國 立 交 通 大 學

電子工程學系 電子研究所碩士班

碩 士 論 文

於非同調解碼轉接中繼模式下空頻編碼系統之最大多重分集分析

Maximum Achievable Diversity of Noncoherent Space-Frequency Coded Systems with Decode-and-Forward Relays

研 究 生：邱頌恩
指導教授：簡鳳村 博士

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研 究 生：邱頌恩　　Student: Sung-En Chiu

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摘　　要

本論文旨在研究在無線合作中繼網路下分散式非同調空頻碼之最佳編解碼準則、編解碼設計與效能分析。主要想探討的課題爲在採用正交分頻多工(OFDM)調變技術的情形下，在中繼站上用解碼-轉交(Decode-and-Forward, DF)模式並於接收站使用非同調解碼方式(Non-coherent Decoding)之分散式系統模型之建立、最大概似度解碼準則、以及其可達之最大多重分集分析。其中特別處理了在 DF 中繼節點解碼錯誤對最大多重分集所造成的影響，即在中繼節點上用了錯誤審查技術以防止錯誤傳遞。我們發現到在理想的審查技術假設之下，此系統可達之多重分集爲通道內所有獨立路徑個數。本篇更進一步的分析了在不完美的審查技術之下，其不準確所造成系統多重分集之影響，並發現在中繼端所使用的審查技術對於最大多重分集是十分要緊的。

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Student：Sung-En Chiu Advisors： Dr. Feng-Tsun Chien

Department of Electronics Engineering & Institute of Electronics
National Chiao Tung University

ABSTRACT

In this thesis, we conduct the analysis of noncoherent cooperative space-frequency coded (SFC) systems operating under the decode-and-forward (DAF) protocol in a two-hop relaying network, where neither the transmitter nor the receiver knows the channel. We assume practically that each of the intermediate relay nodes may fail to decode the message from the source. Each relay use an error detection method to determine whether or not it has reliably decoded the message, and only those relays who think they decode successfully will forward the message to the destination. We investigate the system under both perfect and imperfect error detection. Under perfect error detection, we develop the maximum likelihood (ML) decoding rule, derive the average pairwise error probability (PEP) and establish the code design criteria for achieving full diversity. We conclude that the diversity gain of the non-coherent cooperative SFC under perfect relay error detection is on the average equal to the product of the total number of relays and the channel order in the relay-destination link. Furthermore, we investigate the impact of imperfect relay error detection and find the significance of error detection on relay nodes.

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邱頌恩

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Chapter 1

Introduction

1.1 Motivation and Related Work

It has been shown that the multiple-input-multiple-output (MIMO) systems employing multiple transmit and receive antennas can offer a significant increase in capacity and mitigate the detrimental effects of the channel fading in wireless communication systems. However, in the uplink of a cellular system, the size of mobile handsets makes it impractical to be quipped with geographically separated multiple antennas for ensuring independent fading on the transmit side. Addressing this problem, the strategy built upon the relay channel model, where the source broadcasts a message to several intermediate relays and subsequently these relay nodes forward the message they received to the destination, is considered as one of the promising methods to exploit spatial diversity using a collection of distributed antennas from different users in the network. This form of diversity is referred to as the cooperative diversity [6, 13], in the sense that the relay nodes cooperating with the source node creates a virtual antenna array, or virtual MIMO system, to facilitate the ultimate transmission between the source and the destination.

In conventional MIMO systems, the design of space-time codes have been well investigated and shown to be a very efficient approach to invoking the diversity. An important attribute of the space-time codes is that, for achieving full diversity, the transmitter actually needs to conducts encoding across antennas. That means we need to transmit quite different signals over independent channels. However, for a virtual MIMO system created by relay nodes, it is more difficult to have the relay cooperate. For a practical concern, it is usually assumed that the relay nodes cannot have instantaneous message exchanges between each other. Still, they may cooperate to a level in a way that we pre-assign different coding rule to each relay by a controlling center so that they can form a distributed space-time code cooperatively as suggested in [5, 12]. Such considerations let us exploit the cooperative diversity easier at the cost of lower scalability. We will follow that spirit of partial cooperation in this thesis. On the other hand, there is another interesting mechanism using randomized space-time codes [9]. Randomization allows all relay to encode with a common randomization rule but transmit different and independent signals. Hence, it has good scalability with other problems such as instantaneous power control on each individual relay. A more general consideration on the randomized space-time codes can also be found in [2].

For frequency non-selective flat fading channels, distributed space-time codes have been proposed to effectively exploit the spatial and temporal diversity offered by the virtual antenna array in the cooperative relaying network [5, 7]. On the other hand, when in the frequency-selective channel environment, the presence of multipath channel fading offers another dimension of diversity, *i.e.* the frequency diversity, that the system can further exploit. Combined with the technique of orthogonal frequency-division multiplexing (OFDM) modulation, the design of distributed space-frequency codes with coherent decoding is considered in [8], where the authors employ the decode-and-forward (DAF) protocol and assume that all relays decode correctly. A more realistic scheme which takes into account the condition that all the relays do not always decode reliably is proposed and analyzed in [12], where the authors assume that each relay knows whether or not it has decoded reliably, *i.e.*, an assumption of perfect censoring in each relay. This assumption is reasonable since the censoring method, such as using the cyclic redundancy check, can be quite accurate. However, for the purpose of diversity achieving, we need the error rate of the system vanishing rapidly as the signal-to-noise ratio (SNR) goes to infinity. The main focus is on the high SNR regime and arbitrarily small error rate. In such case, a fixed error probability of censoring could be a decisive factor and should be treated cautiously. To the best of our knowledge, only on work [14] considers the practical scenario that some relays may fail in censoring and forward incorrect signals to the destination. Specifically, in [14], the approach to mitigating the effect of error propagation needs the relay nodes know the instantaneous channel gains.

In the aforementioned work, perfect channel state information (CSI) is assumed to enable coherent detection at the end receiver [2,5,7–9,12,14]. However, performing channel estimation can be costly and very challenging in multiple-hop wireless links and/or in fast-fading environments. Therefore, noncoherent communications not requiring the CSI is of particular interests. An early work [3] of noncoherent communications on MIMO space-time systems suggests the use of unitary modulation, or the unitary space-time code. Such unitary constellation is then generally used in noncoherent MIMO systems. The existence and construction of the unitary space-time codes have been investigated in [15], which presents a very elegant geometric thought for the maximum-likelihood (ML) decoding with unitary modulation and a good interpretation of the diversity as the dimension of the column space of the codeword matrix. On the other hand, the analysis and design for non-coherent space-frequency coded MIMO systems has also been considered in [1], where the authors prove that the maximum achievable diversity gain is given by the product of the number of transmit antennas, the number of receive antennas, and the channel order, which is the same as the diversity gain that can be provided in coherent communications. For cooperative networks, noncoherent communications have also been studied over frequency flat fading channel. For example, noncoherent decoding in amplify-and-forward (AF) relaying scheme is explored in [17].

1.2 Contribution

In this thesis, we focus on the analysis of noncoherent space-frequency coded cooperative OFDM-based relaying systems, which consists of a source node, a destination node and multiple relaying nodes. The channel between each node pair is assumed to be frequency-selective. The DAF protocol at the relays is considered and CSI is unknown to all nodes in the network. Perfect timing synchronization is assumed in this work. In practice, perfect timing synchronization is difficult to achieve. However, with OFDM signaling, timing mismatches can be mitigated by adding appropriate cyclic prefix. An interesting work addressing the issue of timing asynchronism in cooperative relaying networks can be found in [11]. We further assume that there is no direct link from the source node to the destination node, and that each of the relay nodes may fail to decode the message from the source. The relay will first employ a censoring method to determine whether or not it receives informative messages. Only those relays who pass the censoring will they decode and forward the messages to the destination. We investigate the system under both perfect and imperfect relay censoring. For perfect censoring, we assume that each relay can know if it has decoded reliably and thus perfectly prevent error propagations. Such assumption has been made in several studies with DAF protocol [9, 12]. The case of perfect censoring not only yields neat analytic results but also provides insight to the maximum achievable diversity with noncoherent cooperative space-frequency system. For a more realistic scenario, on the other hand, we also investigate the impact of imperfect censoring on the diversity order. That is, we deal with the case when some relays may decode incorrectly but still transmit to destination.

With the assumption of perfect censoring, we first consider the case that the receiver does not have instantaneous CSI, but knows the long-term channel statistics and the instantaneous decoding status of the relays for obtaining the ML decoding rule. Knowing the relay decoding status requires signaling overhead sent by the relays, but yields a simpler decoding rule and thus simplify the analysis. We refer to this case as a partial knowledge receiver or the ML decoder. In condition to that, we also consider a completely noncoherent receiver which needs neither long-term CSI nor the decoding status of the relays. We refer to it as the suboptimum decoder or simply the correlator decoder since it exploit the correlation structure of the codewords. We analyze the PEP under both receiver. The code design criteria is provided based on the derived pairwise error probability (PEP) in high SNR regimes. We conclude that under perfect error detection, the proposed non-coherent cooperative SFC can achieve a diversity of order RL for both the ML decoder and correlator-like decoder, where R is the total number of cooperating relays, regardless whether they can decode correctly or not, and L is the channel order between the relay and destination pair. On the other hand, for the case of imperfect error detection, we find that its impact on maximum diversity is significant. And in such case, there is a large gap on error rate between ML decoder and correlator-like decoder. Simulation results also justify the correctness of our analysis. This demonstrates that the non-coherent cooperative virtual MIMO networks can potentially offer as good performance as that in the conventional MIMO networks in terms of diversity order, while the relay error detection and the error propagation effect should be concerned and controlled carefully.

Notation: Boldface capital letter for matrices, boldface lowercase letter for vectors. $(.)^T$ and $(.)^H$ denote transpose and hermitian respectively. The notation $f(\mathcal{P}) \doteq g(\mathcal{P})$ denote that f and g has same exponent, i.e.,

$$\lim_{\mathcal{P}\to\infty} \frac{\log f(\mathcal{P})}{\log \mathcal{P}} = \lim_{\mathcal{P}\to\infty} \frac{\log g(\mathcal{P})}{\log \mathcal{P}},$$

which may be referred to as diversity equivalence. A bound $f(\mathcal{P}) < g(\mathcal{P})$ is said to be a diversity preserving bound if $f(\mathcal{P}) \doteq g(\mathcal{P})$.

Chapter 2

Relay Transmission Model

Consider a two-hop wireless relay network consisting of a source node, a destination node, and R relay nodes as shown in Fig. 2.1. Each of the $R+2$ nodes has a single antenna. We assume the transmission is accomplished by a two-phase cooperative communication strategy with the decode-and-forward protocol. In Phase I, the source node broadcasts the information message to the R relays, which cooperate for the transmission from the the source to the destination. In Phase II, each relay decodes the message and uses an error detection such as CRC or SNR method to decide whether itself participates in the second phase transmission as suggest in [10]. Then, there are four situations that would happen on each relay.

1. Useful relay

   the relay decodes correctly and decide to participate

2. Useless relay

   the relay decodes correctly but decide not to participate

3. Controlled relay

   the relay decodes incorrectly and decide not to participate

4. Harmful relay

   the relay decodes incorrectly but decide to participate

The probabilities of these four events could be determined by the decoding error rate and the accuracy of error detection on each relay. We denote the error rate on each relay by p_s. And the accuracy of error detection on each relay consist of two types of error. The probability of type I and type II error is denoted by

$$\text{P}(\text{not to participate} \mid \text{decode correctly}) = p_{1|0},$$
$$\text{P}(\text{participate} \mid \text{decode incorrectly}) = p_{0|1},$$

where we think decoding error as hypothesis 1. We assume a symmetric case that all relays have the same p_s, $p_{1|0}$, and $p_{0|1}$. The scale of p_s could be decided by the coding scheme and power used at the source node. While $p_{1|0}$ and $p_{0|1}$ depend on the CRC or the SNR method at each relay (Note that CRC can allow us to do the error detection as good as we require by sacrificing the data rate, while SNR method won't affect the date rate but has limited detection performance). Using these three, we can obtain the previous four cases' probabilities. For example,

$$\text{P}(\text{Useful relay}) = (1 - p_s)(1 - p_{1|0}).$$

If the relay participates, then we called it an active node(case 1,4). Otherwise it would be a silent node(case 2,3). Since source to each relay could be viewed a SISO system whose behavior has been well investigated, we can simply use above model with the parameters p_s, $p_{1|0}$, and $p_{0|1}$ to characterize Phase I transmission which allow us abbreviating the channel/noise modeling. Thus, in the following analysis, we will focus on the non-coherent space-frequency codes applied in Phase II of the communication.

The system is based on OFDM modulation with N subcarriers. Under perfect synchronization, we assume that the baseband frequency-selective fading channel between rth relay and destination has L independent delay paths, written as $h_r(l), l = 0, 2, ..., L$–1. The channel delay path gain $h_r(l)$ is modeled as independent complex Gaussian random variable (independent cross both r and l) with zero mean and covariance $\sigma_{r,l}^2$. We assume a normalized power on each channel between relay to destination with $\sum_{l=0}^{L-1} \sigma_{r,l}^2 = 1$ for all $r = 1, ..., R$.

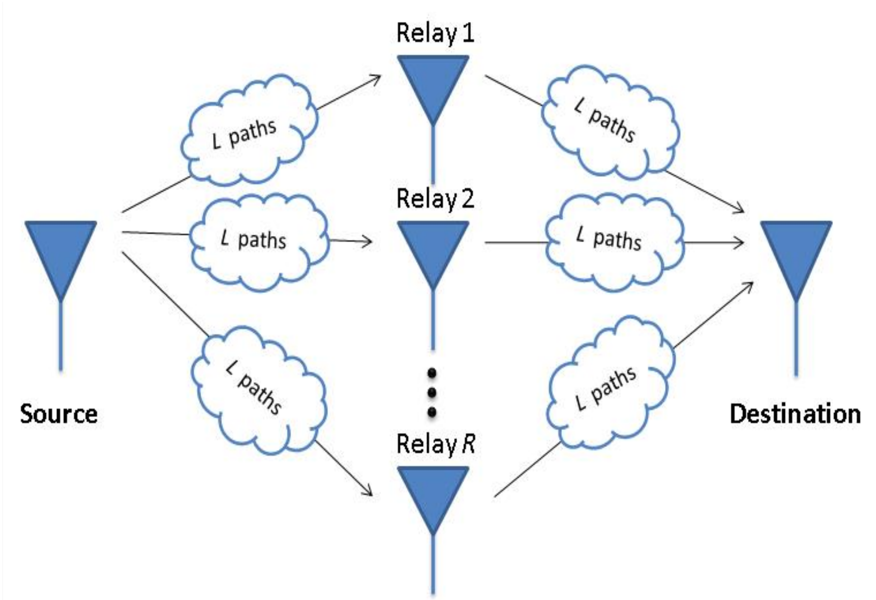

Figure 2.1: The two-hop wireless relay system with R relays.

In Phase II transmission, an active node will re-encode its decoded message using an mapping. Specifically, let the codebook used in phase I be $\mathfrak{S} = \{\mathbf{s}_0 \ \mathbf{s}_1 \ \cdots \ \mathbf{s}_{K-1}\}$, containing K codewords with each an $N \times 1$ OFDM symbol vector. Suppose that the rth relay decode the message as $\hat{\mathbf{s}} = \mathbf{s}_k$, then it will re-encode the message by $\mathbf{s}_k \mapsto \mathbf{c}_k^r$ and transmit the new $N \times 1$ coded OFDM symbol vector $\mathbf{c}_k^r$ to the destination node. The subscript r indicates that different relay would re-encode same message differently. After the channel from relay nodes to destination, the receiver end will then receive the sum of signals from all active relays. Addition by the additive noise, the received signal at destination node after IDFT could be expressed as

$$\mathbf{r}_d = \sqrt{\mathcal{P}} \sum_{r:\ \text{active}} \mathbf{H}_r \mathbf{c}^r + \mathbf{n}, \tag{2.1}$$

where $\mathcal{P}$ is each relay's power scaling factor (we assume a uniform power over all relays) relative to the normalized complex Gaussian noise vector $\mathbf{n} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_N)$, $\mathbf{c}^r$ is the transmitted codeword from rth relay, and $\mathbf{H}_r$ is a diagonal matrix with the diagonal term $(\mathbf{H}_r)_{nn}$ representing the frequency channel gain from rth relay to destination on nth sub-carrier. This frequency channel gain matrix $\mathbf{H}_r$ and the channel delay path gain is related by

$$\mathbf{H}_r = \sum_{l=0}^{L} h_r(l)\mathbf{D}^l, \tag{2.2}$$

where

$$\mathbf{D} = diag_{k=0}^{N-1}\{e^{-2\pi\frac{k}{N}}\}.$$

Substitute (2.2) into (2.1) and reorder the summation, we have

$$\mathbf{r}_d = \sqrt{\mathcal{P}}\sum_{l=0}^{L}\mathbf{D}^l \sum_{r:\ \text{active}} h_r(l)\mathbf{c}^r + \mathbf{n}, \tag{2.3}$$

Now suppose that source transmitted the message $\mathbf{s}_i$. We categorize the status of relay by using relay status matrices $\mathbf{S}_m = diag_{r=1}^{R}\{S_{m,r}\}$, with $S_{m,r}$ signifying the state of the rth relay w.r.t mth codeword. More specifically,

$$S_{m,r} = \begin{cases} 1, & \text{if } r\text{th relay decoded as } \hat{\mathbf{s}} = \mathbf{s}_m \text{ and} \\ & \quad \text{participated in phase II} \\ 0, & \text{o.w.} \end{cases}$$

Using this convention, together with rewriting the r-summation into matrix multiplication, we have

$$\mathbf{r}_d = \sqrt{\mathcal{P}}\sum_{l=0}^{L}\mathbf{D}^l\big(\mathbf{C}_i\mathbf{S}_i\mathbf{h}(l) + \sum_{m\neq i}^{K}\mathbf{C}_m\mathbf{S}_m\mathbf{h}(l)\big) + \mathbf{n},$$

where $\mathbf{C}_i = \begin{pmatrix}\mathbf{c}_i^1 & \mathbf{c}_i^2 & \cdots & \mathbf{c}_i^R\end{pmatrix}$ is the codeword matrix constructed when all relays decode the message to be $\mathbf{s}_i$, and $\mathbf{h}(l) = [h_1(l)\ h_2(l)\ \cdots\ h_R(l)]^T$ is the channel vector of lth delay path. Note that the status matrix $\mathbf{S}_i$ nulls out those relays which didn't participate or didn't decode the message to be $\mathbf{s}_i$.

Next, we further rewrite the l-summation in to matrix multiplication, arriving at

$$\mathbf{r}_d = \sqrt{\mathcal{P}}\mathbf{E}_i\hat{\boldsymbol{S}}_i\mathbf{h} + \sqrt{\mathcal{P}}\sum_{j\neq i}^{K}\mathbf{E}_j\hat{\boldsymbol{S}}_j\mathbf{h} + \mathbf{n}. \tag{2.4}$$

where $\mathbf{E}_i = \begin{pmatrix} \mathbf{C}_i & \mathbf{D}\mathbf{C}_i & \cdots & \mathbf{D}^{L-1}\mathbf{C}_i \end{pmatrix}$ is the pseudocodeword matrix of message i, as named in [1]. $\hat{\boldsymbol{S}}_i = \mathbf{I}_L \otimes \mathbf{S_i}$ is the stacked relay status matrix with $\otimes$ being the Kronecker product operation, and $\mathbf{h} = [\mathbf{h}^T(0)\ \mathbf{h}^T(1)\ \cdots\ \mathbf{h}^T(L-1)]^T$ is the $RL \times 1$ stacked channel vector.

As we can see, the harmful relays, which causing an error propagation and resulting to the second term in (2.4), will play the role of interference in the system.

Chapter 3

Error Probability and Diversity Analysis

3.1 Performance Analysis under Perfect Relay Error Detection

In this section, we analyze the maximum achievable diversity of the system under perfect relay error detection. That is, under the assumption that each relay knows perfectly whether or not it has decoded correctly, as the same assumption suggested in [9, 12]. This corresponds to $p_{1|0} = 0$ and $p_{0|1} = 0$, which avoid any harmful relay and useless relay perfectly. Though unrealistic, such assumption could simplify the analysis largely and let us catch a glimpse of the system behavior. By the assumption, we have $\hat{\boldsymbol{S}}_m = \mathbf{0}\ \forall\, m \neq i$, for which transmitted message from source is $\mathbf{s}_i$. Then, we can simplify equation (2.4) as

$$\mathbf{r}_d = \sqrt{\mathcal{P}}\mathbf{E}_i\hat{\boldsymbol{S}}\mathbf{h} + \mathbf{n},$$

where we abbreviated $\hat{\boldsymbol{S}}_i = \hat{\boldsymbol{S}}$ without ambiguity here.

To analyze the system, we first considered in Sec. 3.1.1 the case that destination has the knowledge about the channel statistic and the relay decoding status. Such knowledge might be acquired by having all relays send additional information to destination in the signaling overhead. To overcome this overhead, in Sec. 3.1.2, we then investigate the ability of a completely noncoherent receiver, *i.e.*, the destination decodes with neither channel statistic nor relay decoding status.

3.1.1 Some Knowledge Known at Receiver

Maximum-Likelihood Decoding

In order to find the maximum achievable diversity, we first need to find out the minimum achievable error probability, which resorting to maximum-likelihood decoding. With channel statistic and relay decoding status, the likelihood function $p(\mathbf{r}_d|\mathbf{C}_i, \hat{\boldsymbol{S}})$ is simply a multivariate complex gaussian with mean zero. The covariance matrix of $\mathbf{r}_d$ could be calculated as

$$\boldsymbol{\Lambda}(\mathbf{C}_i|\hat{\boldsymbol{S}}) = \mathbf{I}_N + \mathcal{P}\mathbf{E}_i\hat{\boldsymbol{S}}\boldsymbol{\Sigma}^2\mathbf{E}_i^H, \tag{3.1}$$

where $\boldsymbol{\Sigma}^2 = E[\mathbf{h}\mathbf{h}^H] = diag_{l=0}^{L-1}\{diag_{r=1}^{R}\{\sigma_{r,l}^2\}\}$ is the covariance matrix of the stacked channel vector $\mathbf{h}$. Note that we have exploited the fact that $\hat{\boldsymbol{S}}$ is idempotent, *i.e.* $\hat{\boldsymbol{S}}^{\mathbf{2}} = \hat{\boldsymbol{S}}$, and that $\boldsymbol{\Sigma}^2$ is diagonal in representing (3.1). The conditional density of the received signal is then given by

$$p(\mathbf{r}_d|\mathbf{C}_i, \hat{\boldsymbol{S}}) = \frac{\exp\left(-\mathbf{r}_d^H\boldsymbol{\Lambda}^{-1}(\mathbf{C}_i|\hat{\boldsymbol{S}})\mathbf{r}_d\right)}{\pi^N\left(\det\ \boldsymbol{\Lambda}(\mathbf{C}_r|\hat{\boldsymbol{S}})\right)}.$$

Consequently, we have the ML decoding rule

$$\hat{\boldsymbol{C}}_{\text{ML}} = \arg\min_{\mathbf{C}_i\in\mathcal{C}}\left(\mathbf{r}_d^H\boldsymbol{\Lambda}_{i,\hat{S}}^{-1}\mathbf{r}_d + \ln\det\boldsymbol{\Lambda}_{i,\hat{S}}\right),$$

where we let $\boldsymbol{\Lambda}(\mathbf{C}_i|\hat{\boldsymbol{S}}) = \boldsymbol{\Lambda}_{i,\hat{S}}$ for notational convenience.

In this work, we restrict ourselves to the case of unitary codebook, *i.e.* $\mathbf{E}_i^H\mathbf{E}_i = \mathbf{I}_{RL}$, for $i = 0, \ldots, K-1$, which also allows the analysis more tractable. This unitary constellation originated from [3] and is commonly used in noncoherent system. The ML decoding rule is therefore simplified to

$$\hat{\boldsymbol{C}}_{\text{ML}} = \arg\max_{\mathbf{C}_i\in\mathcal{C}}\left(\mathbf{r}_d^H\mathbf{E}_i\hat{\boldsymbol{S}}\boldsymbol{\Sigma}^2\left(\mathbf{I}_{RL} + \mathcal{P}\hat{\boldsymbol{S}}\boldsymbol{\Sigma}^2\right)^{-1}\mathbf{E}_i^H\mathbf{r}_d\right) \tag{3.2}$$

by applying the matrix inversion lemma to $\mathbf{\Lambda}_{i,\hat{S}}^{-1}$. It is worthwhile to emphasize that the proposed ML decoding rule indeed requires the destination node knowing the matrix $\hat{\boldsymbol{S}}$, *i.e.* the relay decoding status.

Conditional Pairwise Error Probability

According to this ML decoding rule, we derive the pairwise error probability (PEP) of deciding in favor of $\mathbf{C}_j$ at the receiver while $\mathbf{C}_i$ is the true transmitted codeword. Conditioned on the relay status matrix $\hat{\boldsymbol{S}}$, we have

$$\mathrm{P}(\mathbf{C}_i \to \mathbf{C}_j | \mathbf{C}_i, \hat{\boldsymbol{S}}) = \mathrm{P}(v > 0 | \mathbf{C}_i, \hat{\boldsymbol{S}}) \tag{3.3}$$

where $v = \mathbf{r}_d^H(\mathbf{E}_j \hat{\boldsymbol{S}} \mathbf{\Sigma}^2 \mathbf{T}^{-1} \mathbf{E}_j^H - \mathbf{E}_i \hat{\boldsymbol{S}} \mathbf{\Sigma}^2 \mathbf{T}^{-1} \mathbf{E}_i^H)\mathbf{r}_d$ with $\mathbf{T} = \mathbf{I}_{RL} + \mathcal{P}\hat{\boldsymbol{S}}\mathbf{\Sigma}^2$. It follows by the Chernoff bound that

$$\mathrm{P}(v \geq 0 | \mathbf{C}_i, \hat{\boldsymbol{S}}) leq E[e^{sv} | \mathbf{C}_i, \hat{\boldsymbol{S}}] \stackrel{\text{def}}{=} \phi(s), \forall s > 0 \tag{3.4}$$

where $\phi(s)$ is the moment generating function (MGF) of v.

Following the algebraic approach in [1], the MGF $\phi(s)$ is given by

$$\begin{aligned} \phi(s) = \det{}^{-1}(\mathbf{I}_{RL} + s\hat{\boldsymbol{S}}\mathbf{\Sigma}^2) \cdot \det{}^{-1}\Big(\mathbf{I}_{RL} - s\hat{\boldsymbol{S}}\mathbf{\Sigma}^2\mathbf{T}^{-1} - \\ s(\mathcal{P} - s)\hat{\boldsymbol{S}}\mathbf{\Sigma}^2\mathbf{T}^{-1}\mathbf{E}_j^H\mathbf{E}_i\hat{\boldsymbol{S}}\mathbf{\Sigma}^2\big(\mathbf{I}_{RL} + s\hat{\boldsymbol{S}}\mathbf{\Sigma}^2\big)^{-1}\mathbf{E}_i^H\mathbf{E}_j\Big). \end{aligned} \tag{3.5}$$

We next establish a further upper bound on the above Chernoff bound.

With some algebraic manipulations, the MGF $\phi(s)$ can be decomposed into

$$\phi(s) = \phi_1(s)\phi_2(s) \tag{3.6}$$

where

$$\begin{aligned} \phi_1(s) &= \det{}^{-1}(\mathbf{I}_{RL} + s\hat{\boldsymbol{S}}\mathbf{\Sigma}^2)\det(\mathbf{I}_{RL} + \mathcal{P}\hat{\boldsymbol{S}}\mathbf{\Sigma}^2) \\ \phi_2(s) &= \det{}^{-1}\big(\mathbf{I}_{RL} + (\mathcal{P} - s)\hat{\boldsymbol{S}}\mathbf{\Sigma}^2\Psi\big) \end{aligned}$$

with

$$\begin{aligned} \Psi &= \mathbf{I}_{RL} - s\mathbf{E}_j^H\mathbf{E}_i\hat{\boldsymbol{S}}\mathbf{\Sigma}^2(\mathbf{I}_{RL} + s\hat{\boldsymbol{S}}\mathbf{\Sigma}^2)^{-1}\mathbf{E}_i^H\mathbf{E}_j \\ &= \mathbf{I}_{RL} - \mathbf{E}_j^H\mathbf{E}_i\big(\mathbf{I}_{RL} - (\mathbf{I}_{RL} + s\hat{\boldsymbol{S}}\mathbf{\Sigma}^2)^{-1}\big)\mathbf{E}_i^H\mathbf{E}_j \\ &= \mathbf{I}_{RL} - \mathbf{E}_j^H\mathbf{E}_i\mathbf{E}_i^H\mathbf{E}_j + \mathbf{E}_j^H\mathbf{E}_i(\mathbf{I}_{RL} + s\hat{\boldsymbol{S}}\mathbf{\Sigma}^2)^{-1}\mathbf{E}_i^H\mathbf{E}_j. \end{aligned} \tag{3.7}$$

Then, to give bounds, we first assume that the s we used is a function of $\mathcal{P}$ and $s < \mathcal{P}$, furthermore, $s = \Theta(\mathcal{P})$, i.e., $\lim_{\mathcal{P}\to\infty} \frac{s}{\mathcal{P}} = c > 0$. We will show the validity of these assumptions when the minimizer s^* is found. Let $n_{\mathbf{S}} = \sum_{r=1}^{R} S_{i,r}$ be the number of active relays. We thus arrive at

$$\phi_1(s) < \left(\frac{\mathcal{P}}{s}\right)^{n_{\mathbf{S}} L}. \tag{3.8}$$

Next, we evaluate $\phi_2(s)$ in terms of the eigenvalues of $\hat{\boldsymbol{S}}\boldsymbol{\Sigma}^2\Psi$ as

$$\begin{aligned}\phi_2(s) &= \prod_{r=0}^{RL-1} \Big(1 + (\mathcal{P} - s)\lambda_r\big(\hat{\boldsymbol{S}}\boldsymbol{\Sigma}^2\Psi\big)\Big)^{-1} \\ &= \prod_{r=0}^{RL-1} \Big(1 + (\mathcal{P} - s)\lambda_r\big(\hat{\boldsymbol{S}}\boldsymbol{\Sigma}\Psi\boldsymbol{\Sigma}\hat{\boldsymbol{S}}\big)\Big)^{-1} \end{aligned} \tag{3.9}$$

where the fact that $\hat{\boldsymbol{S}}$ is idempotent is used in deriving (3.9). Substituting (3.7) into (3.9) gives

$$\phi_2(s) = \prod_{r=0}^{RL-1} \Big(1 + (\mathcal{P} - s)\lambda_r\big(\hat{\boldsymbol{S}}\boldsymbol{\Sigma}\mathbf{Q}\boldsymbol{\Sigma}\hat{\boldsymbol{S}} + \mathbf{P}\big)\Big)^{-1},$$

where

$$\begin{aligned}\mathbf{Q} &= \mathbf{I}_{RL} - \mathbf{E}_j^H\mathbf{E}_i\mathbf{E}_i^H\mathbf{E}_j \\ \mathbf{P} &= \hat{\boldsymbol{S}}\boldsymbol{\Sigma}\mathbf{E}_j^H\mathbf{E}_i(\mathbf{I}_{RL} + s\hat{\boldsymbol{S}}\boldsymbol{\Sigma}^2)^{-1}\mathbf{E}_i^H\mathbf{E}_j\boldsymbol{\Sigma}\hat{\boldsymbol{S}}. \end{aligned} \tag{3.10}$$

The case that all relays have decoded correctly, *i.e.* $\hat{\boldsymbol{S}} = \mathbf{I}_{RL}$, is identical to a MISO system, which has been elaborated in [1]. Thus, we shall preconceive that there exists at least one inactive relay node, which results in $\lambda_{min}(\mathbf{P}) = 0$. Further, observe that both $\hat{\boldsymbol{S}}\boldsymbol{\Sigma}\mathbf{Q}\boldsymbol{\Sigma}\hat{\boldsymbol{S}}$ and $\mathbf{P}$ are Hermitian matrices. Hence, by assumption that $s < \mathcal{P}$, applying Weyl's inequality [4], theorem 4.3.1, to bound the eigenvalues in (3.10), yielding

$$\phi_2(s) \le \prod_{r=0}^{RL-1} \big(1 + (\mathcal{P} - s)\lambda_r(\hat{\boldsymbol{S}}\boldsymbol{\Sigma}^2\mathbf{Q})\big)^{-1}. \tag{3.11}$$

It follows, by combining (3.8) and (3.11), that

$$\phi(s) < \left(\frac{\mathcal{P}}{s}\right)^{n_{\mathbf{S}} L} (\mathcal{P} - s)^{-d_{ij}} \prod_{r=0}^{d_{ij}-1} \lambda_r^{-1}(\hat{\boldsymbol{S}}\boldsymbol{\Sigma}^2\mathbf{Q}), \tag{3.12}$$

where $d_{ij} = \text{rank}(\hat{\boldsymbol{S}}\boldsymbol{\Sigma}^2\mathbf{Q})$ with the subscripts corresponding to the codewords $\mathbf{C}_i$ and $\mathbf{C}_i$ contained in $\mathbf{P}$ and $\mathbf{Q}$. Note that the two bounds in (3.8) and (3.11) could be shown to be diversity preserving by using the assumption that $s = \Theta(\mathcal{P})$. This means the bound (3.12) could reflect the exponent of $\mathcal{P}$, *i.e.*, the diversity order, correctly.

Performing the minimization over s in (3.12), we obtain $s^* = \mathcal{P}n_{\mathbf{S}}L/(d_{i,j} + n_{\mathbf{S}}L) < \mathcal{P}$, which validates the initial assumptions on s. Specifically, $s^* = \mathcal{P}/2$ if $\mathbf{Q}$ is nonsingular (which guarantees $d_{ij} = n_{\mathbf{S}}L$). The bound then is simplified to

$$\mathrm{P}(\mathbf{C}_i \to \mathbf{C}_j | \mathbf{C}_i, \hat{\boldsymbol{S}}) < \left(\frac{\mathcal{P}}{4}\right)^{-n_{\mathbf{S}}L} \prod_{r=0}^{n_{\mathbf{S}}L-1} \lambda_r^{-1}(\hat{\boldsymbol{S}}\boldsymbol{\Sigma}^2\mathbf{Q}). \tag{3.13}$$

Therefore, restricting that $\mathbf{Q}$ is of full-rank will guarantee the decay rate of the conditional PEP as $\mathcal{P}^{-n_{\mathbf{S}}L}$ at high SNR.

Average Pairwise Error Probability

Based on the results established so far, we can derive the average PEP at the destination node by averaging over $\hat{\boldsymbol{S}}$. By the model in Sec. 2, the state of the rth relay node $S_{i,r}$ is a Bernoulli random variable with a probability mass function as

$$S_{i,r} = \begin{cases} 0, & \text{with probability } p_s \\ 1, & \text{with probability } 1 - p_s, \end{cases} \tag{3.14}$$

where the error rate p_s depends on the transmit power and the coding scheme at source node in phase I transmission. It can be viewed as error rate of a SISO system. Hence we have $p_s \leq \beta \times \mathcal{P}^{-L}$ achievable with a constant β at each relay node through adequate source node coding. Therefore, $n_{\mathbf{S}}$ follows the binomial distribution

$$\mathrm{P}(n_{\mathbf{S}} = k) = \binom{R}{k}(1 - p_s)^k p_s^{R-k}. \tag{3.15}$$

Thus, the destination average PEP is given by

$$\begin{aligned}
&\mathrm{P}(\mathbf{C}_i \to \mathbf{C}_j|\mathbf{C}_i) \\
&= \sum_{\hat{\boldsymbol{S}}} \mathrm{P}(\hat{\boldsymbol{S}})\mathrm{P}(\mathbf{C}_i \to \mathbf{C}_j|\mathbf{C}_i, \hat{\boldsymbol{S}}) \\
&= \sum_{k=0}^{R} \mathrm{P}(n_{\mathbf{S}} = k) \sum_{\hat{\boldsymbol{S}}:n_{\mathbf{S}}=k} \mathrm{P}(\mathbf{C}_i \to \mathbf{C}_j|\mathbf{C}_i, \hat{\boldsymbol{S}}) \\
&= \sum_{k=0}^{R} \binom{R}{k} (1-p_s)^k p_s^{R-k} \sum_{\hat{\boldsymbol{S}}:n_{\mathbf{S}}=k} \mathrm{P}(\mathbf{C}_i \to \mathbf{C}_j|\mathbf{C}_i, \hat{\boldsymbol{S}}) \\
&< \sum_{k=0}^{R} \binom{R}{k} \mathcal{P}^{-L(R-k)} \sum_{\hat{\boldsymbol{S}}:n_{\mathbf{S}}=k} \left(\frac{\mathcal{P}}{4}\right)^{-n_{\mathbf{S}}L} \prod_{r=0}^{n_{\mathbf{S}}L-1} \lambda_r^{-1}(\hat{\boldsymbol{S}}\boldsymbol{\Sigma}^2\mathbf{Q}) \\
&= \mathcal{P}^{-RL} \cdot \underbrace{\sum_{k=0}^{R} \binom{R}{k} \sum_{\hat{\boldsymbol{S}}:n_{\mathbf{S}}=k} 4^{n_{\mathbf{S}}L} \prod_{r=0}^{n_{\mathbf{S}}L-1} \lambda_r^{-1}(\hat{\boldsymbol{S}}\boldsymbol{\Sigma}^2\mathbf{Q})}_{\triangleq \eta_1},
\end{aligned} \tag{3.16}$$

where we simply upper bound $(1 - p_s)$ by 1. Summarizing, we have the unconditional PEP

$$\mathrm{P}(\mathbf{C}_i \to \mathbf{C}_j|\mathbf{C}_i) < \eta_1 \times \mathcal{P}^{-RL}, \tag{3.17}$$

with η_1 representing the component in (3.16) that is independent with $\mathcal{P}$. The bound used in equation (3.16) is obviously diversity preserving. Combining with the discussion about the bounds in previous section, we actually have

$$\mathrm{P}(\mathbf{C}_i \to \mathbf{C}_j|\mathbf{C}_i) \doteq \eta_1 \times \mathcal{P}^{-RL}, \tag{3.18}$$

i.e., the maximum achievable diversity under the assumption of perfect relay error detection is exactly RL when the receiver has the knowledge of long-term channel statistics and instantaneous decoding status of all relays.

3.1.2 Completely Noncoherent Decoder

Now we show that receiver can still retrieve full diversity even without the knowledge of channel statistics and relay decoding status.

Maximum-Likelihood Decoding

Without conditioning on $\hat{S}$, obtaining the likelihood function involves an average over all possible $\hat{S}$. That is, receiver needs to test all possible relay decoding state and take all of them into account. The likelihood function then becomes a gaussian mixture

$$\begin{aligned} p(\mathbf{r}_d|\mathbf{C}_i) &= \sum_{\hat{\boldsymbol{S}}\in 2^R} \mathrm{P}(\hat{\boldsymbol{S}}) p(\mathbf{r}_d|\mathbf{C}_i, \hat{\boldsymbol{S}}) \\ &= \sum_{\hat{\boldsymbol{S}}\in 2^R} p_s^{n_\mathbf{S}} (1-p_s)^{R-n_\mathbf{S}} \frac{\exp\left(-\mathbf{r}_d^H \boldsymbol{\Lambda}_{i,\hat{S}}^{-1} \mathbf{r}_d\right)}{\pi^N \left(\det \ \boldsymbol{\Lambda}_{i,\hat{S}}\right)}. \end{aligned}$$

Hence without knowledge of relay decoding status, the optimum ML decoding at receiver can be written as

$$\hat{\boldsymbol{C}}_{\mathrm{ML}} = \arg\max_{\mathbf{C}_i\in\mathcal{C}} \sum_{\hat{\boldsymbol{S}}\in 2^R} p_s^{n_\mathbf{S}} (1-p_s)^{R-n_\mathbf{S}} \frac{\exp\left(-\mathbf{r}_d^H \boldsymbol{\Lambda}_{i,\hat{S}}^{-1} \mathbf{r}_d\right)}{\pi^N \left(\det \ \boldsymbol{\Lambda}_{i,\hat{S}}\right)} \tag{3.19}$$

Due to the summation of exponential, this optimum decision rule is hard to be simplified. And the error probability analysis based on (3.19) directly will be mathematically intractable. Thus, we approximate (3.19) using the dominated term in the summand. This will result in a suboptimum decoder. However, for investigating the maximum achievable diversity, we approximate (3.19) tightly such that

$$\mathrm{P}_{\mathrm{subopt}}(\mathrm{error}) \leq \eta_2 \times \mathcal{P}^{-RL} \tag{3.20}$$

and then we can establish

$$\begin{aligned} \eta_1 \times \mathcal{P}^{-RL} &\doteq \mathrm{P}_{\mathrm{ML\_A}}(\mathrm{error}) \\ &\leq \mathrm{P}_{\mathrm{ML\_B}}(\mathrm{error}) \leq \mathrm{P}_{\mathrm{subopt}}(\mathrm{error}) \\ &\leq \eta_2 \times \mathcal{P}^{-RL}, \end{aligned} \tag{3.21}$$

where $\mathrm{P}_{\mathrm{ML\_A}}(\mathrm{error})$ and $\mathrm{P}_{\mathrm{ML\_A}}(\mathrm{error})$ represent the error probability under the ML decision rule in Sec. 3.1.1, equation (3.2) and Sec. 3.1.2, equation (3.19) respectively. The RHS in (3.21) indicate that diversity RL is achievable, and the LHS provides the converse, *i.e.*, the maximum diversity do not exceed RL. Hence, by (3.21), we have the maximum achievable diversity be still RL even without knowledge of relay decoding status for both optimum ML decoder and our suboptimum decoder.

In equation (3.21), the first diversity equivalence has been shown in Sec. 3.1.1. And in second line, the first inequality follows from that the decoder in section 3.1.1 has an additional information, the relay decoding status. Moreover, optimality of ML decoder gives the second inequality. Therefore, in the following, we only focus on describing suboptimum decoder and the corresponding error probability to obtain equation (3.20) for completing the proof.

Suboptimum Decoder

As mentioned, we use the dominated term in the summand of (3.19). Diversity is an SNR asymptotic quantity. Instead of having the receiver compute the largest term exactly, we simply select the one that $\hat{S} = \mathbf{I}_{RL}$ in the summand, which has the largest probability in high SNR, to approximate (3.19) for providing a maximum diversity achieving decoder. And then we have the following,

$$\begin{aligned}\hat{C}_{\text{subopt}*} &= \arg\max_{\mathbf{C}_i\in\mathcal{C}} \frac{\exp\left(-\mathbf{r}_d^H \boldsymbol{\Lambda}_{i,\hat{S}=\mathbf{I}_{RL}} \mathbf{r}_d\right)}{\pi^N\left(\det\ \boldsymbol{\Lambda}_{i,\hat{S}=\mathbf{I}_{RL}}\right)} \\ &= \arg\max_{\mathbf{C}_i\in\mathcal{C}} \left(\mathbf{r}_d^H \mathbf{E}_i \boldsymbol{\Sigma}^2 \left(\mathbf{I}_{RL} + \mathcal{P}\boldsymbol{\Sigma}^2\right)^{-1} \mathbf{E}_i^H \mathbf{r}_d\right).\end{aligned} \tag{3.22}$$

The decoder in (3.22) could do without the relay decoding status but it still need the channel statistic $\boldsymbol{\Sigma}^2$. To have a completely noncoherent decoder, we further simplified (3.22) by substituting $\boldsymbol{\Sigma}^2 = \mathbf{I}_{RL}$ and scaling with $1 + \mathcal{P}$, resulting to

$$\hat{C}_{\text{subopt}} = \arg\max_{\mathbf{C}_i\in\mathcal{C}} \left(\mathbf{r}_d^H \mathbf{E}_i \mathbf{E}_i^H \mathbf{r}_d\right), \tag{3.23}$$

which only exploits the correlation structure of codeword matrices to distinguish them. Next, we use the suboptimum decoder in (3.23), which needs neither the instantaneous CSI nor the long-term channel statistic nor the relay decoding status, to carry out equation (3.20).

Conditional Pairwise Error Probability

Similarly as in section 3.1.1, we analyze the PEP. The pairwise error event that the decoder (3.23) decides in favor of $\mathbf{C}_j$ than $\mathbf{C}_i$ while $\mathbf{C}_i$ is truly transmitted can be written as

$$\{\mathbf{r}_d^H \mathbf{E}_j \mathbf{E}_j^H \mathbf{r}_d - \mathbf{r}_d^H \mathbf{E}_i \mathbf{E}_i^H \mathbf{r}_d > 0\}. \tag{3.24}$$

Using Chernoff bound, the conditional pairwise error probability could be upper bounded by

$$\begin{aligned} \mathrm{P}(\mathbf{C}_i \to \mathbf{C}_j | \mathbf{C}_i, \hat{\boldsymbol{S}}) &= \mathrm{P}(w > 0 | \mathbf{C}_i, \hat{\boldsymbol{S}}) \\ &\leq E[e^{sw} | \mathbf{C}_i, \hat{\boldsymbol{S}}], \quad \forall s \geq 0 \end{aligned} \tag{3.25}$$

where $w = \mathbf{r}_d^H \mathbf{E}_j \mathbf{E}_j^H \mathbf{r}_d - \mathbf{r}_d^H \mathbf{E}_i \mathbf{E}_i^H \mathbf{r}_d$. Unlike in section 3.1.1, applying the algebraic method in [1] here to establish a series of diversity preserving upper bounds would be too involved to manipulate. However, we didn't need to certify the tightness on each individual bound. We could use any upper bound to bound equation (3.25) as long as we could establish (3.20) in the end, which would guarantee the diversity preserved on every bound we had used automatically, as mentioned in (3.21). We start with expanding w by using $\mathbf{r}_d = \sqrt{\mathcal{P}} \mathbf{E}_i \hat{\boldsymbol{S}} \mathbf{h} + \mathbf{n}_d$

$$\begin{aligned} w &= \mathbf{r}_d^H \mathbf{E}_j \mathbf{E}_j^H \mathbf{r}_d - \mathbf{r}_d^H \mathbf{E}_i \mathbf{E}_i^H \mathbf{r}_d \\ &= \mathcal{P} \mathbf{h}^H \hat{\boldsymbol{S}} \mathbf{E}_i^H \mathbf{E}_j \mathbf{E}_j^H \mathbf{E}_i \hat{\boldsymbol{S}} \mathbf{h} - \mathcal{P} \mathbf{h}^H \hat{\boldsymbol{S}} \hat{\boldsymbol{S}} \mathbf{h} \\ &\quad + 2Re\{\sqrt{\mathcal{P}} \mathbf{h}^H \hat{\boldsymbol{S}} \mathbf{E}_i^H \mathbf{E}_j \mathbf{E}_j^H \mathbf{n}_d\} - 2Re\{\sqrt{\mathcal{P}} \mathbf{h}^H \hat{\boldsymbol{S}} \mathbf{E}_i^H \mathbf{n}_d\} \\ &\quad + \mathbf{n}_d^H \mathbf{E}_j \mathbf{E}_j^H \mathbf{n}_d - \mathbf{n}_d^H \mathbf{E}_i \mathbf{E}_i^H \mathbf{n}_d. \end{aligned}$$

To evaluate the conditional expectation in (3.25), we first further conditioned on the channel. Substituting w in, we have

$$
\begin{aligned}
&E[e^{sw}|\mathbf{C}_i, \hat{\boldsymbol{S}}, \mathbf{h}] \\
&\quad = \exp\big(s\mathcal{P}\mathbf{h}^H\hat{\boldsymbol{S}}(\mathbf{E}_i^H\mathbf{E}_j\mathbf{E}_j^H\mathbf{E}_i - \mathbf{I}_{RL})\hat{\boldsymbol{S}}\mathbf{h}\big) \\
&\quad\quad \cdot E\big[\exp\big(2sRe\{\sqrt{\mathcal{P}}\mathbf{h}^H\hat{\boldsymbol{S}}\mathbf{E}_i^H(\mathbf{E}_j\mathbf{E}_j^H - \mathbf{I}_N)\mathbf{n}_d\}\big) \\
&\quad\quad\quad \exp\big(s\mathbf{n}_d^H(\mathbf{E}_j\mathbf{E}_j^H - \mathbf{E}_i\mathbf{E}_i^H)\mathbf{n}_d\big) \,\big|\, \mathbf{C}_i, \hat{\boldsymbol{S}}, \mathbf{h}\,\big] \\
&\quad \leq \exp\big(s\mathcal{P}\mathbf{h}^H\hat{\boldsymbol{S}}(\mathbf{E}_i^H\mathbf{E}_j\mathbf{E}_j^H\mathbf{E}_i - \mathbf{I}_{RL})\hat{\boldsymbol{S}}\mathbf{h}\big) \\
&\quad\quad \cdot\big(E\big[\exp\big(4sRe\{\sqrt{\mathcal{P}}\mathbf{h}^H\hat{\boldsymbol{S}}\mathbf{E}_i^H(\mathbf{E}_j\mathbf{E}_j^H - \mathbf{I}_N)\mathbf{n}_d\}\big) \\
&\quad\quad\quad\quad \big|\, \mathbf{C}_i, \hat{\boldsymbol{S}}, \mathbf{h}\,\big]\big)^{\frac{1}{2}} \\
&\quad\quad \cdot\big(E\big[\exp\big(2s\mathbf{n}_d^H(\mathbf{E}_j\mathbf{E}_j^H - \mathbf{E}_i\mathbf{E}_i^H)\mathbf{n}_d\big) \,\big|\, \mathbf{C}_i, \hat{\boldsymbol{S}}, \mathbf{h}\,\big]\big)^{\frac{1}{2}}
\end{aligned} \tag{3.26}
$$

where the last bound followed from cauchy-schwarz inequality. The first expectation in equation (3.26) could be evaluated by using MGF of Gaussian random variable. And the second expectation is related to the MGF of Hermitian quadratic form in complex Gaussian, for which the closed form solution could be found in [16]. Carrying out the two, we have

$$
\begin{aligned}
&E[e^{sw}|\mathbf{C}_i, \hat{\boldsymbol{S}}, \mathbf{h}] \\
&\quad \leq \exp\big(s\mathcal{P}\mathbf{h}^H\hat{\boldsymbol{S}}(\mathbf{E}_i^H\mathbf{E}_j\mathbf{E}_j^H\mathbf{E}_i - \mathbf{I}_{RL})\hat{\boldsymbol{S}}\mathbf{h}\big) \\
&\quad\quad \cdot \exp\big(2s^2\mathcal{P}\mathbf{h}^H\hat{\boldsymbol{S}}\mathbf{E}_i^H(\mathbf{E}_j\mathbf{E}_j^H - \mathbf{I}_N)(\mathbf{E}_j\mathbf{E}_j^H - \mathbf{I}_N)\mathbf{E}_i\hat{\boldsymbol{S}}\mathbf{h}\big) \\
&\quad\quad \cdot \det{}^{-\frac{1}{2}}\big(\mathbf{I}_N - 2s(\mathbf{E}_j\mathbf{E}_j^H - \mathbf{E}_i\mathbf{E}_i^H)\big) \\
&\quad = \exp\big((s-2s^2)\mathcal{P}\mathbf{h}^H\hat{\boldsymbol{S}}(\mathbf{E}_i^H\mathbf{E}_j\mathbf{E}_j^H\mathbf{E}_i - \mathbf{I}_{RL})\hat{\boldsymbol{S}}\mathbf{h}\big) \\
&\quad\quad \cdot \det{}^{-\frac{1}{2}}\big(\mathbf{I}_N - 2s(\mathbf{E}_j\mathbf{E}_j^H - \mathbf{E}_i\mathbf{E}_i^H)\big).
\end{aligned} \tag{3.27}
$$

This bound holds for all $s > 0$. Instead of finding the optimum s to minimize the bound, we simply use $s = \frac{1}{3}$ as it is neat and enough for providing the bound (3.20). In such case, the determinant term in equation (3.27) could be bounded as

$$
\det{}^{-\frac{1}{2}}\big(\mathbf{I}_N - \frac{2}{3}(\mathbf{E}_j\mathbf{E}_j^H - \mathbf{E}_i\mathbf{E}_i^H)\big) \leq (\frac{1}{3})^{-\frac{1}{2N}},
$$

where we have used Weyl's inequality [4], theorem 4.3.1, that

$$\lambda(\mathbf{E}_j\mathbf{E}_j^H - \mathbf{E}_i\mathbf{E}_i^H) \leq \lambda_{\max}(\mathbf{E}_j\mathbf{E}_j^H) - \lambda_{\min}(\mathbf{E}_i\mathbf{E}_i^H) = 1.$$

Consequently, we now have

$$\begin{aligned}&\mathrm{P}(w > 0|\mathbf{C}_i, \hat{\boldsymbol{S}}, \mathbf{h}) \\ &\quad \leq (\frac{1}{3})^{-\frac{1}{2N}} \exp\big(\frac{\mathcal{P}}{9}\mathbf{h}^H\hat{\boldsymbol{S}}(\mathbf{E}_i^H\mathbf{E}_j\mathbf{E}_j^H\mathbf{E}_i - \mathbf{I}_{RL})\hat{\boldsymbol{S}}\mathbf{h}\big).\end{aligned}$$

Taking expectation over **h** on both side, and using the MGF formula in [16] again, it yields

$$\begin{aligned}&\mathrm{P}(w > 0|\mathbf{C}_i, \hat{\boldsymbol{S}}) \\ &\quad \leq (\frac{1}{3})^{-\frac{1}{2N}} \det{}^{-1}\big(\mathbf{I}_{RL} + \frac{\mathcal{P}}{9}\hat{\boldsymbol{S}}\boldsymbol{\Sigma}^2(\mathbf{I}_{RL} - \mathbf{E}_i^H\mathbf{E}_j\mathbf{E}_j^H\mathbf{E}_i)\big) \\ &\quad = (\frac{1}{3})^{-\frac{1}{2N}} \prod_{k=1}^{d_{i,j}} \big(1 + \frac{\mathcal{P}}{9}\hat{\boldsymbol{S}}\boldsymbol{\Sigma}^2\mathbf{Q}\big)^{-1} \\ &\quad \leq (\frac{1}{3})^{-\frac{1}{2N}} (\frac{\mathcal{P}}{9})^{-d_{i,j}} \prod_{k=1}^{d_{i,j}} \lambda_k^{-1}(\hat{\boldsymbol{S}}\boldsymbol{\Sigma}^2\mathbf{Q}),\end{aligned} \tag{3.28}$$

where $\mathbf{Q} = \mathbf{I}_{RL} - \mathbf{E}_i^H\mathbf{E}_j\mathbf{E}_j^H\mathbf{E}_i$ is the same matrix as in section 3.1.1. From (3.28), we know that assuring nonsingularity of $\mathbf{Q}$ will guarantee the diversity being $n_{\mathbf{S}}L$, which is an identical result as section 3.1.1, equation (3.13).

Average Pairwise Error Probability

Using a similar argument as in 3.1.1, we could obtain

$$\mathrm{P}(\mathbf{C}_i \to \mathbf{C}_j|\mathbf{C}_i) < \eta_2 \times \mathcal{P}^{-RL}$$

with

$$\eta_2 = (\frac{1}{3})^{-\frac{1}{2N}} \sum_{k=0}^{R} \binom{R}{k} \sum_{\hat{\boldsymbol{S}}:n_{\mathbf{S}}=k} 9^{n_{\mathbf{S}}L} \prod_{r=0}^{n_{\mathbf{S}}L-1} \lambda_r^{-1}(\hat{\boldsymbol{S}}\boldsymbol{\Sigma}^2\mathbf{Q}),$$

which provides equation (3.20). Therefore, we can conclude the result.

3.1.3 Code Design Criteria

It follows from (3.13) and (3.28) that, by ensuring the matrix $\mathbf{Q}$ to be nonsingular for any pair of distinct pseudocodeword matrices, the error probability could show a decaying rate scaling with $\mathcal{P}^{-RL}$ no matter whether the receiver has the knowledge of channel statistic and/or the relay decoding status.

The requirement on pseudocodeword matrices resembles that of non-coherent SFC in [1] with identical diversity gain. It follows that using codeword matrices constructed by the same coding criteria as in [1] can also invoke full diversity that resides in a distributed channel. Thus, as in [1], define the diversity product

$$\gamma = \min_{0 \leq i \leq j \leq K-1} \prod_{r=0}^{RL-1} \left(1 - \rho_r^2(i,j)\right) \tag{3.29}$$

where $\rho_r^2(i,j),\ r = 0, 1, \ldots, RL-1$, are the singular values of the matrix $\mathbf{E}_j^H \mathbf{E}_i$. Then, the design criteria for achieving full diversity is to find the pseudocodeword matrices satisfying $\gamma > 0$.

3.2 Performance Analysis under Imperfect Relay Error Detection

In this section, we investigate the impact of imperfect relay error detection. Assume the source transmit message $\mathbf{s}_i$, recall that the received signal can be written as

$$\mathbf{r}_d = \sqrt{\mathcal{P}} \mathbf{E}_i \hat{\boldsymbol{S}}_i \mathbf{h} + \sqrt{\mathcal{P}} \sum_{j \neq i}^{K} \mathbf{E}_j \hat{\boldsymbol{S}}_j \mathbf{h} \ + \mathbf{n}.$$

We start the analysis with an observation about following equation

$$\mathrm{P}(\mathbf{C}_i \to \mathbf{C}_j | \mathbf{C}_i) = \sum_{\hat{\boldsymbol{S}}_1^K} \mathrm{P}(\hat{\boldsymbol{S}}_1^K) \mathrm{P}(\mathbf{C}_i \to \mathbf{C}_j | \mathbf{C}_i, \hat{\boldsymbol{S}}_1^K), \tag{3.30}$$

where we abbreviate $\hat{\boldsymbol{S}}_1, \hat{\boldsymbol{S}}_2, \cdots, \hat{\boldsymbol{S}}_K$ as $\hat{\boldsymbol{S}}_1^K$, and the summation is taken over all possible $\hat{\boldsymbol{S}}_1^K$.

We can see that the diversity, the exponent of LHS, will be dominated by the one with worst exponent in the summand of RHS Thus, we should calculate the worst one of $\mathrm{P}(\hat{\boldsymbol{S}}_1^K)\mathrm{P}(\mathbf{C}_i \to \mathbf{C}_j | \mathbf{C}_i, \hat{\boldsymbol{S}}_1^K)$ in exponent for different realization of $\hat{\boldsymbol{S}}_1^K$. Now, we express

$\mathrm{P}(\hat{\boldsymbol{S}}_1^K)$ as

$$\mathrm{P}(\hat{\boldsymbol{S}}_1^K) = \big[(1-p_s)(1-p_{1|0})\big]^{n_{\mathbf{S}_i}} \prod_{m\neq i} (\frac{p_{0|1}p_s}{K-1})^{n_{\mathbf{S}_m}} \cdot \big[(1-p_s)p_{1|0} + p_s(1-p_{0|1})\big]^{n_{\mathbf{S}_0}} \tag{3.31}$$

where $n_{\mathbf{S}_m} = \sum_{r=1}^{R} S_{m,r}$ is the number of active relay that transmit mth message and $n_{\mathbf{S}_0} = R - \sum_{m=1}^{K} n_{\mathbf{S}_m}$ is the number of silent relay nodes.

For obtaining equation (3.31), aside from the model of detection accuracy in section 2, we further need an assumption that if a relay decode incorrectly, then it decoded message would be uniformly distributed over $\{1, 2, \cdots, K\} \setminus \{i\}$, *i.e.*, $\frac{1}{K-1}$ for each. Such assumption rely on a symmetric design of the codebook used in phase I transmission which is consist of many SISO systems and is not of our focus. In more rigorous words, this assumption would not affect the exponent as long as there is no pair of codeword used in phase I has a different SNR-exponent of PEP than others.

To catch the diversity, we simplify (3.31) to

$$\mathrm{P}(\hat{\boldsymbol{S}}_1^K) \doteq (p_{0|1}p_s)^{\sum_{m\neq i} n_{\mathbf{S}_m}} (p_{1|0} + p_s)^{n_{\mathbf{S}_0}}. \tag{3.32}$$

As we can see, the parameters of detection accuracy $p_{0|1}$ and $p_{1|0}$ will affect the exponent in different manners. Next, to have a thorough analysis, we need to evaluate $\mathrm{P}(\mathbf{C}_i \rightarrow \mathbf{C}_j | \mathbf{C}_i, \hat{\boldsymbol{S}}_1^K)$ for all possible $\hat{\boldsymbol{S}}_1^K$. Unfortunately, the math structure is too complicated and we failed to carry it out. Thus, we only investigate the impact of imperfect error detection on the system diversity by following two facts and one proposition.

Fact1 Even with ML decoding at receiver, fixed positive $p_{1|0}$ leads to zero diversity.

Fact2 Diversity order under fixed positive $p_{0|1}$ do not exceed $\lceil \frac{R}{2} \rceil L$ for any decoder.

Proposition3 Correlator-like decoder proposed in section 3.1.2 can only retrieve diversity of order $\min\{L + L_{01}, RL_{10}, RL\}$ under detection accuracy $p_{0|1} \doteq \mathcal{P}^{-L_{01}}$ and $p_{1|0} \doteq \mathcal{P}^{-L_{10}}$.

Proposition3 indicates that correlator-like decoder might not attain diversity of order RL, especially when L_{01} is small. It would turn out that the argument of (3.21) won't work here. ML decoder would probably do better in diversity. Actually, the one that we are incapable to treat its involved math is the PEP using ML decoder under existence of harmful relays. Hence, unlike the case of correlator-like decoder, we are incapable to give a complete analysis for an ML decoder. For example, in **Fact2**, we do not know whether or not using ML decoder can achieve the diversity $\lceil \frac{R}{2} \rceil L$ under fixed positive $p_{0|1}$. ML decoder takes the form

$$\hat{\boldsymbol{C}}_{\text{ML}} = \arg\max_{\mathbf{C}_i \in \mathcal{C}} \sum_{\text{all possible } \hat{\boldsymbol{S}}_1^K} \mathrm{P}(\hat{\boldsymbol{S}}_1^K) \frac{\exp\left(-\mathbf{r}_d^H \boldsymbol{\Lambda}_{i,\hat{\boldsymbol{S}}_1^K}^{-1} \mathbf{r}_d\right)}{\pi^N \left(\det\ \boldsymbol{\Lambda}_{i,\hat{\boldsymbol{S}}_1^K}\right)}. \tag{3.33}$$

It is again a gaussian mixture and need to do the summation over all possible $\hat{\boldsymbol{S}}_1^K$, which we are not able to tackle with both analytically and practically. For this part, we simulate it by computer and have some discussion in section 4.2. For the rest of this section, we give brief arguments about the two facts and one proposition.

Fixed positive $p_{1|0}$ leads to zero diversity

We aims to prove

$$\mathrm{P}(\mathbf{C}_i \to \mathbf{C}_j | \mathbf{C}_i) \doteq \zeta_1 \tag{3.34}$$

for some constant ζ_1 irrelevant to SNR, which implies the zero diversity. To see this, consider the particular summand in (3.30) that $\hat{\boldsymbol{S}}_1^K = (\mathbf{0}_{RL}, \mathbf{0}_{RL}, \cdots, \mathbf{0}_{RL}) \overset{def}{=} \mathbf{O}$ in equation (3.30), which means that $\hat{\boldsymbol{S}}_0 = \mathbf{I}_{RL}$ and $n_{\mathbf{S}_0} = RL$, *i.e.*, no relay transmit to destination. In this case, since the receiver receives only additive noise, we have

$$\mathrm{P}(\mathbf{C}_i \to \mathbf{C}_j | \mathbf{C}_i, \mathbf{O}) \doteq \zeta_a$$

for some constant ζ_a for any type of decoder used in receiver end. Combing with equation (3.32), we reach

$$\begin{aligned} \mathrm{P}(\hat{\boldsymbol{S}}_1^K = \mathbf{O}) \mathrm{P}(\mathbf{C}_i \to \mathbf{C}_j | \mathbf{C}_i, \mathbf{O}) &\doteq (p_{1|0} + p_s)^{RL} \zeta_a \\ &\doteq (p_{1|0})^{RL} \zeta_a \\ &\overset{def}{=} \zeta_b, \end{aligned} \tag{3.35}$$

where (3.35) follows from $p_s \doteq \mathcal{P}^{-L}$ and $p_{1|0}$ is a constant. And as mentioned, the diversity will be dominated by the worst exponent one summand of (3.30), hence equation (3.34) holds.

This fact demonstrate a result: we do a quite accurate error detection at each relay and have a very small probability of existence of useless relay. Even though, if we have fixed positive $p_{1|0}$, it will be significant on the behavior of error probability at high SNR since the decoding error will be dominated by the event that all relays are useless relay in high SNR regime. And a simple conclusion can be made from equation (3.32) is that, to illuminate such effect, we need at least $p_{1|0} \doteq \mathcal{P}^{-L}$ of error detection accuracy.

Diversity under fixed positive $p_{0|1}$ do not exceed $\lceil \frac{R}{2} \rceil L$

In this part, we consider the summand in (3.30) that

$$\hat{\boldsymbol{S}}_i = \mathbf{I}_L \otimes diag\{\underbrace{1, \cdots, 1}_{\lfloor \frac{R}{2} \rfloor}, 0, \cdots, 0\}$$

$$\hat{\boldsymbol{S}}_j = \mathbf{I}_L \otimes diag\{0, \cdots, 0, \underbrace{1, \cdots, 1}_{\lfloor \frac{R}{2} \rfloor}\}$$

$$\hat{\boldsymbol{S}}_m = \mathbf{0}_{RL} \qquad \forall\, m \neq i, j,$$

and for convenience, we abbreviate such case as $\hat{\boldsymbol{S}}_1^K = \mathbf{V}_{ij}$. By symmetry, the receiver will have same favor of $\mathbf{C}_i$ and $\mathbf{C}_j$. Hence we have

$$\mathrm{P}(\mathbf{C}_i \to \mathbf{C}_j | \mathbf{C}_i, \mathbf{V}_{ij}) = \frac{1}{2}.$$

And again together with the probability of such relay status from equation (3.32), we can obtain

$$\begin{aligned}
\mathrm{P}(\hat{\boldsymbol{S}}_1^K = \mathbf{V}_{ij})&\mathrm{P}(\mathbf{C}_i \to \mathbf{C}_j | \mathbf{C}_i, \mathbf{V}_{ij}) \\
&\doteq \frac{1}{2}(p_{0|1}p_s)^{\lfloor \frac{R}{2} \rfloor L}(p_{1|0} + p_s)^{RL - 2\lfloor \frac{R}{2} \rfloor L} \\
&\geq \frac{1}{2}(p_{0|1}p_s)^{\lfloor \frac{R}{2} \rfloor L} p_s^{RL - 2\lfloor \frac{R}{2} \rfloor L} \\
&\doteq \mathcal{P}^{RL - \lfloor \frac{R}{2} \rfloor L} = \mathcal{P}^{\lceil \frac{R}{2} \rceil L},
\end{aligned} \tag{3.36}$$

where the dot equal in (3.36) follows from $p_s \doteq \mathcal{P}^{-L}$ and $p_{0|1} > 0$ is a constant. Similarly, since diversity is dominated by worst one summand, we conclude that diversity do not exceed $\lceil \frac{R}{2} L \rceil$ under fixed positive $p_{0|1}$, the possibility for existence of harmful relay.

Correlator-like decoder proposed in section 3.1.2 can only retrieve diversity of order $\min\{L + L_{01}, RL_{10}, RL\}$ **under detection accuracy** $p_{0|1} \doteq \mathcal{P}^{-L_{01}}$ **and** $p_{1|0} \doteq \mathcal{P}^{-L_{10}}$

Recall the correlator-like decoder proposed in Sec. 3.1.2,

$$\hat{C}_{\text{subopt}} = \arg\max_{\mathbf{C}_i \in \mathcal{C}} \left(\mathbf{r}_d^H \mathbf{E}_i \mathbf{E}_i^H \mathbf{r}_d \right).$$

And the corresponding pairwise error event,

$$\{\mathbf{r}_d^H \mathbf{E}_j \mathbf{E}_j^H \mathbf{r}_d - \mathbf{r}_d^H \mathbf{E}_i \mathbf{E}_i^H \mathbf{r}_d > 0\}.$$

We consider the summand in (3.30) such that $\hat{\boldsymbol{S}}_i$, $\hat{\boldsymbol{S}}_j \neq \mathbf{0}_{RL}$ and $\hat{\boldsymbol{S}}_m = \mathbf{0}_{RL}$ for all $m \neq i, j$, abbreviated as $\hat{\boldsymbol{S}}_1^K = \mathbf{U}_j$. In such case, the received signal can be written as

$$\mathbf{r}_d = \sqrt{\mathcal{P}} \mathbf{E}_i \hat{\boldsymbol{S}}_i \mathbf{h} + \sqrt{\mathcal{P}} \mathbf{E}_j \hat{\boldsymbol{S}}_j \mathbf{h} + \mathbf{n}.$$

Claim:

$$\mathrm{P}(\mathbf{C}_i \to \mathbf{C}_j | \mathbf{C}_i, \mathbf{U}_j) \doteq \zeta_3.$$

Proof: We write the PEP as

$$\begin{aligned}
\mathrm{P}(\mathbf{C}_i \to & \mathbf{C}_j | \mathbf{C}_i, \mathbf{U}_j) \\
&= \mathrm{P}(\mathbf{r}_d^H \mathbf{E}_j \mathbf{E}_j^H \mathbf{r}_d - \mathbf{r}_d^H \mathbf{E}_i \mathbf{E}_i^H \mathbf{r}_d > 0 | \mathbf{C}_i, \mathbf{U}_j) \\
&= \mathrm{P}(\frac{1}{\mathcal{P}} (\mathbf{r}_d^H \mathbf{E}_j \mathbf{E}_j^H \mathbf{r}_d - \mathbf{r}_d^H \mathbf{E}_i \mathbf{E}_i^H \mathbf{r}_d) > 0 | \mathbf{C}_i, \mathbf{U}_j) \\
&= \mathrm{P}(w_{\mathcal{P}} > 0 | \mathbf{C}_i, \mathbf{U}_j),
\end{aligned}$$

where $w_{\mathcal{P}} = \bar{\mathbf{r}}_d^H \mathbf{E}_j \mathbf{E}_j^H \bar{\mathbf{r}}_d - \bar{\mathbf{r}}_d^H \mathbf{E}_i \mathbf{E}_i^H \bar{\mathbf{r}}_d$ with

$$\bar{\mathbf{r}}_d = \frac{\mathbf{r}_d}{\sqrt{\mathcal{P}}} = \mathbf{E}_i \hat{\boldsymbol{S}}_i \mathbf{h} + \mathbf{E}_j \hat{\boldsymbol{S}}_j \mathbf{h} + \frac{\mathbf{n}}{\sqrt{\mathcal{P}}}.$$

Then, the claim is equivalent to

$$\lim_{\mathcal{P} \to \infty} \mathrm{P}(w_{\mathcal{P}} > 0 | \mathbf{C}_i, \mathbf{U}_j) > 0$$

Define $w_\infty = \tilde{\mathbf{r}}_d^H \mathbf{E}_j \mathbf{E}_j^H \tilde{\mathbf{r}}_d - \tilde{\mathbf{r}}_d^H \mathbf{E}_i \mathbf{E}_i^H \tilde{\mathbf{r}}_d$, where $\tilde{\mathbf{r}}_d = \mathbf{E}_i \hat{\boldsymbol{S}}_i \mathbf{h} + \mathbf{E}_j \hat{\boldsymbol{S}}_j \mathbf{h}$. Then obviously,

$$w_{\mathcal{P}} \longrightarrow w_\infty \quad \text{almost surely as } \mathcal{P} \longrightarrow \infty.$$

Thus, we can prove the claim by showing

$$\mathrm{P}(w_\infty > 0 | \mathbf{C}_i, \mathbf{U}_j) > 0.$$

Expand w_∞ by using $\tilde{\mathbf{r}}_d = \mathbf{E}_i \hat{\boldsymbol{S}}_i \mathbf{h} + \mathbf{E}_j \hat{\boldsymbol{S}}_j \mathbf{h}$, through some manipulation,

$$\begin{aligned} w_\infty &= \tilde{\mathbf{r}}_d^H \mathbf{E}_j \mathbf{E}_j^H \tilde{\mathbf{r}}_d - \tilde{\mathbf{r}}_d^H \mathbf{E}_i \mathbf{E}_i^H \tilde{\mathbf{r}}_d \\ &= \mathbf{h}^H \hat{\boldsymbol{S}}_j (\mathbf{I} - \mathbf{E}_j^H \mathbf{E}_i \mathbf{E}_i^H \mathbf{E}_j) \hat{\boldsymbol{S}}_j \mathbf{h} \\ &\quad - \mathbf{h}^H \hat{\boldsymbol{S}}_i (\mathbf{I} - \mathbf{E}_i^H \mathbf{E}_j \mathbf{E}_j^H \mathbf{E}_i) \hat{\boldsymbol{S}}_i \mathbf{h} \\ &\triangleq X - Y, \end{aligned}$$

where $X \triangleq \mathbf{h}^H \hat{\boldsymbol{S}}_j (\mathbf{I}_{RL} - \mathbf{E}_j^H \mathbf{E}_i \mathbf{E}_i^H \mathbf{E}_j) \hat{\boldsymbol{S}}_j \mathbf{h}$ and $Y \triangleq \mathbf{h}^H \hat{\boldsymbol{S}}_i (\mathbf{I}_{RL} - \mathbf{E}_i^H \mathbf{E}_j \mathbf{E}_j^H \mathbf{E}_i) \hat{\boldsymbol{S}}_i \mathbf{h}$. Without loss of generality, we assume $\mathbf{E}_i \neq \mathbf{E}_j$. (Otherwise, codewords i and j cannot be distinguished by receiver.) Then we have $\mathbf{I}_{RL} - \mathbf{E}_j^H \mathbf{E}_i \mathbf{E}_i^H \mathbf{E}_j$ being positive definite. As we can see, X and Y both follow a generalized chi-square distribution with support $[0, \infty)$. Further note that X and Y are independent, we obtain

$$\begin{aligned} \mathrm{P}(w_\infty > 0 | \mathbf{C}_i, \mathbf{U}_j) &= \mathrm{P}(X - Y > 0 | \mathbf{C}_i, \mathbf{U}_j) \\ &\geq \mathrm{P}(\{X > 1\} \cap \{Y < 1\} | \mathbf{C}_i, \mathbf{U}_j) \\ &= \mathrm{P}(X > 1 | \mathbf{C}_i, \mathbf{U}_j)\, \mathrm{P}(Y < 1 | \mathbf{C}_i, \mathbf{U}_j) \\ &> 0 \end{aligned}$$

Hence the claim is proved.

This claim indicate that under any existence of harmful relay, an error floor occur when employing correlator-like decoder. When there does not exist harmful relay, on the other hand, by the result established in Sec. 3.1.2, the diversity order will equal to the number of useful relays in the system. Again, reexamining (3.30) and using (3.32), the worst summand of (3.30) could be found as

$$\min\{L + L + 01, \min_{1 \leq k \leq R} (R - k)L + kL_{10}\}.$$

Solve the minimization, we can conclude that correlator-like decoder can only retrieve diversity of order $\min\{L + L_{01}, RL_{10}, RL\}$ under detection accuracy $p_{0|1} \doteq \mathcal{P}^{-L_{01}}$ and $p_{1|0} \doteq \mathcal{P}^{-L_{10}}$. Note that when $L + L_{01}$ dominate, the diversity do not scale with R, the number of relays. This gives us some clue how $p_{0|1}$ and $p_{1|0}$ affect the diversity order differently.

Chapter 4

Simulations

4.1 Simulation Setup

We present simulation results in this section. We simulated the system with channel taps $L = 2$ and number of subcarriers $N = 16$. And we assume a symmetry power delay profile between relays, *i.e.*, $\sigma_{r,l}^2 = \sigma_{r',l}^2$ for $l = 0, 1$ and for all $r \neq r'$. Setting the noise power to one, we define the SNR as the total transmit power from R relays per unit frequency, *i.e.* SNR $= \mathcal{P}R/N$. Since our analysis excludes the source node coding, we give the source node an extra power equal to that of each relay in the simulation in order to determine the probability p_s at relay nodes. Accordingly, we assume $p_s = \beta \times \mathcal{P}^{-L}$ and the constant β is irrelevant to the diversity order. Hence we just set it a particular number that ensures $0 < p_s < 1$.

4.2 Simulation Results

4.2.1 Perfect error detection on each relay

We first simulate the system with perfect error detection, which corresponds to $p_{1|0} = 0$ and $p_{0|1} = 0$. We have seen in Sec. 3.1.3 that the diversity achieving code design criteria for the non-coherent cooperative SFC in wireless relay networks are consistent with that of the non-coherent SFC in MIMO-OFDM systems discussed in [1]. Therefore, we follow a similar procedure to construct a code, which is shown in Table 4.1, in our simulation.

Some knowledge known at receiver

Performance curves of four cases with receiver having knowledge of relay decoding status and channel statistics discussed in Sec. 3.1.1 are presented in Fig. 2, where each set of curves corresponds to a combination of the number of relays $R = 2, 4$ and the codebook size $K = 8, 16$. The channel PDP is set to be uniform, *i.e.*, $[\sigma_{r,1}^2 \; \sigma_{r,2}^2] = [0.5 \; 0.5]$, in simulating Fig. 2. The results show that the diversity order of 4 and 8 are achieved for $R = 2$ and $R = 4$, respectively. It implies that the potential diversity of relay network indeed resembles that of MIMO system in a noncoherent space-frequency environment.

Fig. 3 plot the curves under different power delay profile for number of relays $R = 2$. In the figure, 50%, 10%, 5% and 1% represent the PDP being $[\sigma_{r,1}^2 \; \sigma_{r,2}^2] = [0.5\,0.5], [0.9\,0.1], [0.95\,0.05]$ and $[0.99 \; 0.01]$, respectively. As we can see, all of them have same diversity of order 4 although it get worse when we have a more asymmetric PDP. Such phenomenon could be viewed by equation (3.16). Roughly speaking, the magnitude of the term $\lambda^{-1}(\hat{\boldsymbol{S}}\boldsymbol{\Sigma}^2\mathbf{Q})$ is close to $\lambda^{-1}(\boldsymbol{\Sigma}^2) = \prod_{r=1}^{R}(\sigma_{r,1}^2\sigma_{r,2}^2)$. Since we fixed the total power $\sum_{l=0}^{L-1}\sigma_{r,l}^2 = 1$, the term $\sigma_{r,1}^2\sigma_{r,2}^2$ is larger for a more symmetric PDP by A-G inequality. Intuitively, we need more power for an asymmetric channel for invoking the particular weak one into

Table 4.1: Table of code Constructions. $\boldsymbol{\Phi} = diag_{k=0}^{N-1}\{e^{j\frac{2\pi}{K}u_k}\}$ and $\mathbf{f}_i$ represents the ith column of the $N \times N$ DFT matrix.

R	K	Codeword $\mathbf{C}_i$	$[u_0, u_2, \cdots, u_{15}]$
2	8	$\boldsymbol{\Phi}^i[\mathbf{f}_1\mathbf{f}_3]$	[1 0 3 4 1 0 3 4 1 0 3 4 1 0 3 4]
2	16	$\boldsymbol{\Phi}^i[\mathbf{f}_1\mathbf{f}_3]$	[1 4 3 0 1 8 3 12 1 4 3 0 1 8 3 12]
4	8	$\boldsymbol{\Phi}^i[\mathbf{f}_1\mathbf{f}_3\mathbf{f}_5\mathbf{f}_7]$	[1 0 3 4 1 0 3 4 1 0 3 4 1 0 3 4]
4	16	$\boldsymbol{\Phi}^i[\mathbf{f}_1\mathbf{f}_3\mathbf{f}_5\mathbf{f}_7]$	[1 4 3 0 1 8 3 12 1 4 3 0 1 8 3 12]

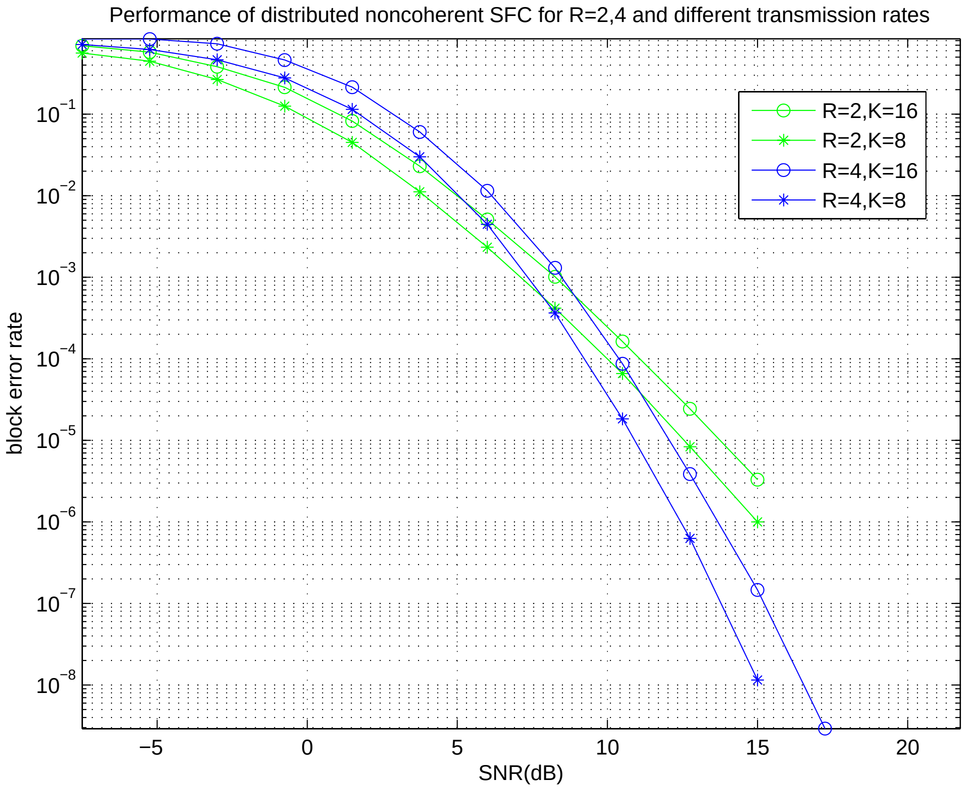

Figure 4.1: The block error rate versus SNR of the non-coherent distributed SFC with $N = 16$ and $L = 2$.

contribution of diversity. As we can see in Fig. 3, the slope of "1%" do not achieve 4 until SNR > 20 dB.

Completely noncoherent receiver

We further simulate the system which use the suboptimum decoder proposed in Sec. 3.1.2 and do some comparison with the ML decoding of Sec. 3.1.1 under different channel power delay profiles. In Fig. 4, we use "ML" for the ML decoding which requires the relay status information as indicated in Sec. 3.1.1, and "subopt" for the completely noncoherent correlator-like decoder proposed in Sec. 3.1.2. Observe that the two decoder have almost the same performance. We can also see that the gap between the two decoder is a little

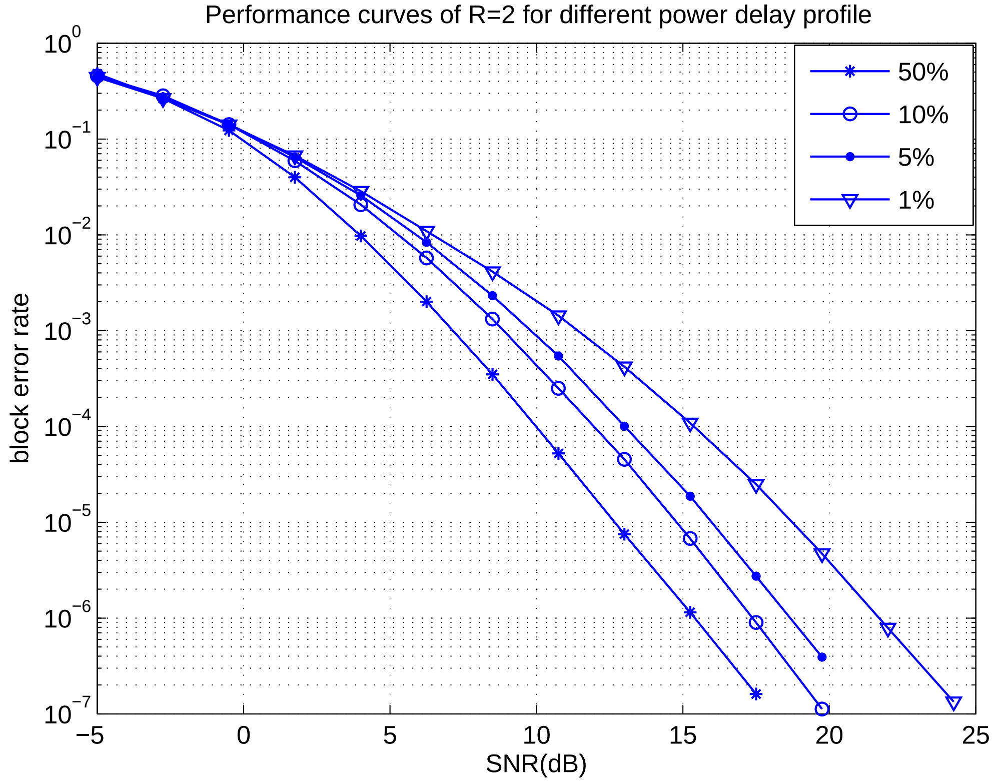

Figure 4.2: The block error rate versus SNR for different PDP.

bit larger at low SNR particularly when the PDP is more asymmetry. This follows from the correlator-like decoder actually assumes a uniform PDP as indicated in the description between equation (3.22) and (3.23). However, they performed almost the same in the high SNR regime. This shows that the suboptimum decoder is good enough for capturing the system under perfect relay error detection.

4.2.2 Imperfect relay error detection

In this subsection, we use simulation to investigate performance of the system under imperfect relay error detection, including the PEP with ML decoder when there exist some harmful relays, for which we are incapable of carrying out an analytical solution. In Fig. 5, we present the block error rate under $p_{0|1} = 0$ and different positive $p_{1|0}$. We can see

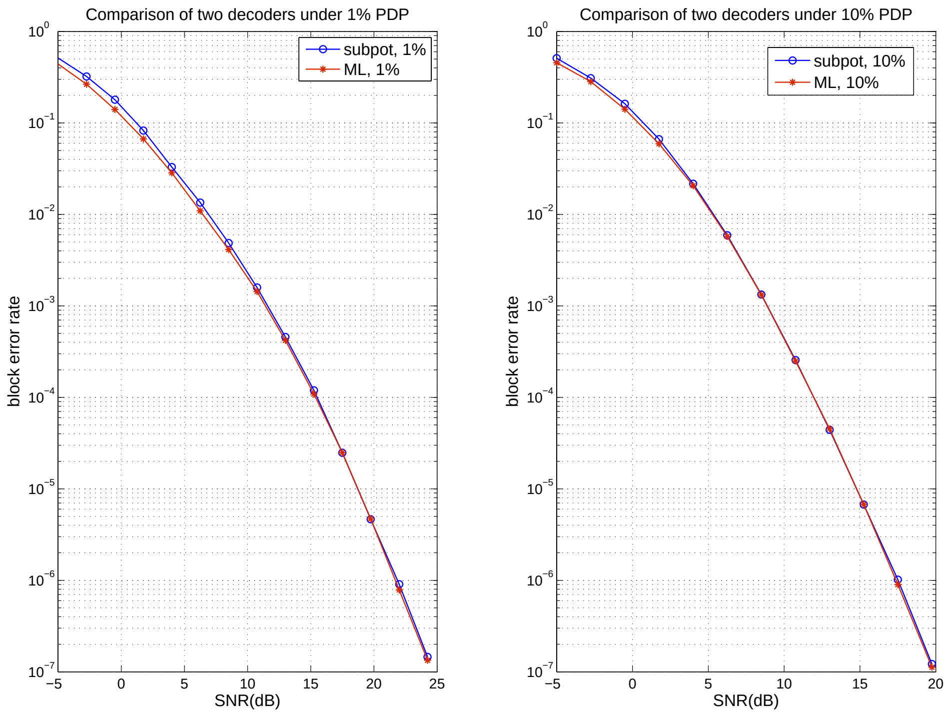

Figure 4.3: The block error rate versus SNR of different PDP under different decoders.

that even with $p_{0|1} = 0$, it still has error floor for all cases, which corresponds to **Fact 1**, fixed positive $p_{1|0}$ leads to zero diversity. However, we may observe that increase the relay number can lower the error floor efficiently. It directly comes from BER $\longrightarrow p_{1|0}^{R}$ as $SNR \longrightarrow \infty$.

Then, in Fig. 6, we present the block error rate under some particular relay status with two different decoders used in receiver. The number of relay is set to be $R = 4$. In the legend, the status of relays is indicated as "1" for useful relay, "0" for useless relay and "2" for harmful relay. For example, "ML1112" represent that three relays are useful and one is harmful with receiver employing ML decoder of equation (3.33). And "subopt1102" stands for two useful relays, one useless and one harmful with receiver use the suboptimum correlator-like decoder. As we can see in Fig. 6, no matter how many useful relays is currently in the system, the suboptimum receiver has a severe error floor

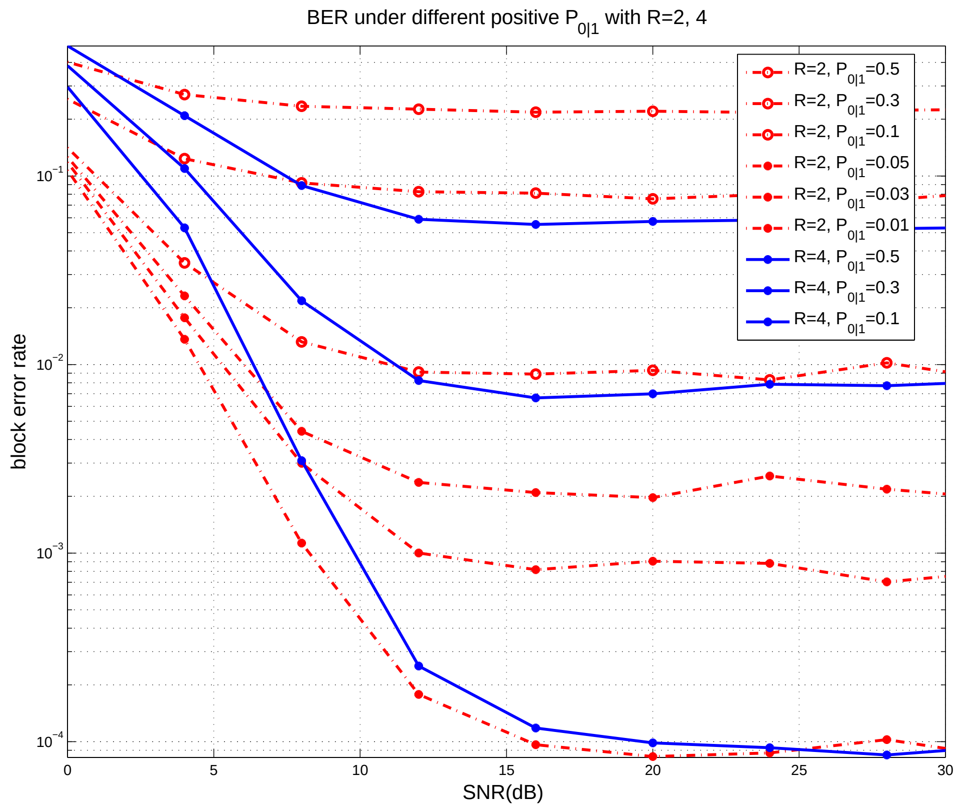

Figure 4.4: The block error rate versus SNR of two decoder under existence of harmful relay.

when there exist any harmful relay. While for ML decoder, there is no error floor for both "1102" and "1112". And from the figure, we see the diversity that ML decoder could retrieve is of order 2 for "1102" and of order 4 for "1112". We might guess that the diversity would be

$$L \times (\#(\text{useful relay}) - \#(\text{harmful relay})) \tag{4.1}$$

for using ML decoder at receiver. And we can also see that the BER of the two decoders has only minor gap at low SNR but it turn to be significant very soon when we increase the SNR. This again demonstrate the importance of the diversity.

To have a realistic scenario, we further simulate the system for the relay employing

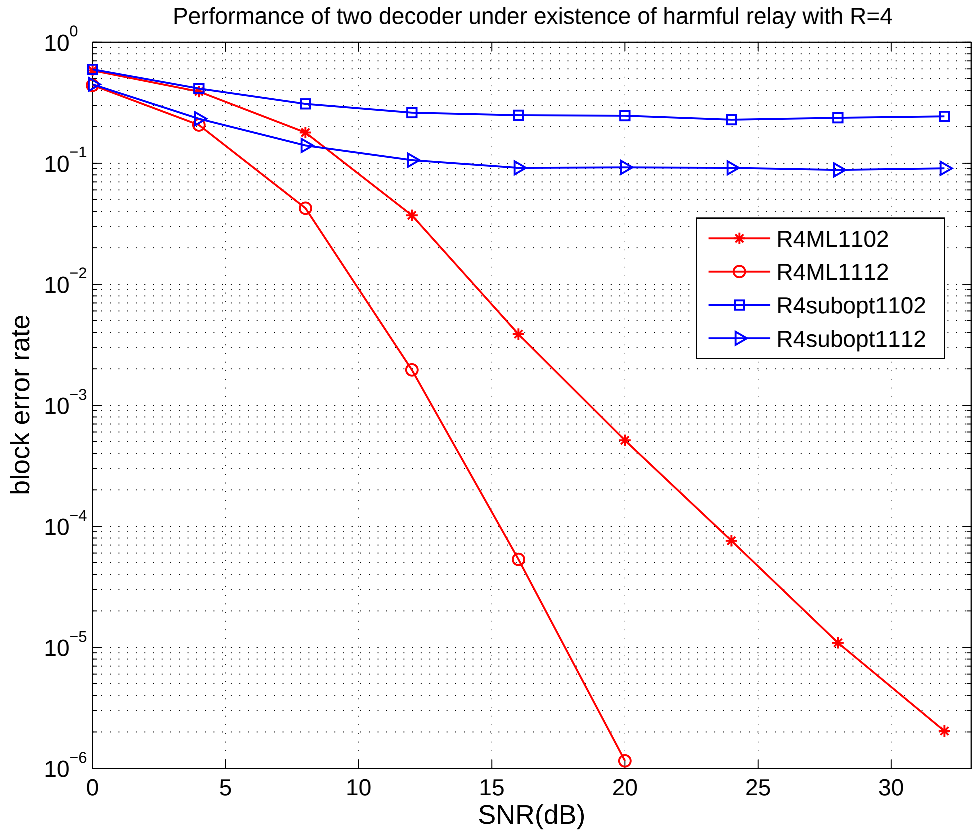

Figure 4.5: The block error rate versus SNR of two decoder under existence of harmful relay.

no error detection, *i.e.*, all relays are always active. This scenario is simpler in implementation since the relay do not need any control protocol. It corresponds to $p_{1|0} = 0$ and $p_{0|1} = 1$ in our model. We simulate it under both number of relay $R = 4$ and $R = 2$. In Fig. 6, "R2nocontrolsubopt" stands for $R = 2$ and that suboptimum correlator decoder is used. Similar for other legend. From the figure, we can observe that the achieved diversity order is 2 for suboptimum decoder for both $R = 2$ and $R = 4$. It verified **Proposition 3** given in Sec. 3.2, *i.e.*, correlator-like decoder can retrieve only diversity of order $\min\{L + L_{01}, RL_{10}, RL\} = L = 2$ under detection accuracy $p_{0|1} =$ positive constant $\doteq \mathcal{P}^{-0}$ and $p_{1|0} = 0 \doteq \mathcal{P}^{-\infty}$. In such case, as we can see, increasing number of relay has only limited help and has no increase in diversity if we employing the correlator-like decoder at receiver. On the other hand, ML decoder achieves diversity 2 and 4 for $R = 2$ and $R = 4$ respectively, as presented in Fig. 7. Comparing with **Fact 2** in Sec. 3.2, diversity under fixed positive $p_{0|1}$ do not exceed $\lceil \frac{R}{2} \rceil L$, we can see that ML decoder achieves $\lceil \frac{R}{2} \rceil L$ in the presented cases. It could be guessed that the maximum diversity under fixed positive $p_{0|1}$ and $p_{1|0} = 0$ would indeed achieve $\lceil \frac{R}{2} \rceil L$ by an ML decoder. Actually, we have following conjecture,

Conjecture 4 For $p_{0|1} \doteq \mathcal{P}^{-L_{01}}$ and $p_{1|0} \doteq \mathcal{P}^{-L_{10}}$, the diversity d achieved by an ML decoder is

$$d = \begin{cases} d_1 = \lfloor \frac{R}{2} \rfloor (L + \min\{L, L_{01}\}) \\ \qquad\qquad + (\lceil \frac{R}{2} \rceil - \lfloor \frac{R}{2} \rfloor) L_{10}, \text{ for } L_{01} < L_{10} \\ d_2 = R \min\{L_{10}, L\}, \text{ for } L_{01} > L_{10} \\ \min\{d_1, d_2\}, \text{ for } L_{01} = L_{10} \end{cases}$$

This conjecture can be proved by a similar argument of finding the worst diversity in summand of (3.30) and solving the minimization, which is

$$\min_{\substack{k_{10}+k_{01} \le R \\ k_{10} \ge 0, k_{01} \ge 0}} k_{10} L_{10} + k_{01}(L + L_{01}) + \max\{R - k_{10} - 2k_{01}, 0\} L,$$

provided the guess (4.1) is true. Finally note that **Conjecture 4** covers both **Fact 1** and **Fact 2**, and as mentioned, the result of ML decoder in Fig. 7 also conforms to it.

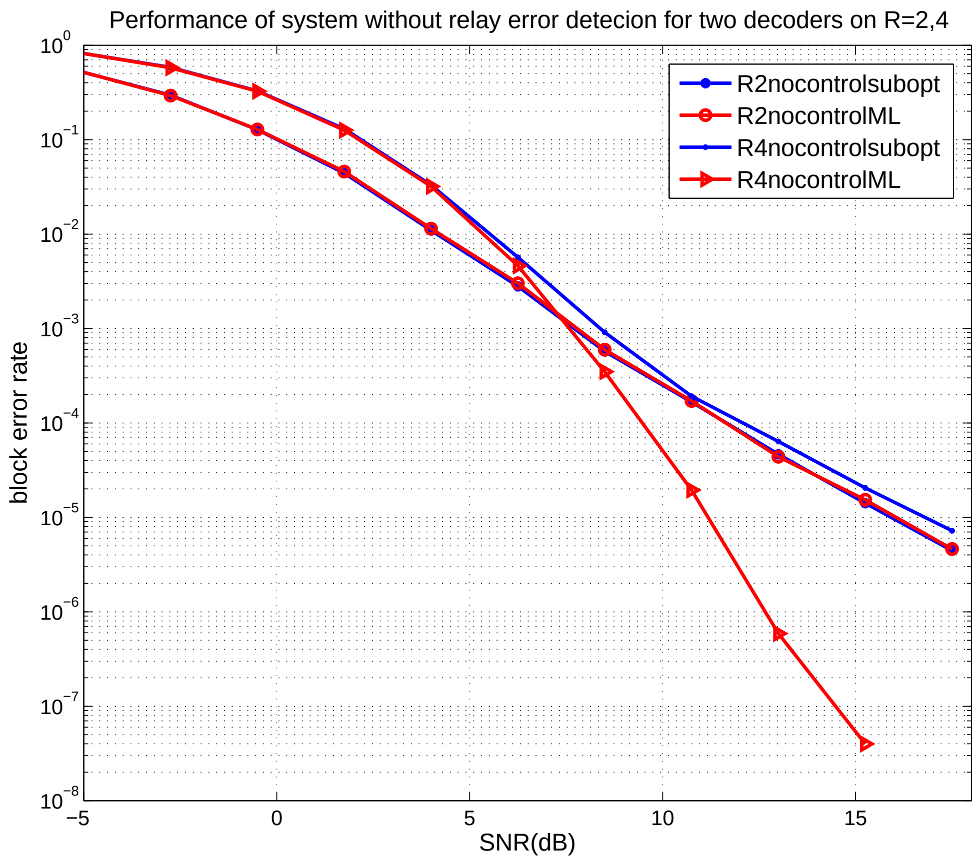

Figure 4.6: The block error rate versus SNR of two decoder under no control protocol.

Chapter 5

Conclusion and Future Work

5.1 Concluding Remarks

In this work, we analyzed the diversity order of noncoherent cooperative SFC in a two-hop wireless relay network with the DAF protocol. We considered a realistic scenario that each relay node may fail to correctly decode the message from the source node, thus modeling the uncertainty naturally arisen in wireless relay networks. We first analyzed the system under a usual assumption of perfect relay censoring and discussed partial knowledge receiver and completely noncoherent receiver. For partial knowledge receiver, we analyzed the PEP based on the ML decoding rule and establish the code design criteria. For completely noncoherent receiver, we proposed a maximum diversity achieving suboptimum correlator-like decoder, and showed that the maximum achievable diversity using the suboptimum decoder is the same as that of the partial knowledge ML decoder. We justified that the diversity gain of the non-coherent cooperative SFC in the relay network under perfect relay error detection is, on the average, equal to the product of the total number of cooperating relays and the channel order in the relay-destination link, a result identical to that of the non-coherent SFC in MIMO-OFDM systems. This provided us with the insight that the noncoherent virtual MIMO networks could potentially offer as good performance, in terms of diversity order, as that promised by the conventional noncoherent MIMO networks. Furthermore, we explored the impact of imperfect censoring and showed that it has significant influence on the achievable diversity, especially when there exists any "harmful" relays. We concluded that, in a DAF relay system, it is crucial to carefully design the relay censoring schemes in order to maintain the achievable diversity that potentially existed in the system.

5.2 Future work

The mathematical analyses on the system under imperfect censoring employing the ML decoder at the receiving end has not completely unveiled. In general, how to accurately analyze the PEP of the ML decoder with mixture gaussian likelihood is an important and interesting problem that is worthwhile probed into. In some cases, for example, the case we considered in Sec. 3.1.2, we can simplify it and have a closed-form solution. However, as the suboptimum decoder fails to achieve the maximum diversity order as indicated in Sec. 3.2, we still need to tackle it in a more general scenario.

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