Chapter 2
Fundamentals of OFDM

2.1 OFDM Basics [9]

The basic principle of OFDM is to divide the high-rate data stream into many low rate streams that each is transmitted simultaneously over its own subcarrier orthogonal to all the others. Due to narrowband property, they experience mostly flat fading, which makes channel equalization very simple. In order to eliminate intersymbol interference (ISI) and intercarrier interference (ICI) as much as possible, it is a good idea to add a trailing portion of each symbol to the head of itself, which is called cyclic prefix extension as shown in Figure 2.1. The guard interval is chosen larger than the expected delay spread, such that multipath components from one symbol will not interfere with its succeeding symbol. After a signal passes through the time-dispersive channel, orthogonality of its subcarrier components can be maintained by the introduction of cyclic prefix.

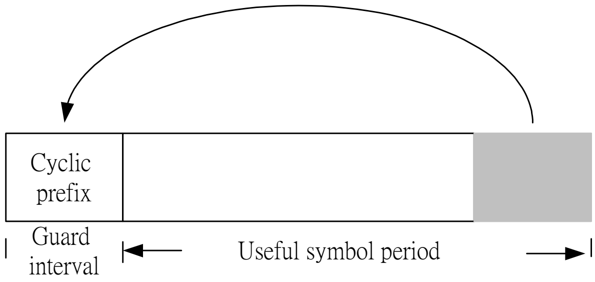

Figure 2.1 Guard interval and cyclic prefix structure of an OFDM symbol

2.1.1 Continuous-time OFDM Model

The continuous-time OFDM model presented below is an ideal OFDM system, which in practice is digitally synthesized. The baseband model is shown in Figure 2.2.

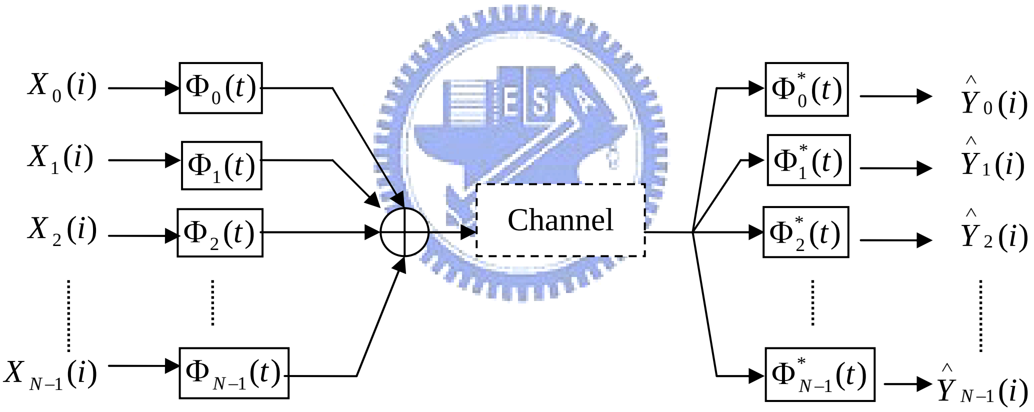

Figure 2.2 Continuous-time OFDM system model

In Figure 2.2, data $X_n(i)$ is modulated by QPSK or QAM and then transmitted by using a set of sinusoidal oscillators. Here, we assume that the OFDM system has N subcarriers and total symbol duration of T_s seconds, of which T_g seconds is the length of the cyclic prefix, and T_u is the useful symbol duration. The transmitter assumes the following waveform at the k-th subcarrier

$$\Phi_k(t) = \begin{cases} e^{\frac{j2\pi(k-1)(t-T_g)}{T}} & ,T_g \le t \le T_s \\ 0 & ,otherwise \end{cases} \tag{2.1}$$

Then after summation of these subcarriers, the baseband signal of the i-th OFDM symbol is

$$S_i(t) = \sum_{k=1}^{N} X_k(i)\Phi_k(t-T_s) \tag{2.2}$$

When an infinite sequence of OFDM symbols is transmitted, the output of the transmitter can be represented as

$$S(t) = \sum_{i=-\infty}^{\infty} S_i(t) = \sum_{i=-\infty}^{\infty}\sum_{k=1}^{N} d_k(i)\Phi_k(t-iT_s) \tag{2.3}$$

The spectrum of each subchannel will be as shown in Figure 2.3. The center subcarrier frequencies of subchannel spectrums are not overlapping with each other, which is the key feature of the OFDM symbol technique.

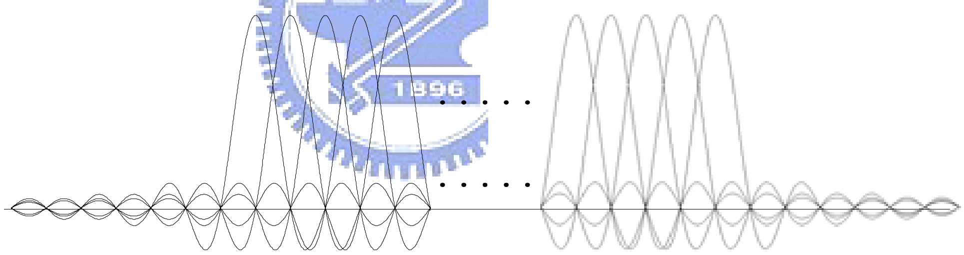

Figure 2.3 Spectrum of an OFDM symbol

2.1.2 Discrete-time OFDM Model

In order to analyze the performance of synchronization techniques easily, we need to understand discrete-time OFDM model as well. Compared with the continuous-time model expressed before, modulation and demodulation in a discrete-time OFDM system are performed by IDFT and DFT, respectively. In practice, DFT/IDFT can be implemented very efficiently by FFT/IFFT. A discrete-time OFDM system can be represented as:

$$\hat{Y}(i) = DFT(IDFT(X(i))) \tag{2.4}$$

As a result, we have the following discrete-time OFDM model as shown in Figure 2.4.

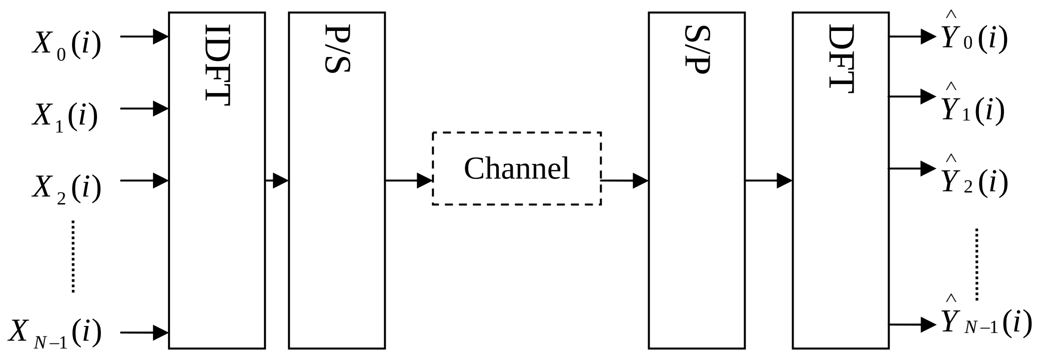

Figure 2.4 Discrete-time OFDM system model

2.2 Effect of the Synchronization Errors on OFDM [10]

One of the arguments against OFDM is that it is highly sensitive to synchronization errors. Synchronization of an OFDM signal requires finding the symbol timing, clock timing and the carrier frequency offset. Here, we will discuss the effects of the carrier frequency and the timing offset to an OFDM system.

2.2.1 Effect of the Carrier Frequency Offset

OFDM systems are very sensitive to carrier frequency offsets, because they can only tolerate offsets which are a fraction of the subcarrier spacing without a large degradation in system performance [9]. Frequency offsets are created by differences in oscillators in transmitter and receiver and Doppler shifts. If the frequency error is an integer multiple n of the subcarrier spacing Δf , then the received frequency-domain data are shifted by n subcarrier positions. Note that the subcarriers are still mutually orthogonal but the received data symbols, which are mapped to the OFDM spectrum are now in the wrong positions in the demodulated spectrum.

If the subcarrier frequency error is not an integer multiple of the subcarrier spacing, then we will sample those non-orthogonal frequency points after the FFT operations. Interference is then observed between the subcarriers, which deteriorates the system performance.

In order to investigate the effect of the frequency offset, we consider one baseband symbol from the transmitter given by [12]

$$s(t) = \frac{1}{N}\sum_{k=0}^{N-1} X_k \Phi_k(t) \tag{2.5}$$

$$\Phi_k = \begin{cases} e^{j2\pi f_k t}, & t \in [0, T_g + T_u) \\ 0, & otherwise \end{cases} \tag{2.6}$$

where X_k is a complex data for the k-th carrier; T_u is the useful symbol time, T_g is the guard interval time, and $f_k = k / T_u$,

We assumed the channel and the sampling clock are ideal, and the received signal is only affected by the carrier frequency offset Δf, as formulated by

$$r(t) = e^{j2\pi\Delta f t} s(t) \tag{2.7}$$

The sampled signal with the sampling period $T = T_u / N$ is

$$\begin{aligned} z_m &= e^{j2\pi m \Delta f T} \frac{1}{N}\sum_{k=0}^{N-1} X_k e^{j2\pi f_k m T} \\ &= \frac{1}{N}\sum_{k=0}^{N-1} X_k e^{j\frac{2\pi m (k+\Delta k)}{N}}; \qquad m = 0,1,\ldots..N-1 \end{aligned} \tag{2.8}$$

where $\Delta k = NT\Delta f$ is the ratio of the frequency offset to the carrier spacing. Thus, the received signal sample on the k-th subchannel is

$$
\begin{aligned}
\hat{X}_{k'} &= \sum_{m=0}^{N-1} z_m e^{-j\frac{2\pi mk'}{N}} \\
&= \sum_{m=0}^{N-1} \frac{1}{N} \sum_{k=0}^{N-1} X_k e^{j\frac{2\pi m(k+\Delta k)}{N}} e^{-j\frac{2\pi mk'}{N}} \\
&= X_{k'} \frac{\sin(\pi\Delta k)}{N\sin(\pi\Delta k/N)} e^{j\pi\Delta k(\frac{N-1}{N})} + I_{k'}
\end{aligned}
\tag{2.9}
$$

The first term shows the received $X_{k'}$ amplitude degradation and phase shift due to the frequency offset. The second term $I_{k'}$ is the ICI caused by the frequency offset.

In [11], if degradation D in SNR caused by a frequency offset that is small relative to the subcarrier spacing, then it can be approximated as

$$
D \cong \frac{10}{3\ln 10} (\pi\Delta f T_u)^2 \frac{E_s}{N_0} \tag{2.10}
$$

where E_s is the average received energy, and $N_0/2$ is the power spectral density of the AWGN.

2.2.2 Effect of the Timing Offset

Finding the symbol timing for an OFDM symbol is equivalent to finding the symbol start. There usually allows some tolerance for symbol timing errors when there is a cyclic prefix in an OFDM symbol. If the receiver's FFT window is shifted in the received sampling stream by m samples, then there will introduce a phase error of $2\pi m/N$, where N is the FFT length. However, these phase errors can be mitigated by channel estimation. If the timing errors are so high that the FFT window covers samples from two consecutive OFDM symbols, both ISI and ICI will occur and will severely affect the system's performance.