Structure of even-even Dy nuclei in the interacting boson model with two-quasiparticle states

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(Received 25 January 1988)

The energy levels of g.s. bands, β bands, and γ bands of $^{154-162}$Dy isotopes are studied in the model of the traditional interacting boson approximation, allowing one boson to break and form a quasiparticle pair. The two quasiparticles are allowed to excite to the $i_{13/2}$ and $h_{11/2}$ orbitals. It was found that the energy levels of the g.s., β, and γ bands of even-even Dy nuclei can be reproduced very well. The backbends of the moment of inertia can also be reasonably described. The yrast $B(E2)$ values are also calculated and compared with the experimental data.

I. INTRODUCTION

The interacting boson approximation (IBA) model[1] has been remarkably successful in the description of the low-lying collective states in many medium to heavy even-even nuclei. Recently, a large amount of high-spin states of nuclei has been accumulated. Among these data some backbending occurs as one plots the moment of inertia versus the square of the angular velocity for the yrast band of a nucleus. Many efforts[2–12] within the framework of IBA have been attempted to understand the mechanism of the sudden change of the moment of inertia. It is believed that the backbending phenomenon is caused by the crossing of the ground-state band and a two-quasiparticle band.[12] Yoshida and co-workers[4,6] extended the *n-p* IBA (IBA-2) to allow one of the bosons to be changed into a pair of nucleons and applied this model to study the Ge isotopes. Alonso *et al.*[8] followed the work of Yoshida and applied this model to reproduce the backbending phenomena of Dy isotopes. Since the number of basic states of IBA-2 is tremendously large as the proton number goes away from the closed shell, one therefore needs to employ some kind of truncation on the basic states. In order to make the problem manageable, Alonso *et al.* used the weak-coupling technique in their calculation, although their calculation yielded satisfactory results for the ground-state band, the abundant experimental data of β and γ bands are still impossible to describe. Recently, Harter *et al.*[13] investigated the relationship between the IBA-1 and IBA-2 and concluded that for $N\pi+N\nu >> |N\pi-N\nu|$ the IBA-1 is a valid approximation. This, in fact, has been reflected in the success of the semimicroscopic model of Morrison *et al.*[11] in a boson basis based on the philosophy of the IBF model.[14] Hence it should be valuable and interesting to study in more detail whether the structure of the whole energy spectrum including g.s., β, and γ bands and the observed backbending phenomena of the deformed nuclei can be described in terms of all of the *whole* basic states of the IBA-1 plus a two-quasiparticle pair. In this work we shall illustrate the model by taking Dy isotopes as a testing example.

In Sec. II we describe the model. In Sec. III we present the results. The final section will give the summary and the discussion.

II. THE MODEL

The Dy isotopes we are interested in have $Z=66$, and N lies between 88 and 96. Thus valence nucleons are in the 50–82 major shell. Taking $Z=50$ and $N=82$ as the core, the traditional IBA assumes the valence boson numbers 11, 12, 13, 14, and 15 for $^{154-162}$Dy isotopes. It is assumed that one boson could break to form a quasiparticle pair, usually assigned to a unique parity intruder orbital with spin j. In the region of well-deformed nuclei the unique parity intruder orbitals such as $h_{11/2}$ and $i_{13/2}$ are the most important because both the Coriolis antipairing effect and the rotation alignment effect increase with increasing angular momentum.[4] In our model the two quasiparticles are allowed to excite to these two orbits. The angular momentum of the nucleon pair takes the values $4,6,\ldots,2j-1$ and is coupled to the rotation of the core. The couplings to angular momentum 0 and 2 are excluded in order to avoid the double counting of states, because they are included through the s and d bosons, respectively.

Our model space includes the IBA space with N bosons and states with $N-1$ bosons plus two nucleons. The model Hamiltonian is

$$H=H_B+H_F+V_{BF} ,$$

where H_B is the IBA boson Hamiltonian

$$H_B=a_0\epsilon_d+a_1p^{\dagger}\cdot p+a_2L\cdot L+a_3Q\cdot Q .$$

The octopole term $T_3\cdot T_3$ and the hexadecapole term $T_4\cdot T_4$ have been omitted in H_B since they are generally believed to be less important. The fermion Hamiltonian H_F is

$$H_F=\sum_m \epsilon_j a_{jm}^{\dagger}a_{jm}+\tfrac{1}{2}\sum_{JM} V^J(a_j^{\dagger}a_j^{\dagger})^{JM}(\tilde{a}_j\tilde{a}_j)^{JM} ,$$

with $a_j^\dagger$ being the nucleon creation operator. The mixing Hamiltonian V_{BF} is assumed,

$$V_{BF}=Q^B\cdot Q-Q^B\cdot Q^B ,$$

where

$$Q^B=(d^\dagger\tilde{s}+s^\dagger\tilde{d})^{(2)}-\frac{\sqrt{7}}{2}(d^\dagger d)^{(2)} ,$$

and

$$Q=Q^B+\alpha(a_j^\dagger a_j)^{(2)}+\beta[(a_j^\dagger a_j^\dagger)^{(4)}\tilde{d}-d^\dagger(\tilde{a}_j\tilde{a}_j)^{(4)}]^{(2)} .$$

For the radial dependence of the fermion potential, the Yukawa type with Rosenfeld mixture is used and an oscillator constant $\nu=0.96A^{-1/3}$ fm^{-2} with $A=160$ assumed. The interaction strength is adjusted so that the $J=0$ state is lower than the $J=2$ state by 2 MeV. The single-particle energy is obtained as a result of fitting. The other parameters contained in the boson Hamiltonian H_B and V_{BF} were chosen to reproduce the energy-level spectra of even Dy isotopes. In the calculation the interaction parameters contained in the boson part for each nuclei are kept to be the same values for either the N boson configuration or the $N-1$ boson plus two-quasiparticle configuration. The interaction strengths and the single-particle energies for each isotope are allowed to be mass number dependent.

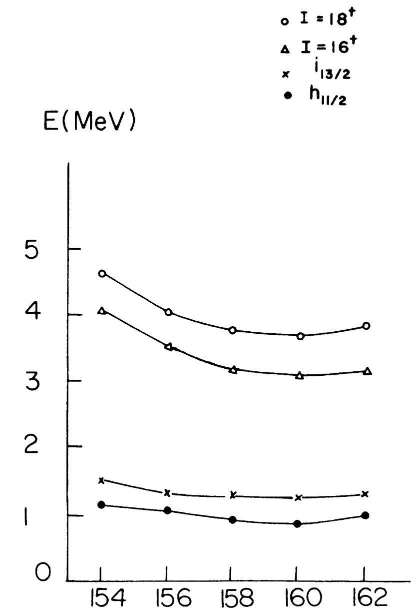

FIG. 1. Variation of calculated single-particle energies of $\epsilon(h_{11/2})$ and $\epsilon(i_{13/2})$ orbits compared with the variation of the energy values of $I=16$ and $I=18$ states for the Dy isotopes.

III. RESULTS

Table I lists the searched interaction strengths and single-particle energies for all isotopes. One can see that the strength of the d boson a_0 and the mixing parameters α and β can be unified for the whole string of even-even Dy isotopes with a mass number between 154 and 162. The magnitudes of the pairing term $P^\dagger\cdot P$, the quadrupole term $Q\cdot Q$, and the term $L\cdot L$ increase as mass number A increases, while the single quasinucleon energies $\epsilon(h_{11/2})$ and $\epsilon(i_{13/2})$ decrease from nucleus ^{154}Dy to nucleus ^{160}Dy and then increase for nucleus ^{162}Dy. The variation for the single-particle energies of the quasinucleons coincides approximately with the variational trend of the $I=16$ and $I=18$ states through the whole isotope string, as can be seen in Fig. 1.

The calculated and observed energy spectra for the string of Dy isotopes are shown in Figs. 2–6. There have been abundant experimental data[15–18] observed in recent years. The different quasibands are displayed in different columns for clear comparison. The energy levels marked with asterisks are not included in the fitting. It can be seen from the figures that the energy levels can be reproduced quite well, especially the g.s. bands. The calculated states of quasibands are all in correct order and agree reasonably with the observed data, except for very few levels. The relative intensities of wave functions for each energy level, corresponding to the N boson and $(N-1)$-boson plus two $h_{11/2}$ or $i_{13/2}$ quasiparticle configurations, are shown in Tables II and III. The total intensity of the N-boson configuration, the $(N-1)$-boson plus two $h_{11/2}$ quasinucleons configurations, and the $(N-1)$-boson plus two $i_{13/2}$ configuration for each state is normalized to 1.00. One can see the lower-lying levels of the β and the γ bands, and the yrast levels with angular momentum up to 14 are dominated by the pure boson configuration. The $(N-1)$-boson plus two $h_{11/2}$ quasiparticle configuration becomes dominant from $I=16_1$ states for g.s. bands. The $(N-1)$-boson plus two $i_{13/2}$ quasiparticle configuration becomes more important for the higher-spin states. Therefore, if we increase the $h_{11/2}$ quasiparti-

TABLE I. The interaction parameters in Mev for IBA-1 plus two-quasiparticle configuration model.

Parameter (Mev) nuclei	a_0	a_1	a_2	a_3	α	β	$\epsilon_{11/2}$	$\epsilon_{13/2}$
^{154}Dy	0.58	−0.002	0.0022	−0.009	0.035	0.025	1.16	1.505
^{156}Dy	0.58	0.028	0.0032	−0.009	0.035	0.025	1.05	1.32
^{158}Dy	0.58	0.048	0.0037	−0.009	0.035	0.025	0.905	1.275
^{160}Dy	0.58	0.066	0.0045	−0.008	0.035	0.025	0.865	1.25
^{162}Dy	0.58	0.066	0.0045	−0.005	0.035	0.025	1.0	1.3

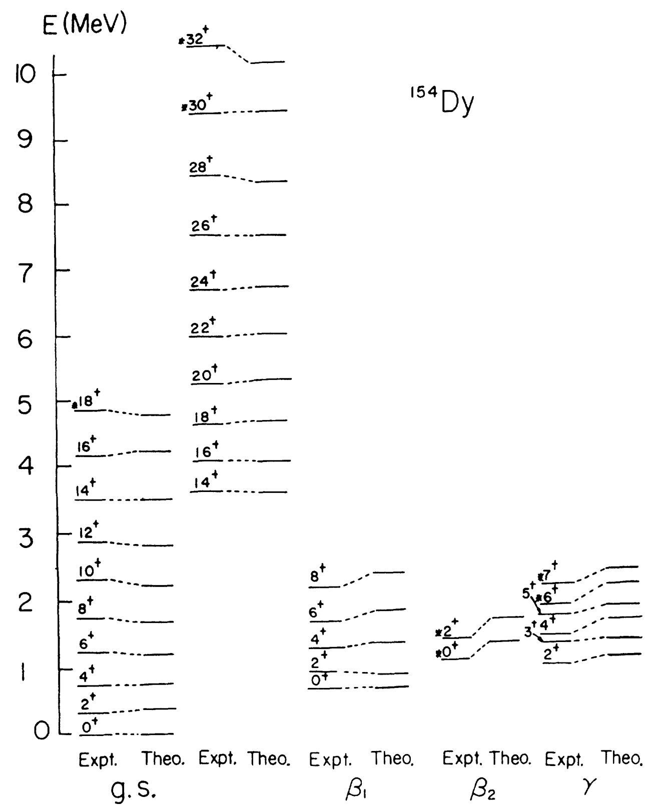

FIG. 2. Calculated and observed energy spectra for ^{154}Dy.

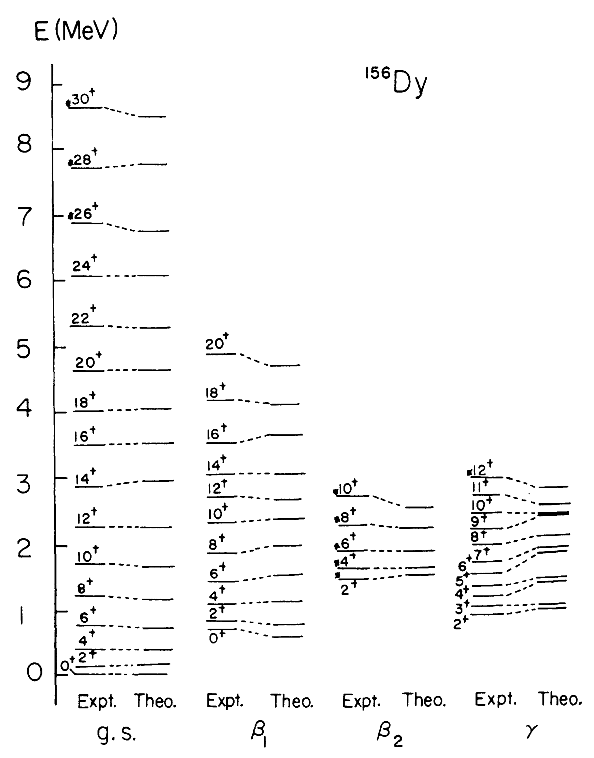

FIG. 3. Calculated and observed energy spectra for ^{156}Dy.

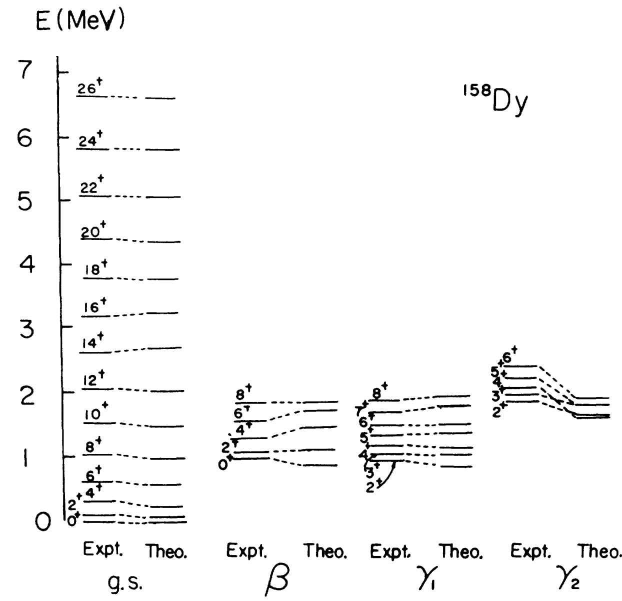

FIG. 4. Calculated and observed energy spectra for ^{158}Dy.

cle orbit in energy so that it becomes effectively irrelevant, then the agreements between the calculated and the observed high spin levels will become worse. Especially, the good coincidence of theoretical and experimental energy levels around the first backbends of the moments of inertia will get worse. Since Dy isotopes are not in the region of U(5) symmetry, the level spacings are almost equal. Although the almost equal spacings between the adjacent energy levels around $I \simeq 14$ in the string of Dy isotopes might be obtained by either increasing the value of the parameter β or lowering the single-

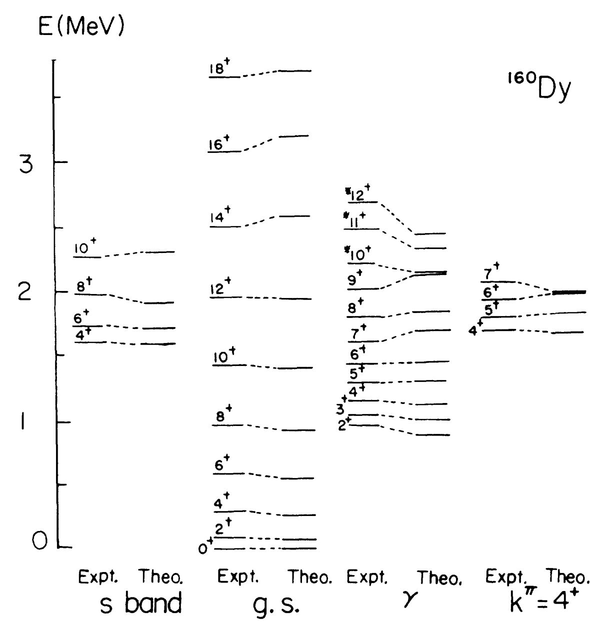

FIG. 5. Calculated and observed energy spectra for ^{160}Dy.

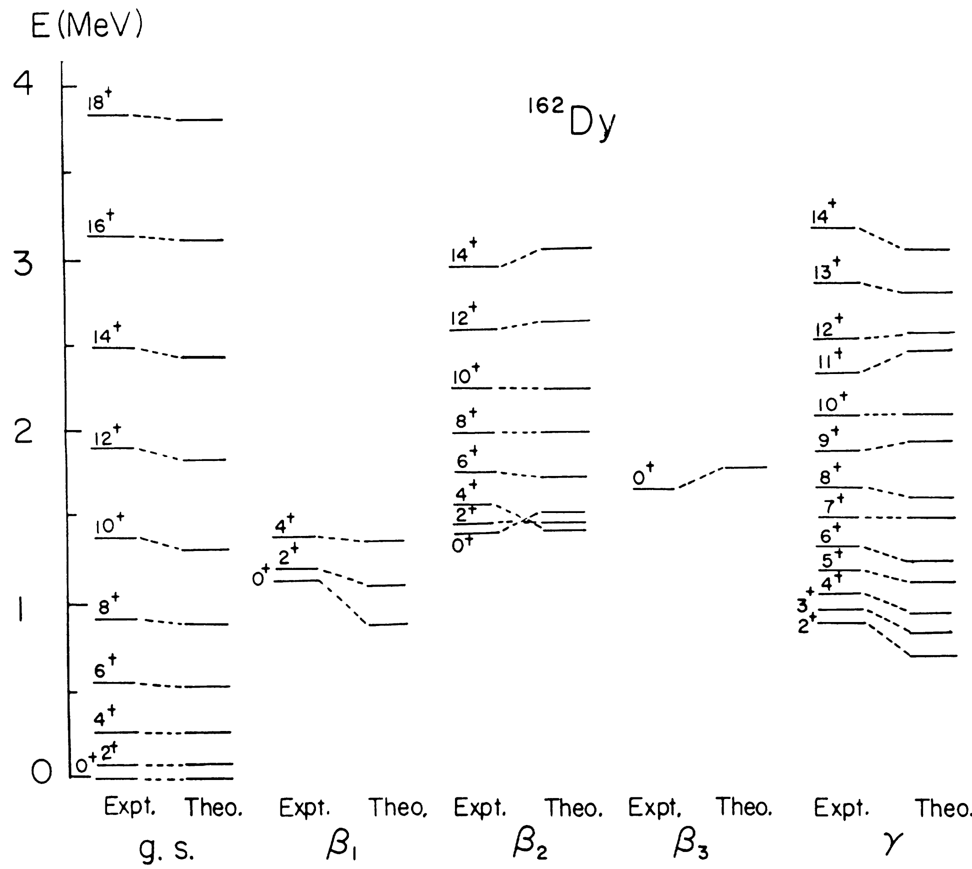

FIG. 6. Calculated and observed energy spectra for ^{162}Dy.

quasiparticle orbit $i_{13/2}$, both approaches will certainly make the agreements between the observed and the calculated higher-spin levels become worse. This shows the statistical significance of the single-quasiparticle energies $\epsilon(h_{11/2})$ and $\epsilon(i_{13/2})$ listed in the Table I. The apparent variation for the intensities of different configurations with the angular momenta shows that two bands of different deformation cross between $I=14$ and $I=18$, and a rotation aligned band originating from the $h_{11/2}$ and $i_{13/2}$ nucleon quasiparticle states stems from the $I=14$ state.

The backbendings in the Dy isotopes are commonly interpreted as the transition from the ground-state rotational band to the aligned two-quasiparticle $i_{13/2}$ nucleon band.[19] Figure 7 shows the results of our calculation. Here we choose the most sensitive expression to plot the conventional $2\vartheta/\hbar^2$ versus $(\hbar\omega)^2$ curves, with

$$2\vartheta/\hbar^2=-\frac{4I-2}{E_{I+2}-E_I}$$

and

$$(\hbar\omega)^2=\left[\frac{E_{I+2}-E_I}{[I(I+1)]^{1/2}-[(I-2)(I-1)]^{1/2}}\right].$$

It can be seen from Fig. 7 the agreements between the calculated and the observed curves are very satisfactory. We also plot the backbending curve for the β band of the ^{156}Dy nucleus. One can notice that the main feature of the observed data can be well reproduced. From the calculated wave-function intensities, as shown in the Tables II and III, one notices that the configurations, including two $h_{11/2}$ quasiparticles and two $i_{13/2}$ quasiparticles, are competitive at the point of the backbending, being of two nucleons $h_{11/2}$ lower in energy.

There are abundant experimental $B(E2)$ values for Dy isotopes.[16,18,20] The study of these values will give us a good test of the model wave functions. The electric quadrupole operator is written as

$$T(E2)=e^BQ+e^F\alpha(a_j^\dagger\tilde{a}_j)^{(2)}+\beta e^B[(a_j^\dagger a_j^\dagger)^{(4)}\tilde{d}-d^\dagger(\tilde{a}_j\tilde{a}_j)^{(4)}]^{(2)},$$

where Q is taken as

$$Q=(d^\dagger\tilde{s}+s^\dagger d)^{(2)}-\kappa(d^\dagger\tilde{d})^{(2)}.$$

TABLE II. The relative intensities of wave functions for energy levels of isotopes ^{154}Dy, ^{156}Dy, and ^{158}Dy.

States \ Nucleus configuration	^{154}Dy 0	^{154}Dy $h_{11/2}^2$	^{154}Dy $i_{13/2}^2$	^{156}Dy 0	^{156}Dy $h_{11/2}^2$	^{156}Dy $i_{13/2}^2$	^{158}Dy 0	^{158}Dy $h_{11/2}^2$	^{158}Dy $i_{13/2}^2$
0_1	1.00	0.00	0.00	0.97	0.02	0.01	0.91	0.06	0.03
2_1	0.99	0.01	0.00	0.95	0.03	0.02	0.90	0.07	0.03
4_1	0.96	0.03	0.01	0.92	0.05	0.03	0.87	0.09	0.04
6_1	0.93	0.05	0.02	0.89	0.08	0.03	0.83	0.12	0.05
8_1	0.90	0.08	0.02	0.85	0.11	0.04	0.79	0.16	0.05
10_1	0.87	0.10	0.03	0.81	0.14	0.05	0.74	0.20	0.06
12_1	0.84	0.12	0.04	0.77	0.17	0.06	0.69	0.25	0.06
14_1	0.80	0.16	0.04	0.73	0.21	0.06	0.63	0.31	0.06
16_1	0.00	0.98	0.02	0.00	0.95	0.05	0.00	0.98	0.02
18_1	0.00	0.97	0.03	0.00	0.87	0.13	0.00	0.98	0.02
20_1	0.00	0.96	0.04	0.00	0.06	0.94	0.00	0.98	0.02
22_1	0.00	0.88	0.12	0.00	0.01	0.99	0.00	0.97	0.03
24_1	0.00	0.04	0.96	0.00	0.00	1.00	0.00	0.67	0.33
26_1	0.00	0.04	0.96	0.00	0.00	1.00	0.00	0.01	0.99
0_2	0.97	0.02	0.01	0.95	0.03	0.02	0.92	0.05	0.03
2_2	0.98	0.02	0.00	0.96	0.03	0.01	0.92	0.06	0.02
4_2	0.95	0.03	0.02	0.93	0.05	0.02	0.88	0.09	0.03
6_2	0.93	0.06	0.01	0.90	0.08	0.02	0.83	0.14	0.03
8_2	0.88	0.10	0.02	0.83	0.13	0.04	0.32	0.60	0.08
0_3	0.95	0.03	0.02	0.94	0.04	0.02	0.91	0.07	0.02
2_3	0.96	0.03	0.01	0.93	0.05	0.02	0.89	0.08	0.03
3_1	0.97	0.02	0.01	0.94	0.04	0.02	0.89	0.08	0.03
4_3	0.94	0.04	0.02	0.90	0.07	0.03	0.86	0.11	0.03
5_1	0.94	0.04	0.02	0.91	0.07	0.02	0.85	0.11	0.04
6_3	0.01	0.91	0.08	0.02	0.77	0.21	0.04	0.78	0.18
7_1	0.91	0.07	0.02	0.87	0.10	0.03	0.79	0.17	0.04

TABLE III. The relative intensities of wave functions for energy levels of isotopes ^{160}Dy and ^{162}Dy.

States \ Nucleus configuration	^{160}Dy 0	^{160}Dy $h_{11/2}^2$	^{160}Dy $i_{13/2}^2$	^{162}Dy 0	^{162}Dy $h_{11/2}^2$	^{162}Dy $i_{13/2}^2$
0_1	0.87	0.08	0.05	0.88	0.08	0.04
2_1	0.86	0.09	0.05	0.86	0.09	0.05
4_1	0.84	0.12	0.04	0.84	0.10	0.06
6_1	0.80	0.15	0.05	0.81	0.13	0.06
8_1	0.76	0.19	0.05	0.77	0.16	0.07
10_1	0.71	0.23	0.06	0.73	0.19	0.08
12_1	0.66	0.28	0.06	0.70	0.22	0.08
14_1	0.60	0.34	0.06	0.66	0.25	0.09
16_1	0.00	0.98	0.02	0.62	0.29	0.09
18_1	0.00	0.98	0.02	0.00	0.96	0.04
0_2	0.90	0.07	0.03	0.91	0.06	0.03
2_2	0.87	0.10	0.03	0.89	0.07	0.04
4_2	0.85	0.11	0.04	0.86	0.09	0.05
6_2	0.80	0.16	0.04	0.83	0.12	0.05
8_2	0.55	0.39	0.06	0.78	0.16	0.06
10_2	0.03	0.90	0.07	0.74	0.20	0.06
2_3	0.87	0.10	0.03	0.88	0.08	0.04
3_1	0.86	0.10	0.04	0.87	0.09	0.04
4_3	0.83	0.13	0.04	0.88	0.08	0.04
5_1	0.82	0.14	0.04	0.84	0.11	0.05
6_3	0.04	0.78	0.18	0.83	0.12	0.05
7_1	0.77	0.19	0.04	0.80	0.14	0.06
8_3	0.24	0.63	0.13	0.02	0.78	0.20
9_1	0.56	0.40	0.04	0.76	0.18	0.06
10_3	0.05	0.90	0.50	0.03	0.85	0.12

For the fermion effective charge e^F, an average value 0.37 of the proton and neutron obtained by Alonso *et al.* is assumed. The boson effective charge in the $T(E2)$ operator has been determined by normalizing the largest calculated $B(E2)$ value for each nucleus to the corresponding observed data. The parameters α and β are assumed to be the same values as used in the mixing Hamiltonian. The value of κ is chosen to be $-\sqrt{7}/2$ which is the generator of the SU(3) group. Figure 8 shows the calculated and observed $B(E2;I \rightarrow I-2)$ versus the spins of the depopulating states. From the figure, it can be seen that important features of the $B(E2)$ values are reproduced well. Especially, the decreasing feature at $I=16$ and the increasing feature at $I=18$ of nucleus ^{156} Dy are obtained nicely. However, the interpretation of the decreasing feature at the $I=6$ state of ^{156}Dy is difficult in our model, since the excitation energy of the first 6^+ state level is only 770 KeV. Therefore the mixing of the two-quasiparticle configuration is very small and there is no contribution to improve the agreement. For the other nuclei, the $B(E2;I \rightarrow I-2)$ values for the yrast states are in reasonable agreement with the observed ones.

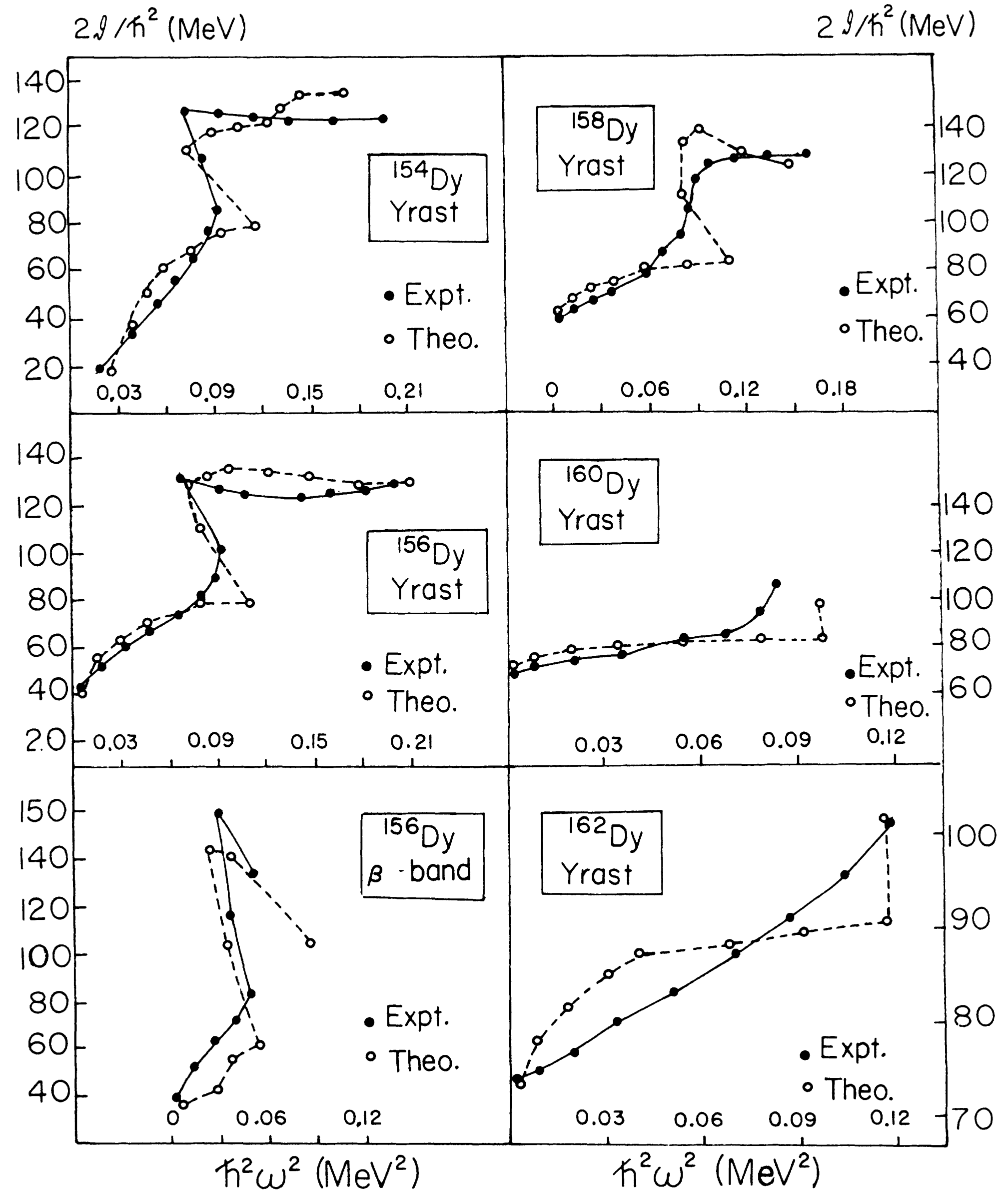

FIG. 7. Calculated and observed moment of inertia $2\theta/\hbar^2$ vs $(\hbar\omega)^2$ for yrast levels of $^{154-162}$Dy and the β band of ^{156}Dy.

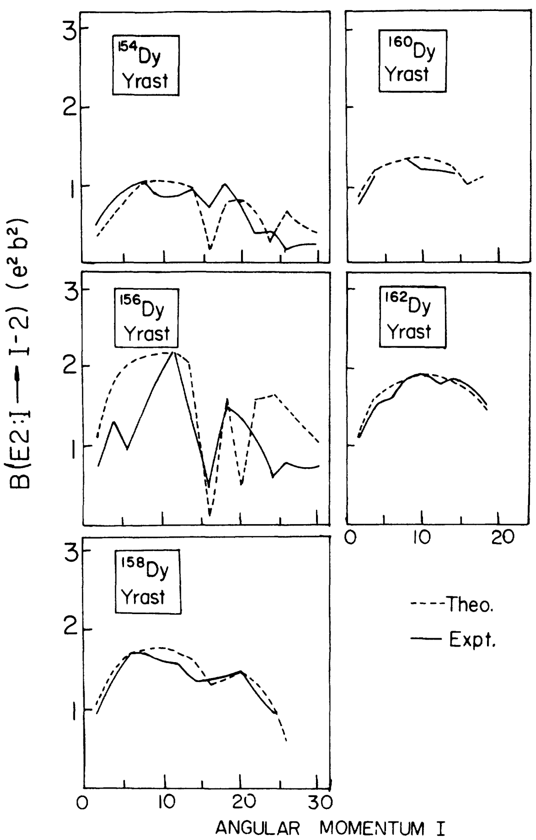

FIG. 8. Calculated and observed $B(E2;I\rightarrow I-2)$ values for the yrast band vs the spins of the depopulating states.

IV. SUMMARY AND DISCUSSION

In summary we have investigated the structure of the energy spectra and the backbending phenomena of the isotope string of Dy with a mass number between 154 and 162. We extend the IBA-1 model to allow a boson to break to form a quasiparticle pair which can occupy the $h_{11/2}$ and $i_{13/2}$ orbitals. The calculated energy levels, including the ground-state, β, and γ bands are in satisfactory agreement with the observed values for the whole string of Dy isotopes. Backbendings of the moment of inertia of the yrast states can be reproduced reasonably. We also plot the backbending curve of the moment of inertia of the β band for ^{156}Dy. The observed data are able to be explained. We also calculated $B(E2)$ values for Dy isotopes. Our model yields satisfactory agreement with the observed data.

The effects of the two-quasiparticle configuration are manifested in the improvement of the energy-level calculation and the fine variations in the calculated $B(E2)$ values. The couplings to angular momenta $J=4,6,8,\ldots$, for the quasiparticles in $h_{11/2}$ and $i_{13/2}$ orbitals might be considered as implicitly including the higher angular momentum bosons, such as the g boson and the I boson, etc., and therefore could make the IBA-1 model space more complete. This is also manifested in the analysis of the wave functions. The high-spin states are usually dominated by the $N-1$ boson plus two-quasiparticle configurations and thus cannot be reproduced by the traditional IBA model.

Recently, very high spin states up to $I\simeq 40$ and a double backbending have been observed in some nuclei in the rare-earth region.[21–23] These phenomena might hopefully be interpreted by considering two or more bosons to break to form more quasiparticle pairs and make more band crossings to form the double backbending. In conclusion, our calculation suggests a feasible model that can be extended very easily to handle the recently observed very-high-spin states and the double backbending phenomena in some rare-earth nuclei.

This work is supported by the National Science Council of the Republic of China under Grant No. NSC77-0208-M009-09.

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