國 立 交 通 大 學

電信工程研究所

碩 士 論 文

網路編碼上行協同多點傳輸系統之功率控制策略

Power Control Strategies for Network Coded Uplink CoMP Systems

研 究 生：蕭永宗

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中 華 民 國 一百 年 五 月

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摘　　　要

下一代通訊系統中要求更高的資料傳輸速率及更可靠的通訊，由於嚴重的路經衰減及干擾，位於蜂巢式網路邊界的用戶很難達到上述的要求。近來，Wang 和 Giannakis 提出 Complex Field Network Coding 來增加多用戶通訊的吞吐量，然而，他們的方法不能為用戶提供差異化服務。其中，差異化服務是指保證基地台內的用戶能穩定的通訊。這些因素促使我們設計上行系統中繼器的功率分配來幫助用戶達到最小的誤碼率，我們更進一步引入最佳化方法來解決這個問題，模擬結果顯示，我們的方法可在蜂巢網路中提供差異化服務，除此之外，這個方法可以達到無干擾下單一使用者的效能。

Power Control Strategies for Network Coded Uplink CoMP Systems

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ABSTRACT

Higher data rates and more reliable communications are required in next generation communication systems. But the subscribers on the cellular boundary are difficult to attain these requirements due to the severe path loss and interference. Recently, the complex field network coding (CNFC) method has been proposed by Wang and Giannakis for multiuser communications to enhance the throughput. However, their method can't provide users with differentiated services. The differentiated services should guarantee that the subscribers have reliable communications in their correspond cellular networks. These considerations motivate us to design a power allocation precoder for a relay node employed in a uplink network to help subscribers, so as to minimize their BERs. We further introduce an optimization method to address this problem. Simulations show that our method can provide the differentiated service in the cellular network. In addition to providing users with differentiated service, our scheme always achieves the single user's bound that is based on a no-interference assumption.

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Higher data rates and more reliable communications are required in next generation communication systems. But the subscribers on the cellular boundary are difficult to attain these requirements due to the severe path loss and interference. Recently, the complex field network coding (CNFC) method has been proposed by Wang and Giannakis for multiuser communications to enhance the throughput. However, their method can't provide users with differentiated services. The differentiated services should guarantee that the subscribers have reliable communications in their correspond cellular networks. These considerations motivate us to design a power allocation precoder for a relay node employed in a uplink network to help subscribers, so as to minimize their BERs. We further introduce an optimization method to address this problem. Simulations show that our method can provide the differentiated service in the cellular network. In addition to providing users with differentiated service, our scheme always achieves the single user's bound that is based on a no-interference assumption.

Chapter 1

Introduction

In view of the demands of the 3GPP LTE-Advanced [1] and WiMax systems for high data rates and reliable communications, we in this work apply a relay-based cooperative scheme in a multi-cell uplink network. Employing relays in the system can have many benefits such as enhancing transmission coverage, exploiting the spatial diversity and other benefits. On the other hand, it also can efficiently improve the destination's received SINRs. In other words, relaying techniques are very useful for wireless communications, especially when the subscribers are located on the cellular boundary. But in this specific environment, there isn't just an intra-cell interference but also inter-cell interference [2]. And the subscribers on the cell boundary usually need to consume more power for communication with their own base station than ones within the cell do. Therefore, the boundary issue about how to control the users' power for improving their transmit SINRs becomes a very important topic.

On the other hand, the topologies of the 4G communication systems are more complicated than other conventional communication systems. If simply applying traditional relaying schemes into the 4G systems, the transmission efficiency will decrease with the number of subscribers, because a relay node equipped with a single antenna can serve only one subscriber each transmission round from the viewpoint of the degree of freedom. But recently, there are a few publications that use the network coding to address this problem [3–6]. Generally, the ideal of network coding was proposed for the noiseless wireless networks to enhance capacity [7]. The publication [8] used the max-flow min-cut theory to derive the network capacity based on linear network coding. In [9], the author proposed a practical network coding and further implemented it. Simulations in [9] show that the scheme with this coding method can almost achieve the proposed theoretical optimal performance. Besides, more and more publications extend the network coding method to applications of wireless cooperative communications, such as XOR network coding [3], nonlinear network coding [4], analog network coding [5], and complex field network coding [6]. Due to the broadcasting nature of wireless networks, network coding becomes more and more useful in the field of cooperative communications.

Not only can the relaying mechanism and network coding scheme improve throughput but also there exists another popular technique that can improve the spectrum efficiency in 4G communication systems called network MIMO [10–14]. This scheme collects some resources from base stations to exploit the MIMO-like potential. In the uplink Network MIMO system, there usually exists a specific center node, via backhauls collecting all the signal packets received by the coordinated BSs or other useful data for performing a multi-user detection. In fact, the scheme can be viewed as a virtual MIMO system, like V-BLAST. However, the idea of virtual MIMO for the uplink Network MIMO will become more and more impractical with the amount of the data that need to be exchanged between the center and the BSs, especially when the users require higher data rates. Therefore, for reality, our CoMP systems only coordinates users' transmissions but doesn't exchange any information among the BSs.

Besides, Wang and Giannakis' publication [6] combines the complex field network coding methods and precoder mechanism. The complex field network coding method can provide higher degree of freedom than other network coding methods. In their scenario, the relay node has users-to-relay and relay-to-destination CSIs to design the precoder by itself. This precoding method perform well and is easy to use when the channel qualities between each user and the destination are equal. But when the channel qualities are not equal anymore, the user's BER performances will become poor, and it's hard to exploit the diversity in finite SNR. In fact, their method can't provide the user's differentiated services. The differentiated services can guarantee the reliable communications for the subscribers on the cellular boundary.

In our work, we concentrate on the scenario of the uplink relaying CoMP systems having users on the cell boundary. We first derive the system BER conditioned on all the channel states. Later, we also derive the subscriber's BER at the corresponding BS which is a marginal case of the system error probability. The above two results can be bounded as a sum of exponential functions. And it's a function of the precoder. However, the function is too complicated to average the channel effect. We apply the optimization technique to solve this complicated problem. The problem can be modeled as minimizing the BER function of the channel state and being subject to the total power constraint at relay node. We have known that the BER function is a sum of exponential functions and the power constraint is a posynomial function. The optimization problem can be dealt with using the geometric programming (GP) [15–17]. However,the GP requires posynomial function in the exponents. But the result in this work is difficult to attain the requirement. To move on, we use another more powerful optimization method which is called signomial programming (SP) [18]. SP is an extension version of GP. However, SP is also not a convex optimization problem and there doesn't exist powerful tools to solve it so far. Fortunately, we apply a technique which is called condensed programming to transform the SP problem into a GP one. The condensed programming is based on the arithmetic-geometric inequality. After the manipulations, our problem can be addressed by the solver CVX [19]. Simulations show that the CVX and exhaustive search have almost the same BER performance. And the subscriber's BER at the corresponding BS isn't influenced by the inter-cell interference in any topology. For comparison, we introduce a single user's BER bound calculated based on the assumption that the subscriber doesn't suffers interference from other sources any more. The subscriber at the corresponding BS in our scheme almost achieves the single user's BER bound. Both on symmetric and asymmetric channel scenarios, our method can provide the differentiated service in the cellular network.

Chapter 2

System Model and Error Analysis

2.0.1 System Model

The system model is showed on Figure 2.1. The wireless relay network consists of two subscribers, two relays, and two base stations. All the nodes are equipped with single antenna, and transmit packets in a half-duplex mode. Besides, the two subscribers belong to different cells and their signals would interfere with each other. They would like to transmit their signals to their own base stations, and can be assisted by the relay nodes, respectively. We apply the complex field network coding (CNFC) scheme in our system. And the precoders at the relay nodes are developed to ensure that the subscribers can get differentiated services. Due to the differentiated services, the performance of the subscribers in the cell are better than the other subscribers. In the first time slot, S_1 and S_2 broadcast their signal b_1x_1 and b_2x_2 to the relays and the base stations simultaneously and the coefficient b_1 and b_2 [20] are assumed to be known at all nodes. And the known coefficients b_1 and b_2 are drawn from the complex field which can make sure $b_1x_1 + b_2x_2 \neq b_1x_2 + b_2x_1$ when $x_1 \neq x_2$. The inequality property ensures that the relays and base stations can detect both x_1 and x_2 only by the received signal that transmitted from the subscribers. This can't be done by the other network coding methods like the XOR network coding and physical layer network coding, that need

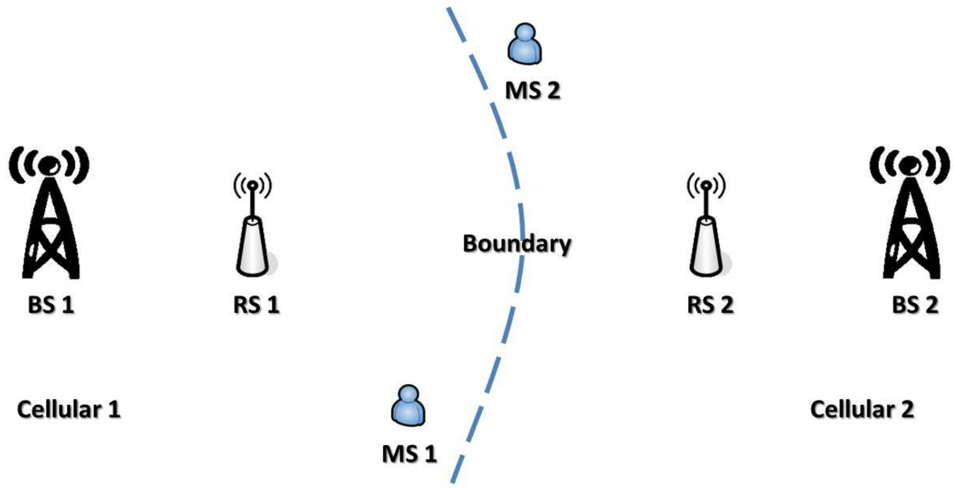

Figure 2.1: The system model has two cellular. Each cellular has one source, one relay, and one destination.

another associated signal to recover the original signal. For example, the XOR signal $x_1 \oplus x_2$ can get the original signal only by XOR x_1 or XOR x_2. The ideal degree of freedom of CNFC [6] is 1/2 symbol per channel use. However, the XOR network coding will take three time slots to perform an entire round for transmitting a message packet to base station and conventional cooperative system need four time slots to do it. So, the ideal degree of freedom of CNFC is better than XOR network coding and conventional cooperative system. The CNFC has good properties both on detection and degree of freedom. This properties are suited for our application. The received signals in the first time slot at relays and base stations are

$$\begin{aligned} y_{R_1,1} &= h_{S_1R_1}b_1x_1 + h_{S_2R_1}b_2x_2 + n_{R_1,1} \\ y_{R_2,1} &= h_{S_1R_2}b_1x_1 + h_{S_2R_2}b_2x_2 + n_{R_2,1} \\ y_{D_1,1} &= h_{S_1D_1}b_1x_1 + h_{S_2D_1}b_2x_2 + n_{D_1,1} \\ y_{D_2,1} &= h_{S_1D_2}b_1x_1 + h_{S_2D_2}b_2x_2 + n_{D_2,1} \end{aligned} \tag{2.1}$$

where $h_{ij} \sim CN\left(0, \sigma_{ij}^2\right)$, $i, j \in S_1, S_2, R_1, R_2, D_1, D_2$ denote the channel coefficient, $n_{i,j} \sim CN\left(0, \sigma_n^2\right)$, $i \in R_1, R_2, D_1, D_2$, $j \in 1, 2$ represent the noise term, and $y_{i_j,k}$ represents the received signal at node i_j in the time slot k. The relays use the maximum likelihood (ML) detection after receiving the combining signal. The ML detection at relay nodes are

$$\begin{aligned}(\hat{x}_1,\hat{x}_2)_{R_1} &= \arg\min_{x_1,x_2\in S(M)}\left\{\|y_{R_1,1}-h_{S_1R_1}b_1x_1-h_{S_2R_1}b_2x_2\|^2\right\}\\(\hat{x}_1,\hat{x}_2)_{R_2} &= \arg\min_{x_1,x_2\in S(M)}\left\{\|y_{R_2,1}-h_{S_1R_2}b_1x_1-h_{S_2R_2}b_2x_2\|^2\right\}\end{aligned} \tag{2.2}$$

where $S(M)$ is the possible constellation points set and M is the constellation size. In the second time slot, the relays transmit the detection signals $\hat{x}_1, \hat{x}_2$ to the base stations without checking the correctness. The received signals in the second time slot at base stations are

$$\begin{aligned}y_{D_1,2} &= h_{R_1D_1}\left(b_1p_1\hat{x}_1+b_2p_2\hat{x}_2\right)+n_{D_1,2}\\y_{D_2,2} &= h_{R_2D_2}\left(b_1p_1\hat{x}_1+b_2p_2\hat{x}_2\right)+n_{D_2,2}\end{aligned} \tag{2.3}$$

where $h_{R_iD_i} \sim CN\left(0,\sigma^2_{R_iD_i}\right), i \in 1,2$ represent the channel coefficient, and $n_{D_1,2}, n_{D_2,2} \sim CN\left(0,\sigma_n^2\right)$ denote the noise term. The p_1 and p_2 are power allocation factor at relay node which will affect the system performance. Designing the power allocation factor is the main problem in our work. However, the system doesn't change the sources power because each source wants transmit their signal to the their base station. If the system set one subscriber's power as zero, the corresponding base station will get poor performance.

2.0.2 Error Analysis

The error propagation will influence the detection result at base station because the relay doesn't check the correctness before transmission. So, the error probability at base station is conditional on the event which the relay detects the subscribers signal correct or not. There are four possible constellation points in the relay node. The four possible constellation points are

$$\begin{aligned}
d_{R_1} &= h_{S_1R}b_1x_1 + h_{S_2R}b_2x_2 \\
d_{R_2} &= h_{S_1R}b_1x_1 - h_{S_2R}b_2x_2 \\
d_{R_3} &= -h_{S_1R}b_1x_1 + h_{S_2R}b_2x_2 \\
d_{R_4} &= -h_{S_1R}b_1x_1 - h_{S_2R}b_2x_2
\end{aligned} \tag{2.4}$$

where the second line in (2.4) means the constellation point which the subscriber 1 transmits x_1 and the subscriber 2 transmits $-x_2$. The probability of the four cases can be calculated by the union bound. For example, if the subscriber 1 and subscriber 2 transmit x_1 and x_2 respectively and the relay detects as x_1 and $-x_2$, the probability can be expressed as

$$\begin{aligned}
P_{(x_1,x_2),(x_1,-x_2)} &= Q\left(\frac{\left|d_{R_1}-d_{R_2}\right|/2}{\sigma_n/\sqrt{2}}\right) \\
&= Q\left(\frac{\left|\sqrt{2}h_{S_2R}b_2x_2\right|}{\sigma_n}\right) \\
&\leq \frac{1}{2}\exp\left(\frac{\left|h_{S_2R}b_2x_2\right|^2}{\sigma_n^2}\right)
\end{aligned} \tag{2.5}$$

where $|d_{R_1} - d_{R_2}|$ means the Euclidean distance between the two constellation point d_{R_1} and d_{R_2}. The last equation in (2.5) uses the Chernoff bound which is $Q(x) \leq \frac{1}{2}\exp\left(-\frac{x^2}{2}\right)$. Similarly, the other three cases are

$$\begin{aligned}
&P_{(x_1,x_2),(-x_1,x_2)} \leq \frac{1}{2}\exp\left(-\frac{\left|h_{S_1R}b_1x_1\right|^2}{\sigma_n^2}\right) \\
&P_{(x_1,x_2),(-x_1,-x_2)} \leq \frac{1}{2}\exp\left(-\frac{\left|h_{S_1R}b_1x_1+h_{S_2R}b_2x_2\right|^2}{\sigma_n^2}\right) \\
&P_{(x_1,x_2),(x_1,x_2)} \approx 3 - P_{(x_1,x_2),(x_1,-x_2)} - P_{(x_1,x_2),(-x_1,x_2)} - P_{(x_1,x_2),(-x_1,-x_2)}
\end{aligned} \tag{2.6}$$

and the last equation uses the property of the total probability equals to one. This isn't a tight bound in low SNR region. But it can present the diversity performance in high SNR region. The union bound only associate with the relative position. So, the other cases can be express as the same method. There are some advantages of the CNFC scheme such as the relay doesn't need the CRC because it doesn't check the correctness before transmission. This can reduce the complexity of the relay node and the relay can process the signal more quickly.

Originally, the base stations use the ML detector to detect the signals which come from the subscriber nodes and relay node. The ML detector at base stations can be expressed as

$$
\begin{aligned}
(\hat{x}_1, \hat{x}_2)_{D_1} = \arg \max_{(x_1,x_2)\in S(M)} \Bigg\{ \sum_{(\tilde{x}_1,\tilde{x}_2)\in S(M)} & P_{(x_1,x_2),(\tilde{x}_1,\tilde{x}_2)} \times \\
& \exp\left(-\frac{|y_{D_1,1} - h_{S_1D_1}b_1x_1 - h_{S_2D_1}b_2x_2|^2 + |y_{D_1,2} - h_{R_1D_1}p_{11}b_1\tilde{x}_1 - h_{R_1D_1}p_{12}b_2\tilde{x}_2|^2}{2\sigma_n^2}\right)\Bigg\} \\
(\hat{x}_1, \hat{x}_2)_{D_2} = \arg \max_{(x_1,x_2)\in S(M)} \Bigg\{ \sum_{(\tilde{x}_1,\tilde{x}_2)\in S(M)} & P_{(x_1,x_2),(\tilde{x}_1,\tilde{x}_2)} \times \\
& \exp\left(-\frac{|y_{D_2,1} - h_{S_1D_2}b_1x_1 - h_{S_2D_2}b_2x_2|^2 + |y_{D_2,2} - h_{R_2D_2}p_{21}b_1\tilde{x}_1 - h_{R_2D_2}p_{22}b_2\tilde{x}_2|^2}{2\sigma_n^2}\right)\Bigg\}
\end{aligned}
\tag{2.7}
$$

where (x_1, x_2) is a candidate of the transmitting symbol, $(\tilde{x}_1, \tilde{x}_2)$ is a candidate of the detection symbol in the relay node, and $S(M)$ is the possible constellation set, where M is the constellation size. The ML detector has four terms (MPSK has M^2 terms) in its equation because the relay give four possible reverse signals to the destination. Based on the detection scheme, the destination must knows all the channel state information which includes sources to relay, sources to destination, and relay to destination. And the detection scheme consider all the possible constellation points which comes from sources and relay.

For convenience, we define the function f as

$$
\begin{aligned}
f = \sum_{(\tilde{x}_1,\tilde{x}_2)\in S(M)} & P_{(x_1,x_2),(\tilde{x}_1,\tilde{x}_2)} \times \\
& \exp\left(-\frac{|y_{D_1,1} - h_{S_1D_1}b_1x_1 - h_{S_2D_1}b_2x_2|^2 + |y_{D_1,2} - h_{R_1D_1}p_{11}b_1\tilde{x}_1 - h_{R_1D_1}p_{12}b_2\tilde{x}_2|^2}{2\sigma_n^2}\right)
\end{aligned}
\tag{2.8}
$$

There are four terms in (2.8). Each term multiply with a probability which represents the constellation point (x_1, x_2) decoding as $(\tilde{x}_1, \tilde{x}_2)$. In high region, the probability

$P_{(x_1,x_2),(x_1,x_2)}$ almost equal to one and the other three probabilities almost equal to zero when comparing with $P_{(x_1,x_2),(x_1,x_2)}$. The $P_{(x_1,x_2),(x_1,x_2)}$ means that the two sources transmit (x_1, x_2) and relay decode as the same constellation point. So, the (2.8) can be approximate as

$$\begin{aligned} f = & \sum_{(\tilde{x}_1,\tilde{x}_2)\in S(M)} P_{(x_1,x_2),(\tilde{x}_1,\tilde{x}_2)} \times \\ & \qquad \exp\left(-\frac{|y_{D_1,1}-h_{S_1D_1}b_1x_1-h_{S_2D_1}b_2x_2|^2+|y_{D_1,2}-h_{R_1D_1}p_{11}b_1\tilde{x}_1-h_{R_1D_1}p_{12}b_2\tilde{x}_2|^2}{2\sigma_n^2}\right) \\ \approx & P_{(x_1,x_2),(x_1,x_2)} \times \exp\left(-\frac{|y_{D_1,1}-h_{S_1D_1}b_1x_1-h_{S_2D_1}b_2x_2|^2+|y_{D_1,2}-h_{R_1D_1}p_{11}b_1x_1-h_{R_1D_1}p_{12}b_2x_2|^2}{2\sigma_n^2}\right) \\ \approx & \exp\left(-\frac{|y_{D_1,1}-h_{S_1D_1}b_1x_1-h_{S_2D_1}b_2x_2|^2+|y_{D_1,2}-h_{R_1D_1}p_{11}b_1x_1-h_{R_1D_1}p_{12}b_2x_2|^2}{2\sigma_n^2}\right) \end{aligned} \tag{2.9}$$

After approximation, there is only one term in the likelihood function. And the detection rule is also changed. The detection rule in destination 1 can be

$$\begin{aligned} (\hat{x}_1, \hat{x}_2)_{D_1} = \arg \min_{(x_1,x_2)\in S(M)} & \{|y_{D_1,1} - h_{S_1D_1}b_1x_1 - h_{S_2D_1}b_2x_2|^2 \\ & + |y_{D_1,2} - h_{R_1D_1}p_{11}b_1x_1 - h_{R_1D_1}p_{12}b_2x_2|^2\} \end{aligned} \tag{2.10}$$

and the detection rule in destination 2 is

$$\begin{aligned} (\hat{x}_1, \hat{x}_2)_{D_2} = \arg \min_{(x_1,x_2)\in S(M)} & \{|y_{D_2,1} - h_{S_1D_2}b_1x_1 - h_{S_2D_2}b_2x_2|^2 \\ & + |y_{D_2,2} - h_{R_2D_2}p_{21}b_1x_1 - h_{R_2D_2}p_{22}b_2x_2|^2\} \end{aligned} \tag{2.11}$$

We can find that the detection rule is only correlated with the constellation distance. For analysis, we define the function

$$\bar{f}\left(y_{D_1,1}, y_{D_1,2}, x_1, x_2\right) = |y_{D_1,1}-h_{S_1D_1}b_1x_1-h_{S_2D_1}b_2x_2|^2+|y_{D_1,2}-h_{R_1D_1}p_{11}b_1x_1-h_{R_1D_1}p_{12}b_2x_2|^2 \tag{2.12}$$

where $y_{D_1,1}$ and $y_{D_1,2}$ are the receiving signals in the first and second time slot at destination 1. Thus, the bit error probability (BEP) at destination 1 is

$$P_e = \frac{1}{4} \sum_{(x_1,x_2)\in S(M)} P_{(x_1,x_2)} \tag{2.13}$$

where $P_{(x_1,x_2)}$ is the error probability when the sources transmit (x_1, x_2) and $\frac{1}{4}$ means each signal transmits with equal probability in the source nodes. The $P_{(x_1,x_2)}$ can be expressed as

$$\begin{aligned} & P_{(x_1,x_2)} = \\ & \sum_{(\hat{x}_1,\hat{x}_2)\in S(M)} \Big[P_{(x_1,x_2),(\hat{x}_1,\hat{x}_2)} \times \\ & \Pr\Big\{ \bar{f}\left(h_{S_1D_1}b_1x_1 + h_{S_2D_1}b_2x_2 + n_{D_1,1}, h_{R_1D_1}b_1p_1\hat{x}_1 + h_{R_1D_1}b_2p_2\hat{x}_2 + n_{D_1,2}, x_1, x_2\right) \\ & \geq \min_{(\tilde{x}_1,\tilde{x}_2)\neq(x_1,x_2)} \left(\bar{f}\left(h_{S_1D_1}b_1x_1 + h_{S_2D_1}b_2x_2 + n_{D_1,1}, h_{R_1D_1}b_1p_1\hat{x}_1 + h_{R_1D_1}b_2p_2\hat{x}_2 + n_{D_1,2}, \tilde{x}_1, \tilde{x}_2\right)\right) \Big\} \Big] \end{aligned} \tag{2.14}$$

Originally, the term before the inequality in (2.14) should be the smallest one when there is no error occurring in the destination 1. Because the detection rule will choose the constellation point which closed to the candidate (x_1, x_2) in this case. So, the error event describes that the distance between transmitted constellation point and the candidate (x_1, x_2) isn't the smallest one.

The minimum function can be expanded as

$$\Pr\{a \geq \min(b, c, d)\} \leq \Pr\{a \geq b\} + \Pr\{a \geq c\} + \Pr\{a \geq d\} \tag{2.15}$$

So, the (2.14) can be rewrote as

$$
\begin{aligned}
&P_{(x_1,x_2)} \le \\
&\sum_{(\hat{x}_1,\hat{x}_2)\in S(M)} \Big[P_{(x_1,x_2),(\hat{x}_1,\hat{x}_2)} \times \\
&\sum_{(\tilde{x}_1,\tilde{x}_2)\neq(x_1,x_2)} \Pr\left\{\bar{f}\left(h_{S_1D_1}b_1x_1 + h_{S_2D_1}b_2x_2 + n_{D_1,1}, h_{R_1D_1}b_1p_1\hat{x}_1 + h_{R_1D_1}b_2p_2\hat{x}_2 + n_{D_1,2}, x_1, x_2\right)\right. \\
&\qquad\qquad \left.\ge \bar{f}\left(h_{S_1D_1}b_1x_1 + h_{S_2D_1}b_2x_2 + n_{D_1,1}, h_{R_1D_1}b_1p_1\hat{x}_1 + h_{R_1D_1}b_2p_2\hat{x}_2 + n_{D_1,2}, \tilde{x}_1, \tilde{x}_2\right)\right\}\Big]
\end{aligned} \tag{2.16}
$$

Now, we are going to analysis the probability function in (2.16).

$$
\begin{aligned}
&\Pr\left\{\bar{f}\left(h_{S_1D_1}b_1x_1 + h_{S_2D_1}b_2x_2 + n_{D_1,1}, h_{R_1D_1}b_1p_1\hat{x}_1 + h_{R_1D_1}b_2p_2\hat{x}_2 + n_{D_1,2}, x_1, x_2\right)\right. \\
&\quad \left.\ge \bar{f}\left(h_{S_1D_1}b_1x_1 + h_{S_2D_1}b_2x_2 + n_{D_1,1}, h_{RD}b_1p_1\hat{x}_1 + h_{R_1D_1}b_2p_2\hat{x}_2 + n_{D_1,2}, \tilde{x}_1, \tilde{x}_2\right)\right\} \\
&= \Pr\left\{|n_{D_1,1}|^2 + |h_{R_1D_1}b_1p_1\left(\hat{x}_1 - x_1\right) + h_{R_1D_1}b_2p_2\left(\hat{x}_2 - x_2\right) + n_{D_1,2}|^2\right. \\
&\qquad \ge |h_{S_1D_1}b_1\left(x_1 - \tilde{x}_1\right) + h_{S_2D_1}b_2\left(x_2 - \tilde{x}_2\right) + n_{D_1,1}|^2 \\
&\qquad \left. + |h_{R_1D_1}b_1p_1\left(\hat{x}_1 - \tilde{x}_1\right) + h_{R_1D_1}b_2p_2\left(\hat{x}_2 - \tilde{x}_2\right) + n_{D_1,2}|^2\right\} \\
&= \Pr\left\{-2((h_{S_1D_1}b_1\left(x_1 - \tilde{x}_1\right) + h_{S_2D_1}b_2\left(x_2 - \tilde{x}_2\right))^* n_{D_1,1})_R\right. \\
&\qquad -2((h_{R_1D_1}b_1p_1\left(x_1 - \tilde{x}_1\right) + h_{R_1D_1}b_2p_2\left(x_2 - \tilde{x}_2\right))^* n_{D_1,2})_R \\
&\qquad \ge |h_{S_1D_1}b_1\left(x_1 - \tilde{x}_1\right) + h_{S_2D_1}b_2\left(x_2 - \tilde{x}_2\right)|^2 \\
&\qquad - |h_{R_1D_1}b_1p_1\left(\hat{x}_1 - x_1\right) + h_{R_1D_1}b_2p_2\left(\hat{x}_2 - x_2\right)|^2 \\
&\qquad \left. + |h_{R_1D_1}b_1p_1\left(\hat{x}_1 - \tilde{x}_1\right) + h_{R_1D_1}b_2p_2\left(\hat{x}_2 - \tilde{x}_2\right)|^2\right\}
\end{aligned} \tag{2.17}
$$

Let $h_{S_1D_1}b_1\left(x_1 - \tilde{x}_1\right) = c_1$, $h_{S_2D_1}b_2\left(x_2 - \tilde{x}_2\right) = c_2$, $h_{R_1D_1}b_1p_1\left(x_1 - \tilde{x}_1\right) = c_3$, $h_{R_1D_1}b_2p_2\left(x_2 - \tilde{x}_2\right) = c_4$, $h_{R_1D_1}b_1p_1\left(\hat{x}_1 - x_1\right) = c_5$, and $h_{R_1D_1}b_2p_2\left(\hat{x}_2 - x_2\right) = c_6$. The above equation can be rewrote as

$$
\begin{aligned}
&\Pr\left\{-2((c_1 + c_2)^* n_{D_1,1})_R - 2((c_3 + c_4)^* n_{D_1,2})_R\right. \\
&\qquad \left.\ge |c_1 + c_2|^2 - |c_5 + c_6|^2 + |c_3 + c_5 + c_4 + c_6|^2\right\} \\
&= \Pr\left\{\frac{-\left((c_1+c_2)^* n_{D_1,1}\right)_R - \left((c_3+c_4)^* n_{D_1,2}\right)_R}{\sqrt{|c_1+c_2|^2 + |c_3+c_4|^2}}\right. \\
&\qquad \left.\ge \frac{|c_1+c_2|^2 - |c_5+c_6|^2 + |c_3+c_5+c_4+c_6|^2}{2\sqrt{|c_1+c_2|^2 + |c_3+c_4|^2}}\right\}
\end{aligned} \tag{2.18}
$$

Let $\Omega = \frac{-\left((c_1+c_2)^* n_{D_1,1}\right)_R - \left((c_3+c_4)^* n_{D_1,2}\right)_R}{\sqrt{|c_1+c_2|^2+|c_3+c_4|^2}}$. The mean and variance of Ω are

$$E\left[\Omega\right] = 0 \tag{2.19}$$

$$\begin{aligned}
&Var\left[\Omega\right] \\
&= \tfrac{1}{|c_1+c_2|^2+|c_3+c_4|^2} Var\left[-\left(\left(c_1+c_2\right)^*{}_R (n_{D_1,1})_R - \left(c_1+c_2\right)^*{}_i (n_{D_1,1})_i\right)\right. \\
&\qquad\qquad \left. - \left(\left(c_3+c_4\right)^*{}_R (n_{D_1,2})_R - \left(c_3+c_4\right)^*{}_i (n_{D_1,2})_i\right)\right] \\
&= \tfrac{1}{|c_1+c_2|^2+|c_3+c_4|^2} \left[\left(\left|\left(c_1+c_2\right)^*{}_R\right|^2 + \left|\left(c_1+c_2\right)^*{}_i\right|^2\right) \tfrac{\sigma_n^2}{2}\right. \\
&\qquad \left. + \left(\left|\left(c_3+c_4\right)^*{}_R\right|^2 + \left|\left(c_3+c_4\right)^*{}_i\right|^2\right) \tfrac{\sigma_n^2}{2}\right] \\
&= \tfrac{\sigma_n^2}{2}
\end{aligned} \tag{2.20}$$

Therefore, the distribution of Ω is $N\left(0, \frac{\sigma_n^2}{2}\right)$. So, the (2.18) can be rewrote as

$$\begin{aligned}
&\Pr\left\{\Omega \geq \tfrac{|c_1+c_2|^2-|c_5+c_6|^2+|c_3+c_5+c_4+c_6|^2}{2\sqrt{|c_1+c_2|^2+|c_3+c_4|^2}}\right\} \\
&= \Pr\left\{\tfrac{\sqrt{2}\Omega}{\sigma_n} \geq \tfrac{|c_1+c_2|^2-|c_5+c_6|^2+|c_3+c_5+c_4+c_6|^2}{\sqrt{2}\sigma_n\sqrt{|c_1+c_2|^2+|c_3+c_4|^2}}\right\} \\
&= Q\left(\tfrac{|c_1+c_2|^2-|c_5+c_6|^2+|c_3+c_5+c_4+c_6|^2}{\sqrt{2}\sigma_n\sqrt{|c_1+c_2|^2+|c_3+c_4|^2}}\right) \\
&\leq \tfrac{1}{2}\exp\left(-\tfrac{\left(|c_1+c_2|^2-|c_5+c_6|^2+|c_3+c_5+c_4+c_6|^2\right)^2}{4\sigma_n^2\left(|c_1+c_2|^2+|c_3+c_4|^2\right)}\right)
\end{aligned} \tag{2.21}$$

where the last inequality use the Chernoff bound which is $Q(x) \leq \frac{1}{2}\exp\left(-\frac{x^2}{2}\right)$. Based on above derivation, we use it in the (2.16). The result can be expressed as

$$\begin{aligned}
P_{(x_1,x_2)} \leq \sum_{(\hat{x}_1,\hat{x}_2)\in S(M)} &\left[P_{(x_1,x_2),(\hat{x}_1,\hat{x}_2)} \times \right. \\
&\left. \sum_{(\tilde{x}_1,\tilde{x}_2)\neq(x_1,x_2)} \tfrac{1}{2}\exp\left(-\tfrac{\left(|c_1+c_2|^2-|c_5+c_6|^2+|c_3+c_5+c_4+c_6|^2\right)^2}{4\sigma_n^2\left(|c_1+c_2|^2+|c_3+c_4|^2\right)}\right)\right]
\end{aligned} \tag{2.22}$$

where $c_1 = h_{S_1D_1}b_1\left(x_1-\tilde{x}_1\right)$, $c_2 = h_{S_2D_1}b_2\left(x_2-\tilde{x}_2\right)$, $c_3 = h_{R_1D_1}b_1p_1\left(x_1-\tilde{x}_1\right)$, $c_4 = h_{R_1D_1}b_2p_2\left(x_2-\tilde{x}_2\right)$, $c_5 = h_{R_1D_1}b_1p_1\left(\hat{x}_1-x_1\right)$, and $c_6 = h_{R_1D_1}b_2p_2\left(\hat{x}_2-x_2\right)$. The result is for destination 1. The destination 2 can use the similar way to get the BER function.

So far, we have derive the system BER for the destination 1 and destination 2.

However, we want to provide the differentiated service to the users in corresponding BSs. Based on (2.12), the BEP of user 1 at BS1 is

$$P_{e_1} = \frac{1}{4} \sum_{(x_1,x_2)\in S(M)} \bar{P}_{(x_1,x_2)} \tag{2.23}$$

where $\bar{P}_{(x_1,x_2)}$ is the error probability of user 1 when the sources transmit (x_1, x_2). The $\bar{P}_{(x_1,x_2)}$ can be expressed as

$$\begin{aligned}
&\bar{P}_{(x_1,x_2)} = \\
&\sum_{(\hat{x}_1,\hat{x}_2)\in S(M)} \Big[P_{(x_1,x_2),(\hat{x}_1,\hat{x}_2)}\times \\
&\Pr\Big\{\min_{\bar{x}_2\in S_2(M)} \bar{f}\left(h_{S_1D_1}b_1x_1 + h_{S_2D_1}b_2x_2 + n_{D_1,1}, h_{R_1D_1}b_1p_1\hat{x}_1 + h_{R_1D_1}b_2p_2\hat{x}_2 + n_{D_1,2}, x_1, \bar{x}_2\right) \\
&\geq \min_{\tilde{x}_1\neq x_1,\tilde{x}_2\in S_2(M)} \bar{f}\left(h_{S_1D_1}b_1x_1 + h_{S_2D_1}b_2x_2 + n_{D_1,1}, h_{R_1D_1}b_1p_1\hat{x}_1 + h_{R_1D_1}b_2p_2\hat{x}_2 + n_{D_1,2}, \tilde{x}_1, \tilde{x}_2\right)\Big\}\Big]
\end{aligned} \tag{2.24}$$

where $S_2(M)$ is the possible constellation set of user 2. The minimum function in (2.24) can be expanded as

$$\Pr\{\min(a,b) \geq \min(c,d)\} \leq \Pr(a \geq c) + \Pr(a \geq d) \tag{2.25}$$

Hence, the (2.24) can be rewrote as

$$\begin{aligned}
&\bar{P}_{(x_1,x_2)} \leq \\
&\sum_{(\hat{x}_1,\hat{x}_2)\in S(M)} \Big[P_{(x_1,x_2),(\hat{x}_1,\hat{x}_2)}\times \\
&\sum_{\tilde{x}_1\neq x_1,\tilde{x}_2\in S_2(M)} \Pr\Big\{\bar{f}\left(h_{S_1D_1}b_1x_1 + h_{S_2D_1}b_2x_2 + n_{D_1,1}, h_{R_1D_1}b_1p_1\hat{x}_1 + h_{R_1D_1}b_2p_2\hat{x}_2 + n_{D_1,2}, x_1, x_2\right) \\
&\qquad \geq \bar{f}\left(h_{S_1D_1}b_1x_1 + h_{S_2D_1}b_2x_2 + n_{D_1,1}, h_{R_1D_1}b_1p_1\hat{x}_1 + h_{R_1D_1}b_2p_2\hat{x}_2 + n_{D_1,2}, \tilde{x}_1, \tilde{x}_2\right)\Big\}\Big]
\end{aligned} \tag{2.26}$$

We can find that the (2.26) is a reduced form of (2.16). Based on the derivation of system error probability, the result can be expressed as

$$\bar{P}_{(x_1,x_2)} \leq \sum_{(\hat{x}_1,\hat{x}_2)\in S(M)} \Big[P_{(x_1,x_2),(\hat{x}_1,\hat{x}_2)} \times \sum_{\tilde{x}_1 \neq x_1, \tilde{x}_2 \in S_2(M)} \frac{1}{2} \exp\left(-\frac{\left(|c_1+c_2|^2 - |c_5+c_6|^2 + |c_3+c_5+c_4+c_6|^2\right)^2}{4\sigma_n^2\left(|c_1+c_2|^2+|c_3+c_4|^2\right)} \right) \Big] \tag{2.27}$$

where $c_1 = h_{S_1D_1} b_1 (x_1 - \tilde{x}_1)$, $c_2 = h_{S_2D_1} b_2 (x_2 - \tilde{x}_2)$, $c_3 = h_{R_1D_1} b_1 p_1 (x_1 - \tilde{x}_1)$, $c_4 = h_{R_1D_1} b_2 p_2 (x_2 - \tilde{x}_2)$, $c_5 = h_{R_1D_1} b_1 p_1 (\hat{x}_1 - x_1)$, and $c_6 = h_{R_1D_1} b_2 p_2 (\hat{x}_2 - x_2)$. Both results in this section are a sum of exponential functions. However, the results are too complicated to average the channel effect. In Chapter 3, we apply the optimization technique to solve this complicated problem.

Chapter 3

Optimization

The result in chapter 2 is a sum of exponential functions. And it's too complicated to average the channel effect. In this chapter, we apply the optimization technique to solve this complicated problem. The optimization of exponential functions can be dealt with by using GP. However,the GP requires a form of posynomial functions in the exponent. But our result is difficult to attain the requirement. To do so, we use another more powerful optimization method which is called SP. SP is an extension version of the GP and it's a nonlinear optimization method. In this section, we introduce the standard form and properties of GP at first. Second, the form of SP and transformation skills are presented.

The standard form of GP is minimizing a posynomial subject to posynomial upper bound inequality constraints and monomial equality constraints. The form can be expressed as

$$\begin{aligned} &minimize\ f_0\left(\mathbf{x}\right) \\ &subject\ to\ f_i\left(\mathbf{x}\right) \leq 1, i = 1, \ldots, m \\ &\qquad\qquad h_l\left(\mathbf{x}\right) = 1, l = 1, \ldots M \\ &variables\ \ \mathbf{x} \end{aligned} \tag{3.1}$$

where $h_l, l = 1, \ldots M$ are monomials

$$h_l(\mathbf{x}) = d x_1^{a_k^{(1)}} x_2^{a_k^{(2)}} \ldots x_n^{a_k^{(n)}} \tag{3.2}$$

where the multiplicative constant $d \geq 0$ and the exponential constants $a^{(j)} \in \Re, j = 1, \ldots, n$ and $x_i \geq 0, i = 1, \ldots, n$. And $f_i, i = 1, \ldots, m$, are posynomials

$$f_i(\mathbf{x}) = \sum_{k=1}^{K} d_{ik} x_1^{a_k^{(1)}} x_2^{a_k^{(2)}} \ldots x_n^{a_k^{(n)}} \tag{3.3}$$

which is a sum of monomials.

Note that the domain of monomials is strictly positive quadrant of $\Re^n$, where the objective functions and constraint functions are writing in terms of monomials, The domain of monomials implies that the optimal variables cannot be zero. GP in standard form isn't a convex optimization problem, because posynomials aren't convex functions. It can be used a logarithmic change of all variables and becomes a convex optimization problem. The details doesn't introduce in this work.

In our work, the objective function isn't a posynomial function. We can't directly use the GP for the problem. In the problem, the objective function and constraint functions are polynomials division. Polynomial is a form of posynomial with negative multiplicative coefficients. It can be divided to two parts which are monomials terms with positive multiplicative coefficients and negative multiplicative coefficients. Each parts are posynomial function and can applied the SP to optimize. The form of SP is

$$\begin{aligned} &minimize\ f_{01}(\mathbf{x}) - f_{02}(\mathbf{x}) \\ &subject\ to\ f_{i1}(\mathbf{x}) - f_{i2}(\mathbf{x}) \leq 1, i = 1, \ldots, m \end{aligned} \tag{3.4}$$

where $f_{i1}(\mathbf{x}), i = 1, \ldots, m$ are separated from those monomial terms with positive multiplicative coefficients and it's a posynomial function. We need to convert the signomial objective function into the form by GP. In Table 3.1, we can see that the major difference

Table 3.1: GP and SP comparison

	Geometric Programming	Signomial Programming
d_{ik}	$\Re_+$	$\Re$
$a^{(j)}$	$\Re$	$\Re$
x_j	$\Re_{++}$	$\Re_{++}$

between GP and SP lies in the multiplicative coefficients and other parameters are almost the same. Let t be an auxiliary variable and transfer the objective to minimization of t.

$$\begin{aligned} &\textit{minimize} \quad t \\ &\textit{subject to}\ f_{01}(\mathbf{x}) - f_{02}(\mathbf{x}) \leq t \\ &\quad f_{i1}(\mathbf{x}) - f_{i2}(\mathbf{x}) \leq 1, i = 1, \ldots, m \end{aligned} \tag{3.5}$$

This problem can be solved by the algorithm that has been proposed by Avriel and Williams. Consider the kth constraint of above signomial:

$$f_{i1}(\mathbf{x}) - f_{i2}(\mathbf{x}) \leq 1 \tag{3.6}$$

This can be rewrote as

$$\frac{f_{i1}(\mathbf{x})}{1 + f_{i2}(\mathbf{x})} \leq 1 \tag{3.7}$$

and the original objective function can be transformed as a constraint :

$$\frac{f_{01}(\mathbf{x})}{t + f_{02}(\mathbf{x})} \leq 1 \tag{3.8}$$

The problem becomes

$$\begin{aligned} &minimize \quad t \\ &subject\ to\ \tfrac{f_{01}(\mathbf{x})}{t+f_{02}(\mathbf{x})} \leq 1 \\ &\qquad\qquad \tfrac{f_{i1}(\mathbf{x})}{1+f_{i2}(\mathbf{x})} \leq 1, i = 1, \ldots, m \end{aligned} \tag{3.9}$$

However, the $\frac{f_{i1}(\mathbf{x})}{1+f_{i2}(\mathbf{x})}$ and $\frac{f_{01}(\mathbf{x})}{t+f_{02}(\mathbf{x})}$ aren't posynomial functions (A posynomial divided by a posynomial isn't a posynomial function). But we can use a technique which is called condensed programs to condense the posynomial function where in the denominator as a monomial function. As a result, the posynomial function divided by a monomial function is a posynomial function. And we can use the function which has been condensed to do the GP.

Before describing the condensed programs, we first introduce the arithmetic-geometric inequality. The condensed program is based on this inequality. The inequality describe that the weighted arithmetic mean of positive numbers $f_1, f_2, \ldots, f_n$ is greater than or equal to the geometric mean. And it can be wrote as follows

$$\sum_{i=1}^{n} f_i \geq \prod_{i=1}^{n} \left(\frac{f_i}{\omega_i}\right)^{\omega_i} \tag{3.10}$$

where

$$\sum_{i=1}^{n} \omega_i = 1 \tag{3.11}$$

$$\omega_i \geq 0, i = 1, 2, \ldots, n \tag{3.12}$$

Equality holds if and only if

$$\frac{f_1}{\omega_1} = \frac{f_2}{\omega_2} = \cdots = \frac{f_n}{\omega_n} \tag{3.13}$$

Based on the above inequality, we introduce the condensed programs. For an any posynomial function

$$f_i(\mathbf{x}) = \sum_{k=1}^{K} d_{ik} x_1^{a_k^{(1)}} x_2^{a_k^{(2)}} \dots x_n^{a_k^{(n)}} = \sum_{k=1}^{K} u_{ik}(\mathbf{x}) \tag{3.14}$$

the definition of condensed posynomial, formed at a point $\tilde{\mathbf{x}}$:

$$f_i(\mathbf{x}, \tilde{\mathbf{x}}) = \prod_{k=1}^{K} \left(\frac{d_{ik} x_1^{a_k^{(1)}} x_2^{a_k^{(2)}} \dots x_n^{a_k^{(n)}}}{\omega_{ik}(\tilde{\mathbf{x}})} \right)^{\omega_{ik}(\tilde{\mathbf{x}})} = \prod_{k=1}^{K} \left(\frac{u_{ik}(\mathbf{x})}{\omega_{ik}(\tilde{\mathbf{x}})} \right)^{\omega_{ik}(\tilde{\mathbf{x}})} \tag{3.15}$$

For a given $\tilde{\mathbf{x}} > 0$ we will choose the set of weights which is based on the arithmetic-geometric inequality :

$$\omega_{ik}(\tilde{\mathbf{x}}) = \frac{u_{ik}(\tilde{\mathbf{x}})}{f_i(\tilde{\mathbf{x}})} \tag{3.16}$$

There is an important property of the above result which the $f_i(\mathbf{x}, \tilde{\mathbf{x}})$ is a monomial function. The condensed posynomial must rule by the arithmetic-geometric inequality :

$$f_i(\mathbf{x}, \tilde{\mathbf{x}}) \le f_i(\mathbf{x}) \tag{3.17}$$

for any positive $\mathbf{x}$ and $\tilde{\mathbf{x}}$.

Back to the original problem in (3.9). We can apply the condensed programs to the denominator of the constraints. And it can be translated as :

$$\begin{aligned} t + f_{02}(\mathbf{x}) &\ge f_0(\mathbf{x}, \tilde{\mathbf{x}}_1) \\ 1 + f_{i2}(\mathbf{x}) &\ge f_i(\mathbf{x}, \tilde{\mathbf{x}}_2) \end{aligned} \tag{3.18}$$

And the optimization problem will be :

$$\begin{aligned} &minimize \quad t \\ &subject\ to\ \tfrac{f_{01}(\mathbf{x})}{f_0(\mathbf{x}, \tilde{\mathbf{x}}_1)} \le 1 \\ &\qquad \tfrac{f_{i1}(\mathbf{x})}{f_i(\mathbf{x}, \tilde{\mathbf{x}}_2)} \le 1, i = 1, \dots, m \end{aligned} \tag{3.19}$$

The program has the following properties :

- (3.19) is a standard form of GP because all constraints are posynomial function.
- At any point, $\tilde{\mathbf{x}}_1$ and $\tilde{\mathbf{x}}_2$ satisfy the constraints of (3.19) will satisfy the constraints of (3.9). This can be observed by the inequality (3.17), i.e.:

$$\begin{array}{l} \frac{f_{01}(\mathbf{x})}{t+f_{02}(\mathbf{x})} \leq \frac{f_{01}(\mathbf{x})}{f_0(\mathbf{x},\tilde{\mathbf{x}}_1)} \leq 1 \\ \frac{f_{i1}(\mathbf{x})}{1+f_{i2}(\mathbf{x})} \leq \frac{f_{i1}(\mathbf{x})}{f_1(\mathbf{x},\tilde{\mathbf{x}}_2)} \leq 1 \end{array} \tag{3.20}$$

- Inequality (3.20) implies that the feasible set of (3.19) is fully constrained in (3.9). So, the optimal solution to (3.19) will be a feasible solution to (3.9).

Based on the above introducing of SP, we want to apply it to our problem. In the chapter 2, we get a BER function which is formed by exponential terms. The general form is like :

$$f\left(p_1, p_2\right) = \sum_i c_i \exp\left(-\frac{w_i\left(p_1, p_2\right) - x_i\left(p_1, p_2\right)}{u_i\left(p_1, p_2\right) - v_i\left(p_1, p_2\right)}\right) \tag{3.21}$$

where the $u_i\left(p_1, p_2\right)$, $v_i\left(p_1, p_2\right)$, $w_i\left(p_1, p_2\right)$, and $x_i\left(p_1, p_2\right)$ are posynomial functions. And the exponential term, $-\frac{w_i(p_1,p_2)-x_i(p_1,p_2)}{u_i(p_1,p_2)-v_i(p_1,p_2)}$, is negative number because we derive it from the Chernoff bound. This can't use the SP directly. In the optimization problem, we use a auxiliary variable to substitute the negative term where in the exponential function. And the negative term becomes a constraint which is upper bounded by the auxiliary variable. This constraint is violate the definition of GP(The constraints are posynomial functions in GP. This implies all the constraints must great than zero.). For this problem, we change the function $f\left(p_1, p_2\right)$ as $\bar{f}\left(p_1, p_2\right)$:

$$\bar{f}\left(p_1, p_2\right) = \mathrm{e}^k \times f\left(p_1, p_2\right) = \sum_i c_i \exp\left(-\frac{w_i\left(p_1, p_2\right) - x_i\left(p_1, p_2\right)}{u_i\left(p_1, p_2\right) - v_i\left(p_1, p_2\right)} + k\right) \tag{3.22}$$

where the k is a constant which ensure the exponential terms in positive domain. And it must follow the rule :

$$k > \max_i \left(\frac{w_i(p_1, p_2) - x_i(p_1, p_2)}{u_i(p_1, p_2) - v_i(p_1, p_2)} \right) \tag{3.23}$$

Now, the original problem is

$$\begin{aligned} &\textit{minimize} \quad && f(p_1, p_2) \\ &\textit{subject to} \quad && p_1^2 + p_2^2 \le 2 \end{aligned} \tag{3.24}$$

and we transform it as

$$\begin{aligned} &\textit{minimize} \quad && \bar{f}(p_1, p_2) \\ &\textit{subject to} \quad && p_1^2 + p_2^2 \le 2 \end{aligned} \tag{3.25}$$

where $p_1^2 + p_2^2 \le 2$ is the total power constraint in the relay node. Introducing auxiliary variables t_i, we transform the above problem to the following equivalent problem :

$$\begin{aligned} &\textit{minimize} \quad && \sum_i c_i \exp(t_i) \\ &\textit{subject to} \quad && p_1^2 + p_2^2 \le 2 \\ & && -\tfrac{w_i(p_1,p_2) - x_i(p_1,p_2)}{u_i(p_1,p_2) - v_i(p_1,p_2)} + k \le t_i, i = 1, \ldots, K \end{aligned} \tag{3.26}$$

The second constraint can be rewrite as

$$\frac{x_i(p_1, p_2) + k u_i(p_1, p_2) + t_i v_i(p_1, p_2)}{t_i u_i(p_1, p_2) + k v_i(p_1, p_2) + w_i(p_1, p_2)} = \frac{x_i(p_1, p_2) + k u_i(p_1, p_2) + t_i v_i(p_1, p_2)}{Q_i(p_1, p_2)} \le 1 \tag{3.27}$$

Let $Q_i(p_1, p_2, \tilde{p}_1, \tilde{p}_2)$ denote the monomial function which obtained by condensing the posynomial function $Q_i(p_1, p_2)$ at the point $\tilde{p}_1, \tilde{p}_2$. The posynomial function will great than or equal to the monomial function.

$$Q_i(p_1, p_2) \ge Q_i(p_1, p_2, \tilde{p}_1, \tilde{p}_2) \tag{3.28}$$

and the weights of condensed program uses the method that is introduced in (3.16). We substitute the $Q_i(p_1, p_2, \tilde{p}_1, \tilde{p}_2)$ for $Q_i(p_1, p_2)$ in (3.26). The optimization becomes

$$\begin{aligned} &\textit{minimize} \quad && \sum_i c_i \exp(t_i) \\ &\textit{subject to} \quad && p_1^2 + p_2^2 \le 2 \\ & && \tfrac{x_i(p_1,p_2)+ku_i(p_1,p_2)+t_i v_i(p_1,p_2)}{Q_i(p_1,p_2,\tilde{p}_1,\tilde{p}_2)} \le 1, i = 1, \ldots, K \end{aligned} \tag{3.29}$$

The (3.29) is a standard form of GP. And the tool CVX can solve this kind of problems.

Algorithm 1 The Application of SP on Power Allocation Precoder for a Relay Node

1). Initialize n = 0
2). Set $k > \max_i \left(\frac{w_i(p_1,p_2)-x_i(p_1,p_2)}{u_i(p_1,p_2)-v_i(p_1,p_2)} \right)$
3). Random peak $p_1(0)$ and $p_2(0)$ in the constraint $p_1^2(0) + p_2^2(0) \le 2$
4). Calculate the weights of condensed program in (3.28)
5). Set Cvx_optimum $(0) = 10$
6). Set $m = 0$
while $m \le 99.9\%$ **do**
  Cvx_begin gp

$$\begin{aligned} &\textit{minimize} \quad && \sum_i c_i \exp(t_i) \\ &\textit{subject to} \quad && p_1^2 + p_2^2 \le 2 \\ & && \tfrac{x_i(p_1,p_2)+ku_i(p_1,p_2)+t_i v_i(p_1,p_2)}{Q_i(p_1,p_2,\tilde{p}_1,\tilde{p}_2)} \le 1, i = 1, \ldots, K \end{aligned} \tag{3.30}$$

  Cvx_end
  m = Cvx_optimum (n) /Cvx_optimum $(n-1)$
  Update the weights of condensed program in (3.28) using $p_1(n)$ and $p_2(n)$
  Set n=n+1
end while

The algorithm 1 described the optimization method in our scheme and the performance will present in the next section.

Chapter 4

Simulation Result

We in this section show the simulations which compare the BERs of the relay-based CoMP systems with the proposed precoder in the different topologies shown in Fig.4.1 and Fig.4.2. Fig.4.1 demonstrates the simulation environment for our multi-cell communications, and Fig.4.2 represents the case that the MS1 doesn't suffers interference from other MSs any more. The D_1, D_2, and D_3 are the distances of BS1-to-RS, BS1-to-MS1, and BS1-to-MS2, respectively. And MS2 isn't belonging to the cellular 1. In our simulations, the average channel power is assumed to be inversely proportional to the cubic of the distance between transmitter and receiver. In Fig.4.3, we set $D_1 = D_2 = D_3 = D/2$ where D is defined as a standard distance that is a distance from the cellular boundary to its BS. The average SNR caused by the path loss of the distance D is assumed as $\gamma = P_x/\sigma_n^2$ where P_x denotes the received power when the MS signals from a cell boundary has been transmitted to the destination, $i.e.$,the BS. Following this assumption, the other average SNRs can be given by $(D/D_i)^3\gamma$ for $i = 1, 2, 3$. Both "ML with Exhaustive Search" and "CVX" (Chapter 3) methods are used to find a pair (p_1, p_2) that can minimize the function (2.22). The curve "ML with Exhaustive Search" and "CVX" show almost the same BEP performance. But the "CVX" is a precise and efficient method to find a solution for the power allocation. Compared with Wang and Giannakis' CNFC method, both "ML with Exhaustive Search" and "CVX have 3dB gain in BER performance. The curve Simplification detection with "$p_1 = 1$ and $p_2 = 1$" shows that the BEPs of the MS1 of the cells without designing the power allocation precoders for the RS. In this situation, the MS can't exploit full diversity at the corresponding BS. The last curve "One user bound" represents a performance metric based on the MS1 doesn't suffer any interference. And our scheme can almost achieve this performance bound.

In Fig.4.4, the network topology is set as $2D_1 = D_2 = D_3 = D$. The topology would happen when the MSs in Figure 4.1 are located on the cellular boundary. In this situation, applying our method the MS1 and MS2 can achieve the same performance as they do by Wang and Giannakis' precoder. As the same result in Fig.4.3, our performance in BER is 3dB better than Wang and Giannakis' CNFC method. In Fig.4.5, the network topology is set as $2D_1 = D_2 = 2D_3/3 = D$. The topology is called as asymmetric topology because the two MSs have different distance to the base station. Besides, the differentiated services are clearly showed on this figure. For example, the MS1 in cellular 1 has better performance than the MS2 in cellular 1. The power allocation precoders at RS1 actually can provide the advantage for the MS1. And compared with Wang and Giannakis' CNFC method, we have 6dB gain in BER performance. In Fig.4.6, the network topology is $2D_1 = D_2 = D_3/3 = D$. The differentiated services are also show up in this figure. The performance of the MS in the corresponding BS is also reliable and closed to the one user bound when the topology becomes more asymmetric than in Fig.4.5. Wang and Giannakis' CNFC method can't exploit diversity in this topology and the performance gap between their scheme and our method is about 10dB. In the last simulation Fig.4.7, the performance of the MS1 on the cell boundary almost remains the same even though the location of MS2 has been changed.

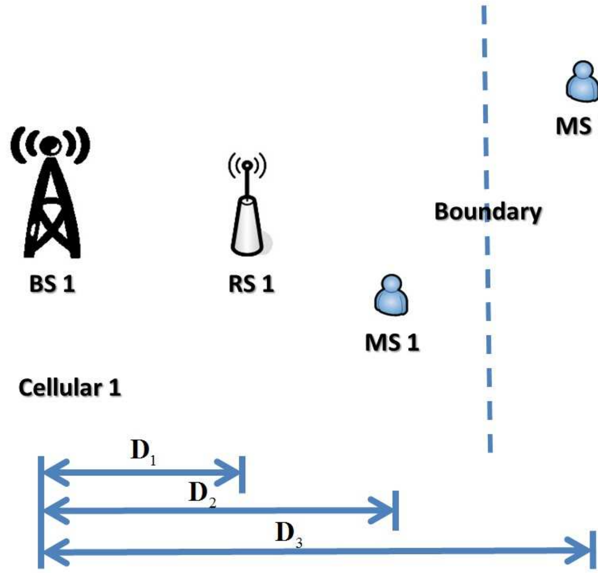

Figure 4.1: The simulation topology in two cellular case.

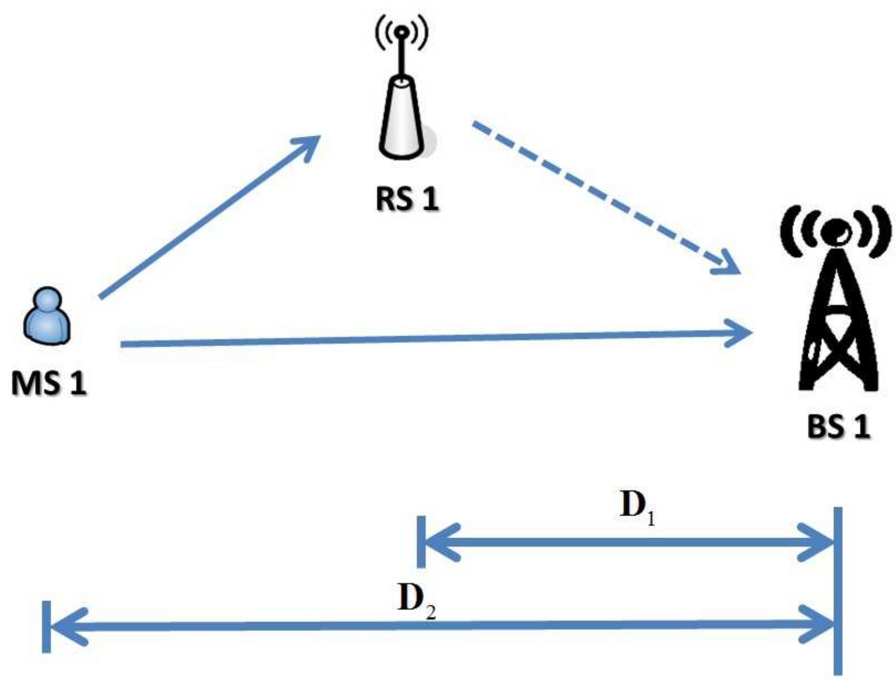

Figure 4.2: The simulation topology in single user environment.

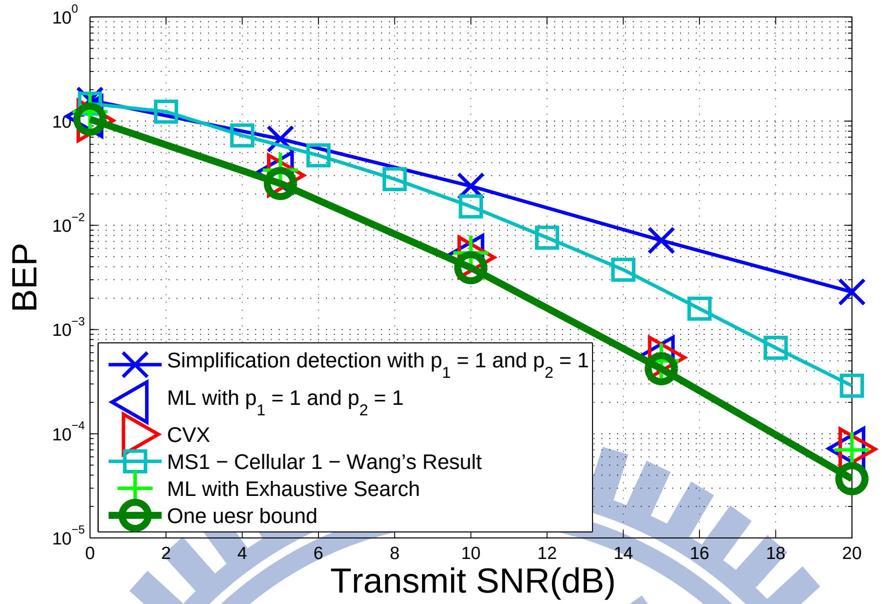

Figure 4.3: The topology setting is $2D_1 = 2D_2 = 2D_3 = D$. Numerical, Simulation, CVX, and One-User-Bound comparison

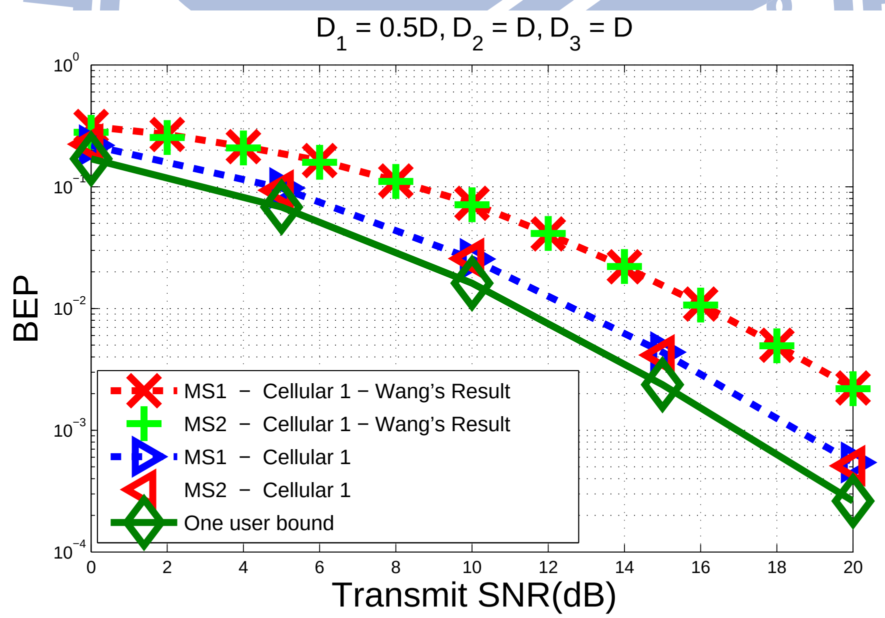

Figure 4.4: The topology setting is $2D_1 = D_2 = D_3 = D$ (Symmetric Case). The BEP performance of all users.

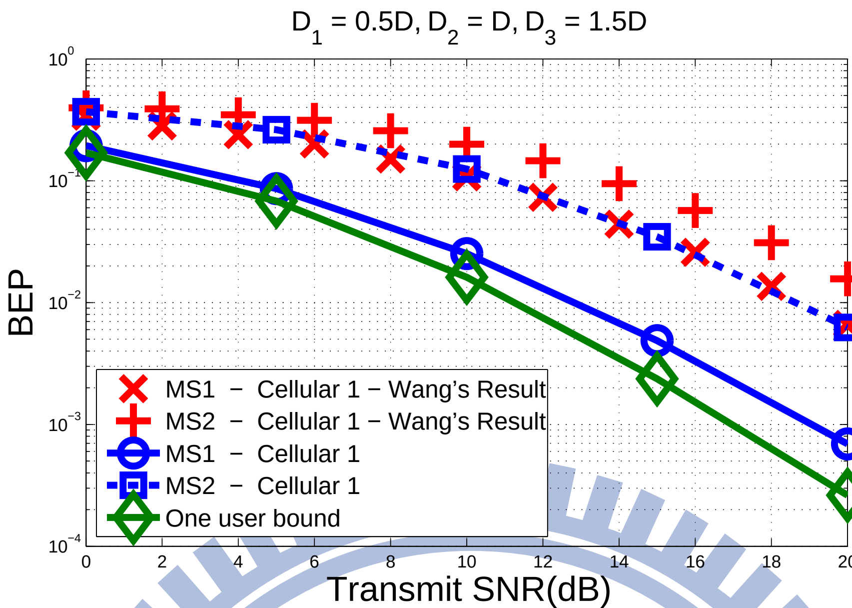

Figure 4.5: The topology setting is $2D_1 = D_2 = 2D_3/3 = D$ (Asymmetric Case). The BEP performance of all users.

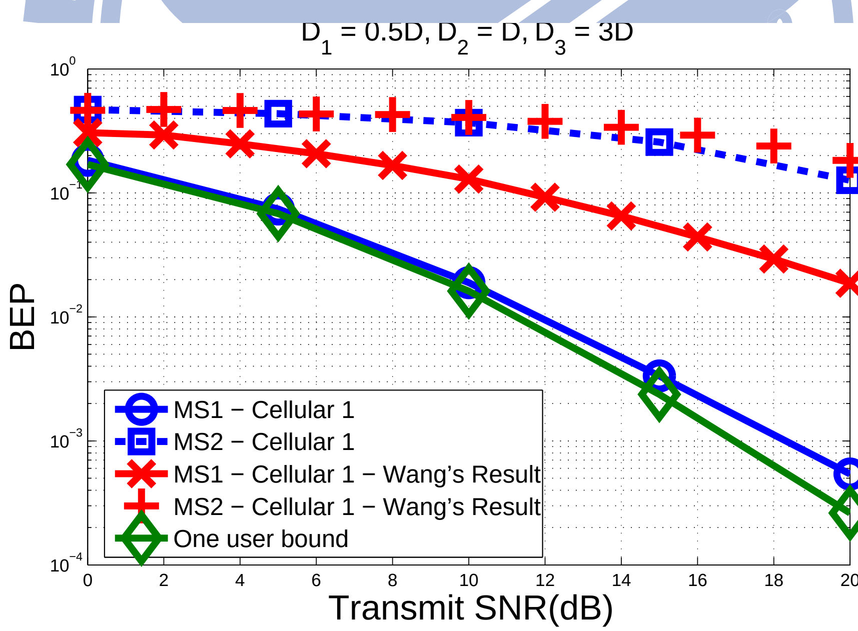

Figure 4.6: The topology setting is $2D_1 = D_2 = D_3/3 = D$ (Asymmetric Case). The BEP performance of all users.

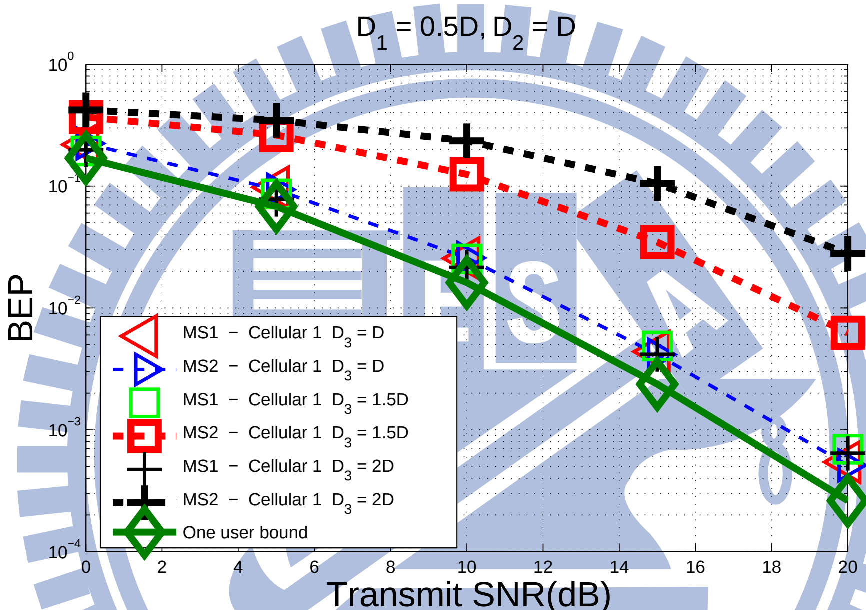

Figure 4.7: The topology setting is $2D_1 = D_2 = D$. And the D_3 is changed.

Chapter 5

Conclusions

We have introduced a power allocation method for the relay node to provide differentiated services in multi-cell communications. The differentiated services are possible in multi-cell communications and subscriber's diversity gain at corresponding BS is guaranteed. Besides, the SP provides an efficiency search for the power allocation method. However, we only considered the simple scenario that consists of two adjacent cells whcih have two individual subscribers both using the BPSK modulation. In the future, the power allocation method would be extended to high order modulation and more complex topologies.

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