國 立 交 通 大 學

電信工程研究所

碩 士 論 文

以基於能量偵測法則的頻譜偵測演算法偵測抵達時間未知的主要使用者訊號

Energy Detection Based Spectrum Sensing with Unknown Primary Signal Arrival Time

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中 華 民 國 九十九 年 七 月

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A Thesis
Submitted to Department of Communications Engineering
College of Electrical and Computer Engineering
National Chiao Tung University
in partial Fulfillment of the Requirements
for the Degree of
Master of Science
in

Communications Engineering

July 2010

Hsinchu, Taiwan, Republic of China

中華民國九十九年七月

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摘要

在下世代的無線感知系統中，次要使用者 (Secondary user) 進行頻譜偵測 (Spectrum sensing) 時會遇到與主要使用者 (Primary user) 時間不同步的情形。因此，本論文假設主要使用者存取頻帶的時間為一均勻分布的隨機變數，進而分析能量偵測器 (Energy detector) 在此環境設定下的效能。其中，本論文推導出準確偵測機率 (Detection probability) 的公式並藉由電腦模擬驗證之。為了進一步提升系統偵測效能，本論文提出一個基於貝氏 (Bayesian) 原則的偵測演算法。此外，當主要使用者存取頻帶的時間被視為一個不變的未知數，本論文則提出另一個以廣義概似比例檢定 (Generalized likelihood ratio test, GLRT) 為基礎的偵測法則，藉以改善系統偵測效能。電腦模擬的結果證實本論文所提出的兩種偵測演算法皆能有效提升系統偵測效能。

Energy Detection Based Spectrum Sensing with Unknown Primary Signal Arrival Time

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Abstract

Spectrum sensing in next-generation wireless cognitive systems, such as overlay femtocell net-works, is typically subject to timing misalignment between the primary transmitter and the secondary receiver. In this thesis we investigate the performance of the energy detector (ED) when the arrival time of the primary signal is modeled as a uniform random variable over the observation interval. The exact formula for the detection probability is derived and corroborated via numerical simulation. To further improve the detection performance, we propose a robust ED based on the Bayesian principle. In addition, when the primary signal arrival time is unknown but fixed, we propose another detection rule based on the generalized-likelihood ratio test (GLRT) to improve the detection performance. Computer simulation confirms the effectiveness of the Bayesian based and the GLRT based solution when compared with the traditional ED.

誌 謝

兩年的研究所生涯即將結束，能夠順利完成學業及論文，最要感謝的就是我的指導教授吳卓諭博士。無論是學術研究上的耐心教導或是為人處事上的諄諄教誨，都使我獲益良多。讓我明白了以嚴謹的態度專注於每一個細節才能將事情做到最好。感謝老師時常關心我的生活，適時在有困難時給予我建議，並提供了舒適的環境和豐富的資源，讓學生們可以安心地學習與研究。在此，我要向吳卓諭老師獻上最誠摯的感謝。

感謝學長李其翰、高健郎，學弟李其峰、徐瑞隆，陪我討論課業和研究上的問題並陪我打球解悶；感謝助理歐陽彣覲幫我處理補助經費和報帳的事情；感謝助理洪思佳幫我分擔打字與網站架設的事情；感謝助理陳力、魏大均與我分享許多見聞；感謝室友陳彥后、黃啟銘、張瑞桓陪伴我的碩一生活，一起吃飯、打球、健身、談天說地、討論課業；感謝高中同學張國煜，在撰寫論文期間陪我度過無數個夜晚，一起宵夜、一起趕論文、一起晚睡早起。陪伴我的這些朋友，你們對我的幫助與情誼，難以磨滅。

最後，感謝養育我長大的父親王永富、母親邱慧玲，你們長久以來的付出與包容，點點滴滴永記在心頭。感謝已故的外公邱欽鐘，外婆邱李金美，自小受到你們的照顧與疼愛。感謝妹妹王文岑、王歆雯陪我嬉鬧並任憑我差遣。感謝女朋友楊雨潔一路走來的扶持、鼓勵與信任，是我能不斷向前的動力。謹以此論文獻給我所有親愛的家人。

王致翔 謹誌

民國九十九年七月二日

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Chapter 1

Introduction

1.1 Overview

Due to tremendous growth in the wireless based systems and the limitations of the natural frequency spectrum, we need innovative techniques that can exploit the available spectrum to accommodate the requirements of higher rate transmissions while the current frequency allocation schemes can't. Even though most of available spectrum has been assigned for various services, such as military communications, broadcast service and telecom service, investigations of spectrum utilization show that many allocated spectrum are not occupied by licensed users for all time. This fact motivates a spectrum allocation scheme that allows secondary users to utilize the idle spectrum licensed to the primary users. It is known as the concept of spectrum reuse. A pictorial description is as follows [1].

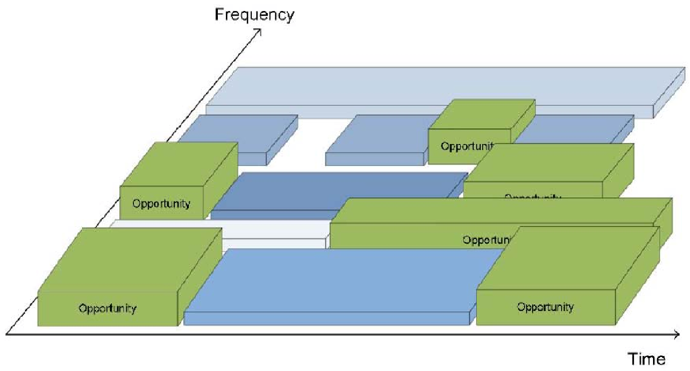

Figure 1.1: Transmission opportunities of specific bands in time

Cognitive radio (CR) is now acknowledged as a tempting solution to reusing the underutilized spectrum in an opportunistic manner [1], [2]. CR is an autonomous system that senses its communication environment, tracks changes and dynamically accesses the unused spectrum [1], [2]. In CR terminology, primary users can be defined as the users who can utilize a specific part of spectrum with higher priority. On the other hand, secondary users have lower priority to access the specific part of spectrum and they can not cause interference to primary users when they exploit the unused part of spectrum.

One of the most essential components for enabling the CR technique is spectrum sensing. The task of spectrum sensing, implemented at the secondary receiver, is to detect the idle frequency bands and monitor the existence of primary users. Challenges, design trade-offs and implementation issues of spectrum sensing are addressed in [2], [3], [4]. The reference [1] provides brief introductions of various sensing techniques. Since enhancing the accuracy of spectrum sensing can not only reduce the possible interference to primary users but also increase the opportunistic access to idle frequency bands, there are many research works that aim to develop new methods to improve the sensing performance.

The most commonly-used techniques of spectrum sensing in the literature can be categorized into the following four classes.

Energy detection (ED): To detect the existence of primary users, ED computes the energy of the received signal, and then compares this energy with a given threshold. If the energy of the received signal exceeds the threshold, the ED claims that the primary user is present. Otherwise, the ED decides that the primary user is absent. ED is the most common method of spectrum sensing when the receiver doesn't have any information about the primary users' signal.

Waveform-Based Sensing: If the known patterns, such as pilot, spreading sequences, preambles etc, are available at the receiver, the spectrum sensing can be performed by correlating the received signal with the known signal pattern.

Cyclostationarity-based detection: Since the modulated signals are generally transmitted by a sinewave carrier, the modulated signals are cyclostationary due to the periodic property. When the noise is wide-sense stationary, certain cyclic autocorrelation function (CAF) of the received signal will be nonzero in the presence of the primary signal. On the other hand, the

CAS of the received signal will be zero since the received signal contains only the noise term. This fact can be exploited for spectrum identification.

Matched-Filtering (MF): Assuming that there is perfect knowledge of primary users' signal and accurate synchronization, MF is known as the optimal solution for spectrum sensing. However, if the mentioned assumptions cannot be satisfied, the performance of MF will be dramatically reduced.

1.2 Research Motivation

In the literature, the detection of idle spectrum is typically considered as a binary hypothesis test, and a commonly used signal model under both hypotheses is [10], [11]

$$\begin{cases} \mathcal{H}_0 : x[n] = v[n], & 0 \le n \le N-1 \quad \text{(idle)} \\ \mathcal{H}_1 : x[n] = s[n] + v[n], & 0 \le n \le N-1 \quad \text{(occupied)} \end{cases} \tag{1.1}$$

where N is the length of the data record, $s[n]$, $x[n]$, $v[n]$ are, respectively, the signal of the primary user, the received signal at the CR receiver, and the measurement noise. The hypothesis model (1.1) implicitly assumes perfect synchronization between the primary transmitter and the CR receiver. Such an assumption, however, is not valid in many practical situations. For example, in an overlay femto cell network [12], the signal of the macro mobile subscriber, synchronized with the macro base station (BS), will arrive at a femto BS asynchronously. The spectrum detection at the femto BS is typically subject to timing misalignment of the primary signal [13], [14]. Thus, in such a case, a more reasonable signal model for the binary hypothesis test is therefore

$$\begin{cases} \mathcal{H}_0 : x[n] = v[n], & 0 \le n \le N-1 \quad \text{(idle)} \\ \mathcal{H}_1 : \begin{cases} x[n] = v[n], & 0 \le n \le n_0 - 1 \\ x[n] = s[n] + v[n], & n_0 \le n \le N-1 \end{cases} & \text{(occupied)} \end{cases} \tag{1.2}$$

where n_0 accounts for the primary signal arrival time. Therefore, in contrast to the spectrum sensing schemes in the literature focusing on the synchronized signal model (1.1) [10], [11], this thesis considers the spectrum detection aimed for tackling signal timing uncertainty under the hypothesis (1.2).

1.3 Thesis Contributions

Unlike the prior researches investigating the performance characteristics of ED based on the idealized model, this thesis studies the detection performance of ED in the presence of unknown primary signal arrival time. Specifically, assuming the time delay is a uniform random variable, the exact formula of average detection probability of ED is derived. Further, in order to improve the detection performance against the timing mismatch, we then propose a Bayesian based detection rule to exploit the prior statistical knowledge of the unknown primary signal arrival time. In addition, when the prior knowledge of unknown primary signal arrival time is not available, we propose a generalized likelihood ratio test (GLRT) based detection rule to deal with the case in which the primary signal arrival time is considered as a deterministic unknown.

1.4 Thesis Organization

The organization of this thesis is as follows. In Chapter 2, the energy detection for spectrum sensing is introduced and the detection performance for the signal model taking account of unknown primary signal arrival time is also provided. In chapter 3, we propose a robust energy detection scheme based on the Bayesian principle to improve the detection performance when primary signal arrival time is uniformly distributed. In Chapter 4, we consider the primary signal arrival time as a deterministic unknown, and then propose another robust energy detection scheme based on the principle of the GLRT. Chapter 5 concludes this thesis and points out some future work. Some proofs are provided in Appendix.

Chapter 2

Detection Performance of Energy Detector in the Presence of Time Delay

2.1 Neyman-Pearson Theorem

Recall the signal model for the considered binary hypothesis test is

$$\mathcal{H}_0 : x[n] = v[n], \qquad 0 \le n \le N-1$$
$$\mathcal{H}_1 : \begin{cases} x[n] = v[n], & 0 \le n \le n_0 - 1 \\ x[n] = s[n] + v[n], & n_0 \le n \le N-1 \end{cases}$$

With this scheme we may make two types of errors. If we decide $\mathcal{H}_1$ but $\mathcal{H}_0$ is true, it can be thought of as a false alarm. On the other hand, if we decide $\mathcal{H}_0$ but $\mathcal{H}_1$ is true, it is a miss detection. Let $P(\mathcal{H}_i; \mathcal{H}_j)$ indicate the probability of deciding $\mathcal{H}_i$ when $\mathcal{H}_j$ is true. Hence, $P(\mathcal{H}_1; \mathcal{H}_0)$ is he probability of false alarm and is denoted by P_{FA}. To design the optimal detector for a given P_{FA}, we would like to minimize the other error $P(\mathcal{H}_0; \mathcal{H}_1)$ or equivalently to maximize $P(\mathcal{H}_1; \mathcal{H}_1)$. The latter is called the probability of detection and is denoted by P_D. In summary, the Neyman-Pearson (NP) approach to hypothesis testing or to signal detection is to maximize $P_D = P(\mathcal{H}_1; \mathcal{H}_1)$ subject to the constraint $P_{FA} = P(\mathcal{H}_1; \mathcal{H}_0) = \alpha$.

Theorem 2.1: Neyman-Pearson Theorem [15]
To maximize P_D for a given $P_{FA} = \alpha$ decide $\mathcal{H}_1$ if

$$L(\mathbf{x}) = \frac{p(\mathbf{x}; \mathcal{H}_1)}{p(\mathbf{x}; \mathcal{H}_0)} > \gamma,$$

where $p(\mathbf{x};\mathcal{H}_1)$ is the probability dencity function (PDF) of $\mathbf{x}$ under $\mathcal{H}_1$, $p(\mathbf{x};\mathcal{H}_0)$ is the PDF $\mathbf{x}$ under $\mathcal{H}_0$, and the threshold γ is found from

$$P_{FA} = \int_{\{\mathbf{x}:L(\mathbf{x})>\gamma\}} p(\mathbf{x};\mathcal{H}_0)d\mathbf{x} = \alpha.$$

□

2.2 Performance Analysis

According to Neyman-Pearson theorem and [15], the NP detector decides $\mathcal{H}_1$ if

$$T(\mathbf{x}) = \sum_{n=0}^{N-1} |x[n]|^2 > \gamma. \tag{2.1}$$

That is, the NP detector computes the energy in the received data and compares it to a threshold. Hence, in this case it is known as an energy detector. This section characterizes the performance of ED under the signal model (2.1).

The following assumptions are made in the sequel.

- The primary signal $s[n]$ is a zero mean, white Gaussian random process with known variance σ_s^2.
- The noise $v[n]$ is a zero mean, white Gaussian random process with known variance σ_v^2.
- $s[n]$ and $v[n]$ are independent.
- The primary signal arrival time n_0 is discrete and uniformly distributed over the observation interval $0 \le n \le N-1$, i.e. the PDF of n_0 is $p(n_0) = 1/N$, for $0 \le n \le N-1$.

2.2.1 False-Alarm Probability

Under the null hypothesis $\mathcal{H}_0$, we have

$$x[n] = v[n], \quad 0 \le n \le N-1 \tag{2.2}$$

The test statistic of the energy detector under $\mathcal{H}_0$ is thus

$$T = \sum_{n=0}^{N-1} |x[n]|^2 = \sum_{n=0}^{N-1} |v[n]|^2. \tag{2.3}$$

and the false-alarm probability P_{FA} is given by

$$\begin{aligned} P_{FA} &= Pr\{T(\mathbf{x}) > \gamma; \mathcal{H}_0\} \\ &= Pr\left\{ \frac{\sum_{n=0}^{N-1} |v[n]|^2}{\sigma_v^2} > \frac{\gamma}{\sigma_v^2}; \mathcal{H}_0 \right\} \\ &\overset{(a)}{=} Q_{\chi_N^2}\left(\frac{\gamma}{\sigma_v^2}\right), \end{aligned} \tag{2.4}$$

where (a) holds directly by definition of the right-tail probability of the Chi-square random variable χ_N^2 with an even degree-of-freedom [15]. However, the probability of detection is much more difficult to compute since its PDF is not as familiar as Chi-square distribution. The detail will be presented in 2.2.2.

To find the threshold of ED according to a given P_{FA}, we represent (2.4) as (for the case of N even) [15]

$$P_{FA} = \exp\left(-\frac{\gamma}{2\sigma_v^2}\right)\left[1 + \sum_{r=1}^{N/2-1} \frac{\left(\frac{\gamma}{2\sigma_v^2}\right)^r}{r!}\right]. \tag{2.5}$$

By letting $\gamma' = \gamma/2\sigma_v^2$ and rearranging terms we have

$$\gamma' = -\ln P_{FA} + \ln\left[1 + \sum_{r=1}^{N/2-1} \frac{\gamma'^r}{r!}\right]. \tag{2.6}$$

To solve for γ' we can use the fixed point iteration

$$\gamma'_{k+1} = -\ln P_{FA} + \ln\left[1 + \sum_{r=1}^{N/2-1} \frac{{\gamma'_k}^r}{r!}\right]. \tag{2.7}$$

Hence, the threshold γ can be obtained by iterating with $\gamma'_0 = 1$.

2.2.2 Exact Detection Probability

Under the alternative hypothesis $\mathcal{H}_1$, we have

$$x[n] = \begin{cases} v[n], & 0 \le n \le n_0 - 1 \\ s[n] + v[n], & n_0 \le n \le N - 1 \end{cases} \tag{2.8}$$

The test statistic of the energy detector under $\mathcal{H}_1$ and conditioned on a fixed n_0 is thus

$$T = \sum_{n=0}^{N-1} |x[n]|^2 = \underbrace{\sum_{n=0}^{n_0-1} |x[n]|^2}_{:=T_1} + \underbrace{\sum_{n=n_0}^{N-1} |x[n]|^2}_{:=T_2} > \gamma. \tag{2.9}$$

Based on (2.9), we shall first derive the conditional detection probability; the average detection probability can then easily obtained by taking the expectation with respect to n_0.

Note that, with T_1 and T_2 defined in (2.9), it is easy to verify $z_1 := T_1/\sigma_v^2 \sim \chi^2_{n_0}$ and $z_2 := T_2/(\sigma_v^2 + \sigma_s^2) \sim \chi^2_{N-n_0}$, and hence the associated probability density functions is

$$f_{z_1}(x) = \frac{x^{(n_0/2)-1} e^{-x/2}}{\sqrt{2^{n_0}}\Gamma(n_0/2)} u(x) \tag{2.10}$$

and

$$f_{z_2}(x) = \frac{x^{[(N-n_0)/2]-1} e^{-x/2}}{\sqrt{2^{(N-n_0)}}\Gamma\left((N-n_0)/2\right)} u(x), \tag{2.11}$$

where $u(t)$ is the unit step function. To simplify notation let us consider the equivalent test statistic

$$\bar{T} = \frac{T}{\sigma_v^2} = \frac{1}{\sigma_v^2}\sum_{n=0}^{N-1} |x[n]|^2 = \frac{T_1}{\sigma_v^2} + \frac{T_2}{\sigma_v^2} = z_1 + \left(\frac{\sigma_s^2 + \sigma_v^2}{\sigma_v^2}\right) z_2 = z_1 + (1+SNR) z_2, \tag{2.12}$$

where $SNR := \sigma_s^2/\sigma_v^2$. Since z_1 and z_2 are independent, the PDF of $\bar{T}$ is given by

$$f_{\bar{T}}(x) = f_{z_1}(x) * \left(\frac{1}{1+SNR}\right) \cdot f_{z_2}\left(\frac{x}{1+SNR}\right), \tag{2.13}$$

where $*$ denotes the convolution. In terms of Laplace transform, (2.13) reads

$$F_{\bar{T}}(s) = F_{z_1}(s) \times \left(\frac{1}{1+SNR}\right) \mathcal{L}\left\{f_{z_2}\left(\frac{x}{1+SNR}\right)\right\} = F_{z_1} \times F_{z_2}\left(s(1+SNR)\right), \tag{2.14}$$

where the second equality follows since $\mathcal{L}\{f(ax)\} = a^{-1}F(s/a)$. To derive an explicit expression for $F_{\bar{T}}(s)$ in (2.14), we need the next lemma.

Lemma 2.2 [16]: For $\lambda > 0$, we have $\mathcal{L}\left\{x^{\lambda-1}e^{-ax}u(x)\right\} = \Gamma(\lambda)(s+a)^{-\lambda}$. □

From (2.10), (2.11), and by means of Lemma 2.1, we immediately have

$$F_{z_1}(s) = \frac{\Gamma\left(n_0/2\right)\left(s+1/2\right)^{-n_0/2}}{\sqrt{2^{n_0}}\Gamma\left(n_0/2\right)} = \frac{\left(s+1/2\right)^{-n_0/2}}{\sqrt{2^{n_0}}} \tag{2.15}$$

and

$$F_{z_2}(s) = \frac{\Gamma\left((N-n_0)/2\right)\left(s+1/2\right)^{-(N-n_0)/2}}{\sqrt{2^{(N-n_0)}}\Gamma\left((N-n_0)/2\right)} = \frac{\left(s+1/2\right)^{-(N-n_0)/2}}{\sqrt{2^{(N-n_0)}}}. \tag{2.16}$$

Based on (2.14), (2.15), and (2.16), direct manipulation shows

$$\begin{aligned} F_{\bar{T}}(s) &= \frac{1}{\sqrt{2^N}}\left(s+\frac{1}{2}\right)^{-n_0/2}\left(s(1+SNR)+\frac{1}{2}\right)^{-(N-n_0)/2} \\ &= \frac{(1+SNR)^{-(N-n_0)/2}}{\sqrt{2^N}}\left(s+\frac{1}{2}\right)^{-n_0/2}\left(s+\frac{1}{2(1+SNR)}\right)^{-(N-n_0)/2}. \end{aligned} \tag{2.17}$$

With the aid of (2.17), the PDF $f_{\bar{T}}(x)$ is given by

$$\begin{aligned} f_{\bar{T}}(x) &= \frac{(1+SNR)^{-(N-n_0)/2}}{\sqrt{2^N}} \times \left\{\mathcal{L}^{-1}\left\{(s+1/2)^{-n_0/2}\right\} * \mathcal{L}^{-1}\left\{(s+1/[2(1+SNR)])^{-(N-n_0)/2}\right\}\right\} \\ &\overset{(b)}{=} \frac{(1+SNR)^{-(N-n_0)/2}}{\sqrt{2^N}} \times \left\{\left[\frac{x^{(n_0/2)-1}e^{-x/2}u(x)}{\Gamma(n_0/2)}\right] * \left[\frac{x^{[(N-n_0)/2]-1}e^{-x/[2(1+SNR)]}u(x)}{\Gamma\left((N-n_0)/2\right)}\right]\right\} \\ &= \frac{(1+SNR)^{-(N-n_0)/2}}{\sqrt{2^N}\Gamma(n_0/2)\Gamma\left((N-n_0)/2\right)} \int_0^x \tau^{(N-n_0)/2-1}e^{-\tau/[2(1+SNR)]}(x-\tau)^{n_0/2-1}e^{-(x-\tau)/2}d\tau \\ &= \frac{(1+SNR)^{-(N-n_0)/2}}{\sqrt{2^N}\Gamma(n_0/2)\Gamma\left((N-n_0)/2\right)} \times e^{-x/2}\int_0^x \tau^{(N-n_0)/2-1}(x-\tau)^{n_0/2-1}e^{SNR\tau/[2(1+SNR)]}d\tau, \end{aligned} \tag{2.18}$$

where (b) holds by using Lemma 2.1. Hence, for a given threshold γ determined according to the prescribed false-alarm probability, the conditional probability of detection can be computed based on (2.18) as

$$P_D(n_0) = \int_\gamma^\infty f_{\bar{T}}(x)dx$$
$$= \frac{(1+SNR)^{-(N-n_0)/2}}{\sqrt{2^N}\Gamma(n_0/2)\Gamma((N-n_0)/2)} \int_\gamma^\infty \underbrace{\left[e^{-x/2}\int_0^x \tau^{(N-n_0)/2-1}(x-\tau)^{n_0/2-1}e^{SNR\tau/[2(1+SNR)]}d\tau\right]}_{:=p(x)} dx. \tag{2.19}$$

To find a closed-form expression of $P_D(n_0)$ in (2.19), we need the next lemma.

Lemma 2.3 [16]: For $\nu > 0$ and $\mu > 0$, it follows

$$\int_0^x t^{\nu-1}(x-t)^{\mu-1}e^{\delta t} = B(\mu,\nu)x^{\mu+\nu-1}\Phi(\nu,\mu_\nu;\delta x), \tag{2.20}$$

where $B(\cdot,\cdot)$ is the beta function, and $\Phi(\cdot,\cdot,\cdot)$ is the confluent hypergeometric function defined by

$$\Phi(\alpha,\gamma,z) = 1 + \frac{\alpha}{\gamma}\cdot\frac{z}{1!} + \frac{\alpha(\alpha+1)}{\gamma(\gamma+1)}\cdot\frac{z^2}{2!} + \frac{\alpha(\alpha+1)(\alpha+2)}{\gamma(\gamma+1)(\gamma+2)}\cdot\frac{z^3}{3!} + \cdots. \tag{2.21}$$

□

Based on Lemma 2.2, (2.18) becomes

$$P_D(n_0) = \frac{(1+SNR)^{-(N-n_0)/2}B\left(\frac{n_0}{2},\frac{N-n_0}{2}\right)}{\sqrt{2^N}\Gamma(n_0/2)\Gamma((N-n_0)/2)} \times \int_\gamma^\infty e^{-x/2}x^{N/2-1}\left[\sum_{i=0}^\infty a_i x^i\right]dx, \tag{2.22}$$

where

$$a_0 = 1, a_1 = \frac{(N-n_0)/2}{N/2}\cdot\frac{\frac{SNR}{2(1+SNR)}}{1!}, a_2 = \frac{[(N-n_0)/2][(N-n_0)/2+1]}{(N/2)(N/2+1)}\cdot\frac{\left(\frac{SNR}{2(1+SNR)}\right)^2}{2!}, \ldots \tag{2.23}$$

Based on (2.22), the exact form of the conditional detection probability can be obtained as

$$P_D(n_0) = \frac{(1+SNR)^{-(N-n_0)/2}B\left(\frac{n_0}{2},\frac{N-n_0}{2}\right)}{\sqrt{2^N}\Gamma(n_0/2)\Gamma((N-n_0)/2)} \times \left[\sum_{i=0}^\infty a_i \int_\gamma^\infty e^{-x/2}x^{N/2+i-1}dx\right]$$
$$\overset{(c)}{=} \frac{(1+SNR)^{-(N-n_0)/2}B\left(\frac{n_0}{2},\frac{N-n_0}{2}\right)}{\sqrt{2^N}\Gamma(n_0/2)\Gamma((N-n_0)/2)} \times \sum_{i=0}^\infty a_i\left[2^{N/2+i}\Gamma\left(\frac{N}{2}+i,\frac{\gamma}{2}\right)\right], \tag{2.24}$$

where (c) follows since $\int_\gamma^\infty x^{\nu-1}e^{-\mu x}dx = \mu^{-\nu}\Gamma(\nu, \mu\gamma)$ [Kay, p-346], and $\Gamma(\alpha, y) := \int_y^\infty e^{-t}t^{\alpha-1}dt$ is the incomplete Gamma function. Based on (2.24), we summarize the main result in the following theorem.

Theorem 2.4: The average detection probability of the ED under the proposed hypothesis test is given by

$$P_D = \frac{1}{N}\sum_{n_0=0}^{N-1} P_D(n_0) = \frac{1}{N}\sum_{n_0=0}^{N-1} \frac{(1+SNR)^{-(N-n_0)/2} B\left(\frac{n_0}{2}, \frac{N-n_0}{2}\right)}{\sqrt{2^N}\Gamma(n_0/2)\Gamma\left((N-n_0)/2\right)} \times \sum_{i=0}^{\infty} a_i \left[2^{N/2+i}\Gamma\left(\frac{N}{2}+i, \frac{\gamma}{2}\right)\right] \tag{2.25}$$

where γ is the threshold determined according to the prescribed false-alarm probability. □

2.2.3 Low-SNR Regime

While the formula (2.25) appears quite involved, in the low-SNR regime it admits a very simple form that is compatible with the existing study of ED [Kay]. To see this, we need the next lemma, which provides an upper and lower bounds for the conditional detection probability $P_D(n_0)$

Lemma 2.5: Let $P_D(n_0)$ be defined in (2.24). Then we have

$$\frac{\Gamma\left(\frac{N}{2}, \gamma\left(\frac{1+SNR}{2}\right)\right)}{(1+SNR)^{n_0/2+1}\Gamma(N/2)} \le P_D(n_0) \le \frac{(1+SNR)^{(N-n_0)/2-1}\Gamma\left(\frac{N}{2}, \frac{\gamma}{2}\right)}{\Gamma(N/2)}. \tag{2.26}$$

[Proof]: See Appendix. □

To gain further insight based on (2.26), let us assume without loss of generality that the total number of samples N is even, so that $N/2$ is a positive integer. In this case, we have $\Gamma(N/2) = (N/2-1)!$ and $\Gamma(N/2, y) = (N/2-1)!e^{-y}\sum_{k=0}^{N/2-1}\frac{y^k}{k!}$ [16]. Hence (2.26) becomes

$$\frac{e^{-\gamma(1+SNR)/2}\sum_{k=0}^{N/2-1}\frac{[\gamma(1+SNR)/2]^k}{k!}}{(1+SNR)^{n_0/2+1}} \le P_D(n_0) \le (1+SNR)^{(N-n_0)/2-1}e^{-\gamma/2}\sum_{k=0}^{N/2-1}\frac{(\gamma/2)^k}{k!}. \tag{2.27}$$

In the low SNR regime, e.g., $SNR \to 0$, we have $1 + SNR \to 1$ and (2.27) then becomes

$$P_D(n_0) \to e^{-\gamma/2} \sum_{k=0}^{N/2-1} \frac{(\gamma/2)^k}{k!} = Q_{\chi^2_N}(\gamma). \tag{2.28}$$

With the aid of (2.28) and since the limiting probability is independent of n_0 , we have the following asymptotic result.

Proposition 2.6: Let P_D be the average detection probability defined in (2.25). Then we have

$$\lim_{SNR \to 0} P_D = Q_{\chi^2_N}(\gamma). \tag{2.29}$$

□

Recall from [15] that $Q_{\chi^2_N}(\gamma)$ is the detection probability for ED when $SNR = \sigma_s^2/\sigma_v^2 \approx 0$. In this case, the performance of ED can be very poor since the energy of the received signal in either hypothesis is very close to the noise floor. To further enhance the detection performance when SNR is low and the signal timing mismatch is present, robust ED schemes based on the Bayesian principle and the GLRT principle will be proposed in next two chapters.

2.3 Simulation Results

In the following simulations we consider the hypothesis signal model (1.2), in which the total number of samples is set to be $N = 200$ and the primary signal arrival time n_0 is uniformly distributed within $0 \leq n_0 \leq 199$. Note that the simulated results are obtained from 5000 Monte-Carlo runs. Figure 2.1 plots the ROC curves of ED (2.1), with SNR set to be -5 dB; Figure 2.2 plots the probability of detection P_D at various SNR levels, assuming that the false-alarm probability $P_{FA} = 0.05$. As can be seen from the figures, the derived analytic formula (2.25) closely matches the simulated results.

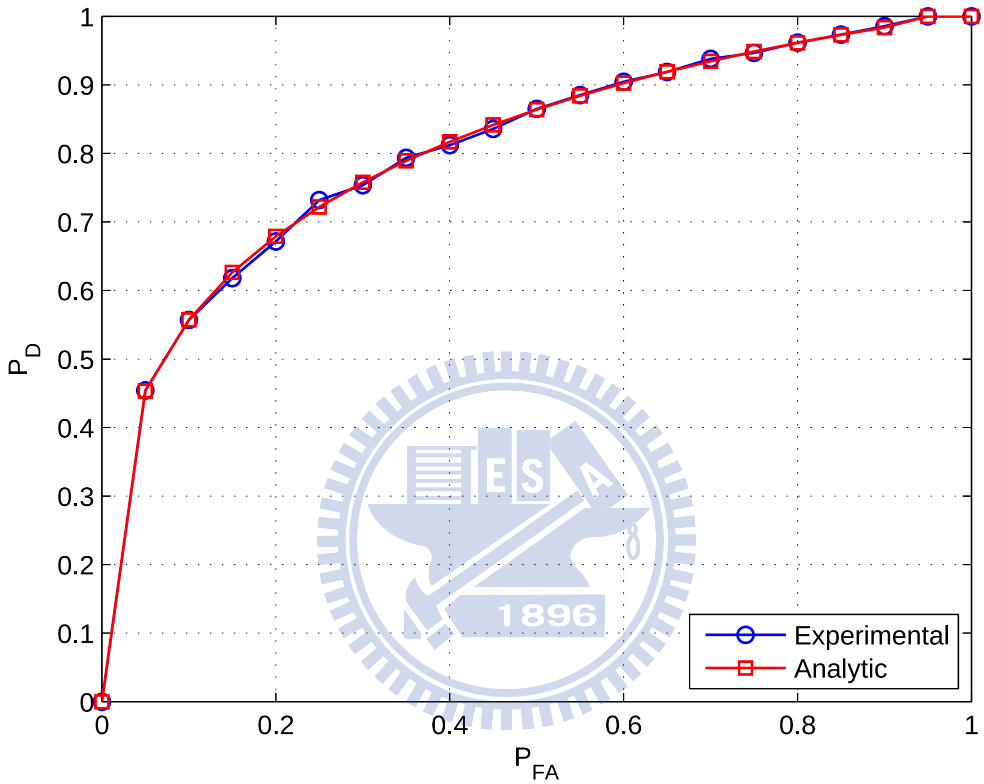

Figure 2.1: Analytic and experimental ROC curves of ED. ($N = 200$, SNR $= -5$ dB)

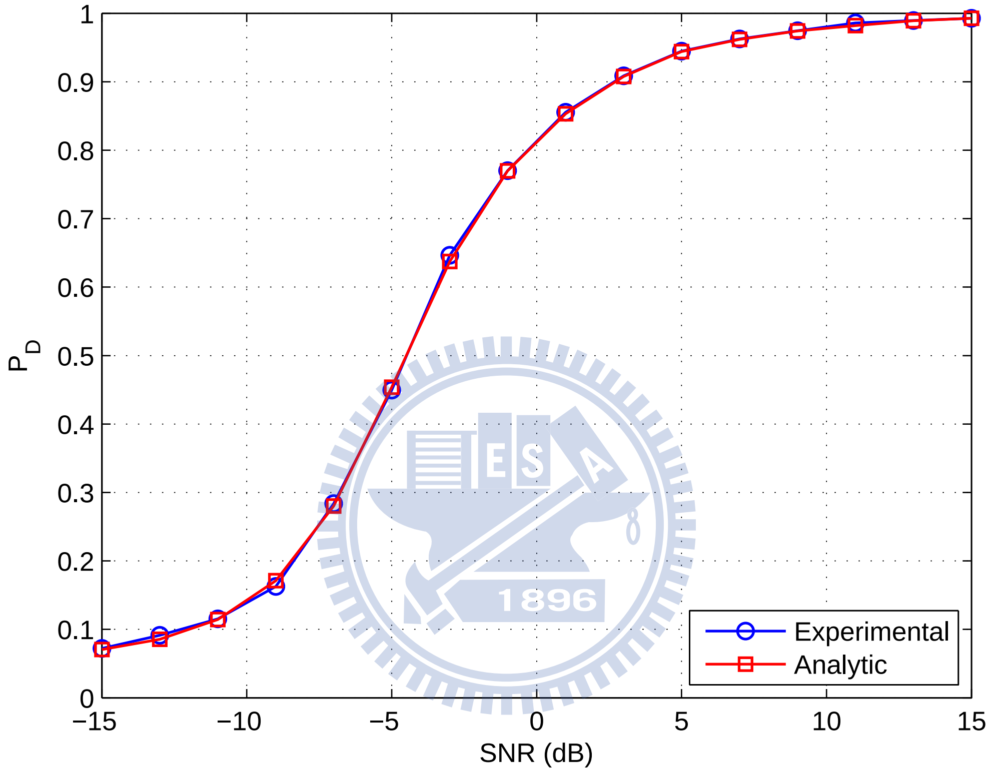

Figure 2.2: Detection probability P_D versus SNR. ($N = 200$, $P_{FA} = 0.05$)

Chapter 3

Bayesian Based Detection

3.1 The Test Statistic of Bayesian Detection

To exploit the prior statistical knowledge of n_0 for enhancing the detection performance, a typical approach is the Bayesian philosophy [15]. The conditional joint PDF of the data samples under two hypotheses $\mathcal{H}_0$ and $\mathcal{H}_1$ are

$$p(\mathbf{x}, \mathcal{H}_0) = \frac{1}{(2\pi\sigma_v^2)^{N/2}} \exp\left[-\frac{1}{2\sigma_v^2}\sum_{n=0}^{N-1}|x[n]|^2\right], \tag{3.1}$$

and

$$p(\mathbf{x}; n_0, \mathcal{H}_1) = \frac{1}{(2\pi\sigma_v^2)^{n_0/2}} \exp\left[-\frac{1}{2\sigma_v^2}\sum_{n=0}^{n_0-1}|x[n]|^2\right] \times \frac{1}{(2\pi(\sigma_v^2+\sigma_s^2))^{(N-n_0)/2}} \exp\left[-\frac{1}{2(\sigma_v^2+\sigma_s^2)}\sum_{n=n_0}^{N-1}|x[n]|^2\right] \tag{3.2}$$

The Bayesian test decides $\mathcal{H}_1$ if [15]

$$\begin{aligned}\frac{p(\mathbf{x}; n_0, \mathcal{H}_1)}{p(\mathbf{x}, \mathcal{H}_0)} &= \frac{\int p(\mathbf{x}|n_0, \mathcal{H}_1)p(n_0)dn_0}{p(\mathbf{x}, \mathcal{H}_0)} \\ &= \frac{\frac{1}{N}\sum_{n_0=0}^{N-1} \frac{1}{(2\pi\sigma_v^2)^{n_0/2}} e^{\left[-\frac{1}{2\sigma_v^2}\sum_{n=0}^{n_0-1}|x[n]|^2\right]} \times \frac{1}{(2\pi(\sigma_v^2+\sigma_s^2))^{(N-n_0)/2}} e^{\left[-\frac{1}{2(\sigma_v^2+\sigma_s^2)}\sum_{n=n_0}^{N-1}|x[n]|^2\right]}}{\frac{1}{(2\pi\sigma_v^2)^{N/2}} e^{\left[-\frac{1}{2\sigma_v^2}\sum_{n=0}^{N-1}|x[n]|^2\right]}} > \gamma.\end{aligned} \tag{3.3}$$

After some manipulations of (3.3), the test statistic of Bayesian detection can be represented as

$$\frac{1}{N}\sum_{n_0=0}^{N-1}\left(\frac{\sigma_v^2}{\sigma_v^2+\sigma_s^2}\right)^{(N-n_0)/2}\exp\left[\left(\frac{1}{2\sigma_v^2}-\frac{1}{2(\sigma_v^2+\sigma_s^2)}\right)\sum_{n=n_0}^{N-1}|x[n]|^2\right]>\gamma. \tag{3.4}$$

3.2 Simulation Results

The following simulation results are obtained from 5000 Monte-Carlo runs under the hypothesis signal model (1.2), in which the total number of samples is set to be $N = 200$ and the primary signal arrival time n_0 is uniformly distributed within $0 \leq n_0 \leq 199$. Figure 3.1 compares the ROC curves of ED (2.1) and the Bayesian based detection rule (3.4). Figures 3.2 and 3.3, respectively, compare P_D and $1 - P_{FA}$ curves (as a function of SNR) of the ED (2.1) and the Bayesian based solution (3.4); note that large values of $1-P_{FA}$ mean better channel utilization efficiency of secondary users [17]. The figures show that the Bayesian based solution (3.4), which takes into account the statistical knowledge of the primary signal arrival time, not only improves P_D but also leads to larger $1 - P_{FA}$, especially when SNR is low.

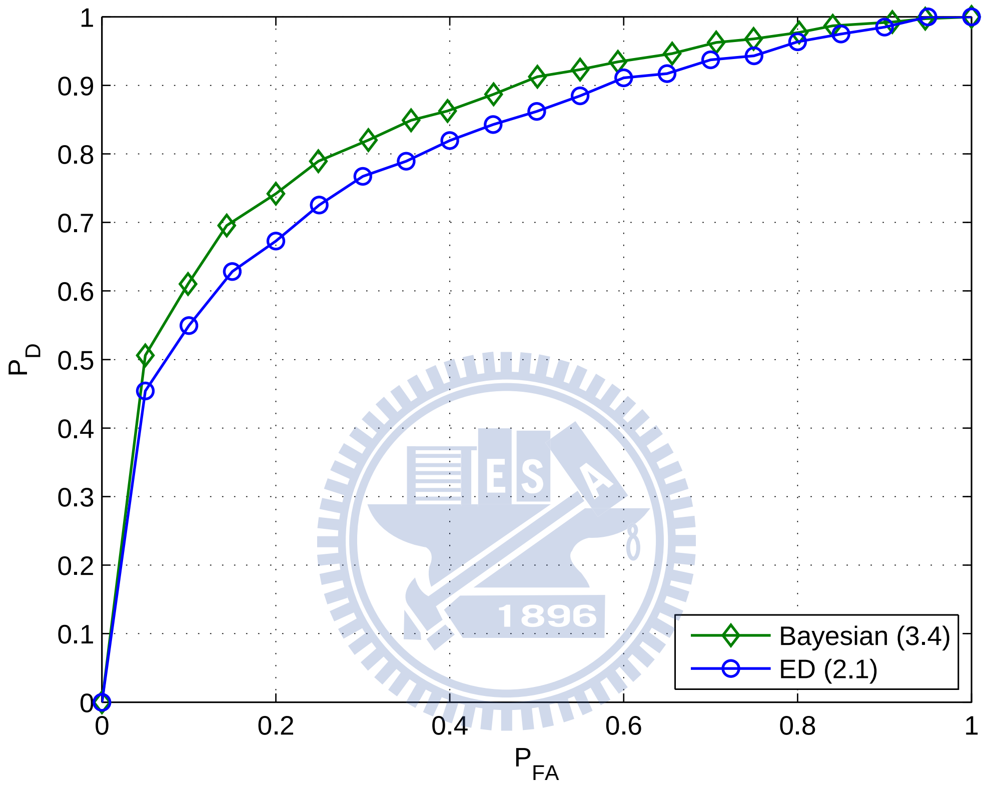

Figure 3.1: Experimental ROC curves of ED and Bayesian ED. ($N = 200$, SNR $= -5$ dB)

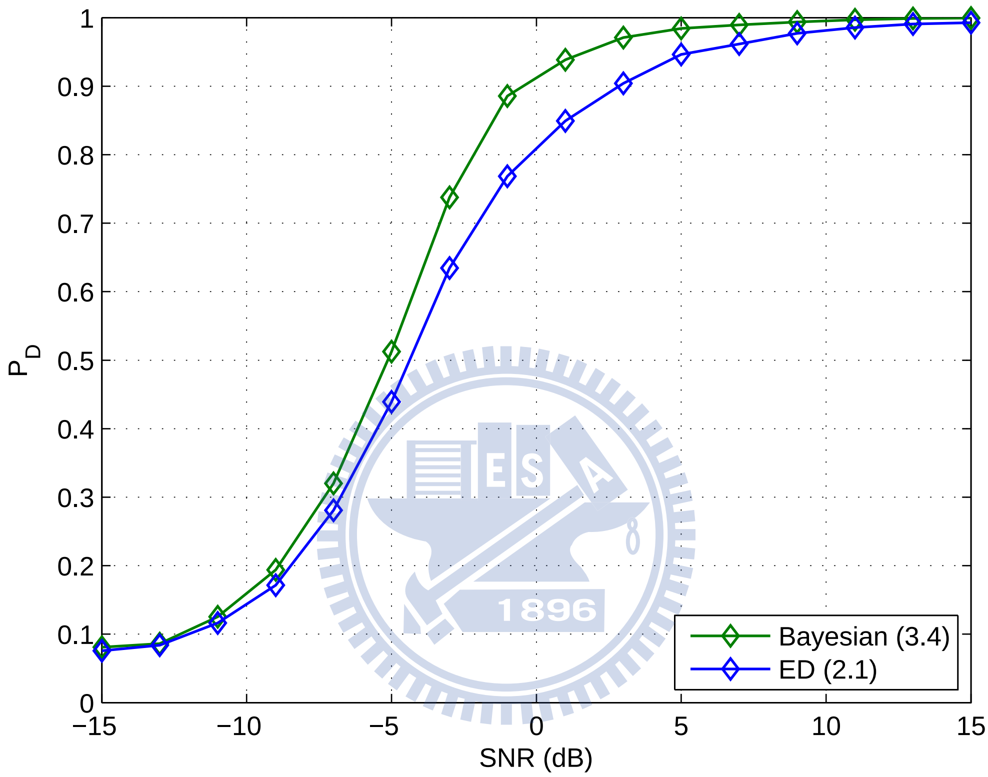

Figure 3.2: Detection probabilities P_D of ED and Bayesian ED versus SNR. ($N = 200$, $P_{FA} = 0.05$)

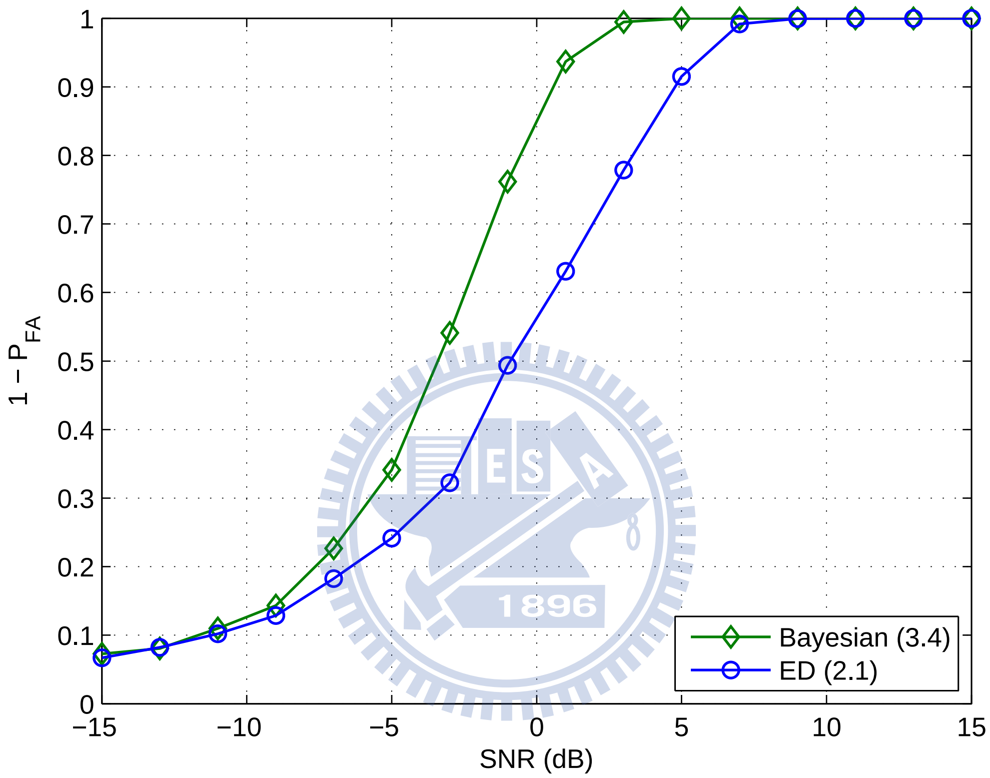

Figure 3.3: $1 - P_{FA}$ of ED and Bayesian ED versus SNR. ($N = 200$, $P_D = 0.95$)

Chapter 4

GLRT Based Detection

4.1 The Test Statistic of GLRT Based Detection

In chapter 3, we consider n_0 as a uniform random variable, and then propose a Bayesian based detection rule to deal with the timing mismatch. However, the prior statistic knowledge of n_0 is not always available at the receiver. Hence, instead of modeling n_0 as a random variable, an alternative approach is to consider time delay n_0 as a deterministic unknown, and resort to the GLRT based test rule for spectrum sensing. Recall the joint PDF of the data samples under the two hypotheses $\mathcal{H}_0$ and $\mathcal{H}_1$ are

$$p(\mathbf{x}, \mathcal{H}_0) = \frac{1}{(2\pi\sigma_v^2)^{N/2}} \exp\left[-\frac{1}{2\sigma_v^2}\sum_{n=0}^{N-1}|x[n]|^2\right], \tag{4.1}$$

and

$$p(\mathbf{x}; n_0, \mathcal{H}_1) = \frac{1}{(2\pi\sigma_v^2)^{n_0/2}} \exp\left[-\frac{1}{2\sigma_v^2}\sum_{n=0}^{n_0-1}|x[n]|^2\right] \times \frac{1}{(2\pi(\sigma_v^2+\sigma_s^2))^{(N-n_0)/2}} \exp\left[-\frac{1}{2(\sigma_v^2+\sigma_s^2)}\sum_{n=n_0}^{N-1}|x[n]|^2\right] \tag{4.2}$$

According to [15] and after some straightforward manipulations, the GLRT decides $\mathcal{H}_1$ if the test statistic exceeds a threshold γ

$$\begin{aligned} L_G(\mathbf{x}) &:= \max_{n_0} \ln \frac{p(\mathbf{x}; n_0, \mathcal{H}_1)}{p(\mathbf{x}, \mathcal{H}_0)} \\ &= \max_{n_0}\left\{\left(\frac{N-n_0}{2}\right)\ln\left(\frac{\sigma_v^2}{\sigma_v^2+\sigma_s^2}\right) + \left(\frac{1}{2\sigma_v^2} - \frac{1}{2(\sigma_v^2+\sigma_s^2)}\right)\sum_{n=n_0}^{N-1}|x[n]|^2\right\} > \gamma. \end{aligned} \tag{4.3}$$

Since the test statistic is maximized over all possible value of n_0, the primary user arrival time can also be estimated.

4.2 Performance Analysis

The probability of false-alarm of the test rule (4.3) is by definition given by

$$\begin{aligned} P_{FA} &= Pr\left\{\max_{n_0} \ln \frac{p(\mathbf{x}; n_0, \mathcal{H}_1)}{p(\mathbf{x}, \mathcal{H}_0)} > \gamma|\mathcal{H}_0\right\} \\ &= Pr\left\{\max_{n_0}\left\{\left(\frac{N-n_0}{2}\right)\ln\left(\frac{\sigma_v^2}{\sigma_v^2+\sigma_s^2}\right) + \left(\frac{1}{2\sigma_v^2} - \frac{1}{2(\sigma_v^2+\sigma_s^2)}\right)\sum_{n=n_0}^{N-1}|x[n]|^2\right\} > \gamma|\mathcal{H}_0\right\} \end{aligned} \tag{4.4}$$

and the probability of detection is

$$\begin{aligned} P_{D} &= Pr\left\{\max_{n_0} \ln \frac{p(\mathbf{x}; n_0, \mathcal{H}_1)}{p(\mathbf{x}, \mathcal{H}_0)} > \gamma|\mathcal{H}_1\right\} \\ &= Pr\left\{\max_{n_0}\left\{\left(\frac{N-n_0}{2}\right)\ln\left(\frac{\sigma_v^2}{\sigma_v^2+\sigma_s^2}\right) + \left(\frac{1}{2\sigma_v^2} - \frac{1}{2(\sigma_v^2+\sigma_s^2)}\right)\sum_{n=n_0}^{N-1}|x[n]|^2\right\} > \gamma|\mathcal{H}_1\right\}. \end{aligned} \tag{4.5}$$

However, neither the exact form of P_{FA} nor the exact form of P_D exist. We then try to derive a lower bound of P_D and that of P_{FA}.

The probability of false-alarm P_{FA} in (4.4) can be expressed as

$$P_{FA} = 1 - Pr\left\{\max_{n_0}\left\{\left(\frac{N-n_0}{2}\right)\ln\left(\frac{\sigma_v^2}{\sigma_v^2+\sigma_s^2}\right) + \left(\frac{1}{2\sigma_v^2} - \frac{1}{2(\sigma_v^2+\sigma_s^2)}\right)\sum_{n=n_0}^{N-1}|x[n]|^2\right\} \le \gamma|\mathcal{H}_0\right\} \tag{4.6}$$

and it will be lower bounded by

$$P_{FA} \ge 1 - \frac{1}{N}\sum_{n=0}^{N-1} Pr\left\{\left(\frac{N-n_0}{2}\right)\ln\left(\frac{\sigma_v^2}{\sigma_v^2+\sigma_s^2}\right) + \left(\frac{1}{2\sigma_v^2} - \frac{1}{2(\sigma_v^2+\sigma_s^2)}\right)\sum_{n=n_0}^{N-1}|x[n]|^2 \le \gamma|n_0, \mathcal{H}_0\right\} \tag{4.7}$$

Since σ_v^2, σ_s^2, and n_0 are known, (4.7) can be further rewritten as

$$\begin{aligned}
P_{FA} &\geq 1 - \frac{1}{N}\sum_{n=0}^{N-1} Pr\left\{\left(\frac{N-n_0}{2}\right)\ln\left(\frac{\sigma_v^2}{\sigma_v^2+\sigma_s^2}\right) + \left(\frac{1}{2\sigma_v^2} - \frac{1}{2(\sigma_v^2+\sigma_s^2)}\right)\sum_{n=n_0}^{N-1}|x[n]|^2 \leq \gamma | n_0, \mathcal{H}_0\right\} \\
&= 1 - \frac{1}{N}\sum_{n_0=0}^{N-1} Pr\left\{\sum_{n=n_0}^{N-1}|x[n]|^2 \leq \frac{\gamma - \left(\frac{N-n_0}{2}\right)\ln\left(\frac{\sigma_v^2}{\sigma_v^2+\sigma_s^2}\right)}{\left(\frac{1}{2\sigma_v^2} - \frac{1}{2(\sigma_v^2+\sigma_s^2)}\right)} | n_0, \mathcal{H}_0\right\} \\
&= 1 - \frac{1}{N}\sum_{n_0=0}^{N-1} Pr\left\{\frac{\sum_{n=n_0}^{N-1}|x[n]|^2}{\sigma_v^2} \leq \frac{\gamma - \left(\frac{N-n_0}{2}\right)\ln\left(\frac{\sigma_v^2}{\sigma_v^2+\sigma_s^2}\right)}{\sigma_v^2\left(\frac{1}{2\sigma_v^2} - \frac{1}{2(\sigma_v^2+\sigma_s^2)}\right)} | n_0, \mathcal{H}_0\right\} \\
&\overset{(d)}{=} 1 - \frac{1}{N}\sum_{n_0=0}^{N-1} P\left\{\frac{N-n_0}{2}, \frac{\gamma - \left(\frac{N-n_0}{2}\right)\ln\left(\frac{\sigma_v^2}{\sigma_v^2+\sigma_s^2}\right)}{2\sigma_v^2\left(\frac{1}{2\sigma_v^2} - \frac{1}{2(\sigma_v^2+\sigma_s^2)}\right)}\right\}.
\end{aligned} \tag{4.8}$$

where (d) follows since $\sum_{n=n_0}^{N-1}\frac{|x[n]|^2}{\sigma_v^2} \sim \chi_N^2$ and $P(\cdot,\cdot)$ is the regular Gamma function.

On the other hand, the probability of detection P_D in (4.5) will be similarly lower bounded by

$$\begin{aligned}
P_{D} &\geq 1 - \frac{1}{N}\sum_{n=0}^{N-1} Pr\left\{\left(\frac{N-n_0}{2}\right)\ln\left(\frac{\sigma_v^2}{\sigma_v^2+\sigma_s^2}\right) + \left(\frac{1}{2\sigma_v^2} - \frac{1}{2(\sigma_v^2+\sigma_s^2)}\right)\sum_{n=n_0}^{N-1}|x[n]|^2 \leq \gamma | n_0, \mathcal{H}_1\right\} \\
&= 1 - \frac{1}{N}\sum_{n_0=0}^{N-1} Pr\left\{\sum_{n=n_0}^{N-1}|x[n]|^2 \leq \frac{\gamma - \left(\frac{N-n_0}{2}\right)\ln\left(\frac{\sigma_v^2}{\sigma_v^2+\sigma_s^2}\right)}{\left(\frac{1}{2\sigma_v^2} - \frac{1}{2(\sigma_v^2+\sigma_s^2)}\right)} | n_0, \mathcal{H}_1\right\} \\
&= 1 - \frac{1}{N}\sum_{n_0=0}^{N-1} Pr\left\{\frac{\sum_{n=n_0}^{N-1}|x[n]|^2}{\sigma_v^2+\sigma_s^2} \leq \frac{\gamma - \left(\frac{N-n_0}{2}\right)\ln\left(\frac{\sigma_v^2}{\sigma_v^2+\sigma_s^2}\right)}{(\sigma_v^2+\sigma_s^2)\left(\frac{1}{2\sigma_v^2} - \frac{1}{2(\sigma_v^2+\sigma_s^2)}\right)} | n_0, \mathcal{H}_1\right\} \\
&= 1 - \frac{1}{N}\sum_{n_0=0}^{N-1} P\left\{\frac{N-n_0}{2}, \frac{\gamma - \left(\frac{N-n_0}{2}\right)\ln\left(\frac{\sigma_v^2}{\sigma_v^2+\sigma_s^2}\right)}{2(\sigma_v^2+\sigma_s^2)\left(\frac{1}{2\sigma_v^2} - \frac{1}{2(\sigma_v^2+\sigma_s^2)}\right)}\right\}.
\end{aligned} \tag{4.9}$$

4.3 Simulation Results

In the following simulations the total number of samples is set to be $N = 100$ and the Monte-Carlo run is 5000. For SNR = 5 dB, Figure 4.1 compares the ROC curves of the ED (2.1) and the GLRT (4.3) for two arrival time $n_0 = 56, 96$. It is seen from the figure that the performance of ED is poor for $n_0 = 96$, and, in this case, the GLRT (4.3) does significantly improve the detection probability. With fixed $n_0 = 96$ and $P_{FA} = 0.1$, Figure 4.2 plots the detection probability of ED (2.1) and the GLRT (4.3) as a function of SNR. As expected, the GLRT performs better over a wide range of SNR. By setting $P_D = 0.9$, Figure 4.3 plots $1 - P_{FA}$ versus SNR (with $n_0 = 96$), whereas Figure 4.4 depicts $1 - P_{FA}$ versus n_0 (with SNR= 0 dB) for ED (2.1) and GLRT (4.3). The figures show that the GLRT does enhance the spectrum utilization efficiency, especially when SNR is small to moderate and is large. Figure 4.5, Figure 4.6, and Figure 4.7 examine the tightness of the lower bound of P_D (4.9) by plotting ROC curves and P_D versus SNR respectively. As we can see, the lower bound is close to the simulated P_D when SNR is large.

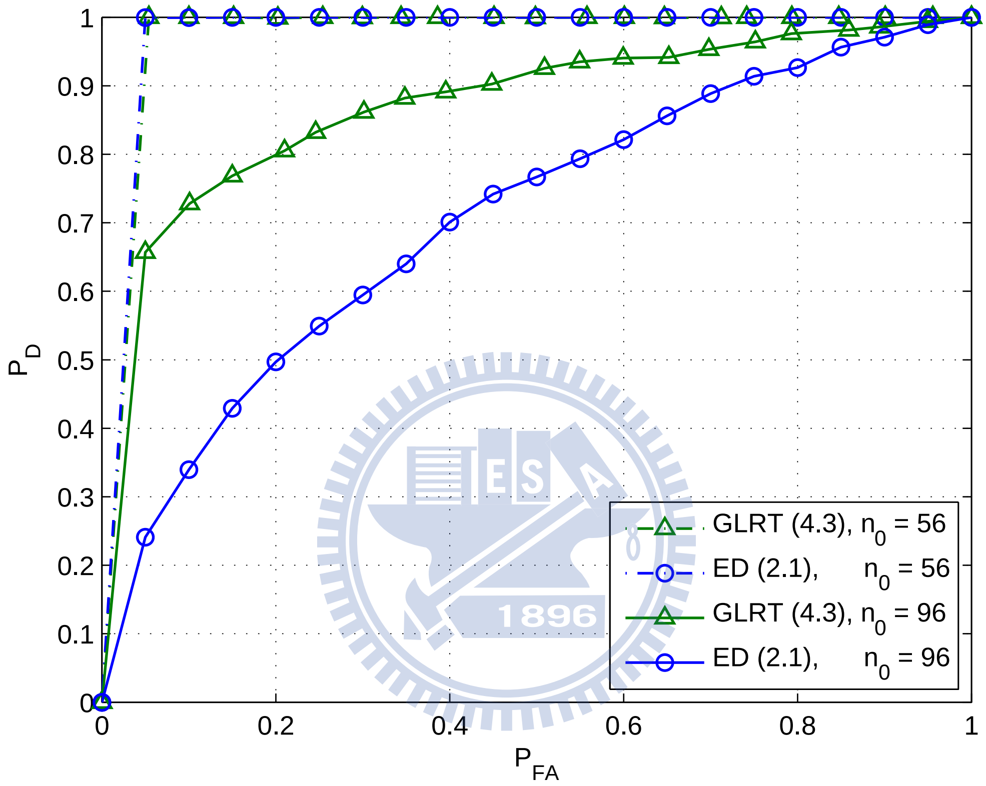

Figure 4.1: Experimental ROC curves of ED and GLRT ED with two different n_0. ($N = 100$, SNR = 5 dB)

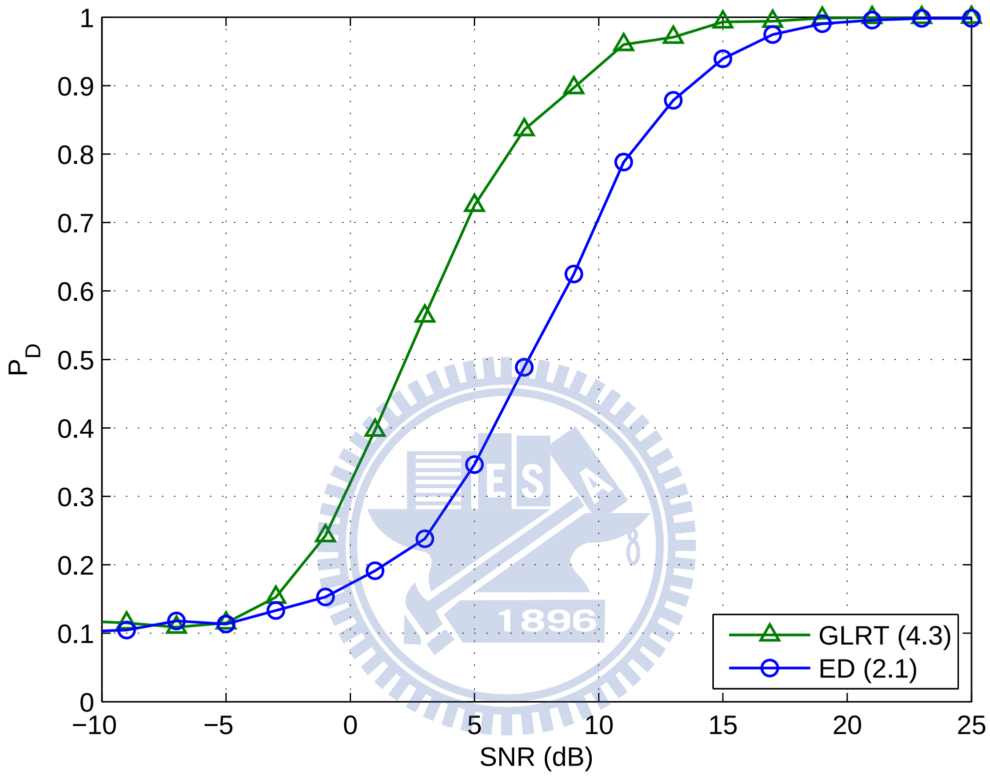

Figure 4.2: Detection probability P_D of ED and GLRT ED versus SNR. ($N = 100$, $n_0 = 96$, $P_{FA} = 0.1$)

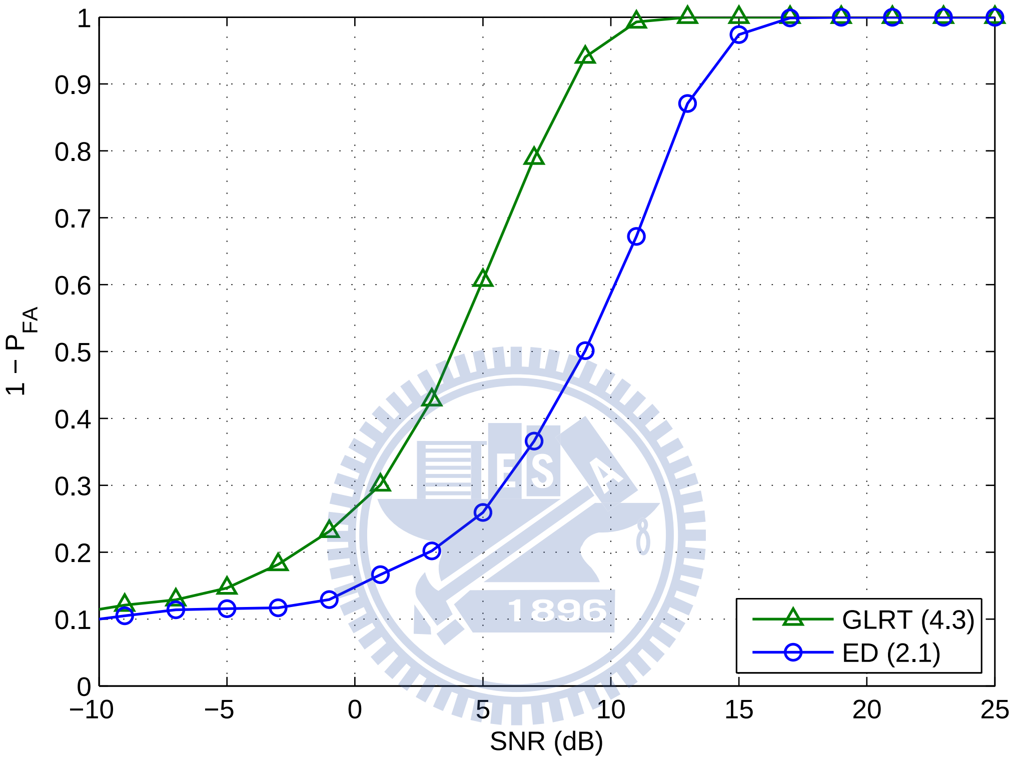

Figure 4.3: $1 - P_{FA}$ of ED and GLRT ED versus SNR. ($N = 100$, $n_0 = 96$, $P_D = 0.9$)

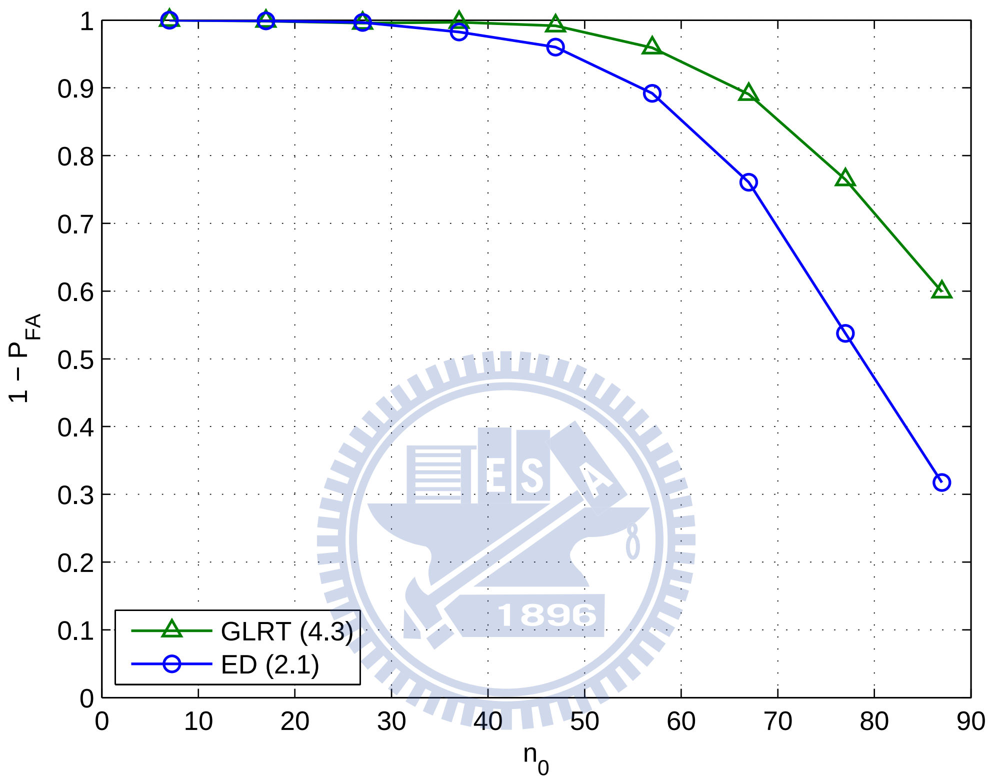

Figure 4.4: $1 - P_{FA}$ of ED and GLRT ED versus n_0. ($N = 100$, SNR = 5 dB, $P_D = 0.9$)

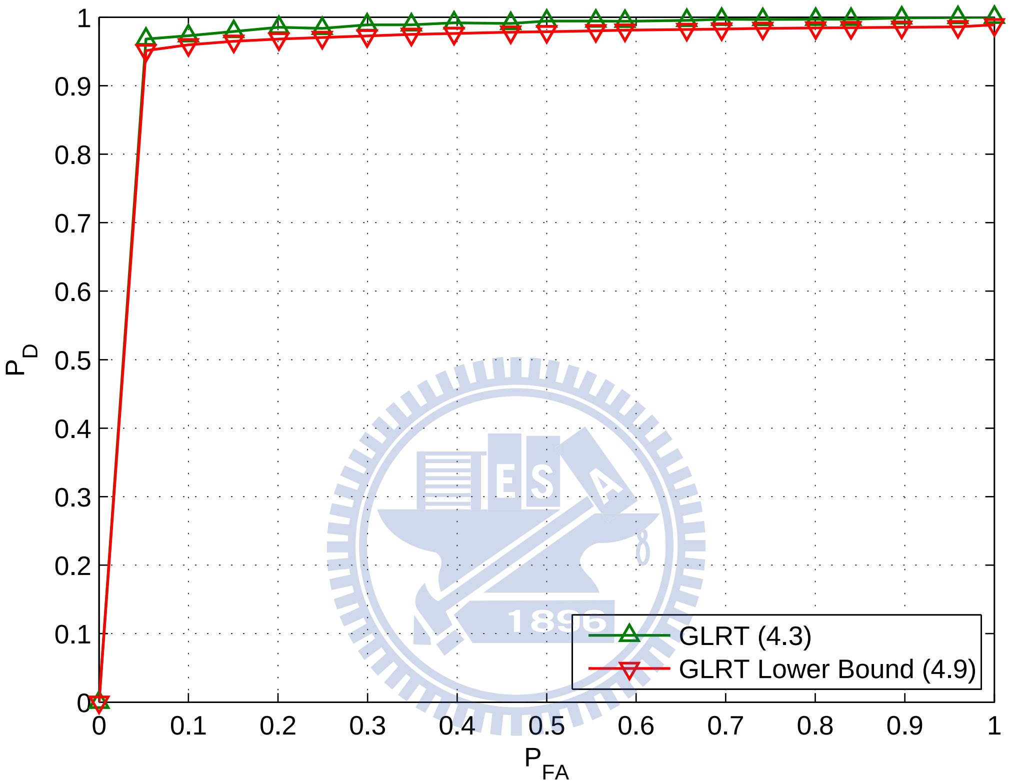

Figure 4.5: Experimental ROC curve and the lower bound of P_D of GLRT ED. ($N = 100$, SNR = 5 dB)

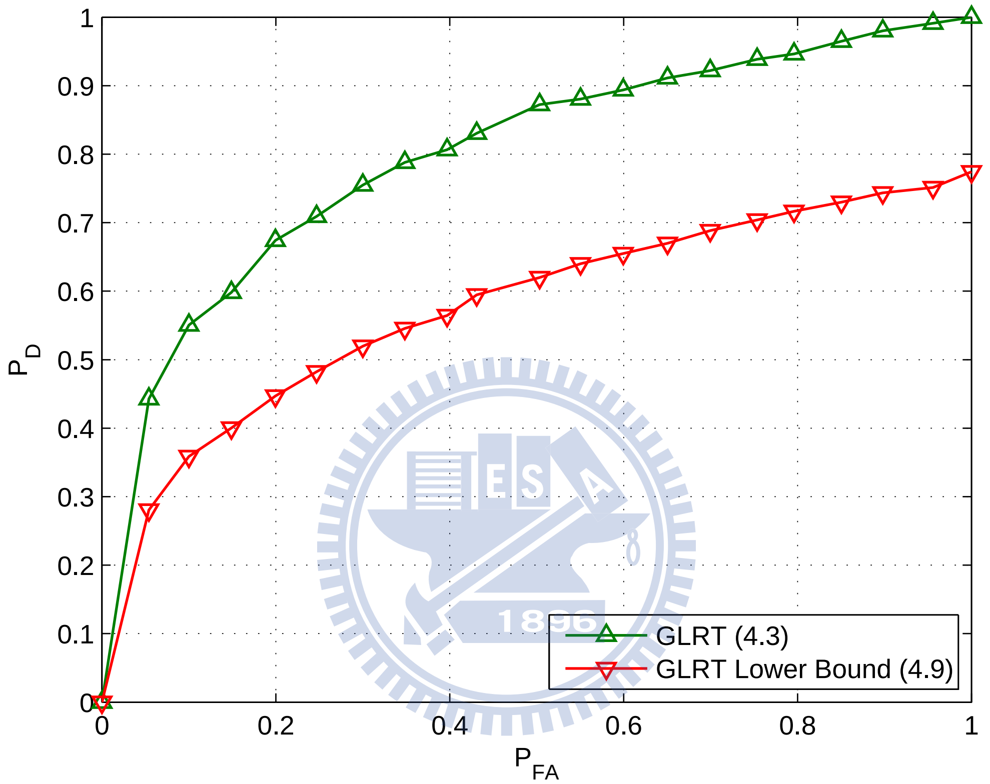

Figure 4.6: Experimental ROC curve and the lower bound of P_D of GLRT ED. ($N = 100$, SNR $= -5$ dB)

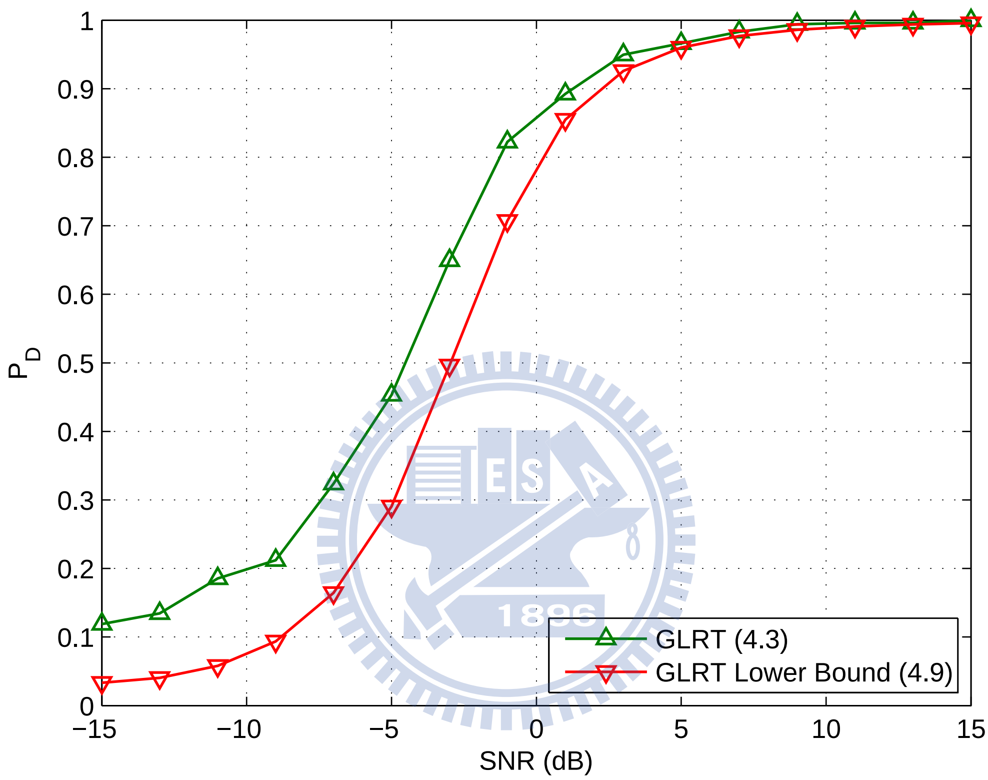

Figure 4.7: P_D and the lower bound of P_D of GLRT ED versus n_0. ($N = 100$, $P_{FA} = 0.1$)

Chapter 5

Conclusion

Spectrum sensing in the presence of unknown arrival time of the primary signal finds applications in many practical system scenarios and is thus an important issue in the study of CR networks. In this thesis we derive the exact formula of conditional detection probability given the primary signal arrival time for ED. when the primary signal arrival time is modeled as a uniform random variable over the observation interval, the exact detection probability for ED can be obtained by averaging the conditional detection probability over all possible arrival time. To further improve the detection performance against the timing uncertainty, we then propose a Bayesian based detection scheme. Moreover, when the prior statistical knowledge of the primary signal arrival time is not available, we consider the time delay as a deterministic unknown, and then proposed a GLRT based detection rule. Simulation results show that the Bayesian ED and the GLRT ED not only improve the detection probability but also reduce the false-alarm probability, thus enhancing the spectrum utilization in the considered asynchronous scenario. Future research will be dedicated to characterizing the ROC performance of the Bayesian scheme and extending the current results to the cooperative sensing scenario.

Appendix A

Proof of Lemma 2.5

We first observe that $p(x)$ in (2.19) satisfies

$$e^{-x/2} \times e^{-SNRx/2} \int_0^x \tau^{(N-n_0)/2-1}(x-\tau)^{n_0/2-1} d\tau \leq p(x) \leq e^{-x/2} \int_0^x \tau^{(N-n_0)/2-1}(x-\tau)^{n_0/2-1} d\tau. \tag{A.1}$$

Since

$$\int_0^x \tau^{(N-n_0)/2-1}(x-\tau)^{n_0/2-1} d\tau = x^{(N-n_0)/2-1}u(x) * x^{n_0/2-1}u(x), \tag{A.2}$$

we have

$$\begin{aligned}
\mathcal{L}\left\{\int_0^x \tau^{(N-n_0)/2-1}(x-\tau)^{n_0/2-1} d\tau\right\} &= \mathcal{L}\left\{x^{(N-n_0)/2-1}u(x)\right\} \times \mathcal{L}\left\{x^{n_0/2-1}u(x)\right\} \\
&= \frac{\Gamma((N-n_0)/2)}{s^{(N-n_0)/2}} \times \frac{\Gamma(n_0/2)}{s^{n_0/2}} \\
&= \frac{\Gamma((N-n_0)/2)\Gamma(n_0/2)}{s^{N/2}}.
\end{aligned} \tag{A.3}$$

By taking the inverse Laplace transform of both sides of (A.3) we have

$$\int_0^x \tau^{(N-n_0)/2-1}(x-\tau)^{n_0/2-1} d\tau = \Gamma((N-n_0)/2)\Gamma(n_0/2)\mathcal{L}^{-1}\left\{\frac{1}{s^{N/2}}\right\} = \frac{\Gamma((N-n_0)/2)\Gamma(n_0/2)}{\Gamma(N/2)} x^{N/2-1} \tag{A.4}$$

where the last equality holds due to Lemma 2.1. With the aid of (A.4), (A.1) becomes

$$\frac{\Gamma((N-n_0)/2)\Gamma(n_0/2)}{\Gamma(N/2)} x^{N/2-1} e^{-(1+SNR)x/2} \leq p(x) \leq \frac{\Gamma((N-n_0)/2)\Gamma(n_0/2)}{\Gamma(N/2)} x^{N/2-1} e^{-x/2}. \tag{A.5}$$

Based on (A.5), we have

$$
\begin{aligned}
P_D(n_0) &= \frac{(1+SNR)^{-(N-n_0)/2}}{\sqrt{2^N}\Gamma(n_0/2)\Gamma\left((N-n_0)/2\right)} \int_\gamma^\infty p(x)dx \\
&\geq \frac{(1+SNR)^{-(N-n_0)/2}}{\sqrt{2^N}\Gamma(N/2)} \int_\gamma^\infty x^{N/2-1} e^{-(1+SNR)x/2} dx \\
&\overset{(a)}{=} \frac{(1+SNR)^{-(N-n_0)/2}}{\sqrt{2^N}\Gamma(N/2)} \left(\frac{1+SNR}{2}\right)^{-N/2} \Gamma\left(\frac{N}{2}, \gamma\frac{1+SNR}{2}\right) \\
&= \frac{\Gamma\left(\frac{N}{2}, \gamma\frac{1+SNR}{2}\right)}{(1+SNR)^{n_0/2+1}\Gamma(N/2)}, \qquad (A.6)
\end{aligned}
$$

where (a) follows since $\int_\gamma^\infty x^{\nu-1}e^{-\mu x}dx = \mu^{-\nu}\Gamma(\nu, \mu\gamma)$ [16]. Similarly we have

$$
P_D(n_0) \leq \frac{(1+SNR)^{(N-n_0)/2-1}}{\Gamma(N/2)} \Gamma\left(\frac{N}{2}, \frac{\gamma}{2}\right). \qquad (A.7)
$$

The assertion follows from (A.6) and (A.7). □

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