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On the Stability Properties of Hexapod Tripod Gait

TSU-TIAN LEE, CHING-MING LIAO, AND TING-KOU CHEN

Abstract—**In this communication, hexapod tripod gaits for straight-line motion and crab walking are derived. Mathematical relations that express the stability margin, the stride length, and the duty factor are formulated for straight-line motion and for crab walking, respectively. The derived results provide tripod gaits of the hexapod for walking with a prescribed stability margin either over perfect terrain or constant slope terrain.**

I. Introduction

It is generally recognized by vehicle designers that the mobility characteristics of terrestrial animals are in many respects superior to those of wheeled or tracked vehicles for off-road locomotion. Indeed, it is commonplace knowledge that much of the earth's surface is accessible only to men on foot or to certain types of multi-legged animals. This fact has motivated considerable attempts to develop the multi-legged walking vehicles. During the past several years, the generalized walking vehicle with multiple degrees of freedom was considered, and the coordinated control of legs by computer to complete the walk by the suitable choice of lifting legs was investigated. As a matter of fact, although the number of the legs of the walking vehicles under research varies from six to one, it seems, that the hexapod attracts more attention. This is justified by the fact that a number of walking vehicles under development is hexapod [2]–[4], [11]–[16].

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T. T. Lee was with the Institute of Control Engineering, National Chiao-Tung University, Hsinchu, Taiwan 300, ROC. He is now with the Department of Electrical Engineering, University of Kentucky, Lexington, KY 40506-0046.

C. M. Liao is with the Institute of Control Engineering, National Chiao-Tung University, Hsinchu, Taiwan 300, ROC.

T. K. Chen is with the Institute of Electronics, National Chiao-Tung University, Hsinchu, Taiwan 300, ROC.

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The basic control problems that all the walking vehicles confront are: 1) How to generate the trajectory and the average speed? 2) How to determine the best sequence for liftoff and placing the feet? 3) What is the suitable distance that each leg should transfer in order to maintain a prescribed statical stability? 4) How to control the body's inclination and height? 5) How to develop a measurement system and information processing method to support the motion planning?

The problem of choosing the best sequence for liftoff and placing the feet of a walking vehicle is the gait selection problem. A considerable amount of previous research has been devoted to the leg sequencing problem [1]–[3], [9], [11], [13], [14]. Central to this work has been the finite characterization of a leg's states in which each leg of a machine is idealized to a two-state device, namely, either those of being on the ground (1-state) or in the air (0-state) [1].

Although the gait control problem of a hexapod has been studied by McGhee [1]–[3], Raibert and Sutherland [4], Orin [11], Sun [13], Ozguner *et al.* [15], Wilson [8], Bessonov and Umnov [14], and Song [19], still, there are some problems which remain unanswered. These include:

1) The tripod gait for crab walking, with a prescribed stability margin, over perfectly flat terrain.

2) The tripod gait for crab walking, with a prescribed stability margin, over constant slope terrain.

3) The tripod gait for moving straightforward, with a prescribed stability margin, over constant slope terrain.

This communication aims at studies of these problems and some results of McGhee [1], [3] and Hirose [9] are extended to cover the case of hexapod crab walking.

Basically, McGhee and Iswandhi [3] developed a nonperiodic gait, namely, the free gait of the hexapod. At each iteration, the free gait determines the legs' movement adaptively that tends to maximize the number of legs in the air. This algorithm yields good mobility and is very useful in locomotion over rough terrain. A similar adaptive gait for quadruped had also been developed by Hirose [9]. However, for the case of even terrain, simple solutions for the gait selection can be found without using the free gait.

The purpose of this communication is to derive a simple periodic gait for the straight-line walking of the hexapod that will have three legs in the air and still ensure the guranteed stability margin during a complete locomotion cycle on the even terrain. Results are then generated for the crab walking tripod gait. Finally, these results are generalized for locomotion on constant slope terrain. It should be noted that all the results derived in this communication are based on the assumptions that 1) the hexapod has a symmetric structure, 2) the reachable area of each leg is a rectangular region, 3) the initial foothold positions should be at the specified locations before the locomotion starts, and 4) unless specified otherwise, the hexapod is moving with constant speed.

II. Problem Statement

Consider the geometry of the hexapod shown in Fig. 1(a) [3], in which each leg has a reachable area in the form of a sector of an annulus. Since overlapping reachable areas raise interference problems [3], one way to solve it is to avoid it altogether by eliminating *a priori* all overlapping reachable areas so that each leg has a distinct region that can be accessed only by it and not by any other leg. Moreover, for simplicity of analysis, we define a rectangular region as the reachable region of each leg as shown in Fig. 1(b).

The problem considered in this communication is that given a reachable region of each leg of the hexapod as shown on Fig. 1(b), determine the tripod gaits for 1) moving straightforward with prescribed stability, and 2) crab walking with prescribed stability. In particular, mathematical equations that relate the stride length,

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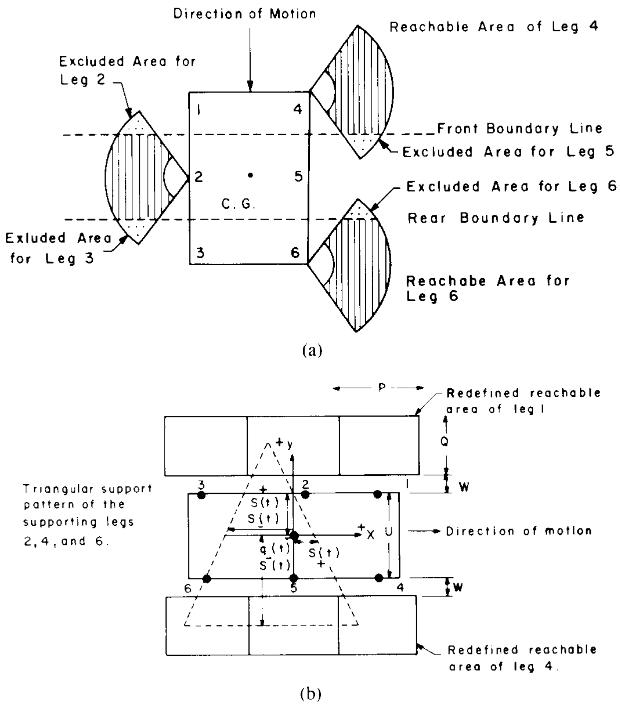


Fig. 1. (a) Schematic top view of a hexapod showing leg numbering, boundary lines for foot placing, and reachable areas of legs. (b) The reachable area of each leg.

duty factor, and stability margin for 1) crab-walking tripod gait, and 2) moving straightforward tripod gait will be formulated, respectively.

III. Tripod Gait for Straight-Line Locomotion on Even Terrain

In the following, we assume the hexapod is moving in the $+X$-axis direction. The following definitions and notations are required for the derivations to be followed.

Let $q(t)$ be the center of gravity of the hexapod, P the length of the reachable area along the X-axis, and Q the length of the reachable area along the Y-axis.

Definition 1

The support pattern of a tripod gait is the convex hull of the point set in a horizontal plane which contains the area of the vertical projections of all supporting legs.

Definition 2 [1]

The support pattern of a tripod gait is statically stable at time t if and only if $q(t)$ is contained inside the support pattern.

Let

- $S_+(t)$ the distance from $q(t)$ to the boundary of support pattern along the forward direction of motion ($+x$ direction).
- $S_-(t)$ the distance from $q(t)$ to the boundary of support pattern along the oppositive direction of motion ($-x$ direction).
- $S^+(t)$ the distance from $q(t)$ to the boundary of the support pattern in a direction that is perpendicular to the motion direction ($+y$ direction).
- $S^-(t)$ the distance from $q(t)$ to the boundary of support pattern in the opposite direction of $s^+(t)$ ($-y$ direction).

Definition 3 [1]

The tripod gait is said to have statical stability margin $S (S > 0)$, if and only if $\min(S_-(t), S_+(t), S^+(t), S^-(t)) = S$, for all $t \geqq 0$.

Let $\lambda(s, \alpha, \theta)$ and $\beta(s, \alpha, \theta)$ denote the stride length and duty factor, respectively, of the hexapod for crab walk tripod gait on a slope plane with a minimum statical stability margin S, where α is the crab angle, and θ is the slope of the plane.

Let $A(s, \alpha, \theta)$ denote the distance that the body moves in order to maintain a minimum statical stability margin S while three legs are in their transfer states, where the hexapod is crab walking with tripod gait on a slope plane.

Remark 1: When $\theta = 0$, i.e., on the flat terrain, the notations

$$\lambda(s, \alpha) = \lambda(s, \alpha, 0)$$

$$\beta(s, \alpha) = \beta(s, \alpha, 0)$$

and

$$A(s, \alpha) = A(s, \alpha, 0)$$

shall be used.

Remark 2: When $\alpha = 0$ and $\theta = 0$, the notations

$$\lambda(s) = \lambda(s, 0, 0)$$

$$\beta(s) = \beta(s, 0, 0)$$

and

$$A(s) = A(s, 0, 0)$$

shall be used.

Assume the hexapod is walking in the $+x$ direction. Let the left side legs of the hexapod be numbered from front to back as 1, 2, 3 and those on the right side (in the same order) as 4, 5, 6. Moreover, assume the initial foothold of each leg is as shown in Fig. 2(a). Then we have the following theorems:

Theorem 1

The hexapod tripod gait for straight-line motion (in the $+x$ direction) with zero stability margin is that the foothold positions, the support states of the legs, and the sequence of transferment of the legs are as shown in Table I with stride length

$$\lambda(s) = 2A(0) = 2A(s)|_{s=0} \tag{1}$$

and duty factor

$$\beta(s) = \frac{1}{2} \tag{2}$$

where $A(0) = P$.

Proof: Note that the "0" and "1" on Fig. 2(a) denote the front-end and rear-end foothold positions of each leg, respectively. The front-end position of leg 2 (denoted "0") is adjacent to the rear-end position of leg 1 (denoted "1"). From Fig. 2(b), it can be observed that the sequence of lifting and placing the feet as given by Table I does indeed guarantee a zero stability margin. Furthermore, the body moves $2A(o)$ during a complete locomotion cycle, hence

$$\lambda(s) = 2A(0) = 2P$$

and the duty factor

$$\beta(s) = \frac{P}{\lambda(s)} = \frac{1}{2}.$$

Q.E.D.

Theorem 1 can be extended to generate the gait for straight-line walking with a prescribed stability margin S. In fact, if the body (center of gravity) moves a distance S before the start of the liftoff of the feet, and the body moves a distance $A(S)$, where $A(S) = P - 2S$, while the transfer legs swing in the air, then the tripod gait with prescribed stability margin S is obtained. Theorem 2 states the result.

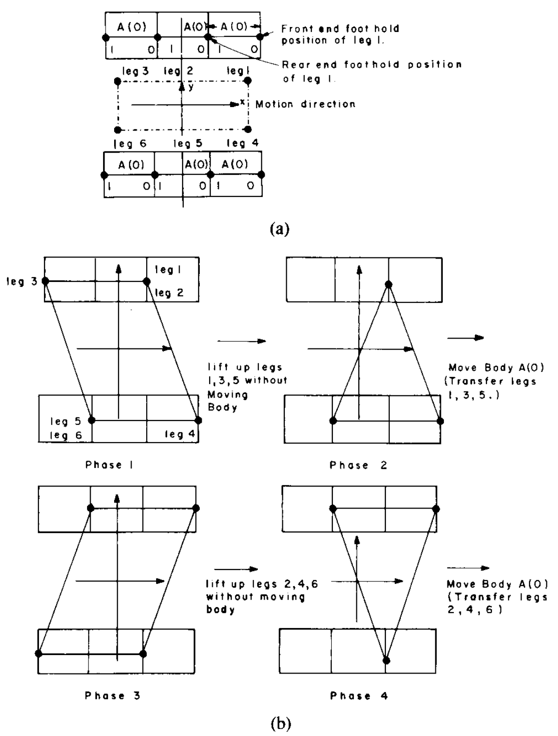


Fig. 2. (a) Schematic top view showing front-end foothold positions (denoted by "0") and rear-end positions (denoted by "1") of each leg. (b) The tripod gait with zero stability margin in a locomotion cycle.

TABLE I
SUPPORT STATES AND SUPPORT SEQUENCE OF TRIPOD GAIT WITH ZERO STABILITY MARGIN

Phase	1	2	3	4
Leg 1	(1, 1)	(1, 0)	(0, 1)	(0, 0)
Leg 2	(0, 1)	(0, 1)	(1, 1)	(1, 1)
Leg 3	(1, 1)	(1, 0)	(0, 1)	(0, 0)
Leg 4	(0, 1)	(0, 1)	(1, 1)	(1, 1)
Leg 5	(1, 1)	(1, 0)	(1, 1)	(1, 1)
Leg 6	(0, 1)	(0, 1)	(1, 1)	(1, 1)
Body move	0	$A(0)$	0	$A(0)$
Transfer legs	×	(1, 3, 5)	×	(2, 4, 6)

Where (i, j) i footholds position $(i = 0, 1)$
j support state
$j = 1$ for support leg
$j = 0$ for transfer leg
× all are support legs.

Theorem 2

The tripod gait of a hexapod for straight-line locomotion (in the $+x$ direction) with stability margin S is that

1) the foothold positions are as shown in Fig. 3(a);
2) the sequence of movement for body (center of gravity) and legs are given by Table II, in which $B_1 = B_3 = S$, and $B_2 = B_4 = A(s)$ where $A(s) = P - 2S$;
3) the stride length

$$\lambda(s) = 2S + 2A(s) \tag{3}$$

4) the duty factor

$$\beta(s) = \frac{P}{2P - 2S} \geqq \frac{1}{2} \tag{4}$$

where

$$P = A(s) + 2S.$$

Proof: Note that the "0," "1," "2," and "3" on Fig. 3(a) denote the foothold positions of each leg. The front-end foothold position of leg 2 (denoted "0") is adjacent to the rear-end foothold position (denoted "3") of leg 1. From Fig. 3(b), it can be observed that the specified sequence of movement of legs and body do provide a tripod gait with the stability margin $S > 0$. In order to characterize the movement of the legs, the movement of the body (center of gravity), and the foothold positions at each instant of this tripod gait, Fig. 3(b), are expressed in a general table, as given by Table II. Indeed, if $B_1 = B_3 = S$, and $B_2 = B_4 = A(s)$, then Table II summarizes all the information required to characterize the hexapod tripod gait with stability margin S.

From Table II and using $B_1 = B_3 = S$ and $B_2 = B_4 = A(s)$, it follows that the stride length

$$\lambda(s) = 2A(s) + 2S$$

since

$$P = A(s) + 2S$$

therefore

$$\lambda(s) = 2P - 2S \tag{5}$$

and the duty factor

$$\beta = \frac{P}{\lambda(s)} = \frac{P}{2P - 2S} \geqq \frac{1}{2} \tag{6}$$

Q.E.D.

Remark 3: The maximal statical stability margin of the hexapod tripod gait occurs when $A(s) = 0$, i.e., $P = 2S$. Hence $S_{max} = P/2$.

IV. Tripod Gait for Crab Walk on Even Terrain

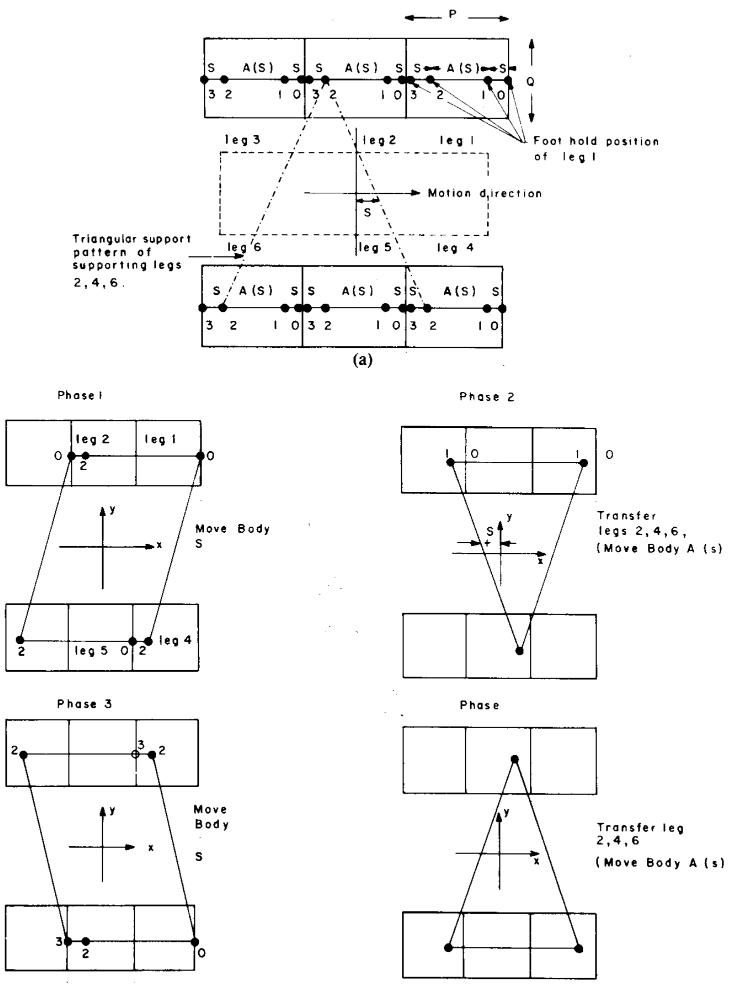
The above results derived for straight-line walking can be further extended to cover the case for crab walk. In this section, the standard gait for crab walking will be derived. Crab walking of a quadruped has been defined and investigated by Hirose [9]. According to [9], crab walking is defined as a walking motion with the direction of locomotion different from, or equal to, the longitudinal axis of the vehicle's body. The angle between the longitudinal axis and the direction of motion is the crab angle, denoted by α. In the case of $\alpha = 0°$, crab walking corresponds to straight-line walking.

Since the configuration of the vehicle is symmetric with respect to both the X and Y axes; it is sufficient to determine the standard gait for a crab angle α in the range $0 \leqq \alpha \leqq 90°$ [9].

From Fig. 4, the stroke length $n(\alpha)$ of a leg is given by

$$n(\alpha) = \begin{cases} P/\cos\alpha, & \text{for } 0 \leqq \alpha \leqq \tan^{-1}(Q/P) \\ Q/\sin\alpha, & \text{for } \tan^{-1}(Q/P) \leqq \alpha \leqq 90°. \end{cases} \tag{7}$$

In order to specify the standard tripod gait for crab walk, we assume that the trajectory of each leg must pass through the central point C_i of the reachable area R_i as shown on Fig. 4. Now, it appears that the reachable area of each leg for the crab walking should be



(b)

Fig. 3. (a) The foothold position of legs with triangular support pattern showing the tripod gait having stability margin S. (b) The tripod gait with stability margin S during a complete locomotion cycle.

TABLE II
THE FOOTHOLD POSITIONS, SUPPORT STATES, AND TRANSFER LEGS OF HEXAPOD TRIPOD GAIT WITH STABILITY MARGIN S

Phase	1	2	3	4
Leg 1	(0, 1)	(1, 1)	(2, 1)	(3, 0)
Leg 2	(2, 1)	(3, 0)	(0, 1)	(1, 1)
Leg 3	(0, 1)	(1, 1)	(2, 1)	(3, 0)
Leg 4	(2, 1)	(3, 0)	(0, 1)	(1, 1)
Leg 5	(0, 1)	(1, 1)	(2, 1)	(3, 0)
Leg 6	(2, 1)	(3, 0)	(0, 1)	(1, 1)
Body move	B_1	B_2	B_3	B_4
Transfer leg	×	(2, 4, 6)	×	(1, 3, 5)

Where (i, j) i foothold position $(i = 0, 1, 2, 3)$
j support state
$j = 1$ for support leg
$j = 0$ for transfer leg
× all are support legs.

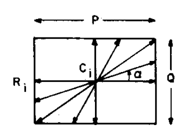


Fig. 4. The stroke motion of a leg at various crab angles.

redefined. To see this, let us consider the critical support pattern ΔDEF as shown on Fig. 5. Notice that when any of the three vertices of triangular support pattern ΔDEF is located inside the shaded regions, denoted by diagonal cross-hatch, the hexapod is unstable. Therefore, these regions denoted by the diagonal cross-hatched area are the forbidden regions for the support pattern ΔDEF. Similarly, there are forbidden areas corresponding to the other three critical support patterns ΔGHI, ΔFDK, and ΔHJG denoted by parallel cross-hatched, vertical cross-hatched, an diagonal cross-hatched areas, respectively, as shown in Fig. 5.

Combining all forbidden regions of these four critical support patterns, one obtains Fig. 6, where shaded areas are forbidden areas of leg 2 (or leg 5) [9]. Note that

$$\delta(\alpha)=\overline{AB}=[(X_A-X_B)^2+(Y_A-Y_B)^2]^{1/2} \tag{8}$$

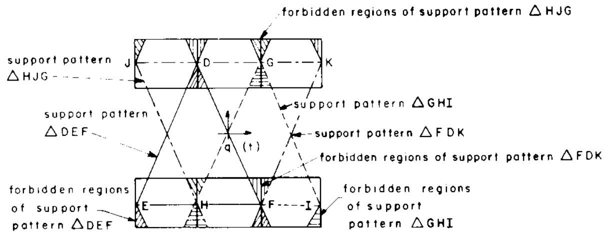


Fig. 5. The triangular critical support patterns of their corresponding forbidden regions of tripod gait while crab walking.

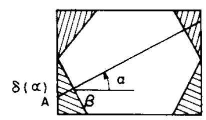


Fig. 6. The forbidden areas for legs 2 and 5, where $\delta(\alpha) = \overline{AB}$.

where (X_A, Y_A) and (X_B, Y_B) denote the Cartesian coordinates of points A and B, respectively. It can be shown that

i) When $\alpha < \tan^{-1}(Q/P)$

$$X_A = -\frac{P}{2}, \quad Y_A = \left(\frac{Y_2}{X_2 - \frac{P}{2}}\right) X_A, \quad X_B = \frac{Y_2 - (\tan\alpha) X_2}{\dfrac{Y_2}{X_2 - \frac{P}{2}} - \tan\alpha}$$

and

$$Y_B = \left(\frac{Y_2}{X_2 - \frac{P}{2}}\right) X_B \tag{9}$$

where X_2 and Y_2 are the positions of the center of gravity of reachable area R_2.

ii) When $\alpha > \tan^{-1}(Q/P)$

$$X_A = \left(\frac{Y_2}{X_2 - \frac{P}{2}}\right)^{-1} Y_A \qquad Y_A = Y_2 - \frac{Q}{2}$$

$$X_B = \frac{Y_2 - (\tan\alpha) X_2}{\dfrac{Y_2}{X_2 - \frac{P}{2}} - \tan\alpha} \qquad Y_B = \left(\frac{Y_2}{X_2 - \frac{P}{2}}\right) X_B. \tag{10}$$

Now we are ready to derive the following theorem for crab walk.

Theorem 3

Assume a hexapod is walking on a flat terrain ($\theta = 0$). The crab walk tripod gait of the hexapod, at crab angle α and with zero stability margin, is as follows:

1) The sequence of the movement of the body (center of gravity) and the transfer legs are as shown in Table II, in which

$$B_1 = B_3 = \delta(\alpha) \quad \text{and} \quad B_2 = B_4 = A(0, \alpha)$$

where $A(0, \alpha) + 2\delta(\alpha) = n(\alpha)$, and the foothold positions denoted by 0, 1, 2, and 3 are as shown on Fig. 7.

2) The stride length

$$\lambda(0, \alpha) = 2A(0, \alpha) + 2\delta(\alpha). \tag{11}$$

3) The duty factor

$$\beta(0, \alpha) = \frac{A(0, \alpha) + 2\delta(\alpha)}{2A(0, \alpha) + 2\delta(\alpha)}. \tag{12}$$

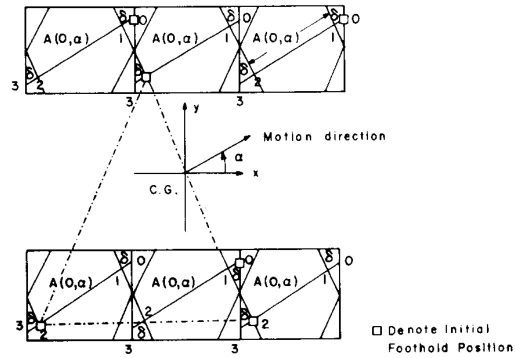


Fig. 7. The foothold positions of each leg of tripod gait having zero stability margin while crab walking. □: Initial foothold position.

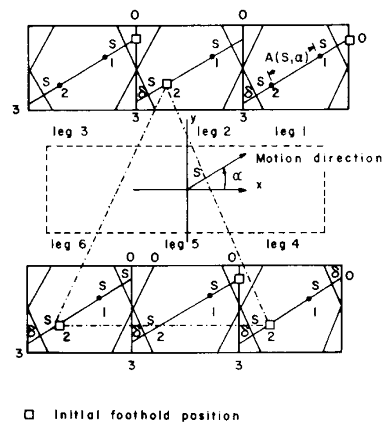


Fig. 8. The foothold positions of tripod gait having stability margin S while crab walking. □: Initial foothold position.

Proof: The proof is similar to that of Theorem 2, expect for the foothold positions as shown on Fig. 7 and $B_1 = B_3 = \delta(\alpha)$, $B_2 = B_4 = A(0, \alpha)$. Hence, the proof is omitted.

Q.E.D.

Note that Theorem 3 can be further extended to generate the tripod gait of the hexapod for crab walking with stability margin S. In fact, if the foothold positions denoted by 0, 1, 2, 3, are as given by Fig. 8, and if the body (center of gravity) moves a distance $B_1 = B_3 = S + \delta(\alpha)$ during phase 1 and phase 3, and a distance $B_2 = B_4 = A(S, \alpha)$ during phase 2 and phase 4 (where $A(s, \alpha) = n(\alpha) - 2S - 2\delta(\alpha)$) then we have the crab-walk tripod gait with stability margin S. Hence, we have the following result.

Theorem 4

Assume a hexapod is to walk on a flat terrain ($\theta = 0$). The crab walk tripod gait of a hexapod, at crab angle α and with stability margin $S > 0$, is as follows:

1) The sequence of movement of the body and legs is as shown on Table II, in which

$$B_1 = B_3 = S + \delta(\alpha) \quad \text{and} \quad B_2 = B_4 = A(S, \alpha)$$

and the foothold positions are as shown on Fig. 8.

2) The stride length

$$\lambda(S, \alpha) = 2A(S, \alpha) + 2S + 2\delta(\alpha). \tag{13}$$

3) The duty factor

$$\beta(S, \alpha) = \frac{A(0, \alpha) + 2\delta(\alpha)}{2A(0, \alpha) + 2\delta(\alpha) - 2S} \tag{14}$$

where

$$A(S, \alpha) = \eta(\alpha) - 2S - 2\delta(\alpha).$$

Remark 4: Similarly to Remark 2, it is easy to show that the maximal stability margin that the crab walking tripod gait can possible achieve is

$$S_{\max} = \frac{A(0, \alpha)}{2}. \tag{15}$$

V. The Tripod Gaits on a Constant-Slope Terrain

Consider a hexapod walking with a crab angle α on a constant-slope terrain, as shown in Fig. 9. Now it is easy to see that the center of gravity of the hexapod will shift a distance $h \tan \theta$, where θ is the slope angle and h is the height between $q(t)$ and the sloped plane, away from the horizontal plane. As a result, the critical support patterns of the tripod gait for crab walking on the sloped plane will also shift a distance $\Delta(\theta) = h \tan \theta$ compared to that on the perfectly flat terrain, as illustrated by Fig. 9. Furthermore, the forbidden areas for the triangular support pattern are redefined as shown on Fig. 10.

Based on the above observations, the tripod gaits for straight-line walking over flat terrain can be generalized to cover the case of tripod gaits on constant slope terrain. Thus we have the following theorem:

Theorem 5

Assume a hexapod is to walk on a constant slope plane with slope angle θ. The tripod gait for straight-line walking with stability margin S is as follows:

1) The foothold positions are as shown on Fig. 3(a).

2) The sequence of movement for body and legs is as given by Table II, in which

$$B_1 = B_3 = S + \Delta(\theta) \quad \text{and} \quad B_2 = B_4 = A(S, 0, \theta) \tag{16}$$

where

$$A(S, 0, \theta) = P - 2S - 2\Delta(\theta). \tag{17}$$

3) The stride length

$$\lambda(S, 0, \theta) = 2A(S, 0, \theta) + 2S + 2\Delta(\theta). \tag{18}$$

4) The duty factor

$$\beta(S, 0, \theta) = \frac{A(S, 0, \theta) + 2\Delta(\theta)}{2A(S, 0, \theta) + 2S + 2\Delta(\theta)}. \tag{19}$$

Similarly, for crab walk on a slope plane, the forbidden areas for the triangular support pattern could be redefined as shown in Fig. 11, and we have the following hexapod tripod gait as given by Theorem 6.

Theorem 6

Assume a hexapod is to walk on a constant slope plane with slope angle θ. The crab-walk tripod gait of a hexapod, at crab angle α and with stability margin S, is as follows:

1) The foothold positions are as shown on Fig. 8.

2) The sequence of movement for body and legs is as given in Table II, in which

$$B_1 = B_3 = S + \delta(\alpha) + \Delta(\theta) \quad \text{and} \quad B_2 = B_4 = A(S, \alpha, \theta) \tag{20}$$

where

$$A(S, \alpha, \theta) = \eta(\alpha) - 2\Delta(\theta) - 2\delta(\alpha) - 2S. \tag{21}$$

3) The stride length

$$\lambda(S, \alpha, \theta) = 2A(S, \alpha, \theta) + 2S + 2\Delta(\theta) + 2\delta(\alpha). \tag{22}$$

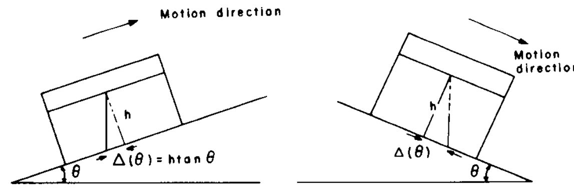


Fig. 9. The hexapod walks on the constant slope plane.

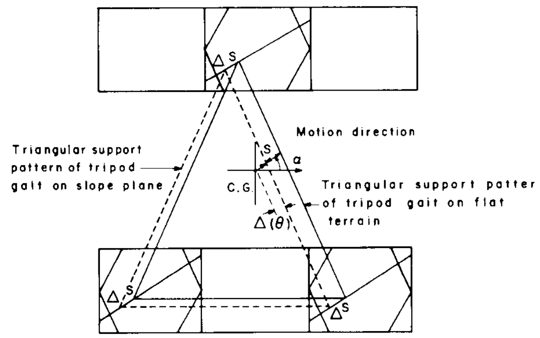


Fig. 10. The support patterns of crab walk tripod gait on constant slope plane and flat terrain with stability matgin S.

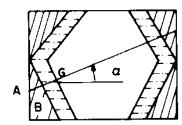


Fig. 11. The forbidden area of crab walk tripod gait on a constant slope plane, where $\delta(\alpha) = \overline{AB}$ and $\Delta(\theta) = \overline{BG}$.

4) The duty factor

$$\beta(S, \alpha, \theta) = \frac{A(0, \alpha, \theta) + 2\Delta(\theta) + 2\delta(\alpha)}{2A(0, \alpha, \theta) + 2\Delta(\theta) - 2S + 2\delta(\alpha)}. \tag{23}$$

Remark 5: Theorem 6 is a very general result that quantitatively characterizes the foothold positions, the stability margin, and the sequence of movement of body and legs. In fact, Theorems 1-5 are all special cases of Theorem 6. Any of Theorems 1-5 can be derived from Theorem 6 by substituting suitable values of θ and α.

Remark 6: The maximal stability margin that the crab-walk tripod gait of a hexapod can possibly achieve is

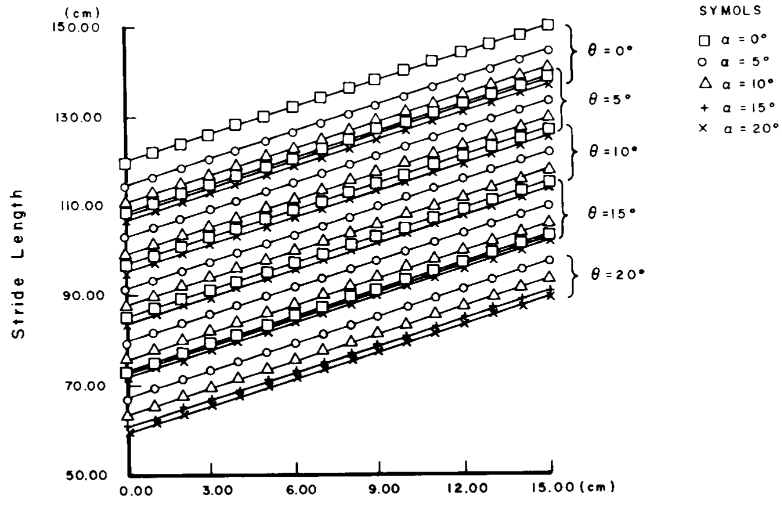
$$S_{\max} = \frac{A(0, \alpha, \theta)}{2}. \tag{24}$$

Fig. 12 shows how the stability margin, the stride length, and duty factor of a hexapod tripod gait are related to one another as a function of θ, the slope angle of the terrain, and α, the crab angle. In this figure the dimensions are $P = 60$ cm, $Q = 40$ cm, $h = 65$ cm, $W = 10$ cm, and $D = 40$ cm. From this figure, the required stride length and duty factor that corresponds to a regular tripod gait for crab walk with a specified stability margin, slope angle, and crab angle can be easily obtained without any further calculation.

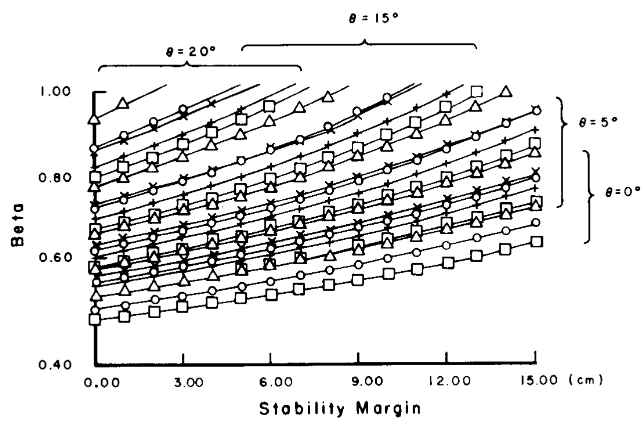
Since the slope angle θ affects the stability margin of the support pattern of the hexapod, there must be some specific value of θ that causes zero stability margin. In the following, we will investigate the maximal slope angle, $\theta_{\max}$, so that the hexapod can walk without causing instability. The following theorem shows the result.

Theorem 7

The maximal slope angle, $\theta_{\max}$, that the crab-walking tripod gait of



(a)



(b)

Fig. 12. (a) The relation between stride length and stability margin as a function of slope angle and crab angle. (b) The relation between duty factor and stability margin as a function of slope angle and crab angle.

a hexapod can walk stably at is

$$\theta_{\max} = \tan^{-1}\left(\frac{\eta(\alpha)-\delta(\alpha)}{2h}\right). \tag{25}$$

Proof: Since the maximal slope angle for a crab-walking tripod gait occurs at $S_{\max} = 0$, therefore

$$A(0, \alpha, \theta_{\max}) = 0$$

or

$$\Delta(\theta)_{\max} = h \tan \theta_{\max} = \frac{\eta(\alpha) - 2\delta(\alpha)}{2}.$$

Thus

$$\theta_{\max} = \tan^{-1}\left(\frac{\eta(\alpha)-\delta(\alpha)}{2h}\right).$$

Remark 7: In the case of walking straighforward (i.e., $\alpha = 0$), then $\delta(\alpha) = 0$, and $\eta(\alpha) = P$. Therefore

$$\theta_{\max} = \tan^{-1}\frac{P}{2h}.$$

VI. Conclusions

Results have been generated which quantitatively characterize the stability margins of the hexapod tripod gaits. In particular, the mathematical expression representing the relations between the stability margin, the stride length, and the duty factor are formulated.

The tripod gait of the hexapod for crab walking over perfectly flat terrain and over constant slope terrain are derived, respectively. We reiterate that these results are obtained based on the assumptions that 1) the hexapod has a symmetrical structure, 2) the reachable area of each leg is a rectangular region, 3) the initial foothold positions should be specified before the locomotion starts, and 4) the sequence for liftoff and placing the feet are as specified. It should be noted that by simply combining derived tripod gaits for moving straightforward and crab walking, one can achieve obstacle avoidence path planning and, possibly, the suitable gaits for a hexapod walking over irregular surface which can be approximated by the piecewise segments of perfect flat terrain and the constant slope terrain. Thus far, some preliminary results have been obtained. The complete solutions to this problem are still under investigation.

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Closed-Loop Manipulator Control Using Quaternion Feedback

JOSEPH S.-C. YUAN

Abstract—**Euler parameters, a form of normalized quaternions, are used here to model the hand orientation errors in resolved rate and resolved acceleration control of manipulators. The quaternion formulation greatly simplifies the stability analysis of the orientation error dynamics. Two types of quaternion feedback have been considered. The first type uses only the vector portion of the quaternion error, while the second one is based on a Euler rotation representation. The quaternion vector approach leads to a linear feedback control law for which the global asymptotic convergence of the orientation error is readily established. The Euler rotation approach also results in asymptotic error convergence in the large except for a singularity where the hand orientation differs from its desired orientation by a rotation of 180°.**

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The author is with Spar Aerospace Ltd., Weston, Ont., Canada M9L 2W7.
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I. Introduction

Manipulators with six or more degrees of freedom are generally required to follow preplanned paths of hand position and orientation defined as a function of time in Cartesian (or task space) coordinates. For closed-loop control of resolved motion, the instantaneous motion of the hand, or end-effector, must be monitored continuously either by using direct endpoint sensing techniques (e.g., [1]) or via a kinematic model of the manipulator which computes the hand position and orientation from the joint variables. This information is, in turn, used to produce corrective control action from the joint actuators in the manipulator (Fig. 1).

The hand position and orientation of a manipulator are typically represented by the position vector and rotation matrix, respectively, between reference coordinate frames fixed to the base and the last link of the manipulator [2]. The rotation matrix has the general form

$$R_{OH}=[\boldsymbol{n}\ \boldsymbol{s}\ \boldsymbol{a}] \tag{1}$$

where $\boldsymbol{n}$, $\boldsymbol{s}$, and $\boldsymbol{a}$ are the ***normal***, ***slide***, and ***approach*** (unit) vectors of the hand frame expressed in base frame coordinates.

It is clear that the position vector $\boldsymbol{p}$ and its derivatives ($\dot{\boldsymbol{p}}$ and $\ddot{\boldsymbol{p}}$ for velocity and acceleration, respectively) completely describe the translational motion of the hand. The position tracking error may be defined as

$$e_p=\boldsymbol{p}-\boldsymbol{p}_d \tag{2}$$

where $\boldsymbol{p}_d$ denote the desired hand position vector. The velocity and acceleration errors can be defined accordingly as $\dot{e}_p=(\dot{\boldsymbol{p}}-\dot{\boldsymbol{p}}_d)$ and $\ddot{e}_p=(\ddot{\boldsymbol{p}}-\ddot{\boldsymbol{p}}_d)$, respectively.

If $\boldsymbol{\omega}$ and $\dot{\boldsymbol{\omega}}$ denote the angular velocity and acceleration of the hand, then the corresponding error terms may be defined as $\dot{e}_0=(\boldsymbol{\omega}-\boldsymbol{\omega}_d)$ and $\ddot{e}_0=(\dot{\boldsymbol{\omega}}-\dot{\boldsymbol{\omega}}_d)$, where $\boldsymbol{\omega}_d$ and $\dot{\boldsymbol{\omega}}_d$ denote the desired angular velocity and angular acceleration, respectively, of the hand. The question that arises now is: What is an appropriate counterpart for $\boldsymbol{p}$ which represents $\int \boldsymbol{\omega}\, dt$ in the following definition of orientation tracking error?

$$e_0=\int \boldsymbol{\omega}\, dt-\int \boldsymbol{\omega}_d\, dt. \tag{3}$$

This question takes on particular significance in the context of closed-loop manipulator control since the position and orientation errors of the hand are used explicitly in the feedback loop.

When the manipulator is controlled in its joint coordinates [3, ch. 7], there is no need to generate the hand orientation error as in (3) since the desired trajectory, represented by the hand position and rotation matrix, may be converted directly into a corresponding trajectory in the joint coordinates. The inverse kinematic models used for such a conversion, however, are typically very complex and exist as closed-form solutions only for manipulators with special configurations, such as parallel adjacent joints or spherical wrists [3, ch. 3].

For orientation error feedback, the rotation matrix representation of (1) is clearly impractical simply because there are too many elements in the matrix. More importantly, not all of its elements (which are direction cosines) are independent due to the requirement of orthogonality among the unit vectors $\boldsymbol{n}$, $\boldsymbol{s}$, and $\boldsymbol{a}$.

Despite their nonuniqueness, Euler angles are frequently used to represent orientation [4, ch. 2.1.1] mainly because of their physical manifestations, such as roll, pitch, and yaw angles. But they are undesirable in feedback control due to singularities and computational complexity [3, ch. 3.2]. Furthermore, because the differential