t)\in\Re^m$ is the control input, $\boldsymbol{d}(t)\in\Re^q$ is the disturbance, $(\boldsymbol{A},\boldsymbol{B})$ is controllable, and that can transform into (2-2) with $\boldsymbol{E}$ replaced by $\boldsymbol{B}$. The reduced-order observer in (2-5) will be used here to estimate the unmeasurable state variables. As usual, the first step in the sliding mode controller design is choosing an appropriate sliding function, such that the system trajectory is steered to the control goal in the sliding mode. Let the sliding function be

$$\boldsymbol{s}(t)=\boldsymbol{C}\hat{\boldsymbol{x}}(t)=\boldsymbol{C}\begin{bmatrix}\boldsymbol{x}_1(t)\\ \hat{\boldsymbol{x}}_2(t)\end{bmatrix} \tag{3-2}$$

where $\det(\boldsymbol{CB})\neq 0$. The matrix $\boldsymbol{C}$ can be determined by the transformation matrix method [13], to guarantee the system stability in the sliding mode $\boldsymbol{s}(t)=0$. For the second step, the control algorithm is designed to drive the system into the sliding mode based on the approaching condition [13]. To derive the control algorithm, differentiating (3-2) yields

$$\dot{\boldsymbol{s}}(t)=\boldsymbol{C}\dot{\hat{\boldsymbol{x}}}(t)=\boldsymbol{CA}\hat{\boldsymbol{x}}(t)+\boldsymbol{CB}\left(\boldsymbol{u}(t)+\boldsymbol{d}(t)\right) \tag{3-3}$$

Then, the equivalent control input $\boldsymbol{u}_{eq}(t)$ in the sliding mode can be found from

$$\dot{\boldsymbol{s}}(t)\Big|_{u=u_{eq}}=\boldsymbol{CA}\hat{\boldsymbol{x}}(t)+\boldsymbol{CB}\left(\boldsymbol{u}_{eq}(t)+\boldsymbol{d}(t)\right)=0 \tag{3-4}$$

which leads to

$$\boldsymbol{u}_{eq}(t)=-(\boldsymbol{CB})^{-1}\boldsymbol{CA}\hat{\boldsymbol{x}}(t)-(\boldsymbol{CB})^{-1}\boldsymbol{CBd}(t)=\boldsymbol{u}_{eq}^{\circ}(t)+\tilde{\boldsymbol{u}}_{eq}(t) \tag{3-5}$$

Note that the equivalent control input is partitioned into two parts shown as

$$\boldsymbol{u}_{eq}^{\circ}(t)=-(\boldsymbol{CB})^{-1}\boldsymbol{CA}\hat{\boldsymbol{x}}(t) \tag{3-6}$$

$$\tilde{\boldsymbol{u}}_{eq}(t)=-(\boldsymbol{CB})^{-1}\boldsymbol{CBd}(t) \tag{3-7}$$

where $\boldsymbol{u}_{eq}^{\circ}(t)$ is the dominant part and $\tilde{\boldsymbol{u}}_{eq}(t)$ is related to the disturbance $\boldsymbol{d}(t)$. Clearly, because of the existence of $\boldsymbol{d}(t)$ in $\tilde{\boldsymbol{u}}_{eq}(t)$, the control input $\boldsymbol{u}$ can not directly adopt the equivalent control $\boldsymbol{u}_{eq}(t)$, instead it is set as

$$\boldsymbol{u}(t)=\boldsymbol{u}_{eq}^{\circ}(t)-\left(\left\|\tilde{\boldsymbol{u}}_{eq}(t)\right\|_{\max}+\sigma\right)sgn(\boldsymbol{s}) \tag{3-8}$$

where the upper bound of $\tilde{\boldsymbol{u}}_{eq}(t)$ is selected as $\left\|\tilde{\boldsymbol{u}}_{eq}(t)\right\|_{\max}=(\boldsymbol{CB})^{-1}\gamma$ with $\gamma\geq\left\|\boldsymbol{CBd}(t)\right\|$ and σ is a positive constant. By substituting (3-8) into (3-3), the derivative of $\boldsymbol{s}(t)$ becomes

$$\dot{\boldsymbol{s}}(t)=-(\gamma+\sigma)sgn(\boldsymbol{s})+\boldsymbol{CBd}(t) \tag{3-9}$$

Further premultiplying $\boldsymbol{s}^T$ results in

$$\boldsymbol{s}^T(t)\dot{\boldsymbol{s}}(t)=-\gamma\|\boldsymbol{s}\|-\sigma\|\boldsymbol{s}\|+\boldsymbol{s}(t)\boldsymbol{CBd}(t)=-\sigma\|\boldsymbol{s}\|-\gamma\|\boldsymbol{s}\|\left(1-\frac{\boldsymbol{s}(t)\boldsymbol{CBd}(t)}{\gamma\|\boldsymbol{s}\|}\right) \tag{3-10}$$

Since $\gamma\geq\left\|\boldsymbol{CBd}(t)\right\|$ and $\sigma>0$, (3-10) becomes

$$\boldsymbol{s}^T(t)\dot{\boldsymbol{s}}(t)<-\sigma\|\boldsymbol{s}\| \tag{3-11}$$

which evidently guarantee the reach and sliding condition. As a consequence, $\boldsymbol{s}(t)\to 0$ in a finite time. It is well known that the chatting exists due to the use of $sgn(s)$ in the control algorithm (3-8). Hence, to ameliorate such undesired chatting, the switching function $sgn(s)$ is often replaced by the saturation function, expressed as

$$sat(\boldsymbol{s},\varepsilon)=\begin{cases}\dfrac{\boldsymbol{s}}{\|\boldsymbol{s}\|} & \|\boldsymbol{s}\|>\varepsilon \\ \dfrac{\boldsymbol{s}}{\varepsilon} & \|\boldsymbol{s}\|\le\varepsilon\end{cases} \tag{3-12}$$

where $\varepsilon>0$ is the thickness of the sliding layer $\|s\|\le\varepsilon$. Therefore, the control algorithm (3-8) is changed into

$$u(t)=-(\boldsymbol{CB})^{-1}\boldsymbol{CA}\hat{\boldsymbol{x}}(t)\text{-}(\boldsymbol{CB})^{-1}(\gamma+\sigma)\mathrm{sat}(s,\varepsilon) \tag{3-13}$$

which will drive the system trajectory into the sliding layer, not in the sliding mode, in a finite time and force it to stay within there. Once the system trajectory is bounded in the sliding layer, it will move toward the control goal and then around there. In other words, the control goal can not be precisely attained and the matched disturbance is not completely eliminated since the system trajectory is not restricted in the sliding mode. The errors are caused by the use of saturation function and depend on the scale of the matched disturbance and the eigenvalues chosen for the sliding mode. The effect of the matched disturbance and the eigenvalues will be demonstrated later by the simulation results in Section 4.1.

3-2 Reduced-order observer-based sliding mode controller design for mismatching disturbance

In this section, the reduced order observer is used to design the mismatching disturbance of LTI system, described as

$$\dot{\boldsymbol{x}}(t)=\boldsymbol{Ax}(t)+\boldsymbol{Bu}(t)+\boldsymbol{Ed}(t) \tag{3-14}$$

where $\boldsymbol{x}(t)\in\Re^n$ is the system state, $\boldsymbol{u}(t)\in\Re^m$ is the control input, $\boldsymbol{d}(t)\in\Re^q$ is the mismatching disturbance since $\boldsymbol{E}\neq\boldsymbol{BQ}$, $(\boldsymbol{A},\boldsymbol{B})$ is controllable, and that can transform into (2-2). The reduced-order observer (2-5) will be used here to estimate the unmeasurable state variables. The first step in the sliding mode controller design is to choose an appropriate sliding function, such that the system trajectory will be moved to the control goal in the sliding mode. Let the sliding function be

$$\boldsymbol{s}(t)=\boldsymbol{C}\hat{\boldsymbol{x}}(t)=\boldsymbol{C}\begin{bmatrix}\boldsymbol{x}_1(t)\\ \hat{\boldsymbol{x}}_2(t)\end{bmatrix} \tag{3-15}$$

where $\det(\boldsymbol{CB}) \neq 0$. The matrix $\boldsymbol{C}$ can be determined by the transformation matrix method [13], such that the system is stabilized in the sliding mode $s(t)=0$. For the second step, the control algorithm is designed to drive the system into the sliding mode based on the approaching condition. In the design process, the derivative of (3-15) is obtained as

$$\dot{\boldsymbol{s}}(t) = \boldsymbol{C}\dot{\hat{\boldsymbol{x}}}(t) = \boldsymbol{CA}\hat{\boldsymbol{x}}(t) + \boldsymbol{CBu}(t) + \boldsymbol{CEd}(t) \tag{3-16}$$

Then, the equivalent control input $\boldsymbol{u}_{eq}(t)$ in the sliding mode can be found by

$$\dot{\boldsymbol{s}}(t)\big|_{u=u_{eq}} = \boldsymbol{CA}\hat{\boldsymbol{x}}(t) + \boldsymbol{CBu}_{eq}(t) + \boldsymbol{CEd}(t) = 0 \tag{3-17}$$

which leads to

$$\boldsymbol{u}_{eq}(t) = -(\boldsymbol{CB})^{-1}\boldsymbol{CA}\hat{\boldsymbol{x}}(t) - (\boldsymbol{CB})^{-1}\boldsymbol{CEd}(t) = \boldsymbol{u}_{eq}^{\circ}(t) + \tilde{\boldsymbol{u}}_{eq}(t) \tag{3-18}$$

Note that the equivalent control input is partitioned into two parts, shown as

$$\boldsymbol{u}_{eq}^{\circ}(t) = -(\boldsymbol{CB})^{-1}\boldsymbol{CA}\hat{\boldsymbol{x}}(t) \tag{3-19}$$

$$\tilde{\boldsymbol{u}}_{eq}(t) = -(\boldsymbol{CB})^{-1}\boldsymbol{CEd}(t) \tag{3-20}$$

where $\boldsymbol{u}_{eq}^{\circ}(t)$ is the dominant part and $\tilde{\boldsymbol{u}}_{eq}(t)$ is related to the disturbance $\boldsymbol{d}(t)$. Clearly, because of the existence of $\boldsymbol{d}(t)$ in $\tilde{\boldsymbol{u}}_{eq}(t)$, the control input $\boldsymbol{u}$ can not directly adopt the equivalent control $\boldsymbol{u}_{eq}(t)$, instead it is set as

$$\boldsymbol{u}(t) = \boldsymbol{u}_{eq}^{\circ}(t) - \left(\left\|\tilde{\boldsymbol{u}}_{eq}(t)\right\|_{\max} + \sigma\right) sgn(\boldsymbol{s}) \tag{3-21}$$

where the upper bound of $\tilde{\boldsymbol{u}}_{eq}(t)$ is $\left\|\tilde{\boldsymbol{u}}_{eq}(t)\right\|_{\max} = (\boldsymbol{CB})^{-1}\gamma$ with $\gamma \geq \left\|\boldsymbol{CEd}(t)\right\|$ and σ is a positive constant. Further substituting (3-21) into (3-16) yields

$$\dot{\boldsymbol{s}}(t) = -(\gamma + \sigma) sgn(\boldsymbol{s}) + \boldsymbol{CEd}(t) \tag{3-22}$$

and premultiplying $\boldsymbol{s}^T$ results in

$$\begin{aligned} \boldsymbol{s}^T(t)\dot{\boldsymbol{s}}(t) &= -\gamma\|\boldsymbol{s}\| - \sigma\|\boldsymbol{s}\| + \boldsymbol{s}(t)\boldsymbol{CEd}(t) \\ &= -\sigma\|\boldsymbol{s}\| - \gamma\|\boldsymbol{s}\|\left(1 - \frac{\boldsymbol{s}(t)\boldsymbol{CEd}(t)}{\gamma\|\boldsymbol{s}\|}\right) \end{aligned} \tag{3-23}$$

Since $\gamma \geq \left\|\boldsymbol{CEd}(t)\right\|$ and $\sigma > 0$, (3-23) becomes

$$\boldsymbol{s}^T(t)\dot{\boldsymbol{s}}(t) < -\sigma\|\boldsymbol{s}\| \tag{3-24}$$

which evidently guarantees the reaching and sliding condition. As a consequence, $\boldsymbol{s}(t) = 0$ in a finite time and the chattering exists due to the use of $sgn(s)$ in (3-21). Hence, to ameliorate such undesired chattering, the switching function $sgn(s)$ is often replaced by the saturation function, expressed as

$$sat(\boldsymbol{s},\varepsilon)=\begin{cases}\dfrac{\boldsymbol{s}}{\|\boldsymbol{s}\|} & \|\boldsymbol{s}\|>\varepsilon \\ \dfrac{\boldsymbol{s}}{\varepsilon} & \|\boldsymbol{s}\|\le\varepsilon\end{cases} \tag{3-25}$$

where $\varepsilon>0$ is the thickness of the sliding layer $\|s\|\le\varepsilon$. Therefore, the control algorithm (3-21) is changed into

$$u(t)=-(\boldsymbol{CB})^{-1}\boldsymbol{CA}\hat{\boldsymbol{x}}(t)-(\boldsymbol{CB})^{-1}(\gamma+\sigma)sat(s,\varepsilon) \tag{3-26}$$

and the system trajectory will be steered into the sliding layer, not in the sliding mode, during a finite time and then stays therein. Once the system trajectory is bounded in the sliding layer, it will move toward the control goal $\boldsymbol{x}\to 0$ and finally go around there. As a result, the control goal can not be precisely attained and the mismatching disturbance can not be eliminated in the sliding layer. The errors are caused by the use of saturation function, the scale of the mismatching disturbance and the eigenvalues chosen for the sliding mode. The effect of the mismatching disturbance and the eigenvalues will be demonstrated later by the simulation results in Section 4.2.

3-3 Reduced-order observer-based sliding mode controller design output tracking control for disturbance

In this section, the reduced-order observer is used to design tracking control for disturbance with LTI system as

$$\dot{\boldsymbol{x}}(t)=\boldsymbol{Ax}(t)+\boldsymbol{Bu}(t)+\boldsymbol{Ed}(t) \tag{3-27}$$

where $\boldsymbol{x}(t)\in\Re^n$ is the system state, $\boldsymbol{u}(t)\in\Re^m$ is the control input, $\boldsymbol{d}(t)\in\Re^q$ is the disturbance, and $(\boldsymbol{A},\boldsymbol{B})$ is controllable. Without loss of generality, the system state $x(t)$ is preprocessed to contain measurable part $\boldsymbol{x}_1(t)$ and unmeasurable part $\boldsymbol{x}_2(t)$, i.e., $\boldsymbol{x}(t)=[\boldsymbol{x}_1(t) \quad \boldsymbol{x}_2(t)]^T$. Here, the reduced-order observer (2-5) will be used here to estimate the unmeasurable state variables. The control goal is to fulfill tracking control for the output

$$y(t) = Gx(t) \tag{3-28}$$

where G is full rank. That means the controller is designed to drive the output to follow the desired trajectory $y_d(t)$, i.e., the tracking error

$$e(t) = y(t) - y_d(t) \tag{3-29}$$

will vanish as $t \to \infty$.

For the sliding mode tracking controller design, first define a new state concerning the tracking error, expressed as

$$h(t) = \int_0^t e(\tau)\, d\tau \tag{3-30}$$

then

$$\dot{h}(t) = e(t) = y(t) - y_d(t) \tag{3-31}$$

Clearly, by combining (3-27) and (3-31), the system can be reconstructed as

$$\dot{p}(t) = A_p p(t) + B_p u(t) + E_p d(t) + f(t) \tag{3-32}$$

where $p(t) = \begin{bmatrix} x(t) \\ h(t) \end{bmatrix}$, $A_p = \begin{bmatrix} A & 0 \\ G & 0 \end{bmatrix}$, $B_p = \begin{bmatrix} B \\ 0 \end{bmatrix}$, $E_p = \begin{bmatrix} E \\ 0 \end{bmatrix}$, $f(t) = \begin{bmatrix} 0 \\ -y_d(t) \end{bmatrix}$. Note that $\begin{pmatrix} A_p & B_p \end{pmatrix}$ is controllable since the pair $\begin{pmatrix} A & B \end{pmatrix}$ is controllable. The first step in the sliding mode controller design is to choose an appropriate sliding function, such that the system trajectory can trace the control goal in the sliding mode. Let the sliding function be

$$s(t) = C\hat{p}(t) = C\begin{bmatrix} \hat{x}(t) \\ h(t) \end{bmatrix} \tag{3-33}$$

where $\det(CB_p) \neq 0$ and C can be determined by the transformation matrix method [13], such that the system is stabilized in the sliding mode $s(t)=0$. For the second step, the control algorithm is designed to drive the system into the sliding mode based on the approaching condition.

Once C is determined in the first step, then the control algorithm is derived by differentiating (3-33) as

$$\dot{s}(t) = C\dot{\hat{p}}(t) = CA_p\hat{p}(t) + CB_p u(t) + CE_p d(t) + Cf(t) \tag{3-34}$$

The equivalent control $u_{eq}(t)$ can be found from

$$\dot{s}(t)\big|_{u=u_{eq}} = CA_p\hat{p}(t) + CB_p u_{eq}(t) + CE_p d(t) + Cf(t) = 0 \tag{3-35}$$

which leads to

$$\boldsymbol{u}_{eq}(t)=\boldsymbol{u}_{eq}^{0}(t)+\tilde{\boldsymbol{u}}_{eq}(t) \tag{3-36}$$

with nominal part

$$\boldsymbol{u}_{eq}^{o}(t)=-\left(\boldsymbol{C}\boldsymbol{B}_{p}\right)^{-1}\boldsymbol{C}\boldsymbol{A}_{p}\hat{\boldsymbol{p}}(t)-\left(\boldsymbol{C}\boldsymbol{B}_{p}\right)^{-1}\boldsymbol{C}\boldsymbol{f}(t) \tag{3-37}$$

and disturbance

$$\tilde{\boldsymbol{u}}_{eq}(t)=-\left(\boldsymbol{C}\boldsymbol{B}_{p}\right)^{-1}\boldsymbol{C}\boldsymbol{E}_{p}\boldsymbol{d}(t) \tag{3-38}$$

Because of the existence of $\boldsymbol{d}(t)$ in $\tilde{\boldsymbol{u}}_{eq}(t)$, the control input $\boldsymbol{u}$ is generally designed as

$$\boldsymbol{u}(t)=\boldsymbol{u}_{eq}^{o}(t)-\left(\left\|\tilde{\boldsymbol{u}}_{eq}(t)\right\|_{\max}+\sigma\right)sat(s,\varepsilon) \tag{3-39}$$

where σ is a positive constant and $\left\|\tilde{\boldsymbol{u}}_{eq}(t)\right\|_{\max}=\left(\boldsymbol{C}\boldsymbol{B}_{p}\right)^{-1}\gamma$ with $\gamma\geq\left\|\boldsymbol{C}\boldsymbol{E}_{p}\boldsymbol{d}(t)\right\|$ is the upper bound of $\tilde{\boldsymbol{u}}_{eq}(t)$. The saturation function is given as

$$sat(\boldsymbol{s},\varepsilon)=\begin{cases}\dfrac{\boldsymbol{s}}{\|\boldsymbol{s}\|} & \|\boldsymbol{s}\|>\varepsilon\\[2mm] \dfrac{\boldsymbol{s}}{\varepsilon} & \|\boldsymbol{s}\|\leq\varepsilon\end{cases} \tag{3-40}$$

where $\varepsilon>0$ is the thickness of the sliding layer $\|s\|\leq\varepsilon$. By substituting (3-39) into (3-34), the derivative of $\boldsymbol{s}(t)$ becomes

$$\dot{\boldsymbol{s}}(t)=-(\gamma+\sigma)\cdot sat(s,\varepsilon)+\boldsymbol{C}\boldsymbol{E}_{p}\boldsymbol{d}(t) \tag{3-41}$$

Further premultiplying $\boldsymbol{s}^{T}$ results in

$$\boldsymbol{s}^{T}(t)\dot{\boldsymbol{s}}(t)=-\gamma\|\boldsymbol{s}\|-\sigma\|\boldsymbol{s}\|+\boldsymbol{s}(t)\boldsymbol{C}\boldsymbol{E}_{p}\boldsymbol{d}(t)=-\sigma\|\boldsymbol{s}\|-\gamma\|\boldsymbol{s}\|\left(1-\frac{\boldsymbol{s}(t)\boldsymbol{C}\boldsymbol{E}_{p}\boldsymbol{d}(t)}{\gamma\|\boldsymbol{s}\|}\right) \tag{3-42}$$

Since $\gamma\geq\left\|\boldsymbol{C}\boldsymbol{E}_{p}\boldsymbol{d}(t)\right\|$ and $\sigma>0$, (3-42) becomes

$$\boldsymbol{s}^{T}(t)\dot{\boldsymbol{s}}(t)<-\sigma\|\boldsymbol{s}\| \tag{3-43}$$

which evidently guarantees the reaching and sliding condition of the sliding mode $\boldsymbol{s}(t)=0$ in a finite time. By substituting (3-36) into (3-32), obtained as

$$\begin{aligned}\dot{\boldsymbol{p}}(t)&=\boldsymbol{A}_{p}\boldsymbol{p}(t)+\boldsymbol{B}_{p}\boldsymbol{u}_{eq}(t)+\boldsymbol{E}_{p}\boldsymbol{d}(t)+\boldsymbol{f}(t)\\&=\left(\boldsymbol{I}-\boldsymbol{B}_{p}\left(\boldsymbol{C}\boldsymbol{B}_{p}\right)^{-1}\boldsymbol{C}\right)\boldsymbol{A}_{p}\boldsymbol{p}(t)+\left(\boldsymbol{I}-\boldsymbol{B}_{p}\left(\boldsymbol{C}\boldsymbol{B}_{p}\right)^{-1}\boldsymbol{C}\right)\left(\boldsymbol{E}_{p}\boldsymbol{d}(t)+\boldsymbol{f}(t)\right)\end{aligned} \tag{3-44}$$

Above equation, consider relationship with eigenvalues of ***C***, disturbance and $\boldsymbol{y}_{d}(t)$. If

$E_p = B_pQ$ is matched disturbance and $y_d(t)$ is constant, then restructures (3-44) as

$$\dot{p}(t) = \left(I - B_p(CB_p)^{-1}C\right)\left(A_p p(t) + f(t)\right) \qquad \text{(3-45)}$$

The design eigenvalues of C can be chosen such that $\left(I - B_p(CB_p)^{-1}C\right)\left(A_p p(t) + f(t)\right)$ is Hurwitz, and then $\dot{p}(t) = \left[\dot{x}(t) \quad \dot{h}(t)\right]^T = 0$ as $t \to \infty$, hence $\dot{h}(\infty) = e(\infty) = y(\infty) - y_d(\infty) = 0$, and then attains the control goal $y(t)$ to $y_d(t)$, that will simulate in Section 4.3 case 1.

If $E_p \neq B_pQ$ is mismatching disturbance and $y_d(t)$ is constant, then restructure (3-44) as

$$\dot{p}(t) = \left(I - B_p(CB_p)^{-1}C\right)\left(A_p p(t) + f(t)\right) + \left(I - B_p(CB_p)^{-1}C\right)E_p d(t) \qquad \text{(3-46)}$$

The mismatching disturbance is not remove, and then will make control error, such that output tracking control not exact tracks $y(t)$ to $y_d(t)$. In order to reduce tracking error that can chooses the design eigenvualues of C distant from origin in left phase plane, but that can make control input have high gain, that will be simulate in Section 4.3 case 2.

If $E_p \neq B_pQ$ is mismatching disturbance and $y_d(t)$ is not constant i.e., $y_d = cos(t)$, then must be affected with mismatching disturbance and y_d, and then the output tracking control not complete tracks $y(t)$ to $y_d(t)$. In order to reduce tracking error that can chooses the design eigenvualues of C distant from origin in left phase plane, but that can make control input have high gain, that will be introduce in Section 4.3 case 3.

Chapter 4

Simulation and Result

In this chapter, consider six case of reduced-order observer-based sliding mode controller design. There are the matched disturbance system and the dimension $p = q$, the matched disturbance system and the dimension $p > q$, the mismatching disturbance system for controller stability, the output tracking with matched disturbance system and tracking trajectory is constant, the output tracking with mismatching disturbance and tracking trajectory is constant, and the output tracking with mismatching disturbance system and tracking trajectory is not constant.

4.1 Observer-based sliding mode controller design for matched disturbance

In this section, using reduced-order observer-based sliding mode controller design to simulates two case, there are matched disturbance system that dimension of measurable equal dimension of disturbance, and matched disturbance system that dimension of measurable greater than dimension of disturbance.

In Case 1, consider a LTI system (3-1) suffering from the matched disturbance, the system state be decomposed into two parts $\boldsymbol{x}(t) = \left[\boldsymbol{x}_1^T(t) \quad \boldsymbol{x}_2^T(t)\right]^T$, where $\boldsymbol{x}_1(t)$ is measurable and $\boldsymbol{x}_2(t)$ is not obtainable. Then the system reconstruct as:

$$\begin{bmatrix} \dot{\boldsymbol{x}}_1(t) \\ \dot{\boldsymbol{x}}_2(t) \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} \boldsymbol{x}_1(t) \\ \boldsymbol{x}_2(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \left(\boldsymbol{u}(t) + \boldsymbol{d}(t)\right) \tag{4-1}$$

The matched disturbance is assumed as

$$d(t) = 0.5\left(\sin(2\pi t) + \cos\left(\frac{\pi t}{4}\right)\right) + 0.1\sin(t)\,x_1 \tag{4-2}$$

Apparently, the upper bound of the matched disturbance is obtained as

$$\|d(t)\| \le \|\boldsymbol{x}(t)\| + 1 \tag{4-3}$$

The system dimension of measurable part and the dimension of disturbance represent as

$$rank\begin{pmatrix}\boldsymbol{A}_{11} & \boldsymbol{A}_{12}\end{pmatrix} = rank\left(\begin{bmatrix}-1 & 1\end{bmatrix}\right) = p = rank(\boldsymbol{B}) = rank\left(\begin{bmatrix}1\\1\end{bmatrix}\right) = q = 1 \tag{4-4}$$

The first step is to design the reduced order observer, and the observer (2-3) represent as

$$\dot{\boldsymbol{z}}(t) = \boldsymbol{F}\boldsymbol{z}(t) + \boldsymbol{J}\boldsymbol{x}_1(t) + \boldsymbol{M}\boldsymbol{u}(t) \tag{4-5}$$

where

$$\begin{aligned}\boldsymbol{F} &= \boldsymbol{A}_{22} - \boldsymbol{L}\boldsymbol{A}_{12} = -3 - \boldsymbol{L}\times 1\\ \boldsymbol{J} &= \boldsymbol{F}\boldsymbol{L} + \boldsymbol{A}_{21} - \boldsymbol{L}\boldsymbol{A}_{11} = \boldsymbol{F}\boldsymbol{L} + 2 - \boldsymbol{L}\times(-1)\\ \boldsymbol{M} &= \boldsymbol{B}_2 - \boldsymbol{L}\boldsymbol{B}_1 = 1 - \boldsymbol{L}\times 1\end{aligned} \tag{4-6}$$

And the reduced order observer should be designed under two conditions of (2-9) and (2-10). Because the dimension $p = q$, hence, the matrix $\boldsymbol{L}$ can be chosen as (2-13) by

$$\boldsymbol{L} = \boldsymbol{B}_2\boldsymbol{B}_1^{-1} = -1 \tag{4-7}$$

where $\boldsymbol{L}$ is not chosen by $\boldsymbol{\Gamma}$. By substituting (4-7) into (4-6) and the observer (4-5) rewrite as

$$\dot{\boldsymbol{z}}(t) = -2\boldsymbol{z}(t) + 3\boldsymbol{x}_1(t) \tag{4-8}$$

In this case, the eigenvalue of $\boldsymbol{F}$ is stable, then it is completed estimate $\boldsymbol{x}_2$ as given in (4-8) successfully. The second step is using this information to design the sliding mode controller, Let the sliding function be

$$\boldsymbol{s}(t) = \boldsymbol{C}\hat{\boldsymbol{x}}(t) \tag{4-9}$$

Because the eigenvalues of $\boldsymbol{A}$ are -0.2679 and -3.7321, then the matrix $\boldsymbol{C}$ can be determined by the transformation matrix method and via the pole-assignement method to appoint the eigenvalue is -4, and $\boldsymbol{C}\boldsymbol{B} = \boldsymbol{I}$, hence, the design $\boldsymbol{C}$ as

$$\boldsymbol{C} = \begin{bmatrix}1 & 0\end{bmatrix} \tag{4-10}$$

The design input $\boldsymbol{u}(t)$ as (3-12) by

$$\boldsymbol{u}(t) = -\boldsymbol{C}\boldsymbol{A}\hat{\boldsymbol{x}}(t) - (\gamma + \sigma)sgn(\boldsymbol{s}) = -\begin{bmatrix}-1 & 1\end{bmatrix}\hat{\boldsymbol{x}}(t) - \left(\|\hat{\boldsymbol{x}}(t)\| + 1\right)sat(\boldsymbol{s}, \varepsilon) \tag{4-11}$$

where $\sigma = 0$ and sliding layer $-0.01 \le \varepsilon \le 0.01$. The simulation results as:

Figure 4.1 to Figure 4.8 are simulation results with initial condition $\boldsymbol{x}(0) = \begin{bmatrix}10 & 7\end{bmatrix}^T$ and $\boldsymbol{z}(0) = 0$. Figure 4.1 shows the MATLAB simulink connection diagram, that contains system, reduced-order observer, sliding mode controller, and disturbance. Figure 4.2 shows the observer state error, that at 4.87s convergence to approach of zero, that after into sliding layer, and the convergence speed from (17, 0s) to (2.3, 1s), conform the $eig(\boldsymbol{F}) = -2$. Figure 4.3 shows the sliding surface, that at 3.115s into the sliding layer, and then it effect the control input $\boldsymbol{u}$ that has a disjunctive part that from -0.9769 to 0.08283 at this time shows in Figure 4.5, it possible two reason that the observer does not completely estimated, and sliding surface must bound in sliding layer, hence, must has higher gain instant to conform two possible reasons. Figure 4.4 shows the sliding surface bound into $\|s\| \le \varepsilon$ after complete estimate, Figure 4.6 shows the disturbance, that the range between -1 and 1.35. Figure 4.7 and 4.8 illustrate the system state variable all converge to $\boldsymbol{x} = 0$.

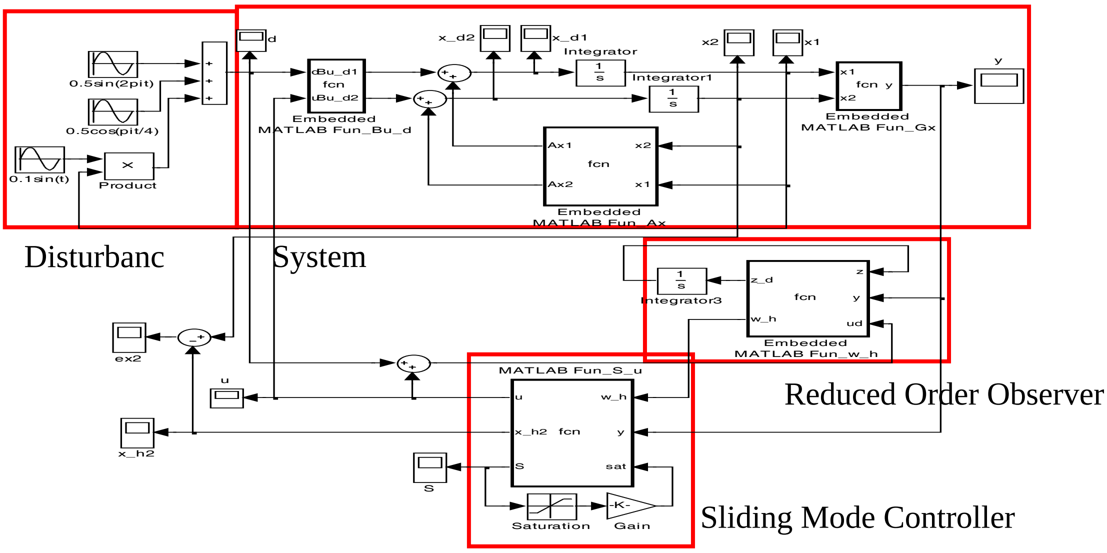

Figure 4.1 The MATLAB slimulink connection diagram

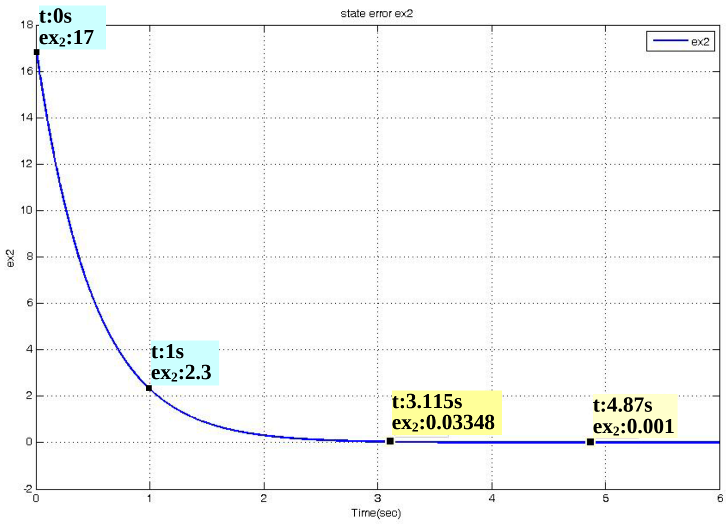

Figure 4.2 The observer state error

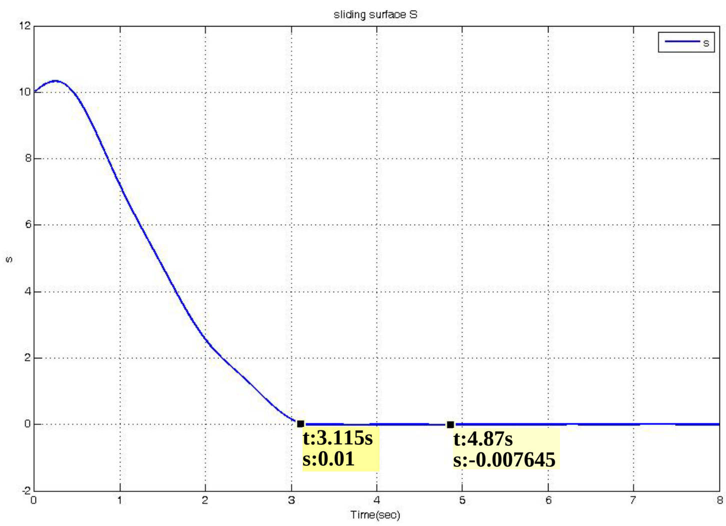

Figure 4.3 The sliding surface $s(t)$

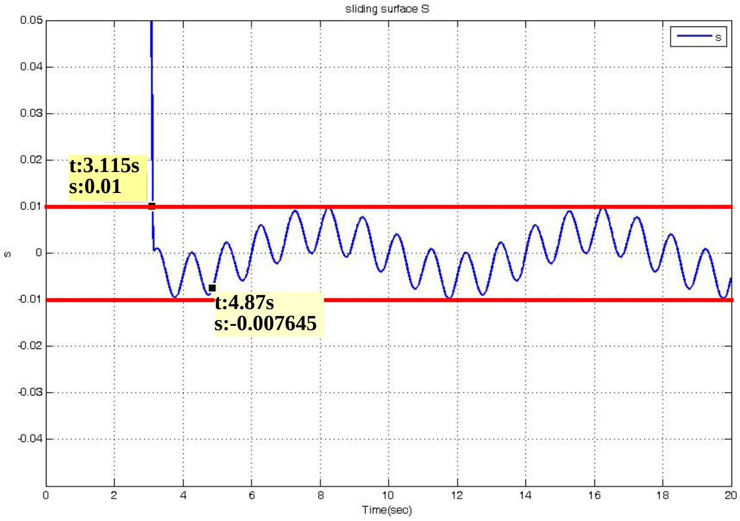

Figure 4.4 The sliding surface $\boldsymbol{s}(t)$ bound in $\|s\| \le \varepsilon$

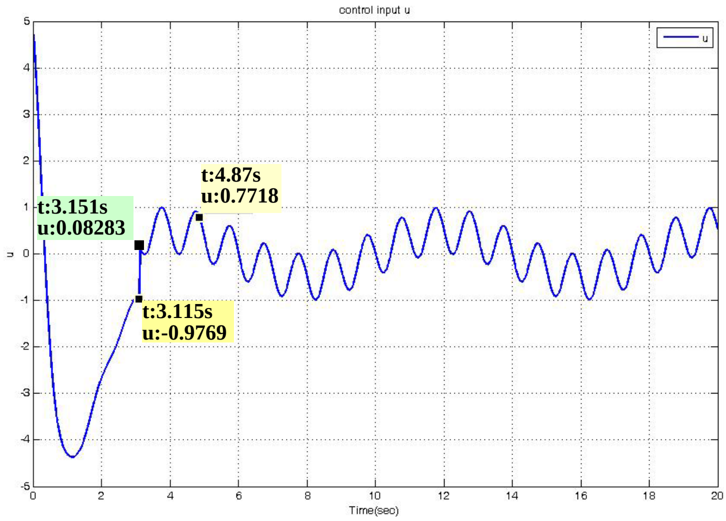

Figure 4.5 The control input $\boldsymbol{u}(t)$

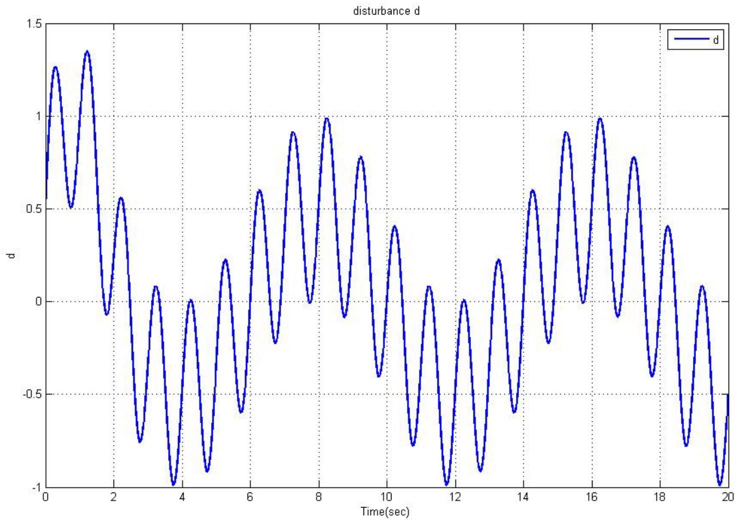

Figure 4.6 The disturbance $\boldsymbol{d}(t)$

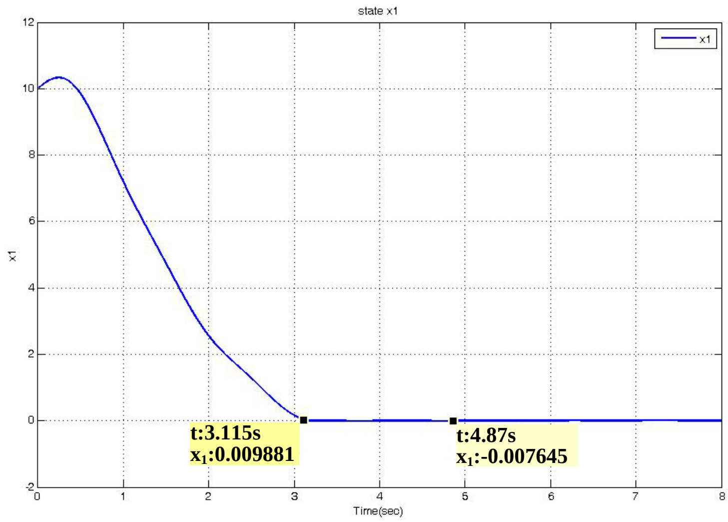

Figure 4.7 The system state $\boldsymbol{x}_1(t)$

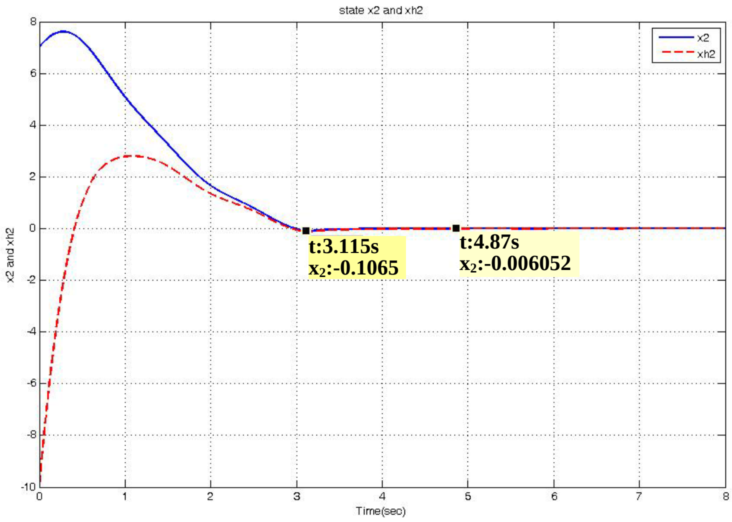

Figure 4.8 The system state $\boldsymbol{x}_2(t)$ and observer state $\hat{\boldsymbol{x}}_2(t)$

In case 2, consider a LTI system (3-1) suffering from the matched disturbance, the system state be decomposed into three parts $\boldsymbol{x}(t)=\begin{bmatrix}\boldsymbol{x}_1^T(t) & \boldsymbol{x}_2^T(t) & \boldsymbol{x}_3^T(t)\end{bmatrix}^T$, where $\boldsymbol{x}_1(t)$ and $\boldsymbol{x}_2(t)$ are measurable and $\boldsymbol{x}_3(t)$ is not obtainable. Then the system reconstruct as:

$$\begin{bmatrix}\dot{\boldsymbol{x}}_1(t)\\ \dot{\boldsymbol{x}}_2(t)\\ \dot{\boldsymbol{x}}_3(t)\end{bmatrix}=\begin{bmatrix}-0.277 & 1 & -0.002\\ -1.71 & -0.178 & -12.2\\ 0 & 0 & -6.67\end{bmatrix}\begin{bmatrix}\boldsymbol{x}_1(t)\\ \boldsymbol{x}_2(t)\\ \boldsymbol{x}_3(t)\end{bmatrix}+\begin{bmatrix}1\\ 0\\ 1\end{bmatrix}\left(\boldsymbol{u}(t)+\boldsymbol{d}(t)\right) \tag{4-12}$$

The matched disturbance is assumed as

$$d(t)=0.5\left(\sin(2\pi t)+\cos\left(\frac{\pi t}{4}\right)\right)+0.1\sin(t)x_1 \tag{4-13}$$

Apparently, the upper bound of the matched disturbance is obtained as

$$\|d(t)\|\le\|\boldsymbol{x}(t)\|+1 \tag{4-14}$$

The system dimension of measurable part and the dimension of disturbance represent as

$$\begin{gathered}rank\begin{pmatrix}\boldsymbol{A}_{11} & \boldsymbol{A}_{12}\end{pmatrix}=rank\left(\begin{bmatrix}-0.277 & 1 & -0.002\\ -1.71 & -0.178 & -12.2\end{bmatrix}\right)=p=2\\ rank(\boldsymbol{B})=rank\left(\begin{bmatrix}1\\ 0\\ 1\end{bmatrix}\right)=q=1\end{gathered} \tag{4-15}$$

where the dimension $p>q$. The first step is to design the reduced order observer, and the observer (2-3) represent as

$$\dot{\boldsymbol{z}}(t)=\boldsymbol{Fz}(t)+\boldsymbol{Jx}_1(t)+\boldsymbol{Mu}(t) \tag{4-16}$$

where

$$\begin{aligned}\boldsymbol{F}&=\boldsymbol{A}_{22}-\boldsymbol{LA}_{12}=-6.67-\boldsymbol{L}\times\begin{bmatrix}-0.002\\ -12.2\end{bmatrix}\\ \boldsymbol{J}&=\boldsymbol{FL}+\boldsymbol{A}_{21}-\boldsymbol{LA}_{11}=\boldsymbol{FL}+\begin{bmatrix}0 & 0\end{bmatrix}-\boldsymbol{L}\times\begin{bmatrix}-0.277 & 1\\ -1.71 & -0.178\end{bmatrix}\\ \boldsymbol{M}&=\boldsymbol{B}_2-\boldsymbol{LB}_1=1-\boldsymbol{L}\times\begin{bmatrix}1\\ 0\end{bmatrix}\end{aligned} \tag{4-17}$$

And the reduced order observer should be designed under two conditions of (2-9) and (2-10). Because the dimension $p>q$, hence, the matrix $\boldsymbol{L}$ can be chosen as (2-12) by

$$\boldsymbol{L}=\boldsymbol{B}_2\boldsymbol{B}_1^{+}+\boldsymbol{\Gamma}\left(\boldsymbol{I}_p-\boldsymbol{B}_1\boldsymbol{B}_1^{+}\right)=\begin{bmatrix}1 & 0\end{bmatrix}+\boldsymbol{\Gamma}\begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix} \tag{4-18}$$

where $\boldsymbol{L}$ can be chosen by $\boldsymbol{\Gamma}$. The matrix $\boldsymbol{L}$ can affect the observer estimate speed, that determine for eigenvalues of $\boldsymbol{F}$. By substituting (4-18) into (4-16) and design the eigenvalues of $\boldsymbol{F}$ are -1, -5, and -10 respectively, then the $\boldsymbol{L}$ represent separately as

$$\begin{aligned} \boldsymbol{L} &= \begin{bmatrix} -1 & 0.4648 \end{bmatrix} \\ \boldsymbol{L} &= \begin{bmatrix} -1 & 0.1369 \end{bmatrix} \\ \boldsymbol{L} &= \begin{bmatrix} -1 & -0.273 \end{bmatrix} \end{aligned} \tag{4-19}$$

By substituting (4-19) into (4-17), then the observer (4-16) rewrite as

$$\begin{aligned} \dot{z}(t) &= -1z(t) - \begin{bmatrix} 1.5192 & 0.6173 \end{bmatrix} \begin{bmatrix} \boldsymbol{x}_1(t) \\ \boldsymbol{x}_2(t) \end{bmatrix} \\ \dot{z}(t) &= -5z(t) - \begin{bmatrix} 4.9589 & 0.3396 \end{bmatrix} \begin{bmatrix} \boldsymbol{x}_1(t) \\ \boldsymbol{x}_2(t) \end{bmatrix} \\ \dot{z}(t) &= -10z(t) - \begin{bmatrix} 9.2588 & 3.6821 \end{bmatrix} \begin{bmatrix} \boldsymbol{x}_1(t) \\ \boldsymbol{x}_2(t) \end{bmatrix} \end{aligned} \tag{4-20}$$

Therefore, it is completed which can successfully estimate $\boldsymbol{x}_3$ as given in (4-20). The second step is using this information to design the sliding mode controller, let the sliding function be

$$\boldsymbol{s}(t) = \boldsymbol{C}\hat{\boldsymbol{x}}(t) \tag{4-21}$$

Because the eigenvalues of $\boldsymbol{A}$ are -0.2275±1.3067i and -6.67, then the matrix $\boldsymbol{C}$ can be determined by the transformation matrix method and via the pole-assignment method to appoint the eigenvalues are -1 and -4, and $\boldsymbol{CB} = \boldsymbol{I}$, hence, the design $\boldsymbol{C}$ as

$$\boldsymbol{C} = \begin{bmatrix} 0.1323 & -0.2659 & 0.8677 \end{bmatrix} \tag{4-22}$$

The design input $\boldsymbol{u}(t)\,\boldsymbol{u}$ as (3-12) by

$$\begin{aligned} \boldsymbol{u}(t) &= -\boldsymbol{CA}\hat{\boldsymbol{x}}(t) - (\gamma + \sigma)\,sgn(\boldsymbol{s}) \\ &= -\begin{bmatrix} 0.4180 & 0.1796 & -2.5438 \end{bmatrix} \hat{\boldsymbol{x}}(t) - \left(\left\| \hat{\boldsymbol{x}}(t) \right\| + 1 \right) sat(\boldsymbol{s}, \varepsilon) \end{aligned} \tag{4-23}$$

where $\sigma = 0$ and sliding layer $-0.01 \le \varepsilon \le 0.01$. The simulation results as:

Figure 4.9 to Figure 4.18 are simulation results with initial condition $\boldsymbol{x}(0) = \begin{bmatrix} 10 & 7 & 3 \end{bmatrix}^T$ and $z(0) = 5$. Figure 4.9 shows the MATLAB simulink connection diagram, that contains system, reduced-order observer, sliding mode controller, and disturbance. Figure 4.10 shows the observer state error, that at 8.453s, 1.771s and 0.9199s convergence to approach of zero respectively, therefore, Figure 4.11 shows the convergence speed from (4.746, 0s) to (1.744, 1s), (7.042, 0s) to (0.04736, 1s), (9.911, 0s) to (0.0004488, 1s), conform the $eig(\boldsymbol{F}) = -1$, $eig(\boldsymbol{F}) = -5$, $eig(\boldsymbol{F}) = -10$ respectively, and the eigenvalues of $\boldsymbol{F}$ distant from origin in left phase plane, the observer accurate estimate soon. Figure 4.12 shows the sliding surface, that at 0.3244s, 0.191s, and 0.2413s into the sliding layer respectively, and then it lead to the control input have disjunctive parts shows in Figure 4.14 at these time, and the sliding surface guarantee bound into sliding layer after accurate estimate show in Figure 4.13. Figure 4.15 shows the disturbance, that the range between -1 and 1.3. Figure 4.16, 4.17 and 4.18 illustrate the system state variable all converge to $\boldsymbol{x} = 0$.

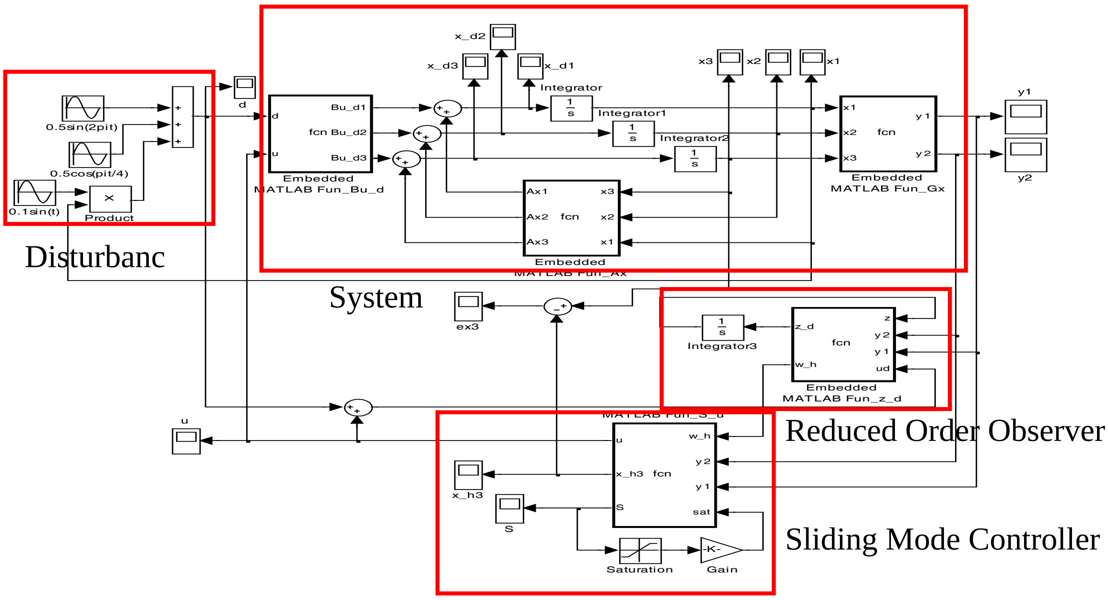

Figure 4.9 The MATLAB slimulink connection diagram

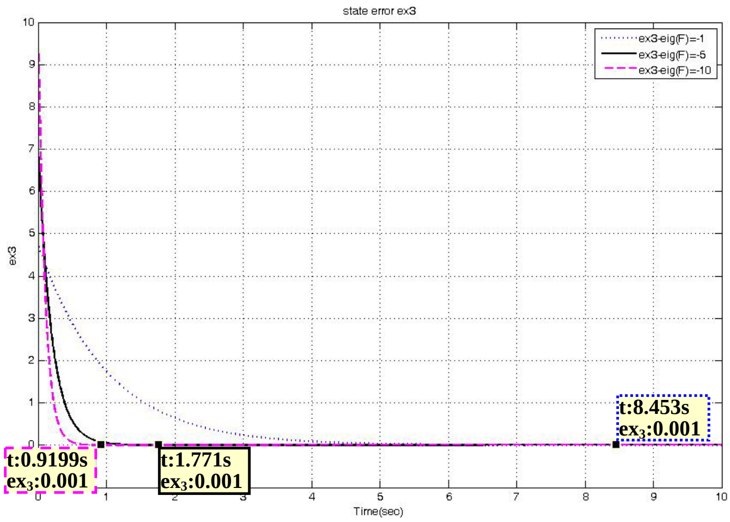

Figure 4.10 The observer state error

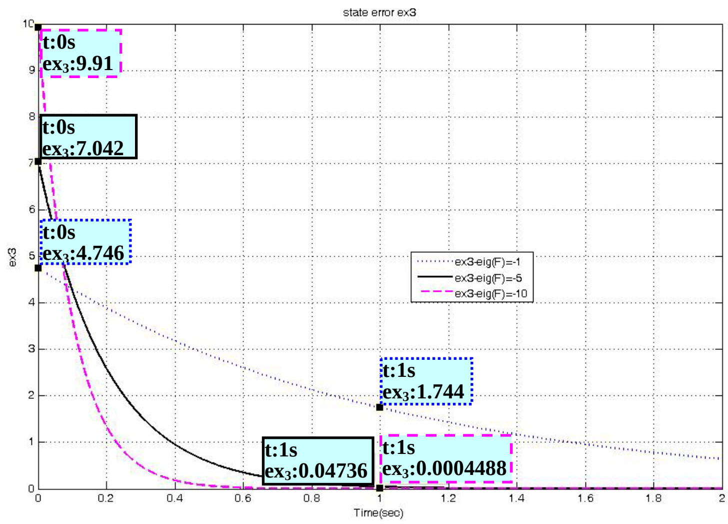

Figure 4.11 The observer state error of convergence speed

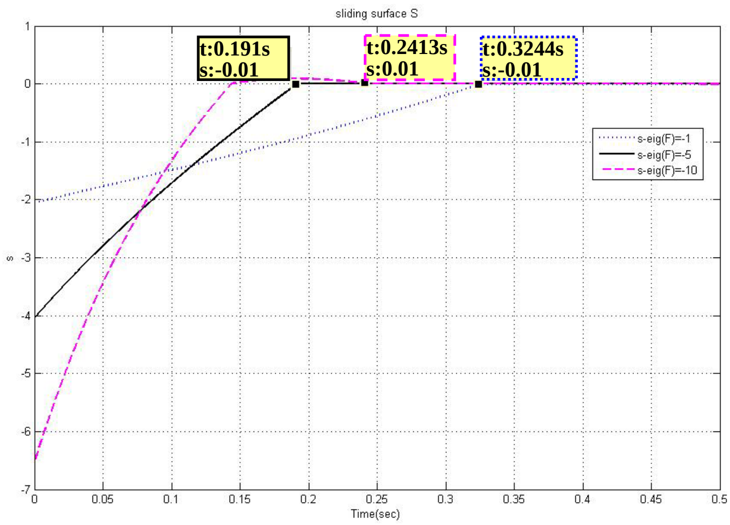

Figure 4.12 The sliding surface $s(t)$

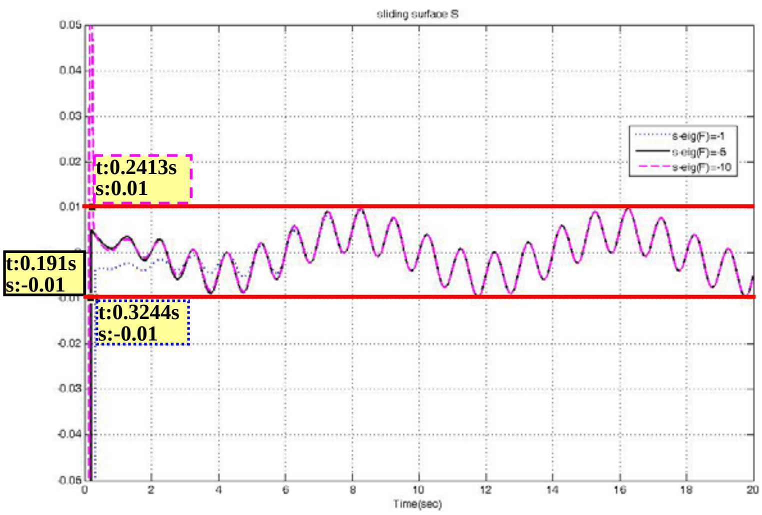

Figure 4.13 The sliding surface $s(t)$ bound in $\|s\| \le \varepsilon$

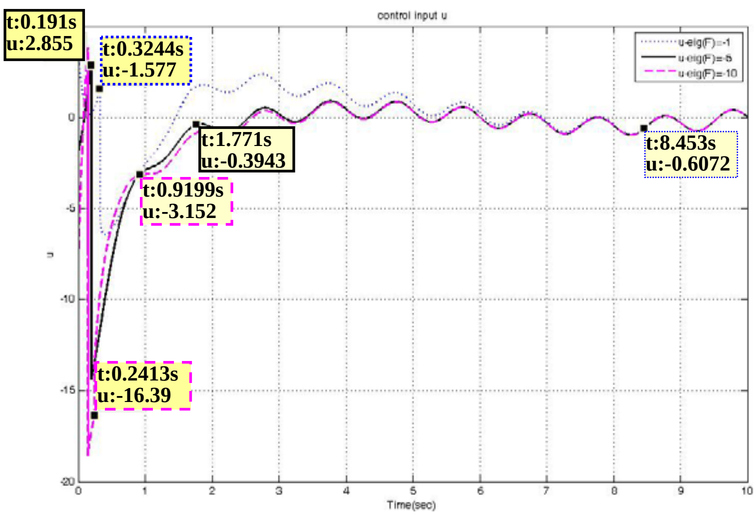

Figure 4.14 The control input $u(t)$

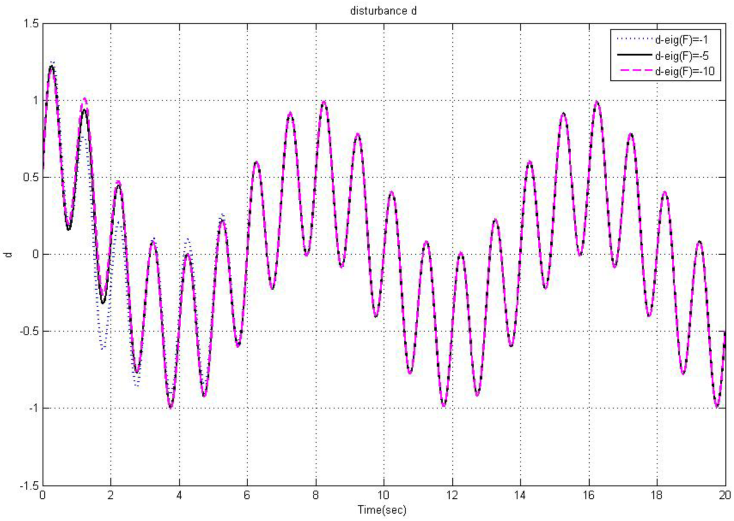

Figure 4.15 The disturbance $\boldsymbol{d}(t)$

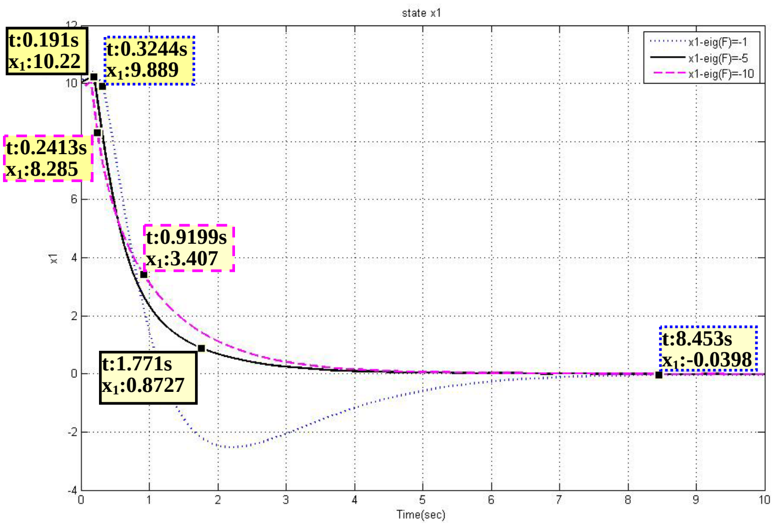

Figure 4.16 The system state $\boldsymbol{x}_1(t)$

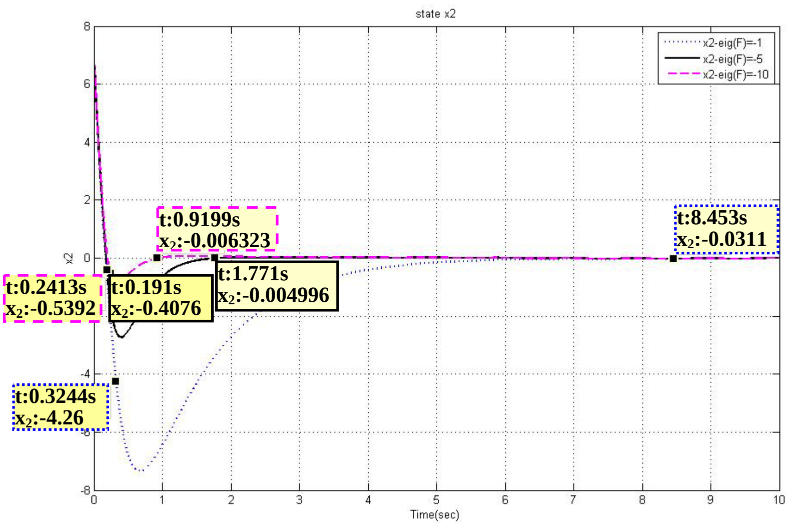

Figure 4.17 The system state $\boldsymbol{x}_2(t)$

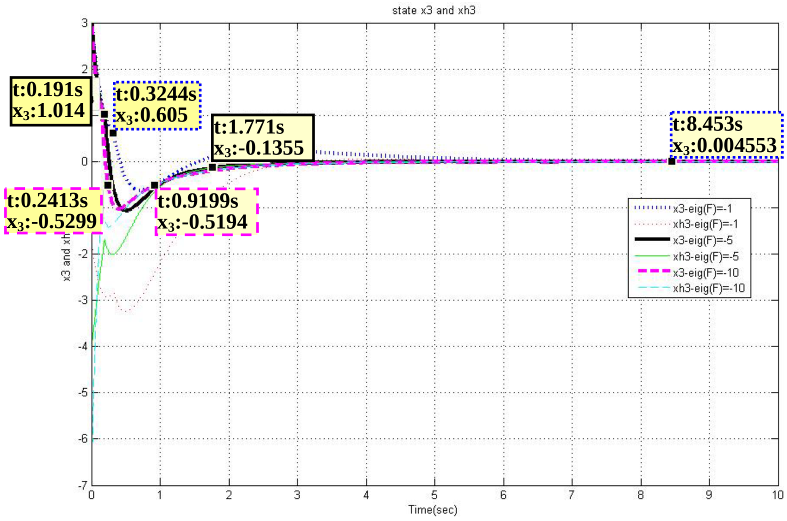

Figure 4.18 The system state $\boldsymbol{x}_3(t)$ and observer state $\hat{\boldsymbol{x}}_3(t)$

4.2 Observer-based sliding mode controller design for mismatching disturbance

In this section, using reduced-order observer-based sliding mode controller design to simulates a mismatching disturbance of LTI system. Consider a LTI system (3-1) suffering from the mismatching disturbance, the system state be decomposed into three parts $\boldsymbol{x}(t)=\begin{bmatrix}\boldsymbol{x}_1^T(t) & \boldsymbol{x}_2^T(t) & \boldsymbol{x}_3^T(t)\end{bmatrix}^T$, where $\boldsymbol{x}_1(t)$ and $\boldsymbol{x}_2(t)$ are measurable and $\boldsymbol{x}_3(t)$ is not obtainable. Then the system reconstruct as:

$$\begin{bmatrix}\dot{\boldsymbol{x}}_1(t)\\ \dot{\boldsymbol{x}}_2(t)\\ \dot{\boldsymbol{x}}_3(t)\end{bmatrix}=\begin{bmatrix}-0.277 & 1 & -0.002\\ -1.71 & -0.178 & -12.2\\ 0 & 0 & -6.67\end{bmatrix}\begin{bmatrix}\boldsymbol{x}_1(t)\\ \boldsymbol{x}_2(t)\\ \boldsymbol{x}_3(t)\end{bmatrix}+\begin{bmatrix}1\\0\\1\end{bmatrix}\boldsymbol{u}(t)+\begin{bmatrix}1\\1\\0\end{bmatrix}\boldsymbol{d}(t) \tag{4-24}$$

The mismatching disturbance is assumed as

$$d(t)=0.5\left(\sin(2\pi t)+\cos\left(\frac{\pi t}{4}\right)\right)+0.1\sin(t)x_1 \tag{4-25}$$

Apparently, the upper bound of the mismatching disturbance is obtained as

$$\|d(t)\|\le\|\boldsymbol{x}(t)\|+1 \tag{4-26}$$

The system dimension of measurable part and the dimension of disturbance represent as

$$\begin{aligned}&rank\begin{pmatrix}\boldsymbol{A}_{11} & \boldsymbol{A}_{12}\end{pmatrix}=rank\left(\begin{bmatrix}-0.277 & 1 & -0.002\\ -1.71 & -0.178 & -12.2\end{bmatrix}\right)=p=2\\ &rank(\boldsymbol{E})=rank\left(\begin{bmatrix}1\\1\\0\end{bmatrix}\right)=q=1\end{aligned} \tag{4-27}$$

where the dimension $p>q$. The first step is to design the reduced order observer, and the observer (2-3) represent as

$$\dot{\boldsymbol{z}}(t)=\boldsymbol{F}\boldsymbol{z}(t)+\boldsymbol{J}\boldsymbol{x}_1(t)+\boldsymbol{M}\boldsymbol{u}(t) \tag{4-28}$$

where

$$\begin{aligned}&\boldsymbol{F}=\boldsymbol{A}_{22}-\boldsymbol{L}\boldsymbol{A}_{12}=-6.67-\boldsymbol{L}\times\begin{bmatrix}-0.002\\ -12.2\end{bmatrix}\\ &\boldsymbol{J}=\boldsymbol{F}\boldsymbol{L}+\boldsymbol{A}_{21}-\boldsymbol{L}\boldsymbol{A}_{11}=\boldsymbol{F}\boldsymbol{L}+\begin{bmatrix}0 & 0\end{bmatrix}-\boldsymbol{L}\times\begin{bmatrix}-0.277 & 1\\ -1.71 & -0.178\end{bmatrix}\\ &\boldsymbol{M}=\boldsymbol{E}_2-\boldsymbol{L}\boldsymbol{E}_1=0-\boldsymbol{L}\times\begin{bmatrix}1\\1\end{bmatrix}\end{aligned} \tag{4-29}$$

And the reduced order observer should be designed under two conditions of (2-9) and (2-10). Because the dimension $p > q$, hence, the matrix $\boldsymbol{L}$ can be chosen as (2-12) by

$$\boldsymbol{L} = \boldsymbol{E}_2\boldsymbol{E}_1^+ + \boldsymbol{\Gamma}\left(\boldsymbol{I}_p - \boldsymbol{E}_1\boldsymbol{E}_1^+\right) = \begin{bmatrix}0 & 0\end{bmatrix} + \boldsymbol{\Gamma}\begin{bmatrix}0.5 & -0.5\\ -0.5 & 0.5\end{bmatrix} \tag{4-30}$$

where $\boldsymbol{L}$ can be chosen by $\boldsymbol{\Gamma}$. The matrix $\boldsymbol{L}$ can affect the observer estimate speed, that determine for eigenvalues of $\boldsymbol{F}$, shows in Section 4.1. By substituting (4-30) into (4-29) and design the eigenvalue of $\boldsymbol{F}$ is -10, that can convergence to approach of zero soon, then the $\boldsymbol{L}$ represent as

$$\boldsymbol{L} = \begin{bmatrix}0.273 & -0.273\end{bmatrix} \tag{4-31}$$

By substituting (4-31) into (4-29), then the observer (4-28) rewrite as

$$\dot{\boldsymbol{z}}(t) = -10\boldsymbol{z}(t) - \begin{bmatrix}-3.1212 & 2.4084\end{bmatrix}\begin{bmatrix}\boldsymbol{x}_1(t)\\ \boldsymbol{x}_2(t)\end{bmatrix} \tag{4-32}$$

Therefore, it is completed which can successfully estimate $\boldsymbol{x}_3$ as given in (4-28). The second step is using this information to design the sliding mode controller, let the sliding function be

$$\boldsymbol{s}(t) = \boldsymbol{C}\hat{\boldsymbol{x}}(t) \tag{4-33}$$

Because the eigenvalues of $\boldsymbol{A}$ are -0.2275±1.3067i and -6.67, then the matrix $\boldsymbol{C}$ can be determined by the transformation matrix method and via the pole-assignment method to appoint the eigenvalues are [-1 -2], [-2 -6.67] and [-2+2i -2-2i] respectively, and $\boldsymbol{CB} = \boldsymbol{I}$, hence, the design $\boldsymbol{C}$ as

$$\begin{aligned}\boldsymbol{C} &= \begin{bmatrix}0.1259 & -0.1251 & 0.8741\end{bmatrix}\\ \boldsymbol{C} &= \begin{bmatrix}-0.1457 & -0.6575 & 1.1457\end{bmatrix}\\ \boldsymbol{C} &= \begin{bmatrix}-0.1264 & -0.3129 & 1.1264\end{bmatrix}\end{aligned} \tag{4-34}$$

The design input $\boldsymbol{u}(t)$ as (3-12) respective represent by

$$\begin{aligned}\boldsymbol{u}(t) &= -\boldsymbol{CA}\hat{\boldsymbol{x}}(t) - (\gamma + \sigma)\,sgn(\boldsymbol{s})\\ &= -\begin{bmatrix}0.1790 & 0.1482 & -4.3043\end{bmatrix}\hat{\boldsymbol{x}}(t) - \left(\left\|\hat{\boldsymbol{x}}(t)\right\| + 1\right) sat(\boldsymbol{s}, \varepsilon)\\ \boldsymbol{u}(t) &= -\begin{bmatrix}1.1647 & -0.0287 & 0.3800\end{bmatrix}\hat{\boldsymbol{x}}(t) - \left(\left\|\hat{\boldsymbol{x}}(t)\right\| + 1\right) sat(\boldsymbol{s}, \varepsilon)\\ \boldsymbol{u}(t) &= -\begin{bmatrix}0.5701 & -0.0707 & -3.6955\end{bmatrix}\hat{\boldsymbol{x}}(t) - \left(\left\|\hat{\boldsymbol{x}}(t)\right\| + 1\right) sat(\boldsymbol{s}, \varepsilon)\end{aligned} \tag{4-35}$$

where $\sigma = 0$ and sliding layer $-0.01 \le \varepsilon \le 0.01$. The simulation results as:

Figure 4.19 to Figure 4.27 are simulation results with initial condition $\boldsymbol{x}(0) = \begin{bmatrix}10 & 7 & 3\end{bmatrix}^T$ and $z(0) = 5$. Figure 4.19 shows the MATLAB simulink connection diagram, that contains system, reduced-order observer, sliding mode controller, and disturbance. Figure 4.20 shows the observer state error, because constant the eigenvalues of $\boldsymbol{F}$, hence, the state error at 0.7944s convergence to approach of zero, and the convergence speed from (-2.819, 0s) to (-0.000128, 1s), conform the $eig(\boldsymbol{F}) = -10$. Figure 4.21 shows the sliding surface, that at 0.3867s, 0.2631s, and 0.123s into the sliding layer respectively, and then it effect the control input that have disjunctive parts in these time show in Figure 4.23. Figure 4.22 shows the sliding surface guarantees bound into sliding layer after accurate estimate. Figure 4.24 shows the disturbance, that the range between -1 and 1.3. Figure 4.25, 4-26 and 4.27 illustrate the system state variable all converge to approach $\boldsymbol{x} = 0$, because the system affected mismatching disturbance, then it uncompleted converge to $\boldsymbol{x} = 0$.

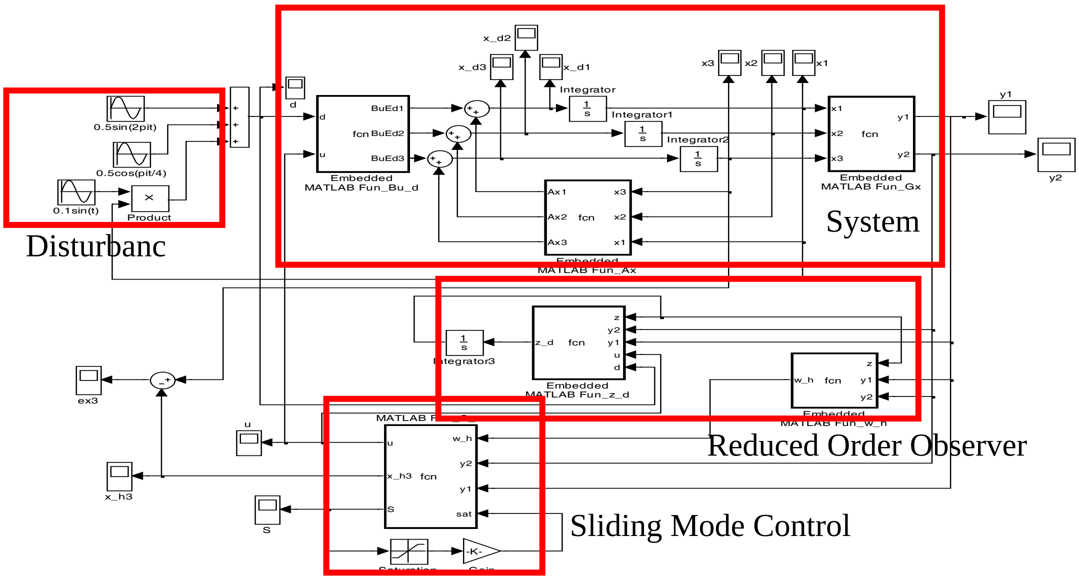

Figure 4.19 The MATLAB slimulink connection diagram

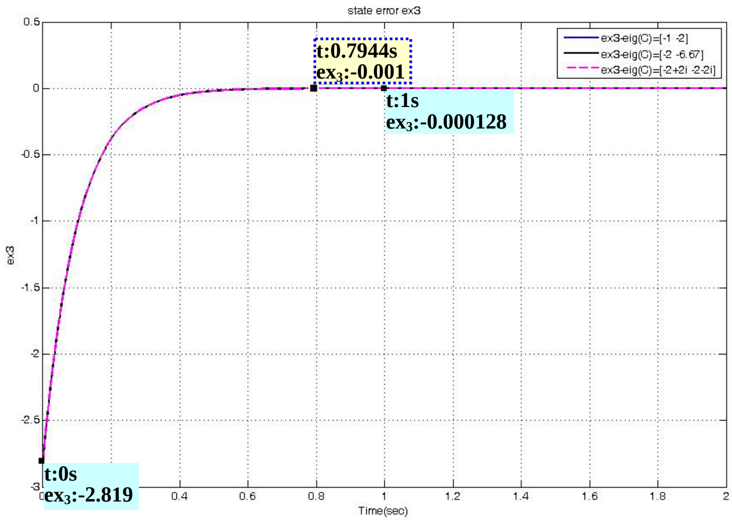

Figure 4.20 The observer state error

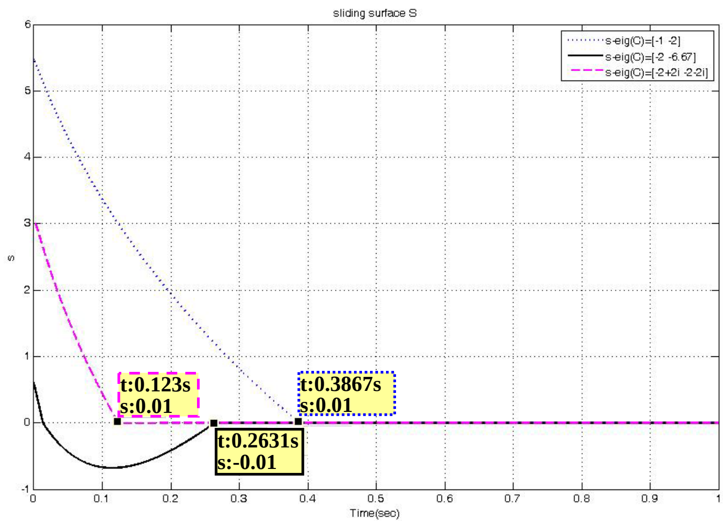

Figure 4.21 The sliding surface $s(t)$

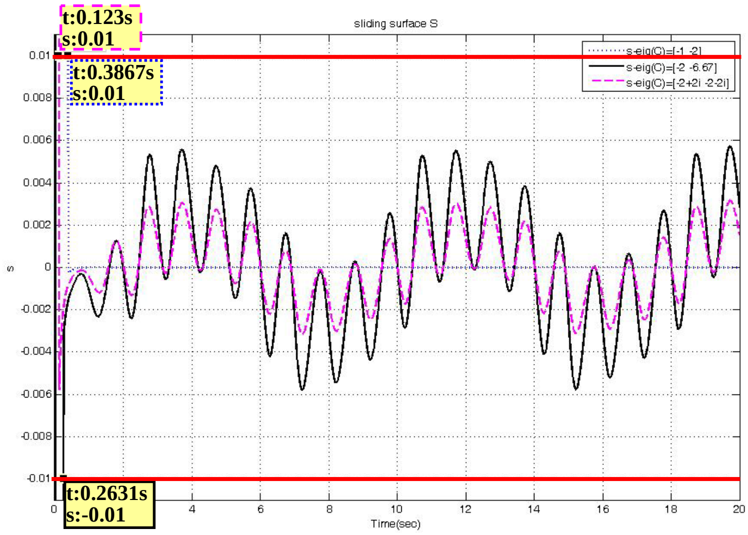

Figure 4.22 The sliding surface $\boldsymbol{s}(t)$ bound in $\|s\| \leq \varepsilon$

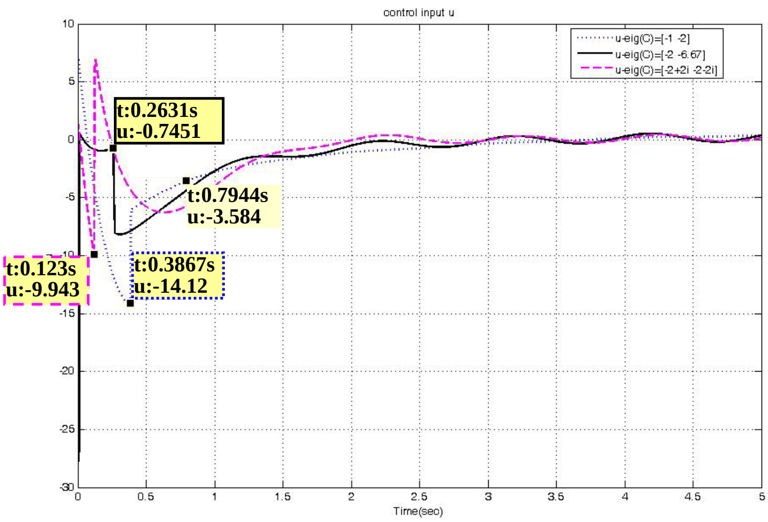

Figure 4.23 The control input $\boldsymbol{u}(t)$

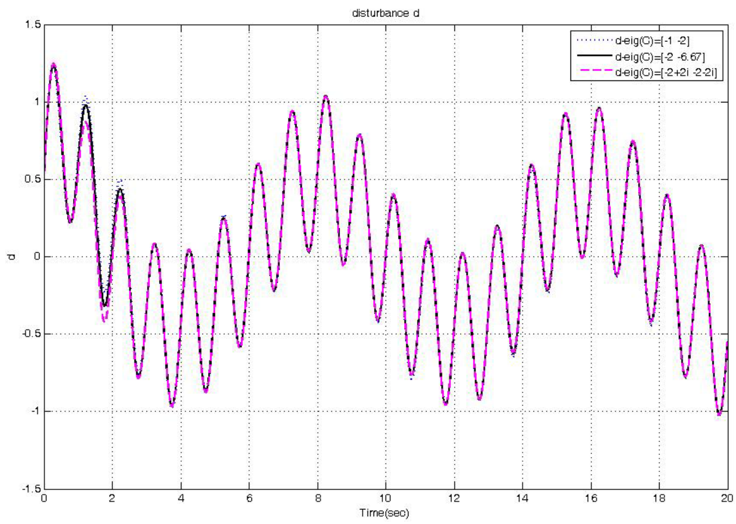

Figure 4.24 The disturbance $\boldsymbol{d}(t)$

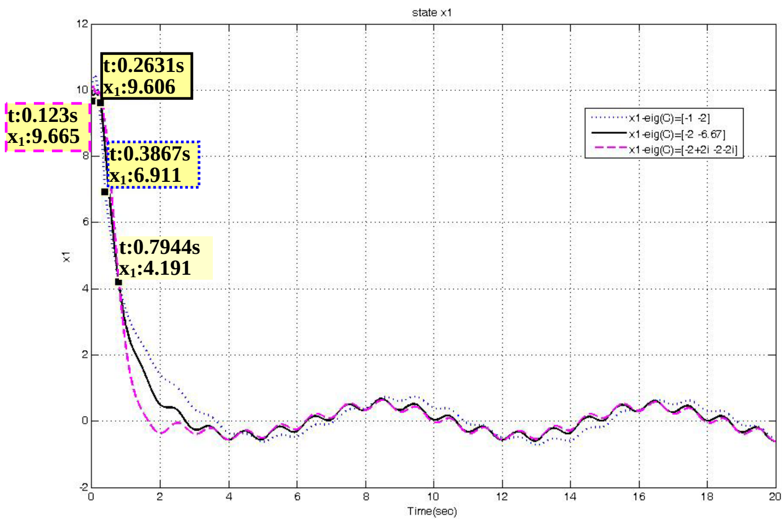

Figure 4.25 The system state $x_1(t)$

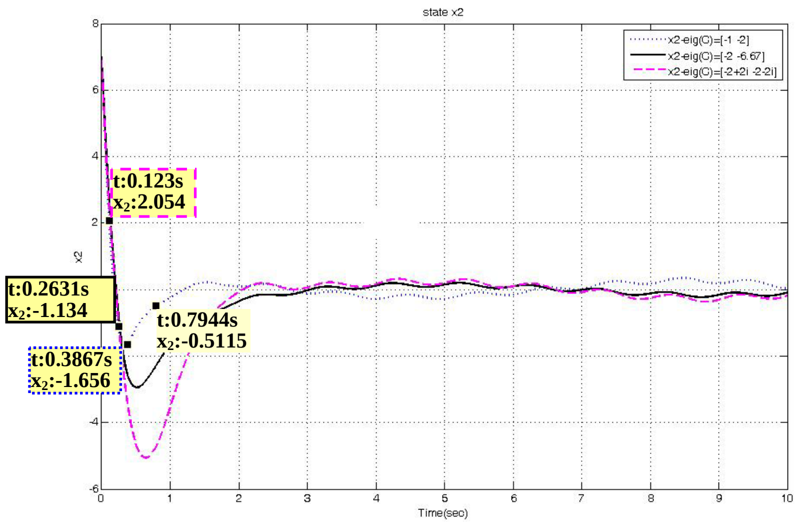

Figure 4.26 The system state $x_2(t)$

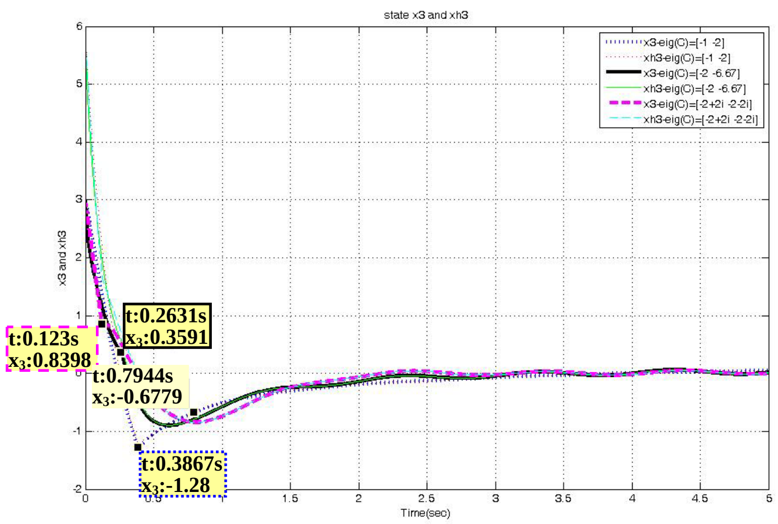

Figure 4.27 The system state $x_3(t)$ and observer state $\hat{x}_3(t)$

4.3 Observer-based sliding mode controller design for output tracking

In this section, using reduced-order observer-based sliding mode controller design to simulates three cases, there are output tracking that tracking trajectory $\boldsymbol{y}_d(t)$ is constant with matched disturbance, output track that tracking trajectory $\boldsymbol{y}_d(t)$ is constant with mismatching disturbance, and output track that tracking trajectory $\boldsymbol{y}_d(t)$ is not constant with mismatching disturbance.

In case 1, use the matched disturbance of LTI system, and output tracks control $\boldsymbol{y}(t)$ to $\boldsymbol{y}_d(t)$. The system and the observer design same with Section 4.1 case 2, hence, it is completed which can successfully estimate $\boldsymbol{x}_3(t)$ as given in (4-20), and the observer rewrites as

$$\dot{\boldsymbol{z}}(t) = -10\boldsymbol{z}(t) - \begin{bmatrix} 9.2588 & 3.6821 \end{bmatrix} \begin{bmatrix} \boldsymbol{x}_1(t) \\ \boldsymbol{x}_2(t) \end{bmatrix} \tag{4-36}$$

The control goal is to fulfill tracking control for the output

$$\boldsymbol{y}(t) = \begin{bmatrix} 1 & 2 & 0 \end{bmatrix} \boldsymbol{x}(t) \tag{4-37}$$

Assume the tracking error $\boldsymbol{e}$ as

$$\boldsymbol{e}(t) = \boldsymbol{y}(t) - \boldsymbol{y}_d(t) \tag{4-38}$$

where $\boldsymbol{y}_d(t)$ is output tracking control trajectory, and design $\boldsymbol{y}_d(t) = 10$ is constant. Hence, the system can be reconstructed as

$$\begin{aligned} \begin{bmatrix} \dot{\boldsymbol{x}}_1(t) \\ \dot{\boldsymbol{x}}_2(t) \\ \dot{\hat{\boldsymbol{x}}}_3(t) \\ \dot{\boldsymbol{H}}(t) \end{bmatrix} &= \begin{bmatrix} -0.277 & 1 & -0.002 & 0 \\ -1.71 & -0.178 & -12.2 & 0 \\ 0 & 0 & -6.67 & 0 \\ 1 & 2 & 0 & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{x}_1(t) \\ \boldsymbol{x}_2(t) \\ \hat{\boldsymbol{x}}_3(t) \\ \boldsymbol{H}(t) \end{bmatrix} \\ &+ \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \left(\boldsymbol{u}(t) + \boldsymbol{d}(t) \right) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ -\boldsymbol{y}_d \end{bmatrix} \end{aligned} \tag{4-39}$$

The first step in the sliding mode controller design is to choose an appropriate sliding function, such that the system trajectory can trace the control goal in the sliding mode. Let the sliding function be

$$s(t) = C\hat{p}(t) = C\begin{bmatrix} x_1(t) \\ x_2(t) \\ \hat{x}_3(t) \\ H(t) \end{bmatrix} \tag{4-40}$$

Because the eigenvalues of A are 0, -0.2275±1.3067i and -6.67, then the matrix C can be determined by the transformation matrix method and via the pole-assignment method to appoint the eigenvalues are $[-1 \quad -4 \quad -6.67]$, and $CB = I$, hence, the design C as

$$C = [-0.4694 \quad -1.0692 \quad 1.4694 \quad -0.6575] \tag{4-41}$$

The design input $u(t)$ as (3-39) represent by

$$\begin{aligned} u(t) &= -CA\hat{x}(t) - (\gamma + \sigma)\, sgn(s) \\ &= -[1.3008 \quad -1.5941 \quad 3.2442 \quad 0]\hat{x}(t) - (\|\hat{x}(t)\| + 1 + \sigma)\, sat(s, \varepsilon) \end{aligned} \tag{4-42}$$

where σ is 8, and sliding layer $-0.01 \le \varepsilon \le 0.01$. The simulation results as: Figure 4.28 to Figure 4.35 are simulation results with initial condition $x(0) = [10 \quad 7 \quad 5]^T$ and $z(0) = 0$. Figure 4.28 shows the MATLAB simulink connection diagram, that contains system, reduced-order observer, sliding mode controller, and disturbance. Figure 4.29 shows the observer state error, that at 0.9773s convergence to approach zero, and the convergence speed from (16.91, 0s) to (0.0007658, 1s), conform the $eig(F) = -10$. Figure 4.30 shows the sliding surface, that at 0.725s into the sliding layer, and then it affect the control input that have disjunctive parts in this time shows in Figure 4.32. Figure 4.31 shows the sliding surface bound into $\|s\| \le \varepsilon$. Figure 4.33 shows the disturbance, that the range between -1.2 and 1.25. Figure 4.34 shows the output tracking control $y(t)$ to $y_d(t)$, because this is matched disturbance system and $y_d(t)$ is constant, then the control output can complete track to $y_d(t) = 10$, that introduced in Section 3.3. Figure 4.35 shows the control output error.

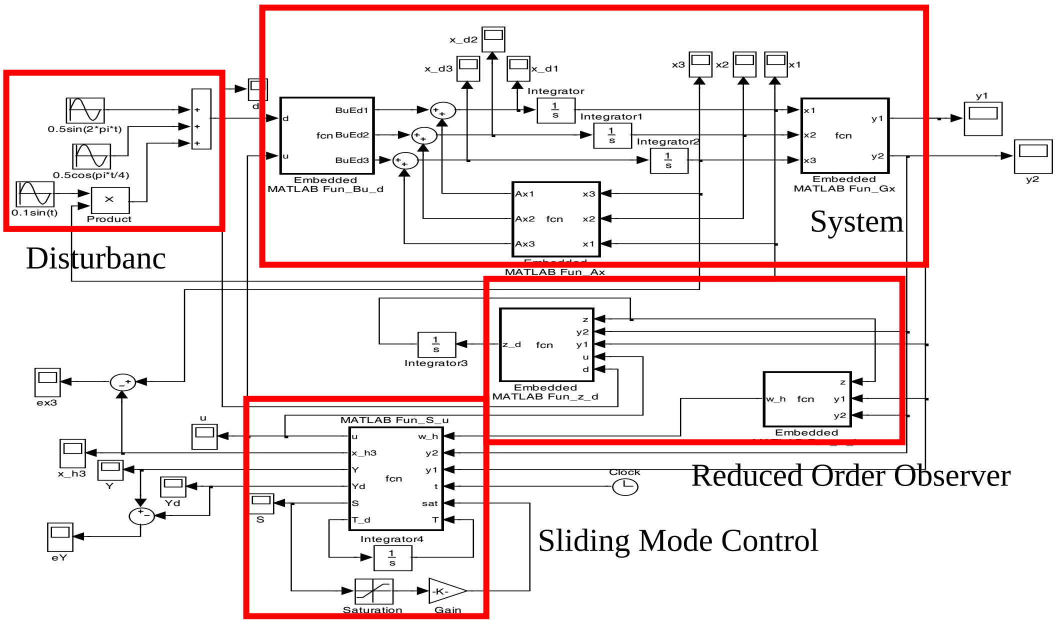

Figure 4.28 The MATLAB slimulink connection diagram

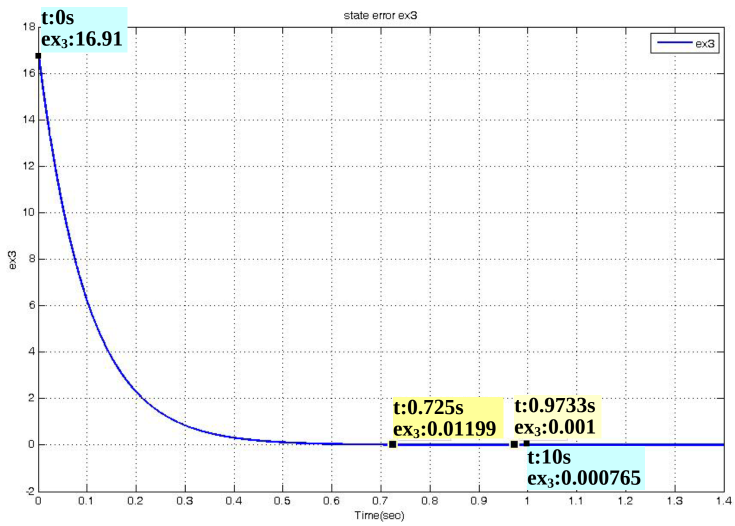

Figure 4.29 The observer state error

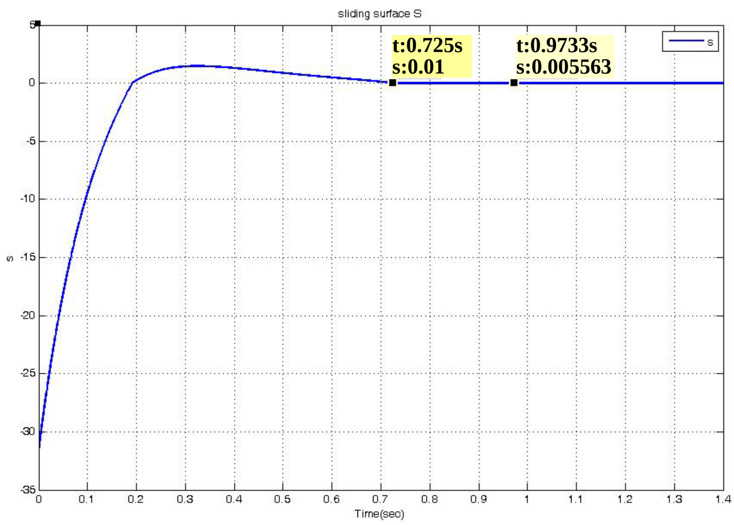

Figure 4.30 The sliding surface $s(t)$

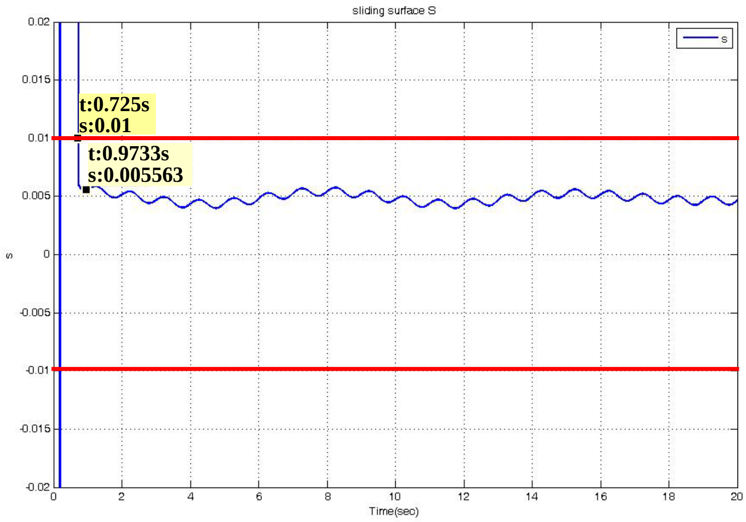

Figure 4.31 The sliding surface $\boldsymbol{s}(t)$ bound in $\|\mathrm{s}\| \leq \varepsilon$

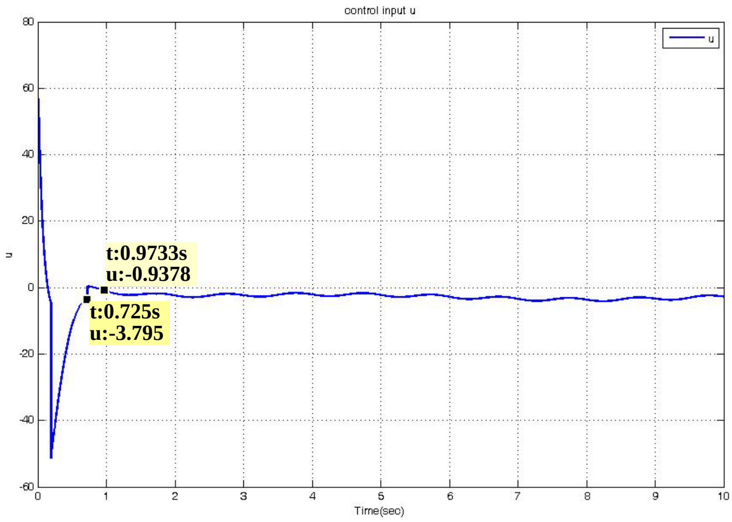

Figure 4.32 The control input $\boldsymbol{u}(t)$

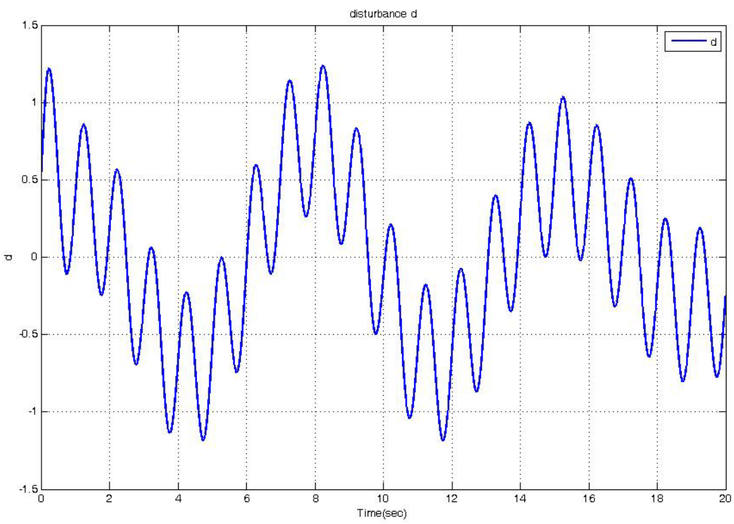

Figure 4.33 The disturbance $\boldsymbol{d}(t)$

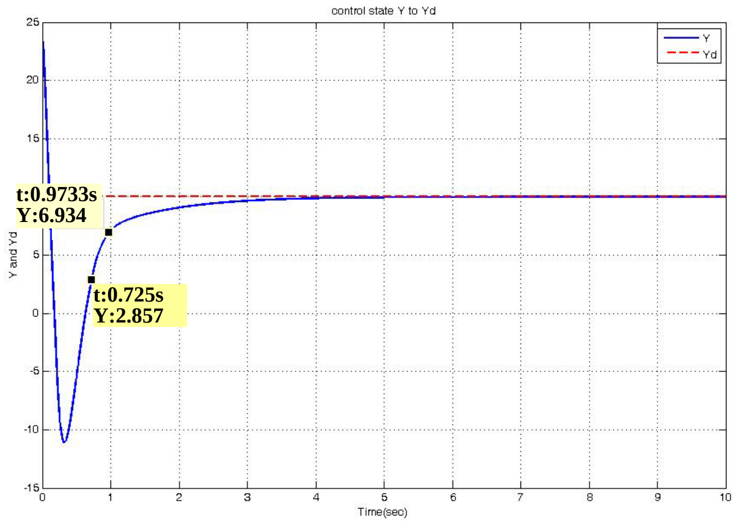

Figure 4.34 The output tracking control $\boldsymbol{y}(t)$ to $\boldsymbol{y}_d(t)$

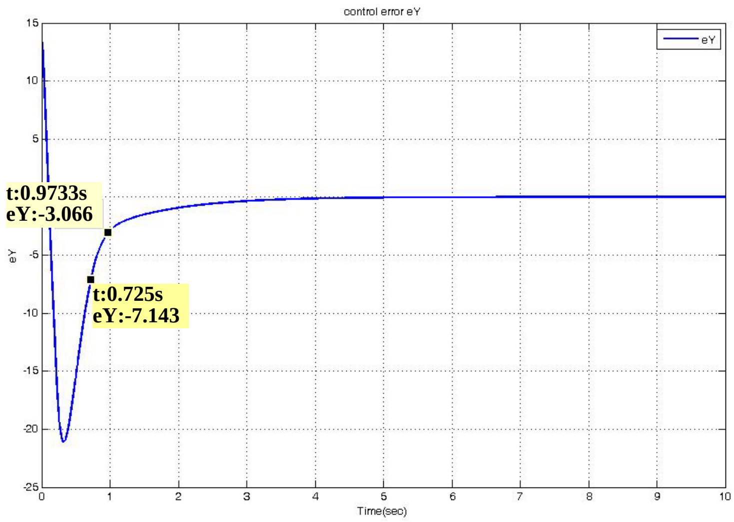

Figure 4.35 The output tracking control error

In Case 2 and Case 3, use the mismatching disturbance of LTI system, and output tracking control $\boldsymbol{y}(t)$ to $\boldsymbol{y}_d(t)$, and then will consider two condition of $\boldsymbol{y}_d(t)=10$ is constant in case 2 and $\boldsymbol{y}_d(t)=cos(t)$ is not constant in case 3. The system and the observer design same with Section 4.2, hence, it is completed that can successfully estimate $\boldsymbol{x}_3(t)$ as given in (4-32), and the observer rewrites as

$$\dot{\boldsymbol{z}}(t)=-10\boldsymbol{z}(t)-\begin{bmatrix}-3.1212 & 2.4084\end{bmatrix}\begin{bmatrix}\boldsymbol{x}_1(t)\\ \boldsymbol{x}_2(t)\end{bmatrix} \tag{4-43}$$

The control goal is to fulfill tracking control for the output

$$\boldsymbol{y}(t)=\begin{bmatrix}1 & 2 & 0\end{bmatrix}\boldsymbol{x}(t) \tag{4-44}$$

Assume the tracking error $\boldsymbol{e}$ as

$$\boldsymbol{e}(t)=\boldsymbol{y}(t)-\boldsymbol{y}_d(t) \tag{4-45}$$

Hence, the system (3-32) can be reconstructed as

$$\begin{bmatrix}\dot{\boldsymbol{x}}_1(t)\\ \dot{\boldsymbol{x}}_2(t)\\ \dot{\hat{\boldsymbol{x}}}_3(t)\\ \dot{\boldsymbol{H}}(t)\end{bmatrix}=\begin{bmatrix}-0.277 & 1 & -0.002 & 0\\ -1.71 & -0.178 & -12.2 & 0\\ 0 & 0 & -6.67 & 0\\ 1 & 2 & 0 & 0\end{bmatrix}\begin{bmatrix}\boldsymbol{x}_1(t)\\ \boldsymbol{x}_2(t)\\ \hat{\boldsymbol{x}}_3(t)\\ \boldsymbol{H}(t)\end{bmatrix}+\begin{bmatrix}1\\0\\1\\0\end{bmatrix}\boldsymbol{u}(t)+\begin{bmatrix}1\\1\\0\\0\end{bmatrix}\boldsymbol{d}(t)+\begin{bmatrix}0\\0\\0\\-\boldsymbol{y}_d\end{bmatrix} \tag{4-46}$$

The first step in the sliding mode controller design is to choose an appropriate sliding function, such that the system trajectory can trace to control goal in the sliding mode. Let the sliding function be

$$\boldsymbol{s}(t)=\boldsymbol{C}\hat{\boldsymbol{p}}(t)=\boldsymbol{C}\begin{bmatrix}\boldsymbol{x}_1(t)\\ \boldsymbol{x}_2(t)\\ \hat{\boldsymbol{x}}_3(t)\\ \boldsymbol{H}(t)\end{bmatrix} \tag{4-47}$$

Because the eigenvalues of A are 0, -0.2275±1.3067i and -6.67, then the matrix $\boldsymbol{C}$ can be determined by the transformation matrix method and via the pole-assignment method to appoint the eigenvalues are $\begin{bmatrix}-1 & -2 & -3\end{bmatrix}$, $\begin{bmatrix}-1 & -4+2i & -4-2i\end{bmatrix}$ and $\begin{bmatrix}-2 & -6.67 & -12\end{bmatrix}$ respectively, and $\boldsymbol{CB}=\boldsymbol{I}$, hence, the design $\boldsymbol{C}$ as

$$\begin{aligned}
\boldsymbol{C} &= \begin{bmatrix} -0.0046 & -0.4114 & -1.0046 & -0.1479 \end{bmatrix} \\
\boldsymbol{C} &= \begin{bmatrix} -0.3219 & -0.7977 & 1.3219 & -0.4929 \end{bmatrix} \\
\boldsymbol{C} &= \begin{bmatrix} -0.3681 & -1.9060 & 1.3681 & -3.9451 \end{bmatrix}
\end{aligned} \tag{4-48}$$

The design input $\boldsymbol{u}(t)$ as (3-12) respective represent by

$$\begin{aligned}
\boldsymbol{u}(t) &= -\boldsymbol{CA}\hat{\boldsymbol{x}}(t) - (\gamma + \sigma)\,sgn(\boldsymbol{s}) \\
&= -\begin{bmatrix} 0.5569 & -0.2272 & -1.6816 & 0 \end{bmatrix}\hat{\boldsymbol{x}}(t) - \left(\left\|\hat{\boldsymbol{x}}(t)\right\| + 1 + \sigma\right) sat(\boldsymbol{s}, \varepsilon) \\
\boldsymbol{u}(t) &= -\begin{bmatrix} 0.9603 & -1.1657 & 0.9155 & 0 \end{bmatrix}\hat{\boldsymbol{x}}(t) - \left(\left\|\hat{\boldsymbol{x}}(t)\right\| + 1 + \sigma\right) sat(\boldsymbol{s}, \varepsilon) \\
\boldsymbol{u}(t) &= -\begin{bmatrix} -0.5839 & -7.919 & 14.1287 & 0 \end{bmatrix}\hat{\boldsymbol{x}}(t) - \left(\left\|\hat{\boldsymbol{x}}(t)\right\| + 1 + \sigma\right) sat(\boldsymbol{s}, \varepsilon)
\end{aligned} \tag{4-49}$$

where σ are 0, 3, 38 respective in Case 2, and σ are 0, 1, 15 respective in Case 3, and sliding layer $-0.01 \le \varepsilon \le 0.01$. The simulation results as:

In case2, Figure 4.36 to Figure 4.43 are simulation results with initial condition $\boldsymbol{x}(0) = \begin{bmatrix} 10 & 7 & 5 \end{bmatrix}^T$ and $\boldsymbol{z}(0) = 0$. Figure 4.36 shows the MATLAB simulink connection diagram that contains system, reduced-order observer, sliding mode controller, and disturbance. Figure 4.37 shows the observer state error, because constant the eigenvalues of $\boldsymbol{F}$, hence, the state error at 0.8338s convergence to zero, and the convergence speed from (4.181, 0s) to (0.0001898, 1s), conform the $eig(\boldsymbol{F}) = -10$. Figure 4.38 shows the sliding surface, that at 0.2092s, 0.2086s, and 0.1964s into the sliding layer respectively, and then it effect the control input that have disjunctive parts in these time shows in Figure 4.40 and the design eigenvalues of $\boldsymbol{C}$ distant from origin in left phase plane, then the input $\boldsymbol{u}$ must be higher gain. Figure 4.39 shows the sliding surface bound into $\|\mathrm{s}\| \le \varepsilon$. Figure 4.41 shows the disturbance, that the range between -1.12 and 1.32. Figure 4.42 shows the output tracking control $\boldsymbol{y}(t)$ to $\boldsymbol{y}_d(t)$, because the system affected mismatching disturbance and constant $\boldsymbol{y}_d(t)$, then the control output can not complete track to $\boldsymbol{y}_d(t) = 10$, conform with introduced Section 3.3. Figure 4.35 shows the control output error, and the design eigenvalues of $\boldsymbol{C}$ distant from origin in left phase plane more approach $\boldsymbol{y}_d(t) = 10$.

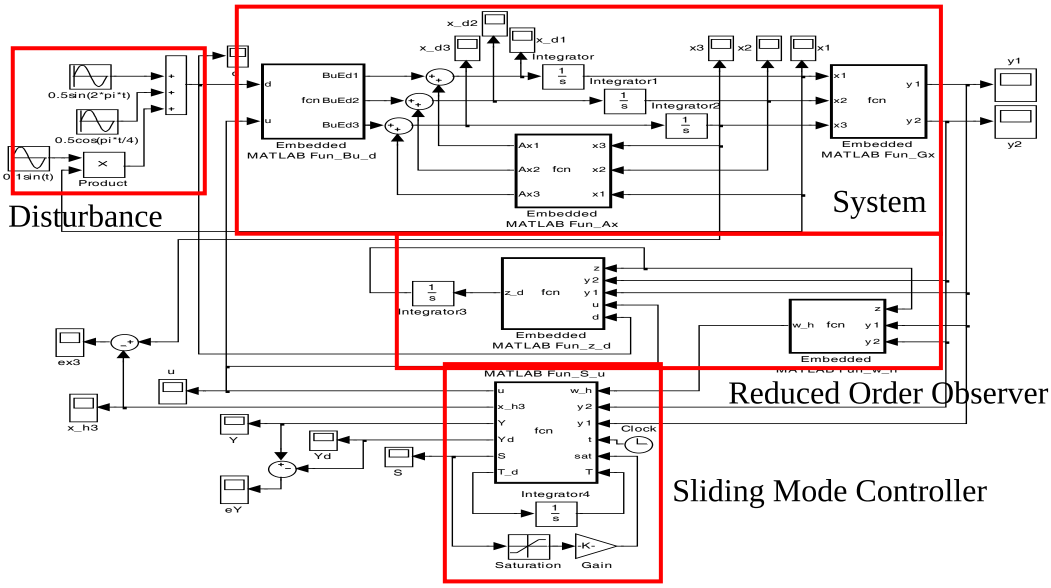

Figure 4.36 The MATLAB slimulink connection diagram

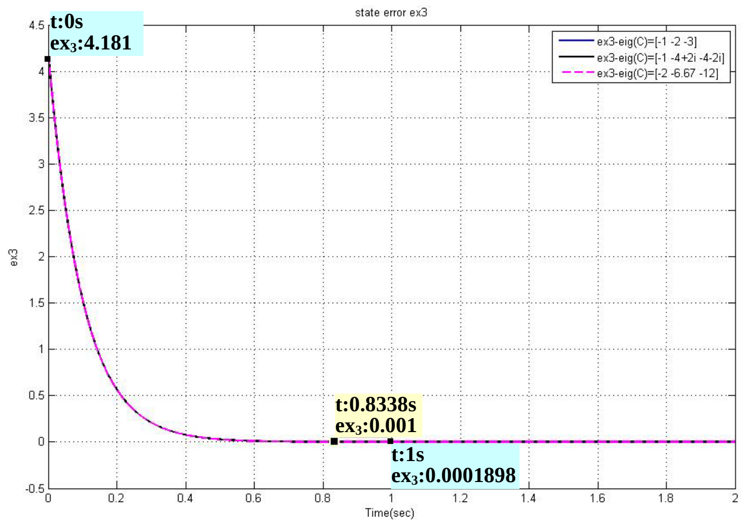

Figure 4.37 The observer state error

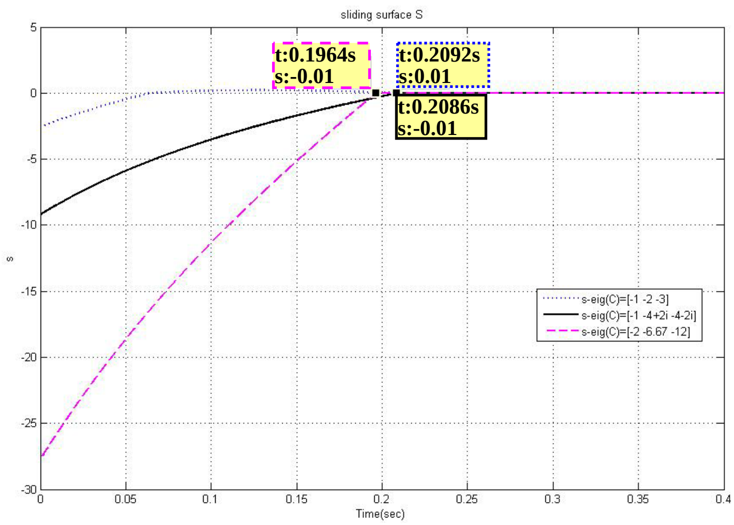

Figure 4.38 The sliding surface $s(t)$

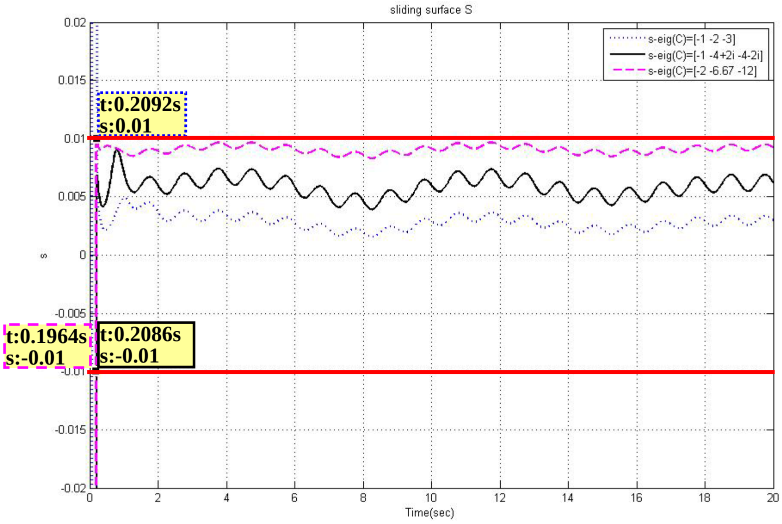

Figure 4.39 The sliding surface $s(t)$ bound in $\|s\| \le \varepsilon$

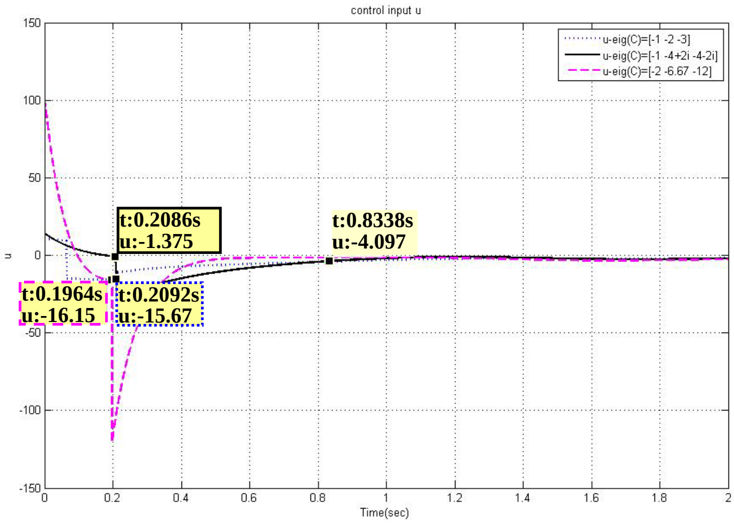

Figure 4.40 The control input $u(t)$

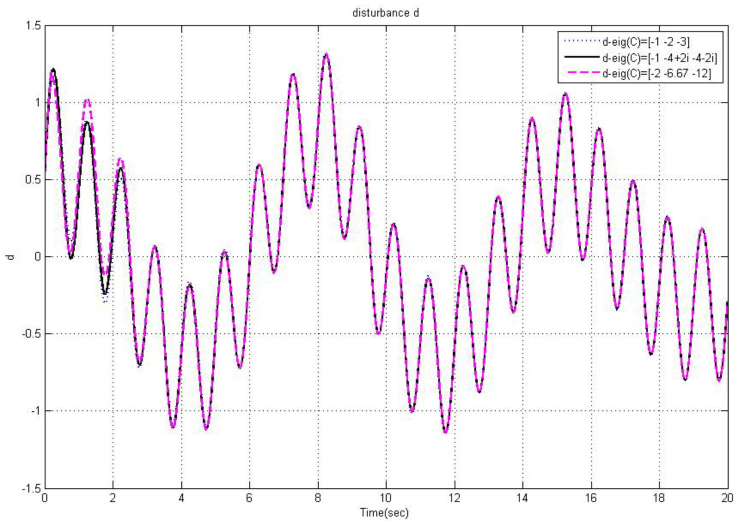

Figure 4.41 The disturbance $d(t)$

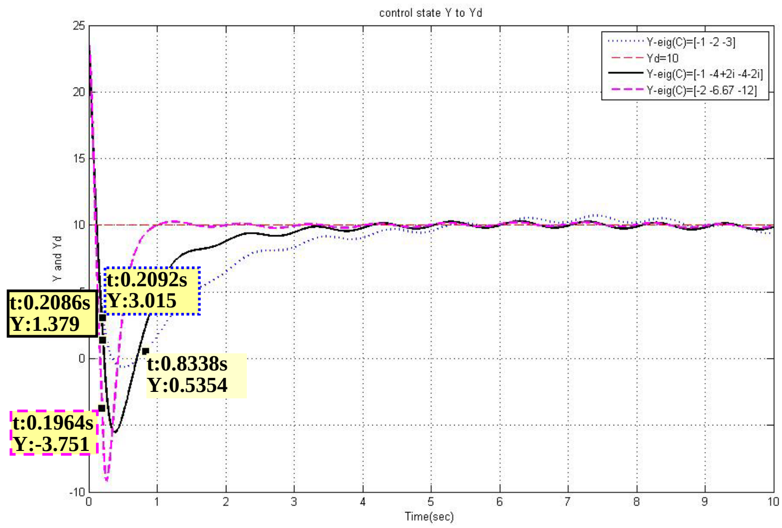

Figure 4.42 The output tracking control $\boldsymbol{y}(t)$ to $\boldsymbol{y}_d(t)$

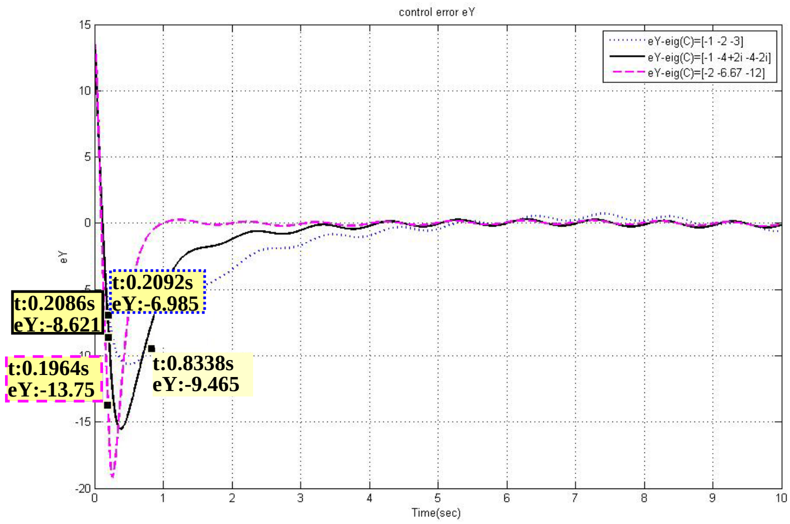

Figure 4.43 The output tracking control error

In case 3, Figure 4.44 to Figure 4.50 are simulation results with initial condition $\boldsymbol{x}(0)=[10 \quad 7 \quad 5]^T$ and $\boldsymbol{z}(0)=0$. Figure 4.44 the observer state error, because constant the eigenvalues of $\boldsymbol{F}$, hence, the state error at 0.8338s convergence to zero, and the convergence speed from (4.181, 0s) to (0.0001898, 1s), conform the $eig(\boldsymbol{F})=-10$. Figure 4.45 shows the sliding surface, that at 0.1643s, 0.2883s, and 0.5745s into the sliding layer respectively, and then it effect the control input have disjunctive parts in these time shows in Figure 4.47, and the design eigenvalues of $\boldsymbol{C}$ distant from origin in left phase plane, then the input $\boldsymbol{u}$ must be higher gain. Figure 4.46 shows the sliding surface bound into $\|s\| \le \varepsilon$. Figure 4.48 shows the disturbance, that the range between -1 and 1.25. Figure 4.49 shows the output tracking control $\boldsymbol{y}(t)$ to $\boldsymbol{y}_d(t)$, because the system affected mismatching disturbance and not constant $\boldsymbol{y}_d(t)$, then the control output can not complete track to $\boldsymbol{y}_d(t)=cos(t)$, conform with introduced Section 3.3. Figure 4.50 shows the control output error, and the design eigenvalues of $\boldsymbol{C}$ distant from origin in left phase plane more approach $\boldsymbol{y}_d(t)=cos(t)$.

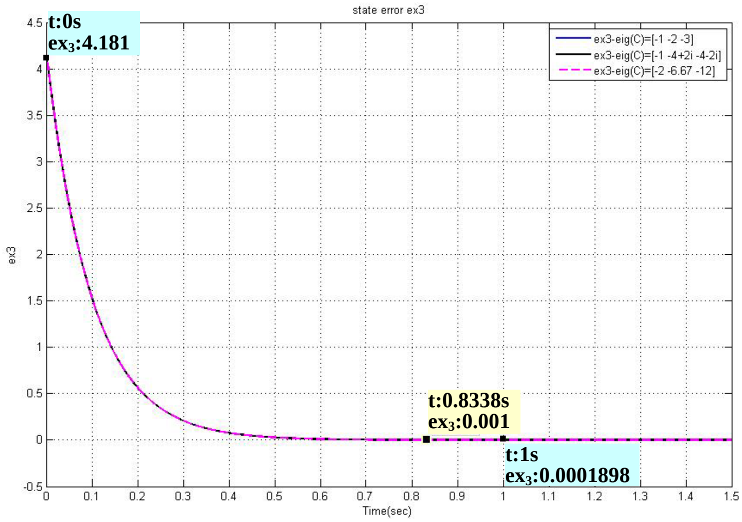

Figure 4.44 The observer state error

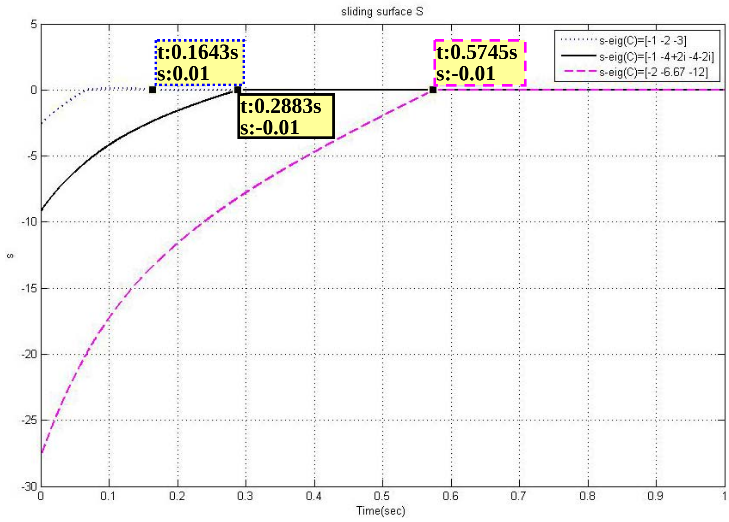

Figure 4.45 The sliding surface $s(t)$

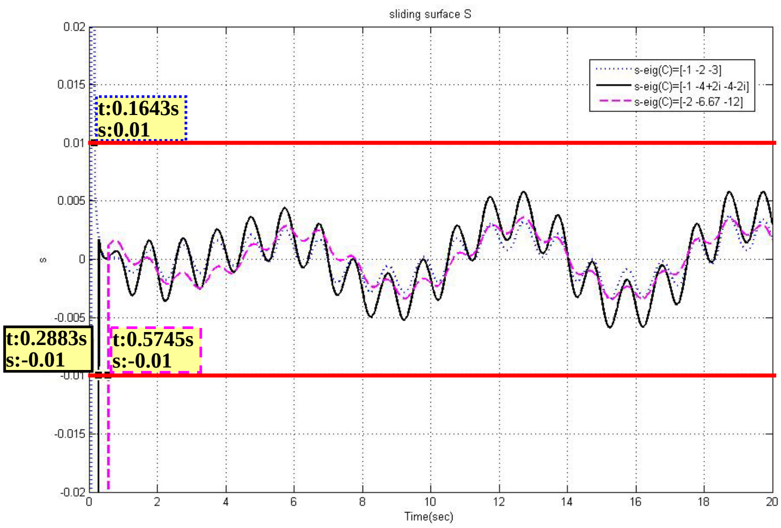

Figure 4.46 The sliding surface $s(t)$ bound in $\|s\| \le \varepsilon$

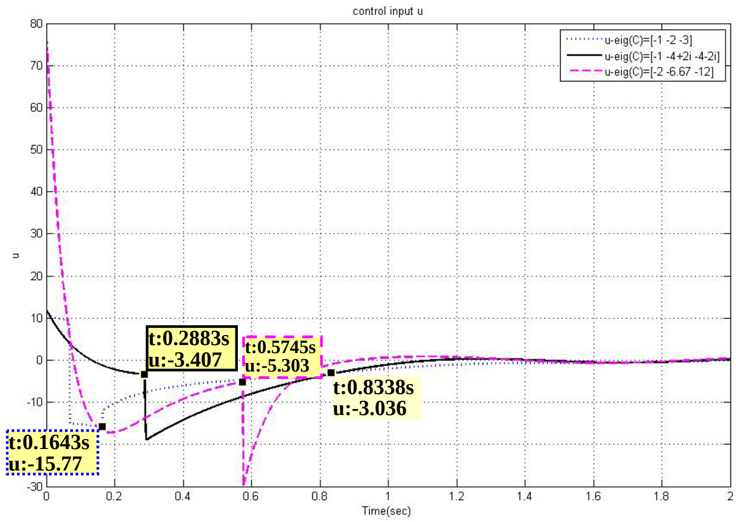

Figure 4.47 The control input $\boldsymbol{u}(t)$

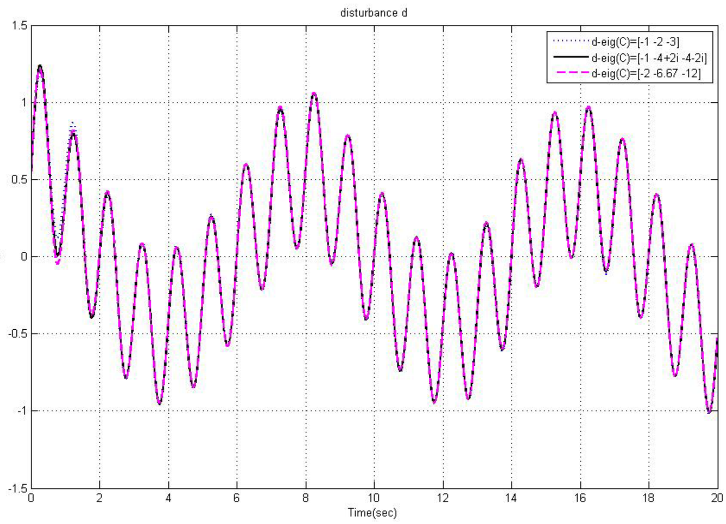

Figure 4.48 The disturbance $\boldsymbol{d}(t)$

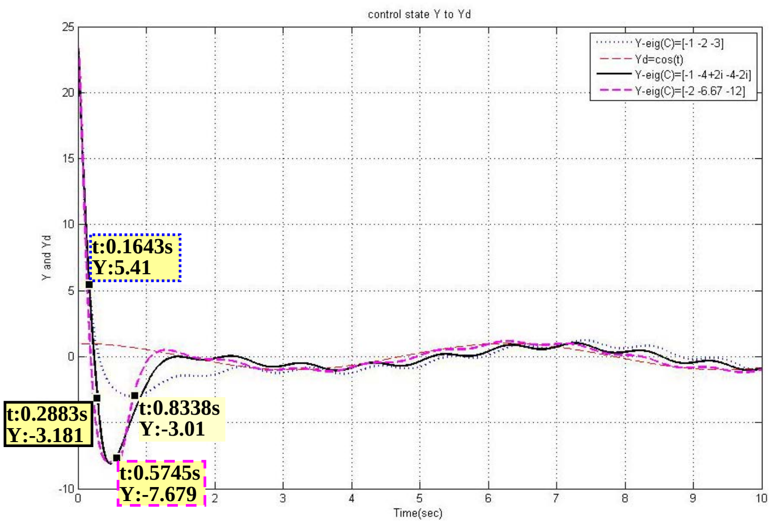

Figure 4.49 The tracking control output $\boldsymbol{y}(t)$ to $\boldsymbol{y}_d(t)$

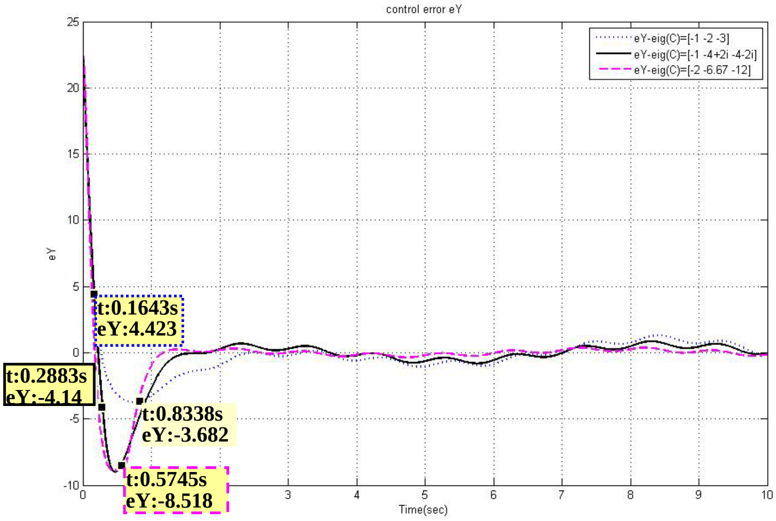

Figure 4.50 The tracking control output error

Chapter 5

Conclusions and future research

The reduced-order observer-based sliding mode controller design combines with reduced-order observer and sliding mode controller. The basic theorem related to the reduced order observer has been introduced in Chapter 2. The reduced-order observer is used to estimate unmeasurable state variables and with these estimated information the sliding mode controller for the LTI system is designed in Chapter 3. There are three cases discussed, including stability control of system with matched disturbance, stability control of system with mismatching disturbance, and output tracking control of disturbance system. Most importantly, all the cases can reach control goal and have good performance.

In Chapter 4, use MATLAB simulink to simulate six cases, which are matched disturbance system with dimension $p = q$, matched disturbance system with dimension $p > q$, mismatching disturbance system for controller stability, output tracking control for constant trajectory, and output tracking control to trace time-varying trajectory for mismatching disturbance. The reduced order observer was given to accurately estimate the unmeasurable state variables and not affected by disturbance [2]. The convergence rate of the estimation depends on the eigenvalues of the observer. The sliding mode controller can completely reject the matched disturbance in the sliding mode [3]. Consequently, the reduced-order observer-based sliding mode controller can successfully eliminate the matched disturbance and reach the control goal.

The problem concerning fast tracking trajectory still exists and the developed ROSMC is not able to trace such trajectory effectively. Besides, the ROSMC is also restricted to the condition of invariant zeros to the unknown inputs. In the future, it is needed to improve the proposed ROSMC to deal with such problems.

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