TREFFTZ METHOD FOR HYDRODYNAMIC PRESSURE ON RIGID DAMS WITH NON-VERTICAL UPSTREAM FACE

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SUMMARY

Based on a two-dimensional potential flow theory, earthquake-induced hydrodynamic pressures on a rigid dam with a non-vertical upstream face are examined by the Trefftz method. The effect of surface waves on the hydrodynamic pressure distribution is discussed in detail. Numerical values are given for different wave effect parameters and different geometries of the dam–water interface.

KEY WORDS   Trefftz method   Hydrodynamic pressure

INTRODUCTION

Earthquake-induced hydrodynamic pressures on the upstream face of a dam are an important factor in the design of dams in seismic regions. By an incompressible fluid assumption, Westergaard[1] first derived the hydrodynamic pressure on a dam with a vertical upstream face when subjected to horizontal harmonic ground motion. Since then many researchers have extended Westergaard's work to different aspects of the problem, such as the compressibility of the fluid in the reservoir, the flexibility of the dam, the reservoir bottom absorption and the effect of gravity surface waves.

The hydrodynamic pressures on a dam also depend on the geometry of the dam–water interface. Using a two-dimensional potential flow theory and ignoring the presence of surface waves, Chwang[2] presented an exact solution for the hydrodynamic pressure on an accelerating dam with an inclined upstream face by assuming that the reservoir has a constant depth. Liu[3] extended Chwang's work to cases where the inclined upstream dam face has a constant slope and the reservoir has a triangular shape. Both Chwang's and Liu's exact solutions are based on the Schwarz–Christoffel theory, which is not valid if the geometry of the two slopes or the effect of surface waves is included in the analysis. Avilés and Sánchez-Sesma[4] applied the Trefftz method to study the influence of the upstream shape and found that the Trefftz method is simple and accurate when compared with Zangar's[5] experimental solutions. The importance of the effect of surface waves on the hydrodynamic pressure distribution was proved by Chwang,[6] who studied a dam with a vertical upstream face.

The objective of this paper is to investigate the effects of surface gravity waves on the hydrodynamic pressure distribution of rigid dams with different shapes of the upstream face during earthquakes by using the Trefftz method. The Trefftz method[7,8] is one of two boundary method approaches; the other is the boundary integral equation method or boundary element method. The Trefftz method used in this research, which is based on the use of complete systems of

solutions, can be described as a linear combination of functions that form *T*-complete functions[9] which are solutions of the governing equation of the problem and satisfy the boundary conditions, except the one at the upstream face of the dam. The unknown coefficents of the combination are found from a least-squares matching of the remaining boundary condition, i.e. by minimizing the quadratic error integrated exactly along the upstream surface of the dam.[4]

Numerical results both including and excluding surface gravity waves are computed and compared in order to discuss and to prove the important role of surface gravity waves in the study of the hydrodynamic pressure on dams with a non-vertical upstream face during short-duration earthquakes.

THEORETICAL FORMULATION

Consider a rigid dam with a non-vertical upstream face as shown in Figure 1. We assume that: the dam–reservoir system is two-dimensional and has constant water depth; the reservoir is infinitely long; the dam and bottom are rigid; the dam is subjected to a horizontal harmonic motion of short duration; the displacements are small compared with the dimensions of the dam; and the fluid is incompressible, inviscid and irrotational. Therefore the flow field can be expressed by a velocity potential function Φ which satisfies the Laplace equation[10]

$$\nabla^2 \Phi = 0 \tag{1}$$

such that

$$u = -\frac{\partial \Phi}{\partial x}, \qquad v = -\frac{\partial \Phi}{\partial y}, \qquad p = \rho \frac{\partial \Phi}{\partial t}, \tag{2}$$

where u and v are the velocities in the x- and y-directions respectively, p is the hydrodynamic pressure, ρ is the mass density of the fluid and t is time.

Since the dam is subjected to a short-duration harmonic motion $X_g(t)$ with frequency ω, as shown in Figure 1, the boundary conditions to be satisfied are[10]

bottom boundary condition

$$\frac{\partial \Phi}{\partial y} = 0 \quad \text{on} \quad y = 0, \tag{3}$$

free surface boundary condition

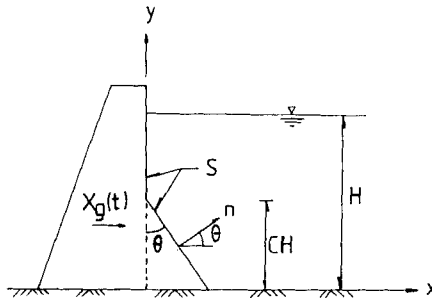$$\frac{\partial \Phi}{\partial y} - \left(\frac{\omega^2}{g}\right)\Phi = 0 \quad \text{on} \quad y = H, \tag{4}$$

Figure 1. Definition sketch of rigid dam with nonvertical upstream face

radiation condition

$$\lim_{x \to \infty} \Phi = 0, \tag{5}$$

upstream face boundary condition

$$\left. \frac{d\Phi}{dn} \right|_S = \dot{X}_g(t) \cos \theta, \tag{6}$$

where H is the water depth, n is the unit normal to the upstream face, S is the dam–water interface and θ is the angle between the normal n and the direction x.

SOLUTION BY THE TREFFTZ METHOD

Using the method of separation of variables, it can be found that the solutions which satisfy equations (3)–(5) are given by

$$\Phi = \dot{X}_g(t) \sum_{n=1}^{\infty} A_i e^{-\lambda_i x} \cos \lambda_i y, \tag{7}$$

where A_i are unknown coefficients to be determined from the remaining boundary condition, equation (6), and λ_i must satisfy the dispersion relation

$$\frac{1}{CW} = -\lambda_i H \tan \lambda_i H, \quad i = 1, 2, \ldots, \infty, \tag{8}$$

where $CW = g/\omega^2 H$ is called the wave effect parameter, defined by Chwang.[6]

Substituting equation (7) into equation (6) yields

$$\sum_{i=1}^{\infty} A_i f_i(x, y) \bigg|_S = 1, \tag{9}$$

where

$$f_i(x, y) = \lambda_i e^{-\lambda_i x} (\cos \lambda_i y + \tan \theta \sin \lambda_i y).$$

Equation (9) will be solved in the least-squares sense. The quadratic error in equation (9) integrated along the dam–water interface is

$$E = \int_S \left(\sum_{i=1}^{\infty} A_i f_i(x, y) - 1 \right)^2 dS, \tag{10}$$

which, in view of the condition of minimum

$$\frac{\partial E}{\partial A_j} = 0, \quad j = 1, 2, \ldots, \infty, \tag{11}$$

yields

$$\sum_{i=1}^{\infty} A_i \int_S f_i(x, y) f_j(x, y) \, dS = \int_S f_j(x, y) \, dS, \quad j = 1, 2, \ldots, \infty. \tag{12}$$

This is an infinite linear system of algebraic equations which can be written in matrix form as

$$[F_{ji}]\{A_i\} = \{G_j\}, \quad i, j = 1, 2, \ldots, \infty, \tag{13}$$

where

$$F_{ji} = F_{ij} = \int_S f_i(x, y) f_j(x, y)\, dS$$

and

$$G_j = \int_S f_j(x, y)\, dS.$$

The corresponding curvilinear integrals are calculated to give

$$F_{ji} = \frac{\lambda_j \lambda_i}{2} \left[\delta_{ji} \left(\frac{\sin 2\lambda_j H}{2\lambda_j} + H(1-C) \right) + (1 - \delta_{ji}) \left(\frac{\sin(\lambda_i + \lambda_j)H}{\lambda_i + \lambda_j} + \frac{\sin(\lambda_i + \lambda_j)CH}{\lambda_i + \lambda_j} \right. \right.$$

$$\left. + \frac{\sin(\lambda_i - \lambda_j)H}{\lambda_i - \lambda_j} - \frac{\sin(\lambda_i - \lambda_j)CH}{\lambda_i - \lambda_j} \right) + \frac{\sec\theta}{\lambda_i + \lambda_j} [(1 - 2\cos\theta)\sin(\lambda_j + \lambda_i)CH - K_{ji}]$$

$$\left. + \frac{\sec^3\theta / (\lambda_j + \lambda_i)}{[(\lambda_j + \lambda_i)\tan\theta]^2 + (\lambda_j - \lambda_i)^2} [(\lambda_j^2 - \lambda_i^2)\sin(\lambda_j - \lambda_i)CH + (\lambda_j + \lambda_i)^2 L_{ji}] \right],$$

$$j, i = 1, 2, \ldots, \infty, \tag{14}$$

$$G_j = \sin \lambda_j H - (1 - \sec\theta)\sin \lambda_j CH, \quad j = 1, 2, \ldots, \infty, \tag{15}$$

where

$$K_{ji} = \tan\theta \{\cos(\lambda_j + \lambda_i)CH - \exp[-(\lambda_j + \lambda_i)CH \tan\theta]\}, \tag{16}$$

$$L_{ji} = \tan\theta \{\cos(\lambda_j - \lambda_i)CH - \exp[-(\lambda_j + \lambda_i)CH \tan\theta]\}, \tag{17}$$

δ_{ji} is the Kronecker delta ($= 1$ if $j = i$; $= 0$ if $j \neq i$) and C is the ratio of the height of the inclined upstream face to the depth of the water.

When $C = 0$ or $\theta = 0$, i.e. a dam with a vertical upstream face, this forms a Sturm–Liouville problem and can be solved analytically.[6]

Since equation (13) includes infinite equations, it cannot be solved exactly. An approximate solution is obtained by truncating the functions appearing in equation (7) to a finite number of terms, I, and solving the resulting linear system of I equations with I unknowns.

RESULTS

Substituting equation (7) into equation (2), the hydrodynamic pressure can be expressed as

$$p = C_s \gamma H C_p, \tag{18}$$

where $C_s = \ddot{X}_g / g$ is the normalized horizontal ground acceleration, g is the acceleration due to gravity, γ is the unit weight of water and C_p is the pressure coefficient, which is given by the equation

$$C_p = \sum_{i=1}^{\infty} \left(\frac{A_i}{H} \right) e^{-\lambda_i x} \cos \lambda_i y. \tag{19}$$

Variations of the hydrodynamic pressure coefficient are presented for different geometries of the upstream face and for different wave effect parameters CW. The number of terms used to calculate the solution is $I = 25$, which is the same as Avilés and Sánchez-Sesma used.

The vertical variation of the pressure coefficient on the dam is shown in Figure 2 for several values of θ, the inclined angle, and CW, the wave effect parameter. The broken curves are Chwang's[2] exact solutions ignoring the presence of surface waves. Avilés and Sánchez-Sesma's[4]
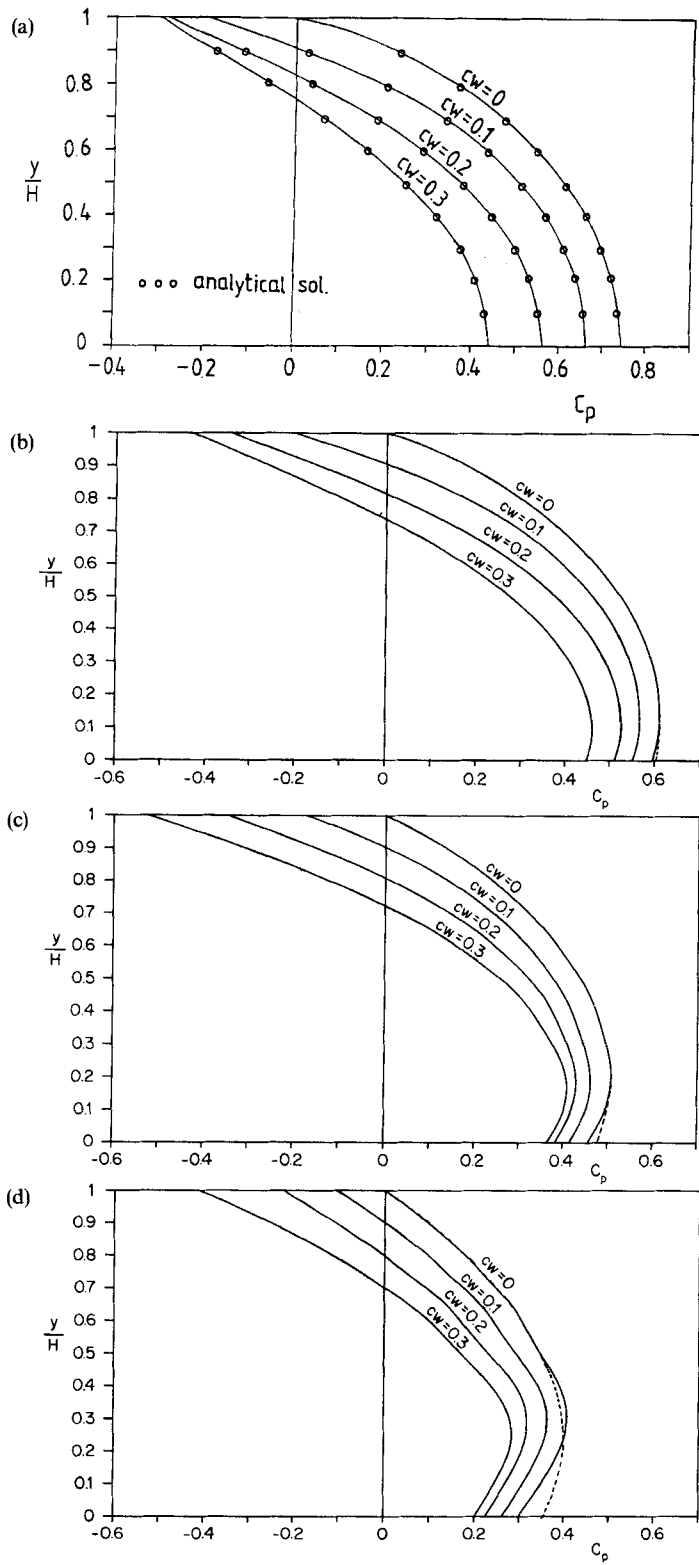
Figure 2. Pressure distribution on inclined upstream face for various values of CW. The broken curves are Chwang's exact solutions. (a) $\theta = 0°$, $C = 1·0$. (b) $\theta = 15°$, $C = 1·0$. (c) $\theta = 30°$, $C = 1·0$. (d) $\theta = 45°$, $C = 1·0$

solutions correspond to the solid curves with $CW=0$. Figure 3 displays the vertical variation of the pressure coefficient for various values of CW and for various combinations of vertical and sloping faces on the upstream side of the dam. The broken curves are the experimental solutions of Zangar.[5]
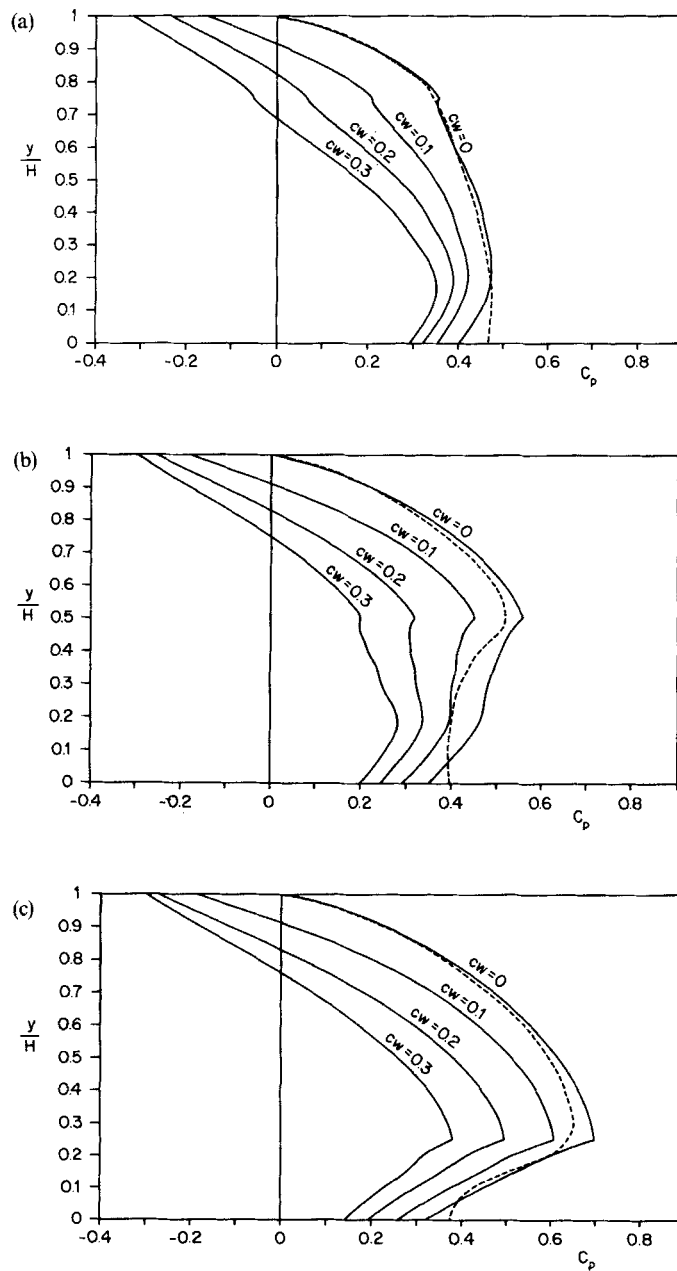

Figure 3. Pressure distribution on upstream faces that consist of two planes for various values of CW. The broken curves are Zangar's experimental solutions. (a) $\theta=37\cdot6°$, $C=0\cdot75$. (b) $\theta=49\cdot1°$, $C=0\cdot50$. (c) $\theta=66\cdot6°$, $C=0\cdot25$.

CONCLUSIONS

The effect of surface gravity waves on the hydrodynamic pressures on dams with non-vertical upstream face due to a short-duration, horizontal harmonic ground acceleration has been analysed by using the Trefftz method. It has been found that for any fixed height the hydrodynamic pressure on the dam decreases as the wave effect parameter CW increases. This phenomenon can be attributed to energy being radiated by waves propagating away from the dam. When CW vanishes, which means harmonic ground motion with a very high frequency, the pressure distribution becomes the same as that given by Aviles and Sánchez-Sesma.[4]

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