Interaction of optical solitons with a forward Raman pump wave

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The interaction between optical solitons and a forward Raman pump wave in fibers is investigated. It is found that when their relative velocity is small and the soliton pulse width is short, the soliton frequency is shifted by the interaction. Near zero relative velocity the pulse shape is distorted, in addition to having a frequency shift.

The optical soliton has been proposed as an information carrier in fiber communication systems.[1] Because of the fiber loss, the soliton disperses as it propagates along the fiber. Use of periodically bidirectional Raman pumps to compensate for the loss has been suggested.[2,3] Numerical calculations showed that the soliton can preserve its pulse shape for thousands of kilometers with this method. The experiment also showed that the dispersion of the soliton due to loss can be reduced by a backward Raman pump.[4] In previous theoretical research, Raman pump waves were assumed to be either uniformly depleted[2] or undepleted.[3] With the assumption of uniform depletion, the pump wave is assumed to be a cw wave along the fiber. This is valid if the relative group velocity between the pump wave and the soliton is sufficiently large. The relative group velocity between a backward pump wave and the soliton is large enough, but this is not in general true for a forward pump wave. In this Letter we show the effects caused by a small relative group velocity between a forward pump wave and the soliton.

The soliton and a forward pump wave in a fiber satisfy the following coupled dimensionless wave equation[2]:

$$i \frac{\partial q_s}{\partial \xi} + \frac{1}{2} \alpha_s \frac{\partial^2 q_s}{\partial \tau^2} + \eta_s (|q_s|^2 + 2|q_p|^2) q_s$$
$$= -i \Gamma_s q_s + i G |q_p|^2 q_s, \quad \text{(1a)}$$

$$i \left(\frac{\partial q_p}{\partial \xi} - V \frac{\partial q_p}{\partial \tau} \right) - \frac{1}{2} \alpha_p \frac{\partial^2 q_p}{\partial \tau^2} + \eta_p (|q_p|^2 + 2|q_s|^2) q_p$$
$$= -i \Gamma_p q_p - i \frac{\lambda_s}{\lambda_f} G |q_s|^2 q_p, \quad \text{(1b)}$$

where the nonlinear coupling of the soliton (q_s) and the forward pump wave (q_p) are due to the Kerr effect and Raman scattering. The Kerr effect causes the phases of q_s and q_p to be mutually modulated. The variables and the coefficients in the above equations are normalized in the following way:

$$\xi = X^{-1} z, \quad \tau = T^{-1}(t - k_s' z), \quad q_s = Q^{-1} \phi_s,$$
$$q_p = Q^{-1} \phi_p \quad \text{(2)}$$

and

$$\alpha_s = -k_s'' X T^{-2}, \quad \alpha_p = k_p'' X T^{-2},$$
$$\eta_s = \frac{\pi n_2}{\lambda_s} Q^2 X, \quad \eta_p = \frac{\pi n_2}{\lambda_f} Q^2 X,$$
$$\Gamma_s = \gamma_s X, \quad \Gamma_p = \gamma_p X, \quad G = \frac{n_0}{4} \sqrt{\epsilon_0/\mu_0} g X Q^2,$$
$$V = (k_p' - k_s') X T^{-1}. \quad \text{(3)}$$

In Eqs. (1)–(3), ϕ_s and ϕ_p are the electric-field envelopes of the soliton and the forward pump wave; λ_s and λ_p are the wavelengths of the soliton and the pump wave; t and z are the time and propagation distance along the fiber; T, X, and Q are the time, distance, and electric-field scale factors; k_s', k_s'' and k_p', k_p'' are the derivatives of the propagation constant k with respect to the angular frequencies at ω_s and ω_p, respectively, where ω_s and ω_p are the angular frequencies of the soliton and the pump wave, the first derivative is reciprocal group velocity, and the second derivative is proportional to group dispersion; γ_s and γ_p are the loss coefficients of the fiber at λ_s and λ_p; n_2 is the Kerr coefficient and equal to 1.22×10^{-22} (V/m)$^{-2}$ for fused silica; g is the Raman gain coefficient, which is maximum when the frequency difference between the pump wave and the carrier wave of the soliton is 440 cm^{-1}.[5] The maximum gain $g = 9.75 \times 10^{-14}/\lambda_p$ m/W, where λ_p is in micrometers. In the following we take $X = 10$ km, $Q = 4.55 \times 10^5$ V/m, and let T be varied (T is in picoseconds). The given electric-field scale corresponds to 20 mW for a 50-μm^2 fiber cross section.

The soliton wavelength is in the negative-dispersion regime ($k_s'' < 0$). We assume, however, that the pump wave is in the positive-dispersion regime ($k_p'' > 0$). Because the reciprocal group velocity is minimum at zero dispersion, the relative reciprocal group velocity $\Delta k'$ ($= k_p' - k_s'$) may be small. Using the dispersion relation in Ref. 6 for fused silica and a 440-cm^{-1} frequency difference between λ_s and λ_p, we plot $\Delta k'$ in Fig. 1. There, the zero-dispersion wavelength is assumed to be at 1.273 μm, and then $\Delta k' = 0$ at $\lambda_s = 1.30885$ μm. This wavelength is denoted by λ_{s0}.

We first consider the interaction of the soliton and a forward pump wave at $\lambda_s = 1.3$ μm and $\lambda_p = 1.23$ μm.
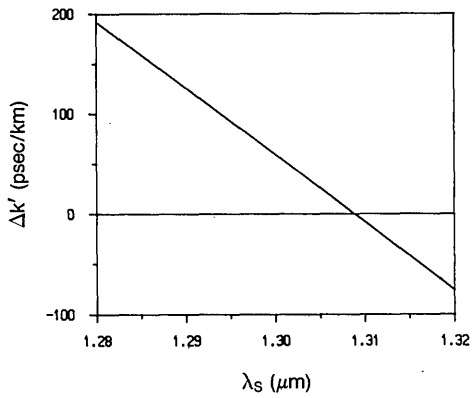
Fig. 1. Relative reciprocal group velocity $\Delta k'$ between the Raman pump wave and the carrier wave of the soliton with a frequency difference of 440 cm^{-1}.
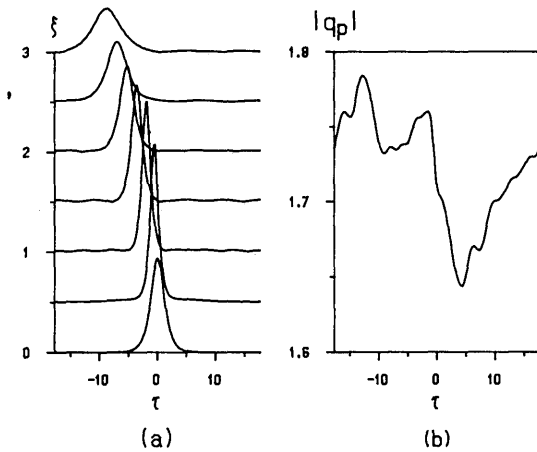

Fig. 2. (a) Envelope evolution of the soliton amplified by the forward pump wave. The soliton has a wavelength of 1.3 μm and an initial amplitude of 6.25. The pump wave has a wavelength of 1.23 μm and an initial amplitude of 3. The time scale $T = 1$ psec, and the relative velocity $V = 590$. (b) Envelope of the forward pump wave at $\xi = 0.5$.

In this case, $k_s'' = -2.37$ psec^2/km, $k_p'' = 3.62$ psec^2/km, and $\Delta k' = 59$ psec/km. The loss coefficients are assumed to be $\gamma_s = 0.3$ dB/km and $\gamma_p = 0.5$ dB/km. The initial conditions are taken to be

$$q_s(\tau, \xi = 0) = \sqrt{\alpha_s/\eta_s}\,\text{sech}(\tau), \qquad q_p(\tau, \xi = 0) = q_{p0}.$$

$$(4)$$

Initially, q_s is the fundamental soliton with amplitude $\sqrt{\alpha_s/\eta_s}$ and q_p is the cw wave with amplitude q_{p0}. The pulse width (FWHM) of the sech pulse is $1.76T$ (psec). Note that, from Eqs. (3) and (4), the pulse amplitude is inversely proportional to the time scale T, which is proportional to the pulse width. It is shown below that the effects caused by the interaction between them is increased as T is decreased. Equations (1) are numerically solved by the propagating-beam method[7] with periodic boundary conditions. We take $\tau = -17.6$ to 17.6, i.e., what we consider is an infinite pulse train with 20 pulse widths between neighboring solitons.

The envelope evolution of the soliton is shown in Fig. 2(a) for $T = 1$ psec ($V = 590$) and $q_{p0} = 3$. The envelope of the forward pump wave at $\xi = 0.5$ is shown in Fig. 2(b). In Fig. 2(b), the pump wave is depleted by the soliton, in addition to the decay due to fiber loss. The depletion makes the envelope of the pump wave fluctuate and leads to the cross-phase modulation[8,9] between the pump wave and the soliton. We have found that the cross-phase modulation shifts the carrier frequency of the soliton. It is seen in Fig. 2(a) that the soliton is at first amplified and finally decays owing to pump depletion and fiber loss. Furthermore, the soliton propagates to the negative direction of the τ frame, which is the moving frame with the velocity of the initial soliton; this shows that the velocity of the soliton is changed. The velocity change is due to the frequency shift of the soliton. δf can be calculated by[3]

$$\delta f = \frac{-1}{2\pi T}\frac{\text{Im}\int q_s^* \frac{\partial}{\partial \tau} q_s \, d\tau}{\int q_s^* q_s \, d\tau}.$$

$$(5)$$

Figure 3 shows the frequency shift δf along the fiber. At approximately $\xi = 2$, δf reaches a steady-state value. After $\xi = 2$, the contribution of the forward pump wave can be neglected. We define the steady-state value to be δf_s. In Fig. 3, $\delta f_s = 22.8$ GHz. δf_s for initial
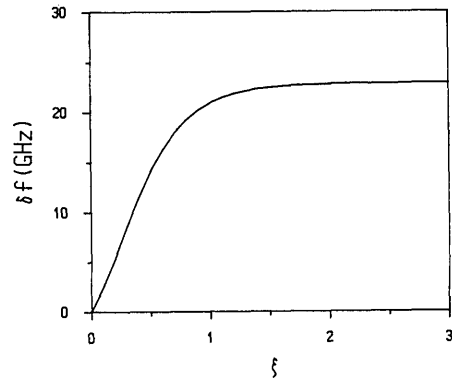

Fig. 3. Frequency shift δf of the soliton along the fiber shown in Fig. 2(a).
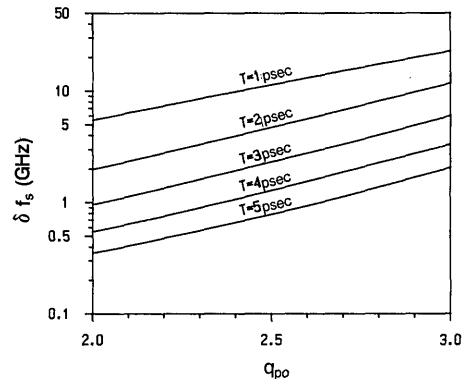

Fig. 4. Steady-state frequency shift δf_s of the soliton with a wavelength of 1.3 μm amplified by a forward Raman pump wave with initial amplitude q_{p0} for several time scales.
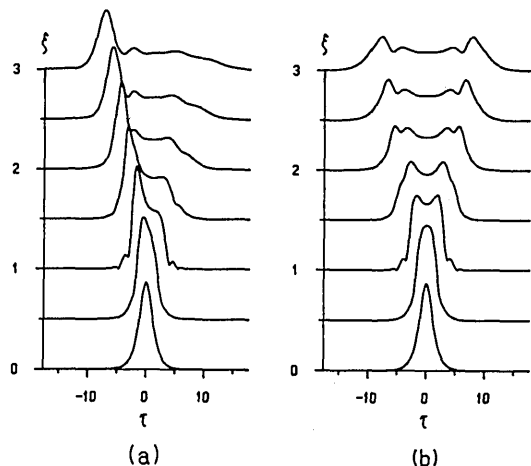
Fig. 5.   Envelope evolution of the soliton with time scale T = 5 psec and a wavelength of (a) 1.3087 μm and (b) 1.3885 μm amplified by the forward pump wave with an initial amplitude of 3. The initial soliton amplitudes are (a) 1.443 and (b) 1.444.
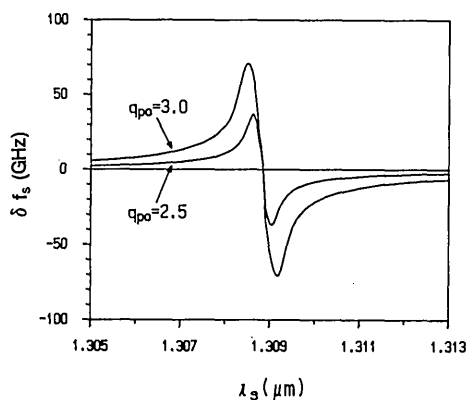

Fig. 6.   Steady-state frequency shift δf_s of the soliton with time scale T = 5 psec and a wavelength near zero relative velocity wavelength λ_{s0} amplified by a forward Raman pump wave with initial amplitudes q_{p0} of 2.5 and 3.

pump amplitudes varied from 2 to 3, and several time scales with λ_s = 1.3 μm are shown in Fig. 4. From the figure we see that δf_s increases with the initial pump amplitude q_{p0}. For T = 1 psec, δf_s increases exponentially with q_{p0}, while, for T = 5 psec, δf_s increases exponentially with q_{p0}^2. For T between 1 and 5 psec, δf_s increases exponentially with q_{p0} by a power between 1 and 2. Note that δf_s increases when T de-

creases. This is due to higher depletion for the larger soliton power with a smaller T or pulse width.

Next we study the cases around λ_{s0}, the wavelength of the soliton at zero relative velocity. For T = 5 psec and q_{p0} = 3, Fig. 5 shows the cases with λ_s = 1.3087 μm and $\lambda_s = \lambda_{s0}$. V = 1.63 for the case with λ_s = 1.3087 μm. From the figure we see that the pulse shapes distort significantly for both cases. Although the corresponding frequency spectrum also distorts greatly, there is a steady-state frequency shift. In such a case, the frequency shift calculated from Eq. (5) can be referred to the average frequency spectrum shift. For fixed T and q_{p0}, the steady-state frequency shift δf_s for the cases with λ_s around λ_{s0} is shown in Fig. 6. The sign of δf_s is reversed from $\lambda_s < \lambda_{s0}$ to $\lambda_s > \lambda_{s0}$ for a different sign of relative velocity. There are extrema on both sides of λ_{s0}. As λ_s is away from the wavelength of the extremum, δf_s decreases rapidly, and the corresponding pulse-shape distortion is reduced. Note that λ_{s0} is away from 1.3 μm by only a few nanometers. This indicates that a slight source frequency shift may lead the soliton to the large pulse-shape distortion regime.

In conclusion, we have considered the interaction between the soliton and a forward pump wave. When their relative velocity is small and the soliton pulse width is short, the soliton's frequency is shifted by the interaction. Near zero relative velocity, the pulse shape is distorted, in addition to its experiencing a frequency shift. Therefore, to avoid frequency shift and pulse-shape distortion, either the pulse width cannot be too short or the soliton and pump frequencies should be properly chosen so that the relative velocity between the forward pump wave and the soliton is not too small.

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