國立交通大學

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碩 士 論 文

多輸入多輸出系統之位元配置有限回饋

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研 究 生：鄭人予

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中 華 民 國 九 十 九 年 六 月

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研 究 生：鄭人予　　Student：Panna Jeng

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摘要

本論文包含兩個部分，在第一個部分我們提出一個有限回饋位元配置的多輸入多輸出系統。我們會證明此系統可以達到全多樣性。在傳輸速率固定的假設下，我們推算出能達到最小錯誤率的最佳位元配置，並證明達到最小錯誤率的位元配置也是使用最低傳輸能量的位元配置。模擬結果顯示我們所提出的系統可以使用較少的回饋位元達到低錯誤率。在第二個部分我們針對傳統單階預先編碼器的有限回饋系統設計低複雜度的編碼器選擇準則。我們也提出一個可以降低複雜度的二階預先編碼器系統。模擬結果會展示出我們所設計的系統的可用性。

MIMO systems with limited feedback of bit allocation

Panna Jeng

Advisor: Dr. Yuan-Pei Lin
Department of Electrical and Control Engineering
National Chiao Tung University

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Abstract

In this thesis we first proposed a limited feedback system which sends back only the bit allocation (BA) information. The system will be termed a BA system. we show that the proposed BA system can achieve full diversity order. we will also derive the optimal bit allocation for minimum bit error rate when the transmission rate is given. Secondly, we develop low-complexity selection criteria for conventional one-step precoder system which feedbacks only the precoder information. A two-step system is proposed to reduce the number of searches. In simulations, the usefulness of the proposed systems will be demonstrated.

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Chapter 1

Introduction

Multiple input multiple output (MIMO) systems with limited feedback have attracted great interest recently [1–4]. These practical systems can improve performance metric such as transmission rate or error rate by sending limited amount of information bits through a reverse channel to transmitter [1]. It is generally assumed that there is no channel state information at the transmitter and only the receiver has the perfect channel knowledge. To obtain complete channel knowledge at the transmitter may be unrealistic since it requires infinite number of bits. In practice the reverse channel can transmit only finite amount of bits and it is desirable to have feedback rate as low as possible.

Various methods have been proposed to exploit the use of feedback bits. For precoded spatial multiplexing systems with finite-rate feedback, the receiver selects a transmitting matrix (or precoder) from a set of matrices (precoder codebook) known to both transmitter and receiver. Then the corresponding index is sent back to the transmitter using finite number of bits. Different criteria of precoder selection and unitary precoder codebook designs are developed in [5]. For the criteria considered in [5], it has been show that with some approximations the design of optimal codebook can be converted to a problem of Grassmannian subspace packing. Randomly generated codebooks known to both transmitter and receiver is proposed in [6], and the method is called random vector quantization (RVQ). In [7], Using bit error rate (BER) as a criterion of selecting precoder matrix from the codebook is proposed and the optimal unitary precoder for infinite feedback rate, i.e., full channel knowledge at transmitter, is given. Generalized Lloyd algorithm is employed to construct precoder codebooks. An iterative approach of searching a codebook for maximum mutual information is proposed in [8]. Capacity loss due to quantized feedback is thoroughly analyzed in [9]. Spatial multiplexing for two substreams using simple rotation is designed in [10]. A special form of precoding systems is the antenna selection system [11, 12] that chooses the best subset of transmitt antennas to minimize BER. In this case the transmitter possess the advantage of low complexity since the precoder is a submatrix of the identity matrix.

In addition to precoder information, quantized power allocation information can be also fed back for improving system performance. In this case there are two codebooks, one for quantized precoder and one for quantized power allocation. The index of precoder and the index of power allocation are both sent back to the transmitter. Usually a higher feedback rate is required. In [13], power loading codebook is designed separately and the performance is significantly improved. In [14], based on parameterizations, two efficient methods for precoder quantization are proposed. Combined with feedback of power loading, the proposed system's capacity is very close to the case when full channel state information is available at the transmitter. In some recent work, bit loading information is also sent back to the transmitter. In [15], the optimal unquantized precoder is factorized via Given's rotations and the parameters in the rotation matrices are quantized. Thus the complexity of precoder quantization is low. Feedback of bit loading, power loading and precoder is considered in [16] to improve the system throughput. In these works, bit loading is not quantized.

In most of the previously mentioned works, the number of subchannels (or substreams) M is fixed and does not change with the channel. Multimode antenna selection [17] allows the number of sucstreams M or "mode" to vary with the channel. The transmission bits are uniformly allocated on the M substreams. It is shown in [17] that with M_t feedback bits, the system can achieve diversity order M_rM_t, where M_r and M_t represent the number of transmit and receive antenna respectively. Similarly, multimode precoding [18] also allows number of substreams M to alter in accordance with the channel. Transmission bits are equally allocated too. In addition, precoder codebooks are constructed for each modes. With judicious design, multimode precoding can achive diversity order $M_r M_t$ with $\log_2 M_t$ bits. The design of codebooks for multimode precoding over spatially correlated channel is developed in [19]. Generalized Lloyd algorithm is applied to design capacity maximizing codebooks for multimode transmission in [20]. In [21] a quantized principal component selection precoding scheme for capacity maximizing is proposed. The achieved performance by [21] can be close to the capacity obtained with full channel state information.

In this thesis, we consider two feedback scenarios. In the first scenario, the receiver feedbacks only bit allocation and in the second scenario the receiver feedbacks only the precoder information. The system that sends back only information of bit allocation (BA) is called BA system. Given a channel realization, receiver selects a bit allocation vector that minimizes the BER from a bit allocation codebook whose codewords satisfy the target transmission rate. The index correspond to this BER-minimizing codeword is sent back to transmitter through a reverse channel. According to the feedback information, the transmitter allocates bits to the modulation symbols and perform spatial multiplexing (precoding) using a unitary precoder known to the transmitter and receiver a priori. We will show that BA system can achieve full diversity order $M_r M_t$ using $\log_2 M_t$ bits. Moreover, we will derive the optimal bit allocation that minimizes the BER when the bit allocation vector is not constrained to be from a codebook and it can be real nembers. In this case, the BER performance of the BA system always outperforms the optimal BER-minimizing unitary precoder system which employs uniform bit loading and has complete channel knowledge at the transmitter. Furthermore, we will show that the unconstrained optimal bit allocation for BER minimization also minimizes the transmission power for a given error rate. To reduce the complexity of bit allocation vector selection, we develop an efficient quantization method. Simulation will be presented to show the usefulness of the proposed BA system, especially for MIMO systems with low feedback rate.

The system that feedbacks only the precoder information is called a precoder system in this thesis. For a given precoder codebook, we propose a simple selection criterion whose BER performance is very close to the method in [7] which requires exact BER computation. In addition, we propose a two-step design. The design is motivated by crucial properties of the optimal unquantized precoder. Namely, the total mean squared error (MSE) is minimized and the subchannel error variances are equalized.

In the proposed two-step design, the precoder $\mathbf{F}$ is a product of the form $\mathbf{F}_V\mathbf{F}_Q$. When there is unlimited feedback, $\mathbf{F}_V$ and $\mathbf{F}_Q$ can be chosen so that $\mathbf{F}$ is the optimal precoder. When the feedback rate is finite, $\mathbf{F}_V$ and $\mathbf{F}_Q$ are chosen from their respective codebooks; $\mathbf{F}_V$ is chosen to minimize total MSE while $\mathbf{F}_Q$ is chosen to equalize subchannel error variances. The indexes of codewords for $\mathbf{F}_V$ and $\mathbf{F}_Q$ are sent back to the transmitter. If the codebooks for $\mathbf{F}_V$ and $\mathbf{F}_Q$ contains respectively 2^{B_V} and 2^{B_Q} codewords, the required number feedback bits is $B = B_V + B_Q$, while the number of searches for selecting the precoder is $2^{B_V} + 2^{B_Q}$.

Simulation results show that the performance of the proposed two-step design is comparable to the conventional design for the same feedback rate but the complexity of selecting precoder is much lower.

1.1 Outline

- Chapter 2: General system model is presented.

- Chapter 3: Previous works are reviewed in this chapter. Section 3.1 introduces a BER criterion and optimal unitary precoder for precoded spatial multiplexing system with infinite feedback rate proposed by S. Zhou and B. Li. In section 3.2, we review multiple antenna selection proposed by R. W. Heath, Jr. and D. J. Love. Section 3.3 introduces multimode precoding which is also proposed by R. W. Heath, Jr. and D. J. Love.

- Chapter 4: The proposed BA system is presented in this chapter. In Section

4.1, we give the MIMO system model for BA system. Feedback of bit allocation is presented in Section 4.2. The diversity order of the proposed system is given in Section 4.3. Optimal bit allocation for minimum BER without constraining the bit allocation vector to be from a codebook is derived in Section 4.4. In Section 4.5, an efficient method of bit allocation vector selection is discussed.

- Chapter 5: We consider the precoder system in this chapter. Section 5.1 introduces the system model for precoder system and the BER optimal precoder. Section 5.2 presents two simple selection criterion for precoder system. Two-step system is given in Section 5.3.
- Chapter 6: Simulation examples are presented in this chapter.
- Chapter 7: A conclusion is given in this chapter.

1.2 Notations

1. Bold face upper case letters represents matrices. Bold face lower case letters represents matrices. The notation $\mathbf{A}^{\dagger}$ denotes transpose-conjugate of $\mathbf{A}$. The notation $\mathbf{A}^{T}$ denotes transpose of $\mathbf{A}$.
2. The function $E[y]$ denotes the expect value of a random variable y.
3. The notation $\mathbf{I}_m$ is used to represent the $m \times m$ identity matrix.
4. The notation $\mathbf{W}_m$ is used to represent the $m \times m$ unitary DFT matrix given by,
$$[\mathbf{W}_m]_{kn} = \frac{1}{\sqrt{m}} e^{-j\frac{2\pi}{m}kn} \quad \text{for} \quad 0 \le k, n \le m-1. \tag{1.1}$$
5. The notation $C(n, k)$ is used to denote the chosen function of n and k.

Chapter 2

General System Model

The finite-rate feedback $M_r \times M_t$ MIMO system is shown in Fig. 2.1. The channel

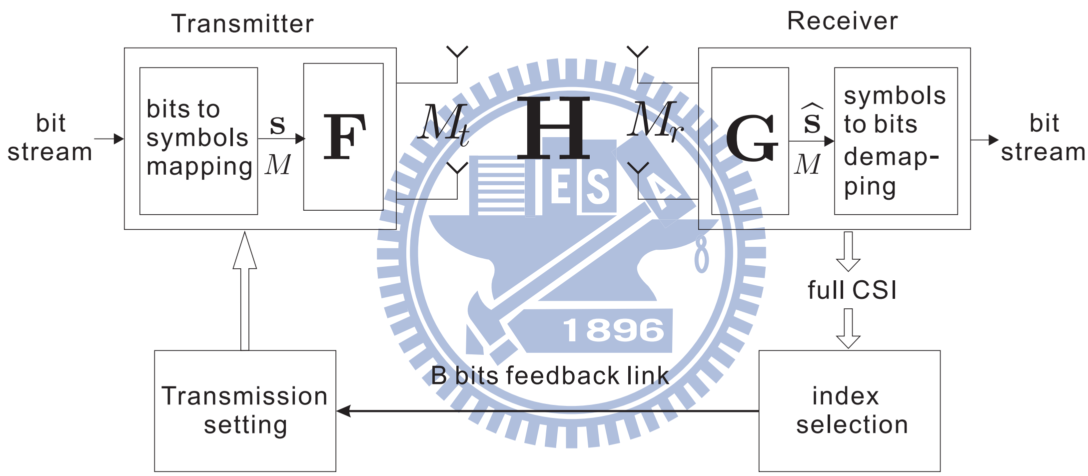

Figure 2.1: MIMO system with limited feedback

is modeled by a $M_r \times M_t$ memoryless matrix with an channel noise vector $\mathbf{q}$ of size $M_r \times 1$. It is supposed that the channel is block fading, which means the channel remains constant over sufficiently long period before independently taking a new realization. The noise vector $\mathbf{q}$ is assumed to be additive white Gaussian with zero mean and variance N_0. The system can process M substreams, where $M \leq \min(M_r, M_t)$. The input vector $\mathbf{s}$ is an $M \times 1$ vector which consists of M modulation symbols. The symbols s_k are assumed to be zero mean and uncorrelated, hence the autocorrelation matrix $\mathbf{R}_s = E[\mathbf{s}\mathbf{s}^\dagger]$ is a diagonal matrix.

R_b is the number of bits transmitted during each symbol period. Assume the total transmission power is P_0 and the precoder $\mathbf{F}$ is an unitary $M_t \times M$ matrix. The total transmission power $P_0 = E[\mathbf{x}^\dagger \mathbf{x}]$ can be written as

$$P_0 = E[\mathbf{x}^\dagger \mathbf{x}] = trace(\mathbf{F}\mathbf{R_s}\mathbf{F}^\dagger) = trace(\mathbf{R_s}) = \sum_{k=0}^{M-1} \sigma_{s_k}^2, \tag{2.1}$$

where we have used the trace property $trace(\mathbf{AB}) = trace(\mathbf{BA})$ for two matrices $\mathbf{A}$ and $\mathbf{B}$ and the fact that $\mathbf{F}^\dagger \mathbf{F} = \mathbf{I}_M$. The channel output vector $\mathbf{r}$ is therefore

$$\mathbf{r} = \mathbf{HFs} + \mathbf{q} \tag{2.2}$$

The $M \times M_r$ receiving matrix $\mathbf{G}$ can be zero forcing receiver or minimum mean square error (MMSE) receiver [22]

$$\mathbf{G} = \begin{cases} (\mathbf{F}^\dagger \mathbf{H}^\dagger \mathbf{HF})^{-1} \mathbf{F}^\dagger \mathbf{H}^\dagger, & \text{zero-forcing receiver,} \\ \mathbf{R_s} \mathbf{F}^\dagger \mathbf{H}^\dagger (\mathbf{HFR_s F}^\dagger \mathbf{H}^\dagger + N_0 \mathbf{I}_{M_r})^{-1}, & \text{MMSE receiver.} \end{cases} \tag{2.3}$$

The error vector $\mathbf{e}$ at the output of receive matrix $\mathbf{G}$ is

$$\mathbf{e} = \hat{\mathbf{s}} - \mathbf{s} = \mathbf{Gr} - \mathbf{s} \tag{2.4}$$

The autocorrelation matrix of error vector $\mathbf{R_e} = E[\mathbf{e}\mathbf{e}^\dagger]$ given by [22] is

$$\mathbf{R_e} = \begin{cases} N_0 (\mathbf{F}^\dagger \mathbf{H}^\dagger \mathbf{HF})^{-1}, & \text{zero-forcing receiver} \\ \mathbf{R_s} - \mathbf{R_s} \mathbf{F}^\dagger \mathbf{H}^\dagger (\mathbf{HFR_s F}^\dagger \mathbf{H}^\dagger + N_0 \mathbf{I}_{M_r})^{-1} \mathbf{HFR_s}, & \text{MMSE receiver} \end{cases} \tag{2.5}$$

Generally, it is assumed the transmitter has no channel state information and the channel is perfectly estimated at the receiver. The reverse link can feedback B bits. At the receiver, transmission information for enhancing desired performance is derived from the full channel knowledge. Based on this information, an index is selected from the codebooks which are known to both the transmitter and the receiver. Then the index is sent back through the reverse channel to the transmitter. According to the feedback information, the transmitter adapts the transmission settings and sends the signals into channel. Transmission information extracted from the full CSI at receiver such as precoding matrix, power loading, and bit allocation are used by different system designs. Using these information, various performance like BER, capacity and transmission bit rate can be improved. In this thesis, the efficiency between BER performance and the amount of feedback bits is the main topic of our work.

In the following we discuss the system model for the precoder system (no bit allocation) and the system model for the BA system (with bit allocation) separately.

Precoder system. In the precoder system there is no bit allocation, the transmitted bits are assumed to be equally allocated on M symbols. Each modulation symbol carries $\frac{R_b}{M}$ bits and $\frac{R_b}{M}$ is assumed to be integer. Assuming QAM modulation, the symbol error rate for k-th subchannel is well approximated by [23]:

$$SER_k = 4(1 - \frac{1}{2^{R_b/2M}})Q\left(\sqrt{\frac{3}{(2^{R_b/M}-1)}\beta_k}\right), \tag{2.6}$$

where

$$Q(y) = \frac{1}{\sqrt{2\pi}}\int_y^{\infty} e^{-t^2/2}dt, \quad y \geq 0,$$

and β_k is the unbiased SNR of the k-th subchannel. For zeroforcing and MMSE linear receiver, β_k can be expressed respectively as,

$$\beta_k = \begin{cases} \frac{\sigma^2_{\mathbf{s}_k}}{\sigma^2_{\mathbf{e}_k}}, & \text{zero-forcing receiver,} \\ \frac{\sigma^2_{\mathbf{s}_k}}{\sigma^2_{\mathbf{e}_k}} - 1, & \text{MMSE receiver.} \end{cases} \tag{2.7}$$

When Gray code is used, the BER for k-th subchannel can be approximated by

$$BER_k \approx \frac{SER_k}{(R_b/M)}.$$

So, when precoder matrix $\mathbf{F}$ is used the average BER for a given channel $\mathbf{H}$ can be approximately expressed as

$$BER(\mathbf{F},\mathbf{H}) \approx \frac{1}{R_b}\sum_{k=0}^{M-1}\frac{R_b}{M}BER_k = \frac{1}{M}\sum_{k=0}^{M-1}SER_k. \tag{2.8}$$

Since the bit allocation is set to be uniformly loaded, the error performance is independent of bit allocation and is decided by the unbiased SNR β_k. In the precoder system, the receiver sends back the information of the precoder back to the transmitter.

BA system. In the BA system, the symbols can carry different number of bits. Suppose b_k bits are carried by the k-th modulation symbols. Thus, the transmitted bits per channel use is

$$R_b = \sum_{k=0}^{M-1} b_k. \tag{2.9}$$

Let $\mathbf{b} = \begin{bmatrix} b_0 & b_1 & \cdots & b_{M-1} \end{bmatrix}^T$ be the bit allocation vector. When the input symbols s_k are b_k-bits QAM symbols, the k-th symbol error rate is approximated by [23]:

$$SER_k = 4(1 - \frac{1}{2^{b_k/2}})Q\left(\sqrt{\frac{3}{(2^{b_k}-1)}\beta_k}\right). \tag{2.10}$$

where β_k is the unbiased SNR of k-th subchannel (2.7). Using Gray code, the BER can be approximated by $BER_k \approx SER_k/b_k$. Given a channel $\mathbf{H}$ and the precoding matrix $\mathbf{F}$, the average BER can be approximately computed using

$$BER(\mathbf{b}, \mathbf{F}, \mathbf{H}) \approx \frac{1}{R_b} \sum_{k=0, b_k \neq 0}^{M-1} b_k BER_k = \frac{1}{R_b} \sum_{k=0, b_k \neq 0}^{M-1} SER_k. \tag{2.11}$$

In addition, the system without bit allocation can be considered as having a uniform bit allocation vector $\overline{\mathbf{b}}$ whose entries

$$\overline{b}_0 = \overline{b}_1 = \cdots = \overline{b}_{M-1} = \frac{R_b}{M}. \tag{2.12}$$

Chapter 3

Previous Works

In this chapter, previous works for minimizing error performance are reviewed. Section 3.1 presented a limited feedback precoder ststem with BER selection criterion and codebook design proposed in [7]. Optimal unitary precoder for infinite feedback rate is also derived. In section 3.2 multomode antenna selection [17] is introduced. Section 3.3 recaps multimode precoding [18].

3.1 Precoder System

This section is organized as follows: Section 3.1.1 introduces the system model and presents the BER-based selection criterion. Optimal precoder for infinite feedback rate is given in Section 3.1.2. And Codebook construction is showed in Section 3.1.3.

3.1.1 System Model

Based on the general system model at chapter 2, the system in [7] assumes the number of subchannels M is fixed and all M subchannels are used. The system is without bit allocation design. Thus, the bit loading is uniform and the target bit rate R_b is divisible for M. Each symbol carries $\frac{R_b}{M}$ bits. The power is also equally allocated for each symbols, $\mathbf{R_s} = \frac{P_0}{M}\mathbf{I}_M$. For the reverse channel, it is constrained to send B bits. In this paper, the feedback information is the precoder matrix. Therefore, a precoder codebook $\mathcal{C}_F$ of size 2^B is prepared. After the estimation of forward channel, a precoder matrix is selected using a BER-based selection criterion from $\mathcal{C}_F$ and the corresponding index is fed back to the transmitter. The BER-based selection criterion will be reviewed as follows.

BER selection criterion. Under the assumption of uniform bit allocation, the average BER for each precoder matrix in $\mathcal{C}_F$ can be computed by (2.8). The BER-base selection criterion is

$$\widehat{\mathbf{F}} = \arg\min_{\mathbf{F}\in\mathcal{C}_F} BER(\mathbf{F}, \mathbf{H}). \tag{3.1}$$

To choose a precoder matrix by BER selection criterion, we need to compute the BER formula (2.8) for each precoder matrix in $\mathcal{C}_F$. Therefore, 2^B computations of (2.8) are required to complete BER selection criterion.

3.1.2 Optimal Precoder for infinite-feedback rate

With infinite feedback bits, it can be assumed that the transmitter has full channel knowledge. The optimal precoder $\mathbf{F}_{opt}$ with BER-based criterion can be derived directly from $\mathbf{H}$. The optimal precoder $\mathbf{F}_{opt}$ can provide a benchmark performance for finite-rate precoder feedback system. Assuming the singular value decomposition of $\mathbf{H} = \mathbf{U}\mathbf{\Lambda}\mathbf{V}^\dagger$, where $\mathbf{U}$ and $\mathbf{V}$ are respectively $M_r \times M_r$ and $M_t \times M_t$ unitary matrices. The $M_r \times M_t$ matrix $\mathbf{\Lambda}$ is a diagonal matrix whose diagonal elements are the singular values of $\mathbf{H}$ in a nonincreasing order. And let β_k be the k-th largest subchannel SNR. The optimal precoders for zero forcing and MMSE receiver are given respectively as follows.

Zero-forcing case. Consider a rectangular/square QAM constellation with size M is applied for $\bar{\mathbf{b}}$. Constellation-specific threshold Γ_{th} is shown in table 3.1.2.

1. When $\beta_1 \leq \Gamma_{th}$, $\mathbf{F}_{opt} = \mathbf{V}_M$, where $\mathbf{V}_M$ is the $M_t \times M$ matrix obtained by keeping the first M columns of $\mathbf{V}$.

2. When $\beta_M \geq \Gamma_{th}$, $\mathbf{F}_{opt} = \mathbf{V}_M\mathbf{Q}_M$, where $\mathbf{Q}_M$ is an $M \times M$ unitary that has equal magnitude property, i.e., $|[\mathbf{Q}_M]_{m,n}| = 1/\sqrt{M}$, for $0 \leq m, n \leq M-1$.

3. When conditions in 1 or 2 do not hold, the optimal precoder $\mathbf{F}_{opt}$ can't be found analytically. Suppose that K_1 subchannels' SNR are larger than Γ_{th}. Then one suboptimal precoder that is better than $\mathbf{V}_M$ can be constructed as

$$\mathbf{F} = \mathbf{V}_M \begin{bmatrix} \mathbf{Q}_{K_1} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{M-k_1} \end{bmatrix} \tag{3.2}$$

MMSE case. Consider a rectangular/square QAM constellation with size M is applied for $\bar{b}$. Two constellation-specific thresholds $\Gamma_{th,l}$, $\Gamma_{th,h}$ are shown in table 3.1.2.

1. When $\Gamma_{th,l} \leq \beta_M$ and $\beta_1 \leq \Gamma_{th,h}$, $\mathbf{F}_{opt} = \mathbf{V}_M$.

2. When $\beta_1 \leq \Gamma_{th,l}$ or $\beta_M \geq \Gamma_{th,h}$, $\mathbf{F}_{opt} = \mathbf{V}_M\mathbf{Q}_M$.

3. When conditions in 1 or 2 do not hold, the optimal precoder $\mathbf{F}_{opt}$ can't be found analytically. Suppose that K_1 subchannels' SNR are larger than $\Gamma_{th,h}$ and K_2 subchannel SNRs are smaller than $\Gamma(th,l)$. Then one suboptimal precoder that is better than $\mathbf{V}_M$ can be constructed as

$$\mathbf{F} = \mathbf{V}_M \begin{bmatrix} \mathbf{Q}_{K_1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{M-K_1-K_2} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{Q}_{K_2} \end{bmatrix} \tag{3.3}$$

M	2	4	8	16	32	64	128	256
Γ_{th}	1.5	3	9.01	14.93	38.46	62.50	166.7	250.0

Table 3.1: Table of Γ_{th}

M	2	4	8	16	32	64	128	256
$\Gamma_{th,l}$	0	0	0.579	0.247	0.326	0.264	0.330	0.271
$\Gamma_{th,h}$	0	0	7.621	13.72	37.46	61.50	165.7	249.0

Table 3.2: Table of $\Gamma_{th,l}$ and $\Gamma_{th,h}$

3.1.3 Codebook construction

From [5] it is shown that the precoder codebook design problem can be related to Grassmanian subspace packing. Thus, in [7], generalized Lloyd algorithm is used to construct a precoder codebook by minimizing a chordal distance cost function. The chordal distance between two unitary M_t by M matrices, $\mathbf{F}_i$ and $\mathbf{F}_j$ is

$$d_c(\mathbf{F}_i, \mathbf{F}_j) = \frac{1}{\sqrt{2}} \left\| \mathbf{F}_i \mathbf{F}_i^{\dagger} - \mathbf{F}_j \mathbf{F}_j^{\dagger} \right\|_F , \tag{3.4}$$

where $\| \cdot \|_F$ denotes Frobenius norm. Suppose that $\mathbf{V}$ is an isotropically distributed $M_t \times M$ matrix. The following algorithm quantizes $\mathbf{V}$ to 2^B matrices. Starting with an initial codebook $\mathcal{C}_F = \{\mathbf{F}_0, \mathbf{F}_1, \cdots, \mathbf{F}_{2^B-1}\}$ (obtained from random computer search or using the currently best codebook if available), the codebook design steps are as follows.

1. Generate a training set with N_{tr} samples $\{\mathbf{V}_n\}_{n=1}^{N_{tr}}$.

2. Iterate following steps until it converges.

   (a) Assign $\mathbf{V}_n$ to one of the regions $\{\mathcal{R}_i\}_{i=0}^{2^B-1}$ using the rule

   $$\mathbf{V}_n \in \mathcal{R}_i, \quad \text{if} \quad d_c(\mathbf{V}_n, \mathbf{F}_i) < d_c(\mathbf{V}_n, \mathbf{F}_j), \forall j \neq i. \tag{3.5}$$

   (b) For each region $\mathcal{R}_i$, find the centroid as

   $$\begin{aligned} \mathbf{F}_i^{centroid} &= \arg\min_{\mathbf{F}} \frac{1}{N_{tr}} \sum_{\mathbf{V}_n \in \mathcal{R}_i} d_c^2(\mathbf{V}_n, \mathbf{F}) && (3.6) \\ &= \arg\min_{\mathbf{F}} \frac{1}{N_{tr}} \sum_{\mathbf{V}_n \in \mathcal{R}_i} trace(\mathbf{I}_M - \mathbf{F}^{\dagger} \mathbf{V}_n \mathbf{V}_n^{\dagger} \mathbf{F}) && (3.7) \\ &= \arg\max_{\mathbf{F}} trace(\mathbf{F}^{\dagger} \mathbf{R} \mathbf{F}) && (3.8) \end{aligned}$$

   where $\mathbf{R}$ is defined as

   $$\mathbf{R} = \frac{1}{N_{tr}} \sum_{\mathbf{V}_n \in \mathcal{R}_i} \mathbf{V}_n \mathbf{V}_n^{\dagger}. \tag{3.9}$$

   Let the eigendecomposition of $\mathbf{R}$ as

   $$\mathbf{R} = \mathbf{U}_R \mathbf{\Lambda}_R \mathbf{U}_R^{\dagger}. \tag{3.10}$$

$\mathbf{\Lambda}_R$ is a diagonal matrix whose diagonal elements are in nonincreasing order. It is easy to show that $\mathbf{F}_i^{centroid}$ is a $M_t \times M$ matrix obtained by keeping the first M columns of $\mathbf{U}_R$.

(c) Set $\mathcal{C}_F = \{\mathbf{F}_i^{centroid}\}_{i=1}^{2^B-1}$. During each iteration, The codebook will be record if the minimum chordal distance of $\mathcal{C}_F$

$$\min_{0 \leq i < j \leq 2^B-1} d_c(\mathbf{F}_i, \mathbf{F}j)$$

is larger than the currently best.

3. Go back to 1, generate another training set, then execute the next steps. The algorithm will stop if there is no further improvement on the minimum chordal distance.

3.2 Multimode Antenna Selection

This section is organized as follows. Section 3.2.1 introduces the system model and the diversity of multimode antenna selection system. Section 3.2.2 presents the selection criteria.

3.2.1 System Model

Based on the general model in chapter 2, multimode antenna selection design a system whose number of subchannels M varies according to the channel $\mathbf{H}$ and $M \leq \min(M_r, M_t)$. Assuming target transmission rate R_b is unchanged and independent of channel $\mathbf{H}$, the bit loading for each subchannel is $b_k = \frac{R_b}{M}$ and the power is uniformly divided among M symbols, $R_s = \frac{P_0}{M}\mathbf{I}_M$. For sending M symbols, antenna selection system selects M antennas from M_t transmit antennas to perform transmission. Therefore, there are $C(M_t, M)$ possible antenna combinations. This is equivalent to select a precoder matrix from a set $\mathcal{W}_M$, where the matrices in $\mathcal{W}_M$ are generated by choosing M columns from $\mathbf{I}_{M_t}$. For example, assume $M_t = 3$,

$$\mathcal{W}_1 = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}, \mathcal{W}_2 = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \right\},$$
$$\text{and } \mathcal{W}_3 = \left\{ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right\}.$$

And $\mathcal{W}_M = \{\mathbf{W}_{M,1}, \mathbf{W}_{M,2}, \cdots, \mathbf{W}_{M,C(M_t,M)}\}$. For each M, $\mathcal{W}_M$'s size is $C(M_t, M)$. Suppose it is allowed to select from the complete precoder codebook $\mathcal{C}_F = \{\mathcal{W}_M\}_{M=1}^{M_t}$, the total number of precoder matrices is

$$\sum_{m=1}^{M_t)} C(M_t, M) = 2^{M_t} - 1 \tag{3.11}$$

which requires M_t bits to feedback.

Given a channel, the receiver decide what the number of subchannels M is and which precoder should be chosen from $\mathcal{W}_M$. Then the corresponding index is sent back to transmitter. The transmission is adapt based on this information.

Diversity. Selection diversity provides full diversity M_rM_t [11]. Since selection diversity is equivalent to selecting a precoder matrix from $\mathcal{W}_1$ which is included in the complete precoder codebook $\mathcal{C}_F = \{\mathcal{W}_M\}_{M=1}^{M_t}$, the diversity gain of multimode antenna selection can only be better than selection diversity system. Thus, the diversity order of multimode antenna selection is M_rM_t.

3.2.2 Selection Criteria

Various selection criteria is designed in this paper [17]. Simulations in [17] shows that these selection criteria all yield approximately identical performance. Here we introduce a suboptimal, low-complexity selection criterion that is proposed in this paper. This selection criterion decides M^*, number of using subchannels, first, then selects precoder matrix $\mathbf{F}^*$ from $\mathcal{W}_M$.

Eigenmode Based Selection. Choose M^* such that

$$M^* = \arg \max_{1 \le M \le M_t} \lambda_M^2(\mathbf{H}) d_{\min}^2(M, R_b) \tag{3.12}$$

where $\lambda_k(\mathbf{H})$ is the k-th largest singular value of $\mathbf{H}$, and $d^2_{\min}(M, R_b)$ is the normalized minimun distance in QAM constellation defined as

$$d^2_{\min}(M, R_b) = \frac{6}{(2^{R_b/M} - 1)}/M.$$

After the M^* is determined, $\mathbf{F}^*$ is chosen as

$$\mathbf{F}^* = \arg \max_{\mathbf{F} \in \mathcal{W}_{M^*}} \lambda^2(\mathbf{HF}). \tag{3.13}$$

3.3 Multimode Precoding

This section is organized as follows. In Section , we show the system model and diversity of multimode precoding system. The selection criteria are given in Section 3.3.2. And Section 3.3.3 reviews the criteria of codebook size allocation and construction.

3.3.1 System Model

Founded on the general system model in chapter 2, multimode precoding assumes R_b is the fix target transmission rate, the bit loading is uniformly allocated $b_k = \frac{R_b}{M}$, for $k = 1 \cdots M$, and transmission power is equally divided for M symbols, $\mathbf{R_s} = \frac{P_0}{M}\mathbf{I}_M$. Similar to multimode antenna selection in section 3.2, the multimode precoding system allows the number of subchannels M to vary according to the channel $\mathbf{H}$ and $M \leq \min(M_r, M_t)$. In addition, a codebook $\mathcal{F}_M$ is prepared for each mode M. Since multimode precoding requires $\frac{R_b}{M}$ to be integer, thus only some modes can support transmission. The set of these supported modes is denoted as $\mathcal{M}$. For example, if $R_b = 8$ bits and $M_r = M_t = 4$, then $\mathcal{M} = \{1, 2, 4\}$.

Based on the channel $\mathbf{H}$, the receiver determines the number of subchannels M and selects the precoder matrix from the complete precoder codebook $\mathcal{C}_F = \{\mathcal{F}_M\}_{M=1}^{M_t}$. Subsequently, the index represented this selection is fed back to the transmitter. The transmitter adjusts the transmission setting according to the feedback information.

Diversity. let N_M denoted the number of precoder matrices in $\mathcal{F}_M$. It is proved in [18] that multimode precoding provides full diversity order M_rM_t, if N_1, the codebook size of $\mathcal{F}_1$, is greater than or equal to M_t and the vectors in $\mathcal{F}_1$ span $\mathbb{C}^{M_t}$. Selecting vector from $\mathcal{F}_1 = \{\mathbf{f}_1, \mathbf{f}_2, \cdots, \mathbf{f}_{N1}\}$ is equal to a beamforming system with finite beamforming feasible set [24]. From [24], we know that such a beamforming system has full diversity order equal to $M_r M_t$ if the span of $\mathcal{F}_1$ is equal to $\mathbb{C}^{M_t}$. Therefore, the multimode precoding has full diversity order if above mentioned condition is satisfied.

3.3.2 Selection Criteria

Two selection criteria are proposed in this paper. One is for minimizing probability of error. The other is for maximizing capacity.

Probability of Error Selection Criterion. The selection is divided in two step. For every $M \in \mathcal{M}$, first step selects the F_M^* from each precoder codebooks $\mathcal{F}_M$ using the following selection criterion,

$$F_M^*(\mathbf{H}) = \arg \max_{\mathbf{F} \in \mathcal{F}_M} \lambda_M^2(\mathbf{HF}), \tag{3.14}$$

where $\lambda_k(H)$ is the k-th largest singular value of $\mathbf{H}$. The second step determines the number of subchannels M^* by

$$M^*(\mathbf{H}) = \arg \max_{M \in \mathcal{M}} \frac{\lambda_M^2\{\mathbf{HF}^*_M(H)\}}{M} d_{\min}^2(M, R_b), \tag{3.15}$$

where $d_{\min}^2(M, R_b)$ is defined as

$$d_{\min}^2(M, R_b) = \frac{6}{M(2^{R_b/M} - 1)}.$$

Capacity Selection Criterion. Assuming uncorrelated Gaussian signaling on each substream, the mutual information is known to be

$$C_{UT}(\mathbf{F}_M) = \log_2 \det \left(\mathbf{I}_M + \frac{P_0}{M N_0} \mathbf{F}_M^\dagger \mathbf{H}^\dagger \mathbf{H} \mathbf{F}_M \right). \tag{3.16}$$

Similar to above selection criterion, for every $M \in \mathcal{M}$, first step select the F_M^* from each precoder codebooks $\mathcal{F}_M$ using the following selection criterion,

$$\mathbf{F}_M^* = \arg \max_{\mathbf{F} \in \mathcal{F}_M} C_{UT}(\mathbf{F}). \tag{3.17}$$

Then, M^* is decided by

$$M^* = \arg \max_{M \in \mathcal{M}} C_{UT} \left(\mathbf{F}_M^* \right). \tag{3.18}$$

3.3.3 Allocation Criterion and Codebook Construction

Given B feedback bits, there are total 2^B codewords for complete codebook $\mathcal{C}_F$. Some criterions are designed in [18] to distribute 2^B codewords among the modes in $\mathcal{M}$. Under the assumption that the probabilities of selecting each mode in $\mathcal{M}$ are equal, the codeword allocation criteria for maximizing capacity and minimizing probability of error are given as follows.

Probability of Error Allocation Criterion. Define the cost function as

$$A(N_1, \cdots, N_{M_t}) = \sum_{M \in \mathcal{M}} \frac{d_{\min}^2(M, R_b)}{M} N_M^{\frac{-2}{M_t(M_t+1)}} \tag{3.19}$$

- For $B \leq \log_2(M_t + 1)$, set $N_{Mt} = 1$ and $N_1 = 2^B - 1$.
- For $B > \log_2(M_t+1)$, find the $(N_1, \cdots, N_{M_t})$ that minimizes $A(N_1, \cdots, N_{M_t})$ such that $N_1 \geq M_t$, $N_{M_t} = 1$, and $\sum_{M \in \mathcal{M}} N_M = 2^B$ This minimization can be done using a numerical search or by using convex optimization techniques.

Capacity Allocation Criterion.

- For $B \leq \log_2(M_t + 1)$, set $N_{Mt} = 1$ and $N_1 = 2^B - 1$.
- For $B > \log_2(M_t+1)$, if $B \leq \log_2(M_t(|\mathcal{M}|-1)+1)$, set $N_{M_t} = 1$, $N_1 = M_t$, and $N_k = \frac{(2^B - M_t - 1)}{|\mathcal{M}|-2}$, for $k \in \mathcal{M}, k \neq 1, M_t$. If $B > \log_2(M_t(|\mathcal{M}| - 1) + 1)$, set $N_{M_t} = 1$ and $N_k = \frac{2^B - 1}{|\mathcal{M} - 1|}$ for $k \in \mathcal{M}, k \neq M_t$.

After the sizes for each modes' codebooks are allocated. The codebook for each mode is construct using the method in [5]. The work in [5] can approximately convert the problem of precoder codebook construction into Grassmannian subspace packing. As a result, the codebook design criteria are presented as follows.

Probability of Error Design Criterion. From [5], the projection two-norm distance is defined as

$$d_{proj}(\mathbf{F}_i, \mathbf{F}_j) = \|\mathbf{F}_i\mathbf{F}_i^\dagger - \mathbf{F}_j\mathbf{F}_j^\dagger\|_2,$$

where $\|\cdot\|$ denotes 2-norm of a matrix. For minimizing probability of error, design $\mathcal{F}_M$ such that

$$\delta_{proj} = \min_{\mathbf{F}_i,\mathbf{F}_j \in \mathcal{F}_M:\mathbf{F}_i \neq \mathbf{F}_j} d_{proj}(\mathbf{F}_i, \mathbf{F}_j)$$

is maximized.

Capacity Design Criterion. The Fubini-Study distance is defined in [5] as

$$d_{FS}(\mathbf{F}_i, \mathbf{F}_j) = \arccos|\det(\mathbf{F}_i^\dagger\mathbf{F}_j)|.$$

For maximizing capacity, design $\mathcal{F}_M$ such that

$$\delta_{FS} = \min_{\mathbf{F}_i,\mathbf{F}_j \in \mathcal{F}_M:\mathbf{F}_i \neq \mathbf{F}_j} d_{FS}(\mathbf{F}_i, \mathbf{F}_j)$$

is maximized.

Chapter 4

The Proposed BA system

In this chapter we propose the feedback of only bit allocation (BA) for MIMO systems with limited feedback. The proposed system will be termed a BA system. We show that the proposed BA system can achieve full diversity order. We also derive the optimal bit allocation for minimum BER when the transmission rate is given and the bit allocation vector is not constrained to be from a codebook. It turns out that the optimal bit allocation that minimizes the BER is also the optimal solution for minimizing the transmission power. Using the optimal unconstrained bit allocation, an efficient method for selection BA is developed.

4.1 System Model

Based on the general system model in chapter 2, we assume the total transmission power P_0 is equally divided among all symbols carrying nonzero bits. So s_k has variance given by

$$\sigma_{\mathbf{s}}^2 = \begin{cases} P_0/M_0, & b_k > 0, \\ 0. & b_k = 0, \end{cases} \tag{4.1}$$

where M_0 is the number of symbols carrying nonzero number of bits. As the power is equally divided among symbols with nonzero bits, the autocorrelation matrix of the error vector for the MMSE case (2.5) can be simplified. Removing the symbols with zero bits from $\mathbf{s}$, we obtain a reduced vetor $\mathbf{s}_0$ of size $M_0 \times 1$. If we remove the corresponding columns of $\mathbf{F}$, the result is an $M_t \times M_0$ matrix, say $\mathbf{F}_0$. Then using precoder $\mathbf{F}_0$ with input $\mathbf{s}_0$ gives the same transmitter output

($\mathbf{x} = \mathbf{F}_0\mathbf{s}_0 = \mathbf{F}\mathbf{s}$). The vector $\mathbf{s}_0$ has the autocorrelation matrix $\mathbf{R}_{\mathbf{s}_0} = \frac{P_0}{M_0}\mathbf{I}_{M_0}$. The autocorrelation matrix of the corresponding error vector $\mathbf{e}_0$ is

$$\mathbf{R}_{\mathbf{e}_0} = \begin{cases} N_0(\mathbf{F}_0^\dagger \mathbf{H}^\dagger \mathbf{H}\mathbf{F}_0)^{-1}, & \text{zero-forcing receiver,} \\ (\frac{1}{N_0}\mathbf{F}_0^\dagger \mathbf{H}^\dagger \mathbf{H}\mathbf{F}_0 + \frac{1}{P_0/M_0}\mathbf{I}_{M_0})^{-1}, & \text{MMSE receiver.} \end{cases} \tag{4.2}$$

In our proposed system, the precoder matrix $\mathbf{F}$ in the transmitter is determined beforehand. Therefore, when the channel $\mathbf{H}$ is given, the average BER formula in (2.11) depends only on the bit allocation vector $\mathbf{b}$, which can be optimized to minimize BER.

$$BER(\mathbf{b}, \mathbf{H}) \approx \frac{1}{R_b} \sum_{k=0, b_k \neq 0}^{M-1} b_k BER_k = \frac{1}{R_b} \sum_{k=0, b_k \neq 0}^{M-1} SER_k, \tag{4.3}$$

The receiver feedbacks only the bit allocation vector $\mathbf{b}$ to the transmitter. When the bit allocation vector $\mathbf{b}$ has integer entries, in principle the whole vector can be sent back to the transmitter using finite-rate feedback. However, in a system with low feedback rate it may not be possible to feedback the complete information of $\mathbf{b}$ without quantization. In this case the bit allocation vector is chosen from a codebook $\mathcal{C}_b$ and the index of the bit allocation vector is fed back to the transmitter as we will see in the next section.

4.2 Feedback of Bit Allocation

In the proposed BA system, only bit allocation will be sent back to the transmitter. The information of the precoder is not fed back to the transmitter. We discuss the feedback of bit allocation for two cases (i) precoder is square with $M = M_t$ (implicitly $M_t \leq M_r$), and (ii) precoder is rectangular with $M \leq M_t$, separately in Section 4.2.1 and Section 4.2.2. Although the first case is a special case of the second, it is more convenient to discuss the simpler case $M = M_t$ first.

4.2.1 $M = M_t$ Case

In this case the precoding matrix $\mathbf{F}$ in the transmitter of the BA system shown in Fig. 4.1(a) is a fixed $M_t \times M_t$ matrix. When we consider bit allocation in practical

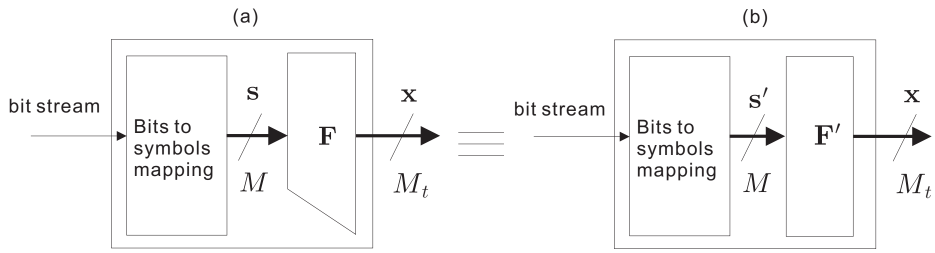

Figure 4.1: The transmitter of the BA system with (a) precoder $\mathbf{F}$, and (b) augmented precoder $\mathbf{F}'$

applications, the bits assigned to the symbols are typically integer-valued. When the number of bits transmitted per channel use R_b is given, the components of the bit allocation vector $\mathbf{b}$ satisfies

$$b_0 + b_1 + \cdots + b_{M-1} = R_b, \quad \text{where} \quad b_i \in \mathcal{Z}^+, \tag{4.4}$$

where $\mathcal{Z}^+$ denotes the set of nonnegative integers. The number of such nonnegative integer bit allocation vector is (pp. 337, [25])

$$C(R_b + M_t - 1, R_b), \tag{4.5}$$

where $C(\cdot, \cdot)$ denotes the choose function. Feedback of all these possible bit allocation vectors requires

$$B_0 = \lceil \log_2(C(R_b + M_t - 1, R_b)) \rceil, \tag{4.6}$$

where $\lceil x \rceil$ denotes the smallest integer larger than or equal to x. For example $R_b = 8$, $M = M_t = 4$, the required number of feedback bits is $B_0 = 8$. To reduce the number of feedback bits, we can quantize the bit allocation vector.

Quantization of bit allocation. Suppose we are given B feedback bits and a codebook $\mathcal{C}_b$ of 2^B bit allocation vectors. The vectors in $\mathcal{C}_b$ satisfy the transmission rate constraint in (4.4) so that the number of bits transmitted for each channel use is R_b. We can choose the best bit allocation vector $\hat{\mathbf{b}} \in \mathcal{C}_b$ that minimizes the BER. The BER expression in (4.3) is a function of bit allocation vector and we can choose

$$\hat{\mathbf{b}} = \arg \min_{\mathbf{b} \in \mathcal{C}_b} BER(\mathbf{b}, \mathbf{H}). \tag{4.7}$$

The actual number of transmitted symbols can be smaller than M as some of the symbols may be assigned with 0 bits. The selection criterion in (4.7) requires the computation of BER for all possible bit allocation vectors in the codebook, so $BER(\mathbf{b}, \mathbf{H})$ is evaluated 2^B times. When the codebook size is small (i.e. low feedback rate), for example, $B = 2, 3$, the number of searches is small as well. As we will see in the simulation examples, we can get good BER performance using a small codebook size.

4.2.2 $M \leq M_t$ Case

For $M \leq M_t$, we can start off with an augmented initial precoder $\mathbf{F}'$ of size $M_t \times M_t$. The corresponding augmented input vector $\mathbf{s}'$ and bit allocation vector $\mathbf{b}'$ are of size $M_t \times 1$. For a given M, we can choose M columns out of $\mathbf{F}'$ to form the actual $M_t \times M$ precoder $\mathbf{F}$, i.e., $(M - M_t)$ columns of $\mathbf{F}'$ are removed. As we choose M columns from $\mathbf{F}'$, there are $C(M_t, M)$ possible choices. The entries of $\mathbf{s}'$ and $\mathbf{b}'$ corresponding to the removed columns of $\mathbf{F}'$ are equal to zero. $\mathbf{s}$ and $\mathbf{b}$ are $M \times 1$ vectors which is formed by removing the zero entries of $\mathbf{s}'$ and $\mathbf{b}'$ so that $\mathbf{F}'\mathbf{s}' = \mathbf{F}\mathbf{s}$. The transmitter with the augmented precoder and augmented input vector $\mathbf{s}'$ is shown in Fig. 4.1(b). The augmented bit allocation vector $\mathbf{b}'$ satisfies

$$b'_0 + b'_1 + \cdots + b'_{M-1} = R_b, \quad \text{where} \quad b'_i \in \mathcal{Z}^+, \tag{4.8}$$

with the additional constraint that at most M of the components can be nonzero as it is assumed that the transmitter and receiver can process at most M substreams. In this case the number of symbols transmitted is at most M, carrying a total of R_b bits. To count the number of integer bit allocation vectors satisfy (4.8), let us first consider the case that $\mathbf{b}'$ has exactly k zeros, where $k \geq M_t - M$. Then R_b will be distributed among $M_t - k$ symbols, each with at least one bit. There are $C(M_t, k)C(R_b - 1, M_t - 1 - k)$ such combinations [25]. Thus the total number of possible integer bit allocation vectors satisfying (4.8) is

$$\sum_{k=M_t-M}^{M_t-1} C(M_t, k)C(R_b - 1, M_t - 1 - k). \tag{4.9}$$

For example, when $M_t = 4$, $M = 3$ and $R_b = 8$, the number is 130. To feedback all these vectors requires 8 bits. To have a smaller feedback rate, we can use a codebook $\mathcal{C}'_b$ of augmented bit allocation vectors. Each $\mathbf{b}' \in \mathcal{C}'_b$ satisfies (4.8). The BER can be obtained by a slight change of the summation in (4.3),

$$BER(\mathbf{b}', \mathbf{H}) = \frac{1}{R_b} \sum_{k=0, b'_k \neq 0}^{M_t - 1} SER_k. \tag{4.10}$$

We can choose the best bit allocation vector from $\mathcal{C}'_b$ to minimize BER,

$$\widehat{\mathbf{b}'} = \arg \min_{\mathbf{b}' \in \mathcal{C}'_b} BER(\mathbf{b}', \mathbf{H}). \tag{4.11}$$

Note that there is no need to feedback the information of the actual precoder $\mathbf{F}$ used. The information is embedded in the augmented bit allocation vector $\mathbf{b}'$. For $i = 0, 1, \cdots, M_t - 1$, the transmitter removes the i-th column from $\mathbf{F}'$ if $b'_i = 0$. The transmitter can then use the resulting $M_t \times M_0$ submatrix as the precoder, where M_0 is the number of nonzero entries in $\mathbf{b}'$.

The optimal augmented precoder. In the BA system, the augmented precoder $\mathbf{F}'$ is a fixed square unitary matrix. It does not vary with the channel; only the bit allocation does. A question that arises naturally here is this: What is the optimal channel-independent augmented precoder? It turns out that any $M_t \times M_t$ unitary matrix will yield the same performance if the entries of the channel matrix $\mathbf{H}$ are independent, identically distributed circularly symmetric Gaussian random variables with zero mean. For example, choosing $\mathbf{F}'$ as the normalized DFT matrix in (1.1) or the identity matrix will give us the same result. To see this let us view the BA system as having precoder $\mathbf{F}'$ and input $\mathbf{s}'$. (In the case $M = M_t$, $\mathbf{F}' = \mathbf{F}$ and $\mathbf{s}' = \mathbf{s}$). Let the auto correlation matrix of $\mathbf{s}'$ be $\mathbf{R}_{\mathbf{s}'}$. It can be verified that the corresponding $M_t \times M_t$ error autocorrelation matrix $R_{\mathbf{e}'}$ can be obtained from (2.5) by replacing $\mathbf{F}$ with $\mathbf{F}'$ and $\mathbf{R}_\mathbf{s}$ with $\mathbf{R}_{\mathbf{s}'}$,

$$\mathbf{R}_{\mathbf{e}'} = \begin{cases} N_0(\mathbf{F}'^\dagger \mathbf{H}^\dagger \mathbf{H} \mathbf{F}')^{-1}, & \text{zero-forcing receiver,} \\ \mathbf{R}_{\mathbf{s}'} - \mathbf{R}_{\mathbf{s}'} \mathbf{F}'^\dagger \mathbf{H}^\dagger (\mathbf{H}\mathbf{F}'\mathbf{R}_{\mathbf{s}'}\mathbf{F}'^\dagger \mathbf{H}^\dagger + N_0 \mathbf{I}_{M_r})^{-1} \mathbf{H}\mathbf{F}'\mathbf{R}_{\mathbf{s}'}, & \text{MMSE receiver.} \end{cases} \tag{4.12}$$

We see that $\mathbf{R}_{\mathbf{e}'}$ depends on $\mathbf{H}\mathbf{F}'$ as a whole. From [26], we know that when $\mathbf{F}'$ is a deterministic square unitary matrix, $\mathbf{H}\mathbf{F}'$ has the same distribution as

$\mathbf{H}$. That is, the entries of $\mathbf{HF}'$ are independent, identically distributed circularly symmetric Gaussian random variables with zero mean. Therefore, for any fixed unitary $\mathbf{F}'$, $\mathbf{HF}'$ is statistically equivalent to $\mathbf{H}$ and hence the same performance is achieved.

Fixed $M_t \times M$ precoder. In the above discussion, we have used augmented initial precoder when $M < M_t$. The actual precoder $\mathbf{F}$ is not a fixed $M_t \times M$ matrix. The reason for not using a fixed precoder $\mathbf{F}$ is as follows: If the channel matrix is such that the column space of $\mathbf{F}$ is contained in the null space of $\mathbf{H}$, then there is zero signal power at the receiver. This can be avoided by allowing $\mathbf{F}$ to be an arbitrary $M_t \times M$ submatrix of $\mathbf{F}'$. There is no such problem for the case $M = M_t$ because the column space of any $M_t \times M_t$ unitary $\mathbf{F}$ is $\mathbb{C}^{M_t}$, where $\mathbb{C}^{M_t}$ is the set of all $M_t \times 1$ vectors of complex numbers. Note that with B feedback bits, for a given channel, using augmented precoder $\mathbf{F}'$ is not guaranteed to be better than using a fixed $\mathbf{F}$. This is because for a given number of feedback bits B, the codebook $\mathcal{C}_b'$ for BA system with augmented $\mathbf{F}'$ is different from $\mathcal{C}_b$ for a fixed $M_t \times M$ precoder. Suppose $\mathbf{F}$ is a submatrix of $\mathbf{F}'$. Let us consider the special case that the codewords of $\mathcal{C}_b'$ is obtained by inserting appropriate zeros in the codewords of $\mathcal{C}_b$. Then the system with augmented precoder has the same performance as the one with a fixed precoder, but not better. Nonetheless the simulations will demonstrate that when $M < M_t$ the system of augmented precoder outperforms the one with a fixed precoder for the same number of feedback bits.

The case $\mathbf{F}' = \mathbf{I}_{M_t}$. When the initial precoder is the identity matrix, the BA system implicitly employs a form of antenna selection at the transmitter [12], in which the best M antenna are chosen to minimize the BER. But unlike conventional antenna selection, the symbols transmitted on the chosen antennas do not carry the same amount of bits. For the BA system, the feedback of antenna selection at the transmitter is embedded in the feedback of bit allocation. There is no need to tell the transmitter which antennas to use other than the index of bit allocation vector. When $\mathbf{F}' = \mathbf{I}_{M_t}$, we can also view the BA system as a extension of the multimode antenna selection [17], which also chooses a subset of transmit antennas, but the number of antenna used is allowed to vary with the channel. As the bits are uniformly loaded [17], the number of antenna used should divided R_b. There is no such condition for the BA system.

4.3 Diversity Gain of BA System

In this section, we show that the BA system can achieve diversity order $M_r M_t$ for a system with M_r receive antennas and M_t transmit antennas if the codebook is properly designed and has at least M_t codewords. Let the initial precoder $\mathbf{F}'$ be an $M_t \times M_t$ unitary matrix ($\mathbf{F}' = \mathbf{F}$ and $M = M_t$). The number of bits to be transmitted in each channel use is R_b, which is distributed among M symbols ($M \leq \min(M_t, M_r)$). The augmented bit allocation vector $\mathbf{b}'$ is of size $M_t \times 1$. It has at most M nonzero entries and $\sum_{i=0}^{M_t-1} b'_i = R_b$. Suppose the bit allocation codebook is $\mathcal{C}'_b$. The minimum achievable BER is

$$BER_{\min}(\mathbf{H}) = \min_{\mathbf{b}' \in \mathcal{C}'_b} BER(\mathbf{b}', \mathbf{H}), \tag{4.13}$$

where $BER(\mathbf{b}', \mathbf{H})$ is the BER in (4.3). Assume the bit allocation codebook $\mathcal{C}'_b$ contains the set of codewords

$$\mathcal{C}^*_b = \{R_b \mathbf{e}_0, R_b \mathbf{e}_1, \cdots, R_b \mathbf{e}_{M_t-1}\}, \tag{4.14}$$

where $\mathbf{e}_i$ are standard vectors of size $M_t \times 1$, i.e., $[\mathbf{e}_i]_i = 1$ and $[\mathbf{e}_i]_j = 0$ for $j \neq i$. The following theorem shows that the BA system can achieve full diversity order using the bit allocation vectors in $\mathcal{C}^*_b$. Therefore to achieve a diversity order of $M_r M_t$ we can use a codebook of size M_t, which requires only $\log_2 M_t$ feedback bits.

Theorem 1. *For a finite-rate feedback MIMO channel with M_r receive antennas and M_t transmit antennas, the BA system with an $M_t \times M_t$ augmented unitary precoder $\mathbf{F}'$ achieves diversity order $M_r M_t$ if the bit allocation codebook $\mathcal{C}'_b$ contains the M_t vectors in* (4.14).

Proof. As $\mathcal{C}_b^*$ is a subset of $\mathcal{C}_b'$, we have

$$BER_{\min}(\mathbf{H}) = \min_{\mathbf{b}' \in \mathcal{C}_b'} BER(\mathbf{b}', \mathbf{H}) \leq \min_{\mathbf{b}' \in \mathcal{C}_b^*} BER(\mathbf{b}', \mathbf{H}). \qquad (4.15)$$

The BER averaged over the channel $\mathbf{H}$ is denoted as $\overline{BER} = E[BER_{\min}(\mathbf{H})]$. Using (4.15), it is bounded by

$$\overline{BER} \leq E[\min_{\mathbf{b}' \in \mathcal{C}_b^*} BER(\mathbf{b}', \mathbf{H})].$$

When the bit allocation $\mathbf{b}'$ is chosen from $\mathcal{C}_b^*$, all the R_b bits are allocated to the same symbol and this becomes a beamforming system. For example, when $\mathbf{b}' = [\ R_b \ \ 0 \ \ \cdots \ \ 0\ ]^T$, the beamforming vector is the 0-th column of $\mathbf{F}'$. When we choose $\mathbf{b}' \in \mathcal{C}_b^*$ to minimize the BER, we are actually choosing the best beamforming vector from the columns of $\mathbf{F}'$ to maximize the received SNR. In other words, the equivalent codebook of beamforming vectors is $\mathcal{C}_f = \{\mathbf{f}_0', \mathbf{f}_1', \cdots, \mathbf{f}_{M_t-1}'\}$, where $\mathbf{f}_i'$ is the i-th column of $\mathbf{F}'$. The zero-forcing receiver in (2.3) performs maximal ratio combining. From [24], we know such a beamforming system has diversity order equal to $M_r M_t$ if the span of $\mathcal{C}_f$ is equal to $\mathbb{C}^{M_t}$. Therefore the BA system has diversity order $M_r M_t$ as well. The result holds as long as the codebook $\mathcal{C}_b'$ contains the simple vectors in (4.14).

■

We have shown that the BA system achieves full diversity order if the codebook has the codewords in (4.14). However, when $\mathcal{C}_b'$ has only M_t codewords, the codewords in $\mathcal{C}_b^*$ are not necessarily the best choices as we will see in the simulations.

Alternative proof of Theorem 1. Suppose the initial precoder $\mathbf{F}' = \mathbf{I}_{M_t}$ and bit allocation vector is chosen from $\mathcal{C}_b^*$. As $\mathbf{F}'$ has only one nonzero entry in each column and $\mathbf{b}' \in \mathcal{C}_b^*$ has only one nonzero entry, only one transmit antenna is used and this becomes an antenna selection system that chooses only one antenna. The right hand side of (4.15) corresponds to the BER of the system in which the transmitter chooses the best transmit antenna and receiver uses maximal ratio combining. Such a system has been shown to achieve a diversity order equal to

M_rM_t [27]. So the BA system with identity $\mathbf{F}'$ achieves diversity order M_rM_t. From the discussion of optimal augmented precoder in the previous section we know any $M_t \times M_t$ unitary $\mathbf{F}'$ lead to the same average performance. Therefore we can arrive at the result given in Theorem 1.

4.4 BA system with Unconstrained Bit Allocation

In this section, we will consider the BA system when there is no integer constraint on bit allocation. For a given precoder, we will derive the optimal bit allocation that minimizes the BER when the bit allocation is not constrained. Although the unconstrained optimal bit allocation requires infinite feedback rate, the corresponding BER performance provides insightful observations as we shall see. The optimal bit allocation is given in section 4.4.1. The connection of zero-forcing BA system with $M = M_t$ to precoder system and power-minimizing BA system are given in Section 4.4.2.

4.4.1 Optimal Bit Allocation

We first consider the case when the precoder $\mathbf{F}$ is a fixed $M_t \times M$ unitary matrix. The number of bits transmitted per channel use is R_b and $b_0+b_1+\cdots+b_{M-1} = R_b$. Assume the transmission rate is high and b_k is large enough so that $1-2^{\frac{b_k}{2}} \approx 1$ and $1-2^{b_k} \approx 1$, then the symbol error rate expression in (2.10) can be approximated by

$$SER_k \approx 4Q\left(\sqrt{\frac{3}{2^{b_k}}\beta_k}\right). \tag{4.16}$$

With the high bit rate assumption, $b_k > 0$, for all k and thus $\mathbf{R_s} = P_0/M\mathbf{I}_M$. For the convenience of derivation, we define the function

$$f(y) = Q(\tfrac{1}{\sqrt{y}}), \quad y > 0. \tag{4.17}$$

The function $f(y)$ is monotone increasing and it can be verified that $f(y)$ is convex for $y \leq 1/3$. Using $f(\cdot)$, we can express SER_k as

$$SER_k \approx 4f(\frac{2^{b_k}}{3\beta_k}). \tag{4.18}$$

Therefore the average BER in (4.3) can be written as

$$BER(\mathbf{b}) \approx \frac{4}{R_b}\sum_{k=0}^{M-1} f(\frac{2^{b_k}}{3\beta_k}). \tag{4.19}$$

where we have dropped the dependence of BER function on the channel $\mathbf{H}$ for convenience. Assume the arguments of $f(\cdot)$ are smaller than 1/3 so that the convexity of $f(\cdot)$ holds (we will see later why this assumption is reasonable). Using the convexity of $f(\cdot)$, we have

$$\frac{1}{M}\sum_{k=0}^{M-1} f(\frac{2^{b_k}}{3\beta_k}) \geq f(\frac{1}{M}\sum_{k=0}^{M-1}\frac{2^{b_k}}{3\beta_k}). \tag{4.20}$$

It follows that

$$\begin{aligned} BER(\mathbf{b}) &\approx \frac{4}{(R_b/M)}\frac{1}{M}\sum_{k=0}^{M-1} f\left(\frac{2^{b_k}}{3\beta_k}\right) && (4.21)\\ &\geq \frac{4}{(R_b/M)} f\left(\frac{1}{3M}\sum_{k=0}^{M-1}\frac{2^{b_k}}{\beta_k}\right) && (4.22)\\ &\geq \frac{4}{(R_b/M)} f\left(\frac{2^{R_b}/M}{3}\left(\prod_{k=0}^{M-1}\frac{1}{\beta_k}\right)^{1/M}\right) && (4.23)\\ &\triangleq BER_{BA}. && (4.24) \end{aligned}$$

The second inequality is obtained by using the fact that $R_b = b_0 + b_1 + \cdots + b_{M-1}$ and the AM-GM (arithmetic mean-geometric mean) inequality

$$\frac{1}{M}\sum_{k=0}^{M-1}\frac{2^{b_k}}{\beta_k} \geq \left(\prod_{k=0}^{M-1}\frac{2^{b_k}}{\beta_k}\right)^{1/M} = 2^{R_b/M}\left(\prod_{k=0}^{M-1}\frac{1}{\beta_k}\right)^{1/M}. \tag{4.25}$$

and also using the monotone increasing property of $f(\cdot)$.

Notice that the lower bound in (4.23) is independent of bit allocation. The optimal bit allocation is such that the two inequalities in (4.22) and (4.23) become equalities. Due to convexity of $f(\cdot)$, the first inequality (4.22) holds if and only if $2^{b_k}/(3\beta_k)$ are of the same value for all k. The same set of conditions is also necessary and sufficient for equality to hold in the second inequality as $f(\cdot)$ is monotone increasing. When both inequalities hold, the lower bound in (4.23) is achieved. Therefore the optimal bit allocation for minimizing the BER is such that $2^{b_k}/\beta_k = 2^{R_b/M}(\prod_{l=1}^{M-1} 1/\beta_l)^{1/M}$, i.e.,

$$b_k = \log_2(\beta_k) + \frac{R_b}{M} - \frac{1}{M}\sum_{l=0}^{M-1}\log_2\beta_l. \tag{4.26}$$

We can see that the symbols with larger SNR β_k are allocated with more bits. We have denoted the BER lower bound in (4.23) as BER_{BA}, where the subscript is a reminder which notifies that it is the BER of the BA system. Note that BER_{BA} is obtained when the bits are allocated as in (4.26) and there is no integer constraint on bit allocation in the above derivation. The bit allocation b_k computed in (4.26) are not integers in general. Nonetheless BER_{BA} gives useful insight on the performance of the BA system and connections with other system as we will see in Section 4.4.2

Remarks

1. In the above derivation, we have assumed that the argument of $f(\cdot)$ in (4.19) is larger than 1/3 so that the convexity of $f(\cdot)$ can be used in (4.20). We now examine the validity of such an assumption. When the argument $2^{b_k/(3\beta_k)} = 1/3$, the corresponding SER_k is $SER_k \approx 4Q(\sqrt{3}) \approx 0.17$, a large symbol error rate that may not be useful. In practical applications, it is more reasonable to have smaller error rate, which requires $2^{b_k}/(3\beta_k) < 1/3$.

2. When bits allocated optimally as in (4.26), $2^{b_k/\beta_k}$ are the same for all k. This means the symbol error rates are equalized for all transmitted symbols.

3. The actual number of symbols transmitted may be smaller than M if some symbols are allocates with 0 bits. However the number of bits transmitted for each channel use will be maintained at R_b.

Now let us consider the case $\mathbf{F}$ is not fixed, but an $M_t \times M_t$ augmented precoder $\mathbf{F}'$ (implicitly $M < M_t$ in this case). The imput $\mathbf{s}'$ is an augmented $M_t \times 1$ vector and bit allocation vector $\mathbf{b}'$ is $M_t \times 1$ as in Section 4.2.2. For a given M, we can choose M columns out of $\mathbf{F}'$ to form the actual $M_t \times M$ precoder. As we choose M columns of $\mathbf{F}'$, there are $C(M_t, M)$ possible choices. For each of these choices, we can compute the optimal bit allocation and the corresponding BER using (4.23), and choose the best precoder. In this case the BA system with augmented precoder $\mathbf{F}'$ is always better than the BA system with a fixed precoder $\mathbf{F}$ if $\mathbf{F}$ us a submatrix of $\mathbf{F}'$.

Bit allocation for optimal number of substreams

In the above discussion of optimal bit allocation, we assumed all symbols carry nonzero bits and transmission power is loaded on all M symbols. In the end some of the symbols may be assigned zero bits while take up $1/M$ of the total power. To make efficient use of power, we can allocate power to only the symbols that carry nonzero bits. To do this, we can compute the optimal bit allocation for all possible number of symbols with nonzero bits and choose the best one. To be more specific, let us illustrate this in another viewpoint. We start out with an $M_t \times M_t$ initial precoder $\mathbf{F}'$ as before. The precoder $\mathbf{F}$ can be any $M_t \times M_0$ submatrix of $\mathbf{F}'$, where $M_0 = 1, 2, \cdots, M$. There are $\sum_{M_0=1}^{M} C(M_t, M_0)$ possible precoders. We collect all these possible precoder in a set $\mathcal{S}_F$. For each $\mathbf{F} \in \mathcal{S}_F$, we can use (4.29) to compute the BER under optimal bit allocation. The error rate BER_{BA} given in (4.23) depends on the precoder used. For convenience let us use the notation $BER_{BA}(\mathbf{F})$ to indicate the dependence on $\mathbf{F}$. The best $\mathbf{F}$ is

$$\mathbf{F}_{opt} = \arg \min_{\mathbf{F} \in \mathcal{S}_{\mathbf{F}}} BER_{BA}(\mathbf{F}). \tag{4.27}$$

The resulting minimum BER is given by

$$BER_{BA,opt} = \min_{\mathbf{F} \in \mathcal{S}_F} BER_{BA}(\mathbf{F}). \tag{4.28}$$

When the optimal precoder is obtained this way, all the symbols will carry nonzero bits. The reason is as follows: Let the optimal precoder $\mathbf{F}_{opt}$ be $M_t \times l$, and the optimal $l \times 1$ bit allocation be $\mathbf{b}_{opt}$. Suppose one of the symbols is assigned with zero bits. The actual number of symbols transmitted is $l-1$. Let us remove from $\mathbf{F}_{opt}$ the column corresponding to the symbol with zero bit and call the remaining $M_t \times (l-1)$ submatrix $\mathbf{F}_0$. Also remove from $\mathbf{b}_{opt}$ the element equal to zero and call the reduced vector $\mathbf{b}_0$. Then using precoder $\mathbf{F}_0$ with bit allocation $\mathbf{b}_0$ gives a smaller BER for the same transmission power as the power is now distributed among $(l-1)$ symbols instead of l symbols. So $\mathbf{F}_{opt}$ can not be optimal if one symbol is assigned 0 bits. We can therefore conclude that all symbols carry nonzero bits in the optimal system that uses $\mathbf{F}_{opt}$ as precoder.

4.4.2 BER performance of Zero-forcing BA system When $M = M_t$

In this subsection, we will examine the BER lower bound BER_{BA} derived in (4.24) when the receiver is zero forcing and $M = M_t$. Connection between the BA system with two other systems, the precoder system [7] and power minimizing BA system, will be studied.

For $M = M_t$, the precoder $\mathbf{F}$ is an $M_t \times M_t$ unitary matrix. When the receiver is zero-forcing, the k-th SNR β_k is equal to $P_0/(M\sigma^2_{e_k,BA})$, where we have added a subscript to the error variances to indicate that these are the error variances in the BA system. The BER lower bound BER_{BA} in (4.24) can be written as

$$BER_{BA} = \frac{4}{R_b/M} Q\left(\sqrt{\frac{3P_0/M}{2^{R_b/M}} \frac{1}{(\prod_{l=0}^{M-1} \sigma^2_{e_l,BA})^{1/M}}}\right). \tag{4.29}$$

We can see from the above expression that BER_{BA} depends on the geometric mean of $\{\sigma^2_{e_l,BA}\}_{l=0}^{M-1}$, which is in turn determined by the given precoder. Although the geometric mean of $\{\sigma^2_{e_l,BA}\}_{l=0}^{M-1}$ depend on the choice of precoder, the arithmetic mean does not. This is due to unitary property of the precoder. To see this, we can use the expression $\mathbf{R}_e = N_0(\mathbf{F}^\dagger\mathbf{H}^\dagger\mathbf{H}\mathbf{F})^{-1}$ given in (4.12) for zero-forcing receiver. It follows that the average error $\mathcal{E}_{rr}$ is

$$\mathcal{E}_{rr} = \frac{1}{M}\sum_{l=0}^{M-1} \sigma^2_{e_l,BA} = \frac{1}{M} trace(\mathbf{R}_e) = \frac{1}{M} N_0 trace(\mathbf{F}^\dagger\mathbf{H}^\dagger\mathbf{H}\mathbf{F})^{-1}. \tag{4.30}$$

Using $\mathbf{F}\mathbf{F}^\dagger = \mathbf{I}_M$ and $trace(\mathbf{AB}) = trace(\mathbf{BA})$, we have

$$\mathcal{E}_{rr} = \frac{1}{M} N_0 trace(\mathbf{H}^\dagger\mathbf{H})^{-1}. \tag{4.31}$$

As $N_0 trace(\mathbf{H}^\dagger\mathbf{H})^{-1}$ does not depend on the precoder $\mathbf{F}$, we come to the conclusion that the average error $\mathcal{E}_{rr}$ does not depend on the precoder. It is the same quantity for any square unitary precoder regardless of bit allocation. This property allows us to show that the BER of the BA system is always smaller than the BER-minimizing precoder system, which is briefly reviewed below.

BER-minimizing precoder system [7,22]. In the precoder system [7], the precoder is optimized to minimize BER. Referring to Chapter 3.1, the power and bits are uniformly loaded on all M symbols ($M \leq \min(M_t, M_r)$). Suppose R_b bits are transmitted using total power P_0 for each channel use; each s_k, for $k = 0, 1, \cdots, M-1$, is a QAM symbol with variance P_0/M that carries R_b/M bits. Let the singular value decomposition of $\mathbf{H}$ be $\mathbf{U\Lambda V}^\dagger$, where $\mathbf{U}$ and $\mathbf{V}$ are respectively $M_r \times M_r$ and $M_t \times M_t$ unitary matrices. The $M_r \times M_t$ matrix $\mathbf{\Lambda}$ is diagonal, whose diagonal elements are singular values of $\mathbf{H}$ in a nonincreasing order. From Section 3.1.2, the optimal $M_t \times M$ unitary precoder that minimizes the BER for large SNR is given by [7, 22]

$$\mathbf{F}_{eq} = \mathbf{V}_M\mathbf{Q}, \tag{4.32}$$

where V_M is the $M_t \times M$ matrix obtained by keeping the first M columns of $\mathbf{V}$ and $\mathbf{Q}$ is an $M \times M$ unitary matrix that has the equal magnitude property.

For the optimal precoder given in (4.32), the subchannel error $e_k = \widehat{s_k} - s_k$ has the property that variance $\sigma^2_{e_k}$ are equalized [7, 22],

$$\sigma^2_{e_0} = \sigma^2_{e_1} = \cdots = \sigma^2_{e_{M-1}}. \tag{4.33}$$

Now consider the case $M = M_t$ and the receiver is zero forcing. We know from (4.31) that all square unitary precoders lead to the same average error variance. That is, the BER-minimizing precoder yields the same average error variance as the BA system. Therefore when the optimal precoder in (4.32) is used, all error variances are equal to $\mathcal{E}_{rr}$ given in (4.31) and hence identical BER for all symbols transmitted. Using the approximation in (4.16), the minimized BER of the precoder system can be expressed as

$$BER \approx \frac{4}{R_b/M} Q\left(\sqrt{\frac{3P_0/M}{2^{R_b/M}}\frac{1}{\mathcal{E}_{rr}}}\right) \triangleq BER_{precoder}. \tag{4.34}$$

Using the fact that $\mathcal{E}_{rr}$ is also equal to $\frac{1}{M}\sum_{l=0}^{M-1}\sigma_{e_l,BA}^2$ and applying AM-GM inequality to $\{\sigma_{e_l,BA}^2\}_{l=0}^{M-1}$, we get

$$\mathcal{E}_{rr} = \frac{1}{M}\sum_{l=0}^{M-1}\sigma_{e_l,BA}^2 \geq \left(\prod_{l=0}^{M-1}\sigma_{e_l,BA}^2\right)^{1/M}. \tag{4.35}$$

As Q-function is monotone decreasing we arrive at

$$BER_{precoder} = \frac{4}{R_b/M} Q\left(\sqrt{\frac{3P_0/M}{2^{R_b/M}}\frac{1}{\frac{1}{M}\sum_{l=0}^{M-1}\sigma_{e_l,BA}^2}}\right) \tag{4.36}$$

$$\geq \frac{4}{R_b/M} Q\left(\sqrt{\frac{3P_0/M}{2^{R_b/M}}\frac{1}{(\prod_{l=0}^{M-1}\sigma_{e_l,BA}^2)^{1/M}}}\right) = BER_{BA} \tag{4.37}$$

We recognized that the right hand side of the above inequality is the BER of the BA system given in (4.29). Therefore when $M = M_t$ the BA system with optimal bit allocation and an arbitrary fixed precoder has a smaller BER than the precoder system with an optimal precoder.

Unlike the $M = M_t$ case, the BER of the BA system for $M < M_t$ is not guaranteed to be smaller than the precoder system. We can see this using the case $M = 1$ as an example, i.e., beamforming transmission. When $M = 1$, the precoder system corresponds to the beamforming system with maximal ratio transmission [28] at the transmitter and maximal ratio combining at the receiver, which achieves the smallest error rate among all beamforming systems. As $M = 1$, all R_b bits are loaded on one symbol. For the BA system, all the bits are allocated to only one symbol as well but the choices of the beamforming vectors are limited to the M_t columns of $\mathbf{F}_{\prime}$. If the number of symbols transmitted in each channel use can not exceed one, the precoder system is better than BA system.

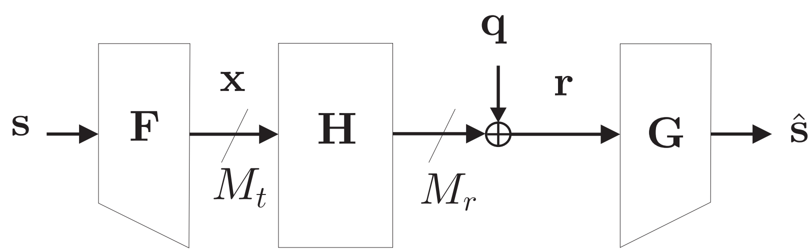

Figure 4.2: MIMO wireless system with M_t transmit antennas and M_r receive antennas

Connection with power-minimizing BA system

In Section 4.4 bit allocation is optimized to minimize BER. Suppose, instead of BER criterion, we optimize the bit allocation to minimize the transmission power for a given symbol error rate constraint ϵ and transmission rate R_b. When the receiver is zero-forcing, we now show that the optimal bit allocation derived in Section 4.4 for minimum BER is also optimal for minimizing transmission power. Consider the MIMO system in Fig. 4.2. Let the total transmission power be P_T. From (2.1), we have $P_T = trace(E[\mathbf{x}\mathbf{x}^\dagger]) = \sum_{k=0}^{M-1} \sigma_{s_k}^2$. Suppose the k-th symbols s_k is loaded with b_k and $b_0 + b_1 + \cdots + b_{M-1} = R_b$. In the power minimization problem, we allow P_T and symbol variance $\sigma_{s_k}^2$ to vary so that the given symbol error rate constraint ϵ can be satisfied. For a zero forcing receiver, the error variance $\sigma_{e_k}^2$ can be computed using $\mathbf{R}_e = N_0(\mathbf{F}^\dagger\mathbf{H}^\dagger\mathbf{H}\mathbf{F})^{-1}$ in (2.5). If $\sigma_{s_k}^2$ and $\sigma_{e_k}^2$ are given, the number of bits that can be loaded is well approximated by [29]

$$b_k = \log_2\left(1 + \frac{\sigma_{s_k}^2}{\sigma_{e_k}^2\Gamma}\right), \tag{4.38}$$

where Γ, called SNR gap, depends on the given symbol error rate ϵ. In our problem $\sigma_{s_k}^2$ is not given. Let us rearrange the above equation to get $\sigma_{s_k}^2 = \Gamma(2^{b_k} - 1)\sigma_{e_k}^2$, which gives the required symbol variance when the k-th symbol is loaded with b_k bits. Using high bit rate assumption $2^{b_k} - 1 \approx 2^{b_k}$, we have

$$\sigma_{s_k^2} \approx \Gamma 2^{b_k}\sigma_{e_k}^2. \tag{4.39}$$

Using the approximation in (4.39), we have

$$P_T \approx \Gamma \sum_{k=0}^{M-1} 2^{b_k} \sigma_{e_k}^2. \tag{4.40}$$

Applying the AM-GM inequality to the above summation, we get

$$P_T \approx \Gamma \sum_{k=0}^{M-1} 2^{b_k} \sigma_{e_k}^2 \geq M\Gamma \left(\prod_{k=0}^{M-1} 2^{b_k} \sigma_{e_k}^2 \right)^{1/M} = M\Gamma 2^{R_b/M} \left(\prod_{k=0}^{M-1} \sigma_{e_k}^2 \right)^{1/M} \tag{4.41}$$

The right hand side is a lower bound that is independent of bit allocation. The minimum transmission power can be achieved by allocating the bits b_k such that AM-GM inequality becomes an equality, i.e., $2^{b_k}\sigma_{e_k}^2$ are equalized. This in turns means $\sigma_{s_k}^2$ are identical and thus $2^{b_k}\sigma_{e_k}^2/\sigma_{s_k}^2 = 2^{b_k}/\beta_k$ are the same for all k. It follows that b_k are as given in (4.26). So the optimal bit allocation for minimizing BER of zero forcing BA system is also optimal for minimizing transmission power.

4.5 Efficient Method of Selecting Bit Allocation Vector

In this section we consider efficient search of bit allocation vector from the codebook C_b'. Suppose the feedback bits is B, so the codebook size is 2^B. To obtain $\widehat{\mathbf{b}'} = \arg\min_{\mathbf{b}' \in \mathcal{C}_b'} BER(\mathbf{b}', \mathbf{H})$ given in (4.11), exhaustive search can be applied by computing BER formula (4.3) for each bit allocation vector in $\mathcal{C}_b'$, thus $BER(\mathbf{b}', \mathbf{H})$ is evaluated 2^B times. When B is large, such an exhaustive search requires lots of computations. Using the unconstrained optimal bit allocation in (4.26), an efficient method is developed to reduce the complexity of selecting bit allocation in $\mathcal{C}_b'$. The development of our method can be easier to understand if we explain the basic idea first.

Quantization of $\mathbf{b}'_{opt}$**.** The basic idea of the proposed method is described as follows. Rather than evaluating BER formula for 2^B bit allocation vectors, we can compute the optimal unconstrained bit allocation $\mathbf{b}'_{opt}$ first. Let $\mathbf{b}'_{opt}$ be the unconstraint bit allocation that achieves $BER_{BA,opt}$ in (4.28). From Section 4.4.1,

$BER_{BA,opt}$ is obtained by computing BER_{BA} for each possible submatrix of the initial precoder $\mathbf{F}'$. The collection of all these possible submatrices is $\mathcal{S}_F$. The size of $\mathcal{S}_F$ is $\sum_{M_0=1}^{M} C(M_t, M_0)$. $\mathbf{F}_{opt}$ is the best BER-minimizing precoder matrix in $\mathcal{S}_F$. And $\mathbf{b}'_{opt}$ is the corresponding optimal unconstrained bit allocation when $\mathbf{F}_{opt}$ is used. After $\mathbf{b}'_{opt}$ is computed, we then quantize $\mathbf{b}'_{opt}$ to the integer bit allocation vectors in $\mathcal{C}'_b$ by minimizing a distance function $D(\mathbf{b}', \mathbf{b}'_{opt})$,

$$\widehat{\mathbf{b}} = \arg \min_{\mathbf{b}' \in C'_b} D(\mathbf{b}', \mathbf{b}'_{opt}), \tag{4.42}$$

where $D(\mathbf{b}', \mathbf{b}'_{opt})$ is a measure of the distance between $\mathbf{b}'$ and $\mathbf{b}'_{opt}$. From the remarks in Section 4.4.1, the symbol error rates are equalized when the bits are optimally allocated,

$$SER_k(b'_{k,opt}) = SER_{BA}, \quad \text{for all} \quad b'_{k,opt} > 0 \ ,$$

where SER_{BA} denotes the average SER. From (2.10), when the k-th symbol carries b'_k bits, the SER is

$$SER_k(b'_k) = 4(1 - \frac{1}{2^{b'_k/2}})Q\left(\sqrt{\frac{3}{(2^{b'_k} - 1)}\beta_k}\right).$$

If $b'_k > b'_{k,opt}$, then $SER_k(b'_k) > SER_{BA}$, and the error rate performance will be dominated by the worst subchannel. The largest difference between b'_k and $b'_{k,opt}$ is corresponded to the worst subchannel symbol error rate, SER_{worst}. Moreover, from (4.10), the BER formula can be upper bound by

$$BER(\mathbf{b}', \mathbf{H}) = \frac{1}{R_b} \sum_{k=0, b'_k \neq 0}^{M_t - 1} SER_k \leq \frac{M_0}{R_b} SER_{worst}, \tag{4.43}$$

where M_0 is the number of nonzero entries in $\mathbf{b}'$. Thus, we employ the following distance measure,

$$D(\mathbf{b}', \mathbf{b}'_{opt}) = \|(\mathbf{b}' - \mathbf{b}'_{opt})^+\|_\infty, \tag{4.44}$$

where $\|\cdot\|_\infty$ denotes infinity norm and the vector $(\mathbf{a})^+$ is formed by extracting the positive entries from the vector $\mathbf{a}$. For example, if $\mathbf{a} = \begin{bmatrix} 1 & 0 & 2 & -1 \end{bmatrix}^T$, then $(\mathbf{a})^+ = \begin{bmatrix} 1 & 2 \end{bmatrix}^T$. Minimizing the distance measure $D(\mathbf{b}', \mathbf{b}'_{opt})$ is equal to minimizing the worst subchannel's SER, because $(\mathbf{b}' - \mathbf{b}'_{opt})^+$ corresponds to the SER_k which are larger than SER_{BA} and $\|(\mathbf{b}' - \mathbf{b}'_{opt})^+\|_\infty$ picks out the largest $\mathbf{b}'_k - \mathbf{b}'_{opt}$ which causes the worst subchannel SER, SER_{worst}. The smaller the $D(\mathbf{b}', \mathbf{b}'_{opt})$, the better the SER_{worst}. So $\mathbf{b}_q$ is the bit allocation vector in $\mathcal{C}'_b$ that minimizes the BER upper bound in (4.43). $D(\mathbf{b}', \mathbf{b}'_{opt})$ provides a low-complexity measure for bit allocation vector quantization.

Quantization of $\mathbf{b}'_{opt,\mathcal{C}}$**.** If we want to obtain $\mathbf{b}'_{opt}$, the unconstrained bit allocation equation (4.26) needs to be computed $\sum_{M_0=1}^{M} C(M_t, M_0)$ times, since there are $\sum_{M_0=1}^{M} C(M_t, M_0)$ possible precoder matrices (submatrices of $\mathbf{F}'$) in $\mathcal{S}_F$. However not all matrices in $\mathcal{S}_F$ are possible precoders. This is because $\mathcal{C}'_b$ may not contain all possible combination of bit allocation vectors. For example suppose $M_t = 4$, $M = 4$, $B = 2$ and $\mathcal{C}'_b$ is equal to $\mathcal{C}^*_b$ in (4.14). The number of possible precoders is 4 while the number of matrices in $\mathcal{S}_F$ is 15. So not all matrices in $\mathcal{S}_F$ have corresponding bit allocation vectors in $\mathcal{C}'_b$. Notice that if the optimal BA vector $\mathbf{b}'_{opt}$ is quantized to $\widehat{\mathbf{b}} \in \mathcal{C}'_b$ and $\widehat{b}_i = 0$, it is likely that $b'_{i,opt}$ is also equal to zero. That is, the i-th symbol carries zero bits and thus the i-th column of $\mathbf{F}'$ is removed in forming the precoder $\mathbf{F}$. Therefore when we compute $\mathbf{b}'_{opt}$, skipping the matrices in $\mathcal{S}_F$ that have no corresponding BA vectors in $\mathcal{C}_b$ will reduce the number of searches. Let $\mathcal{S}_{F,\mathcal{C}}$ denotes the collection of precoders that have bit allocation vectors in $\mathcal{C}'_b$. $\mathcal{S}_{F,\mathcal{C}}$ is a subset of $\mathcal{S}_F$ and the size of $\mathcal{S}_{F,\mathcal{C}}$ is smaller than $\sum_{M_0=1}^{M} C(M_t, M_0)$ in general. We use $\mathbf{b}'_{opt,\mathcal{C}}$ to represent the unconstrained bit allocation who achieves the best BER_{BA} of the precoder matrices in $\mathcal{S}_{F,\mathcal{C}}$, $\min_{\mathbf{F} \in \mathcal{S}_{F,\mathcal{C}}} BER_{BA}(\mathbf{F})$. The subscript $\mathcal{C}$ is used to remind that $\mathbf{b}'_{opt,\mathcal{C}}$ is computed from the precoder matrix in $\mathcal{S}_{F,\mathcal{C}}$. Once $\mathbf{b}'_{opt,\mathcal{C}}$ is obtained, we quantize $\mathbf{b}'_{opt,\mathcal{C}}$ to the closest integer bit allocation vector in $\mathcal{C}'_b$,

$$\widehat{\mathbf{b}}_{\mathcal{C}} = \arg \min_{\mathbf{b}' \in C'_b} D(\mathbf{b}', \mathbf{b}'_{opt,\mathcal{C}}). \tag{4.45}$$

By using $\mathbf{b}'_{opt,\mathcal{C}}$, the number of times (4.26) is computed can be reduced, thus the complexity of selecting bit allocation vector is decreased. Simulation result shows the proposed efficient method provides performance close to the exhaustive search. The about algorithm is summarized below.

Efficient method for selecting optimal bit allocation vector.

1. Compute unconstrained bit allocation by (4.26) for the cases in $\mathcal{S}_{F,\mathcal{C}}$.

2. Choose $\mathbf{b}'_{opt,\mathcal{C}}$, the unconstrained bit allocation vector whose BER_{BA} is the smallest among all the unconstraint bit allocations obtained from first step.

3. Use $D(\mathbf{b}', \mathbf{b}'_{opt,\mathcal{C}})$ (4.44) as distance measure, and quantize $\mathbf{b}'_{opt,\mathcal{C}}$ to the bit allocation vector in $\mathcal{C}'_b$ as in (4.45).

Chapter 5

Precoder System with Limited Feedback

In this chapter, we discuss the selection criteria and codebook design of MIMO precoder systems with limited feedback. First, the BER optimal precoder is presented. Then we will discuss the precoder system in two separate cases: (i) simple selection criteria and (ii) proposed two-step system.

5.1 BER minimizing optimal precoder

Referring to the BER optimal precoder, $\mathbf{F}_{eq} = \mathbf{V}_M\mathbf{Q}$, in Section 4.4.2, two insightful observations of the BER optimal precoder with infinite-rate are described as follows.

- **Total MSE minimized.** $\mathbf{V}_M\mathbf{T}$ is the optimal unitary matrix of minimizing total MSE, where $\mathbf{T}$ is an arbitrary $M \times M$ unitary matrix and the total mean squared error (MSE) is $\sum_{l=0}^{M-1} \sigma_{e_l}^2 = trace(\mathbf{R_e})$.

- **Error covariance equalized.** $\mathbf{V}_M\mathbf{Q}$ equalizes the error covariance $\sigma_{e_k}^2$.

$$\sigma_{e_0}^2 = \sigma_{e_1}^2 = \cdots = \sigma_{e_{M-1}}^2 = \frac{1}{M}\sum_{l=0}^{M-1}\sigma_{e_l}^2, \tag{5.1}$$

These two properties can be related to the BER formula (2.8). When β_k is sufficiently large, SER_k can be expressed as a convex function for $\sigma_{e,k}^2$. Thus,

$$BER(\mathbf{F},\mathbf{H}) = \frac{M}{R_b}\frac{1}{M}\sum_{l=0}^{M-1} SER_k(\sigma_{e_l}^2) \geq \frac{M}{R_b} SER(\frac{\sum_{l=0}^{M-1}\sigma_{e_l}^2}{M}) = BER_{lb}, \quad (5.2)$$

where BER_{lb} denotes the lower bound of BER. The equality of the lower bound in (5.2) holds if $\sigma_{e_k}^2$ are equalized. The lower bound BER_{lb} is a increasing function of $\frac{1}{M}\sum_{l=0}^{M-1}\sigma_{e,l}^2$. So BER_{lb} will become smaller if we minimize the total MSE.

The diversity order of the BER optimal precoder is given here.

Diversity of the BER optimal precoder. It was shown in [22], without unitary constraint, the BER optimal precoder matrix without unitary constraint can be written as $\mathbf{F}_{opt,wf} = \mathbf{V}_M\mathbf{P}\mathbf{Q}$, where $\mathbf{P}$ is the water filling power loading matrix, $\mathbf{V}_M$ is obtained by keeping the first M columns of $\mathbf{V}$ and $\mathbf{Q}$ is the equal magnitude matrix as mentioned in Section 4.4.2. From [30], the diversity order of using $\mathbf{F} = \mathbf{F}_{opt,wf}$ are $(M_r - M + 1)(M_t - M + 1)$. And from [31], the diversity order of using $\mathbf{F} = \mathbf{V}_M$ is also $(M_r - M + 1)(M_t - M + 1)$. Since the BER performance of $\mathbf{F}_{eq}$ is

$$BER(\mathbf{F}_{opt,wf},\mathbf{H}) < BER(\mathbf{F}_{eq},\mathbf{H}) < BER(\mathbf{V}_M,\mathbf{H}).$$

Thus, we can conclude that the diversity order of BER optimal precoder $\mathbf{F}_{eq}$ is $(M_r - M + 1)(M_t - M + 1)$.

5.2 Simple Selection Criterion

When the feedback rate is finite in the precoder system, the receiver selects a precoder from the precoder codebook $\mathcal{C}_F$ and sends the index back to the transmitter. The codebook $\mathcal{C}_F$ is assumed to be known to the transmitter as well. With the BER-minimizing criterion, the precoder is chosen by (3.1),

$$\widehat{\mathbf{F}} = \arg\min_{\mathbf{F}\in\mathcal{C}_F} BER(\mathbf{F},\mathbf{H}),$$

If the codebook size is 2^B, we need to compute 2^B times the BER formula in (2.8).

$$BER(\mathbf{F}, \mathbf{H}) = \frac{1}{M} \sum_{k=0}^{M-1} SER_k.$$

To reduce the complexity, we propose two selection criteria. One criterion is for $M = M_t$ case (square precoder). the other criterion is for $M < M_t$ case (rectangular precoder).

5.2.1 $M = M_t$ case

Since the precoder matrix $\mathbf{F}$ is square and unitary, the total MSE, $\sum_{l=0}^{M-1} \sigma_{e_l}^2$, is independent of $\mathbf{F}$ as we showed in Section 4.4.2.

As the total MSE is independent of $\mathbf{F}$, from (5.2), the smallest BER is achieved when the subchannel error covariance $\sigma_{e,k}^2$ are equalized. We propose to find the precoder matrix in $\mathcal{C}_F$ that equalizes the error covariances $\sigma_{e_k}^2$ the most. To indicate the level of equalization, we employ AM-GM inequality,

$$\frac{1}{M} \sum_{l=0}^{M-1} \sigma_{e_k}^2 \geq \left(\prod_{l=0}^{M-1} \sigma_{e_k}^2 \right)^{1/M} \tag{5.3}$$

since the inequality becomes equality when all parameters are equalized. The measure A_{AM-GM} is defined as

$$A_{AM-GM}(\mathbf{F}) = \prod_{l=0}^{M-1} \sigma_{e_l}^2. \tag{5.4}$$

The larger the measure A_{AM-GM}, the better the error covariance's level of equalization. Because the arithmetic mean is the same for all precoder, we only need to choose the precoder matrix corresponding to the largest product of $\sigma_{e_k}^2$. the AM-GM selection criterion is

$$\widehat{\mathbf{F}} = \arg \max_{\mathbf{F} \in \mathcal{C}_F} A_{AM-GM}(\mathbf{F}). \tag{5.5}$$

The exhaustive search method reviewed in Section 3.1 requires computation of the BER formula for each precoder matrix in $\mathcal{C}_F$. The complexity of our method is lower, since the computation effort of BER formula in (2.11) is higher than computing the product of $\sigma_{e_k}^2$. Simulation shows that the performance of our method is very close to the performance of exhaustive search in [7].

5.2.2 $M < M_t$ case

Unlike the square precoder case, the total MSE of the rectangular precoder is not the same for all precoders. Therefore, total MSE and error covariances equalization should both be considered in selecting the precoder. We propose to use the following simple cost function,

$$A_{SS}(\mathbf{F}) = \sum_{l=0}^{M-1} (\sigma_{e_l}^2)^2, \tag{5.6}$$

where the subscript SS is a reminder that it is the summation of the squared error covariance. Note that the squared function $(\cdot)^2$ is convex. Thus,

$$A_{SS}(\mathbf{F}) = \sum_{l=0}^{M-1} (\sigma_{e_l}^2)^2 \geq \left(\frac{\sum_{l=0}^{M-1} \sigma_{e_l}^2}{M} \right)^2 . \tag{5.7}$$

The equality of (5.7) holds if $\sigma_{e_k}^2$ is equalized. This corresponds to the error covariance equalizing property of the BER optimal precoder. In addition, $(\sum_{l=0}^{M-1} \sigma_{e_l}^2 / M)^2$ is a monotone increasing function of total MSE, $\sum_{l=0}^{M-1} \sigma_{e_l}^2$. And $A_{SS}(\mathbf{F})$ is a upper bound of $(\sum_{l=0}^{M-1} \sigma_{e_l}^2 / M)^2$. So minimizing $A_{SS}(\mathbf{F})$ is equal to minimizing the upper bound of the total MSE, which corresponds to the total MSE minimizing property in some degree. A selection criterion of squared error covariance's summation is

$$\widehat{\mathbf{F}} = \arg \min_{\mathbf{F} \in \mathcal{C}_F} A_{SS}(\mathbf{F}). \tag{5.8}$$

The computation effort of A_{SS} is the squared function for each $\sigma_{e_k}^2$ and the summation. The complexity is lower than the BER criterion in [7]. The simulation result will show that the performance of summation of squared error covariance criterion (SS criterion) is very close to that of the BER criterion.

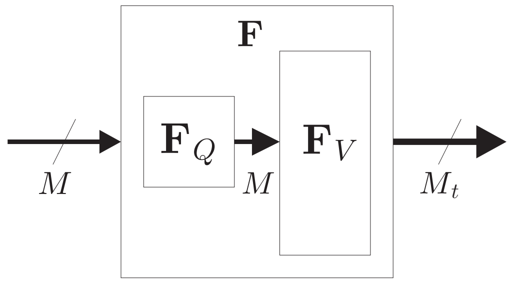

Figure 5.1: Two step transmitter

5.3 Two-steps Design of Precoder System

In this subsection, we proposed a two-step system which is modified from the conventional one-step precoder system [5,7]. The proposed two-steps system can lower the complexity by reducing the number of searches.

Referring to the system model in Section 5.1, equal bit allocation and uniform power distribution are assumed. The transmission rate per channel use is R_b and the feedback bits is B. M is assumed to be smaller than M_t. As depicted in Fig. 5.1, the spatial multiplexing is performed by two stages of precoding in two-steps system. Each stage has a precoding matrix. $\mathbf{F}_V$ is a $M_t \times M$ unitary matrix chosen from codebook $\mathcal{C}_{F_Q}$ and $\mathbf{F}_Q$ is a $M \times M$ square unitary matrix selected from codebook $\mathcal{C}_{F_Q}$. $\mathcal{C}_{F_V}$ and $\mathcal{C}_{F_Q}$ are the precoder codebooks designed for $\mathbf{F}_V$ and $\mathbf{F}_Q$. The codebook sizes of $\mathcal{C}_{F_V}$ and $\mathcal{C}_{F_Q}$ are respectively 2^{B_V} and 2^{B_Q}, where $B = B_V + B_Q$. Given a channel, the receiver selects $\mathbf{F}_V$ from $\mathcal{C}_{F_V}$ first. After $\mathbf{F}_V$ is chosen, $\mathbf{F}_Q$ is selected from $\mathcal{C}_{F_Q}$ based on the choice of $\mathbf{F}_V$. The corresponding indexes are sent back to the transmitter through the reverse channel together. M modulation symbols are precoded by the equivalent precoder matrix $\mathbf{F} = \mathbf{F}_V\mathbf{F}_Q$. Receiving is performed by zero-forcing receiver or MMSE receiver of $\mathbf{F}$ as given in (2.3).

Two-steps system has lower complexity compared to that of one-step system. For one-step precoder system, the size of codebook $\mathcal{C}_F$ is 2^B. Thus, the number of searches is 2^B. For two-steps system, the total amount of matrices in $\mathcal{C}_{F_V}$ and $\mathcal{C}_{F_Q}$ is $2^{B_V} + 2^{B_Q}$. So the number of searches is $2^{B_V} + 2^{B_Q}$, which is always fewer than 2^B if $B_V > 0$ and $B_Q > 0$. When B bits are equally allocated for $\mathbf{F}_V$ and $\mathbf{F}_Q$, $B_V = B_Q = \frac{B}{2}$, the number of searches is the fewest. For example, when feedback bitts $B = 8$, conventional one-step system [5,7] requires $2^B = 256$ searches. For two-steps system, only $2^{B/2} + 2^{B/2} = 2 \times 2^4 = 32$ searches is needed.

5.3.1 Selection Criteria

Since there are two codebooks in two-steps system, two selection criteria are designed for choosing $\mathbf{F}_V$ and $\mathbf{F}_Q$. Motivated by the properties of BER optimal precoder $\mathbf{F}_{eq} = \mathbf{V}_M\mathbf{Q}$, the selection criteria expect to select the total MSE minimizing precoder in $\mathcal{C}_{F_V}$ and the error covariance equalizing precoder in $\mathcal{C}_{F_Q}$. Let the BER optimal precoder be written as $\mathbf{F} = \mathbf{V}_M\mathbf{T}\mathbf{T}^\dagger\mathbf{Q}$, where $\mathbf{T}$ is an arbitrary $M \times M$ unitary matrix. The selection criterion for each stage is as follows.

- For $\mathbf{F}_V$, we choose the precoder matrix that achieves the smallest total MSE, or the trace of $\mathbf{R_e}$

$$\widehat{\mathbf{F}}_V = \arg\min_{\mathbf{F}_V \in \mathcal{C}_{F_V}} trace(\mathbf{R_e}). \tag{5.9}$$

  Thus, $\widehat{\mathbf{F}}_V$ is selected to be as close to $\mathbf{V}_M\mathbf{T}$ as possible.

- For $\mathbf{F}_Q$, after $\mathbf{F}_V$ is chosen, we choose the precoder matrix that equalized the error covariance the most. Note that the total MSE is the same for all possible $\mathbf{F}_Q$. Let $\mathbf{R}_{e,v}$ denotes the error autocorrelation matrix of $\mathbf{F} = \widehat{\mathbf{F}}_V$. Then,

$$\mathbf{R_e} = \mathbf{F}_Q^\dagger \mathbf{R}_{e,v} \mathbf{F}_Q.$$

  Since $\mathbf{F}_Q$ is square and unitary, $trace(\mathbf{R_e})$ is independent of $\mathbf{F}_Q$. Thus, we can employ the AM-GM criterion (5.5),

$$\widehat{\mathbf{F}}_Q = \arg\max_{\mathbf{F}_Q \in \mathcal{C}_{F_Q}} \prod_{l=0}^{M-1} \sigma_{e_l}^2.$$

  Since $\widehat{\mathbf{F}}_V$ is close to $\mathbf{V}_M\mathbf{T}$, $\widehat{\mathbf{F}}_Q$ is chosen to be as close to $\mathbf{T}^\dagger\mathbf{Q}$ as possible.

5.3.2 Codebook Design

The codebook designs of $\mathcal{C}_{F_V}$ and $\mathcal{C}_{F_Q}$ are presented in this subsection. By observing the statistic characteristic of $\mathbf{V}_M$, the codebook constructions of $\mathcal{C}_{F_V}$ and $\mathcal{C}_{F_Q}$ are given as follows.

Design of $\mathcal{C}_{F_V}$. From [26], the random matrx $\mathbf{V}_M$ is uniformly distributed in $\mathcal{V}(M_t, M)$ and the column space of $\mathbf{V}_M$ is also uniformly distributed in $\mathcal{G}(M_t, M)$. Suppose $n \geq p$, $\mathcal{V}(n,p)$ is the set of $n \times p$ complex unitary matrices and $\mathcal{G}(M_t, M)$ is the collection of all column spaces of the matrices in $\mathcal{V}(n,p)$. Thus, the codebook entries can be generated randomly following a uniform distribution. This is called random vector quantization (RVQ) in [6].

Design of $\mathcal{C}_{F_V}$. The matrix $\widehat{\mathbf{F}}_Q$ is chosen to be as close to $\mathbf{TQ}$ as possible, where $\mathbf{T}$ is an arbitrary square unitary matrix. We assume that $\mathbf{T}$ is uniformly distributed in $\mathcal{V}(M, M)$. It follows that $\mathbf{T}^{\dagger}\mathbf{Q}$ also has a uniform distribution in $\mathcal{V}(M, M)$ [26]. So we can also use a RVQ for design of $\mathcal{C}_{F_V}$.

Chapter 6

Simulations

In our simulations, the elements of the $M_r \times M_t$ channel matrix $\mathbf{H}$ are complex circularly symmetric Gaussian random variable with zero mean and unit variance. We have used 10^6 channel realizations in the Monte Carlo simulations. The receivers are MMSE in all the system considered.

6.1 The BA system

6.1.1 Distribution of Bit Allocation Vectors

$M = M_t$ ***case.*** In this example $M_r = 4$, $M_t = 4$ and $M = 4$. The number of bits transmitted per channel use is $R_b = 8$. The preocer $\mathbf{F}$ is the identity matrix. Using (4.5), the number of possible integer bit allocation vector is 165. The codebook contains all 165 integer bit allocation vectors. For a given channel realization, the best bit allocation vector in the codebook is chosen using the BER criterion in (4.7). Fig. 6.1 shows the distribution of the bit allocation vectors, where the indexes of the vectors are ordered so that the probabilities are in decreasing order. The cumulative distribution function (cdf) is shown in Fig. 6.2. We can see that some bit allocation vectors are far more probable than others. The probability of the 43 most probable bit allocation vectors is more than 99.96%. In the following subsections we will choose the most probable 2^B bit allocation vectors obtained in experiments like this example and use them as codewords when the number of feedback bits is B.

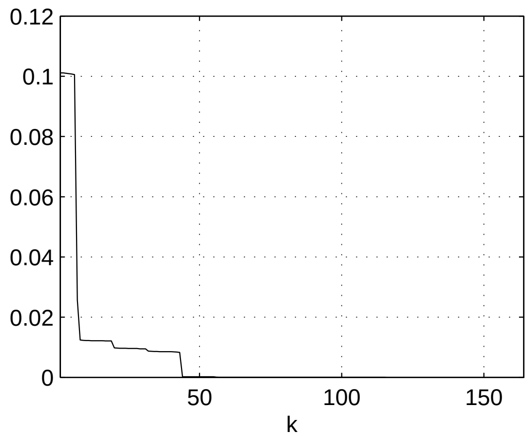

Figure 6.1: Probability mass function of the bit allocation vectors, where the indexes of the vectors are ordered so that the probabilities are in nonincreasing order for $M_r = 4$, $M_t = 4$, $M = 4$ and $R_b = 8$

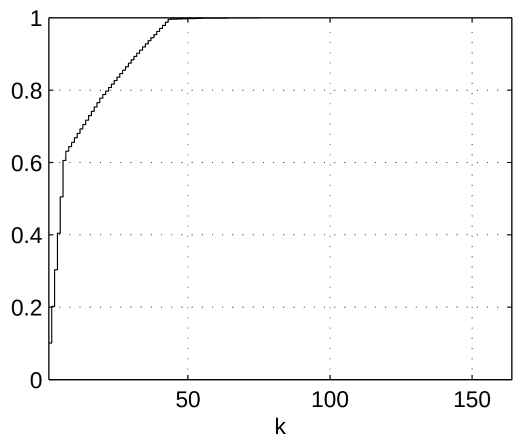

Figure 6.2: Corresponding CDF

$M < M_t$ ***case.*** In this example $M_r = 3$, $M_t = 4$ and $M = 3$. The number of bits transmitted per channel use is $R_b = 8$. The augmented preocer $\mathbf{F}$ is the identity matrix. Using (4.9), the number of possible integer bit allocation vector is 130. The codebook contains all 130 integer bit allocation vectors. For a given channel realization, the best bit allocation vector in the codebook is chosen using the BER criterion in (4.7). Fig. 6.3 shows the distribution of the bit allocation vectors, where the indexes of the vectors are ordered so that the probabilities are in decreasing order. The cumulative distribution function is shown in Fig. 6.4. We can see that some bit allocation vectors are far more probable than others. The probability of the 78 most probable bit allocation vectors is more than 99.96%. Like $M = M_t case$, we will choose the most probable 2^B bit allocation vectors and use them as codewords when the number of feedback bits is B.

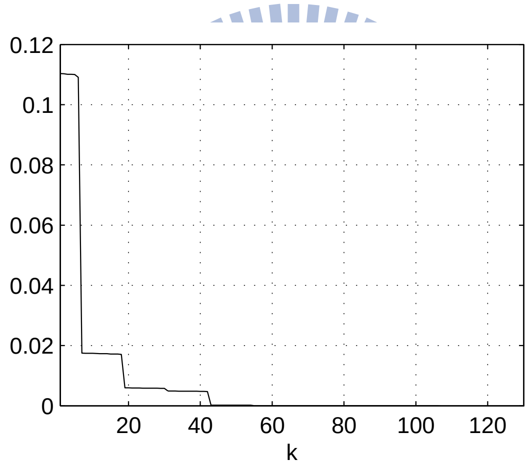

Figure 6.3: Probability mass function of the bit allocation vectors, where the indexes of the vectors are ordered so that the probabilities are in nonincreasing order for $M_r = 3$, $M_t = 4$, $M = 3$ and $R_b = 8$

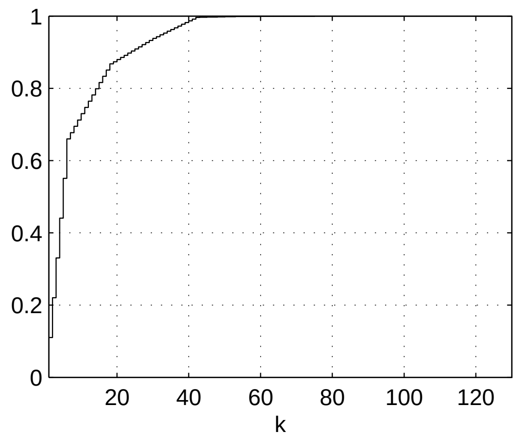

Figure 6.4: Corresponding CDF

6.1.2 BER of BA System

The case $M_r = 4$***,*** $M_t = 4$***,*** $M = 4$ ***and*** $R_b = 8$***.*** The square unitary precoder is the identity matrix $\mathbf{I}_4$. Using (4.5), the number of all possible bit allocation vectors in this case is 165, which corresponds to 8 bits. The BERs of the BA system for different B are shown in Fig. 6.5. When the number of feedback bits is equal to B, the codebook $\mathcal{C}_b$ is constructed by choosing the most probable 2^B bit allocation vectors as mentioned in previous example. The BER improves as the number of feedback bits B increases. For the case $B = 8$, there are 165 vectors in $\mathcal{C}_b$ and the best integer bit allocation vector is chosen to minimize the BER. We can see that the error rate of $B = 3$ is close to that of $B = 8$, i.e., the performance of the best integer bit allocation vector. Observe that the curves correspond to $B = 6$ and $B = 8$, are indistinguishable in the figure. We can understand this by examining the distribution plot in Fig. 6.2. The cdf is very close to one for $k \geq 50$. When we increase B from 6 to 7 to 8, the added codewords are almost never chosen and there is no improvement. The figure also shows the BER when $B = 2$ and the codebook $\mathcal{C}_b^*$ in (4.14) is used. The performance is not as good as the case that use the most probable 4 bit allocation vectors as codewords.

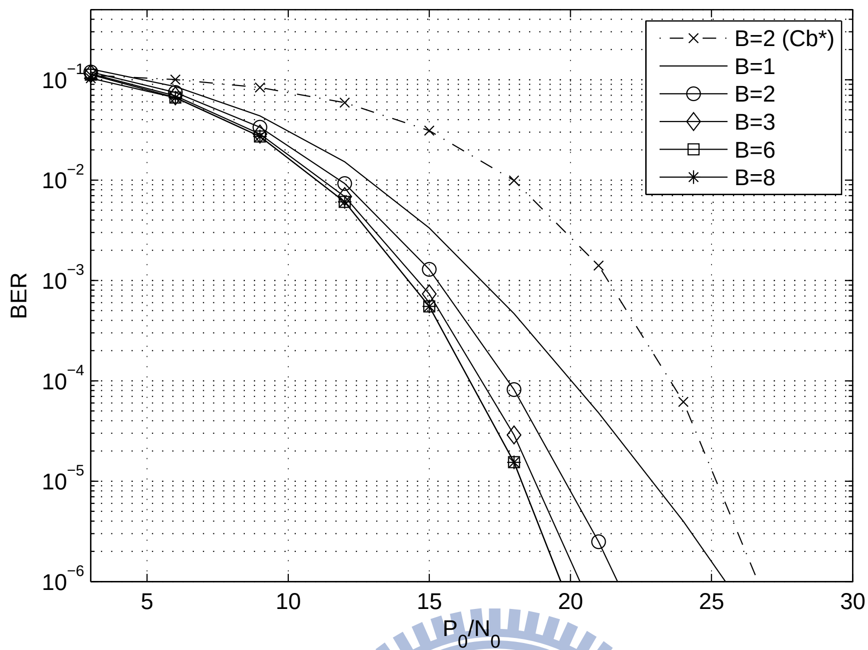

Figure 6.5: Bit error rate of BA system for $M_r = 4$, $M_t = 4$, $M = 4$ and $R_b = 8$

The case $M_r = 2$***,*** $M_t = 4$***,*** $M = 2$ ***and*** $R_b = 4$***.*** The augmented precoder $\mathbf{F}' = \mathbf{I}_4$. The total number of possible integer bit allocation vectors computed using (4.9) is 22. The BERs of the BA system using different number of feedback bits are shown in Fig. 6.6. The codebooks for different B are generated as we did in $M = M_t$ case. The curves for $B = 4, 5$ are almost identical because when B goes from 4 to 5 the added codewords have very small probability, like in the previous example. We also see that the gap between the error rate of $B = 3$ and that of $B = 5$ (the performance of the best integer bit allocation vector) is a small one. For $B = 2$, the codebook designed by choosing the most probable bit allocation vectors is the same as $\mathcal{C}_b^*$, so the two curves overlap. In the figure we have also shown the performance when the precoder is a fixed 4×2 matrix obtained by retaining the first 2 columns of the 4×4 normalized unitary DFT matrix. The performance of augmented precoder is much better than that of a fixed precoder.

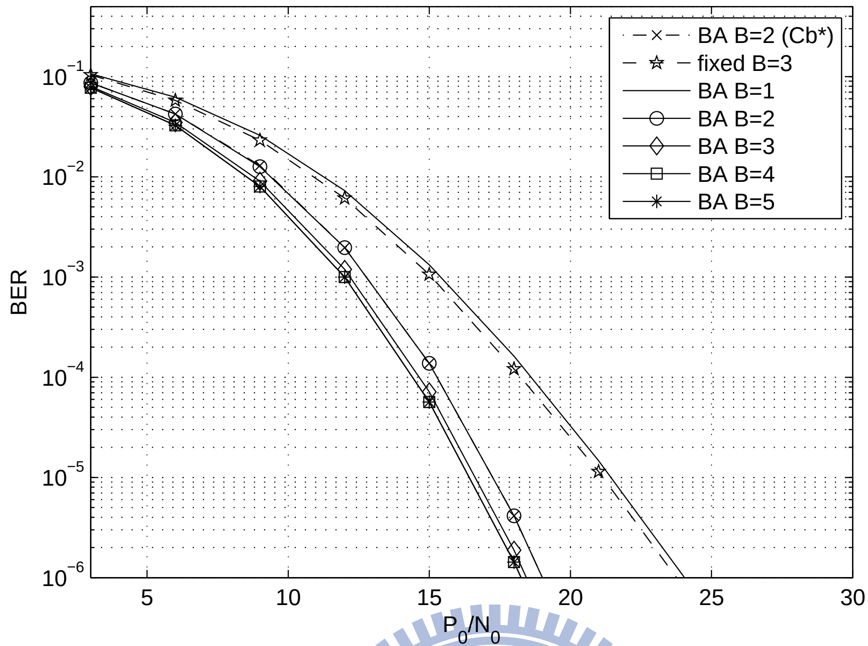

Figure 6.6: Bit error rate of BA system for $M_r = 2$, $M_t = 4$, $M = 2$ and $R_b = 4$

The case $M_r = 3$***,*** $M_t = 3$***,*** $M = 3$ ***and*** $R_b = 8$***.*** The square unitary precoder is the identity matrix $\mathbf{I}_3$. Using (4.5), the number of all possible bit allocation vectors in this case is 45, which corresponds to 6 bits. The BERs of the BA system for different B are shown in Fig. 6.7. Like previous examples, the codebook $\mathcal{C}_b$ is constructed by choosing the most probable 2^B bit allocation vectors as mentioned in previous example. The BER improves as the number of feedback bits B increases. For the case $B = 6$, there are 45 vectors in $\mathcal{C}_b$ and the best integer bit allocation vector is chosen to minimize the BER. Observe that the curves correspond to $B = 5$ and $B = 6$, are indistinguishable in the figure. The figure also shows the BER when $B = 2$ and the codebook $\mathcal{C}_b^*$ in (4.14) is used. The performance is not as good as the case that use the most probable 4 bit allocation vectors as codewords.

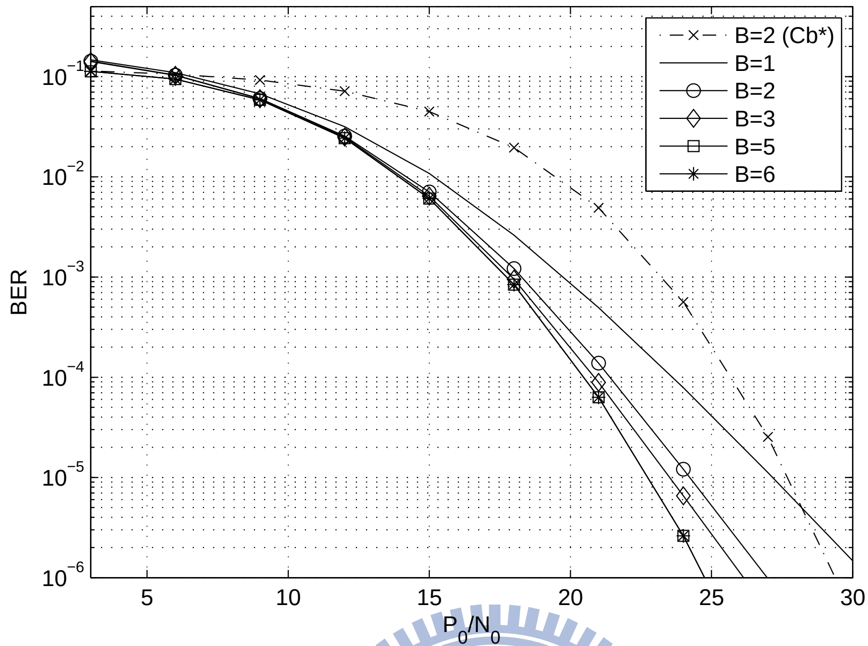

Figure 6.7: Bit error rate of BA system for $M_r = 3$, $M_t = 3$, $M = 3$ and $R_b = 8$

***The case** $M_r = 3$**,** $M_t = 4$**,** $M = 3$ **and** $R_b = 8$.* The augmented precoder $\mathbf{F}' = \mathbf{I}_4$. The total number of possible integer bit allocation vectors computed using (4.9) is 130. The BERs of the BA system using different number of feedback bits are shown in Fig. 6.8. The codebooks for different B are generated as we mentioned before. The curves for $B = 6$ to $B = 8$ are almost identical because when B goes from 6 to 8 the added codewords have very small probability. We also see that the gap between the error rate of $B = 3$ and that of $B = 8$ (the performance of the best integer bit allocation vector) is a small one. The performance of using $\mathcal{C}_b^*$ is not as good as using the most probable 4 bit allocation vectors as codewords. In the figure we have also shown the performance when the precoder is a fixed 4×3 matrix obtained by retaining the first 3 columns of the 4×4 normalized unitary DFT matrix. The performance of augmented precoder is much better than that of a fixed precoder.

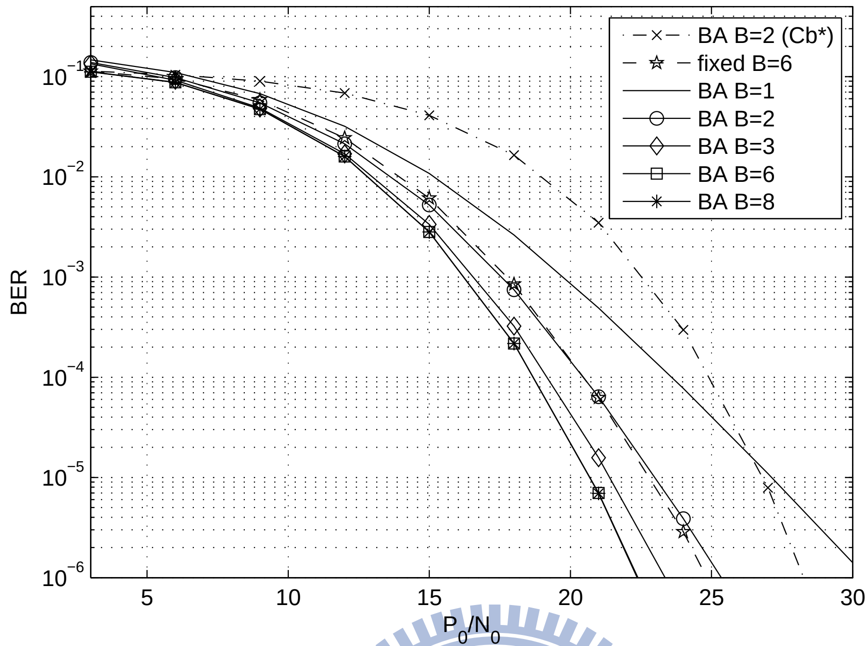

Figure 6.8: Bit error rate of BA system for $M_r = 3$, $M_t = 4$, $M = 3$ and $R_b = 8$

6.1.3 Comparisons of BER

In thisexample we will compare the BA system with the precoding system [7], in which the feedback is the index of the optimal precoder in the codebook and bits uniformly loaded on all M symbols transmitted. In addition, we will compare with multimode antenna selection (MMAS) [17] introduced in Section 3.2, and multimode precoding (MMP) [18] reviewed in Section 3.3.

The case $M_r = 4$***,*** $M_t = 4$***,*** $M = 4$ ***and*** $R_b = 8$***.*** The result are shown in Fig 6.9. The BA, MMAS and MMP systems with finite rate feedback are better than the precoder system with unquantized optimal precoder (infinite feedback bits). Also shown in the figure is BA with unquatized bit allocation computed in (4.28). It is below the curve of the precoder system with unquantized precoder, as shown in section 4.4.2. The performance of BA system with 3 feedback bits is similar to that of MMAS with 4 bits of feedback for small SNR, and slightly better for large

SNR. When there are more feedback bits, e.g., $B = 8$, MMP system outperforms the rest.

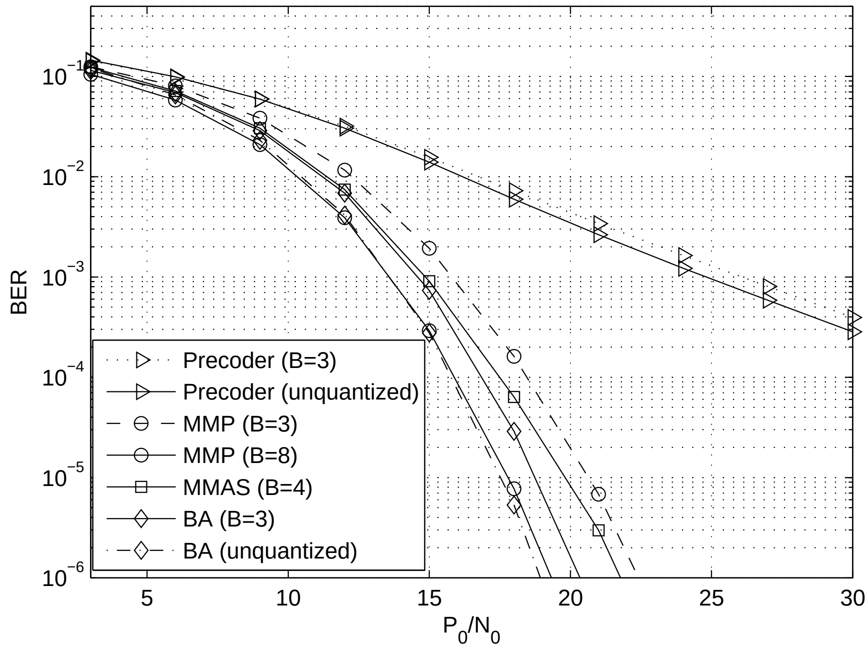

Figure 6.9: Comparison of BER for $M_r = 4$, $M_t = 4$, $M = 4$ and $R_b = 8$

The case $M_r = 2$***,*** $M_t = 4$***,*** $M = 2$ ***and*** $R_b = 4$***.*** Fig. 6.10 shows the comparison for $M_r = 2$, $M_t = 4$, $M = 2$ and $R_b = 4$. As $M < M_t$, the precoding system with unquatized precoder can be better than BA system, as we have explained in section 4.4.2. We see that for low SNR the precoder system with unquantized precoder is the best among all system shown in the figure. If we consider finite feedback rate, the BA, MMAS and MMP systems are better. Similar to Fig. 6.9, the BA system with $B = 3$ is slightly better than MMAS with $B = 4$ and the MMP system outperforms the other 3 systems if more feedback bits are available.

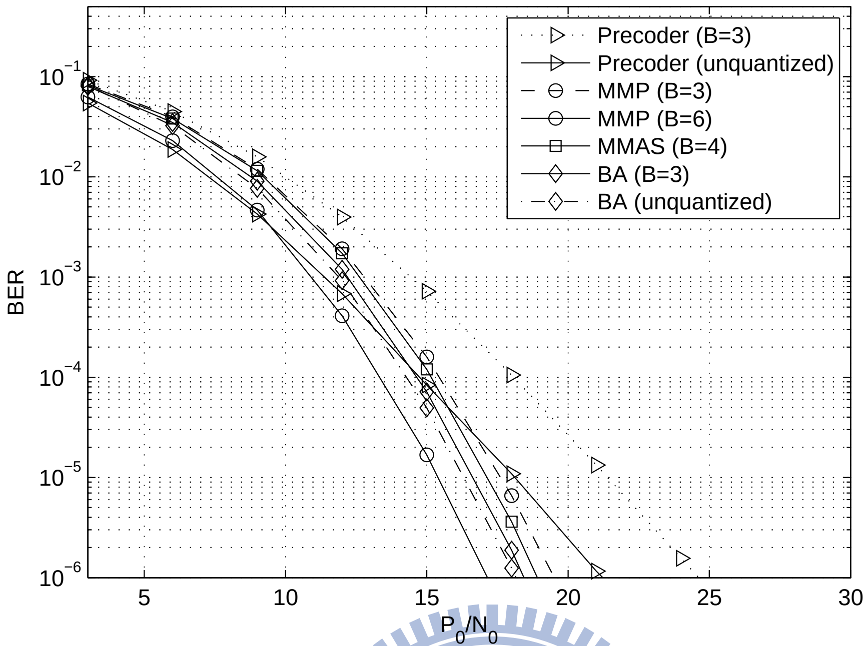

Figure 6.10: Comparison of BER for $M_r = 2$, $M_t = 4$, $M = 2$ and $R_b = 4$

The case $M_r = 3$***,*** $M_t = 3$***,*** $M = 3$ ***and*** $R_b = 8$***.*** The result are shown in Fig 6.11. Since R_b is not divisible for M, it is unrealizable to implement precoder system with uniform bit loading. The BA, MMAS and MMP systems with finite rate feedback are present in this figure. Also shown in the figure is BA with unquatized bit allocation computed in (4.28). The performance of BA system with $B = 3$ is similar to that of MMP with $B = 6$ and is slightly better than MMP with $B = 3$ and MMAS with $B = 3$.

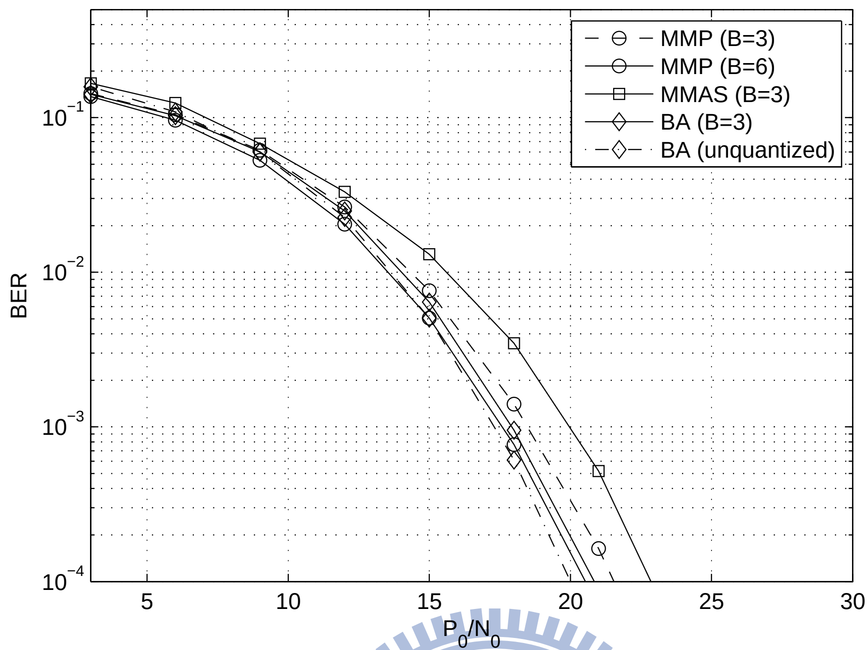

Figure 6.11: Comparison of BER for $M_r = 3$, $M_t = 3$, $M = 3$ and $R_b = 8$

The case $M_r = 3$***,*** $M_t = 4$***,*** $M = 3$ ***and*** $R_b = 8$***.*** The result are shown in Fig 6.12. Since R_b is not divisible for M, it is unrealizable to implement precoder system with uniform bit loading. The BA, MMAS, MMP systems with finite rate feedback and BA with unquatized bit allocation computed in (4.28) is shown. The performance of BA system with 3 feedback bits is close to that of MMP with $B = 4$ and is better than that of MMAS with $B = 4$. When there are more feedback bits, e.g., $B = 8$, MMP system outperforms the rest.

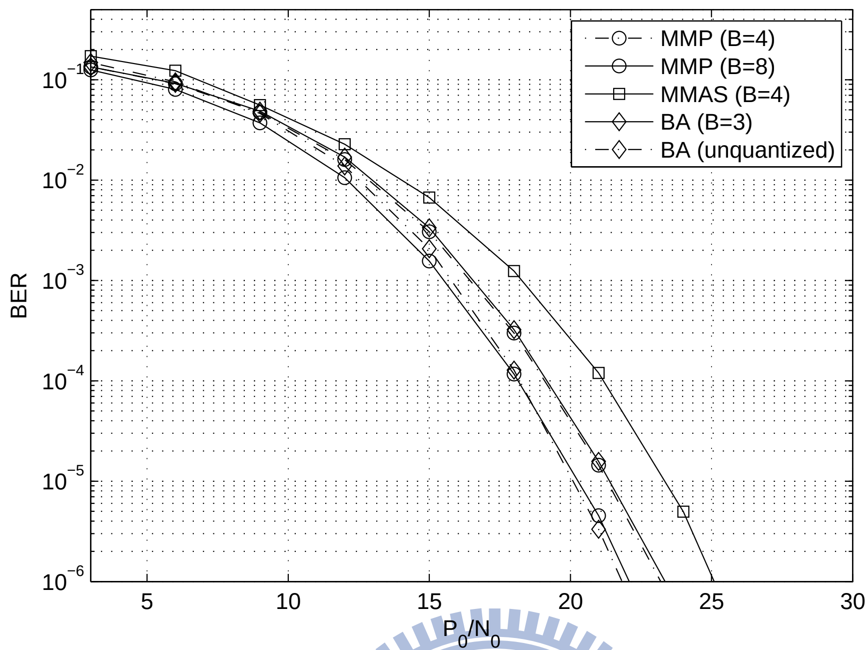

Figure 6.12: Comparison of BER for $M_r = 3$, $M_t = 4$, $M = 3$ and $R_b = 8$

6.1.4 BER for Different Precoders

The BER plots are given for three different types of $M_t \times M_t$ precoders, (i) the identity matrix, (ii) the normalized DFT matrix $\mathbf{W}_{M_t}$ and (iii) the precoder is pulled out one by one from a sequence of $M_t \times M_t$ random unitary matrices known to both the transmitter and receiver, as in RVQ [6]. In Fig. 6.13, we use $M_r = 3$, $M_t = 3$, $M = 3$ and $R_b = 8$. We can see that the curves of all three types of precoders overlap for the same B. Fig. 6.14 shows the performance for $M_r = 3$, $M_t = 4$, $M = 3$ and $R_b = 8$. We use the same three types of matrices as initial precoders. Again the performance is the same regardless of the augmented precoder. The simulation corroborates the result in section 4.2.2 that the performance of the BA system is not affected by the choice of augmented precoder. To have a lower computational complexity at the transmitter, we can simply choose the identity matrix.

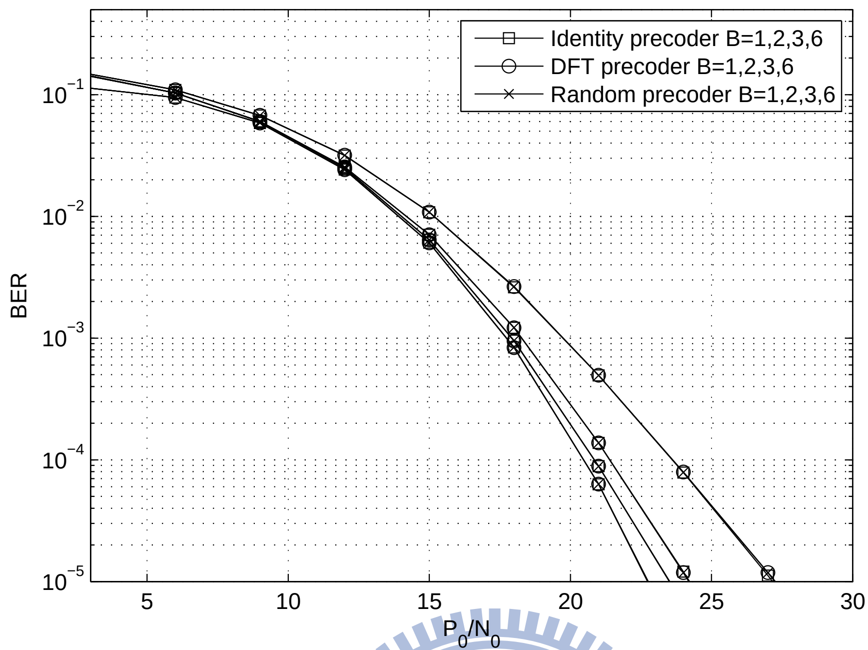

Figure 6.13: BER of the BA system with different precoders for $M_r = 3$, $M_t = 3$, $M = 3$ and $R_b = 8$

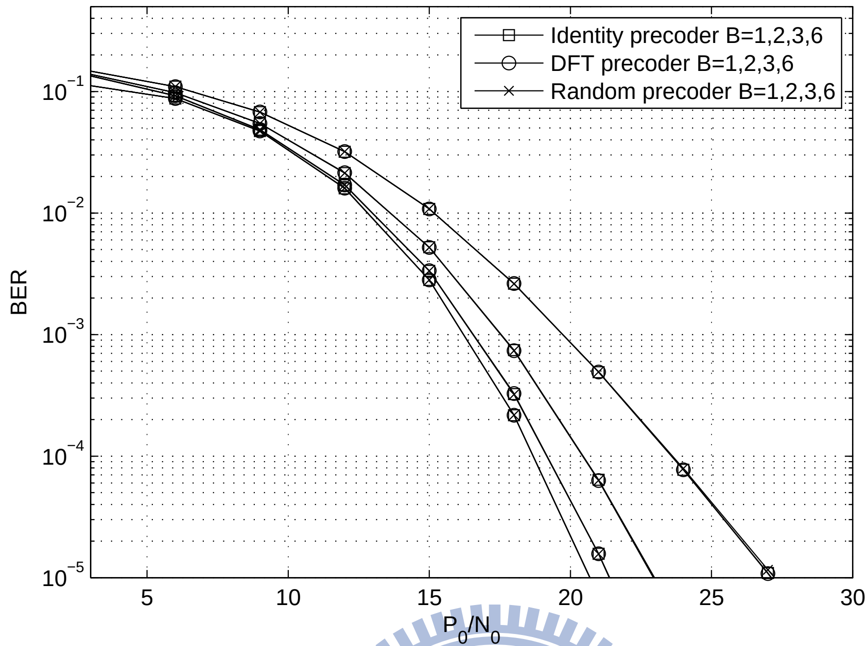

Figure 6.14: BER of the BA system with different precoders for $M_r = 3$, $M_t = 4$, $M = 3$ and $R_b = 8$

6.1.5 Efficient Method of Selecting Optimal Bit Allocation Vector

In this example, we show the usefulness of the efficient searching method proposed in Section 4.5. is also plotted to provide comparison. In figure 6.15, the bit error rate obtained using the exhaustive search in Section 3.1 is denoted by BA and that obtained using the efficient method in Section 4.5 is denoted as BA_2. The precoder $\mathbf{F} = \mathbf{I}_4$. The bit allocation codebook C_b for B is constructed as in previous examples and the same $\mathcal{C}_b$ is used for both BA and BA_2. Curves in Fig. 6.15 shows that the quantization loss of our proposed searching method is small, which can be observed from the small gaps between the BA and BA_2. From Section 4.5, the size of $\mathcal{S}_F$ or $\mathcal{S}_{F,\mathcal{C}}$ indicates the number of times (4.26) is computed. Using the efficient method, the sizes of $\mathcal{S}_{F,\mathcal{C}}$ is 2 for $B = 1$, 8 for $B = 3$, and 14 for $B = 6$. If $\mathbf{b}'_{opt}$ is used, the size of $\mathcal{S}_F$ is 15.

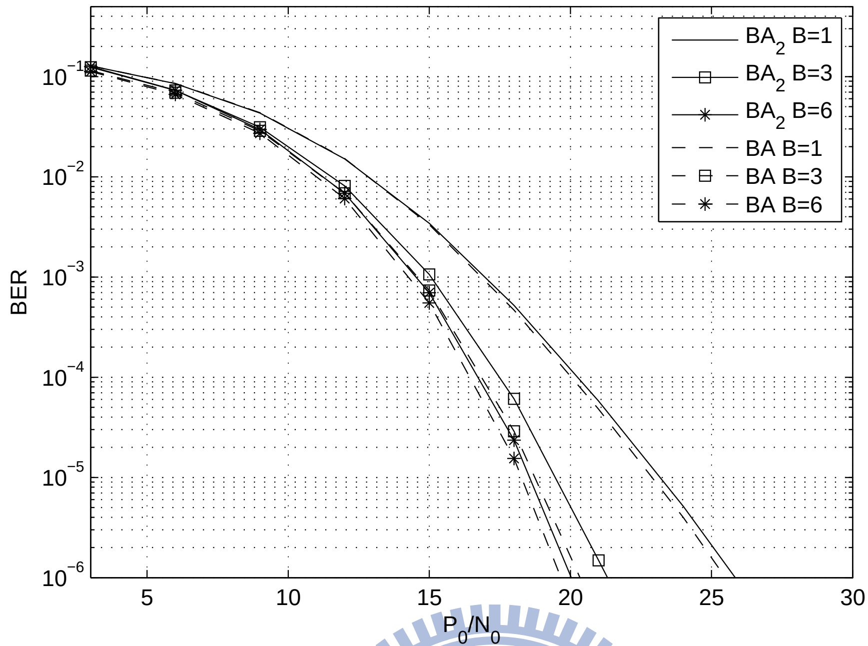

Figure 6.15: BER of the efficient method proposed in Sec. 4.5 for $M_r = 4$, $M_t = 4$, $M = 4$ and $R_b = 8$

6.2 Criteria and Two-step Design for Precoder System

In this subsection, we presented the simulations of the selection criteria and the two-step system design for the precoder system discussed in section 5.2 and section 5.3. The MIMO limited feedback system here is designed to send back information of precoder matrix. Bits and power is assumed to be equally allocated.

6.2.1 Simple Selection Criteria

Criteria for square precoder. In this example, various selection criteria for square precoder are compared. These selection criteria include the AM-GM selection (AMGM) criterion developed in (5.5), trace function of $\mathbf{R_e}$ criterion (TrMSE) proposed in [5] and the BER-based selection criterion (BER) introduced in section 3.1. The BER optimal unitary precoder with infinite feedback bits is also plotted to provide benchmark performance. Due to the precoder independent property of summation of error covariance mentioned in section 4.4.2, the trace function of $\mathbf{R_e}$ selection criterion can't pick a precoder matrix from precoder codebook. Thus, the trace function selection criterion will randomly selects a precoder matrix from precoder codebook. In Fig. 6.16, $M_r = 4$, $M_t = 4$, $M = 4$, and $R_b = 8$, the feedback bits $B = 8$ shows that the AM-GM selection criterion and BER-based criterion both yield BER performance very close to that of infinite feedback case. Note that the AM-GM selection criterion requires computation of the product of error covariance.Thus, the complexity is lower compared to BER-based criterion.

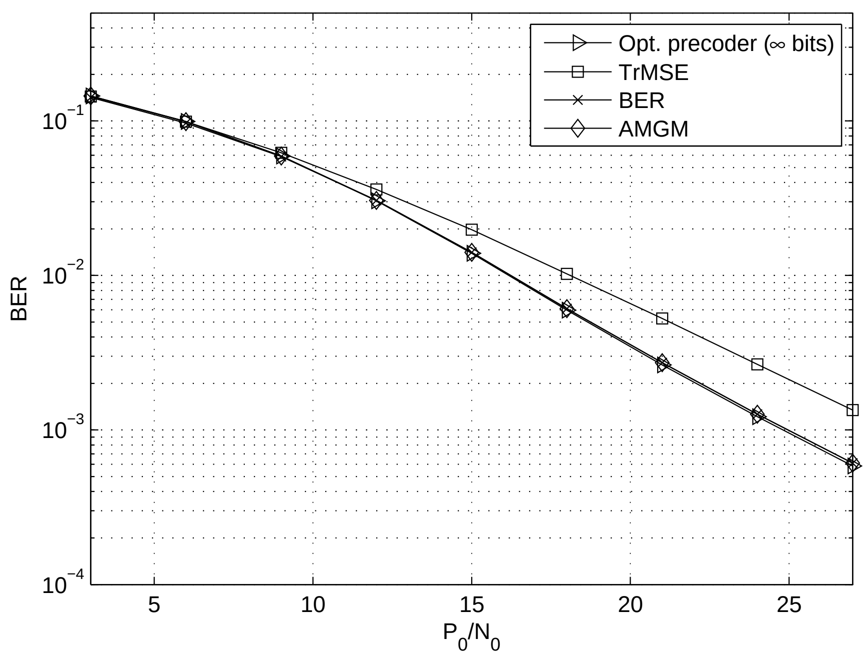

Figure 6.16: BER of different selection criterion for $M_r = 4$, $M_t = 4$, $M = 4$ and $R_b = 8$

Criteria of Rectangular Precoder. In this example, we compare the criterion of summation of squared error covariance (SS) in (5.8), selection criterion of trace function of $\mathbf{R_e}$ (TrMSE) in [5], and BER-based selection criterion (BER) in [7]. Optimal precoder with infinite feedback bits in (4.32) is also plotted. For feedback bits $B = 6$, Fig. 6.17 shows that SS criterion and BER criterion are better than TrMSE criterion. In addition the BER performance of the lower-complexity SS criterion is very close to that of using BER criterion. This observation shows the usefulness of the selection criterion of summation of squared error covariance.

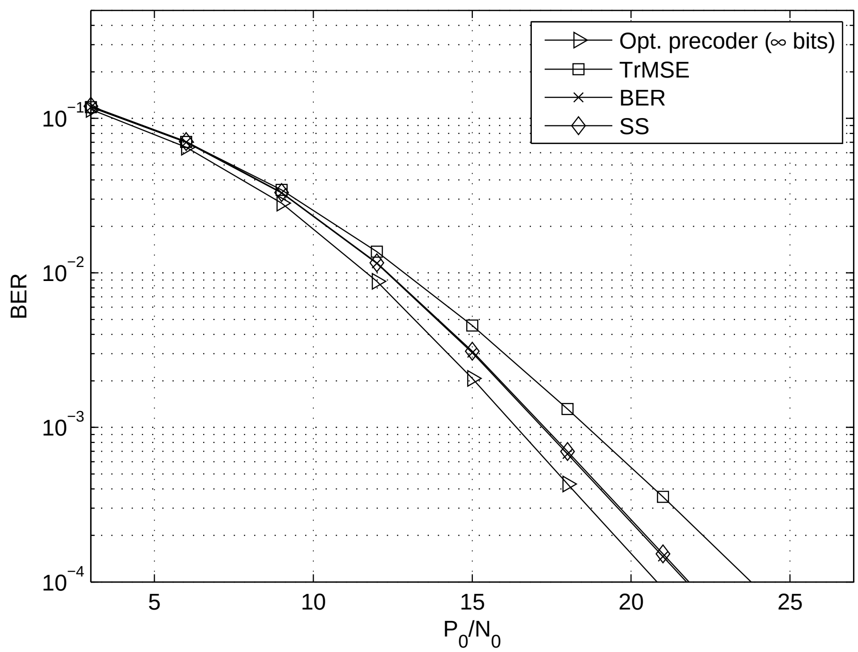

Figure 6.17: BER of different selection criterion for $M_r = 4$, $M_t = 5$, $M = 4$ and $R_b = 8$

6.2.2 Proposed Two-step System

In this example, we show the BER performance of two step system in section 5.3. The two step system has separate selection criteria for $\mathbf{F}_V$ and $\mathbf{F}_Q$. We first select $\mathbf{F}_V$. The criteria we considered for $\mathbf{F}_V$ are sum of squared error covariance criterion in (5.8) (SS), trace function of $\mathbf{R_e}$ criterion in [5] (TrMSE) and BER criterion in [7] (BER). After $\mathbf{F}_V$ is chosen, $\mathbf{F}_Q$ is selected from $\mathcal{C}_{F_Q}$ for the chosen $\mathbf{F}_V$. The selection criteria for choosing $\mathbf{F}_Q$ include the AM-GM selection criterion (AMGM), BER criterion, and sum of squared error covariance criterion (SS). Figure 6.18 shows the BER performance using different selection criterion for $\mathbf{F}_Q$ when $\mathbf{F}_V$ is chosen using TrMSE criterion. In this example $M_r = 4$, $M_t = 5$, $M = 4$, $R_b = 8$ and $B_V = 4$ and $B_Q = 4$. Both $\mathcal{C}_{F_V}$ and $\mathcal{C}_{F_Q}$ are generated by RVQ method [6]. It can be observed that the difference between AM-GM criterion, BER criterion and SS criterion is almost negligible. Hence, we can use the low complexity AM-GM selection criterion for $\mathbf{F}_Q$.

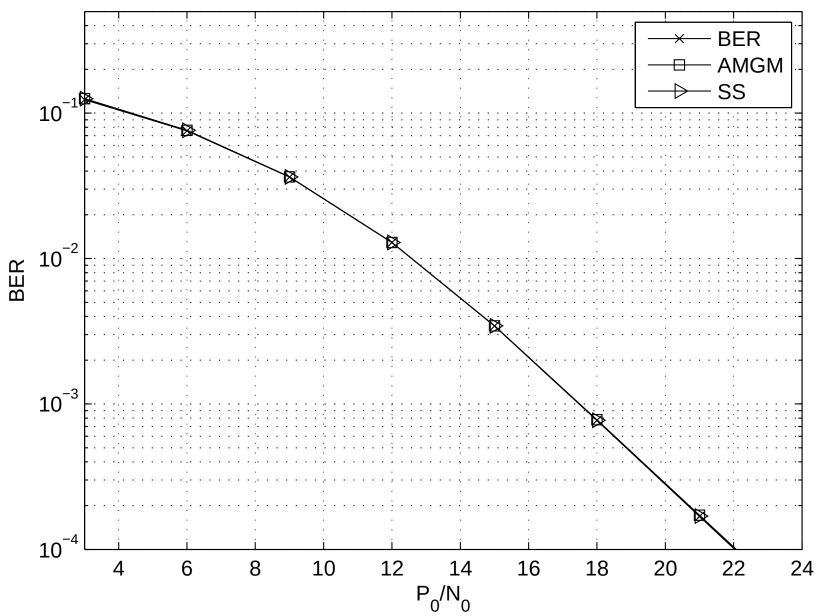

Figure 6.18: BER of different selection criterion of $\mathbf{F}_Q$ for $M_r = 4$, $M_t = 5$, $M = 4$ and $R_b = 8$

The BER performance for different $\mathbf{F}_V$ selection criteria is shown in Fig. 6.19. The system has $M_r = 4$, $M_t = 5$, $M = 4$ and $R_b = 8$. The matrix $\mathbf{F}_Q$ is chosen using the AM-GM criterion. The feedback bits $B = 8$ are equally divided for $\mathcal{C}_{F_V}$ and $\mathcal{C}_{F_Q}$, $B_V = 3$ and $B_Q = 3$. We compare sum of squared error covariance criterion, trace function of $\mathbf{R_e}$ criterion and BER-based selection criterion for choosing $M_t \times M$ precoder matrix $\mathbf{F}_V$. The BER curves are close but trace function of $\mathbf{R_e}$ (total MSE) criterion is slightly better than the other two selection criteria. TrMSE criterion selects the total MSE minimizing $\mathbf{F}_V$ but the other two criteria consider total MSE minimizing and error covariance equalizing together. Therefore, the AM-GM selection criterion for $\mathcal{C}_{F_Q}$ combined with the trace function of $\mathbf{R_e}$ for $\mathcal{C}_{F_V}$ provides a low-complexity design for two step system.

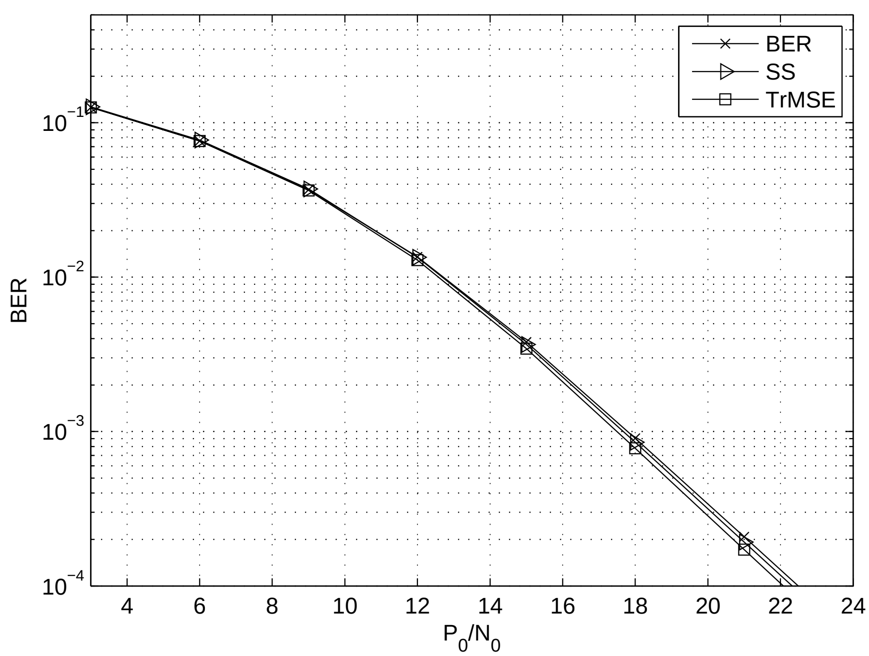

Figure 6.19: BER of different selection criterion of $\mathbf{F}_V$ for $M_r 4 =$, $M_t = 5$, $M = 4$ and $R_b = 8$

Comparison between one-step and two-step system. In figure 6.20 the BER performance of one-step system and two-step system are compared. In this example $M_r = 4$, $M_t = 5$, $M = 4$ and $R_b = 8$. The feedback bit $B = 8$. For one-steps system, a precoder codeook of size 2^B from [7] is prepared. The BER criterion in (3.1) is employed. For two-step system, $B_V = 4$ and $B_Q = 4$. The codebooks $\mathcal{C}_{F_V}$ and $\mathcal{C}_{F_Q}$ are generated using RVQ method [6]. TrMSE criterion is used for $\mathbf{F}_V$ selection and AMGM criterion is used for $\mathbf{F}_Q$ selection. A similar hierarchical system (hierarchical) with two precoder codebooks [32] is also provided. From [32], the feedback bits allocation of hierarchical system is 6 bits for one Grassmanian codebook and 2 bits for one rotational based codebook. The performance of BER optimal precoder with infinite feedback bits is also plotted. Observing from figure 6.20, the BER optimal precoder has the best BER, and the curve of the two-step system is close to that of one-step system. In addition, The performance of two-steps system is slightly better than hierarchical system. Note that the number of searches for two-step system is 32, the numbers of searches for one-step system and hierarchical system are respectively 256 and 68.

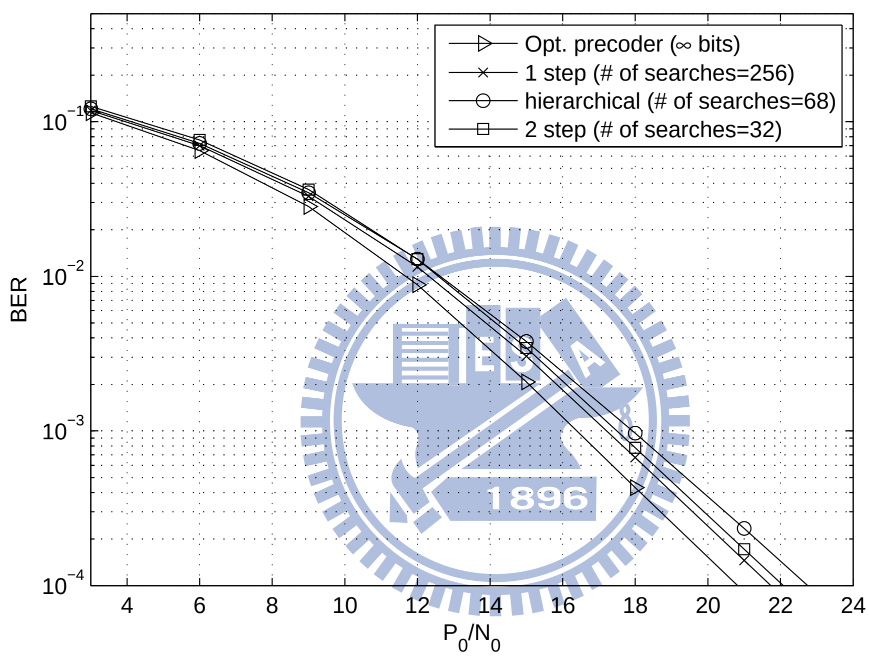

Figure 6.20: BER comparison of one-step and two-steps systems for $M_r = 4$, $M_t = 5$, $M = 4$ and $R_b = 8$

Chapter 7

Conclusions

In this paper we first proposed to feedback only bit allocation for MIMO systems with limited feedback and the system is called a BA systems. Secondly, for precoder system with limited feedback, we describe two insightful properties of the BER optimal precoder. Motivated by these two properties, we develop two selection criteria for conventional one-step system and propose a two-steps design

In proposed BA system, the augmented precoder is assumed to be known to both transmitter and receiver. With the BA scheme, the bits can be nonuniformly loaded. By allowing general bit allocation, bits can be allocated according to the channel. We have also shown that the proposed BA system can achieve diversity order of $M_r M_t$ using $log_2(M_t)$ bits. The optimal augmented precoder can be any square unitary matrix. Furthermore, the unconstrained bit allocation is derived. Using the unconstrained bit allocation, we develop an efficient method for selecting the BER-minimizing bit allocation vectors from the codebook.

For precoder feedback system, two simple selection criterion are developed for square and rectangular precoders respectively, and a two-step system is designed for reducing the complexity. These selection criteria for one-step system are easy to compute and provide BER performance close to the BER-based selection criterion. The two-step system contains two precoder matrices in transmitter and lower the number of searches. Many interesting problem remain to be solved, such as the design of bit allocation codebook and the feedback bits allocation for two-step system.

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