國　立　交　通　大　學

電子工程學系　電子研究所碩士班

碩　士　論　文

感知無線網路下基於累和式演算法之頻譜變化即時偵測研究

CUSUM-Based Quickest Spectrum Detection in Cognitive Radio Networks

研 究 生: 劉藹璇

指導教授: 簡鳳村 博士

中 華 民 國 九 十 九 年 七 月

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感知無線網路下基於累和式演算法之頻譜變化即時偵測研究

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摘要

在本篇論文，我們從序列式即時變化偵測的觀點來探討感知無線電網路頻帶偵測問題。為了能增進感知網路中次要使用者與主要使用者之間的共存性，我們提出以「累積和程序」為基礎之演算法，在利用已知主要使用者之初始訊號結構下，能有效且即時地偵測到主要使用者的重新運作並歸還次要使用者投機使用之頻帶資源以避免對主要使用者造成嚴重干擾。我們特別考慮感知使用者與主要使用者之間存在衰變通道且於接收端僅知通道統計特性下累和式即時變化偵測的可行性。

我們首先考慮一個由單一主要使用者和單一次要使用者所構成的感知無線電網路。在考慮衰變通道效應之下，我們從非同調與同調的角度出發各別提出以累積和程序為基礎的即時偵測演算法。並於第二階段，在考慮各種分散式協作機制之下，將先前所提出的累和式演算法推演於由多個次要使用者共同參與之合作式偵測，以期能進一步地藉由多重使用者所提供的空間多樣性以增進即時偵測主要使用者活動的效能表現。在模擬中，我們在主要使用者的訊號模型上採取了 IEEE 802.16e 的長同步碼做為序列偵測的目標，並且在各種衰變環境下驗證並比較所提之累和式演算法的可行性與效能表現。

CUSUM-Based Quickest Spectrum Detection in Cognitive Radio Networks

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Abstract

In this thesis, we study the problem of spectrum sensing in cognitive radio networks from the view of sequential change-point detection (also called quickest detection). Aiming at avoiding interference to licensed primary users, which can help promote the willingness of the primary systems to accept the idea of coexistence with cognitive users, we propose several cumulative-sum (CUSUM)-based algorithms for detecting as quickly as possible the event that the dormant primary systems start reclaiming the use of the spectrum by exploiting the feature of the incipient part of the primary signals. Particularly, we are interested in the applicability of CUSUM-based tests to the coexistence problem when considering the fading effects between the cognitive and the primary user, given only the statistical channel information.

In the first part of the thesis, we consider the single-user scenario and propose four CUSUM-based algorithms according to different perspectives and manners on the fading effects under flat fading or frequency-selective fading environments. In the second part of the thesis, we extend the proposed CUSUM-based algorithms to the case of cooperative quickest detection, where a number of cognitive users provide decision strategies and collaboratively detect the beginning of the reclaims of the primary signals. In the

simulations, we demonstrate the effectiveness of the proposed algorithms with the settings defined in IEEE 802.16e as the primary signal model. Comparisons of the proposed CUSUM-based algorithms are also provided in the simulations.

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劉謞璇

誌於新竹

民國九十九年七月

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Chapter 1

Introduction

1.1 Motivation

Recently, cognitive radio has emerged as a feasible approach to alleviate the problem that most of the allocated radio spectrum are used sporadically and utilized inefficiently in wireless communications applications [2]. Generally speaking, a cognitive radio is a software-defined radio aware of its environment and autonomously adapting its operations to achieve desired objectives in response to unexpected variations. Many kinds of possible mechanisms have been proposed for promising better system efficiency and spectrum utilization. One main approach is to concede unlicensed cognitive users to opportunistically use those frequency spectrum yielded by underlying idle licensed primary users to improve the efficiency of current spectral utilization. In this case, under the concern of possible interference to primary users, cognitive users should assure that they could fairly detect the activities of primary users and try to avoid interference to the primary licensed users as much as possible, which can help promote the willingness of the primary systems to accept the idea of coexistence with unlicensed cognitive users. Specifically, the effectiveness of detection strategy applying in cognitive radio networks has been one of the key factors that the opportunistic mechanism is workable of increasing the efficiency in two main aspects. One is that the cognitive users have to know whether there exist primary users transmitting or not before opportunistic uses, and the other issue is to vacate frequency bands as quickly as possible when primary user reclaiming the use of the spectrum. This corresponds to the following two types of spectrum detection (or commonly termed as ***spectrum sensing***) problems in cognitive radio networks:

- Detecting the existence of spectrum holes in order to opportunistically use that spectrum vacancy.
- Detecting the retransmission of primary users in a spectrum currently used by a cognitive user and determine the time instant at which the retransmission starts.

There have been plenty of research studies that apply the conventional block-based detection to the detection problems mentioned above in cognitive radio networks [3]. However, the fact that the efficiency of speculative spectrum utilization would be enhanced if we could detect as quickly as possible the idling bands, and that the interference to the primary users could be avoided as much as possible if a cognitive user is capable of perceiving the restart of transmission of the primary user excites our interest in the applicability of sequential detection on cognitive radios.

On the other hand, prior information about primary user's incipient signaling structureis sometimes available to public in existing licensed systems such as WiMAX systems. In this situation, it is possible to enhance the efficacy in awareness of the reoccupying of primary user by exploiting features of the frame structure in cognitive detection. In the thesis, combined with the aim to responding as quickly as possible the activities level of primary users, we are interested in contriving effective detection strategies that make use of known feature of primary signaling in sequential manner for promoting the cognitive coexistence.

1.2 Why Quickest Detection?

Most research and development about spectrum sensing in cognitive radios concentrate on classical block-based detection schemes such as energy detection, feature detection or matched filtering [3]. In these schemes, cognitive users always collect a succession of observations within a fixed sensing time window, and then calculate corresponding test statistics for decisions. Most of them put emphasis on maximizing ***probability of de-***

tection while maintaining an acceptable level of ***false alarm rate***, particularly in detecting whether there is spectrum hole existing in the environment for possible opportunistic cognitive transmissions. On the other hand, when focusing on detecting primary user's retransmission activity and striving at avoiding interference to primary users by vacating frequency bands that are occupied by cognitive users as soon as retransmission of underlying primary users occurs, the delay between the estimated time instant and the true primary user's retransmission time instant becomes a crucial index to the feasibility of cognitive mechanisms. However, due to the inherent nature of block-based detection, this delay performance of detection has not been addressed in the conventional methods [4], [5], [6], and [7]. Thus, this motivates us to study the applicability of sequential-type detection, especially the change-point detection (also called quickest detection), in dealing with the detection problem in cognitive networks.

Quickest detection is a branch topic of sequential-type detection [8], [1]. Conceptually, the idea is to detect changes in distribution of observations as quickly as possible, which coincides the aim that secondary users should detect the change in the activity level of the primary users immediately. In contrast to block-based approaches, mean delay of detection is an essential performance index in sequential change-point detection. This property makes the quickest detection as an appropriate framework for dynamic spectrum sensing in cognitive radio networks.

1.3 Related Work

The problem of detecting an abrupt change was first studied by Page in the context of quality control [9]. In the conventional formulation of the change-point detection problem, there is a sequence of observations whose distribution changes at some unknown point in time, and the goal is to detect this change as quickly as possible subject to false alarm constraints. In the simplest situation where the observations are independent and identically distributed (i.i.d.) with known distributions before and after the change, the problem is well understood and has been solved under a variety of criteria since the seminal work by Page. Under minimax formulation, which is first proposed by Lorden [10], the well-known Page's cumulative sum (CUSUM) algorithm has been proved to be optimal [1] in the sense of minimizing the mean delay of detection while maintaining a certain level of false alarm rate [11] and [12].

The extension to composite hypotheses testing problems where the distributions of observations before or after change are not completely specified could be found in [10], [13] and [14]. In [15], the authors consider the problem of detecting a change from one given stationary and ergodic stochastic process to another such process. Change-point detection involving dependent observations is discussed in [16], where the authors shows that Page's CUSUM procedure is still asymptotically minimax optimal for dependent observations under some conditions which are difficult to verify in general. Although it is easy to extend the CUSUM decision rule for dependent observations by replacing statistics by conditional density as proposed in [15] and [16], it has been an open problem concerning whether the asymptotic optimality[2] of the CUSUM rule still holds as commented in [17].

The first generalization of the CUSUM detection procedure regarding multichannel and distributed systems is proposed in the work by Tartakovsky [18] and further extended and discussed in [19] where asymptotically optimal procedures for two distributed scenarios are presented based on i.i.d. local observations before and after change. In work [20], also concerning the simplest case with i.i.d. local observations, the author proves that a CUSUM procedure based on binary-quantized data with a monotone likelihood ratio quantizer (MLRQ) is asymptotically optimal under a condition on second moments in the system with limited local memory and develops asymptotic theory in the system with full local memory. The case that there exists unknown parameter in the post-change detection as further extension in the distributed multisensor setting with binary quantization is ad-

[1]In the sense that the stopping time of CUSUM procedure minimizes the worst average conditional delay (t_0 denotes the change time and t_a denotes the alarm time) $\bar{\tau}^* = \sup_{t_0 \geq 1} \text{esssup} \mathbf{E}_{\theta_1}(t_a - t_0 + 1 | t_a \geq t_0, \mathcal{F}_1^{t_0-1})$, where $\mathcal{F}_1^{t_0-1}$ is the filtration, namely the smallest σ-field with respect to observations $y_1, \ldots, y_{t_0-1}$ and the essential supremum (esssup) means the worst case detection delay, for a fixed mean time between false alarm $\bar{T} = \mathbf{E}_{\theta_0}(t_a)$

[2]An optimal algorithm for change detection is any algorithm that minimizes the worst mean conditional delay for detection $\bar{\tau}^*$ for a fixed mean time between false alarm $\bar{T}$ and an algorithm is *asymptotically optimal* if it reaches this optimal property asymptotically when $\bar{T} \to \infty$

dressed in [21]. In [22], Tartakovsky proposes nonparametric multi-chart CUSUM test for the rapid intrusion detection applied to general stochastic models in multichannel sensor systems and show that the proposed multi-chart detection procedure typically performs significantly better than single-channel counterparts.

Under cognitive radios setup, the authors in [23] introduce using generalized likelihood ratio (GLR) test combined with parallel CUSUM algorithm and propose a successive refinement to tackle the problem with unknown amplitude of primary signals, which consist of mutually independent sequence before and after change. And, scenarios with different information levels about primary users known by cognitive users are considered in [24], in which GLRT-based algorithm and non-parametric approach are developed based on mutually independent homogeneous gaussian distributed signal model. The work in [25] deals with the spectrum detection problem by introducing the CUSUM-based quickest detection with hidden Markov Models (HMMs). Related work of cooperative spectrum sensing based on CUSUM procedure can be found in [26] and [27], both based on i.i.d. local observations. In [26], collaborative quickest detection in an ad hoc network, where no data fusion center is needed and collaboration among sensors is through information exchange, is proposed for multi-node case. Cooperative spectrum sensing schemes applying linear test on CUSUM statistics for exempting the need of estimation of unknown parameters in post-change distribution are provided in [27] under different distributed scenarios.

1.4 Contributions of the Research

Aiming at avoiding interference to licensed primary users, which can help promote the willingness of the primary systems to accept the idea of coexistence with cognitive users, we propose several cumulative-sum (CUSUM)-based algorithms that exploits the feature of the incipient part of the primary signals for detecting as quickly as possible the event that the dormant primary systems start reclaiming the use of the spectrum. Particularly, our formulation captures possible fading effects between the cognitive and the primary user given only the statistical channel information at the receiver end.

Contrast to the homogenous-distributed and independent observations after change that are commonly assumed in conventional quickest detection, the detection problem we deal with involves non-homogenous and innately dependent observations after the reoccupying of the coexisting primary system. To tackle the problem, we first consider the single-user scenario and propose four CUSUM-based algorithms depending on different assumptions on the fading environments. Specifically, we call the four proposed algorithms as the classical CUSUM, weighted CUSUM, GLRT-based CUSUM, and MMSE-based CUSUM algorithms, which are briefly described as follows. In the classical CUSUM algorithm, we treat the unknown channel factors as random variables with known prior statistics and calculate the likelihood ratio between joint probability density functions of observations under the conditions before and after the reclaiming occurs. While in weighted CUSUM and GLRT-based CUSUM algorithm, we consider the unknown channel coefficient as deterministic but unknown constant during the detection process. We weight the likelihood ratio by applying prior information as weighting function and estimate the unknown parameter through all available observations. The estimates are then substituted into the likelihood. Depart from the philosophy employed by the GLRT-based CUSUM algorithm, the MMSE-based CUSUM algorithm is to estimate the unknown fading coefficient by incorporating prior knowledge. We also examine the required length of backward observations that keeps comparable efficacy with the one without any curtailment of observational window.

Further, we extend the proposed CUSUM-based algorithms to the case of cooperative quickest detection, where a number of cognitive users provide decision strategies and collaboratively detects instantaneously the beginning of the reclaims of the primary signal under three different distributed frameworks. The first distributed scenario is in centralized setting, which means that the original data received at sensors are sent completely to a fusion center where a final decision is made based on all sensor massages for global CUSUM test. In the cases considering decentralized framework, we resort to hard fusion of local CUSUM and global CUSUM with quantized local decision. In hard fusion of local CUSUM scheme, we assume that each of the cooperative sensors has sufficient memory to individually perform CUSUM-based quickest detection then the fusion center makes final decision based on local decisions sent by sensors according to hard-decision combining rules. In the decentralized scheme considering global CUSUM with quantized local decisions, we propose using an approximation on distributions of the received signal after reoccupying to tackle the quantization at the local sensors, and the CUSUM-based algorithm is performed at the fusion center while the local sensors are assumed memory-less and send quantized version of their observations for decision making.

In the simulations, we demonstrate the effectiveness of the proposed algorithms with the settings defined in IEEE 802.16e as the primary signal model. Comparisons of the proposed CUSUM-based algorithms are also provided in the simulations.

To sum up, the contributions of the research include:

- We deal with the spectrum sensing problem under fading environments in a sequential detection viewpoint, which involves non-homogenous distributed and innately mutually dependent observations after the reoccupying of the coexisting primary system. This problem has not been studied before. Although the work in [25] also discusses quickest spectrum detection with dependent observations after change, the dependency among the observations lies on the sampling of the wideband power spectrum density. They first train the corresponding HMM parameters of specific primary signal and then perform quickest pattern cognition, which heavily depends on the Markov properties in calculating statistics, to detect the appearance of a predefined pattern as quickly as possible. While in our proposed schemes, the dependency among observation sequence is due to possible frequency selective fading effects.

- We develop several effective change detection strategies based on CUSUM procedure with practical assumptions. In addition, we also propose cooperative schemes as further extension. By simulation, we demonstrate the effectiveness of the proposed algorithms either under flat fading or frequency-selective fading case.

Chapter 2

Cognitive Radio and CUSUM-Based Quickest Detection Preliminary

2.1 Cognitive Radio

Cognitive radio technology, which is first proposed by Mitola in [2], has emerged as a potential candidate to revolutionize spectrum utilization. In general, cognitive radio is defined as a software-defined radio that is aware of its surrounding and autonomously adapting its operations to achieve desired objectives in response to unexpected variations, based on the active monitoring of several factors in the external and internal radio environment, such as radio frequency spectrum, user behavior and network state. The need for CRs is motivated by various factors. Early works focus on the capability of enhancing the flexibility of personal services in a way that supports automated reasoning about the needs of the anticipated user. The radio seeks out the required information and provides the user with instructions or the desired service. Fig. 2.1[1] illustrates the cognition cycle which consists of Observe, Orient, Plan, Decide, Learn and Act phases, has been widely used to understand and analyze the performance of cognitive processes in cognitive radios and cognitive networks. More recently, the problem of spectrum under-utilization urges the need for intelligent radios to tackle the dynamic allocations efficiently. Although the

[1]This figure is adapted From Mitola, "Cognitive Radio: An Integrated Agent Architecture for Soft-ware Defined Radio", Doctor of Technology, Royal Inst. Technol. (KTH), 2000, pp 48

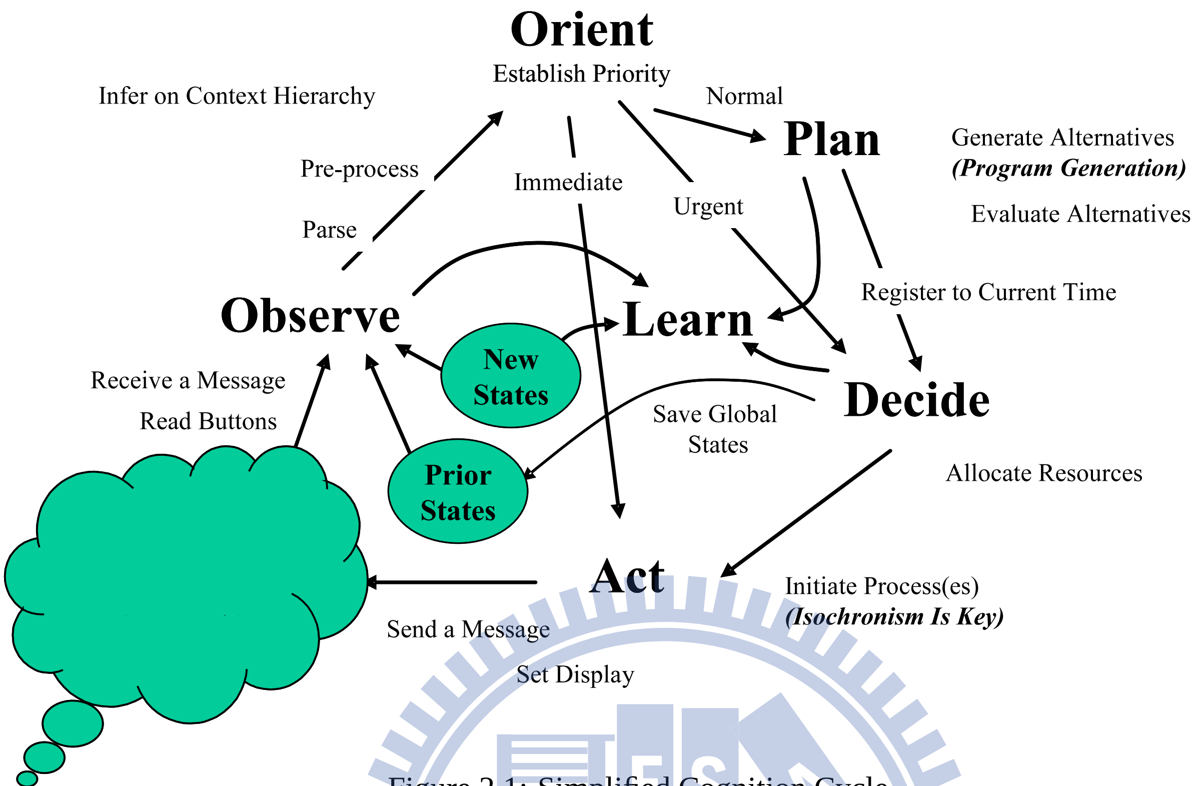


Figure 2.1: Simplified Cognition Cycle.

initial aim of CR not directly lies on promoting the utilization of spectrum resource, it does serves as a potential candidate to alleviate this problem since cognitive users could either opportunistically utilize idled spectrum by detecting the spectrum hole or actively negotiate with primary users, *i.e* the existing licensed users, to access the spectrum. There have been plenty of researches on CR-related topic, which could be classified into three fundamental tasks [3]: 1. Radio-scene analysis, which includes estimation of interference temperature of the radio environment and detection of spectrum holes. 2. Channel state estimation and predictive modeling, which encompasses estimation of channel-state information and prediction of channel capacity for use by the transmitter. 3. Transmit power control and dynamic spectrum management.

Our work is focus on detecting the activity level of primary users under fading environments, aiming at avoiding interference to licensed users for promoting the coexistence with underlying primary system, which we adopt an alternative view in sequential sense contrary to conventional block-based detection to tackle with.

2.2 CUSUM-Based Quickest Detection

Change detection is a fundamental problem arising across various branches of science, finance and engineering. By taking the change-point as deterministic but unknown parameter, we focus ourselves on the minimax formulation of change-point detection under the simplest case and introduce the corresponding efficient detection scheme, Page's cumulative sum (CUSUM) algorithm, with its conceptual derivations and optimal properties for background understanding. Extensive and comprehensive studies could be referred to [1] and [8] for deeper materials.

2.2.1 A Simple Case - Concept and Page's CUSUM Algorithm

Fundamental Concept

Started by a very important concept in analysis of mathematical statistics, the logarithm of the likelihood ratio, defined by

$$l(y) = \ln \frac{p_{\theta_1}(y)}{p_{\theta_0}(y)} \tag{2.1}$$

and referred to as the log-likelihood ratio, CUSUM algorithm is developed from the key statistical property of this ratio as following:

Given that $\mathbf{E}_{\theta_0}$ and $\mathbf{E}_{\theta_1}$ denote the expectations of the random observation under the two distributions p_{θ_0} and p_{θ_1}, respectively. Then, it can be easily verified

$$\mathbf{E}_{\theta_0}(l) < 0 \text{ and } \mathbf{E}_{\theta_1}(l) > 0. \tag{2.2}$$

Namely, *a change in the parameter θ is reflected as a change in the sign of the mean value of the log-likelihood ratio*, which can be regarded as a kind of detectability of change in distribution [1].

Page's CUSUM Algorithm

Consider a sequence of independent random variables $\{y_k\}$ with a probability density $p_\theta(y)$ depending upon only one scalar parameter. Before the unknown change time t_0, the parameter θ is equal to θ_0, and after the change it is equal to θ_1. A decision strategy to

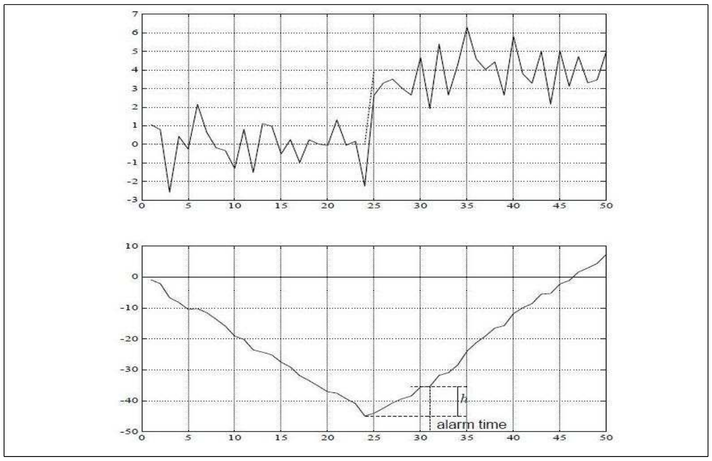


Figure 2.2: Typical behavior of the log-likelihood ratio S_k corresponding to a change in the mean of a Gaussian sequence with constant variance : negative drift before and positive drift after the change. (Fig.2.5 of [1])

raise an alarm of the presence of change can be regarded as a stopping time t_a, which is characterized only by the past observations at each time instant. Let $S_j^k = \sum_{i=j}^{k} l_i$, with $l_i = \ln \frac{p_{\theta_1}(y_i)}{p_{\theta_0}(y_i)}$, be the log-likelihood ratio for the observations from y_j to y_k. Intuitively, the typical behavior of the log-likelihood ratio S_1^k shows a negative drift before change, and a positive drift after change, as shown in Fig. 2.2. Therefore, the relevant information, as far as the change is concerned, lies in the difference between the value of the log-likelihood ratio and its current minimum value; and the corresponding decision rule is then, at each time instant, to compare this difference to some threshold said $\hbar$ as follows:

$$g_k = S_1^k - m_k \geq \hbar, \text{ where } m_k = \min_{1 \leq j \leq k} S_1^j \tag{2.3}$$

which leads to the following equivalent decision function

$$g_k = \max_{1 \leq j \leq k} S_j^k \text{ and } t_a = \min\{k : g_k \geq \hbar\}. \tag{2.4}$$

It could be easily verified that the stopping time t_a is equal to the one determined by Page's procedure (also known as CUSUM algorithm) introduced and derived as follows, which has been proved optimal in the sense that it minimizes the worst average conditional delay

$\bar{\tau}^* = \sup_{t_0 \geq 1} \text{esssup}\mathbf{E}_{\theta_1}(t_a - t_0 + 1 | t_a \geq t_0, \mathcal{F}_1^{t_0-1})$ [2] for a fixed mean time between false alarm $\bar{T} = \mathbf{E}_{\theta_0}(t_a)$ [8].

In specific, Page suggested the use of repeated testing of the two simple hypotheses:

$$\begin{aligned} \mathbf{H}_0 &: \theta = \theta_0 \\ \mathbf{H}_1 &: \theta = \theta_1 \end{aligned} \tag{2.5}$$

with the aid of the sequential probability ratio test (SPRT). The SPRT is defined with the aid of the pair (d, T) where d is the decision rule and T is a stopping time. The definition of the SPRT is thus

$$T = \min\{k : S_1^k \geq \hbar \text{ or } S_1^k \leq 0\} \tag{2.6}$$

and $d = 1$ if $S_1^T \geq \hbar$; otherwise $d = 0$.

The key idea of Page was to restart the SPRT algorithm as long as the previously taken decision is $d = 0$. The first time at which $d = 1$, we stop observation and do not restart a new cycle of the SPRT. This time is then the alarm time at which the change is detected. The resulting decision rule can be rewritten in a recursive manner as

$$g_k = \begin{cases} g_{k-1} + \ln \frac{p_{\theta_1}(y_k)}{p_{\theta_0}(y_k)}, \text{ if } g_{k-1} + \ln \frac{p_{\theta_1}(y_k)}{p_{\theta_0}(y_k)} > 0 \\ 0, \text{ if } g_{k-1} + \ln \frac{p_{\theta_1}(y_k)}{p_{\theta_0}(y_k)} \leq 0 \end{cases} \tag{2.7}$$

i.e.,

$$g_k = (g_{k-1} + s_k)^+ \text{ with } g_0 = 0, \tag{2.8}$$

and the stopping rule is $t_a = \min\{k : g_k \geq \hbar\}$, which is equivalent to other forms presented before.

On the other hand, for deriving the asymptotical optimality of the CUSUM algorithm, it is convenient if we interpret the CUSUM stopping time t_a by using a set of parallel open-ended SPRT, which are activated at each possible change time $j = 1, \ldots, k$, and with upper threshold h and lower threshold equals to $-\infty$. Each of these SPRT stops at time k if, for some $j \leq k$, the observations $y_j, \ldots, y_k$ are significant for accepting the hypothesis about change.

[2] $\mathcal{F}_1^{t_0-1}$ is the filtration, namely the smallest σ-field with respect to observations $y_1, \ldots, y_{t_0-1}$; the essential supremum (esssup) means the worst case detection delay.

Let T_j be the stopping time for the open-ended SPRT activated at time j:

$$T_j = \min\{k \geq j : S_j^k \geq \hbar\} \tag{2.9}$$

where we use the convention that $T_j = \infty$ when this minimum is never reached. Lorden [10] defines the following extended stopping time as the minimum of the $\{T_j\}$:

$$T^* = \min_{j=1,2,\dots} \{T_j\} \tag{2.10}$$

It also can be showed that $t_a = T^*$.

Optimal Properties

First, deduced from the properties of a set of parallel open-ended SPRT, the relation between the lower bound for the mean time between false alarms and the upper bound for the worst average conditional delay for detection under the i.i.d. assumption is stated as follows

Theorem 1 (Thm 5.2.1 in [1]) *Let T be a stopping time with respect to $\{y_1, y_2, \dots\}$ such that*

$$\mathbf{P}_{\theta_0}(T < \infty) \leq \alpha$$

For $k = 1, 2, \dots$, let $\tilde{T}_k$ be the stopping time obtained by applying T to $\{y_k, y_{k+1}, \dots\}$ and let $T_k = \tilde{T}_k + k - 1$.

Define the extended stopping time by

$$T^* = \min\{T_k | k = 1, 2, \dots\}$$

Then, T^* is such that

$$\mathbf{E}_{\theta_0}(T^*) \geq \frac{1}{\alpha}$$

$$\bar{\mathbf{E}}_{\theta_1}(T^*) \leq \mathbf{E}_{\theta_1}(T)$$

where

$$\bar{\mathbf{E}}_{\theta_1}(T^*) = \sup_{k \geq 1} \mathit{esssup} \mathbf{E}_k(T^* - k + 1 | y_1, \dots, y_{k-1})$$

Applying the above theorem to the case that T_k corresponds to an open-ended SPRT with upperthreshold $\hbar$:

$$T_k = \begin{cases} \min\{n \geq k : \sum_{i=k}^{n} \ln \frac{p_{\theta_1}(y_i)}{p_{\theta_0}(y_i)}\} \\ \infty \qquad \text{if no such } n \text{ exists} \end{cases}$$

Then, the extended stopping time T^* is Page's CUSUM stopping time t_a and

$$t_a = T^* = \min\{T_k | k = 1, 2, \ldots\}$$

In this case, it follows from Wald's identity (see **Thm.** 4.3.2 of [1]) that when $\hbar$ goes to infinity

$$\mathbf{E}_{\theta_1}(T) \sim \frac{\hbar}{\mathbf{K}(\theta_1, \theta_0)}$$

where

$$\mathbf{K}(\theta_1, \theta_0) = \mathbf{E}_{\theta_1}[\ln \frac{p_{\theta_1}(y_i)}{p_{\theta_0}(y_i)}]$$

is the Kullback information. Second, from the Wald's inequality, we have

$$\mathbf{P}_{\theta_0}(T < \infty) \leq e^{-\hbar} = \alpha.$$

Thus, the worst mean delay is given by

$$\bar{\tau}^* = \bar{\mathbf{E}}_{\theta_1}(T^*) \sim \frac{\ln \bar{T}}{\mathbf{K}(\theta_1, \theta_0)}, \text{ as } \hbar \to \infty$$

where $\bar{T}$ denotes the *mean time between false alarm*

$$\bar{T} = \mathbf{E}_{\theta_0}(t_a).$$

The above approximal equation gives the basic relation between the delay for detection and the mean time for false alarm for the CUSUM algorithm in the simplest situation.

Secondly, Lorden proved that the infimum of the worst mean delay among a class of extended stopping times is precisely given by this relation. The main results of Lorden concerning the asymptotically optimal solution of change detection problems are briefly described as below.

Theorem 2 (Thm 5.2.2 in [1]) *Let* $\{T(\alpha) | 0 < \alpha < 1\}$ *be a class of open-ended SPRT such that*

$$\mathbf{P}_{\theta_0}[T(\alpha) < \infty] \leq \alpha$$

and for all real θ_1

$$\mathbf{E}_{\theta_1}[T(\alpha)] \sim \frac{\ln(\alpha^{-1})}{\mathbf{K}(\theta_1, \theta_0)}.$$

For $\gamma > 1$*, let* $\alpha = \gamma^{-1}$ *and let* $T^*(\gamma)$ *be the associated extended stopping time defined by*

$$T^*(\gamma) = \min\{T_k(\alpha) | k = 1, 2, \ldots\}$$

Then,

$$\mathbf{E}_{\theta_0}[T^*(\gamma)] \geq \gamma$$

and for all real θ_1*,* $T^*(\gamma)$ *minimizes* $\bar{\mathbf{E}}_{\theta_1}[\bar{T}(\gamma)]$ *among all stopping times* $\bar{T}(\gamma)$ *satisfying the above constraint.*

Furthermore,

$$\bar{\mathbf{E}}_{\theta_1}(T^*(\gamma)) \sim \frac{\ln \gamma}{\mathbf{K}(\theta_1, \theta_0)} \text{ as } \gamma \to \infty$$

This theorem shows the optimality of the CUSUM algorithm from an asymptotic view, what is often called *first-order optimality* [28]. More precisely, CUSUM algorithm is optimal, with respect to the worst average conditional delay, when the mean time between false alarms goes to infinity. Based upon the same criterion of worst average conditional delay, another optimality result for CUSUM algorithm is proven in [11] and [12], in a nonasymptotic framework: The CUSUM algorithm minimizes the worst average conditional delay for all $\bar{T} \geq \bar{T}_0$, where $\bar{T}_0$ is small for most cases of practical interest. Generally, it is difficult to obtain explicit expressions for performance analysis in the finite case. This asymptotic point of view is convenient in practice because a low rate of false alarms is always desirable.

Note that, the CUSUM algorithm is optimal when it is tuned with the true values of the parameters before and after change. When the algorithm is used in situations where the actual parameter values are different from the preassigned values, this optimality is lost.

2.2.2 Extension to Composite Hypothesis Cases

Consider the case where the parameter before change θ_0 is assumed to be known while θ_1 is unknown based on the assumption that observations are independent of one another conditioned on the change-point. There are two main approaches as described below.

- Weighted-CUSUM Algorithm:

  It is an algorithm comes from the idea of *weighting the likelihood ratio with respect to all possible values of the parameter* θ_1*, using a weighting function* $dF(\theta_1)$, where $F(\theta_1)$ may be interpreted as the cumulative distribution function of a probability measure. It was derived for change detection in [29], and is a direct extension of the CUSUM stopping time defined as follows. Let

$$\tilde{\Lambda}_j^k = \int_{-\infty}^{\infty} \frac{p_{\theta_1}(y_j, \ldots, y_k)}{p_{\theta_0}(y_j, \ldots, y_k)} dF(\theta_1) \tag{2.11}$$

  be the weighted likelihood ratio for the observations from time j up to time k. Then the stopping time is

$$t_a^{Weighted} = \min\{k : \max_{1 \leq j \leq k} \ln \tilde{\Lambda}_j^k \geq \hbar\}. \tag{2.12}$$

  The most simple choices involve using the uniform distribution over a specified interval that contains all possible values of the parameter θ_1, or Dirac masses on some specified values. Another useful choice is the Gaussian distribution. Note that this type of algorithm cannot be written in a recursive manner as the classical CUSUM algorithm described before. Some asymptotic properties related to weighted-CUSUM algorithm could be found in Section 5.2.3 of [1].

- General Likelihood Ratio Test (GLRT) based CUSUM Algorithm:

  In this approach, the unknown parameter θ_1 is replaced by its maximum likelihood estimate as

$$\hat{\Lambda}_j^k = \frac{\sup_{\theta_1} p_{\theta_1}(y_j, \ldots, y_k)}{p_{\theta_0}(y_j, \ldots, y_k)} \tag{2.13}$$

  with

$$t_a^{GLRT} = \min\{k : \max_{1 \leq j \leq k} \ln \hat{\Lambda}_j^k \geq \hbar\}. \tag{2.14}$$

  The properties of the case that consider hypotheses $\mathbf{H}_0 : \{\theta = \theta_0\}$ and $\mathbf{H}_1 : \{\theta \geq \underline{\theta}, \theta_0 < \underline{\theta}\}$ with the aid of an exponential family of distributions (*i.e.*, $p_\theta(y) = h(y) \exp^{\theta y - d(\theta)}$) are derived in Section 5.3.1 of [1].

As the weighted-CUSUM algorithm, the GRLT-based approach still has no recursive expression thus needs to store all the observations and re-estimate the unknown parameter in all time slots.

2.2.3 Dependent Observations

In this subsection, we consider the situation in which we have dependent observations and briefly introduce the result in generalization of Lorden's asymptotical optimality on the CUSUM algorithm for dependent models derived by Lai [16].

Suppose that the conditional density function of y_k conditioned on $t_0 = j$ given $y_1, \dots, y_{k-1}$ is $p_{0,j}(\cdot|y_1, \dots, y_{k-1})$ for $k < t_0$ and $p_{1,j}(\cdot|y_1, \dots, y_{k-1})$ for $k \geq t_0$. Let

$$\begin{aligned} \bar{S}_j^k &:= \ln \frac{\mathbf{P}^{(j)}(y_1, \cdots, y_k|H_j)}{\mathbf{P}^{(0)}(y_1, \cdots, y_k|H_\infty)} \\ &= \sum_{i=j}^{k} \ln \frac{p_{1,j}(y_i|y_1, \dots, y_{i-1})}{p_{0,j}(y_i|y_1, \dots, y_{i-1})}, \quad k \geq j \end{aligned} \tag{2.15}$$

where $\mathbf{P}^{(j)}$ and H_j denotes the probability measure and the hypothesis with respect to change-point $t_0 = j$ and $\mathbf{P}^{(0)}$ and H_∞ are used for the situation when there is no change occurs. Then, a natural generalization of the CUSUM rule (2.4) is

$$\bar{t}_a = \min\{k : \max_{1 \leq j \leq k} \bar{S}_j^k \geq \hbar\} \tag{2.16}$$

Under the condition that the conditional likelihood ratio satisfies

$$\lim_{k\to\infty} \sup_{t_0 \geq 1} \text{esssup} \mathbf{P}^{(t_0)} \left[\max_{t \leq k} \sum_{i=t_0}^{t_0+t} \ln \frac{p_{1,t_0}(y_i|y_1, \dots, y_{i-1})}{p_{0,t_0}(y_i|y_1, \dots, y_{i-1})} \geq kI(1+\delta) \mid y_1, y_2, \dots, y_{t_0-1} \right] = 0 \quad \forall \delta > 0, \tag{2.17}$$

where $n^{-1} \sum_{i=t_0}^{t_0+n} \ln \frac{p_{1,t_0}(y_i|y_1,\dots,y_{i-1})}{p_{0,t_0}(y_i|y_1,\dots,y_{i-1})}$ is assumed to converge in probability under $\mathbf{P}^{(t_0)}$ to some positive constant I, Lai provides the asymptotic lower bound for the worst case average conditional delay subject to mean time between false alarm constraint and proves that the generalized CUSUM rule (2.16) with suitably chosen threshold $\hbar$ and certain window-limited modification thereof attain this asymptotic lower bound. However, it has been an open problem concerning whether the asymptotic optimality of the CUSUM rule (2.16) still holds in general as commented in [17].

Chapter 3

CUSUM-Based Quickest Detection for Cognitive Coexistence

3.1 Problem Setup

Under a primary communication system (e.g., WiMAX systems, as our explanatory example throughout the thesis) as shown in Fig. 4.1, we begin with modeling the spectrum sensing problem into a quickest detection with one single cognitive user. To detect the presence of primary user's signals as quickly as possible, one approach is to exploit features of the inceptive part of frame structure if prior knowledge about primary system is available. Take the widespread WiMAX system as example in our proposed scheme, we make use of the long preamble, which consists of two WiMAX OFDM symbols and is transmitted at the beginning of the frame. Due to the periodicity of the preamble structure and the aim that we want to detect the presence at first hand, we may only consider the simplified transmitted signal model composed of repeated segments in the leading preamble for symbolic convenience. Note that the feasibility of our proposed strategies depends on the prior knowledge of leading signals of primary user, not only applicable to periodic structure.

In particular, under the concern of flat fading channel between cognitive and primary

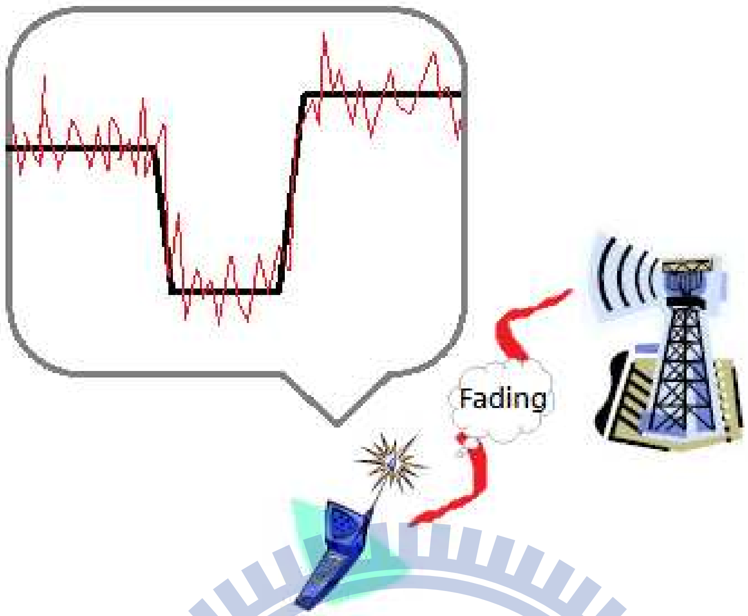


Figure 3.1: A sketch map of the spectrum sensing problem aiming to detect the reoccupying of underlying primary system as quickly as possible.

user (e.g., WiMAX base station), the received signal in time-domain could be modeled as

$$y_k = h\Upsilon(k, t_0) + n_k \tag{3.1}$$

with

$$\Upsilon(k, t_0) = \begin{cases} s((k - t_0)\text{mod}N_s), \text{ as } k \geq t_0 \\ \theta_0 = 0, \text{ as } k < t_0. \end{cases} \tag{3.2}$$

In the above equation, t_0 denotes the unknown presence time instant of primary signal, N_s denotes the length of the repeated segment of the preamble signal, n_k models the complex white gaussian noise with variance σ_n^2 at time k, and $s(i)$ is retrieved from the ith element of $\mathbf{s}$, which collects the periodic fragment of preamble symbol $\mathbf{s} = [s(0), s(1), \ldots, s(N_s - 1)]^T$. Note that h is the fading effect over the start signals of the new frame and might be treated either as deterministic but unknown constant or as random variable of a stochastic process, depending on which philosophy we take.

Thus, Our goal is to determine a decision strategy to fairly detect the beginning of the primary signaling prefixed by known preamble structure as quickly as possible, for the sake that the interference caused by the cognitive emitter to the existing primary systems must be avoided.

Contrast to the case of independent observations characterized by only one parameter, the detection problem we deal with involves *non-homogenous and innately dependent distributed* observations after the reoccupying of the coexisting primary system. In particular, the difficulties lie on the following aspects. First, the feature of preamble packet signaling results in non-homogeneous observations after change, which means the received signals are time-varying distributed even if we ignore the effects of fading channel. In the second place, due to the fact that the signals are transmitted through a fading channel, we have to tackle with the fading effects in the received signals after change. Two approaches are considered in the following two section in non-coherent and coherent sense, respectively.

In non-coherent approaches, as the literal meaning, it is assumed that the receiver only has knowledge of the statistics of channel and no estimation of realization about the unknown channel coefficient is needed. We also discuss and design from coherent sense, which means we resort to channel estimation of the realization of fading coefficient at the receiver end in our proposed strategies.

3.2 Non-Coherent Approaches

In non-coherent approach, we contrive to two CUSUM-based strategies without any estimation of realization about the unknown channel fading coefficients in the receiver end. One is called classical CUSUM algorithm, and another is termed to be weighted CUSUM algorithm. In classical CUSUM algorithm, we treat the unknown channel factor as a random variable with known prior statistics and calculate the likelihood ratio between joint probability density functions of observations under the conditions before and after change occurs, while in weighted CUSUM algorithm we take the unknown channel coefficient as deterministic but unknown constant during the detection process and then weight the likelihood ratio by applying prior information as weighting function.

After restricting ourselves to purely Rayleigh-fading channels, we may assume that the flat fading channel is modeled as $h \sim \mathcal{CN}(0, \sigma_h^2)$. The more general case under frequency-selective channel will be further discussed in later subsection.

3.2.1 Classical CUSUM Algorithm

Given σ_h^2 at the receiver end, the joint distribution of received signals in specific interval of observation time before or after change can be fully specified with the channel effect averaged. Whereas, the observation sequence after the change time $\{y_{t0}, y_{t0+1}, \ldots\}$ is dependent due to the presence of fading channel. Due to the non-homogeneous feature of observations after reoccupying time t_0, it seems implicit in the validity of applying CUSUM algorithm. Thus, we need to study the natural trend of the log-likelihood before and after change.

First, before the change time t_0, it can be verified from the Kullback information that

$$\begin{aligned} \mathbf{E}_{\theta_0}[\ln \frac{p_{\Theta(k,i)}(y_i, y_{i+1}, \cdots, y_k)}{p_{\theta_0}(y_i, y_{i+1}, \cdots, y_k)}] \le \\ \mathbf{E}_{\theta_0}[\ln \frac{p_{\Theta(k,j)}(y_j, y_{j+1}, \cdots, y_k)}{p_{\theta_0}(y_j, y_{j+1}, \cdots, y_k)}] \le 0, \\ \forall i \le j \le k \le t_0, \end{aligned} \tag{3.3}$$

where $p_{\Theta(k,j)}$ denotes the joint distribution of received signals from time j to k given $t_0 = j$ and p_{θ_0} denotes the joint distribution before change occurs. From the above inequality, we can observe a negative drift of the expected log-likelihood before change, which indicates the absence of the primary signaling. Similarly, we have

$$\begin{aligned} 0 \le \mathbf{E}_{\Theta(k,j)}[\ln \frac{p_{\Theta(k,j)}(y_j, y_{j+1}, \cdots, y_k)}{p_{\theta_0}(y_j, y_{j+1}, \cdots, y_k)}] \le \\ \mathbf{E}_{\Theta(k,i)}[\ln \frac{p_{\Theta(k,i)}(y_i, y_{i+1}, \cdots, y_k)}{p_{\theta_0}(y_i, y_{i+1}, \cdots, y_k)}], \\ \forall k \ge j \ge i \ge t_0, \end{aligned} \tag{3.4}$$

which indicates the positive tendency as the change has occurred. Therefore, we might apply the idea of CUSUM algorithm to detect the beginning of the reoccupying signals by the discrimination property. Although the log-likelihood ratio here is not additive due to the dependency among the observations after change, we still adopt the term "CUSUM" to represent the increasing amount on expected log-likelihood ratio. Then, the decision rule is given by

$$g_k = \max_{1\le j\le k} \ln \frac{p_{\Theta(k,j)}(y_j, y_{j+1}, \cdots, y_k)}{p_{\theta_0}(y_j, y_{j+1}, \cdots, y_k)} \tag{3.5}$$

$$= \max_{1\le j\le k} \ln \frac{\frac{1}{\pi^{k-j+1}\det \mathbf{K}_j^k} \exp\left(-\mathbf{y}_j^{k\,H}\mathbf{K}_j^{k-1}\mathbf{y}_j^k\right)}{\frac{1}{(\sigma_n^2\pi)^{k-j+1}} \exp\left(-\frac{1}{\sigma_n^2}\mathbf{y}_j^{k\,H}\mathbf{y}_j^k\right)} \tag{3.6}$$

and

$$t_a = \min\{k : g_k \ge \hbar\}, \tag{3.7}$$

where $\mathbf{K}_j^k$ represents the covariance matrix of received signals from time j to k under $t_0 = j$, and $\mathbf{y}_j^k$ collects observations $y_j, y_{j+1}, \ldots, y_k$.

To be more specific, we can view the received signals from time j to k under $t_0 = j$ alternatively as

$$\mathbf{y}_j^k|_{t_0=j} = \begin{bmatrix} hx_j + n_j \\ hx_{j+1} + n_{j+1} \\ \vdots \\ hx_k + n_k \end{bmatrix} \tag{3.8}$$

where $x_i = s((i - t_0)\mathrm{mod}N_s)$. We could recognize that $\mathbf{y}_j^k|_{t_0=j}$ is a random vector whose real part and imaginary part are collectively jointly Gaussian. Further, it is a circular symmetric complex Gaussian random vector with its joint density function denoted as $\mathcal{CN}(0, \mathbf{K}_j^k)$, where

$$\mathbf{K}_j^k = \begin{bmatrix} \sigma_h^2||x_j||^2 + \sigma_n^2 & \sigma_h^2 x_j x_{j+1}^* & \cdots & \sigma_h^2 x_j x_k^* \\ \sigma_h^2 x_{j+1} x_j^* & \sigma_h^2||x_{j+1}||^2 + \sigma_n^2 & \cdots & \sigma_h^2 x_{j+1} x_k^* \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_h^2 x_k x_j^* & \sigma_h^2 x_k x_{j+1}^* & \cdots & \sigma_h^2||x_k||^2 + \sigma_n^2 \end{bmatrix}. \tag{3.9}$$

In conclusion, we treat the unknown channel factor as a random variable with known prior statistics and calculate the likelihood ratio between joint probability density functions of observations under the conditions before and after change occurs. At each time instant, we search for the time at which the backward accumulated likelihood ratio is maximum, in other words, the instant the reoccupying most likely takes place. Then, we raise alarm to declare the change at the first time the resultant accumulated likelihood ratio is larger than a well-chosen threshold. Appropriate threshold $\hbar$ can be determined by numerical simulation in advance of detection.

3.2.2 Extension to Frequency-selective Fading Case

Under the concern of frequency-selective fading channel between cognitive user and underlying primary user or base station, the received signal in time-domain could be relatively modeled as

$$y_k = \theta(k, t_0) + n_k, \text{ where } \theta(k) = \begin{cases} \Upsilon(k, t_0) \circledast h_{eff}(k), \text{ as } k \geq t_0 \\ 0, \text{ as } k < t_0 \end{cases} \tag{3.10}$$

with

$$\Upsilon(k, t_0) = \begin{cases} s((k - t_0)\text{mod}N_s), \text{ as } k \geq t_0 \\ 0, \text{ as } k < t_0. \end{cases} \tag{3.11}$$

and

$$h_{eff}(k) = \begin{cases} h(k), \text{ as } 0 \leq k \leq L-1 \\ 0, \text{ Otherwise.} \end{cases} \tag{3.12}$$

Similar to previous flat fading case, t_0 denotes the unknown presence time instant of primary signal, N_s denotes the length of the repeated segment of the preamble signal, n_k models the AWGN with variance σ_n^2 at time k, and $s(i)$ is retrieved from the ith element of the periodic fragment of preamble symbol $\mathbf{s} = [s(0), s(1), \ldots, s(N_s - 1)]^T$. Note that the fading effects are caused by the channel $\mathbf{h}$ consists of L uncorrelated taps $h(l), l = 0, 1, \ldots, L-1$, where each element of $\mathbf{h}$ are modeled as purely Rayleigh-fading with variance σ_l^2 with uniform power constraint $\Sigma_{l=0}^{L}\sigma_l^2 = 1$.

Now, since the natural tendency is still reserved in the case of frequency-selective fading, we could design the corresponding decision strategy from the idea of CUSUM algorithm in a similar way for detecting the beginning of the reoccupying of underlying primary system in multipath environments. Specifically, the decision rule is in the form of

$$g_k^{FS} = \max_{1\le j\le k} \ln \frac{p_{\Theta(k,j)}(y_j, y_{j+1}, \cdots, y_k)}{p_{\theta_0}(y_j, y_{j+1}, \cdots, y_k)} \tag{3.13}$$

$$= \max_{1\le j\le k} \ln \frac{\frac{1}{\pi^{k-j+1}\det \mathbf{C}_j^k} \exp\left(-\mathbf{y}_j^{k^H} \mathbf{C}_j^{k^{-1}} \mathbf{y}_j^k\right)}{\frac{1}{(\sigma_n^2\pi)^{k-j+1}} \exp\left(-\frac{1}{\sigma_n^2} \mathbf{y}_j^{k^H} \mathbf{y}_j^k\right)} \tag{3.14}$$

and

$$t_a^{FS} = \min\{k : g_k^{FS} \ge \hbar\}, \tag{3.15}$$

where $\mathbf{y}_j^k$ collects observations $y_j, y_{j+1}, \ldots, y_k$, and $\mathbf{C}_j^k$ represents the covariance matrix of received signals convoluted with L channel taps from time j to k under $t_0 = j$.

In detail, $\mathbf{y}_j^k$ conditioned on $t_0 = j$ can be decomposed as

$$\mathbf{y}_j^k|_{t_0=j} = \begin{bmatrix} \mathbf{x}_j^T\mathbf{h} + n_j \\ \mathbf{x}_{j+1}^T\mathbf{h} + n_{j+1} \\ \vdots \\ \mathbf{x}_k^T\mathbf{h} + n_k \end{bmatrix} = \begin{bmatrix} x_{j,0} & x_{j,1} & \cdots & x_{j,L-1} \\ x_{j+1,0} & x_{j+1,1} & \cdots & x_{j+1,L-1} \\ \vdots & \vdots & \ddots & \vdots \\ x_{k,0} & x_{k,1} & \cdots & x_{k,L-1} \end{bmatrix} \begin{bmatrix} h(0) \\ h(1) \\ \vdots \\ h(L-1) \end{bmatrix} + \begin{bmatrix} n_j \\ n_{j+1} \\ \vdots \\ n_k \end{bmatrix} \tag{3.16}$$

where the L by 1 vector $\mathbf{x}_i$ collects the symbols convoluted with $\mathbf{h}$ at time instant i. After such rearrangement, we could assure that $\mathbf{y}_j^k|_{t_0=j}$ is also a random vector whose real part and imaginary part are collectively jointly Gaussian with its joint density function denoted as $\mathcal{CN}(0, \mathbf{C}_j^k)$, where

$$\mathbf{C}_j^k = \begin{bmatrix} \mathbf{x}_j^T\mathbf{\Sigma}^2\mathbf{x}_j^* + \sigma_n^2 & \mathbf{x}_j^T\mathbf{\Sigma}^2\mathbf{x}_{j+1}^* & \cdots & \mathbf{x}_j^T\mathbf{\Sigma}^2\mathbf{x}_k^* \\ \mathbf{x}_{j+1}^T\mathbf{\Sigma}^2\mathbf{x}_j^* & \mathbf{x}_{j+1}^T\mathbf{\Sigma}^2\mathbf{x}_{j+1}^* + \sigma_n^2 & \cdots & \mathbf{x}_{j+1}^T\mathbf{\Sigma}^2\mathbf{x}_k^* \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{x}_k^T\mathbf{\Sigma}^2\mathbf{x}_j^* & \mathbf{x}_k^T\mathbf{\Sigma}^2\mathbf{x}_{j+1}^* & \cdots & \mathbf{x}_k^T\mathbf{\Sigma}^2\mathbf{x}_k^* + \sigma_n^2 \end{bmatrix} \tag{3.17}$$

with $\mathbf{\Sigma}^2$ being the diagonal power delay profile matrix as $diag\{\sigma_0^2, \sigma_1^2, \ldots, \sigma_{L-1}^2\}$. Note that the elements of covariance $\mathbf{C}_j^k$ only depends on the value $k - j$ and can be calculated and stored in advance.

3.2.3 Modified Window-limited Version

Although the resultant accumulated likelihood ratio of the proposed classical CUSUM decision strategy could not be calculated in recursive way, we could resort to examine the required length of backward observations that keeps comparable efficacy with the one without any curtailment of observational window. On the other hand, we also curious about the influence on the case with regard to limited data storage in the receiver equipment.

Thus, we can simply replace the decision strategy in previous proposed classical CUSUM algorithm with a modified window-limited version, which is thus given as

$$g_k^{WL} = \max_{\max(1,k-W+1)\leq j\leq k} \ln \frac{p_{\Theta(k,j)}(y_j, y_{j+1}, \cdots, y_k)}{p_{\theta_0}(y_j, y_{j+1}, \cdots, y_k)} \tag{3.18}$$

$$t_a^{WL} = \min\{k : g_k^{WL} \geq \hbar\}, \tag{3.19}$$

where W is predetermined window size according to available temporary buffer.

Surprisingly, the effectiveness of classical CUSUM algorithm after truncating the length of needed backward observations is pretty nearly comparable with the original one with full memory of past observations, which lowers the complexity and required storage during implement and is demonstrated by simulation results.

3.2.4 Weighted CUSUM Algorithm

Consider another view about the unknown fading factor, we take the unknown channel coefficient h as deterministic but unknown constant during the detection process in our proposed weighted CUSUM algorithm. The main idea is to weight the likelihood ratio with respect to all possible values of the fading coefficient by using a well-chosen weighting function and take the resultant weighted likelihood ratio as an indicator about whether the reoccupying has occurs or not. Once the resultant weighted likelihood exceeds a particular predetermined threshold, which reveal a distinct possibility of primary user's activity, we stop taking observations and raise an alarm to declare that the change has very likely occurred.

Specifically, the form of the decision strategy of weighted CUSUM algorithm is as following:

$$g_k^{weighted} = \max_{1\leq j\leq k} \ln \int_{-\infty}^{\infty} \frac{p_{\Upsilon_{(j,j)}|h}(y_j) \cdots p_{\Upsilon_{(k,j)}|h}(y_k)}{p_{\theta_0}(y_j, \cdots, y_k)} p_h(h) dh \tag{3.20}$$

and

$$t_a^{weighted} = \min\{k : g_k^{weighted} \geq \hbar\} \tag{3.21}$$

That is, for every time instant k, we calculate the weighted likelihood ratio from time $j = 1, 2, \ldots, k$ to determine the most possible change point and compare the resultant log-likelihood ratio to some determined threshold. Once exceeding, we raise an alarm to declare the reoccupying of underlying primary user.

Provided statistical information about the fading coefficient, it is fairly reasonable to choose the weighting function p_h as $\mathcal{CN}(0, \sigma_h^2)$. So, we could calculate the weighted likelihood ratio as following

$$\begin{aligned}
&\int_{-\infty}^{\infty} \frac{p_{\Upsilon_{(j,j)}|h}(y_j)\cdots p_{\Upsilon_{(k,j)}|h}(y_k)}{p_{\theta_0}(y_j,\cdots,y_k)} p_h(h)dh \\
=&\int_{-\infty}^{\infty} \exp\{\frac{h}{\sigma_n^2}\sum_{n=j}^{k} y_n^H \Upsilon_{(n,j)} + \frac{h^H}{\sigma_n^2}\sum_{n=j}^{k} \Upsilon_{(n,j)}^H y_n - \frac{\|h\|^2}{\sigma_n^2}\sum_{n=j}^{k}\|\Upsilon_{(n,j)}\|^2\} p_h(h)dh \\
=&\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp\{u(\frac{\sum_{n=j}^{k} y_n^H \Upsilon_{(n,j)} + \sum_{n=j}^{k} \Upsilon_{(n,j)}^H y_n}{\sigma_n^2}) - u^2 \sum_{n=j}^{k}\|\Upsilon_{(n,j)}\|^2 \\
&+ vi(\frac{\sum_{n=j}^{k} y_n^H \Upsilon_{(n,j)} - \sum_{n=j}^{k} \Upsilon_{(n,j)}^H y_n}{\sigma_n^2}) - v^2 \sum_{n=j}^{k}\|\Upsilon_{(n,j)}\|^2\} p_{h_R}(u)du p_{h_I}(v)dv \\
=&\frac{1}{\sigma_h\sqrt{\pi}}\int_{-\infty}^{\infty} \exp\{u(\frac{\sum_{n=j}^{k} y_n^H \Upsilon_{(n,j)} + \sum_{n=j}^{k} \Upsilon_{(n,j)}^H y_n}{\sigma_n^2}) - u^2 \sum_{n=j}^{k}\|\Upsilon_{(n,j)}\|^2\} \exp\{-\frac{u^2}{\sigma_h^2}\}du \\
&\times \frac{1}{\sigma_h\sqrt{\pi}}\int_{-\infty}^{\infty} \exp\{vi(\frac{\sum_{n=j}^{k} y_n^H \Upsilon_{(n,j)} - \sum_{n=j}^{k} \Upsilon_{(n,j)}^H y_n}{\sigma_n^2}) - v^2 \sum_{n=j}^{k}\|\Upsilon_{(n,j)}\|^2\} \exp\{-\frac{v^2}{\sigma_h^2}\}dv
\end{aligned} \tag{3.22}$$

After integrating manipulations, we can get the close form of the weighted log-likelihood ratio

$$\begin{aligned}
&\ln\int_{-\infty}^{\infty} \frac{p_{\Upsilon_{(j,j)}|h}(y_j)\cdots p_{\Upsilon_{(k,j)}|h}(y_k)}{p_{\theta_0}(y_j,\cdots,y_k)} p_h(h)dh \\
=&\frac{\sigma_n^2}{2(2\sigma_n^2 l + 1)}[(\widetilde{S_{a_j}}^k)^2 + (\widetilde{S}_{b_j}^k)^2] - \ln(\sigma_h^2 l + 1)
\end{aligned} \tag{3.23}$$

with $\widetilde{S_{a_j}}^k = \sum_{n=j}^{k} y_n^H \Upsilon_{(n,j)} + y_n \Upsilon_{(n,j)}^H$, $\widetilde{S}_{b_j}^k = i(\sum_{n=j}^{k} y_n^H \Upsilon_{(n,j)} - y_n \Upsilon_{(n,j)}^H)$ and $l = \sum_{n=j}^{k} \|\Upsilon_{(n,j)}\|^2$.

Then we could represent the resultant weighted log-likelihood ratio at time instant k as

$$g_k^{weighted} = \max_{1 \leq j \leq k} \frac{\sigma_n^2}{2(2\sigma_n^2 l + 1)} [(\widetilde{S_{a_j}}^k)^2 + (\widetilde{S}_{b_j}^k)^2] - \ln(\sigma_h^2 l + 1) \quad (3.24)$$

and the stopping time to raise an alarm is

$$t_a^{weighted} = \min\{k : g_k^{weighted} \geq \hbar\} \quad (3.25)$$

From the closed-form of weighted log-likelihood ratio (3.23), we can see that the computing complexity of weighted CUSUM algorithm is less than the one of classical CUSUM algorithm, which involves more multiplications during detection process. On the other hand, at low-SNR region, the manipulation of directly weighting over the log-likehood ratio might alleviate the impact of low resolution due to fixed noisy power before and after change.

3.3 Coherent Approaches

In this section, we resort to another route to tackle the fading factor in the received signals after reoccupying. In coherent sense, we take the fading coefficient as unknown but deterministic constant or realization of random variable with known prior knowledge needed to be estimated, which turns the observations after change to independent sequence with a common unknown parameter h.

Now, since there exists unknown factor in the distribution of signals after change, we adopt two reasonable ways to refine the statistics we need for on-line detection. One is the GLRT-based CUSUM algorithm in which we estimate the unknown parameter through all available observations then substituting the result into likelihood ratio; another one is to estimate the realization of unknown fading coefficient incorporating with prior knowledge, which lead to MMSE-based CUSUM algorithm.

3.3.1 GLRT-Based CUSUM Algorithm

Conceptually, the decision strategy of GLRT-based CUSUM algorithm is given by

$$g_k^{GLRT} = \max_{1\le j\le k} \ln \frac{\max_h p_{\Theta_{(k,j)}}(y_j, y_{j+1}, \cdots, y_k; h)}{p_{\theta_0}(y_j, y_{j+1}, \cdots, y_k)} \tag{3.26}$$

and

$$t_a^{GLRT} = \min\{k : g_k^{GLRT} \geq \hbar\}, \tag{3.27}$$

where the unknown parameter h is hence replaced by its maximum likelihood estimate (MLE).

To derive the MLE of unknown parameter h over observations $y_j, y_{j+1}, \ldots, y_k$ under $t_0 = j$ is equivalent to find the estimator which minimize the least-square (LS) error

$$\mathbf{J}_j^k(h) = \Sigma_{n=j}^k \|y_n - h\Upsilon_{(n,j)}\|^2 \tag{3.28}$$

over h.

After decomposing all complex quantities into their real and imaginary parts, we turns the LS error into the following alternatively quadratic form in real variables h_R and h_I

$$\begin{aligned}\mathbf{J}_j^{k'}(h_R, h_I) =&\Sigma_{n=j}^k \|y_{n,R} + iy_{n,I} - (h_R + ih_I)(\Upsilon_{(n,j),R} + i\Upsilon_{(n,j),I})\|^2 \\ =&\Sigma_{n=j}^k (y_{n,R} - h_R\Upsilon_{(n,j),R} + h_I\Upsilon_{(n,j),I})^2 + (y_{n,I} - h_R\Upsilon_{(n,j),I} - h_I\Upsilon_{(n,j),R})^2\end{aligned} \tag{3.29}$$

We can rearrange $\mathbf{J}_j^{k'}(h_R, h_I)$ by setting $\mathbf{y}_{j\,R}^k = [y_{j,R}\, y_{j+1,R} \ldots y_{k,R}]^T$, $\mathbf{y}_{j\,I}^k = [y_{j,I}\, y_{j+1,I} \ldots y_{k,I}]$, $\mathbf{x}_R = [\Upsilon_{(j,j),R}\, \Upsilon_{(j+1,j),R} \ldots \Upsilon_{(k,j),R}]^T$, and $\mathbf{x}_I = [\Upsilon_{(j,j),I}\, \Upsilon_{(j+1,j),I} \ldots \Upsilon_{(k,j),I}]^T$ so that

$$\begin{aligned}\mathbf{J}_j^{k'}(h_R, h_I) = &(\mathbf{y}_{j\,R}^k - h_R\mathbf{x}_R + h_I\mathbf{x}_I)^T(\mathbf{y}_{j\,R}^k - h_R\mathbf{x}_R + h_I\mathbf{x}_I) \\ &+ (\mathbf{y}_{j\,I}^k - h_R\mathbf{x}_I - h_I\mathbf{x}_R)^T(\mathbf{y}_{j\,I}^k - h_R\mathbf{x}_I - h_I\mathbf{x}_R)\end{aligned} \tag{3.30}$$

or letting $\mathbf{x}_1 = [\mathbf{x}_R \;\; -\mathbf{x}_I]$, $\mathbf{x}_2 = [\mathbf{x}_I \;\; -\mathbf{x}_R]$ and $\mathbf{h} = [h_R\; h_I]^T$

$$\mathbf{J}_j^{k'}(h_R, h_I) =(\mathbf{y}_{j\,R}^k - \mathbf{x}_1\mathbf{h})^T(\mathbf{y}_{j\,R}^k - \mathbf{x}_1\mathbf{h}) + (\mathbf{y}_{j\,I}^k - \mathbf{x}_2\mathbf{h})^T(\mathbf{y}_{j\,I}^k - \mathbf{x}_2\mathbf{h}) \tag{3.31}$$

Taking the gradients yields

$$\frac{\partial \mathbf{J}_j^{k'}}{\partial \mathbf{h}} = -2\mathbf{x}_1^T\mathbf{y}_{j\,R}^k + 2\mathbf{x}_1^T\mathbf{x}_1\mathbf{h} - 2\mathbf{x}_2^T\mathbf{y}_{j\,I}^k + 2\mathbf{x}_2^T\mathbf{x}_2\mathbf{h}, \tag{3.32}$$

then setting this equal to zero and solving produces

$$\begin{aligned}\widehat{\mathbf{h}}_j^k =&(\mathbf{x}_1^T\mathbf{x}_1+\mathbf{x}_2^T\mathbf{x}_2)^{-1}(\mathbf{x}_1^T\mathbf{y}_{j\,R}^k+\mathbf{x}_2^T\mathbf{y}_{j\,I}^k)\\ =&\begin{bmatrix}\frac{\mathbf{x}_R^T\mathbf{y}_{j\,R}^k+\mathbf{x}_I^T\mathbf{y}_{j\,I}^k}{\mathbf{x}_R^T\mathbf{x}_R+\mathbf{x}_I^T\mathbf{x}_I}\\ \\ \frac{\mathbf{x}_R^T\mathbf{y}_{j\,I}^k-\mathbf{x}_I^T\mathbf{y}_{j\,R}^k}{\mathbf{x}_R^T\mathbf{x}_R+\mathbf{x}_I^T\mathbf{x}_I}\end{bmatrix}=\begin{bmatrix}\widehat{h}_{j\,R}^k\\ \\ \widehat{h}_{j\,I}^k\end{bmatrix}\end{aligned} \tag{3.33}$$

which is the minimizing solution. However, if we rewrite $\widehat{\mathbf{h}}_j^k$ in complex form as $\widehat{h}_{j\,R}^k + i\widehat{h}_{j\,I}^k = \hat{h}_j^k$, we have

$$\begin{aligned}\hat{h}_j^k =&\frac{\mathbf{x}_R^T\mathbf{y}_{j\,R}^k+\mathbf{x}_I^T\mathbf{y}_{j\,I}^k+i(\mathbf{x}_R^T\mathbf{y}_{j\,I}^k-\mathbf{x}_I^T\mathbf{y}_{j\,R}^k)}{\mathbf{x}_R^T\mathbf{x}_R+\mathbf{x}_I^T\mathbf{x}_I}\\ =&\frac{(\mathbf{y}_{j\,R}^k+i\mathbf{y}_{j\,I}^k)^T(\mathbf{x}_R-i\mathbf{x}_I)}{\mathbf{x}_R^T\mathbf{x}_R+\mathbf{x}_I^T\mathbf{x}_I}\\ =&\frac{\sum_{n=j}^k y_n\Upsilon_{(n,j)}^H}{\sum_{n=j}^k\|\Upsilon_{(n,j)}\|^2}\end{aligned} \tag{3.34}$$

which is the MLE of h over observations $\{y_j, y_{j+1}, \ldots, y_k\}$.

By substituting the MLE $\hat{h}_j^k$ into the above equation, we arrive at

$$g_k^{GLRT} = \max_{1\le j\le k} \ln \frac{p_{\Theta_{(k,j)}}(y_j, y_{j+1}, \cdots, y_k; \hat{h}_j^k)}{p_{\theta_0}(y_j, y_{j+1}, \cdots, y_k)} \tag{3.35}$$

3.3.2 MMSE-based CUSUM algorithm

Analogous to replacing the unknown parameters by their maximum likelihood estimators in GLRT, we could estimate the realization of unknown fading coefficient incorporating with prior knowledge if applicable, which leads to the use of minimum mean square estimator (MMSE) that minimizes the Bayesian MSE.

Depart from the philosophy employed by GLRT, we assume that the unknown fading coefficient h is a random variable whose particular realization we must estimate. The motivation for doing so is, if we have available some prior knowledge about h, we can incorporate it into our estimator. It is somehow difficult to make use of any prior knowledge in classical estimation such as MLE. The mechanism for doing this requires us to assume our unknown factor h as a random variable, which is complex gaussian distributed and represented as $h \sim \mathcal{CN}(0, \sigma_h^2)$.

After rearranging observations $\{y_j, y_{j+1}, \ldots, y_k\}$ conditioned on $t_0 = j$ into complex linear model, we have

$$\mathbf{y}_j^k = \begin{bmatrix} y_j \\ y_{j+1} \\ \vdots \\ y_k \end{bmatrix} = h \begin{bmatrix} \Upsilon(j,j) \\ \Upsilon(j+1,j) \\ \vdots \\ \Upsilon(k,j) \end{bmatrix} + \begin{bmatrix} n_j \\ n_{j+1} \\ \vdots \\ n_k \end{bmatrix} = h\mathbf{x}_j^k + \mathbf{n}_j^k \tag{3.36}$$

where $\mathbf{x}_j^k = [\Upsilon(j,j)\,\Upsilon(j+1,j)\,\ldots\,\Upsilon(k,j)]^T$ and $\mathbf{n}_j^k \sim \mathcal{CN}(0, \sigma_n^2 \mathbf{I}_{k-j+1})$.

Since it can be easily verified that $\mathbf{y}_j^k$ and h are jointly complex gaussian, we have the MMSE estimator for the complex Bayesian linear model given by

$$\begin{aligned} \widehat{h}_j^{k(MMSE)} =& \mathbf{E}(h|\mathbf{y}_j^k) \\ =& ((\sigma_h^2)^{-1} + \frac{\|\mathbf{x}_j^k\|^2}{\sigma_n^2})^{-1} \frac{\mathbf{x}_j^{kH}\mathbf{y}_j^k}{\sigma_n^2} \\ =& \frac{\mathbf{x}_j^{kH}\mathbf{y}_j^k}{\frac{\sigma_n^2}{\sigma_h^2} + \|\mathbf{x}_j^k\|^2} \end{aligned} \tag{3.37}$$

.

By substituting the MMSE of h over observations $\{y_j, y_{j+1}, \ldots, y_k\}$, we arrive at

$$g_k^{MMSE} = \max_{1 \le j \le k} \ln \frac{p_{\Theta_{(k,j)}}(y_j, y_{j+1}, \cdots, y_k; \hat{h}_j^{k(MMSE)})}{p_{\theta_0}(y_j, y_{j+1}, \cdots, y_k)} \tag{3.38}$$

and

$$t_a^{MMSE} = \min\{k : g_k^{MMSE} \ge \hbar\}. \tag{3.39}$$

Chapter 4

Cooperative CUSUM-Based Quickest Detection in Cognitive Sensors Network

4.1 Problem Setup

In this chapter, we are interested in developing cooperative CUSUM-based quickest detection algorithms applied to multiuser scenario. As shown in Fig. 4.1, we restrict ourselves to the case in which the information available for local decision-making is in distributed way, not shared among cooperative users. Assume there is a set of $R \geq 1$ geographically distributed sensors, denoted as $S_1,\ S_2, \ldots,\ S_R$, cooperatively detecting the reactivities of underlying primary systems. The reoccupying of primary signals occurs at an unknown time instant t_0 for all sensors simultaneously. Similar to single-user scenario, we may only consider the simplified transmitted signal model for symbolic convenience. Again, the applicability of our proposed strategies depends on the prior knowledge of primary signals, not only confined to periodic structure.

Specifically, under the concern of fading effects between collaborative cognitive sensors and primary user, the received signal in time-domain at sensor S_r could be modeled as

$$y_{r,k} = h_{r,k} \Upsilon(k, t_0) + n_{r,k} \tag{4.1}$$

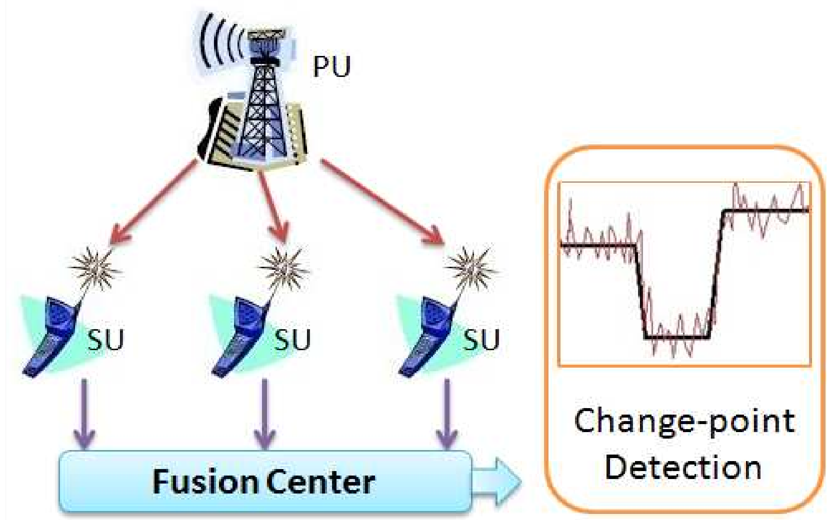


Figure 4.1: A sketch map of the spectrum sensing problem aiming to detect the reoccupying of underlying primary system as quickly as possible in a cooperative multiuser network.

with

$$\Upsilon(k, t_0) = \begin{cases} s((k - t_0) \mathrm{mod} N_s), \text{ as } k \geq t_0 \\ \theta_0 = 0, \text{ as } k < t_0. \end{cases} \tag{4.2}$$

where t_0 denotes the unknown point in time of the presence of primary signal, $n_{r,k}$ models the complex white gaussian noise with variance σ_n^2 at time instant k in the receiver end of S_r, N_s denotes the length of the repeated segment of the preamble signal, and $s(i)$ is retrieved from the ith element of $\mathbf{s}$, which collects the periodic fragment of preamble symbol $\mathbf{s} = [s(0), s(1), \ldots, s(N_s - 1)]^T$. In flat fading case, $h_{r,k}$ is modeled as purely Rayleigh-fading between primary user and sensor S_r with variance $\sigma_{r,h}^2$, remaining constant during observation time and could be simplified as h_r. While in frequency-selective fading case, we take $h_{r,k}$ as the fading effect at time k caused by the channel $\mathbf{h_r} = [h_r(0), h_r(1), \ldots, h_r(L-1)]^T$ consists of L resolvable paths, and each tap of $\mathbf{h_r}$ is mutually uncorrelated and modeled as purely Rayleigh-fading with variance $\sigma_{r,l}^2$.

In general, there are two kinds of scenario as considering detection problems of distributed sensor network, termed of centralized scenario and decentralized scenario. In the centralized setting, the original data received at sensors $y_{r,k}$, $r = 1, \ldots, R$, are sent completely to a fusion center where a final decision is made based on all sensor massages. In the decentralized framework, quantized version of observations or local decisions are forwarded to the fusion center for making a final decision. The decentralized scenario is usually more practical due to communication bandwidth constraint between sensors and the fusion center.

Next, under considering three different distributed frameworks, we will extend previously proposed CUSUM-based algorithms to multiuser quickest detection and provide decision strategies to collaboratively detect the beginning of primary signals as quickly as possible to avoid possible interference to primary systems.

4.2 Centralized Case: Global CUSUM Algorithm

In centralized sense, we might assume that the pairwise channels between the sensors and the fusion center are error-free. The original data received at each sensor is thus sent completely to the fusion center for global CUSUM test. That is, the fusion center collects all messages received by sensors and performs the CUSUM-based quickest detection based on the whole set of observations.

Under the assumption that the fading effects experienced by each sensor are mutually independent, we could simply extend the result strategies derived in single-user case as following:

▶ Classical CUSUM Algorithm (flat fading case):

$$g_k = \max_{1 \le j \le k} \sum_{r=1}^{R} \ln \frac{p_{\Theta(k,j)}(y_{r,j}, y_{r,j+1}, \cdots, y_{r,k})}{p_{\theta_0}(y_{r,j}, y_{r,j+1}, \cdots, y_{r,k})} \tag{4.3}$$

$$= \max_{1 \le j \le k} \sum_{r=1}^{R} \ln \frac{\frac{1}{\pi^{k-j+1} \det \mathbf{K}_{\mathbf{r}_j^k}} \exp\left(-{\mathbf{y}_{\mathbf{r}_j^k}}^H {\mathbf{K}_{\mathbf{r}_j^k}}^{-1} \mathbf{y}_{\mathbf{r}_j^k}\right)}{\frac{1}{(\sigma_n^2 \pi)^{k-j+1}} \exp\left(-\frac{1}{\sigma_n^2} {\mathbf{y}_{\mathbf{r}_j^k}}^H \mathbf{y}_{\mathbf{r}_j^k}\right)} \tag{4.4}$$

and

$$t_a = \min\{k : g_k \ge \hbar\} \tag{4.5}$$

where ${\mathbf{y}_\mathbf{r}}_j^k$ collects observations $\{y_{r,j},\ \ldots,\ y_{r,k}\}$, and ${\mathbf{K}_\mathbf{r}}_j^k$ denotes the covariance matrix of observation vector ${\mathbf{y}_\mathbf{r}}_j^k$ conditioned on $t_0 = j$.

▶ Classical CUSUM Algorithm (frequency-selective fading case):

$$g_k^{FS} = \max_{1\le j\le k} \sum_{r=1}^{R} \ln \frac{p_{\Theta(k,j)}(y_{r,j}, y_{r,j+1}, \cdots, y_{r,k})}{p_{\theta_0}(y_{r,j}, y_{r,j+1}, \cdots, y_{r,k})} \tag{4.6}$$

$$= \max_{1\le j\le k} \sum_{r=1}^{R} \ln \frac{\frac{1}{\pi^{k-j+1} \det {\mathbf{C}_\mathbf{r}}_j^k} \exp\left(-{\mathbf{y}_\mathbf{r}}_j^{kH} {\mathbf{C}_\mathbf{r}}_j^{k-1} {\mathbf{y}_\mathbf{r}}_j^k\right)}{\frac{1}{(\sigma_n^2 \pi)^{k-j+1}} \exp\left(-\frac{1}{\sigma_n^2} {\mathbf{y}_\mathbf{r}}_j^{kH} {\mathbf{y}_\mathbf{r}}_j^k\right)} \tag{4.7}$$

and

$$t_a^{FS} = \min\{k : g_k^{FS} \ge \hbar\}, \tag{4.8}$$

where ${\mathbf{C}_\mathbf{r}}_j^k$ denotes the covariance matrix of observations $\{y_{r,j},\ \ldots,\ y_{r,k}\}$ conditioned on $t_0 = j$ with power delay profile matrix $\mathbf{\Sigma}_r^2 = diag\{\sigma_{r,0}^2, \sigma_{r,1}^2, \ldots, \sigma_{r,L-1}^2\}$.

▶ Weighted CUSUM algorithm:

$$g_k^{weighted}$$

$$= \max_{1\le j\le k} \sum_{r=1}^{R} \ln \int_{-\infty}^{\infty} \frac{p_{\Upsilon_{(j,j)}|h_r}(y_{r,j}) \cdots p_{\Upsilon_{(k,j)}|h_r}(y_{r,k})}{p_{\theta_0}(y_{r,j}, \cdots, y_{r,k})} p_{h_r}(h_r) dh_r \tag{4.9}$$

$$= \max_{1\le j\le k} \sum_{r=1}^{R} \frac{\sigma_n^2}{2(2\sigma_n^2 l + 1)} [(\widetilde{S_{r,a_j}}^k)^2 + (\widetilde{S_{r,b_j}}^k)^2] - \ln(\sigma_{r,h}^2 l + 1) \tag{4.10}$$

with $\widetilde{S_{r,a_j}}^k = \sum_{n=j}^k y_{r,n}^H \Upsilon_{(n,j)} + y_{r,n} \Upsilon_{(n,j)}^H$, $\widetilde{S_{r,b_j}}^k = i(\sum_{n=j}^k y_{r,n}^H \Upsilon_{(n,j)} - y_{r,n} \Upsilon_{(n,j)}^H)$ and $l = \sum_{n=j}^k \|\Upsilon_{(n,j)}\|^2$. The stopping time to raise an alarm is

$$t_a^{weighted} = \min\{k : g_k^{weighted} \ge \hbar\} \tag{4.11}$$

▶ GLRT-based CUSUM algorithm:

$$g_k^{GLRT} = \max_{1\le j\le k} \sum_{r=1}^{R} \ln \frac{\max_{h_r} p_{\Theta_{(k,j)}}(y_{r,j}, y_{r,j+1}, \cdots, y_{r,k}; h_r)}{p_{\theta_0}(y_{r,j}, y_{r,j+1}, \cdots, y_{r,k})} \tag{4.12}$$

$$= \max_{1\le j\le k} \sum_{r=1}^{R} \ln \frac{p_{\Theta_{(k,j)}}(y_{r,j}, y_{r,j+1}, \cdots, y_{r,k}; \hat{h}_{r_j}^k)}{p_{\theta_0}(y_{r,j}, y_{r,j+1}, \cdots, y_{r,k})} \tag{4.13}$$

with $\hat{h}_{r_j}^k$ being the MLE over observations $\{y_{r,j},\ \ldots,\ y_{r,k}\}$ conditioned on $t_0 = j$

$$\hat{h}_{r_j}^k = \frac{\sum_{n=j}^k y_{r,n} \Upsilon_{(n,j)}^H}{\sum_{n=j}^k \|\Upsilon_{(n,j)}\|^2} \tag{4.14}$$

and

$$t_a^{GLRT} = \min\{k : g_k^{GLRT} \geq \hbar\} \tag{4.15}$$

▶ MMSE-based CUSUM algorithm:

$$g_k^{MMSE} = \max_{1\leq j\leq k} \sum_{r=1}^{R} \ln \frac{p_{\Theta_{(k,j)}}(y_{r,j}, y_{r,j+1}, \cdots, y_{r,k}; \hat{h}_{rj}^{k(MMSE)})}{p_{\theta_0}(y_{r,j}, y_{r,j+1}, \cdots, y_{r,k})} \tag{4.16}$$

where the MMSE over observations $\{y_{r,j},\ \ldots,\ y_{r,k}\}$ is given by

$$\hat{h}_{rj}^{k(MMSE)} = \frac{\mathbf{x}_j^{kH} \mathbf{y_r}_j^k}{\frac{\sigma_n^2}{\sigma_{r,h}^2} + \|\mathbf{x}_j^k\|^2} \tag{4.17}$$

.

and

$$t_a^{MMSE} = \min\{k : g_k^{MMSE} \geq \hbar\} \tag{4.18}$$

4.3 Decentralized Case: Hard Fusion of Local CUSUM

In this section, we first resort to a straightforward cooperative decentralized scheme termed of *hard fusion of local CUSUM*. Assume that each of the cooperative sensors has sufficient memory to individually perform CUSUM-based quickest detection. Then, the fusion center makes final decision based on local decisions sent by sensors according to well-known hard-decision combining rules, such as AND, OR or M-out-of-R rules in general. Further, we assume that each of the sensors might either send their local decisions at each time instant for updating information at the fusion center or update just for once at the first time the local statistic reaches predetermined threshold.

Specifically, if the sensors updates their decisions of local quickest detection constantly until the fusion center makes the final decision, the stopping time by the general M-out-of-R rule at the fusion center is given by

$$t_a^{HardFusion} = \min\{k : \sum_{r=1}^{R} U_{r,k} \geq M\} \tag{4.19}$$

where

$$U_{r,k} = \begin{cases} 1, & \text{if } g_{r,k}^{Local} \geq \hbar_r \\ 0, & \text{Otherwise.} \end{cases} \tag{4.20}$$

In the above equation, $g^{Local}_{r,k}$ could be chosen from algorithms previously discussed in single-user scenario.

As to the one-shot scheme, the corresponding stopping time by the general M-out-of-R rule at the fusion center is in the form of

$$t_a^{HardFusion*} = \min\{k : \sum_{r=1}^{R} U_{r,k} \geq M\} \tag{4.21}$$

where

$$U_{r,k} = \begin{cases} 1, & \text{if } k \geq t_a(r) = \min\{n : g^{Local}_{r,n} \geq \hbar_r\} \\ 0, & \text{Otherwise.} \end{cases} \tag{4.22}$$

4.4 Decentralized Case: Global CUSUM with Quantized Local Decision

Now, rather than performing quickest detection at local sensors, we are interested in the decentralized scenario in which the CUSUM test is performed at the fusion center while the local sensors are assumed memoryless and send quantized version of their observations for decision making.

However, due to the memorylessness of local sensor and the need of quantization, it is reluctant to exploit the known signal structure about primary user as before. That is, if we still adopt the signal model in (4.1), the sensor has no ability to do quantization by optimal local mapping, the monotone likelihood ratio quantizer, without the knowledge about the exact timing index if the reoccupying has occurs. Thus, we resort to *approximate* the received observation sequence $\{y_{r,k}, y_{r,k+1}, \cdots\}$ conditioned on $t_0 = k$ to be circularly symmetric complex gaussian random variables with variance $\sigma^2_{r,x}$ and mutually independent. In specific, observations at the rth sensor at time k are approximately distributed as follows:

$$y_{r,k} \begin{cases} {}^{a}\sim \mathcal{CN}(0, \sigma^2_{r,x}), & \text{if } k \geq t_0 \\ \sim \mathcal{CN}(0, \sigma^2_n), & \text{Otherwise.} \end{cases} \tag{4.23}$$

Under this approximation, we might take the quickest detection in the view as suggested by Page, as repeated sequential probability ratio testing of two simple hypotheses:

$$\begin{aligned}\mathbf{H}_0 : y_{r,k} &\sim f_1(y_{r,k}) = \mathcal{CN}(0,\sigma_n^2)\\ \mathbf{H}_1 : y_{r,k} &\sim f_0(y_{r,k}) = \mathcal{CN}(0,\sigma_{r,x}^2) \quad \forall r = 1,2,\cdots,R.\end{aligned} \tag{4.24}$$

Thus, in the mapping of monotone likelihood ratio quantizer, the local message $U_{r,k}$ at time instant k produced by sensor S_r based on the kth observation $y_{r,k}$ is given by

$$U_{r,k} = p \quad \text{if} \quad d_{r,p} \leq \frac{f_1(y_{r,k})}{f_0(y_{r,k})} \triangleq \Lambda(y_{r,k}) < d_{r,p+1} \tag{4.25}$$

where $0 = d_{r,0} < d_{r,1} < \cdots < d_{r,p_{mql-1}} < d_{r,p_{mql}} = \infty$ represents the quantization threshold set used by sensor S_r.

Then, if we let $z_{r,k}$ represents the received signal at the fusion center when $U_{r,k}$ is transmitted by sensor S_r, we have the likelihood ratio of $z_{r,k}$ as following:

$$\Lambda_{r,k}(z_{r,k}) = \frac{\sum_{p=0}^{p_{mql}} f(z_{r,k}|p)P\{U_{r,k}=p|\mathbf{H}_1\}}{\sum_{p=0}^{p_{mql}} f(z_{r,k}|p)P\{U_{r,k}=p|\mathbf{H}_0\}}. \tag{4.26}$$

By applying Page's CUSUM algorithm, the decision strategy of the fusion center is given by

$$g_k^Q = \max_{1\leq j\leq k} \sum_{n=j}^{k}\sum_{r=1}^{R} \ln \Lambda_{r,k}(z_{r,k}) \tag{4.27}$$

$$= \max(g_{k-1}^Q, 0) + \sum_{r=1}^{R} \ln \Lambda_{r,k}(z_{r,k}) \tag{4.28}$$

and

$$t_a^Q = \min\{k : g_k^Q \geq \hbar\}. \tag{4.29}$$

In detail, it is convenient to calculate $P\{U_{r,k} = p|\mathbf{H}_0\}$ and $P\{U_{r,k} = p|\mathbf{H}_1\}$ by transforming thresholds in monotone likelihood ratio quantizer into thresholds related directly to observations $y_{r,k}$. Start from

$$\Lambda(y_{r,k}) = \frac{\sigma_n^2}{\sigma_{r,x}^2} \exp\left(\frac{\sigma_{r,x}^2 - \sigma_n^2}{\sigma_{r,x}^2\sigma_n^2}\|y_{r,k}\|^2\right), \tag{4.30}$$

we have

$$\Lambda(y_{r,k}) \geq d_p \equiv \|y_{r,k}\|^2 \geq \left(\frac{\sigma_{r,x}^2\sigma_n^2}{\sigma_{r,x}^2 - \sigma_n^2}\right) \ln\left(\frac{d_p}{\sigma_n^2/\sigma_{r,x}^2}\right) \tag{4.31}$$

$$\equiv \|y_{r,k}\| \geq \sqrt{\left(\frac{\sigma_{r,x}^2\sigma_n^2}{\sigma_{r,x}^2 - \sigma_n^2}\right) \ln\left(\frac{d_p}{\sigma_n^2/\sigma_{r,x}^2}\right)} \triangleq d_p' \tag{4.32}$$

As we know, given

$$y_{r,k} \sim \begin{cases} \mathcal{CN}(0,\sigma_{r,x}^2), \text{ under } \mathbf{H}_1 \\ \mathcal{CN}(0,\sigma_n^2), \text{ under } \mathbf{H}_0, \end{cases} \tag{4.33}$$

the distributions of absolute value of $y_{r,k}$ under $\mathbf{H}_0$ and $\mathbf{H}_1$ are Rayleigh distributed as

$$\|y_{r,k}\| \sim \begin{cases} \frac{y}{\sigma_{r,x}^2}\exp\left(\frac{-y^2}{2\sigma_{r,x}^2}\right), \text{ under } \mathbf{H}_1 \\ \frac{y}{\sigma_n^2}\exp\left(\frac{-y^2}{2\sigma_n^2}\right), \text{ under } \mathbf{H}_0. \end{cases} \tag{4.34}$$

Thus, we could calculate the conditional probabilities as

$$P\{U_{r,k}=p|\mathbf{H}_0\} = P\{d_{r,p} \le \Lambda(y_{r,k}) < d_{r,p+1}|\mathbf{H}_1\} \tag{4.35}$$

$$= \exp\left(\frac{-\|d'_p\|^2}{2\sigma_{r,x}^2}\right) - \exp\left(\frac{-\|d'_{p+1}\|^2}{2\sigma_{r,x}^2}\right) \tag{4.36}$$

and $$P\{U_{r,k}=p|\mathbf{H}_0\} = P\{d_{r,p} \le \Lambda(y_{r,k}) < d_{r,p+1}|\mathbf{H}_0\} \tag{4.37}$$

$$= \exp\left(\frac{-\|d'_p\|^2}{2\sigma_n^2}\right) - \exp\left(\frac{-\|d'_{p+1}\|^2}{2\sigma_n^2}\right) \tag{4.38}$$

and rewrite the global likelihood ratio (4.26) as

$$\Lambda_{r,k}(z_{r,k}) = \frac{\sum_{p=0}^{p_{mql}} f(z_{r,k}|p)\left[\exp\left(\frac{-\|d'_p\|^2}{2\sigma_{r,x}^2}\right) - \exp\left(\frac{-\|d'_{p+1}\|^2}{2\sigma_{r,x}^2}\right)\right]}{\sum_{p=0}^{p_{mql}} f(z_{r,k}|p)\left[\exp\left(\frac{-\|d'_p\|^2}{2\sigma_n^2}\right) - \exp\left(\frac{-\|d'_{p+1}\|^2}{2\sigma_n^2}\right)\right]} \tag{4.39}$$

By the general form of global likelihood ratio (4.39), we focus on three kinds of scenarios based on different channel conditions between sensors and the fusion center with 1-bit or 2-bit local quantization. The discussion and comparison among these schemes would be further shown in Chapter 5.

Chapter 5

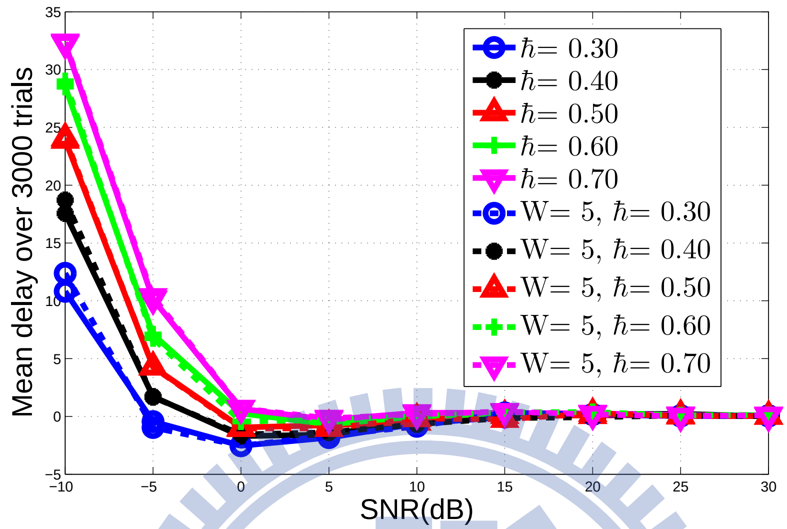
Simulations

5.1 Simulation Setup

In this chapter, we present some numerical results and provide comparisons to demonstrate the effectiveness of our proposed algorithms. As to primary signal model adopted in the simulations, we choose the settings defined in IEEE 802.16e, the long preamble which contains two OFDM symbols and each of them consists of four replications in time domain and constitutes the incipient part of the new frame of primary user, as the target we want to recognize as quickly as possible. We set the finite horizon as 50 and the reoccupying time is uniformly distributed on the time instants $1, 2, \ldots, 15$ in one trial. Fix the noise power at cognitive receivers end to one, we define the SNR as the transmit power from primary user. In flat-fading case, the variance of the Rayleigh distributed channel effect h is set to be $\sigma_h^2 = 1$. While in frequency-selective fading case, we set the channel order to be $L = 4$ with uniform power delay profile matrix as $\Sigma = diag\{0.25, 0.25, 0.25, 0.25\}$. Note that some parameters may vary across the different simulation scenarios thus not stated here, the remaining details will be specified in each case.

5.2 Effectiveness of Proposed CUSUM-based Algorithms

In the first subsection, we focus on the behaviors of different proposed approaches and show the effectiveness of our proposed algorithms, under either flat-fading case or frequency-



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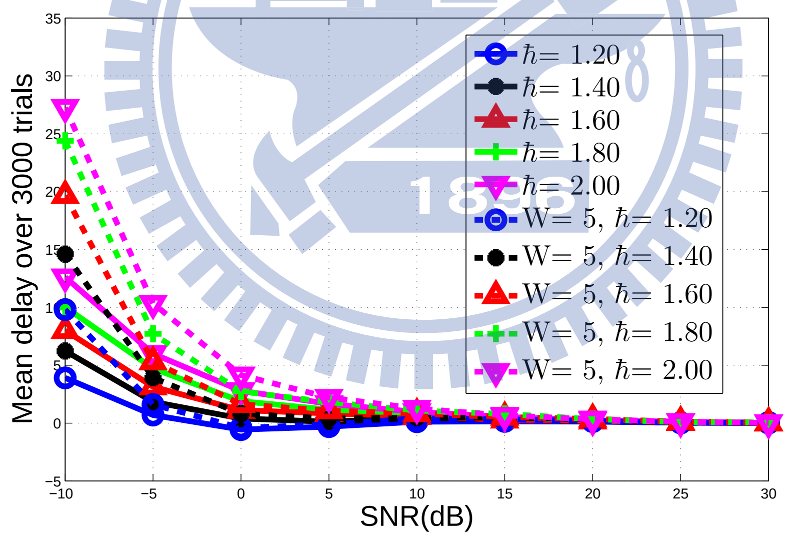


(b)

Figure 5.1: Performances of non-coherent approaches in single-user scenario under flat-fading environment. (a) Classical CUSUM algorithm. (b) Weighted CUSUM algorithm.

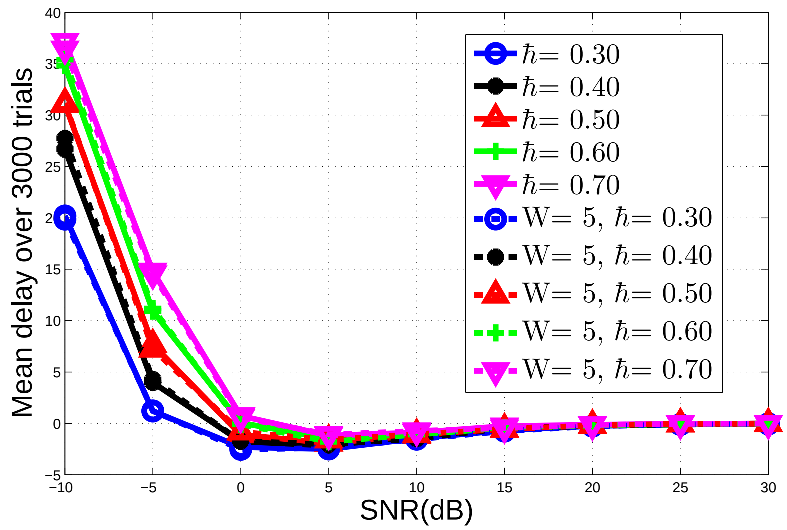
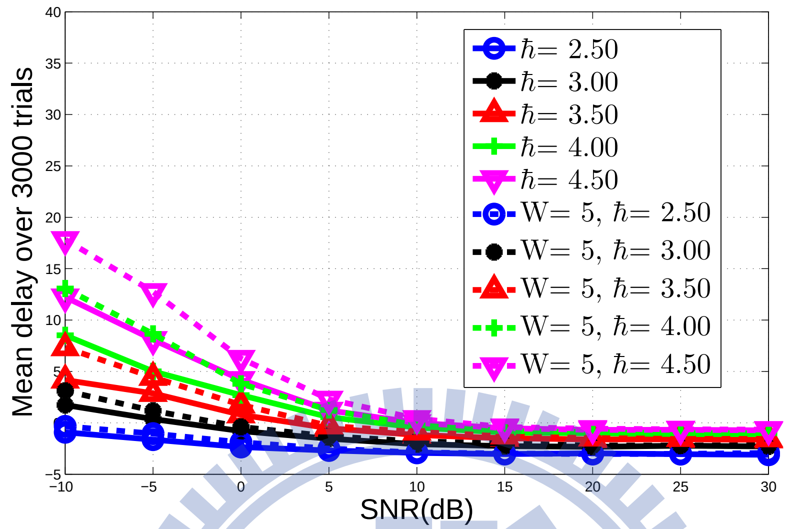


Figure 5.2: Performances of non-coherent approaches in single-user scenario: Classical CUSUM algorithm in frequency-selective-fading environment.

selective fading case. In the second subsection, we simulate the proposed CUSUM-based quickest detection applied to different distributed frameworks and examine the influence when the number of involved cognitive users increases.

5.2.1 Single-user Scenario

Fig. 5.1(a) and Fig. 5.2 show the performances of classical approach as well as the modified truncated version with window size $W = 5$. We can see that the classical method performs fairly effectively by using a well-chosen threshold and converges to zero mean delay as SNR grows in both the flat-fading case and the frequency-selective fading case. Furthermore, the performance curves of window-limited version show that it is enough to keep the detectability in only five backward data because we take the innately dependency among the received sequence after change into account as designing the decision strategy. Another method of non-coherent approaches, the weighted CUSUM algorithm, also performs well as shown in Fig. 5.1(b). Given the statistical information about underlying fading channel, the weighted CUSUM algorithm performs well and converges as quickly as classical method does. Especially at low-SNR region, the manipulation of



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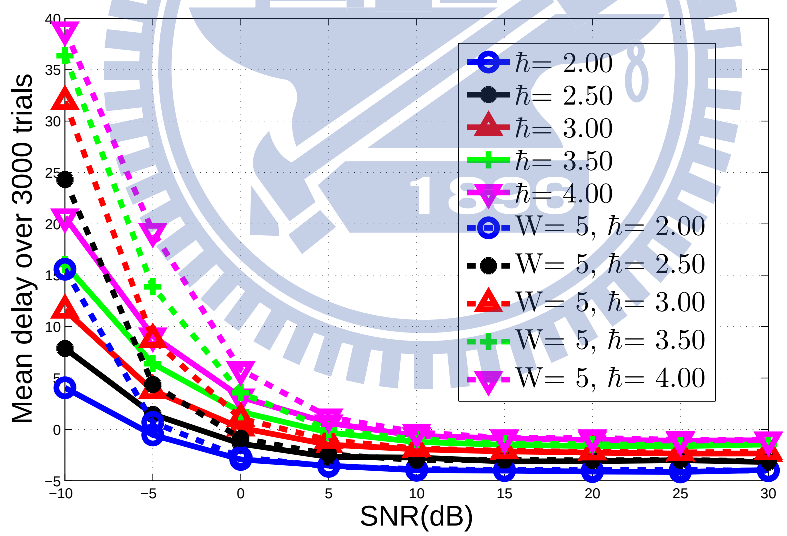

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Figure 5.3: Performances of coherent approaches in single-user scenario under flat-fading environment. (a) GLRT-based CUSUM algorithm. (b) MMSE-based CUSUM algorithm.

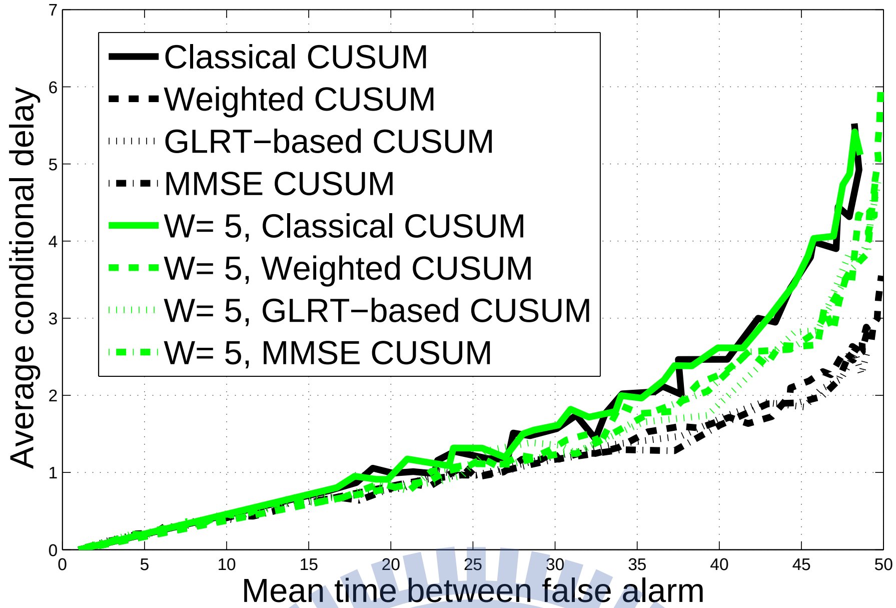


Figure 5.4: Comparison of ROC curves of four proposed algorithms under SNR= 10 in flat-fading case.

directly weighting over the log-likelihood ratio alleviates the impact of low resolution due to fixed noisy power before and after change thus improves the corresponding mean delay. However, we can see that weighted CUSUM algorithm might relatively degrade if we truncate the available past observations due to ignorance of inherent dependency. Specifically, since the mechanism of weighted CUSUM algorithm could be seem as weighting the log-likelihood ratio of each time instant individually and then summing them up for calculating the resultant statistics, it would be relatively hard to accumulate sufficient amount to reach the predetermined threshold at low resolution if we limit the length of observational window.

The performances of coherent approaches GLRT-based CUSUM algorithm and MMSE-based CUSUM algorithm are demonstrated in Fig. 5.3. We could find that the GLRT-based method performs much steadily at low-SNR region due to the fact that it estimates the fading coefficient according to cumulative realizations. The same phenomenon could be observed in the comparison with the performance of the MMSE-based algorithm. It could be inferred from the fact that when there is little observations, the MMSE is approximated by the mean of h according to prior knowledge instead of the realization value of h. On the other hand, contrary to non-coherent approaches, we could observe that the mean delay converges to some specific small value varying with chosen threshold rather than to zero. This might be attributed to the fact that there exists essential error due to estimation especially under the situation that we aim to response to the change immediately. And similar to weighted CUSUM algorithm in non-coherent approaches, the GLRT-based method and MMSE-based method deteriorate to some extent as we concern the limitation of the accessible data buffer. Since if we curtail the observational window, it would result in larger estimation error and insufficient amount on resultant statistics in low resolution condition and thus lead to comparatively larger mean delay.

In Fig. 5.4, we depict the operating curves of four proposed CUSUM-based algorithms under SNR= 10dB in flat-fading case. We can see that if we constraint ourselves work on small rate of false alarm for assuring the efficiency of opportunistic accessing, weighted, GLRT-based and MMSE-based CUSUM algorithms perform better than the classical one. This could be inferred from the fact that former three methods are adaptive to the realization of fading channel and yet the non-coherent classical approach always attributes to the effect in the long run. On the other hand, classical method is less sensitive to the curtailment of observational window due to property that the inherent dependency among the observations after change is reserved in the statistics.

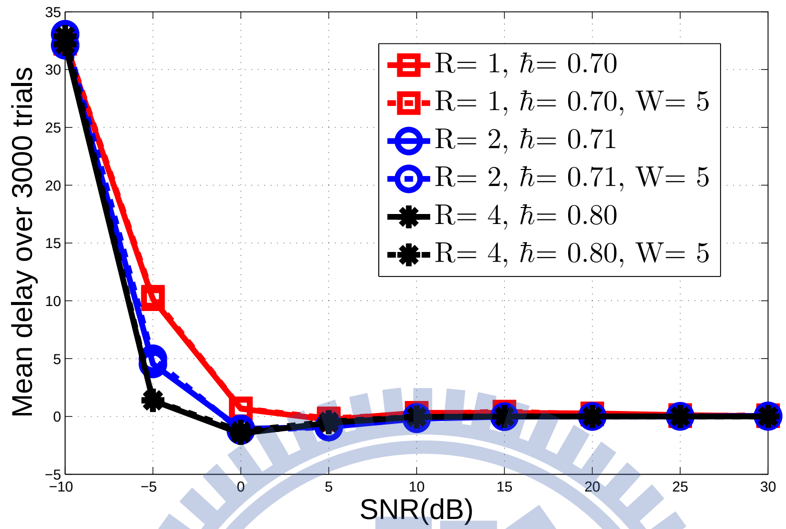
5.2.2 Multiuser Scenario

First, we simulate the cooperative CUSUM-based quickest detection algorithms under centralized framework. Consider a symmetric multiuser scenario where for each sensor the channel conditions are identical and the observations are also assumed independent across the sensors, the performance of proposed algorithms under the case of $R = 2$ and $R = 4$ are shown in Fig. 5.5, Fig. 5.6, and Fig. 5.7. Under non-coherent approaches, we choose the thresholds that result in the same performance value at SNR= -10 dB for comparison of the performances with different number of involved sensors. As to coherent approaches, we set the thresholds that lead to zero mean delay for comparing the effect of increasing set in cooperative sensors. In both flat and frequency-selective fading channels, we could see a growing tendency of the convergent behavior in mean delay as the number of cooperative sensors increasing. But at the same time, the performance gain would gradually saturate as the number of cooperative sensors increases. We could also find that both the GLRT-based and MMSE-based method have smaller mean delay under low resolution condition with growing number of secondary sensors.

Secondly, we present the performances of decentralized schemes with hard fusion of local CUSUM. Similarly, we assume that all of the cooperative sensors are identical and the observations are mutually independent. We set $M = 1$ and $M = 2$ as making final decision by M-out-of-R combing rule under both the cases of $R = 2$ and $R = 4$. In both the schemes that sensors updates just for once and at each time instant, we could observe analogous tendency in convergence of mean delay as we see in the centralized case from Fig. 5.8-5.13. On the other hand, the performance gain of cutting down mean delay is relatively smaller as comparing with centralized cases due to bandwidth constraint of communication between sensors and the fusion center.

Finally, we discuss the decentralized framework adopting global CUSUM with quantized local decisions. Three kinds of scenarios based on different channel conditions between sensors and the fusion center with 1-bit or 2-bit local quantization are presented. Note that under the assumption of fixed noise power at sensors and variance of fading effects, the distribution after reoccupying of the primary user within low-SNR region, especially below 0 dB, results in a small difference with the one before reclaiming. This property also leads to low discrimination between the approximated distributions before and after change and affects the effectiveness of quantization. Fig. 5.14 shows the performance curves under perfect channels between sensors and the fusion center. We also model the channels between sensors and the fusion center as binary symmetric channels with bit-cross-error equals to 0.2 and the corresponding performance curves are given in Fig. 5.15. Further, we consider additive white gaussian noise channels with received SNR at the fusion center being 10 dB and the performance curves are shown in 5.16.

Fixed the number of cooperative sensors, we could see that the case using 2 bits in quantization generally achieves superior performance to the counterpart with only 1-bit quantization in low-SNR region under each channel condition. On the other hand, under the cases considering perfect channel and AWGN channel, we could see that the performance curves converges to zero in mean delay as SNR increases. Whereas, we could observe that the mean delay converges to some specific small value close to zero and varying with bit-cross-error under the cases with BSC channels. Since the likelihood ratio in the fusion center is concentrated at some specific value due to quantization and could not be smoothed out by averaging the channel effect in the second phase due to the discrete property of BSC, the cross-error in the second phase of transmission thus dominates the convergent value in mean delay of global CUSUM test. Similarly to previous two distributed frameworks, we could see the increasing amount in the cooperative sensors expedite the convergence in mean delay and thus improves the effectiveness of detection.



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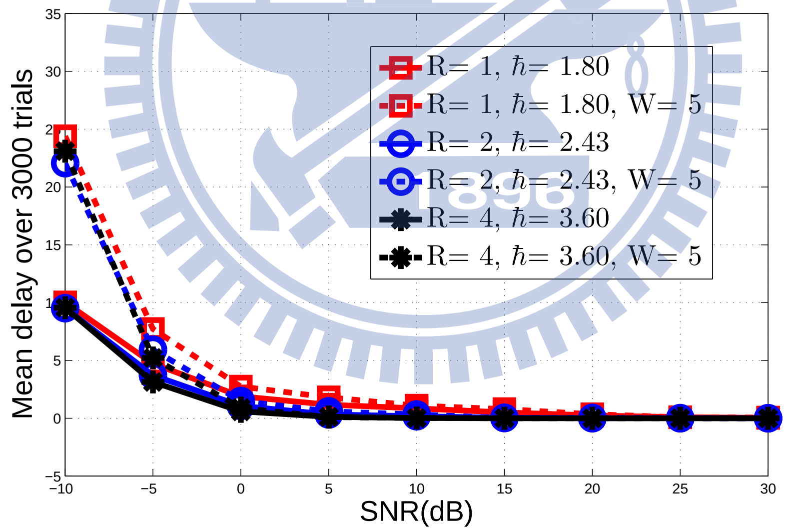
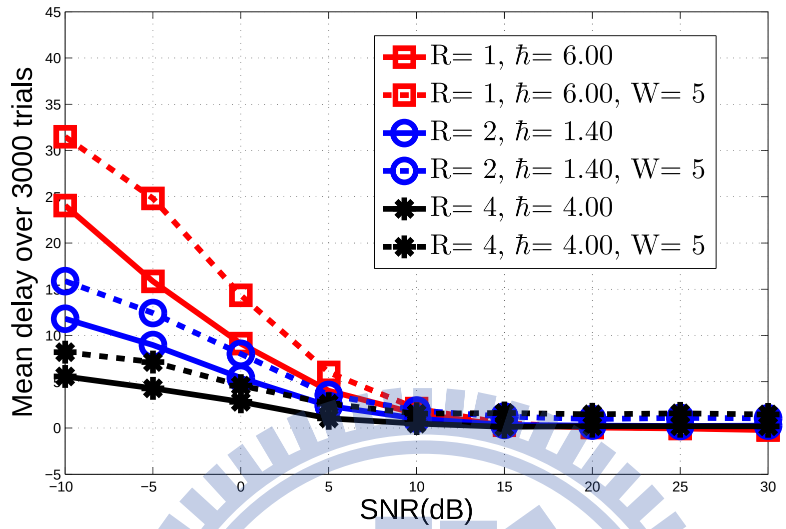


(b)

Figure 5.5: Performances of cooperative schemes by non-coherent approaches in centralized case under flat-fading environment. (a) Classical CUSUM. (b) Weighted CUSUM.



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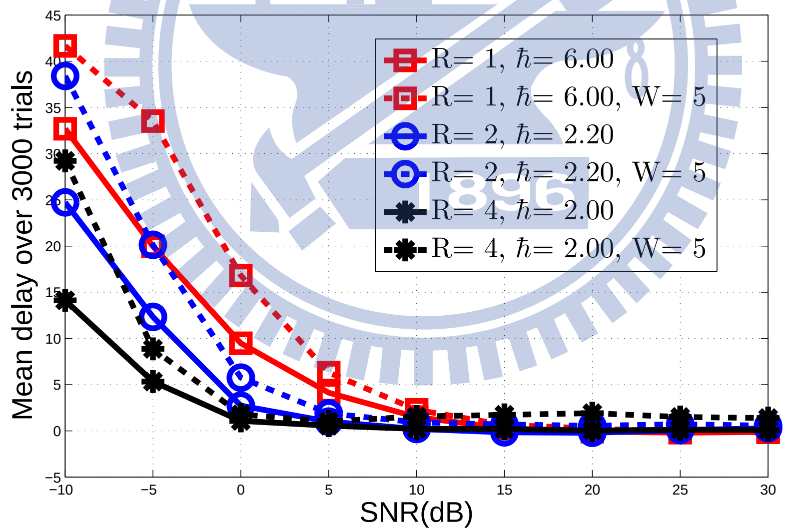

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Figure 5.6: Performances of cooperative schemes by coherent approaches in centralized case under flat-fading environment. (a) GLRT-based CUSUM. (b) GLRT-based CUSUM.

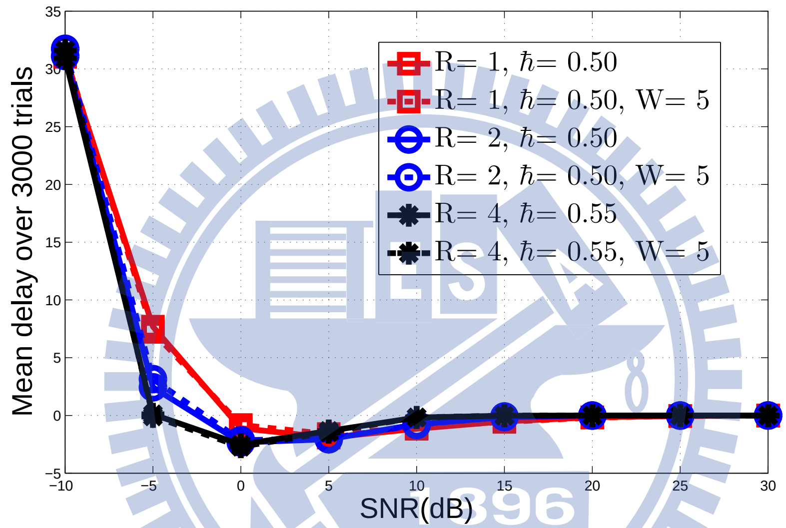
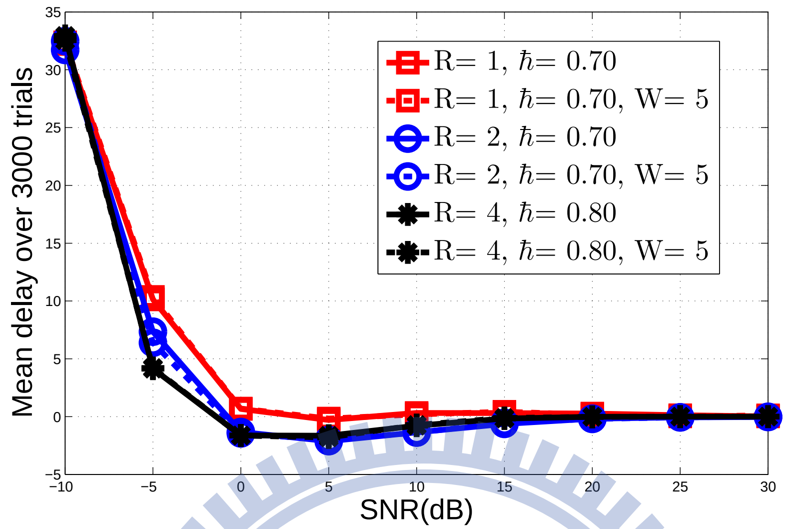


Figure 5.7: Performances of cooperative classical CUSUM algorithm in centralized case under frequency-selective fading environment.



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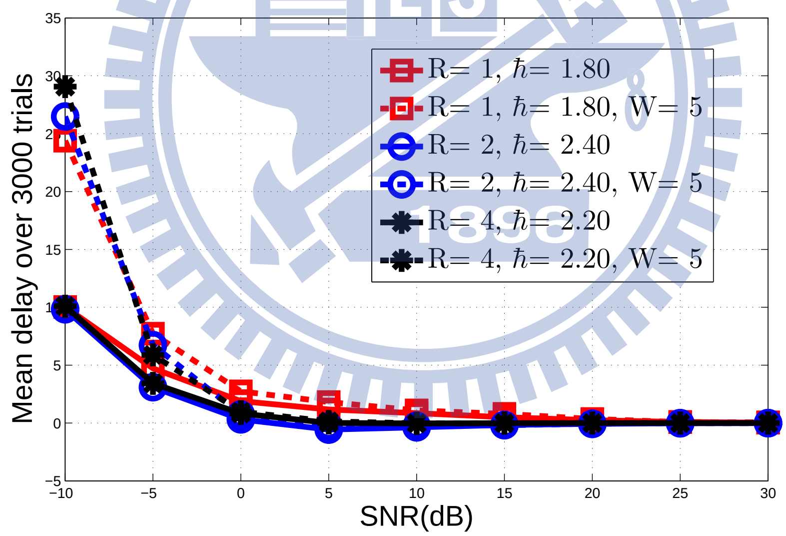

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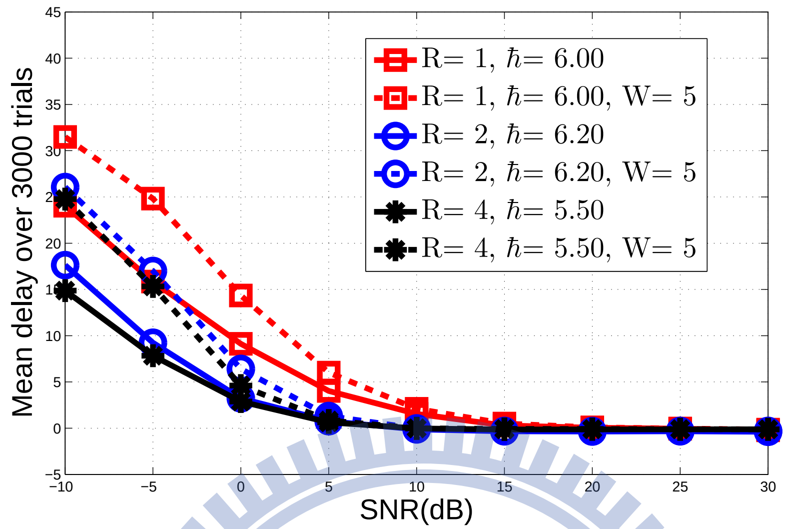
Figure 5.8: Performances of cooperative schemes by non-coherent approaches in decentralized case by hard-fusion of local CUSUM under flat-fading environment in the scheme that sensors updates just for once. (a) Classical CUSUM. (b) Weighted CUSUM.



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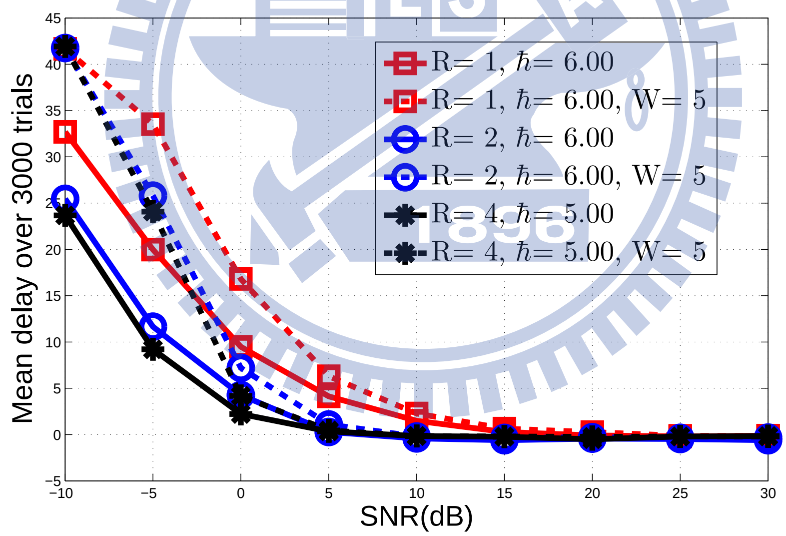

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Figure 5.9: Performances of cooperative schemes by coherent approaches in decentralized case by hard-fusion of local CUSUM under flat-fading environment in the scheme that sensors updates just for once. (a) GLRT-based CUSUM. (b) GLRT-based CUSUM.

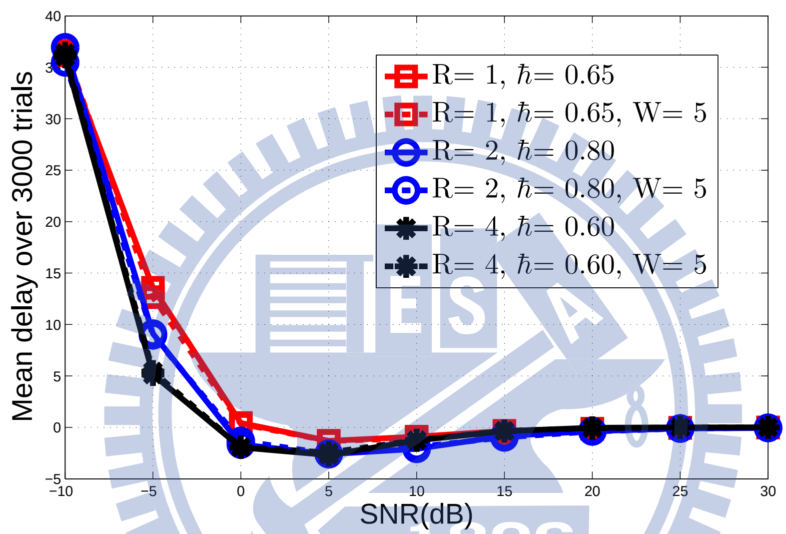
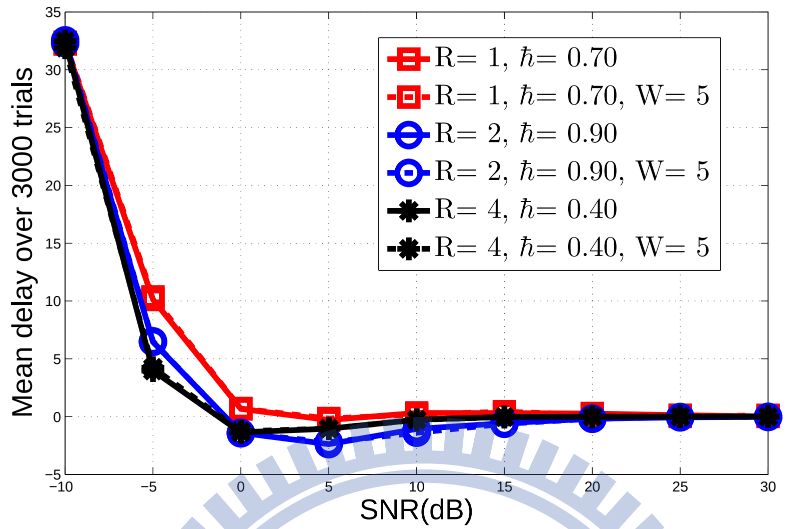


Figure 5.10: Performances of cooperative classical CUSUM algorithm in decentralized case by hard-fusion of local CUSUM under frequency-selective fading environment in the scheme that sensors updates just for once.



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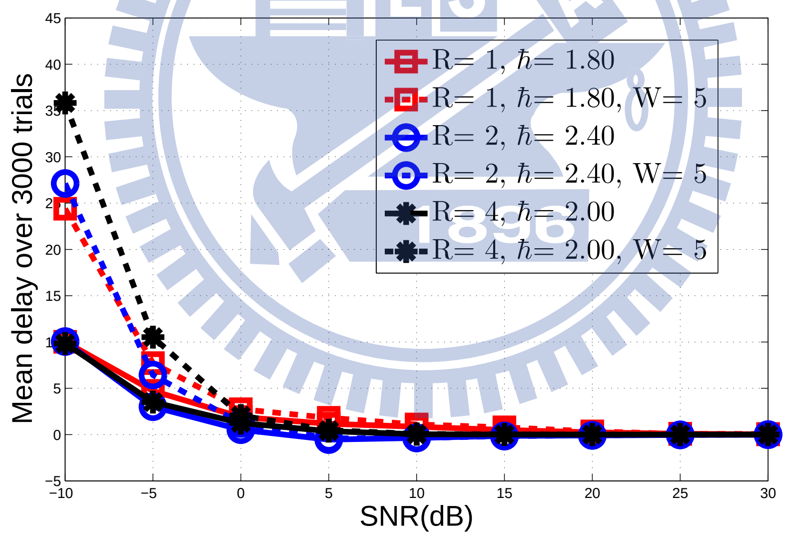

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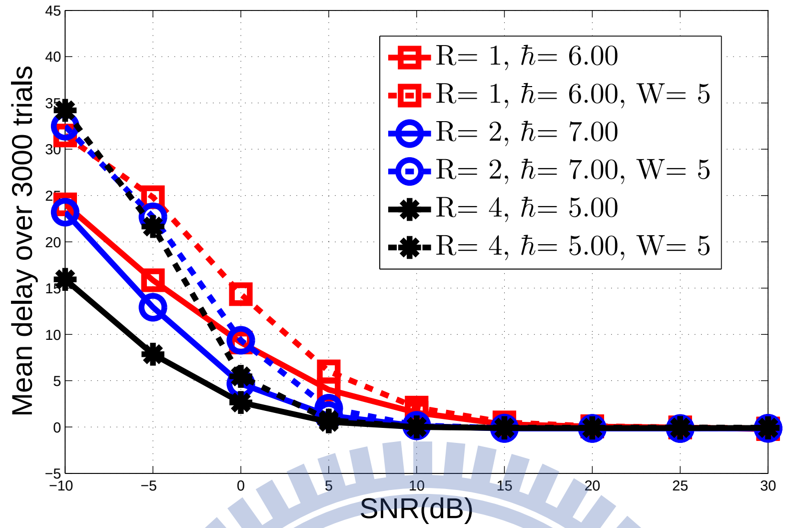
Figure 5.11: Performances of cooperative schemes by non-coherent approaches in decentralized case by hard-fusion of local CUSUM under flat-fading environment in the scheme that sensors updates at each time instant. (a) Classical CUSUM. (b) Weighted CUSUM.



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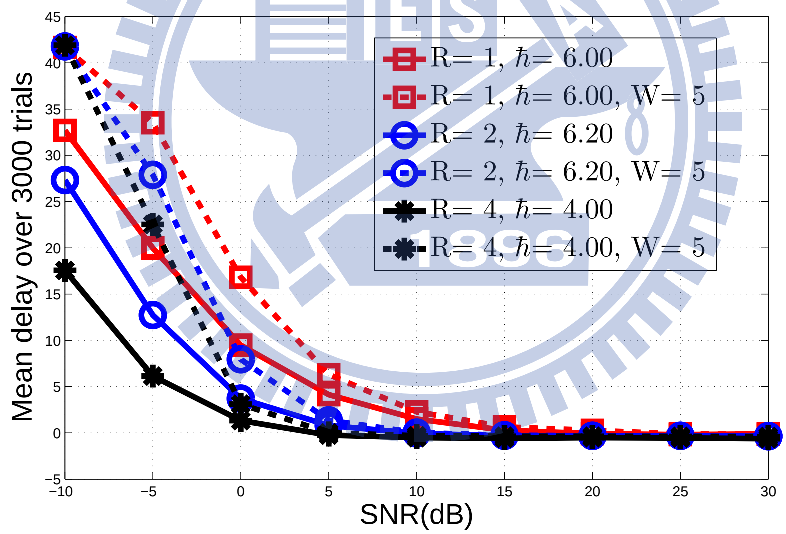

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Figure 5.12: Performances of cooperative schemes by coherent approaches in decentralized case by hard-fusion of local CUSUM under flat-fading environment in the scheme that sensors updates at each time instant. (a) GLRT-based CUSUM. (b) GLRT-based CUSUM.

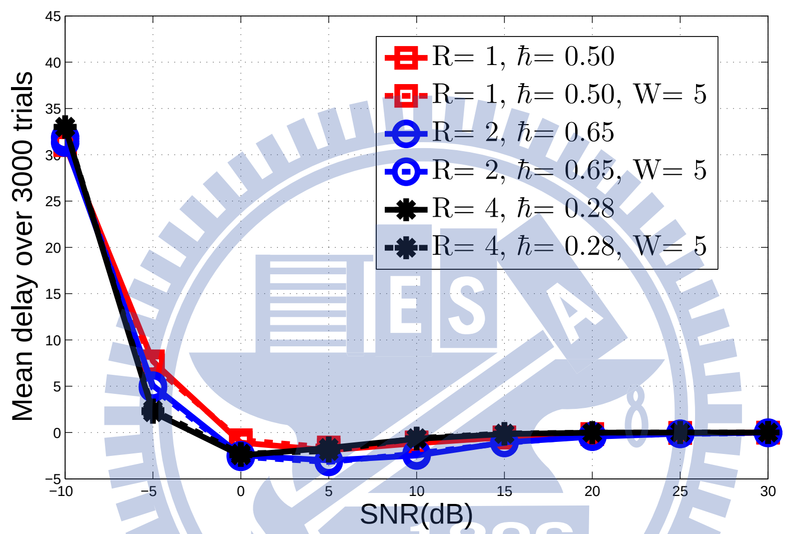


Figure 5.13: Performances of cooperative classical CUSUM algorithm in decentralized case by hard-fusion of local CUSUM under frequency-selective fading environment in the scheme that sensors updates at each time instant.

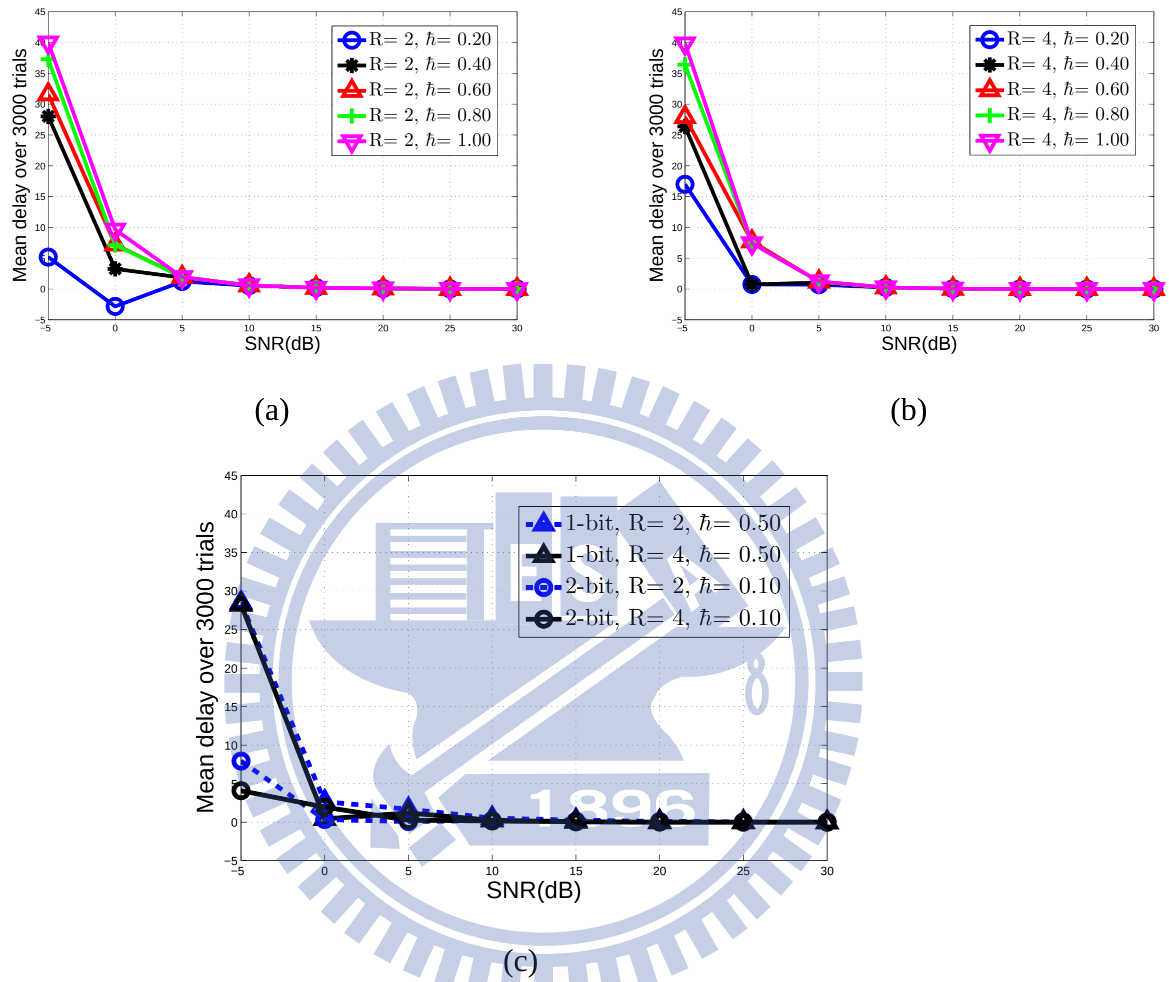


Figure 5.14: Performances of approximation method in decentralized case applying global CUSUM with quantized local decision under perfect channel between sensors and the fusion center. (a) 1-bit quantization, R=2. (b) 1-bit quantization, R=4. (c) Comparison of the cases (1) R=2 with 1-bit quantization (2) R=4 with 1-bit quantization (3) R=2 with 2-bit quantization (4) R=4 with 2-bit quantization.

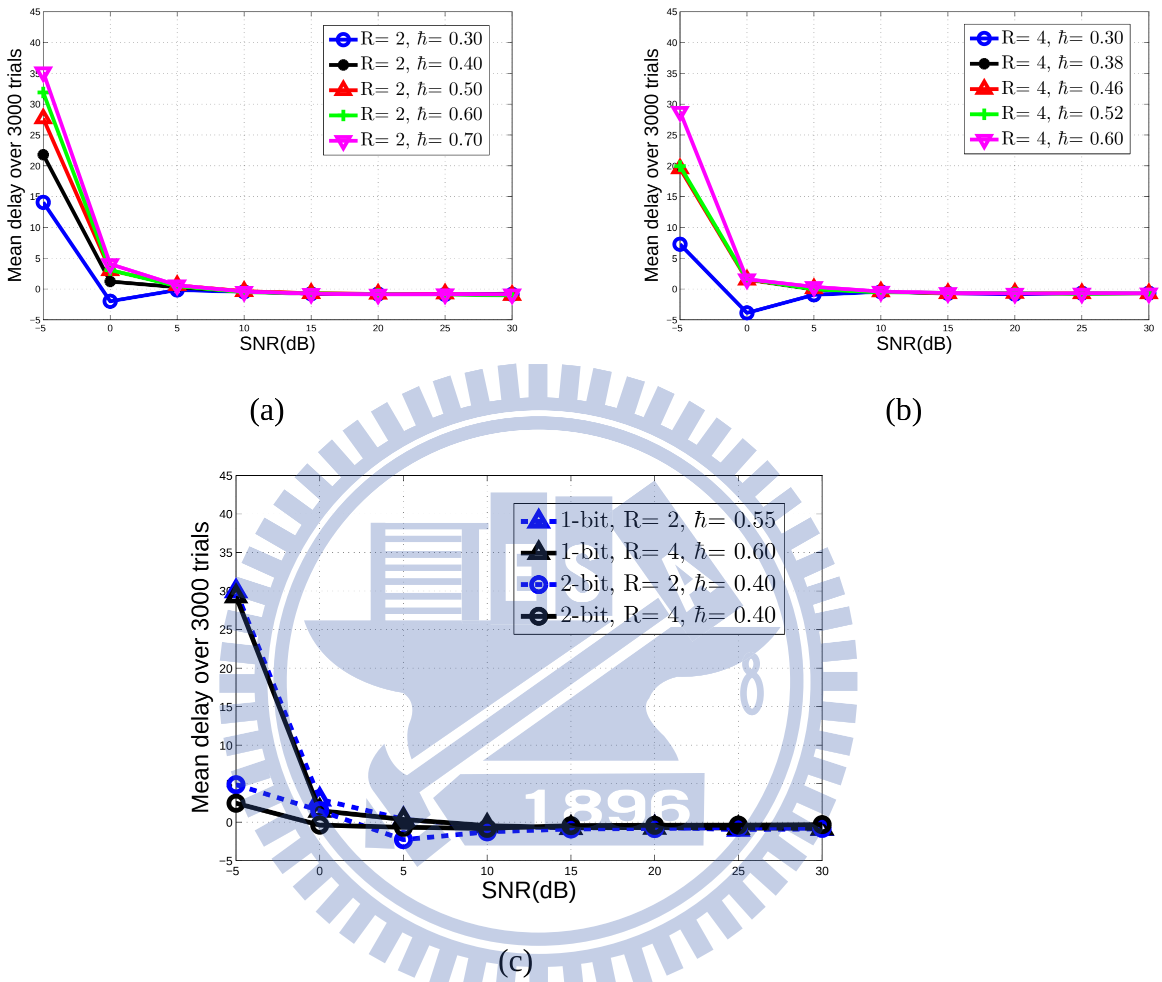
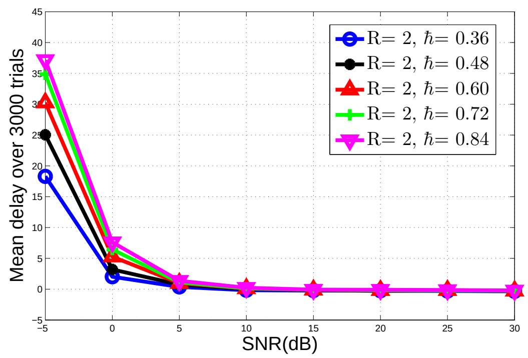


Figure 5.15: Performances of approximation method in decentralized case applying global CUSUM with quantized local decision under binary symmetric channel with cross-error= 0.2 between sensors and the fusion center. (a) 1-bit quantization, R=2. (b) 1-bit quantization, R=4. (c) Comparison of the cases (1) R=2 with 1-bit quantization (2) R=4 with 1-bit quantization (3) R=2 with 2-bit quantization (4) R=4 with 2-bit quantization.



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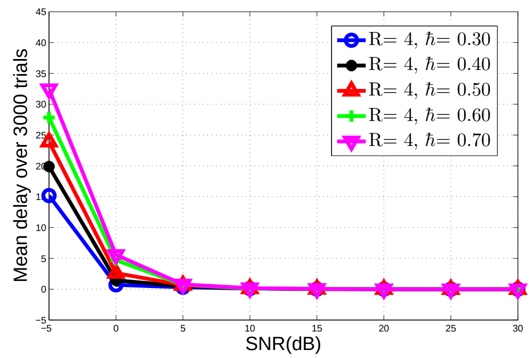

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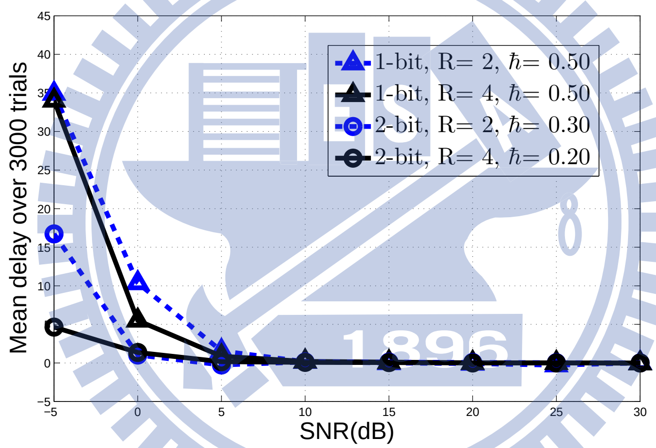

(c)

Figure 5.16: Performances of approximation method in decentralized case applying global CUSUM with quantized local decision under AWGN channel with received SNR= 10dB between sensors and the fusion center. (a) 1-bit quantization, R=2. (b) 1-bit quantization, R=4. (c) Comparison of the cases (1) R=2 with 1-bit quantization (2) R=4 with 1-bit quantization (3) R=2 with 2-bit quantization (4) R=4 with 2-bit quantization.

Chapter 6

Conclusion and Future Work

6.1 Conclusion

We have studied the spectrum sensing problem under the concern of possible fading environments in a sequential view for promoting the coexistence of cognitive applications with existing primary systems. Aiming at avoiding interference to licensed primary users, we have proposed several cumulative-sum (CUSUM)-based algorithms, termed of classical CUSUM, weighted CUSUM, GLRT-based CUSUM and MMSE-based CUSUM algorithm respectively, for detecting as quickly as possible the event that the dormant primary systems start reclaiming the use of the spectrum given only statistical information about the channel condition between cognitive and primary users. We have demonstrated that all of the four proposed algorithms promise agility of detecting the beginning of reoccupying primary signals as for the case with flat-fading. Even under multipath environments, the proposed classical CUSUM algorithm performs fairly well within limited backward observational window.

Further, we have also studied the case of cooperative quickest detection where a number of cognitive users provide decision strategies and collaboratively detects the beginning of the reclaims of the primary signal under three different distributed frameworks. We consider decentralized schemes including hard fusion of local CUSUM test and global CUSUM test with quantized version of local observations in addition to centralized case. In the simulations, we have justified the effectiveness of the proposed CUSUM-based quickest detection applied to different distributed frameworks and examine how the increasing number of involved cognitive users influences the performance gain.

6.2 Future Work

In this thesis, although we have proposed effective CUSUM-based algorithms to tackle the spectrum sensing problem in a sequential manner, the efficiency of proposed algorithms has not been analyzed. Since our formulation captures possible fading effects between the cognitive and the primary user, the detection problem we deal with involves non-homogenous and innately dependent observations after the reoccupying of the coexisting primary system contrast to homogenous-distributed and independent observations after change in conventional quickest detection. Besides, we also emphasize the situation where we could make use of known feature of primary signaling during detection process. These properties exclude existing available analysis about CUSUM procedures under minimax formulation on the optimality or asymptotic behavior to the best of our knowledge as yet.

As for cooperation schemes, future work might consider the design of the quantization strategy on the local sensors with memory or take the dependency among quantized versions of observations into account as designing the decision rule of global CUSUM test. Another aspect for future work is to include the design of power allocation under cooperative schemes as we consider more practical situation where the levels of channel conditions among involved cognitive users are not identical.

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作者簡歷

學生劉譪璇，民國七十五年五月二十日出生於台北市。民國九十三年九月自新竹女中以申請入學管道進入國立交通大學電子工程學系，期間得過電工系書卷獎四次、潘文淵文教基金會九十六年度獎學金，擔任過系學會學術組組長，並於民國九十七年六月以班上第五名畢業於國立交通大學電子工程學系。民國九十七年九月以推甄入學就讀國立交通大學電子研究所，跟隨簡鳳村教授從事即時變化檢測於無線感知通訊網路之相關研究，碩士論文題目為《CUSUM-Based Quickest Spectrum Detection in Cognitive Radio Networks》，期間得過電子所系統組書卷獎一次，並獲選為九十九年度中華民國斐陶斐交通大學分會之榮譽學員。