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Superfield form of discrete gauge states in $\hat{c} = 1$ 2d supergravity

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Abstract. A general formula for the discrete states (Neveu-Schwarz sector) in $N = 1$ 2D super-Liouville theory is written down in the world-sheet supersymmetric form. We then derive a set of gauge states at the discrete momenta. These discrete gauge states are shown to carry the ω_∞ charges and serve as the symmetry parameters in the old covariant quantization of the theory.

1 Introduction

One of the main motivations to study 2d quantum gravity was to understand the non-perturbative information of string theory. The discretized matrix model [1] approach developed so far has been very successful. On the other hand, the continuum Liouville theory serves [2] as an important consistency check for the matrix model approach. Since little is now known for supersymmetric matrix model [3], it would be interesting to develop 2D super-Liouville theory [4, 5] and compare its results directly with the high dimensional critical string theory.

In the Liouville theory, in addition to the massless tachyon mode, an infinite number of massive discrete states were discovered [6, 7] and the target space-time ω_∞, symmetry [8–10] and Ward identities [11] were then identified. In a previous paper [12], we introduced the concept of discrete gauge states (DGS) and gave a general formula for them. These DGS were then shown to carry the ω_∞ charges and can be considered as the symmetry parameters in the old covariant quantization of the theory. This is in parallel with the BRST approach [4, 8, 5, 13] appeared in the literature, and can be compared with the works of 10D critical string theory [14], where a complete gauge state analysis turns out to be extremely difficult to attain. In this paper, we will generalize our results in [12] to $N = 1$ super-Liouville theory in the worldsheet supersymmetric way. We will work out the DGS of the Neveu-Schwarz sector in the zero ghost picture. We organize the paper as following. In Sect. 2, we discuss the $N = 1$ super-Liouville theory and set up the notations. In Sect. 3, we calculate the general formula for discrete states in a worldsheet superfield form which seems missing in the literature. The DGS and ω_∞ charges were then given in Sect. 4. A brief conclusion was summarized in the final section.

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2 2D Super-Liouville theory

The $N = 1$ two dimensional supersymmetric Liouville action is given by [15]

$$S = \frac{1}{8\pi}\int d^2\mathbf{z}[g^{\alpha\beta}(\partial_\alpha \mathbf{X}\partial_\beta \mathbf{X} + \partial_\alpha \boldsymbol{\Phi}\partial_\beta \boldsymbol{\Phi}) - Q\hat{\mathbf{Y}}\boldsymbol{\Phi}] , \quad (2.1)$$

where $\boldsymbol{\Phi}$ is the super-Liouville field, $\hat{\mathbf{Y}}$ the superfield curvature, $d\mathbf{z} = dz d\theta$ and with $\mathbf{X}^\mu = \begin{pmatrix}\boldsymbol{\Phi}\\ \mathbf{X}\end{pmatrix}$,

$$\mathbf{X}^\mu(z,\theta,\bar{z},\bar{\theta}) = X^\mu + \theta\psi^\mu + \bar{\theta}\bar{\psi}^\mu + \theta\bar{\theta}F^\mu . \quad (2.2)$$

Bold faced variables denote superfields hereafter.

For $\hat{c} = 1 = \frac{2}{3}c$ theory Q, which represents the background charge of the super-Liouville field, is set to be 2 so that the total conformal anomaly cancels that from conformal and superconformal ghost contribution.

The equations of motion show that the left and right-moving components of $\mathbf{X}^\mu$ decouple, and the auxiliary fields F^μ vanish. As a result, we need to consider only one of the chiral sectors, while the other (anti-holomorphic) sector has a similar formula. The stress energy tensor is

$$\mathbf{T}_{zz} = -\frac{1}{2}\mathbf{D}\mathbf{X}^\mu\mathbf{D}^2\mathbf{X}_\mu - \frac{1}{2}Q\mathbf{D}^3\boldsymbol{\Phi} = T_F + \theta T_B , \quad (2.3)$$

with

$$\begin{aligned} T_F &= -\tfrac{1}{2}\partial X^\mu\partial X_\mu - \tfrac{1}{2}Q\partial^2 X^0 + \tfrac{1}{2}\psi^\mu\partial\psi_\mu \\ T_B &= -\frac{1}{2}\psi^\mu\partial X_\mu - \frac{1}{2}Q\partial\psi^0 , \end{aligned} \quad (2.4)$$

where $\mathbf{D} = \partial_\theta + \theta\partial_z$, and now $\mathbf{X}^\mu = X^\mu(z) + \theta\psi^\mu(z)$.

For the Neveu-Schwarz sector, if we define the mode expansion by

$$\partial_z X^\mu = -\sum_{n=-\infty}^{\infty} z^{-n-1}(\alpha_n^0, i\alpha_n^1) , \quad (2.5)$$

$$\psi^\mu = -\sum_{r\in\mathbb{Z}+\frac{1}{2}} z^{-r-\frac{1}{2}}(b_r^0, ib_r^1)\,, \tag{2.6}$$

then we have

$$[\alpha_m^\mu, \alpha_n^\nu] = n\eta^{\mu\nu}\delta_{m+n}\,, \quad \{b_r^\mu, b_s^\nu\} = \eta^{\mu\nu}\delta_{r+s}\,. \tag{2.7}$$

With the Minkowski metric $\eta_{\mu\nu} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$, $Q^\mu = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$ and the zero modes $\alpha_0^\mu = f^\mu = \begin{pmatrix} \epsilon \\ p \end{pmatrix}$, we find the super-Virasoro generators as modes of T_F and T_B,

$$L_n = \left(\frac{n+1}{2}Q^\mu + f^\mu\right)\alpha_{\mu,n} + \frac{1}{2}\sum_{k\neq 0} :\alpha_{\mu,-k}\alpha_{n+k}^\mu: + \frac{1}{2}\sum_{r\in\mathbb{Z}+\frac{1}{2}}\left(r+n+\frac{1}{2}\right) :b_{-r}^\mu b_{n+r,\mu}:$$

$$L_0 = \frac{1}{2}(Q^\mu + f^\mu)f_\mu + \sum_{k=1}^{\infty} :\alpha_{\mu,-k}\alpha_k^\mu: + \frac{1}{2}\sum_{r\in\mathbb{Z}+\frac{1}{2}}\left(r+\frac{1}{2}\right) :b_{-r}^\mu b_{r,\mu}:\,,$$

$$G_r = \sum_{s\in\mathbb{Z}+\frac{1}{2}} \alpha_{r-s}^\mu b_{\mu,s} + \left(r+\frac{1}{2}\right)Q^\mu b_{\mu,r} \tag{2.8}$$

The vacuum $|0\rangle$ is annihilated by all α_n^μ and b_r^μ with $n > 0$ and $r > 0$. In the old covariant quantization of the theory, physical states $|\psi\rangle$ are those satisfying the conditions

$$G_{\frac{1}{2}}|\psi\rangle = G_{\frac{3}{2}}|\psi\rangle = 0 \quad \text{and } L_0|\psi\rangle = \frac{1}{2}|\psi\rangle\,. \tag{2.9}$$

3 World-sheet superfield form of the discrete states

With (2.3) one can easily check that the two branches of massless "tachyon"

$$T^\pm(p) = \int d\mathbf{z} e^{ip\mathbf{X}+(\pm|p|-1)\boldsymbol{\Phi}} \tag{3.1}$$

are positive norm physical states. It was also known that there exists discrete momentum physical states. Writing $\int d\mathbf{z}\boldsymbol{\Psi}_{J,\pm J}^{(\pm)} = T^{(\pm)}(\pm J)$, the discrete states in the "material gauge" are

$$\boldsymbol{\Psi}_{J,M}^{(\pm)} \sim (H^-)^{J-M}\boldsymbol{\Psi}_{J,J}^{(\pm)} \sim (H^+)^{J+M}\boldsymbol{\Psi}_{J,-J}^{(\pm)}\,. \tag{3.2}$$

where

$$H^\pm = \sqrt{2}\int d\mathbf{z}e^{\pm i\mathbf{X}(z)}\,, \quad H^0 = \int d\mathbf{z}\mathbf{D}\mathbf{X} \tag{3.3}$$

are zero modes of the level 2 $SU(2)_{\kappa=2}$ Kac-Moody algebra in $\hat{c} = 1$ 2d superconformal field theory. Here we note that the NS sector corresponds to states with $J \in \mathbb{Z}$ while the Ramond sector corresponds to those with $J \in \mathbb{Z} + \frac{1}{2}$.

To find the explicit expressions for the discrete states, we first define the super-Schur polynomials,

$$\mathbf{S}_k(-i\mathbf{X}) = \frac{\mathbf{D}^k e^{-i\mathbf{X}}}{[k/2]!}e^{i\mathbf{X}}\,, \tag{3.4}$$

where $[\frac{k}{2}]$ denotes the integral part of $\frac{k}{2}$, as the $N = 1$ generalization to the Schur polynomial S_k, which is defined as

$$Exp\left(\sum_{k=1}^{\infty} a_k x^k\right) = \sum_{k=0}^{\infty} S_k(\{a_k\})x^k\,. \tag{3.5}$$

Note that $S_k(\{-i\partial^m\mathbf{X}/m!\}) = \mathbf{S}_{2k}(-i\mathbf{X})$. Direct integration shows that

$$\int d\mathbf{z}_1 \frac{1}{(z_1 - z - \theta_1\theta)^n} f(\mathbf{X}_1) = \frac{\mathbf{D}^{2n-1}f(\mathbf{X})}{(n-1)!} = \frac{\partial_z^{n-1}(f'(X)\psi) + \theta\partial_z^n f(X)}{(n-1)!}\,. \tag{3.6}$$

Using (3.6) we obtain

$$\boldsymbol{\Psi}_{J,J-1}^\pm \sim \mathbf{S}_{2J-1}(-i\mathbf{X})e^{i(J-1)\mathbf{X}+(\pm J-1)\boldsymbol{\Phi}} = \frac{1}{(J-1)!}\left[-i\partial^{J-1}(e^{-iX^1}\psi^1) + \theta\partial^{J-1}e^{-iX^1}\right] \times e^{iJ\mathbf{X}+(\pm J-1)\boldsymbol{\Phi}}\,. \tag{3.7}$$

For example, by

$$(-\mathbf{D}^{2r}\mathbf{X}^0, i\mathbf{D}^{2r}\mathbf{X}^1) \to b_{-r}^\mu\,, \quad (-\mathbf{D}^{2n}\mathbf{X}^0, i\mathbf{D}^{2n}\mathbf{X}^1) \to \alpha_{-n}^\mu \tag{3.8}$$

we have

$$\boldsymbol{\Psi}_{1,0}^+ = \mathbf{D}\mathbf{X} \to b_{-\frac{1}{2}}^1|f^\mu = (0,0)\rangle \tag{3.9}$$

and

$$\boldsymbol{\Phi}_{2,\pm 1}^+ = [-i\mathbf{D}^3\mathbf{X} - \mathbf{D}\mathbf{X}\mathbf{D}^2\mathbf{X}]e^{\pm i\mathbf{X}+\boldsymbol{\Phi}} \to [-b_{-\frac{3}{2}}^1 + b_{-\frac{1}{2}}^1\alpha_{-1}^1]|f^\mu = (1,\pm 1)\rangle\,. \tag{3.10}$$

They can be checked to satisfy the physical state conditions in (2.9).

Performing the operator products in (3.2), the discrete states $\boldsymbol{\Phi}_{J,M}^\pm$ are

$$\boldsymbol{\Psi}_{J,M}^\pm \sim \prod_{i=1}^{J-M}\int d\mathbf{z}_i\mathbf{z}_{i0}^{-J}\prod_{j<k}^{J-M}\mathbf{z}_{jk} \times Exp\left[\sum_{i=1}^{J-M}[-i\mathbf{X}(\mathbf{z}_i)] + (iJ\mathbf{X}(\mathbf{z}_0) + (-1\pm J)\boldsymbol{\Phi}(\mathbf{z}_0))\right]\,, \tag{3.11}$$

where $\mathbf{z}_{ab} = z_a - z_b - \theta_a\theta_b$. If we write $\mathbf{z}_{ab} = \mathbf{z}_{a0} - \mathbf{z}_{b0} - (\theta_a - \theta_0)(\theta_b - \theta_0)$, and use $\int d\mathbf{z}_a(\theta_a - \theta_0)\mathbf{z}_{a0}^{-n}f(\mathbf{X}_a) = \mathbf{D}^{2n-2}f(\mathbf{X}_0)/(n-1)!$, we get, for $M = J - 2$,

$$\boldsymbol{\Psi}_{J,J-2}^\pm \sim [2\mathbf{S}_{2J-3}\mathbf{S}_{2J-1} + \mathbf{S}_{2J-2}\mathbf{S}_{2J-2}]e^{i(J-2)\mathbf{X}+(\pm J-1)\boldsymbol{\Phi}}\,. \tag{3.12}$$

The vertex operators correspond to the upper components of (3.12), i.e.,

$$\int d\theta\boldsymbol{\Psi}_{J,J-2}^\pm \sim [(iJ\psi^1 + (\pm J-1)\psi^0)(S_{J-1}^2 + 2S_{J-\frac{3}{2}}^{NS}S_{J-\frac{1}{2}}^{NS}) - 2J(S_J S_{J-\frac{3}{2}}^{NS} - S_{J-1}S_{J-\frac{1}{2}}^{NS}] \times e^{i(J-2)X^1+(\pm J-1)X^0}\,, \tag{3.13}$$

where $S_J = S_J(\{-i\partial^m \mathbf{X}/m!\})$ and

$$S^{NS}_{k+\frac{1}{2}} = \sum_{m=0}^{k} \frac{-iS_m\partial^{k-m}\psi^1}{(k-m1)!} \ .$$

Using (3.8) and (3.12) it is found that

$$\boldsymbol{\Psi}^+_{2,0} \to [2b^1_{-\frac{1}{2}}b^1_{-\frac{3}{2}} + \alpha^1_{-1}\alpha^1_{-1}]|f^\mu = (1,0)\rangle \ . \qquad (3.14)$$

It can be checked that it satisfies the physical state conditions (2.9).

For $M = J-3$, a straighforward calculation gives

$$\begin{aligned}\boldsymbol{\Psi}^\pm_{J,J-3} \sim\ & [3!\mathbf{S}_{2J-1}\mathbf{S}_{2J-3}\mathbf{S}_{2J-5} + 3!\mathbf{S}_{2J-2}\mathbf{S}_{2J-3}\mathbf{S}_{2J-4} \\ & - \frac{3!}{1!2!}\mathbf{S}_{2J-1}\mathbf{S}^2_{2J-4} - \frac{3!}{2!1!}\mathbf{S}^2_{2J-2}\mathbf{S}_{2J-5}] \\ & \times e^{i(J-3)\mathbf{X}+(\pm J-1)\boldsymbol{\Phi}} \ . \end{aligned} \qquad (3.15)$$

It is now easy to write down an expression for general M,

$$\boldsymbol{\Psi}^\pm_{J,M} \sim \begin{vmatrix} \mathbf{S}_{2J-1} & \mathbf{S}_{2J-2} & \cdots & \mathbf{S}_{J+M} \\ \mathbf{S}_{2J-2} & \mathbf{S}_{2J-3} & \cdots & \mathbf{S}_{J+M-1} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{S}_{J+M} & \mathbf{S}_{J+M-1} & \cdots & \mathbf{S}_{2M+1} \end{vmatrix}' \times Exp[(iM\mathbf{X}(\mathbf{z}_0) + (-1\pm J)\boldsymbol{\Phi}(\mathbf{z}_0))] \ , \qquad (3.16)$$

with $\mathbf{S}_k = \mathbf{S}_k(-i\mathbf{X}(\mathbf{z}_0))$ and $\mathbf{S}_k = 0$ if $k < 0$. We will denote the rank $(J-M)$ "primed"-determinant in (3.16) as $\boldsymbol{\Delta}'(J, M, -i\mathbf{X})$, which (by definition) has all the signed terms in the normal determinant, except with a multiplicity of the multinomial coefficient $\frac{(J-M)!}{n_a!n_b!\ldots}$ for the term $\mathbf{S}^{n_a}_a\mathbf{S}^{n_b}_b \ldots$ (where $\sum_a n_a = J-M$).

4 DGS and ω_∞ charges

It was known [4, 5] that the discrete states in (3.2) satisfy the ω_∞ algebra

$$\begin{aligned}&\int d\mathbf{z}\boldsymbol{\Psi}^+_{J_1M_1}(\mathbf{z})\boldsymbol{\Psi}^+_{J_2,M_2}(\mathbf{0}) \\ &\quad = (J_2M_1 - J_1M_2)\boldsymbol{\Psi}^+_{J_1+J_2-1,M_1+M_2}(\mathbf{0}) \ ,\end{aligned} \qquad (4.1)$$

$$\int d\mathbf{z}\boldsymbol{\Psi}^-_{J_1,M_1}(\mathbf{z})\boldsymbol{\Psi}^-_{J_2,M_2}(\mathbf{0}) \sim 0 \ , \qquad (4.2)$$

where the RHS is defined up to a DGS.

In general, there are two types of gauge states in the old covariant quantization of the theory,

Type I:

$$|\psi\rangle = G_{-\frac{1}{2}}|\chi\rangle \quad \text{where} \quad G_{\frac{1}{2}}|\chi\rangle = G_{\frac{3}{2}}|\chi\rangle = L_0|\chi\rangle = 0 \qquad (4.3)$$

Type II:

$$\begin{aligned}|\psi\rangle &= (G_{-\frac{3}{2}} + 2L_{-1}G_{-\frac{1}{2}}|\tilde\chi\rangle \\ &\quad \text{where} G_{\frac{1}{2}}|\tilde\chi\rangle = G_{\frac{3}{2}}|\tilde\chi\rangle = 0 \\ &\qquad (L_0+1)|\tilde\chi\rangle = 0 \ .\end{aligned} \qquad (4.4)$$

They satisfy the physical state conditions (2.9), and have zero norm. There is an infinite number of continuum momentum gauge state solutions for (4.3) and (4.4). However, as far as the dynamics is concerned, we are only interested in those with discrete momentum.

At mass level one, $f_\mu(f^\mu + Q^\mu) = 0$, only gauge states of type I are found: $f_\mu\alpha^\mu_{-1}|f\rangle$, where $|f\rangle =: e^{ip\mathbf{X}+\epsilon\boldsymbol{\Phi}} : |0\rangle$. The DGS $G^-_{1,0} =: \mathbf{D}\boldsymbol{\Phi}e^{-2\boldsymbol{\Phi}} : |0\rangle$ corresponds to the momentum of $\boldsymbol{\Psi}^-_{1,0}$. There is no corresponding DGS for $\boldsymbol{\Psi}^+_{1,0} =: \mathbf{D}\mathbf{X}:$.

At the next mass level, $f_\mu(f^\mu + Q^\mu) = -2$, $N_{\mu\nu} = -N_{\nu\mu}$ and $M_\mu = 2N_{\mu\nu}(f^\nu + Q^\nu)$, the type I gauge state is found to be

$$|\psi\rangle = [(M_\mu f_\nu\alpha^\mu_{-1}b^\nu_{-\frac{1}{2}} + M_\mu b^\mu_{-\frac{3}{2}} + 2N_{\mu\nu}\alpha^\mu_{-1}b^\nu_{-\frac{1}{2}}]|f\rangle \ , \qquad (4.5)$$

while the type II state is

$$|\psi\rangle = [(2f_\mu f_\nu + \eta_{\mu\nu})\alpha^\mu_{-1}b^\nu_{-\frac{1}{2}} + (3f_\mu - Q_\mu)b^\mu_{-\frac{3}{2}}]|f\rangle \ . \qquad (4.6)$$

As in the bosonic Liouville theory [12], the gauge states corresponding to the discrete momenta of $\boldsymbol{\Psi}^+_{2,\pm1}$, are degenerate, i.e., the type I and type II gauge states are linearly dependent:

$$\begin{aligned}G^+_{2,\pm1} &\sim \left[\begin{pmatrix} 1 & \mp2 \\ \mp2 & 3 \end{pmatrix}\right]\alpha^\mu_{-1}b^\nu_{-\frac{1}{2}} + \begin{pmatrix} -1 \\ \pm3 \end{pmatrix} b^\mu_{-\frac{3}{2}}|f^\mu \\ &= (1,\pm1)\rangle \ . \end{aligned} \qquad (4.7)$$

For the minus sector, type I DGS is

$$\begin{aligned}G^{-,I}_{2,\pm1} &\sim \left[\begin{pmatrix} 3 & \pm2 \\ \pm2 & 1 \end{pmatrix}\alpha^\mu_{-1}b^\nu_{-\frac{1}{2}} + \begin{pmatrix} 1 \\ \pm1 \end{pmatrix} b^\mu_{-\frac{3}{2}}\right]|f^\mu \\ &= (-3,\pm1)\rangle \ , \end{aligned} \qquad (4.8)$$

and type II DGS is

$$\begin{aligned}G^{-,II}_{2,\pm1} &\sim \left[\begin{pmatrix} 17 & \pm6 \\ \pm6 & 3 \end{pmatrix}\alpha^\mu_{-1}b^\nu_{-\frac{1}{2}} \begin{pmatrix} 11 \\ \pm3 \end{pmatrix} b^\mu_{-\frac{3}{2}}\right]|f^\mu \\ &= (-3,\pm1)\rangle \ . \end{aligned} \qquad (4.9)$$

Note that $3G^{-,I}_{2,\pm1} - G^{-,II}_{2,\pm1}$ is a "pure $\boldsymbol{\Phi}$" DGS, similar to the DGS in the bosonic Liouville theory.

We now apply the scheme used in [12] to derive a general formula for the DGS. From (4.2), the DGS in the minus sector can be written down explicitly as follows

$$\begin{aligned}\mathbf{G}^-_{J,M} &\sim \left[\int d\mathbf{z}e^{-\boldsymbol{\Phi}}(\mathbf{z})\right)\boldsymbol{\Psi}^-_{J-1,M}(\mathbf{0}) \\ &\sim \mathbf{S}_{2J-1}(-\boldsymbol{\Phi})\boldsymbol{\Delta}'(J-1, M, -iX)e^{[iM\mathbf{X}+(-1-J)\boldsymbol{\Phi}]} \ . \end{aligned} \qquad (4.10)$$

We thus have explicitly obtained a DGS for each $\boldsymbol{\Psi}^-$ discrete momentum. However, there are still other DGS in this sector, for example, the states

$$\mathbf{G}'^-_{J,M} \sim \left[\int d\mathbf{z}e^{-\boldsymbol{\Phi}(\mathbf{z})}\right]^{J-M} \boldsymbol{\Psi}^-M, M(\mathbf{0}) \qquad (4.11)$$

can be shown to satisfy the physical state condtions. Since they are "pure $\boldsymbol{\Phi}$" states, they are also DGS. For example, $\mathbf{G}^-_{1,0} = \mathbf{D}\boldsymbol{\Phi}e^{-\boldsymbol{\Phi}}$ and $\mathbf{G}^-_{2,\pm1} = [-\mathbf{D}^3\boldsymbol{\Phi} + \mathbf{D}\boldsymbol{\Phi}\mathbf{D}^2\boldsymbol{\Phi}]e^{\pm i\mathbf{X}-3\boldsymbol{\Phi}}$, which is a linear combination of (4.8) and (4.9).

For the plus sector, we can subtract two (distinct) positive norm discrete states at the same momentum to obtain a pure gauge state

$$\mathbf{G}^{+}_{J,M} = (J+M+1)^{-1} \int d\mathbf{z} \Big[\boldsymbol{\Psi}^{+}_{1,-1}(\mathbf{z}) \boldsymbol{\Psi}^{+}_{J,M+1}(\mathbf{0}) - \boldsymbol{\Psi}^{+}_{J,M+1}(\mathbf{z}) \boldsymbol{\Psi}^{+}_{1,-1}(\mathbf{0}) \Big] \ . \quad (4.12)$$

As an example, with (4.12) one finds

$$G^{+}_{2,\pm 1} = [\pm 3i\mathbf{D}^3\mathbf{X} + \mathbf{D}^3\boldsymbol{\Phi} + 2i\mathbf{D}^2\mathbf{X}\mathbf{D}\mathbf{X} \pm 2i\mathbf{D}^2\mathbf{X}\mathbf{D}\boldsymbol{\Phi} \pm 2i\mathbf{D}\mathbf{X}\mathbf{D}^2\boldsymbol{\Phi} + \mathbf{D}\boldsymbol{\Phi}\mathbf{D}^2\boldsymbol{\Phi}]e^{\pm i\mathbf{X}+\boldsymbol{\Phi}}, \quad (4.13)$$

which is exactly the state we found in (4.7). We thus have explicitly obtained a DGS for each $\boldsymbol{\Psi}^{+}$ momentum.

By construction in (4.12) one can see that $\mathbf{G}^{+}_{J,M}$ carry the w_∞ charges and serve as the symmetry parameters of the theory. In fact, their operator products form the same w_∞ algebra

$$\int d\mathbf{z}\mathbf{G}^{+}_{J_1,M_1}(\mathbf{z})\mathbf{G}^{+}_{J_2,M_2}(\mathbf{0}) = (J_2M_1 - J_1M_2)\mathbf{G}^{+}_{J_1+J_2-1,M_1+M_2}(\mathbf{0}) \ , \quad (4.14)$$

where the RHS is defined up to another DGS.

5 Conclusion

We have demonstrated that the space-time w_∞ symmetry parameters in the 2D superstring theory come from solution of equations (4.3) and (4.4). This phenomenon should survive in the more realistic 10D heterotic string theory [14], although it would be difficult to find the general solution (due to the high dimensionality of space-time). The DGS in the old covariant quantization of the theory seems to be related to the ground ring structure in the BRST approach. Finally, the GSO projection can be easily imposed on the spectrum.

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