Multimode Antenna Selection with Reduced Complexity for Zero-forcing Receiver

Wan-Chen Yeh

Advisor: Dr. Shang-Ho Tsai
Department of Electrical and Control Engineering
National Chiao Tung University

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Abstract

In this thesis, we proposed reduced-complexity algorithms to select appropriate transmit antennas for MIMO system with zero-forcing receiver. Two criteria for selecting antennas are derived: achievable data rate maximization and average symbol error rate minimization. In addition, bit allocation is also applied to improve the performance. The computational complexity can be reduced by simplifying search strategy and matrix inversion. The computational complexity analysis and simulation results are given to see that the advantages of the proposed algorithm.

Contents

List of Figures

List of Tables

Chapter 1

Introduction

Multiple-input multiple-output (MIMO) technique have been widely employed in wireless systems. MIMO systems were first investigated in the 1980s by simulations on computers [1]. MIMO signaling can improve wireless systems mainly in two aspects: one is diversity [16] and the other is spatial multiplexing.

Diversity To ensure the reliability of a wireless system, the strength of a single signal path should be as large as possible. However in practice, this path may be in a deep fade. When the path is in deep fade, the communication system may suffer from errors. The problem can be conquered in spatial domain. Antenna diversity or spatial diversity, is obtained by placing multiple antennas at the transmitter or/and receiver which are spaced sufficiently. Multiple-input single-output (MISO) systems can be used to obtained transmit diversity and single-input multiple-output (SIMO) systems can be used to obtained receive diversity. With multiple antennas, independent copies of the same signal are available, and they are combined into a signal with high quality, even though there may be some copies of low quality due to fading. Therefore, even in challenging wireless environments, a high-quality transmission is possible for MIMO system.

Spatial multiplexing The data stream to be transmitted is demultiplexed into several substreams modulated independently and transmitted on different transmit antennas [2], [3]. The multiple received antennas with channel knowledge are used for distinguishing different data streams at receiver. The advantage of spatial multiplexing is the transmission rate can be increased by transmitting independent symbols in different transmit antennas.

Spatial multiplexing shows that the transmission rate can be increased by transmitting independent data streams in different antennas. To achieve the maximum capacity, water-filling power allocation can be used [16]. Water-filling power allocation can be used for different data streams to achieve the maximum channel capacity. With water-filling power allocation, using all transmit antennas always leads to the best performance. However, equal power allocation instead of water-filling power allocation is usually used in practical wireless systems since it does not require to send back the power allocation table to the transmit side. Unlike water-filling power allocation, using all transmit antenna in equal power allocated systems does not always lead to the best performance; that is, some transmit antennas may have bad conditions, redistributing the power originally for antennas with bad condition to that with good condition can significantly improve the overall performance, thus selecting proper transmit antennas is needed. This concept was first discovered in [4]. Similar concept to select proper transmit antennas to minimize vector symbol error rate (VSER) was proposed by [10]; and the concept to select proper transmit antennas was called *multimode antenna selection* in [10]. Multimode antenna selection provides additional array gain. The selection criteria in [10] focus on the minimization of nearest neighbor union bound (NNUB) to minimize VSER for linear receivers. The constellation size is the same for each substream, *i.e.* equal bit allocation. Several related topics have been discussed for different criteria. In [9], precoding with codebook design together with multimode antenna selection was proposed to minimize the probability of error and maximize the mutual information for independent and identically distributed (i.i.d.) Gaussian signaling. Other topics for multimode antenna selection such as [5] and [6] were discussed for power control and relay systems respectively.

In MIMO systems, the problem of maximizing transmission data rate is a popular topic [8], [9]. However in practice, bit budget is usually given in advance to assign bits; here the constellation size is adaptive varied by the number of assigned bits. This is known as bit allocation or bit loading [15]. With this technique, the well-behaved channel will be allocated more bits to transmit signal. Bit allocation is usually designed jointly with transceiver designs for different kinds of optimization problems. For instance, used in multicarrier or Discrete Multitone (DMT) systems [11], [12]. For MIMO systems, bit allocation was considered in [7], [8].

In this thesis, we consider multimode antenna selection for zero-forcing (ZF) receiver in optimizing the following two criteria: i.) to achieve maximum data rate, and ii.) to attain minimum symbol error rate (SER). In problem i.), the total transmission power is fixed and the power for each substream is equal. Therefore, the achievable data rate can be increased by multimode antenna selection. In practical systems, bit budget is given in advance by well allocating the given bits to the selected substreams rather than equally allocating the bits [10] can further reduce the SER. The optimal bit allocation used in this thesis is water-filling bit allocation [15]. We investigate the performance of bit allocation for the selected substreams obtained by problem i.). In problem ii.), the average SER is minimized by selecting appropriate transmit antennas. By using different design criteria, the performance of the proposed scheme slightly outperforms that in [10].

However, for optimal multimode antenna selection for problem i.) and ii.) mentioned above or even that in [10], there are two issues for computational complexity. First, the strategy for optimal selection is exhaustive search which demands large number of iterations. Second, with zero-forcing (ZF) receiver, the objective functions in each iteration for problem i.) and ii.) need matrix inversion and matrix multiplications. To simplify the computational complexity: first, in order to reduce the number of iterations, greedy search [13] is used. The greedy search determines the locally optimal selection at each stage, and hopes to obtain the global optimal selection. Second, in order to simplify the matrix multiplication and matrix inversion, the inversion property of a partitioned matrix in [20] is applied. Although the computation is simplified, the results of matrix inversion and matrix multiplication can be equally obtained. With greedy search, the simplification of matrix inversion and matrix multiplications in every iterations can be reduced to vector multiplication and scalar division. The proposed algorithm can greatly reduce the computational complexity with little performance loss compared to that in exhaustive search. The comparison will be presented in latter sections by complexity analysis and simulation results.

This thesis is organized as follows. In Chapter 2, we introduce the system model, the corresponding mathematical models, and some assumptions. Then, in Chapter 3, we present the proposed algorithms for the problem of maximizing achievable data rate, and water-filling bit allocation is applied. In Chapter 4, the proposed algorithm is applied to the problem of minimizing average SER. Chapter 5 illustrates the performance improvements for the proposed algorithms in Chapter 3 and 4 via Monte Carlo simulations. In Chapter 6, we make some conclusions of that we declare in this thesis.

Notations

s : italics denote scalars

$\mathbf{v}$: boldface lower-case letters denotes column vectors

$\mathbf{A}$: boldface upper-case letters denotes matrices

$\mathbf{A}_{i,j}$: the (i, j)-th element of the matrix $\mathbf{A}$

$\mathbf{A}^H$: conjugate transpose matrix of $\mathbf{A}$

$\mathbf{A}^\dagger$: pseudo inverse matrix of $\mathbf{A}$

$E_s[a]$: expectation of a with respect to random variable $\mathbf{s}$

$CN(\mu, \sigma)$: complex Gaussian distribution with mean μ and variance σ

Chapter 2

System model

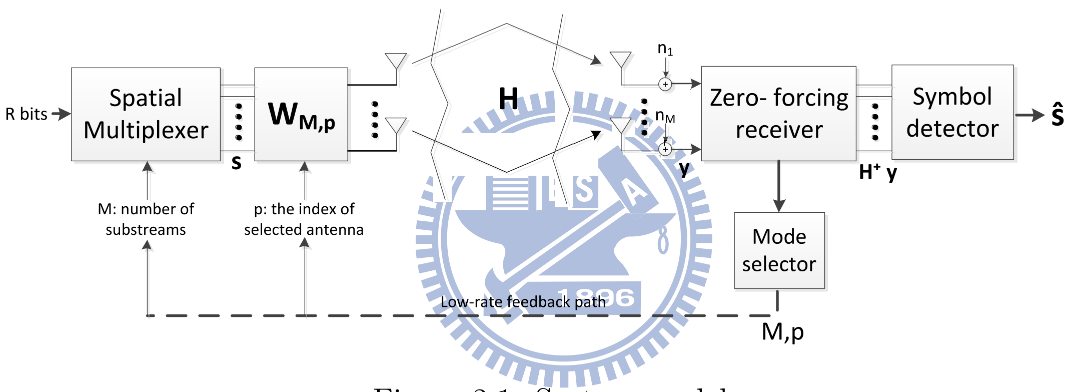

Figure 2.1: System model.

Consider a MIMO system with M_t-transmit antennas and M_r-receive antennas illustrated in Fig. 1. R bits are transmitted per symbol time. The system consists of a spatial multiplexer that produces M-dimentional symbol vector, a symbol mapper that maps the M-dimentional symbol vector to selected antenna subsets, a matrix that is a function of wireless environment, a ZF receiver and symbol detector. The low-rate feedback path sends back the information of the number M of substreams and the index of selected antennas p to spatial multiplexer and symbol mapper respectively.

There are R bits demultiplexed into M substreams, and modulated independently using quadrature-amplitude modulation (QAM). The symbol vector $\mathbf{s}$ is $\begin{bmatrix} s_1 & s_2 & \dots & s_M \end{bmatrix}^T$, the number of bits of the substreams can be either equal allocated or allocated by different subchannel status. We assume that the total transmit power is normalized to one, and each substream has equal power, *i.e.* $E_s[\mathbf{s}\mathbf{s}^H] = (1/M)\mathbf{I}_M$, where $\mathbf{I}_M$ is an $M \times M$ indentity matrix.

Given the number M of substreams, the symbol vector $\mathbf{s}$ is divided into M substremas and mapped to the corresponding antennas. Let $\mathcal{W}_M$ be the set of $\binom{M_t}{M}$ submatrices taken by choosing M columns from $M_t \times M_t$ identity matrix. For each M, $\mathcal{W}_M$ can be written as $\left\{\mathbf{W}_{M,1}, \cdots, \mathbf{W}_{M,\binom{M_t}{M}}\right\}$. $\mathbf{W}_{M,p}$ is called symbol mapper which maps the substreams to the selected antennas and consists of M column vectors of $M_t \times M_t$ identity matrix. The subscripts of $\mathbf{W}_{M,p}$: M represents the number of substreams, also it is called *mode* in [9], [10]; p represents the index of selected transmit antennas (index of selected columns of $\mathbf{H}$). For a 3×3 MIMO system,

$$\begin{aligned}\mathcal{W}_1 &= \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}, \\ \mathcal{W}_2 &= \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \right\}, \\ \text{and } \mathcal{W}_3 &= \left\{ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right\}. \end{aligned} \tag{2.1}$$

The channel $\mathbf{H}$ is a matrix with entry $h_{r,q}$, where r and q represent the indices of receive and transmit antenna respectively. The entries are modeled as proper complex Gaussian random variable with distribution $CN(0, 1)$. The received symbol vector is

$$\mathbf{y} = \mathbf{H}_{eq} \cdot \mathbf{s} + \mathbf{n} \tag{2.2}$$

where $\mathbf{n} = \begin{bmatrix} n_1 & n_2 & \ldots & n_M \end{bmatrix}^T$ is complex Gaussian distributed with $CN(0, N_0)$, and the equivalent channel after transmit antenna selection is

$$\mathbf{H}_{eq} = \mathbf{H}\mathbf{W}_{M,p}. \tag{2.3}$$

At the receiver, we use zero-forcing (ZF) receiver $\mathbf{G} = (\mathbf{H}\mathbf{W}_{M,p})^{\dagger}$ [15], to process the received vector $\mathbf{y}$. The input of symbol detector is $\mathbf{G} \cdot \mathbf{y}$ and the output is $\hat{\mathbf{s}} = \mathbf{s} + \mathbf{G} \cdot \mathbf{n}$.

In this paper, we assume that the transmitter has no perfect channel knowledge, while the channel $\mathbf{H}$ is known perfectly at the receiver. Therefore, the optimal selection of the mode M^* $(1 \leq M^* \leq M_t)$ and the index of selected antenna $p(1 \leq p^* \leq \binom{M_t}{M})$ can be chosen at the receiver and sent back to the transmitter through a zero-delay limited feedback link.

Chapter 3

Multimode antenna selection to maximize achievable data rate

3.1 Problem formulation and optimal solution

In this section, we select the optimal transmit antenna subsets to maximize the achievable data rate. Suppose that a $2b$-bit-QAM symbol with power $\mathcal{E}_s$ is transmitted through a zero-mean complex additive white Gaussian noise (AWGN) channel with noise variance N_0. A bound for SER of QAM symbol is then given by [14]

$$P_{e,2b-QAM} \leq 4(1-\frac{1}{2^b})Q(\sqrt{\frac{3\mathcal{E}_s}{(2^{2b}-1)N_0}}), \tag{3.1}$$

where $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty \exp\left(-\frac{\tau^2}{2}\right)d\tau$ is Gaussian-Q function. For large number of 2^{2b} and high SNR per bit, the upper bound given in (3.1) is quite tight. By rearranging (3.1), we obtain the data rate [15]

$$b = \log_2\left(1 + \frac{3\frac{\mathcal{E}_s}{N_0}}{\left[Q^{-1}\left(\frac{SER}{4(1-2^{-b})}\right)\right]^2}\right), \tag{3.2}$$

The capacity of an additive Gaussian noise channel with power $\mathcal{E}_s$ and noise variance N_0 is given by [16]

$$\mathcal{C} = \log_2\left(1+\frac{\mathcal{E}_s}{N_0}\right). \tag{3.3}$$

Consider (3.2) and (3.3), the former shows the bit rate that can be safely transmitted without exceeding a given SER for QAM while the latter shows the achievable bit rate with arbitrarily small error rate. For above equations, the SNR $\mathcal{E}_s/N_0$ of uncoded QAM symbol in (3.2) should be higher than that in (3.3) if the data rate in (3.2) is equal to the capacity in (3.3). The reason is that for small enough SER (SER $\leq 10^{-2}$), the factor $3/\left[Q^{-1}\left(SER/4(1-2^{-b})\right)\right]^2$ is less than one [15], [19]. To achieve capacity for QAM, *i.e.* $b = \mathcal{C}$, the SNR for QAM should be $\left[Q^{-1}\left(SER/4(1-2^{-b})\right)\right]^2/3$ times larger than that in (3.3), the factor then is defined as *SNR gap*:

$$\Gamma = \frac{1}{3}\left[Q^{-1}\left(\frac{SER}{4(1-2^{-b})}\right)\right]^2. \tag{3.4}$$

For small enough error rates, the inverse Q function becomes flat, thus the SNR gap can be approximated accurately by

$$\Gamma = \frac{1}{3}\left[Q^{-1}\left(\frac{SER}{4}\right)\right]^2. \tag{3.5}$$

For example, if $SER = 10^{-5}$ and b is large enough, the factor is then

$$\frac{\left[Q^{-1}\left(SER/4\right)\right]^2}{3} \cong 6.95 = 8.42\text{dB}. \tag{3.6}$$

In our system model, the maximum achievable data rate for a substream in (3.2) is a function of the post-processing SNR, which is the SNR at the detector. For ZF receiver, the post-processing SNR for the i-th substream can be expressed as [17]

$$SNR_i^{(ZF)} = \gamma_0 \frac{1}{\left[\mathbf{H}_{eq}^H \mathbf{H}_{eq}\right]_{i,i}^{-1}}, \tag{3.7}$$

where $\gamma_0 = \mathcal{E}_s/N_0$. Therefore, by (3.5) and (3.7), the achievable data rate of the i-th substream can be written as

$$b_i = \log_2\left(1 + \frac{SNR_i^{(ZF)}}{\Gamma}\right) = \log_2\left(1 + \frac{\gamma_0}{\Gamma\left[\mathbf{H}_{eq}^H \mathbf{H}_{eq}\right]_{i,i}^{-1}}\right). \tag{3.8}$$

It is worth emphasizing the desired data rate in (3.8) is the same as the following equation for $\mathbf{F} = \mathbf{W}_{M,p}$ and $\mathbf{G} = \mathbf{H}_{eq}^{\dagger}$ derived in [18],

$$b_i = D - \log_2 c_i - \log_2[\mathbf{F}^H\mathbf{F}]_{i,i} - \log_2[\mathbf{G}\mathbf{G}^H]_{i,i}, \tag{3.9}$$

where $\mathbf{F}$ is linear percoder matrix, $\mathbf{G}$ is linear equalizer, and c_i is given by

$$\Gamma N_0 = \frac{N_0}{3}\left[Q^{-1}\left(SER/4\right)\right]^2. \tag{3.10}$$

The proof is in Appendix A.

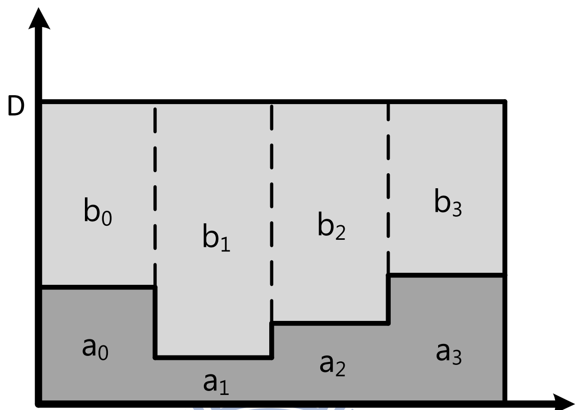

Figure 3.1: A water filling interpretation of the optimal bit allocation.

Let us illustrate (3.9) in Fig. 3.1, which is the bit allocation for a fixed power and equal substream power. It is like pouring water into a tank. The water level D in Fig. 3.1 and (3.9) is the logarithm of substream power, *i.e.* $\log_2(1/M)$. The subscripts of a and b represent the indices of substreams. With uneven floor a_i and the water level D, b_i bits are allocated to achieve water height D for the i-th substream. By (3.9), the floor a_i for each substream is related to the noise and the SNR gap, *i.e.*

$$a_i = \log_2 c_i + \log_2[\mathbf{F}^H\mathbf{F}]_{i,i} + \log_2[\mathbf{G}\mathbf{G}^H]_{i,i} \tag{3.11}$$

Therefore, for an arbitrary SER for each substream, the problem of maximizing the achievable data rate subject to the ZF constraint can be written as follows:

$$\begin{aligned} b &= \max_{M=M^*,p=p^*} \sum_{i=1}^{M} b_i \\ &= \max_{M=M^*,p=p^*} \sum_{i=1}^{M} \log_2\left(1 + \frac{\gamma_0}{\Gamma\left[\mathbf{H}_{eq}^H\mathbf{H}_{eq}\right]_{i,i}^{-1}}\right) &(3.12)\\ &\text{s.t. } \mathbf{G} = (\mathbf{H}\mathbf{W}_{M^*,p^*})^\dagger &(3.13)\end{aligned}$$

Note that b_i is a function of M and p, and p is the indices of selected antennas defined in (2.3).

With transmit antenna selection by (3.12), the achievable transmission data rate can be more than that without antenna selection; the reason is that the transmit power for a data stream is limited to one, and the power of each substream is averaged and decreased as number of transmit antennas is increased. In order to allocate the power efficiently to each substream, the worse substreams are dropped. Then, the power originally for the dropped antennas can be redistributed to all the selected antennas to improve the performance.

Exhaustive search. Exhaustive search is a general problem-solving technique that computes all candidates for the solution and decide which candidate matches the objective function. For the optimal multimode antenna selection of maximizing achievable data rate in (3.12), we must calculate all the achievable data rates under the ZF constraint for all possible equivalent channels. The equivalent channels in (2.3) are calculated by all possible symbol mappers $\mathbf{W}_{M,p}$ from the sets $\mathcal{W}_M$, where $1 \leq M \leq M_t$, *e.g.* in (2.1). The optimal multimode antenna selection is obtained from the maximal achievable data rate.

However, the computational complexity is dominated by the number of transmit antennas in $O(\sqrt{M_t^{M_t}})$. As the number of transmit antenna is increased, the complexity is then prohibited. To overcome the complexity issue, we propose a simplified algorithm in the following section.

3.2 Proposed multimode antenna selection

3.2.1 Greedy selection algorithm

Instead of using exhaustive search in previous section which demands a huge computations when M_t is large, we propose a suboptimal greedy search strategy which has lower computational complexity than exhaustive search. The algorithm is described in Algorithm 1. The procedure of Algorithm 1 is explained as follows: In Step 1, $\mathcal{S}$ and $\mathbf{H}_{\mathcal{S}}$ are define as the indices set of transmit antennas and the channel matrix that contains transmit antennas in set $\mathcal{S}$ respectively. The sum rate for mode M_t is obtained in Step 2. In Step 4 , we remove one antenna from present antennas, and corresponding sum rates are calculated to obtain the maximal sum rate for mode M. In Steps 5-7, if the maximal data rate for mode M is larger than mode $(M+1)$ is, then the set $\mathcal{P}$ is updated to $\mathcal{S}_m$. In Step 8, we remove one antenna from present antennas according to the result in Step 4 that leads to maximal sum rate in mode M, and $\mathcal{S}$ can be used to the next iteration. Steps 3-9 are repeated until the number of elements in $\mathcal{S}$ is less than two. The transmit antenna set $\mathcal{P}$ is determined in Step 10.

Algorithm 1: Proposed greedy multimode antenna selection to maximize data rate.

1: Define the transmit antenna indices set $\mathcal{S} = \{1, 2, \cdots, M_t\}$.
2: Obtain $b(\mathcal{S})$ which is the sum rate for mode $M = M_t$ by (3.2).
3: **while** $|\mathcal{S}| \geq 2$ **do**
4: $\quad m = \arg\max_r b(\mathcal{S}_r)$, where $\mathcal{S}_r = \mathcal{S} - \{r\}$, $r \in \mathcal{S}$.
5: $\quad$ **if** $b(\mathcal{S}_m) > b(\mathcal{S})$ **then**
6: $\quad\quad \mathcal{P} = \mathcal{S}_m$
7: $\quad$ **end if**
8: $\quad \mathcal{S} = \mathcal{S}_m$
9: **end while**
10: The determined transmit antenna set is $\mathcal{P}$.

3.2.2 Simplified greedy algorithm

For exhaustive search, the matrix inversion in (3.7) should be calculated every iteration for different transmit antenna sets. Instead of large computational complexity due to matrix inversion in all iterations, we can reduce the computations of matrix inversion. To solve (3.12) using greedy algorithm, (3.7) must be calculated for obtaining (3.8). From (2.3), the equivalent channel is a submatrix of $\mathbf{H}$ obtained by extracting M columns from $\mathbf{H}$. Therefore, the relationship among all the matrix inversions in (3.7) for different transmit antenna sets can be applied to simplify the computational complexity.

Lemma. 1. Given an $m \times m$ nonsingular matrix $\mathbf{A}$ that can be partitioned as follow:

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{bmatrix}, \tag{3.14}$$

where $\mathbf{A}_{11}, \mathbf{A}_{12}, \mathbf{A}_{21}$, and $\mathbf{A}_{22}$ are submatrices of $\mathbf{A}$; $\mathbf{A}_{11}$ and $\mathbf{A}_{22}$ are nonsingular matrices. We write $\mathbf{B} = \mathbf{A}^{-1}$ for notational convenience

$$\mathbf{B} = \begin{bmatrix} \mathbf{B}_{11} & \mathbf{B}_{12} \\ \mathbf{B}_{21} & \mathbf{B}_{22} \end{bmatrix},$$

where the submatrices of $\mathbf{B}$ are the same sizes as the corresponding submatrices in $\mathbf{A}$. If $\mathbf{A}_{11} - \mathbf{A}_{12}\mathbf{A}_{22}^{-1}\mathbf{A}_{21}$ and $\mathbf{A}_{22} - \mathbf{A}_{21}\mathbf{A}_{11}^{-1}\mathbf{A}_{12}$ are nonsingular, we have [20]

$$\begin{aligned} \mathbf{B}_{11} &= \left(\mathbf{A}_{11} - \mathbf{A}_{12}\mathbf{A}_{22}^{-1}\mathbf{A}_{21}\right)^{-1}, \\ \mathbf{B}_{22} &= \left(\mathbf{A}_{22} - \mathbf{A}_{21}\mathbf{A}_{11}^{-1}\mathbf{A}_{12}\right)^{-1}. \end{aligned} \tag{3.15}$$

■

Because of the equivalent channel in (2.3) is a submatrix of $\mathbf{H}$, the matrix inversion $\left[\mathbf{H}_{eq}^H\mathbf{H}_{eq}\right]^{-1}$ is a submatrix of $\left[\mathbf{H}^H\mathbf{H}\right]^{-1}$ and an $M \times M$ nonsingular matrix $(M \leq M_t)$ in (3.7). Therefore, we can simplify the matrix inversion computation by using lemma 1.

Theorem. 1. Consider an $M_r \times M_t$ channel $\mathbf{H}$ $(M_r = M_t)$ in (3.7)

$$\left[\mathbf{H}^H\mathbf{H}\right]^{-1} = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{bmatrix} \text{ and } \mathbf{H}^H\mathbf{H} = \begin{bmatrix} \mathbf{B}_{11} & \mathbf{B}_{12} \\ \mathbf{B}_{21} & \mathbf{B}_{22} \end{bmatrix}. \tag{3.16}$$

If the number of selected transmit antennas is m_1 $(m_1 \leq M_t)$, then $\mathbf{H}_{eq}$ is an $M_r \times m_1$ matrix and $\mathbf{H}_{eq}^H\mathbf{H}_{eq}$ is an $m_1 \times m_1$ submatrix of $\mathbf{H}^H\mathbf{H}$. The inverse of $\mathbf{H}_{eq}^H\mathbf{H}_{eq}$ can be obtained from (3.15) and (3.16) given by

$$\left[\mathbf{H}_{eq}^H\mathbf{H}_{eq}\right]^{-1} = \mathbf{B}_{11}^{-1} = \left(\mathbf{A}_{11} - \mathbf{A}_{12}\mathbf{A}_{22}^{-1}\mathbf{A}_{21}\right). \tag{3.17}$$

Therefore, we can obtain $\left[\mathbf{H}_{eq}^H\mathbf{H}_{eq}\right]^{-1}$ by the submatrices of $\left[\mathbf{H}^H\mathbf{H}\right]^{-1}$.

■

Because of matrix inversion demands larger computations especially for M_t is large, we proposed Algorithm 2 based on Theorem 1 and Algorithm 1 for simplifying matrix inversion and exhaustive search respectively. The procedure of Algorithm 2 is similar to Algorithm 1, except for the matrix inversion computation. In Step 1, we define $\mathbf{H}_{\mathcal{S}}$ as the channel matrix that contains transmit antennas in set $\mathcal{S}$, and let $\mathbf{A}$ be $\left(\mathbf{H}_{\mathcal{S}}^H \mathbf{H}_{\mathcal{S}}\right)^{-1}$. Note that $\mathbf{A}$ can be partitioned by Theorem 1:

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_r & \mathbf{a}_r \\ \mathbf{a}_r^H & \alpha_r \end{bmatrix}. \tag{3.18}$$

Because we remove an antenna at a time, α_r is a scalar and $\mathbf{a}_r$ is a column vector. $\mathbf{A}_r$ is an $M \times M$ submatrix of $\mathbf{A}$. In Step 4, the matrix $\mathbf{B}_r$ which is equal to $\left[\mathbf{H}_{eq}^H \mathbf{H}_{eq}\right]^{-1}$ in (3.8) can be obtained by the submatrices of $\mathbf{A}$ without computing matrix inversion. In Step 5, the maximal bit rate for mode M can be obtained from $\mathbf{B}_r$ in Step 4. In Step 9, $\mathbf{A}$ is updated to $\mathbf{A}_m$ for next iteration, and the subscript m is obtained from Step 5.

Algorithm 2: Proposed multimode antenna selection to maximize data rate.

1: Define the transmit antenna indices set $\mathcal{S} = \{1, 2, \cdots, M_t\}$,
   $\mathbf{H}_{\mathcal{S}} = [\mathbf{h}_1 \ \mathbf{h}_2 \cdots \mathbf{h}_{M_t}]$, and $\mathbf{A} = \left(\mathbf{H}_{\mathcal{S}}^H \mathbf{H}_{\mathcal{S}}\right)^{-1}$.
2: Obtain $b(\mathcal{S})$ which is the sum rate for mode $M = M_t$ by (3.2).
3: **while** $|\mathcal{S}| \geq 2$ **do**
4:     $\mathbf{B}_r = \mathbf{A}_r - \frac{1}{\alpha_r}\mathbf{a}_r \mathbf{a}_r^H$, where $\mathcal{S}_r = \mathcal{S} - \{r\}$, $r \in \mathcal{S}$.
5:     $m = \arg\max_r b(\mathcal{S}_r) = \arg\max_r \sum_{i=1}^{M} \log_2\left(1 + \frac{\gamma_0}{\Gamma \mathbf{B}_r}\right)$
6:     **if** $b(\mathcal{S}_m) > b(\mathcal{S})$ **then**
7:         $\mathcal{P} = \mathcal{S}_m$
8:     **end if**
9:     $\mathcal{S} = \mathcal{S}_m$, and $\mathbf{A} = \mathbf{A}_m$.
10: **end while**
11: The determined transmit antenna set is $\mathcal{P}$.

Note that in Algorithm 2, only an $M_t \times M_t$ matrix inversion is needed for initialization. As a result, the computational complexity is dramatically reduced due to the remove of one antenna at a time.

3.3 Bit allocation for proposed selections with fixed bit budget

In previous section, we proposed a suboptimal solution to maximize achievable data rate. In practice, we have integer bit budget for the data rate, thus bit allocation is needed. Therefore, we propose a bit allocation scheme for a given bit budget. The proposed bit allocation aims to equalize the SER for individual subchannel. It turns out the equalized SER of each subchannel leads to a significant VSER improvement. The VSER is the probability that at least one subchannel is in error. Using the proposed transmit antenna set $\mathcal{P}$ obtained in Algorithm 2, the bits are allocated to subchannels by water-filling algorithm [15] under a fixed bit budget. Instead of allocating equal number of bits to all the subchannels in [10], the bits can be allocated dynamically to subchannels in water-filling algorithm. The water-filling algorithm is described as follow:

Algorithm 3: The water-filling algorithm for optimal bit allocation

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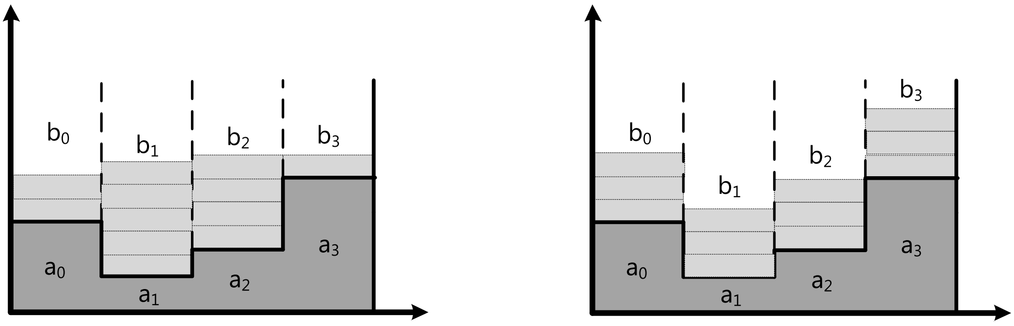
1: Define $|\mathcal{P}|$ in Algorithm 2. Let a_i be calculated from (3.11), and R be the bit budget.
2: **for** $k = 1 : R$ **do**
3: $\quad j = \arg\min_{1 \leq i \leq |\mathcal{P}|} a_i$
4: $\quad b_j \leftarrow b_j + 1$
5: $\quad a_j \leftarrow a_j + 1$
6: **end for**
7: The set b_j for $1 \leq i \leq M$ is the optimal bit allocation by water-filling.

The procedure of water-filling for optimal bit allocation is described as follows: Step 1 contains definitions, $|\mathcal{P}|$ is the determined mode, the height of floor a_i is obtained from (3.11), and a pre-define bit budget R. In Step 3, the index of the subchannel which has the lowest floor height is obtained as j. Then in Step 4, the j-th subchannel is allocated one bit. After adding one bit to the j-th subchannel, the height of floor a_j is added one bit in Step 5. Steps 2-6 are repeated until the number of allocated bits is equal to the total bit budget.

Fig. 3.2(a) and Fig. 3.2(b) are examples for water-filling bit allocation and



(a) (b)

Figure 3.2: (a) Water filling interpretation of optimal bit allocation with $R = 12$; (b) Equal bit allocation with $R = 12$.

equal bit allocation with $R = 12$ respectively. The power of each substream is equalized to $1/M$ no matter how the bits are allocated. From (3.9), we can see that the water level D which is related to the substream power is $(a_i + b_i)$. In Fig. 3.2(a), b_i is related to the height of floor a_i: by Algorithm 2, $(a_i + b_i)$ are nearly the same for all the substreams, which leads nearly the same SER for all substreams due to the assumption of equal substream power. However in Fig. 3.2(b), every substream has the same allocated bits and it results in the different water levels, *e.g.* the fourth substream has much higher floor than that of the second substream. Thus the water level of the fourth subchannel is much higher than the second subchannel. The highest water level of fourth subchannel leads to the highest SER among all the substreams and this degrades VSER.

The advantage of water-filling algorithm described above is that the better the subchannel is, the more the bits can be allocated to this subchannel. The tradeoff is that a bit allocation table is required.

3.4 Complexity analysis

In this section, we analyze and compare the computational complexity of exhaustive search and Algorithm 2. We use the number of multiplications to qualify the complexity of an algorithm. Suppose $\mathbf{X}$ and $\mathbf{Y}$ are $m \times n$ matrices, $\mathbf{Z}$ is an $n \times p$ matrix, and $\mathbf{W}$ is a $q \times q$ nonsingular matrix. The complexity for matrix multiplication and matrix inverse are shown in Table 3.1.

Table 3.1: Complexity orders of matrix operations [21].

	Operation	Complexity order
Matrix multiplication	$\mathbf{X} \cdot \mathbf{Z}$	$O(mnp)$
Matrix inverse	$\mathbf{W}^{-1}$	$O(q^3)$

3.4.1 Exhaustive search

For exhaustive search mentioned in Sec. 3.1, the number of iterations for searching all the possible transmit antenna sets is

$$\begin{aligned}\sum_{k=1}^{M_t} C_k^{M_t} &= \sum_{k=1}^{M_t} \frac{M_t!}{k!(M_t-k)!} \\ &= \sum_{k=1}^{M_t} \frac{M_t(M_t-1)\cdots(M_t-k+1)}{k!} \\ &= \sum_{k=1}^{M_t} O\left(\frac{M_t^k}{k!}\right) \qquad (3.19)\end{aligned}$$

The complexity of the optimal solution of (3.12) from (3.8) is dominated by matrix inversion and multiplication in $\left[\mathbf{H}_{eq}^H \mathbf{H}_{eq}\right]^{-1}$. In order to compare the complexity of exhaustive search with the proposed algorithm, the number of iterations in (3.19) is multiplied by the complexity order of matrix inversion and multiplication from Table 3.1, *i.e.*

$$\sum_{k=1}^{M_t} O\left(\frac{M_t^k}{k!}\right) O(k^3) = \sum_{k=1}^{M_t} O\left(\frac{M_t^k}{k!} \cdot k^3\right) \qquad (3.20)$$

Note that for matrix inversion and multiplication, the order are both $O(k^3)$ in every iteration. Hence the total complexity is $O(k^3)$ as well.

3.4.2 Algorithm 2

For Algorithm 2 mentioned in Sec. 3.2.2, the number of iterations for searching the possible transmit antenna sets by removing one antenna at a time is given by

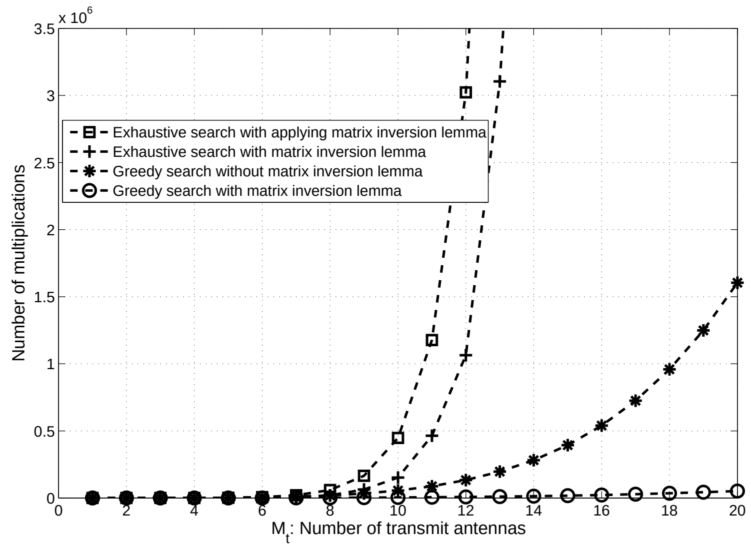
$$\sum_{k=1}^{M_t} C_1^k = \sum_{k=1}^{M_t} k \tag{3.21}$$

The computational complexity of Algorithm 2 is also related to the simplification of matrix inversion and matrix multiplication in (3.15). Note that the matrix inversion in (3.15) is reduced to a scalar division because we remove one antenna at a time. Compare to (3.20), the computational complexity of matrix computations for (3.21) is given by

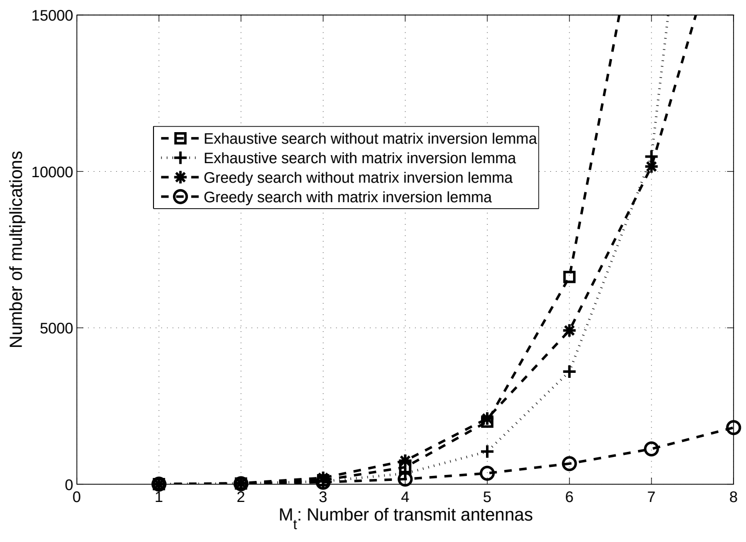
$$\begin{aligned} & \sum_{k=2}^{M_t} k[O(k^2)] + O(M_t^3) \\ = & \sum_{k=2}^{M_t} O(k^3) + O(M_t^3). \end{aligned} \tag{3.22}$$

Note that each iteration needs $O(k^2)$ multiplications, observed from (3.17). Hence the first term in line 1 of (3.22) is $\sum_{k=2}^{M_t} k[O(k^2)]$. The last term $O(M_t^3)$ in line 1 of (3.22) is because in Algorithm 2, we have to calculate the inversion an of $M_t \times M_t$ matrix at first.

Fig. 3.3 shows the comparison of computational complexity between exhaustive search in (3.20) and the proposed algorithm in (3.22). Besides, the computational complexity of greedy search without matrix inverse lemma and exhaustive search with matrix inverse lemma are also plotted. The computational complexity of exhaustive search with matrix inverse lemma is lower than optimal selection (exhaustive search without matrix inverse lemma) but higher than greedy search without matrix inverse lemma. Furthermore, the computational complexity of Algorithm 2 that greedy search and matrix inverse lemma are both applied is dramatically reduced compared to optimal selection. Fig. 3.4 is zoom up of (a) which shows the difference between with/without matrix inverse lemma: when M_t is large, the difference is larger; *e.g.* $M_t = 20$, the difference of the number of multiplications is about 10^9.



(a)



(b)

Figure 3.3: (a) Comparison of computational complexity for optimal selection and proposed selection; (b) Zoom up of (a).

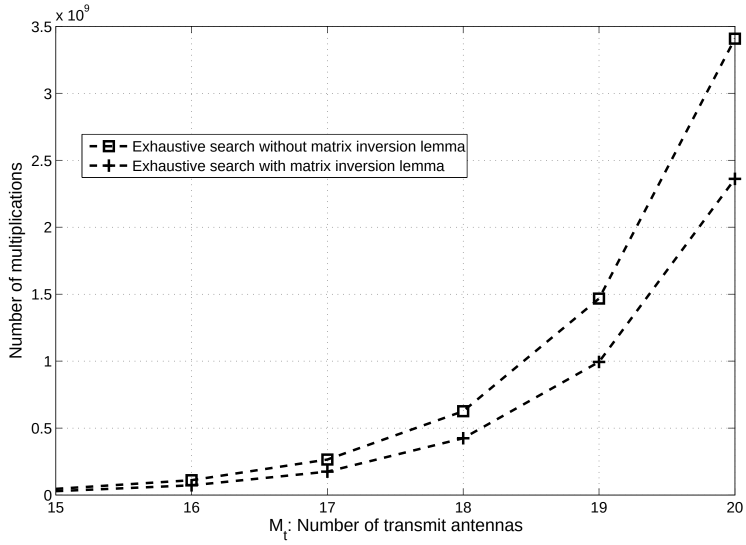


Figure 3.4: Zoom up of Fig. 3.3(a) with/without matrix inversion lemma.

Chapter 4

Multimode antenna selection to minimize symbol error rate

4.1 Problem formulation and optimal solution

According to the system model in Chapter 2, the input R bits are demultiplexed into M streams and each substream has equal number of bits (R/M). With minimum distance d_{min} between adjacent constellation points, the average power of a $2b$-bit QAM constellation is given by [15]

$$\mathcal{E}_{avg,QAM} = \frac{(2^{2b}-1)d_{min}^2}{6}. \tag{4.1}$$

To adjust the power of each individual substream, d_{min} can be chosen appropriately. The power of each substream is $(1/M)$. Thus, the d_{min} for desired substream power can be obtained by arranging (4.1):

$$d_{min} = \sqrt{\frac{6}{M(2^{2b}-1)}}. \tag{4.2}$$

Therefore, the SER of the i-th substream for $2b$-bit QAM symbol in (3.1) can be rewritten as:

$$P_{e_i} = 4\left(1 - \frac{1}{2^{R/2M}}\right) Q\left(\sqrt{\frac{d_{min}^2}{2}\frac{1}{\left[\mathbf{H}_{eq}^H\mathbf{H}_{eq}\right]_{i,i}^{-1} N_0}}\right). \tag{4.3}$$

Although (4.3) is obtained for square constellations, for large $2^{R/M}$ it can be applied for general QAM constellations which are either in rectangular shape (even number of bits) or in cross shape (odd number of bits). The above equation shows that the SER for each substream is related to $\mathbf{H}_{eq}$ and the minimum distance d_{min}. Our goal is to reduce the average SER. Under equal bit allocation and ZF constraint, the problem of minimizing average SER can be described as

$$\bar{P}_{e,min} = \min_{1 \leq M \leq M_t, 1 \leq p \leq \binom{M_t}{M}} \frac{1}{M} \sum_{i=1}^{M} P_{e_i}$$
$$\text{s.t.} \sum_{i=1}^{M} b_i = R \text{ and } \mathbf{G} = (\mathbf{H}\mathbf{W}_{M^*,p^*})^{\dagger} \tag{4.4}$$

Note that the average SER is a function of M and p. The strategy for obtaining optimal multimode antenna selection is exhaustive search mentioned in Sec. 3.1.

Exhaustive search. For the optimal multimode antenna selection of minimizing average SER in (4.4), we must calculate all the average SER under the ZF constraint and fixed bit budget for all possible equivalent channels. The equivalent channels in (2.3) are calculated by all possible symbol mappers $\mathbf{W}_{M,p}$ from the sets $\mathcal{W}_M$, where $1 \leq M \leq M_t$, *e.g.* in (2.1). The optimal multimode antenna selection is obtained from the minimal average SER.

As in Chapter 3, the computational complexity grows exponentially with the number of transmit antennas, *i.e.* $O(\sqrt{M_t^{M_t}})$. As the number of transmit antenna is increased, the complexity is then prohibited. To overcome the complexity issue, we propose a simplified algorithm in the following section.

4.2 Proposed multimode antenna selection

The minimization for the average SER is related to the product of substream SNR and d_{min}, observed from (4.3). The proposed multimode antenna in ZF receiver for minimizing SER is described in Algorithm 4. The procedure of Algorithm 4 is similar to Algorithm 2. In Step 1, we define $\mathbf{H}_{\mathcal{S}}$ as the channel matrix that contains transmit antennas in set $\mathcal{S}$, and $\mathbf{A}$ is $\left(\mathbf{H}_{\mathcal{S}}^H \mathbf{H}_{\mathcal{S}}\right)^{-1}$. Note that the number of substreams should be chosen appropriately for the reason that the number of bits (R/M) allocated to each substream is an integer. If (R/M) is not an integer, then the number of removed antennas v is added one for next iteration. For our case, v is either one or two. If (R/M) is an integer, we set $v = 1$ in Step 11 for next iteration; otherwise, we set $v = 2$ in Step 13. For example, if $M = 4$ and $v = 2$, then the set for removed antenna indices is given by

$$\{(1,2),(1,3),(1,4),(2,3),(2,4),(3,4)\}$$

In Step 6, the matrix $\mathbf{B_r}$ which is equal to $\left[\mathbf{H}_{eq}^H \mathbf{H}_{eq}\right]^{-1}$ in (4.3) can be obtained by

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Algorithm 4: Proposed multimode antenna selection to minimize SER.

1: Define the transmit antenna indices set $\mathcal{S} = \{1, 2, \cdots, M_t\}$, $\mathbf{H}_\mathcal{S} = [\mathbf{h}_1 \ \mathbf{h}_2 \cdots \mathbf{h}_{M_t}]$, and $\mathbf{A} = \left(\mathbf{H}_\mathcal{S}^H \mathbf{H}_\mathcal{S}\right)^{-1}$.
2: Obtain $P_e(\mathcal{S})$ which is the sum rate for mode $M = M_t$ by (3.2).
3: **while** $|\mathcal{S}| \geq 2$ **do**
4: $\quad M = |\mathcal{S}| - 1$
5: $\quad$ **if** $(R/M) \in N$ **then**
6: $\quad\quad \mathbf{B_r} = \mathbf{A_r} - \mathbf{A}_t \mathbf{A}_p^{-1} \mathbf{A}_t^H$, where

$$\begin{cases} \text{for } v = 1, & \mathcal{S}_\mathbf{r} = \mathcal{S} - \{\mathbf{r}\}, \quad \mathbf{r} \in \mathcal{S},\ |\mathbf{r}| = 1. \\ \text{for } v = 2, & \mathcal{S}_\mathbf{r} = \mathcal{S} - \{\mathbf{r}\}, \quad \mathbf{r} = (m,n) \in \mathcal{S}, \text{ and } m \neq n. \end{cases}$$

7: $\quad\quad \mathbf{m} = \arg\min_\mathbf{r} P_e(\mathcal{S}_\mathbf{r}) = \arg\min_\mathbf{r} \frac{1}{M}\sum_{i=1}^{M} 4\left(1 - \frac{1}{2^{R/2M}}\right) Q\left(\sqrt{\frac{d_{min}^2}{2}\frac{1}{\mathbf{B_r} N_0}}\right)$
8: $\quad\quad$ **if** $P_e(\mathcal{S}_\mathbf{m}) > P_e(\mathcal{S})$ **then**
9: $\quad\quad\quad \mathcal{P} = \mathcal{S}_\mathbf{m}$
10: $\quad\quad$ **end if**
11: $\quad\quad \mathcal{S} = \mathcal{S}_\mathbf{m}$, $\mathbf{A} = \mathbf{A}_\mathbf{m}$, and $v = 1$.
12: $\quad$ **else**
13: $\quad\quad v = 2$.
14: $\quad$ **end if**
15: **end while**
16: The determined transmit antenna set is $\mathcal{P}$.

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the submatrices of $\mathbf{A}$ without computing matrix inversion. Note that $\mathbf{r}$ represents the removed elements: for $v = 1$, $|\mathbf{r}| = 1$; for $v = 2$, $|\mathbf{r}| = 2$ and the number of possible sets is $\binom{M}{2}$. In Step 7, the minimal SER for mode M can be obtained from $\mathbf{B_r}$ in Step 6. Note that the transmit antenna set $\mathcal{S}_\mathbf{r}$ is varied by v. According to Theorem 1, if we remove one antenna at a time ($v = 1$), then $\mathbf{A}$ in Algorithm 4 can be partitioned as in (3.18); however if we remove two antennas at a time

($v = 2$), then $\mathbf{A}$ can be partitioned as:

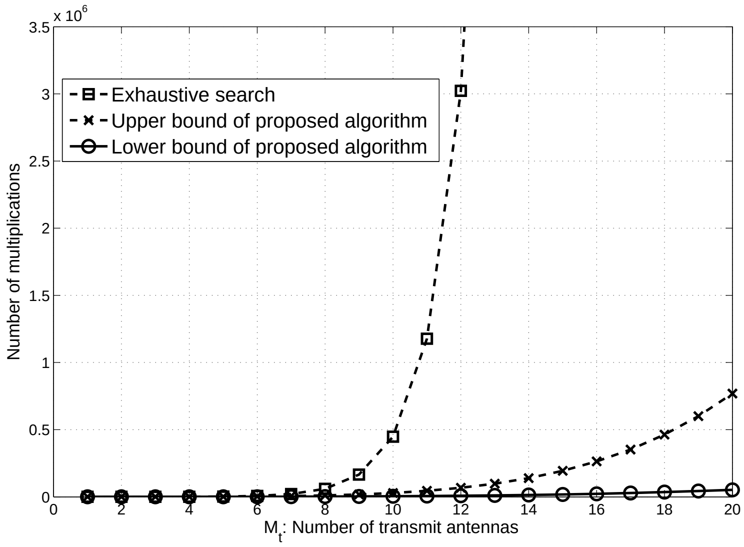
$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_{\mathbf{r}} & \mathbf{A}_t \\ \mathbf{A}_t^H & \mathbf{A}_p \end{bmatrix}, \quad (4.5)$$

where $\mathbf{A}_{\mathbf{r}}$ is an $(M-2) \times (M-2)$ submatrix of $\mathbf{A}$, and $\mathbf{A}_p$ is a 2×2 submatrix of $\mathbf{A}$.

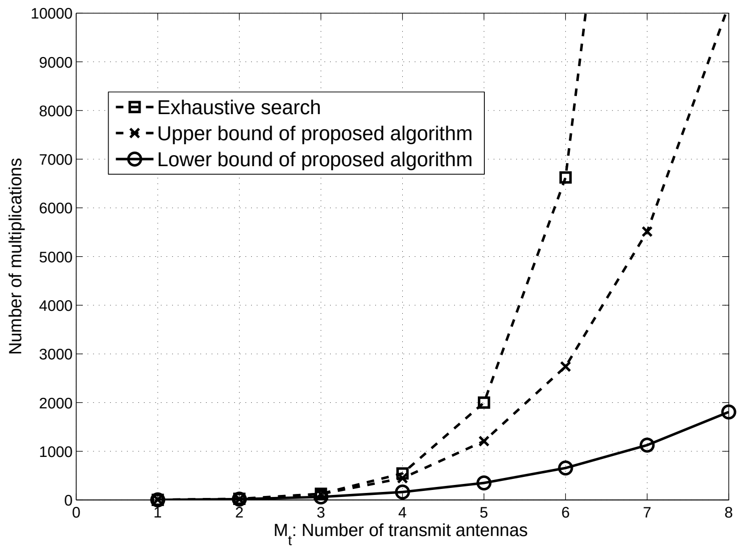
Note that in (4.5), if we remove one antenna at a time, then the matrix inversion can be reduced to a scalar division as mentioned in (3.18). However, not all the modes can be applied since (R/M) should be an integer. The number of removed antennas may be more than one at a time. For instance, an 8×8 MIMO system with $R = 24$, the available modes are $\{1, 2, 4, 6, 8\}$. To obtain the optimal selection for $M = 6$, we have to remove two antennas from eight transmit antennas. In this case, $\mathbf{A}_{\mathbf{r}}^{-1}$ in (4.5) leads to an 2×2 matrix inversion. Although sometimes 2×2 matrix inversion is needed, the computational complexity is still lower than optimal selection, since a 2×2 matrix inversion only require two multiplications and one addition.

4.3 Complexity Analysis

Fig. 4.1(a) illustrates the computational complexity for exhaustive search and proposed algorithm. For exhaustive search , the computational complexity is the same as (3.20). For the proposed algorithm, there are upper bound and lower bound of computational complexity because the number of removed antennas may be one or two at a time. The lower bound is the same as that in Fig. 3.3 which represents removing one antenna at a time. The upper bound represents removing two antennas at a time which needs a 2×2 matrix inversion. The computational complexity of the proposed algorithm which is in the region between the upper bound and the lower bound is greatly reduced compared to that of exhaustive search.



(a)



(b)

Figure 4.1: (a) Comparison of computational complexity for exhaustive search and Algorithm 4; (b) Zoom up of (a).

Chapter 5

Simulation Results

In this chapter, we show the Monte Carlo simulation results of the proposed algorithm and compare them with the optimal solutions in maximum achievable data rate and minimum SER mentioned in Chapters 3 and 4 respectively. QAM is applied and the channel model is i.i.dcomplex Gaussian with distribution $CN(0, 1)$; 10000 MIMO channel realizations were conducted in the simulations.

Experiment 1: In this experiment, we simulated maximum achievable data rate with and without multimode antenna selection for $M_r \times M_t$ ($M_r = M_t$) MIMO systems with $M_t = \{3, \cdots, 8\}$ in Figs. 5.1-5.6. The assigned SER for each substream are 10^{-4}, 10^{-5}, and 10^{-6}. The SNR E/N_0 is total power of a symbol vector over noise power.

The optimal multimode antenna selection for achievable data rate maximization by exhaustive search is compared to Algorithm 2 and without transmit antenna selection. We have several interesting observations from the figures: first the performance with multimode antenna selection greatly outperform that without antenna selection. The improvement becomes more pronounced when M_t is large. For instance, in Fig. 5.6 for an 8×8 MIMO system with SER$= 10^{-4}$, the achievable data rate difference between with and without multimode antenna selection is able to achieve 12 bits. Moreover, the performance with the proposed search in Algorithm 2 is very close to that with the optimal exhaustive search observed from all the figures of this experiment. Hence the computational complexity can be greatly reduced using our proposal in Fig. 3.3 with little

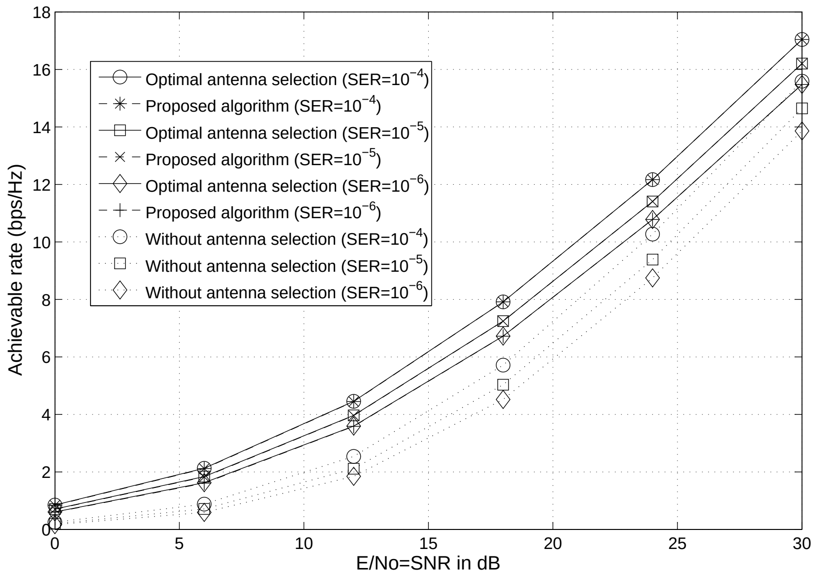


Figure 5.1: Maximization of achievable data rate with antenna selection and without antenna selection for 3×3 MIMO system.

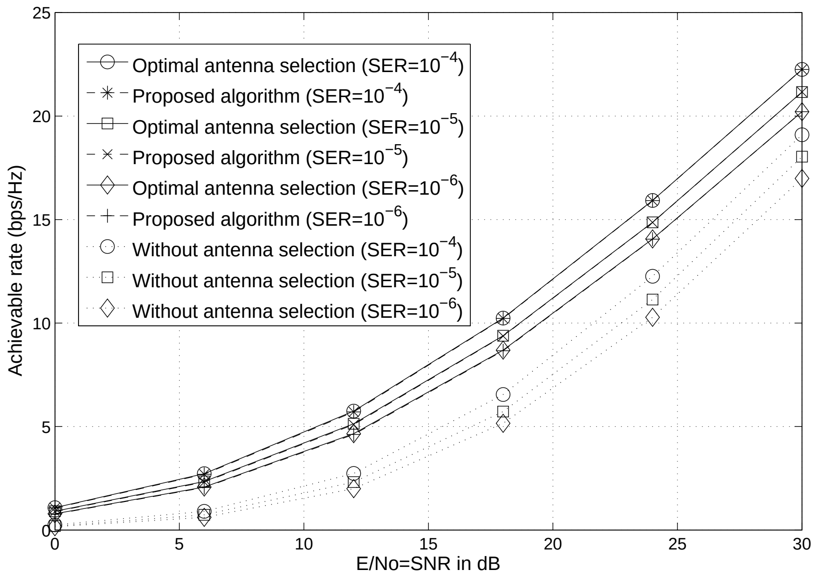


Figure 5.2: Maximization of achievable data rate with antenna selection and without antenna selection for 4×4 MIMO system.

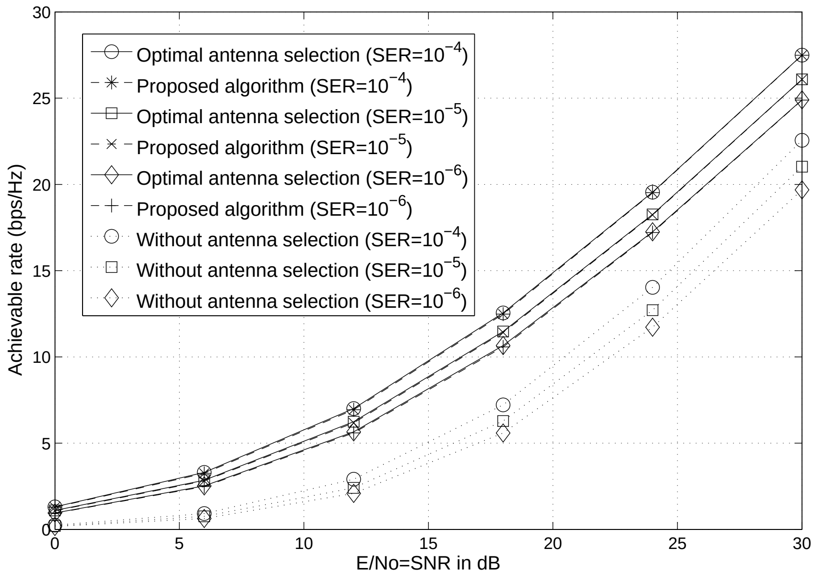


Figure 5.3: Maximization of achievable data rate with antenna selection and without antenna selection for 5×5 MIMO system.

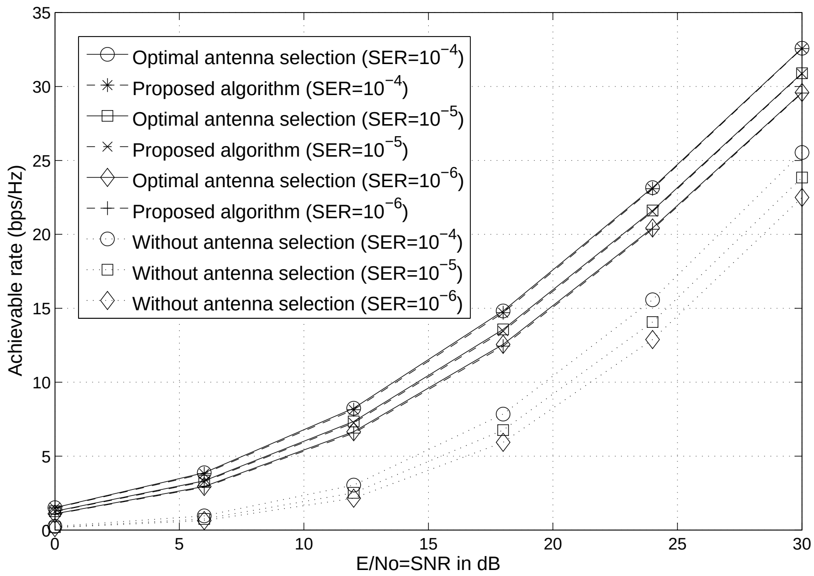


Figure 5.4: Maximization of achievable data rate with antenna selection and without antenna selection for 6×6 MIMO system.

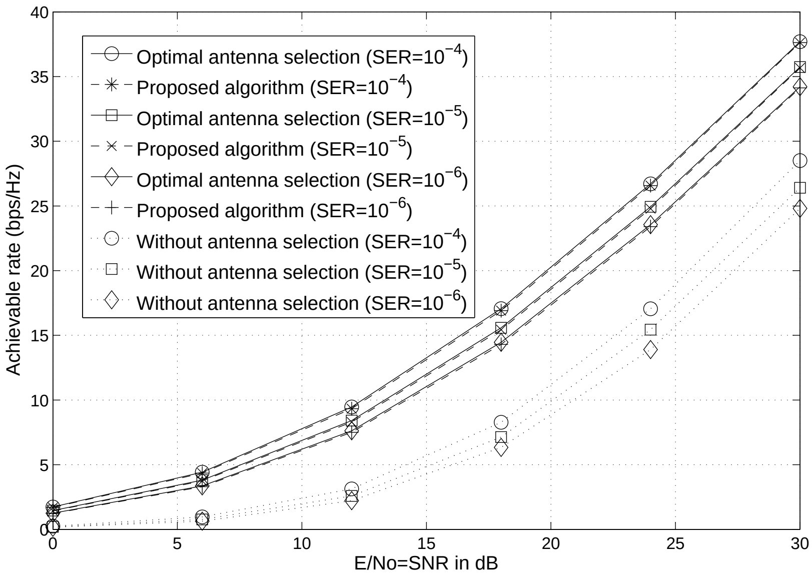


Figure 5.5: Maximization of achievable data rate with antenna selection and without antenna selection for 7×7 MIMO system.

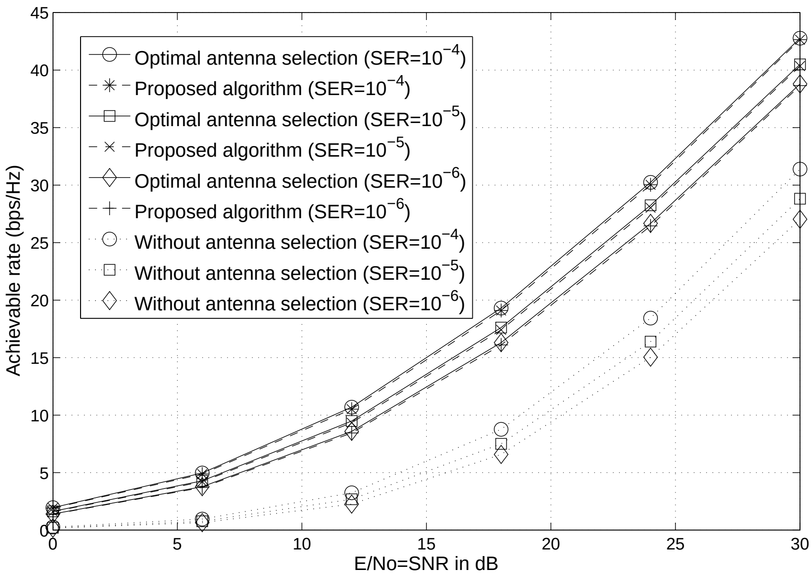


Figure 5.6: Maximization of achievable data rate with antenna selection and without antenna selection for 8×8 MIMO system.

performance loss. Furthermore, when SER is small, the achievable data rate is limited. This is observed from (3.8).

Experiment 2: In this experiment, we simulated the VSER of water-filling bit allocation in Algorithm 3 and based on Algorithm 2 and compare it with that of minimizing the NNUB Criterion 4 in [10]. This experiment contains 3×3 and 4×4 MIMO system and using different number of bit budget to see the performance improvement. The x-axies of these plots use SNR per bit, *i.e.* $\mathcal{E}_b/N_0 = E_s/(N_0(R/M))$.

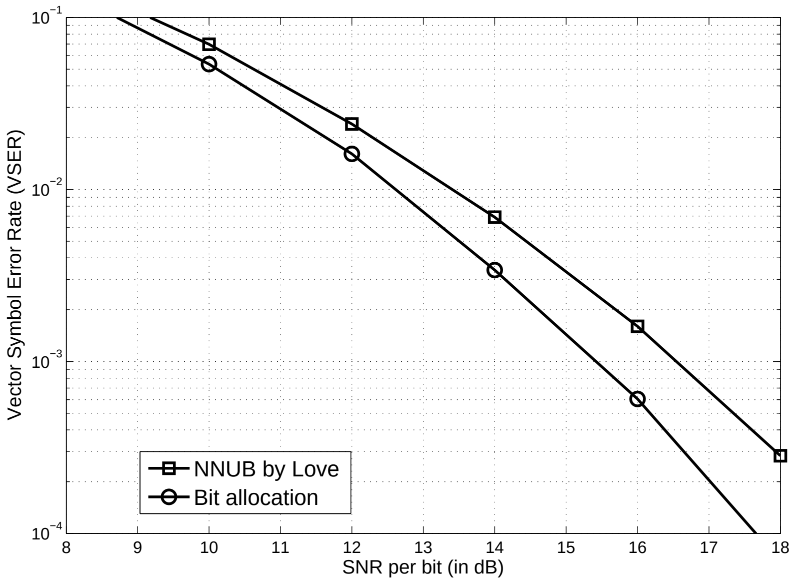


Figure 5.7: Antenna selection with bit allocation and with NNUB for 3×3 MIMO system, $R = 12$.

We have several interesting observations from the figures: first, for the VSER of minimizing NNUB in [10], b_i is equal allocated for all substreams. That is different from our proposed algorithm. The VSER improvement of water-filling bit allocation can be observed from Figs. 5.7-5.10. Moreover, the performance is improved especially when R is increased for the same M_t and M_r. It is can be observed from Fig. 5.8-5.10, 4×4 MIMO systems with bit budget $R = 8$, $R = 12$, and $R = 16$. For $R = 16$, the performance is observable for high SNR compared with that of Love, and the diversity is increased. Under reasonable bit

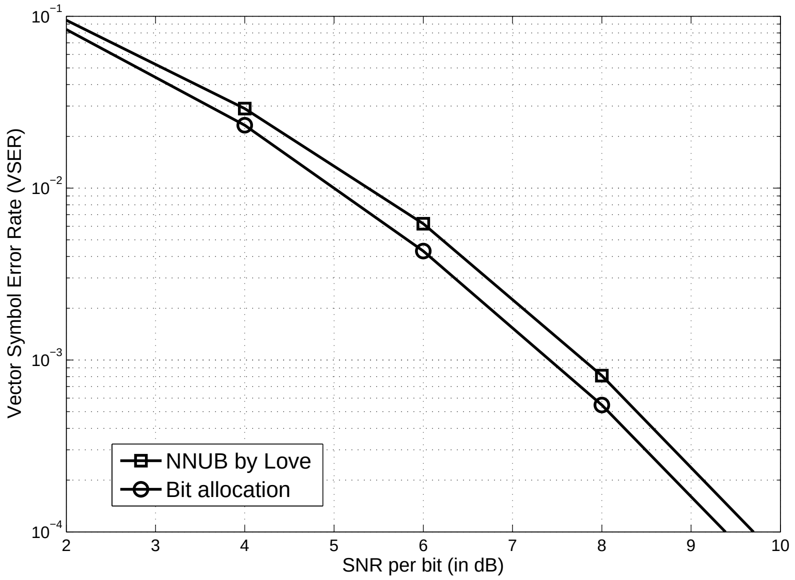


Figure 5.8: Antenna selection with bit allocation and with NNUB for 4×4 MIMO system, $R = 8$.

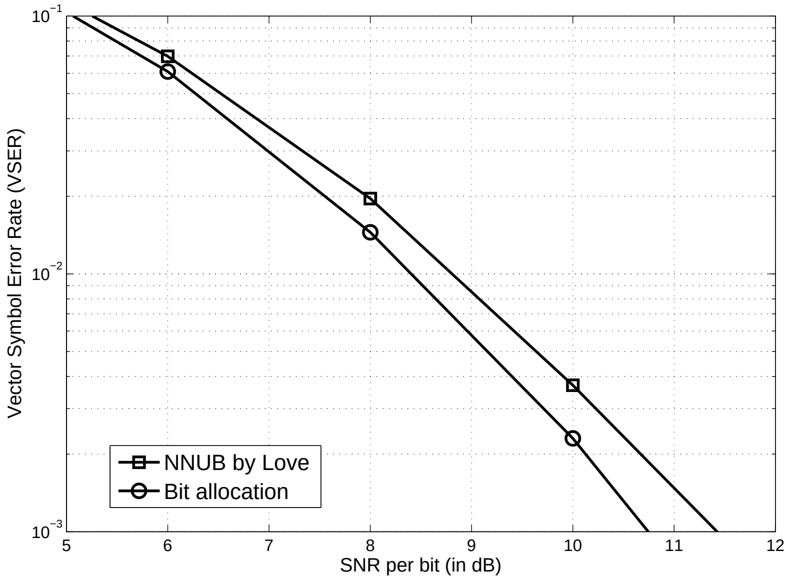


Figure 5.9: Antenna selection with bit allocation and with NNUB for 4×4 MIMO system, $R = 12$.

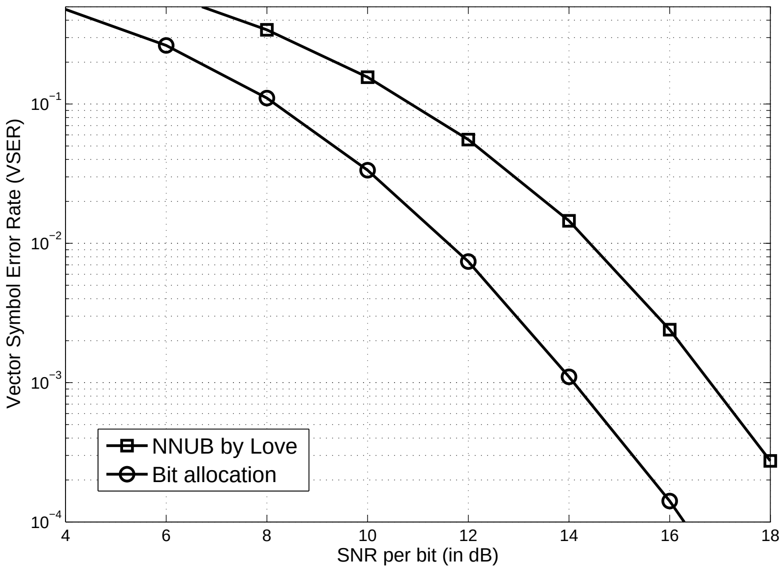


Figure 5.10: Antenna selection with bit allocation and with NNUB for 4×4 MIMO system, $R = 16$.

budget, with more bits budget to be allocated, more bit can be allocated to the substream which has better condition. Furthermore, in Fig. 5.7 for 3×3 MIMO system with $R = 12$, the VSER improvement for bit allocation compared to that of Love is observable but the VSER for both are higher than that in Fig. 5.9 for 4×4 MIMO system with $R = 12$. The constellation size affects the error rate performance.

Experiment 3: In this experiment, the VSER simulations contain the proposed optimal selection mentioned in Sec. 4.1, Algorithm 4 in Sec. 4.2, and Criterion 4 in [10]. This experiment contains 4×4 and 6×6 MIMO systems with different bit budgets. The x-axies of these plots use SNR per bit, *i.e.* $\mathcal{E}_b/N_0 = E_s/(N_0(R/M))$. We have several interesting observations from the figures: first, the VSER of average SER minimization by exhaustive search outperforms that of Love's scheme in high SNR regime observed by Figs 5.12 and 5.13. Moreover, the VSER of Algorithm 4 is close to that of exhaustive search in Fig. 5.11. In Figs. 5.12 and 5.13, the VSER of Algorithm 4 has little performance degradation compared to exhaustive search. Nevertheless, the computational complexity is dramatically reduced by Algorithm 4 illustrated in Fig. 4.1. Furthermore, for the same bit budget, the VSER is reduced when M_t and M_r is large observed by Figs. 5.12 and 5.13.

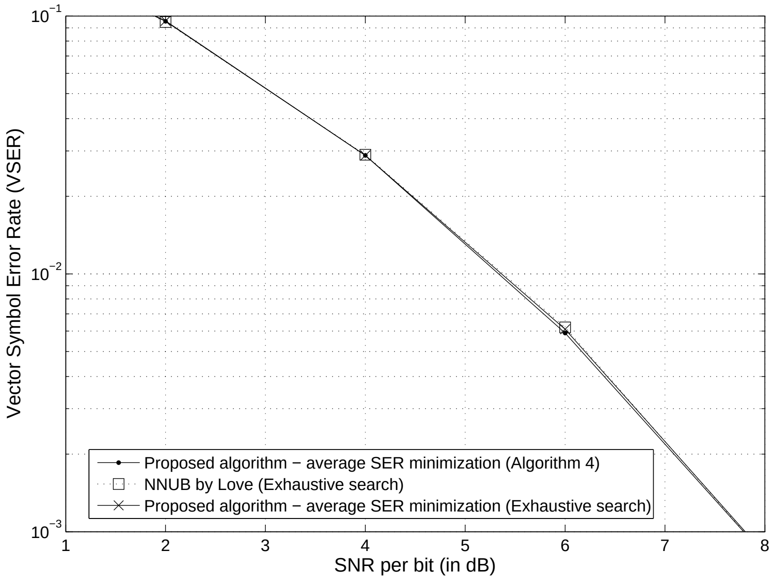


Figure 5.11: Minimization of average symbol error rate with antenna selection for 4×4 MIMO system with $R = 8$ and $M = \{1, 2, 4\}$.

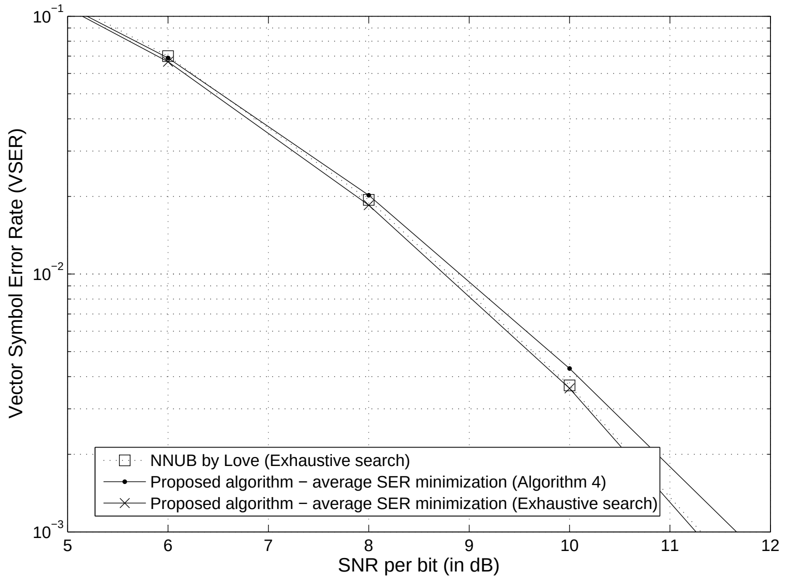


Figure 5.12: Minimization of average symbol error rate with antenna selection for 4×4 MIMO system with $R = 12$ and $M = \{1, 2, 3, 4\}$.

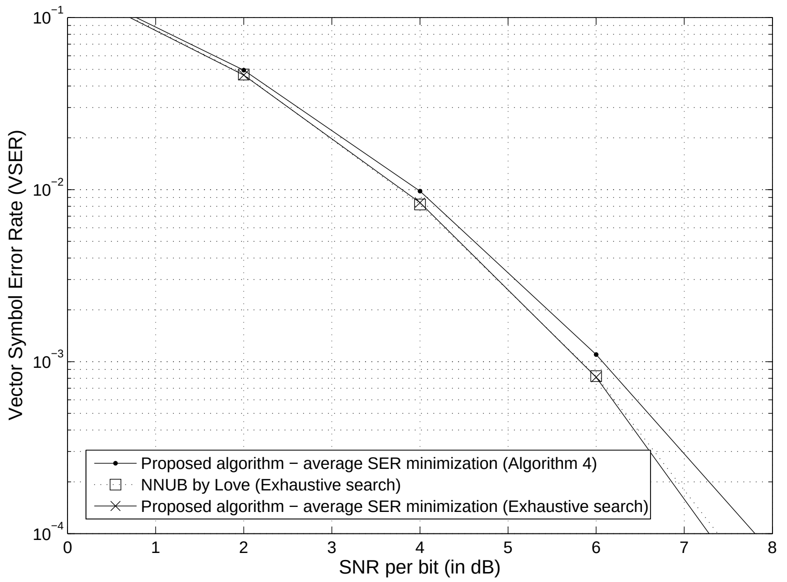


Figure 5.13: Minimization of average symbol error rate with antenna selection for 6×6 MIMO system with $R = 12$ and $M = \{2, 3, 4, 6\}$.

Chapter 6

Conclusion

We have presented a low-complexity multimode antenna selection algorithm for ZF receiver. By greedy search with the simplification of matrix inversion in Theorem 1, the computational complexity is dramatically reduced and with a near-optimal performance in maximizing achievable data rate.

It is also found that the improvement of water-filling bit allocation based on the selection in Algorithm 2. The bit allocation is decided by the relative gains of all substreams. When the bit budget is large enough, the diversity improvement is observable.

In Chapter 4 we proposed multimode antenna selection of minimizing the average of SER which leads to VSER minimization. The relative channel conditions can be more accurate calculated by averaging the performance of substreams. The greedy search with Theorem 1 are also applied to the minimization problem in Algorithm 4.

Fig. 3.3, 3.4 and 4.1 for achievable data rate maximization and average SER minimization shows the computational complexity improvement for the proposed algorithms compared to exhaustive search. By removing one or two antennas at a time, the matrix inversion can be reduced to a scalar division or a 2×2 matrix inversion in (3.17). When M_t is large, the order of computational complexity is grows exponentially with the number of transmit antennas. The proposed algorithms are suitable especially for large number of antennas.

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Appendix

Appendix A: Proof of (3.9)

By rewriting (3.8) under the high bit assumption ($b_i \gg 1$), we can obtain

$$b_i = \log_2 \left(\frac{\mathcal{E}_s}{\Gamma \left[\mathbf{H}_{eq}^H \mathbf{H}_{eq} \right]_{i,i}^{-1} N_0} \right) = \log_2 \mathcal{E}_s - \log_2 \left(\Gamma \left[\mathbf{H}_{eq}^H \mathbf{H}_{eq} \right]_{i,i}^{-1} N_0 \right) \quad \text{(A-1)}$$

$$= \log_2 \mathcal{E}_s - \log_2 \Gamma - \log_2 \left[\mathbf{H}_{eq}^H \mathbf{H}_{eq} \right]_{i,i}^{-1} - \log_2 N_0 \quad \text{(A-2)}$$

$$= \log_2 \mathcal{E}_s - \log_2 N_0 \Gamma - \log_2 \left[\mathbf{H}_{eq}^H \mathbf{H}_{eq} \right]_{i,i}^{-1} \quad \text{(A-3)}$$

For (3.9), the precoder $\mathbf{F}$ is $\mathbf{W}_{M,p}$, and the logarithm of the term $\left[\mathbf{F}^H \mathbf{F} \right]_{i,i}$ is zero for all i due to the diagonal terms always are one for $\mathbf{F} = \mathbf{W}_{M,p}$. The equalizer $\mathbf{G}$ is the pseudo inverse of $\mathbf{H}_{eq}$ given by

$$\mathbf{G} = \mathbf{H}_{eq}^{\dagger} = \left(\mathbf{H}_{eq}^H \mathbf{H}_{eq} \right)^{-1} \mathbf{H}_{eq}^H, \quad \text{(A-4)}$$

the term $\mathbf{G}\mathbf{G}^H$ (3.9) is given by

$$\mathbf{G}\mathbf{G}^H = \left(\mathbf{H}_{eq}^H \mathbf{H}_{eq} \right)^{-1} \mathbf{H}_{eq}^H \left[\left(\mathbf{H}_{eq}^H \mathbf{H}_{eq} \right)^{-1} \mathbf{H}_{eq}^H \right]^H \quad \text{(A-5)}$$

$$= \left(\mathbf{H}_{eq}^H \mathbf{H}_{eq} \right)^{-1} \mathbf{H}_{eq}^H \mathbf{H}_{eq} \left(\mathbf{H}_{eq}^H \mathbf{H}_{eq} \right)^{-H} \quad \text{(A-6)}$$

$$= \left(\mathbf{H}_{eq}^H \mathbf{H}_{eq} \right)^{-H} \quad \text{(A-7)}$$

$$= \left(\mathbf{H}_{eq}^H \mathbf{H}_{eq} \right)^{-1} \quad \text{(A-8)}$$

Therefore, (3.9) for our problem, b_i for $i-th$ substream is given by

$$b_i = D - \log_2 c_i - \log_2 \left[\mathbf{H}_{eq}^H \mathbf{H}_{eq} \right]_{i,i}^{-1}, \quad \text{(A-9)}$$

where D is the logarithm of substream power, and c_i is the product of SNR gap and noise power mentioned in Sec. 3.2. Therefore, for our problem, (A-3) are the same as (A-9).

■