國 立 交 通 大 學
電 機 與 控 制 工 程 學 系
碩 士 論 文

前置編碼多重輸入輸出區塊傳輸系統

Precoded MIMO Block Transmission System

研 究 生：王 彥 棋
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中華民國九十三年六月

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前置編碼多重輸入輸出區塊傳輸系統

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摘　　要

在這篇論文裡，我們提出前置編碼多重輸入輸出區塊傳輸(PMBT)系統。假設有 M 個傳送端天線和 M 個接收端天線的情況下，將 M 個調變訊號經由一個 M×M 傳輸矩置作為前置編碼並且經由 M 個傳送天線傳送出去。我們將使用零強制、最小均方誤差接收器和最佳位元分配的 PMBT 系統來做誤差率的分析並且和廣義的最大傳輸率(GMRT)系統的效率做比較。在使用零強制和最小均方誤差接收器的案例中, GMRT 和 PMBT 系統之間的關係是依據通道矩陣的特徵值而有所不同。對於使用最佳位元分配的案例中，PMBT 系統總是比 GMRT 系統有較好的效率。　.

Precoded MIMO Block Transmission System

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Abstract

In this thesis, we present the precoded MIMO block transmission (PMBT) system. Assuming there are M transmitting antennae and M receiving antennae, a block of M modulation are precoded using a $M \times M$ transmitting matrix and sent to the M Tx antennae. We analyze the BER performance of PMBT with zero forcing (ZF), minimum mean square error (MMSE) receiver and optimum bit allocation and also compare the performance with Generalized MRT (GMRT). In the case of ZF and MMSE receiver, the relationship between GMRT and PMBT depend on the eigenvalues of channel matrix. For optimum bit allocation case, PMBT always has better performance than GMRT.

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這篇論文的完成，首先我要感謝我的指導老師 林源倍 教授，謝謝她在兩年之內的教導，在這兩年的學習中不僅學習到通訊與訊號處理方面的知識，且學習到研究時所須嚴謹的精神與態度。

同時我也要感謝口試委員，林清安 教授與鄭木火 教授，對於我的論文指導與建議，使這篇論文的內容更加完整。

當然我也要感謝陪著我一起學習的伙伴立人、昶誠、重慶、孟良、禮涵、建樟、沛如和裕彬，與你們相處使我的學習更增添色彩。

最後要感謝雙親、姐姐與女友 渟慧，有了他們的陪伴與鼓勵，使我的學習更無後顧之憂，最後要感謝我大學同學 全章，有你在新竹的陪伴真的非常愉快。還有 世昌、志宏、孝勇、揚欣、士偉、學德以及認識我的同學及朋友們，讓我回到台北能有愉快的週末假期，使我的學習上更有衝勁，總之，謝謝大家。

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Chapter 1

Introduction

To improve the performance of a wireless transmission system in which the channel quality fluctuates, researchers suggested that the receiver be provided with multiple received signals generated by the same underlying data. The suggestions are referred to as diversity which exists in different forms including temporal diversity, frequency diversity, and antenna diversity.

Temporal diversity includes channel coding in conjunction with time interleaving which involve redundancy in the time domain. Frequency diversity refers to transmission on different frequencies which provides redundancy in frequency domain. Antenna diversity can be viewed as redundancy in spatial domain and implemented by using multiple antennae at both the transmit side (base station) and receiver side (mobile units). Antenna diversity, as one of the most effective techniques to improve the performance of a wireless communication system in fading environment, is usually achieved by employing multiple element antenna array at either transmitter or receiver, or at both transmitter and receiver.

Recently, transmit diversity technique is becoming more attractive various transmit diversity techniques were proposed in [1], [2], [3], and [4]. For example, the simple transmit diversity is proposed by Alamouti [1]. where a pair of symbols is transmitted using two antennae at first, and the transformed version of the pair is transmitted to obtain the MRC-like diversity. Another example of space-time block coding scheme was proposed by Tarokh [2], [3] and space-time trellis coding scheme was proposed by Tarokh [4]. Space-time block codes (STBC) operate on a block of input symbols, producing a matrix output whose columns represent time and rows represent antennae. Their main feature is the provision of full diversity with a very simple decoding scheme. On the other hand, space-time trellis codes (STTC) operate on one input symbol at a time, producing a sequence of vector symbols whose length represents antennae. STTC is like traditional TCM (trellis coded modulation). Their main feature they can provide full diversity gain and coding gain. Their disadvantage is that they are extremely hard to design and generally require high complexity encoders and decoder.

Most of methods were built on implementation to achieve the diversity rather than to maximize the signal-to-noise ratio (SNR). In [5], Titus K. Y. LO proposed a scheme of the maximum ratio transmission call MRT, which is similar to maximum ratio combining technique at the receiver end. This method use to select the transmitter and receiver weighting vectors to maximize SNR. We consider the selection of the weighting vector under the normalized constraint condition, and derive bounds of the overall SNR gain. But MRT method can not find the optimum SNR. In [6], Pingyi Fan find the generalized weighting vector to maximize SNR. It is proved in [6] that the generalized maximal SNR gain of MRT is always greater than or equal to that presented by Titus K. Y. LO under the normalized constraint condition on weighting vector for any given flat fading channel.

In this thesis, we present the precoded MIMO block transmission (PMBT) system. We assume there are M transmitting antennae and M receiving antennae. A block of M modulation are precoded using a $M \times M$ transmitting matrix and sent to the M Tx antennae. This is different from the MRT and Generalized MRT (GMRT) scheme in [5] and [6], in which multiple antennae are used to transmitted single modulation symbol with different weighting used for different antenna. We can design transceiver matrices of precoded MIMO block transmission to minimize BER. Assuming this is no bit allocation, we will optimum transmitting and receiving matrices to minimize BER. We will use this results to compare with MRT. We can first use zero-forcing (ZF) and minimum mean square error (MMSE) solution to find receiving matrix; then we choose transmitting matrix to minimize BER. In the MMSE case, we show that when the modulation symbols are 4-QAM, the optimum transmitting matrix is not unique. Example of the optimum transmitting matrix include the DFT matrix. In the case of zero forcing receiver, solution of optimal transmitting matrix are SNR dependent. For higher SNR, there also exists a class of channel-independent optimal transmitting matrix. The optimal solution are the same as those of the MMSE receivers. Final, we use optimum bit allocation solution to find the optimum transmitting matrix and to use this result to compare with MRT and GMRT. In the optimum bit allocation case, we show that when the channel noise is white Gaussian noise, the optimum transmitting matrix is the identity matrix and receiving matrix respectively the unitary matrices that diagonalize the channel matrix. We show that with optimal bit allocation, the PMBT requires a smaller transmission power than the GMRT system for the same transmission bit rate. Simulation examples will be given to corroborate the results.

1.1 Outline

- Chapter 2: The system model used in the study is described in Sec. 2.1. In Sec. 2.2, the MRT concept is presented. Discussions are given in terms of average SNR and the order of diversity in Sec. 2.3.

- Chapter 3: The system model used in the study is described in Sec. 3.1. In Sec. 3.2, we shall investigate the principle of generalize maximum ratio transmission (GMRT). In Sec. 3.3, we present generalized bounds of the overall SNR gain.

- Chapter 4: The system model used in the study is described in Sec. 4.1. In Sec. 4.2, we find optimum transmitter matrix for ZF receiver and derive the SNR β_{G_ZF} and the BER P_{G_ZF}. In Sec. 4.3, we find optimum transmitter matrix for MMSE receiver and derive the SNR β_{G_MMSE} and the BER P_{G_MMSE}.

- Chapter 5: In Sec. 5.1, we derive the optimal bit allocation formula for a given target transmission bit rate. In Sec. 5.2, We use ZF, MMSE receiver and optimum bit allocation case to compare GMRT.

- Chapter 6: In Sec. 6.1, it show some numerical examples of Maximum ratio transmission (MRT). In Sec. 6.2, we compare the average performance of the systems with different selection methods of the weighting vectors by some simulation. In Sec. 6.3, we compare PMBT, which use ZF, MMSE receiver and optimum bit allocation with GMRT by some simulation.
- Chapter 7: Conclusion.

1.2 Notations

1. **Bold face** are used to represent the matrices or the vectors.
2. $\mathbf{A}^H$ denotes transpose-conjugate of $\mathbf{A}$.
3. The notation $\mathbf{I}_M$ is used to represent the $M \times M$ identity Matrix.
4. The notation $diag(\lambda_1, \lambda_2, \cdots, \lambda_L)$denotes an $M \times M$ diagonal matrix with the diagonal element equal to λ_k.
5. The notation $\mathbf{W}_M$ is used to represent the normalized $M \times M$ DFT matrix given be
$$[\mathbf{W}_M]_{kn} = \frac{1}{\sqrt{M}} e^{-j\frac{2\pi}{M}kn}$$
where $0 \le k, n \le M-1$.

Chapter 2

Maximum Ratio Transmission (MRT)

A review of MRT given in [5], where multiple antennae are used for both transmission and reception. We will present the concept, principles, and analysis of MRT for wireless communication, where multiple antennae are used for both transmission and reception. This concept shows that the average overall SNR is proportional to the cross correlation between channel vectors and that error probability decrease inversely with the $(K \times L)$th power of the average SNR.

2.1 MRT System Model

A system is considered, which consists of K antennae for transmission and L antennae for reception. The channel consists of $K \times L$ statistically independent coefficients. It can be conveniently represented by a matrix

$$\mathbf{H} = \begin{bmatrix} h_{11} & \cdots & h_{1K} \\ \vdots & \ddots & \vdots \\ h_{L1} & \cdots & h_{LK} \end{bmatrix} = \begin{bmatrix} \mathbf{h_1} \\ \vdots \\ \mathbf{h_L} \end{bmatrix} \tag{2.1}$$

where the entry h_{pk} represents the channel coefficient for antenna k and antenna p. It is assumed that the channel coefficients are available to both the transmitter and receiver.

The system model shown in Fig. 2.1. The symbol s to be transmitted is weighted with a transmit weighting vector $\mathbf{v}$ to from the transmitted signal vector.

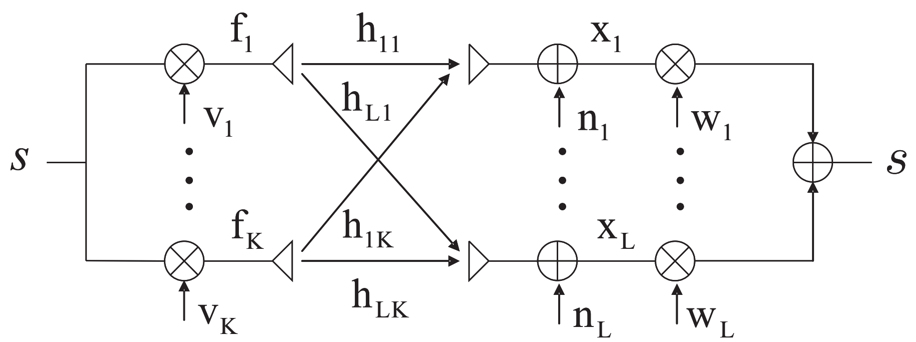

Figure 2.1: Maximum ratio transmission (MRT) system model

The received signal vector is the product of the transmitted signal vector and the channel plus the noise. that is

$$\mathbf{x} = \mathbf{Hf} + \mathbf{n} \tag{2.2}$$

where the transmitted signal s is given by

$$\mathbf{f} = [f_1 \cdots f_k]^T = s[v_1 \cdots v_k]^T$$

where s is the transmitted symbol, $[v_1 \cdots v_k]^T$ is the weighting factor in vector form. The noise vector is expressed as $\mathbf{n} = [n_1 \cdots n_L]^T$. Noise is assumed to be white Gaussian and uncorrelated with the signals. The received signals are weighted and summed to produce the estimate of the symbol.

2.2 Theoretical Analysis on Principle MRT

In order to generate the $K \times 1$ transmission weight. Let the transmission weight be a linear transformation of the channel matrix; that is

$$\mathbf{v} = \frac{1}{a}(\mathbf{gH})^H \tag{2.3}$$

where $\mathbf{g} = [g_1 \cdots g_L]$, and $a = \|\mathbf{gH}\| = \left(\sum_{p=1}^{L} \sum_{q=1}^{L} g_p g_q^* \sum_{k=1}^{K} h_{pk} h_{qk}^*\right)^{1/2}$. The transmitted signal vector is expressed as

$$\mathbf{f} = \frac{s}{a}(\mathbf{gH})^H \tag{2.4}$$

The received signal vector is, given by

$$\mathbf{x} = \frac{s}{a}\mathbf{H}(\mathbf{gH})^H + \mathbf{n} \tag{2.5}$$

To estimate the transmitted symbol, we let $\mathbf{w}$ set to be $\mathbf{g}$. The estimate of the symbol is given by

$$\hat{s} = \mathbf{gx} = \frac{s}{a}\mathbf{gH}(\mathbf{gH})^H + \mathbf{gn} = as + \mathbf{gn} \tag{2.6}$$

with the overall SNR given by

$$r = \frac{a^2}{\|\mathbf{g}\|^2}r_0 = \frac{a^2}{\sum\limits_{p=1}^{L} |g_p|^2}r_0 \tag{2.7}$$

where $r_0 = \sigma_s^2/\sigma_n^2$ denote the average SNR for a single transmitting antenna. From (2.10), we know the overall SNR is a function of $\mathbf{g}$. So we choose the appropriate value for $\mathbf{g}$. If we let $|g_1| = |g_2| = \cdots |g_L| = 1/\sqrt{L}$, the overall SNR is rewritten as

$$r = a^2 r_0 \tag{2.8}$$

which is maximized if a^2 maximized. a^2 reaches the maximum value if we set

$$\left(g_p g_q^*\right)^* = \frac{\sum\limits_{k=1}^{K} h_{pk}h_{qk}^*}{L\left|\sum\limits_{k=1}^{K} h_{pk}h_{qk}^*\right|}. \tag{2.9}$$

That is

$$a_{opt}^2 = \frac{1}{L}\sum_{p=1}^{L}\sum_{q=1}^{L}\left|\sum_{k=1}^{K} h_{pk}h_{qk}^*\right| = \frac{1}{L}\sum_{p=1}^{L}\sum_{q=1}^{L}\left|\mathbf{h_p}\mathbf{h_q^H}\right|. \tag{2.10}$$

2.3 The Bounds of the Average Overall SNR

The summation term with respect to K in (2.13) is the inner product of different pair of channel vectors if $\mathbf{h_p}$ and $\mathbf{h_q}$ are mutually orthogonal (i.e., $\mathbf{h_p}\mathbf{h_q^H} = 0$), a^2 takes on the smallest value; that is

$$a_{opt}^2 = \frac{1}{L}\sum_{p=1}^{L}\sum_{k=1}^{K}|h_{pk}|^2. \tag{2.11}$$

and

$$E[a_{opt}^2] = K\overline{r^2}. \tag{2.12}$$

If $\mathbf{h_p}$ and $\mathbf{h_q}$ are fully correlated (i.e., $\mathbf{h_p}\mathbf{h_q^H} = \|\mathbf{h_q}\|^2$), a^2 takes on the largest value; that is

$$a_{opt}^2 = \frac{1}{L}\sum_{p=1}^{L}\sum_{q=1}^{L}\sum_{k=1}^{K}|h_{qk}|^2. \tag{2.13}$$

and

$$E[a_{opt}^2] = LK\overline{r^2}. \tag{2.14}$$

Therefore, the average overall SNR is bounded by

$$K\overline{r^2}r_0 \leq \overline{r} \leq LK\overline{r^2}r_0. \tag{2.15}$$

For a system consisting of $K \times L$ antennae, it is expected that the order of diversity be $K \times L$; the probability of error decrease inversely with the $(K \times L)$th power of the average SNR. To determine P, the probability of error conditioned on a set of channel coefficients h_{pk} must be obtained first. The conditional error probability is averaged over the probability density function (pdf). We consider a special case with BPSK modulation scheme, the conditional error probability is expressed as

$$P(r) = Q(\sqrt{2r}). \tag{2.16}$$

The pdf $p(r)$ is χ^2-distribution with $2 \times K \times L$ degrees of freedom. It follows that $p(r)$ is given by

$$p(r) = \frac{r^{LK-1}e^{-r/\overline{r}_a}}{(LK-1)!\overline{r}_a^{LK}}. \tag{2.17}$$

where

$$\overline{r}_a = r_0E[|h_{pk}|^2] = r_0\overline{r^2}. \tag{2.18}$$

The error probability is them given by the following integral

$$P = \int_0^\infty P(r)p(r)dr. \tag{2.19}$$

For $\overline{r}_a \gg 1$

$$P = (\frac{1}{4\overline{r}_a})^{LK}\frac{(2LK-1)!}{(LK)!(LK-1)!} \tag{2.20}$$

which indicates the probability of error decrease inversely with the $(K \times L)$th power of the average SNR.

Chapter 3

Generalized Maximum Ratio Transmission (GMRT)

In this chapter, we consider the selection of the weighting vector in [6]. We will give the results. Furthermore, we also present that our generalized upper and lower bound of SNR can be achieved by selecting the weighting vector properly.

3.1 GMRT System Model

A system is considered, which consists of K antennae for transmission and L antennae for reception. The channel consists of $K \times L$ statistically independent coefficients. It can be conveniently represented by a matrix

$$\mathbf{H} = \begin{bmatrix} h_{11} & \cdots & h_{1K} \\ \vdots & \ddots & \vdots \\ h_{L1} & \cdots & h_{LK} \end{bmatrix} = \begin{bmatrix} \mathbf{h_1} \\ \vdots \\ \mathbf{h_L} \end{bmatrix} \tag{3.1}$$

where the entry h_{pk} represents the channel coefficient for antenna k and antenna p. It is assumed that the channel coefficients are available to both the transmitter and receiver.

The GMRT system model is same as MRT shown in Fig. 2.1. Next, by considering the following equivalent baseband model, the received signal vector is

$$\mathbf{x} = \mathbf{Hf} + \mathbf{n} \tag{3.2}$$

The transmitted signal s is given by

$$\mathbf{f} = [f_1 \cdots f_k]^T = s[v_1 \cdots v_k]^T$$

where s is the transmitted symbol, $[v_1 \cdots v_k]^T$ is the weighting factor in vector form. The noise vector is expressed as $\mathbf{n} = [n_1 \cdots n_L]^T$. Noise is assumed to be white Gaussian and uncorrelated with the signals.

3.2 Theoretical Analysis on Principle GMRT

In order to generate the $K \times 1$ transmission weight. Let the transmission weight be a linear transformation of the channel matrix; that is

$$\mathbf{v} = \frac{1}{a}(\mathbf{gH})^H \tag{3.3}$$

where $\mathbf{g} = [g_1 \cdots g_L]$, and $a = \|\mathbf{gH}\| = \left(\sum_{p=1}^{L}\sum_{q=1}^{L} g_p g_q^* \sum_{k=1}^{K} h_{pk}h_{qk}^*\right)^{1/2}$. The received signal vector is, given by

$$\mathbf{x} = \frac{s}{a}\mathbf{H}(\mathbf{gH})^H + \mathbf{n} \tag{3.4}$$

To estimate the transmitted symbol, we let $\mathbf{w}$ set to be $\mathbf{g}$. The estimate of the symbol is given by

$$\hat{s} = \mathbf{gx} = \frac{s}{a}\mathbf{gH}(\mathbf{gH})^H + \mathbf{gn} = as + \mathbf{gn} \tag{3.5}$$

with the overall SNR given by

$$r = \frac{a^2}{\|\mathbf{g}\|^2} r_0 = \frac{\|\mathbf{gH}\|^2}{\|\mathbf{g}\|^2} r_0 = \|\mathbf{gH}\|^2 r_0. \tag{3.6}$$

where $\|\mathbf{g}\| = 1$, and $r_0 = \sigma_s^2/\sigma_n^2$ denote the average SNR for a single transmitting antenna. The maximum ratio transmission problem can be converted into finding an optimal weighting vector $\mathbf{g}$. An optimal weighting vector makes the overall SNR be maximized. This is equivalent to following maximization problem

$$\max_{\mathbf{g}, \|\mathbf{g}\|=1} \{\mathbf{g}^H \mathbf{H}\mathbf{H}^H \mathbf{g}\} \tag{3.7}$$

By using the singular value decomposition of Hermitian symmetric matrix,

$$\mathbf{H}\mathbf{H}^H = \mathbf{U}\mathbf{\Lambda}^2\mathbf{U}^H. \tag{3.8}$$

where $\mathbf{U} = [\mathbf{u}_1, \mathbf{u}_2, \cdots, \mathbf{u}_L]$ is an unitary matrix, u_i is the i-th column vector, and $\mathbf{\Lambda} = diag(\lambda_1, \lambda_2, \cdots, \lambda_L)$ is a diagonal matrix. The diagonal matrix with elements in decremental order $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_L \geq 0$. We can use Rayleigh principle get $\|\mathbf{g}_{max}\mathbf{H}\|^2 = \lambda_1^2$ and choose $\mathbf{g}$ to be the corresponding eigenvector $\mathbf{u}_1$. Thus, the overall SNR can write

$$r_{max} = \|\mathbf{g}_{max}\mathbf{H}\|^2 r_0 = \lambda_1^2 r_0. \tag{3.9}$$

3.3 Generalized Bounds of the Overall SNR

We set $\|\mathbf{g}\| = 1$, by using the equality

$$\sum_{p=1}^{L} \lambda_p^2 = trace(\mathbf{H}\mathbf{H}^H) = \sum_{p=1}^{L}\sum_{q=1}^{K} |h_{pq}|^2. \tag{3.10}$$

So, we can get

$$\frac{1}{L}\sum_{p=1}^{L}\sum_{q=1}^{K} |h_{pq}|^2 \leq \lambda_1^2 \leq \sum_{p=1}^{L}\sum_{q=1}^{K} |h_{pq}|^2. \tag{3.11}$$

Overall SNR is bounded by

$$\frac{1}{L}\sum_{p=1}^{L}\sum_{q=1}^{K} |h_{pq}|^2 r_0 \leq \lambda_1^2 r_0 \leq \sum_{p=1}^{L}\sum_{q=1}^{K} |h_{pq}|^2 r_0. \tag{3.12}$$

In the chapter 2, we presented the maximum SNR gain under the condition $|g_1| = |g_2| = \cdots |g_L| = 1/\sqrt{L}$, as follows

$$a_{opt}^2 = \frac{1}{L}\sum_{p=1}^{L}\sum_{q=1}^{L}\left|\sum_{k=1}^{K} h_{pk}h_{qk}^*\right|. \tag{3.13}$$

It is easy to see the generalized maximal SNR gain of GMRT is always greater than or equal to that presented in the chapter 2 under the normalized constraint condition on weighting vector. Thus we have

$$\lambda_1^2 \geq a_{opt}^2. \tag{3.14}$$

We consider a special case with a given $\mathbf{H}$ and N-ary QAM modulation scheme, the corresponding bit error rate can be calculated by

$$P_e \simeq 2(1 - \frac{1}{\sqrt{N}})Q(\sqrt{\frac{3}{N-1}\lambda_1 r_0}). \tag{3.15}$$

Suppose the transmitted symbol has variance σ_s^2, then the transmission power is $E_{GMRT} = \sigma_s^2$. For a given probability of error P_e, the required transmission power is

$$E_{GMRT} = c\left(2^b - 1\right)\frac{N_0}{\lambda_1^2}. \tag{3.16}$$

where $c = 1/3\left[Q^{-1}\left(\frac{Pe}{2(1-2^{-M})}\right)\right]^2$.

Chapter 4

Precoded MIMO Block Transmission (PMBT)

In this chapter, we consider of M antennae for transmission and M antennae for reception and the minimization of probability error for precoded MIMO block transmission with an unitary precoding matrix. We analyze the probability error of MRBT with zero forcing (ZF) and minimum mean squared error (MMSE) receiver. Finally, we use optimum bit allocation to maximize performance at a given fixed data rate .

4.1 PMBT System Model

The channel consists of $M \times M$ statistically independent coefficients. It can be conveniently represented by a matrix

$$\mathbf{H} = \begin{bmatrix} h_{11} & \cdots & h_{1M} \\ \vdots & \ddots & \vdots \\ h_{M1} & \cdots & h_{MM} \end{bmatrix} = \begin{bmatrix} \mathbf{h_1} \\ \vdots \\ \mathbf{h_M} \end{bmatrix} \tag{4.1}$$

where the entry h_{pk} represents the channel coefficient for antenna k and antenna p. It is assumed that the channel coefficients are available to both the transmitter and receiver. The system model shown in Fig. 4.1. The transmitter is a unitary matrix $M \times M$ $\mathbf{G}$ with $\mathbf{G}^H\mathbf{G} = \mathbf{I}_M$. The receiver is an $M \times M$ matrix $\mathbf{A}$. Next, by considering the following equivalent model, the received signal vector is

$$\mathbf{r} = \mathbf{HGs} + \mathbf{n} \tag{4.2}$$

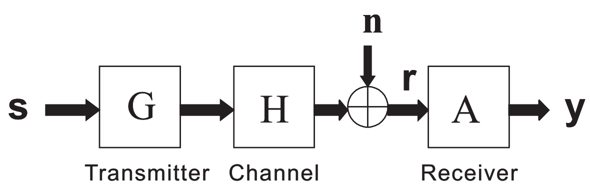

Figure 4.1: Precoded MIMO Block Transmission (PMBT) system model

The transmitted vector $\mathbf{s}$ is given by $\mathbf{s} = [s_1 \cdots s_M]^T$. Suppose the inputs have zero mean and variance E_s, with real and imaginary parts having equal variances $E_s/2$. The noise vector is expressed as $\mathbf{n} = [n_1 \cdots n_M]^T$. The channel noise n_i is uncorrelated complex white Gaussian noise with zero mean and variance N_0.

4.2 Zero Forcing (ZF) Receiver

Suppose the receiver is a zero forcing one. The overall transfer matrix $\mathbf{T_{overall}} = \mathbf{I}$ in the absence of the channel noise $\mathbf{n}$. So we set $\mathbf{A} = \mathbf{G}^H\mathbf{H}^{-1}$. We get the system model of PMBT which uses ZF receiver represented as in Fig. 4.2.

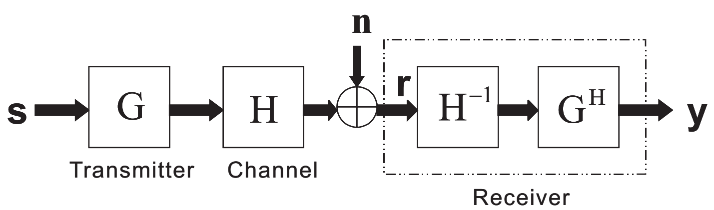

Figure 4.2: The PMBT System Model using Zero forcing Receiver

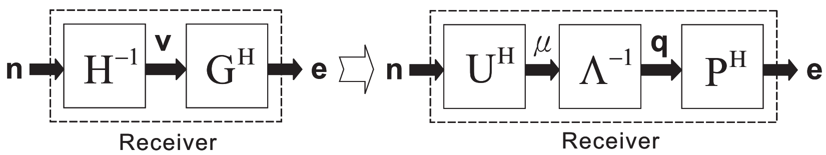

Figure 4.3: Illustration of noise path at a zero-forcing receiver

Let received vector be $\mathbf{r}$ be indicated in Fig. 4.2 ; then, the error vector is

$\mathbf{e} = \mathbf{Ar} - \mathbf{s}$. The noise come entirely from channel noise. The noise vector $\mathbf{e}$ can be analyzed by considering the receiver block diagram in Fig. 4.3. The channel matrix $\mathbf{H} = \mathbf{U\Lambda V}$, which by using singular value decomposition. $\mathbf{\Lambda} = diag(\lambda_1, \lambda_2, \cdots, \lambda_M)$ is a diagonal matrix. The diagonal matrix with elements in decremental order $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_M \geq 0$. Therefore, the output noise vector $\mathbf{e}$ has autocorrelation given by

$$\mathbf{R_e} = N_0 \mathbf{G}^H \mathbf{H}^{-1} \mathbf{H}^{-H} \mathbf{G} = N_0 \mathbf{G}^H \mathbf{V}^H \mathbf{\Lambda}^{-2} \mathbf{V} \mathbf{G} = N_0 \mathbf{P}^H \mathbf{\Lambda}^{-2} \mathbf{P}. \tag{4.3}$$

where $\mathbf{P} = \mathbf{VG}$. The elements of $\mathbf{n}$ are uncorrelated Gaussian random variables with variance N_0. The elements of $\mu = \mathbf{U}^H \mathbf{n}$ continue to be uncorrelated Gaussian random variables with variance N_0, due to the unitary property of $\mathbf{U}^H$. Therefore, the k-th element of the noise vector $\mathbf{q}$ has variance given by $\sigma_{q_k}^2 = N_0/\lambda_k^2$. The output noise $\mathbf{e}$ is related to $\mathbf{q}$ by

$$e_i = \sum_{k=1}^{M} p_{k,i}^* q_k. \tag{4.4}$$

where $p_{k,i}$ denote the (k,i)th element of $\mathbf{P}$. As q_k are uncorrelated, the i-th subchannel noise variance $\sigma_{G-ZF}^2(i) = \sum\limits_{k=1}^{M} |p_{k,i}|^2 \sigma_{q_k}^2$. That is

$$\sigma_{G-ZF}^2(i) = N_0 \sum_{k=1}^{M} \frac{|p_{k,i}|^2}{\lambda_i^2}. \tag{4.5}$$

Let $\beta_{G-ZF}(i) = E_s/\sigma_{G-ZF}^2(i)$, which is the SNR of the ith subchannel; then

$$\beta_{G-ZF}(i) = \frac{r}{\sum\limits_{k=1}^{M} |p_{k,i}|^2 / \lambda_k^2}. \tag{4.6}$$

where $r = E_s/N_0$. As $\mathbf{G}$ is unitary, we have $\sum_{i=1}^{M} |p_{k,i}|^2 = 1$. Using this fact, we can write the average mean square error (MSE) $\Upsilon = 1/M \sum_{i=1}^{M} \sigma_{G-ZF}^2(i)$ as

$$\Upsilon = \frac{1}{M} \sum_{i=1}^{M} \frac{1}{\lambda_i^2}. \tag{4.7}$$

The computation of BER depends on the modulation scheme used. We will use 4-QAM as an example. Assume the subchannel errors have equal variances in real and imaginary parts; therefore, the real and imaginary parts of the 4-QAM symbols have equal probability of error. Furthermore we assume 4-QAM symbols have equal probability. For 4-QAM modulation, the BER of the ith subchannel given by

$$P_{G-ZF}(i) = Q(\sqrt{\beta_{G-ZF}(i)}). \tag{4.8}$$

The average BER is

$$P_{G-ZF} = \frac{1}{M}\sum_{i=1}^{M} Q(\sqrt{\beta_{G-ZF}(i)}). \tag{4.9}$$

- The $\mathbf{G} = \mathbf{V}^H$ Case: The unitary matrix $\mathbf{P} = \mathbf{I}$. The ith subchannel noise variance represented as

$$\sigma^2_{V^H-ZF}(i) = N_0 \sum_{k=1}^{M} \frac{|p_{k,i}|^2}{\lambda_i^2} = \frac{N_0}{\lambda_i^2}. \tag{4.10}$$

  The SNR of the ith subchannel can write as

$$\beta_{V^H-ZF}(i) = \lambda_i^2 r. \tag{4.11}$$

  The average BER is

$$P_{V^H-ZF} = \frac{1}{M}\sum_{i=1}^{M} Q(\sqrt{\lambda_i^2 r}). \tag{4.12}$$

- The $\mathbf{G} = \mathbf{V}^H\mathbf{W}$ Case: The unitary matrix $\mathbf{P} = \mathbf{W}$. The ith subchannel noise variance represented as

$$\sigma^2_{V^HW-ZF} = \frac{N_0}{M}\sum_{k=1}^{M} \frac{1}{\lambda_k^2}. \tag{4.13}$$

  The SNR of the ith subchannel can write as

$$\beta_{V^HW-ZF} = \frac{r}{\frac{1}{M}\sum_{k=1}^{M}\frac{1}{\lambda_k^2}}. \tag{4.14}$$

  The average BER is

$$P_{V^HW-ZF} = Q(\sqrt{\beta_{V^HW-ZF}}). \tag{4.15}$$

For the convenience of subsequent discussion, we introduce the function

$$f(y) = Q(\frac{1}{\sqrt{y}}). \tag{4.16}$$

In terms of $f(\cdot)$ and subchannel SNR, we have

$$P_{G-ZF}(i) = Q(\sqrt{E_s/\sigma^2_{G-ZF}(i)}) = f(\frac{1}{\beta_{G-ZF}(i)}). \tag{4.17}$$

The average BER is give by

$$P_{G-ZF} = \frac{1}{M}\sum_{i=1}^{M} f(\frac{1}{\beta_{G-ZF}(i)}). \tag{4.18}$$

From the lemma 2 in [7] , we know $f(y)$ is monotone increasing. It is convex when $y \leq 1/3$ and it is concave when $y > 1/3$. Then three useful SNR quantities is defined for boundaries to distinguish operation region. They are r_0 , $\overline{r}$ and r_1 as follows:

$$r_0 = \min_i \frac{3}{\lambda_i^2} \ , \quad \overline{r} = \frac{1}{M}\sum_{i=1}^{M}\frac{3}{\lambda_i^2} \ , \quad r_1 = \max_i \frac{3}{\lambda_i^2}. \tag{4.19}$$

Obviously, it is true $r_0 \leq \overline{r} \leq r_1$. Three SNR regions are define as

$$R_{low} = \{r | r \leq r_0\}, \quad R_{mid} = \{r | r_0 \leq \overline{r} \leq r_1\}, \quad R_{high} = \{r | r \geq r_1\}. \tag{4.20}$$

After defining the SNR region, we can introduce the Theroem 1 in [7].

Theorem 1 : The average BER P_{G-ZF} is bounded by

$$P_{V^H-ZF} \leq P_{G-ZF} \leq P_{V^H W-ZF}, \qquad for\ r \in R_{low},$$
$$P_{V^H-ZF} \geq P_{G-ZF} \geq P_{V^H W-ZF}, \qquad for\ r \in R_{high}.$$

Each of two inequalities relating P_{G-ZF} and $P_{V^H W-ZF}$ becomes an equality if and only if subchannel noise variance $\sigma^2_{G-ZF}(i)$ are equal, i.e., $\sigma^2_{G-ZF}(i) = \Upsilon$, where Υ is as given in (4.7). The results in Theorem 1 imply that the $\mathbf{P} = \mathbf{I}$ is the optimal solution for r in R_{low} and the $\mathbf{P} = \mathbf{W}$ is the optimal solution for r in R_{high}. The SNR region R_{low} corresponds to a high error rate, whereas R_{high} corresponds to a more useful range of BER. So we choose the $\mathbf{P} = \mathbf{W}$ is the optimal solution for high SNR. The optimum transmitting matrix $\mathbf{G}$ is equal to $\mathbf{V^H W}$.

4.3 Minimum Mean Square Error (MMSE) Receiver

In this subsection, we consider that the receiver is an MMSE receiver. We will see that using an MMSE receiver improves the system performance, especially when the channel has spectral nulls. Let receiver vector is $\mathbf{r}$ be indicated in Fig.4.4; then, the error vector is $\mathbf{e} = \mathbf{Ar} - \mathbf{s}$. For a given unitary transmitting matrix $\mathbf{G}$, the optimal receiver matrix $\mathbf{A}$ that minimizes $E[\mathbf{e}^H\mathbf{e}]$. By the orthogonality principle, $\mathbf{e}$ should be orthogonal to the observation vector $\mathbf{r}$, i.e., $E[(\mathbf{Ar} - \mathbf{s})\mathbf{r}^H] = 0$. This yields $\mathbf{A}E[\mathbf{rr}^H] = E[\mathbf{sr}^H]$; then

$$E[\mathbf{sr}^H] = E[\mathbf{s}(\mathbf{s}^H\mathbf{G}^H\mathbf{H}^H + \mathbf{n}^H)] = \sigma_s^2\mathbf{G}^H\mathbf{H}^H. \tag{4.21}$$

$$E[\mathbf{rr}^H] = E[(\mathbf{HGs} + \mathbf{n})(\mathbf{s}^H\mathbf{G}^H\mathbf{H}^H + \mathbf{n}^H)] = \sigma_s^2\mathbf{HGG}^H\mathbf{H}^H + N_0\mathbf{I}. \tag{4.22}$$

The channel matrix $\mathbf{H} = \mathbf{U\Lambda V}$, which by using singular value decomposition (SVD). $\mathbf{\Lambda} = diag(\lambda_1, \lambda_2, \cdots, \lambda_M)$ is a diagonal matrix. The diagonal matrix with elements in decremental order $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_M \geq 0$. Solving this equation, we get

$$\begin{aligned} \mathbf{A} &= \sigma_s^2\mathbf{G}^H\mathbf{H}^H(\sigma_s^2\mathbf{HGG}^H\mathbf{H}^H + N_0\mathbf{I})^{-1} && (4.23)\\ &= \sigma_s^2\mathbf{G}^H\mathbf{V}^H\mathbf{\Lambda U}^H(\sigma_s^2\mathbf{U\Lambda}^2\mathbf{U}^H + N_0\mathbf{I})^{-1} && (4.24)\\ &= \mathbf{G}^H\mathbf{V}^H\mathbf{\Lambda}\sigma_s^2(\sigma_s^2\mathbf{\Lambda}^2 + N_0\mathbf{I})^{-1}\mathbf{U}^H. && (4.25) \end{aligned}$$

Let $\mathbf{\Gamma} = \mathbf{\Lambda}\sigma_s^2(\sigma_s^2\mathbf{\Lambda^2} + N_0\mathbf{I})^{\mathbf{-1}}$ is a diagonal matrix, and the diagonal element represent as $[\mathbf{\Gamma}]_i = \sigma_s^2\lambda_i/(\sigma_s^2\lambda_i^2 + N_0)$.

$$\begin{aligned} \mathbf{A} &= \mathbf{G}^H\mathbf{V}^H\mathbf{\Gamma U}^H && (4.26)\\ &= \mathbf{G}^H\mathbf{F}. && (4.27) \end{aligned}$$

where $\mathbf{F} = \mathbf{V}^H\mathbf{\Gamma U}^H$. We get the system model of PMBT which uses MMSE receiver represented as in Fig. 4.4. Using $\mathbf{A} = \mathbf{G}^H\mathbf{F}$, $\mathbf{y} = \mathbf{Ar} + \mathbf{n}$ can be written as

$$\mathbf{y} = \mathbf{G}^H\mathbf{FHGs} + \mathbf{n} = \mathbf{Ts} + \mathbf{n}. \tag{4.28}$$

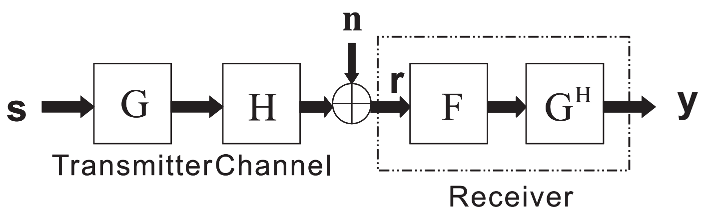

Figure 4.4: The PMBT System Model using MMSE Receiver

where $\mathbf{T} = \mathbf{G}^H\mathbf{F}\mathbf{H}\mathbf{G}$. We can further verify that $\mathbf{y} = \mathbf{G}^H\mathbf{V}^H\mathbf{\Gamma}\mathbf{\Lambda}\mathbf{V}\mathbf{G}\mathbf{s} + \mathbf{n}$ Now we define $\mathbf{P} = \mathbf{V}\mathbf{G}$, and $\mathbf{y}$ can be represented as

$$\mathbf{y} = \mathbf{P}^H\mathbf{\Gamma}\mathbf{\Lambda}\mathbf{P} + \mathbf{n} = \mathbf{P}^H\mathbf{J}\mathbf{P} + \mathbf{n}. \tag{4.29}$$

where $\mathbf{J} = \mathbf{\Gamma}\mathbf{\Lambda}$, the diagonal element represent as

$$[\mathbf{J}]_{ii} = \frac{\sigma_s^2\lambda_i^2}{(\sigma_s^2\lambda_i^2 + N_0)}. \tag{4.30}$$

The autocorrelation matrix of the vector $\mathbf{y}$ is represented as

$$\mathbf{R}_{\mathbf{yy}} = E_s\mathbf{P}\mathbf{J}^2\mathbf{P} + N_0\mathbf{P}\mathbf{\Gamma}^2\mathbf{P}. \tag{4.31}$$

The i-th receiver output y_i can be expressed as

$$y_i = a_i s_i + \tau_i. \tag{4.32}$$

where $a_i = \sum_{k=1}^{M-1} |p_{k,i}|^2 \frac{\lambda_k^2 r}{1+\lambda_k^2 r}$, and $\tau_i = \sum_{j\neq i} s_j \sum_{k=1}^{M-1} p_{k,i}^* p_{k,j} \frac{\lambda_k^2 r}{1+\lambda_k^2 r} + [\mathbf{P}^H\mathbf{\Gamma}\mathbf{U}^H\mathbf{n}]_i$.
The variance of y_i is

$$\begin{aligned} E\left[|y_i|^2\right] &= E_s \sum_{k=1}^{M} |p_{k,i}|^2 \frac{(\lambda_k^2 r)^2}{(1+\lambda_k^2 r)^2} + N_0 \sum_{k=1}^{M} |p_{k,i}|^2 \frac{\lambda_k^2 r^2}{(1+\lambda_k^2 r)^2} && (4.33) \\ &= E_s \sum_{k=1}^{M} |p_{k,i}|^2 \frac{\lambda_k^2 r}{1+\lambda_k^2 r} && (4.34) \\ &= a_i E_s. && (4.35) \end{aligned}$$

The variance of τ_i is

$$E\left[|\tau_i|^2\right] = E\left[|y_i|^2\right] - a_i^2 E_s = a_i E_s (1 - a_i). \tag{4.36}$$

The subchannel SINR $\beta_{G-MMSE}(i) = a_i^2 E_s / E\left[|\tau_i|^2\right]$ is given by

$$\beta_{G-MMSE}(i) = \frac{a_i^2 E_s}{a_i E_s (1 - a_i)} = \frac{a_i}{1 - a_i} = \frac{\sum_{k=1}^{M} |p_{k,i}|^2 \frac{\lambda_k^2 r}{1+\lambda_k^2 r}}{\sum_{k=1}^{M} |p_{k,i}|^2 \frac{1}{1+\lambda_k^2 r}}. \tag{4.37}$$

The computation of BER depends on the modulation scheme used. We will use 4-QAM as an example. Assume the subchannel errors have equal variances in real and imaginary parts; therefore, the real and imaginary parts of the 4-QAM symbols have equal probability of error. Furthermore we assume 4-QAM symbols have equal probability. For 4-QAM modulation, the BER of the i-th subchannel is given by

$$P_{G-MMSE}(i) = Q(\sqrt{\beta_{G-MMSE}(i)}). \tag{4.38}$$

The average BER is

$$P_{G-MMSE} = \frac{1}{M} \sum_{i=1}^{M} Q(\sqrt{\beta_{G-MMSE}(i)}). \tag{4.39}$$

- The $\mathbf{G} = \mathbf{V}^H$ Case: The unitary matrix $\mathbf{P} = \mathbf{I}$. The SNR of the i-th subchannel is

  $$\beta_{V^H-MMSE}(i) = \lambda_i^2 r. \tag{4.40}$$

  Comparing with (4.10), we see that it is the same as $\beta_{V^H-ZF}(i)$. The average BER is

  $$P_{V^H-MMSE} = \frac{1}{M} \sum_{i=1}^{M} Q(\sqrt{\lambda_i^2 r}). \tag{4.41}$$

- The $\mathbf{G} = \mathbf{V}^H\mathbf{W}$ Case: The unitary matrix $\mathbf{P} = \mathbf{W}$. The SNR of the i-th subchannel can be written as

  $$\beta_{V^H W-MMSE} = \frac{\sum_{k=1}^{M} \frac{\lambda_k^2 r}{1+\lambda_k^2 r}}{\sum_{k=1}^{M} \frac{1}{1+\lambda_k^2 r}}. \tag{4.42}$$

  The average BER is

  $$P_{V^H W-MMSE} = Q(\sqrt{\beta_{V^H W-MMSE}}). \tag{4.43}$$

For the convenience of subsequent discussion, we introduce the function

$$h(y) = Q(\sqrt{y^{-1} - 1}). \tag{4.44}$$

The subchannel BER is

$$Q\left(\sqrt{\beta_{G-MMSE}(i)}\right) = h(\frac{1}{1 + \beta_{G-MMSE}(i)}). \tag{4.45}$$

Using (4.37),

$$\frac{1}{1 + \beta_{G-MMSE}(i)} = \sum_{k=1}^{M} \frac{|p_{k,i}|^2}{1 + \lambda_k^2 r}. \tag{4.46}$$

Therefore, we have

$$P_{G-MMSE} = \frac{1}{M}\sum_{i=1}^{M} P_{G-MMSE}(i) \ , \quad P_{G-MMSE}(i) = h\left(\sum_{k=1}^{M} \frac{|p_{k,i}|^2}{1 + \lambda_k^2 r}\right). \tag{4.47}$$

Using the definition of $h(\cdot)$, the BERs of two cases are given, respectively, by

$$P_{V^H-MMSE} = \frac{1}{M}\sum_{i=1}^{M} h\left(\frac{1}{1 + \lambda_i^2 r}\right). \tag{4.48}$$

$$P_{V^H W-MMSE} = h\left(\frac{1}{M}\sum_{i=1}^{M} \frac{1}{1 + \lambda_i^2 r}\right) \tag{4.49}$$

write is as a lemma from the lemma 4 in [7] , the function $h(y) = Q(\sqrt{y^{-1} - 1})$, defined for $0 < y < 1$, is convex. Using the above lemma, we can show the following result

$$P_{V^H W-MMSE} \leq P_{G-MMSE} \leq P_{V^H-MMSE} \tag{4.50}$$

Each of two inequalities relating P_{G-MMSE} and $P_{V^H W-MMSE}$ becomes an equality if and only if subchannel noise variance $\sigma^2_{G-MMSE}(i)$ are equal. So the choice $\mathbf{G} = \mathbf{V}^H\mathbf{W}$ is the optimal solution.

Chapter 5

Precoded MIMO Block Transmission with Bit Allocation

In this chapter, we consider bit allocation the precoded block transmission system in chapter 4. We first derive the bit allocation formula for PMBT which uses zero forcing receiver such that the transmitting power can be minimized for a given bit rate.

Let the bit rate in the k-th channel be b_k, then the total bit rate is $b = \sum_{k=1}^{M} b_k$ per block. The input power of the k-th channel is $\sigma_{s_k}^2$, which is also the output signal power of the k-th channel at the receiver end due to $\mathbf{AHG} = \mathbf{I}_M$. Suppose the output noise power of the k-th channel is $\sigma_{e_k}^2$, given by

$$\sigma_{e_k}^2 = N_0 \sum_{i=1}^{M} |p_{k,i}|^2 / \lambda_i^2. \tag{5.1}$$

For QAM modulation schemes under high bit rate assumption, we have

$$\sigma_{s_k}^2 \simeq \frac{1}{3} \left[Q^{-1} \left(\frac{Pe}{2} \right) \right]^2 2^{b_k} \sigma_{e_k}^2 = c 2^{b_k} \sigma_{e_k}^2. \tag{5.2}$$

where $c = \frac{1}{3} \left[Q^{-1} \left(\frac{Pe}{2} \right) \right]^2$, which depends on the given probability of symbol error P_e. Define P(b) as the transmitting power needed for transmitting b bits. Then we have

$$P(b) = \sum_{k=1}^{M} \sigma_{s_k}^2 = c \sum_{k=1}^{M} 2^{b_k} \sigma_{e_k}^2 \tag{5.3}$$

$$\geq cM \left(\prod_{k=1}^{M} 2^{b_i} \right)^{1/M} \left(\prod_{k=1}^{M} \sigma_{e_k}^2 \right)^{1/M} \tag{5.4}$$

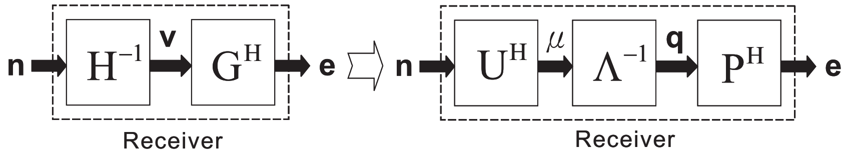

Figure 5.1: Block diagram of the receiver

$$= cM2^{b/M}\left(\prod_{k=1}^{M}\sigma_{e_k}^2\right)^{1/M} = cM2^{b/M}E_0^{1/M}. \quad (5.5)$$

where $E_0 = \prod_{k=1}^{M}\sigma_{e_k}^2$. The equality holds if and only if the bits are optimally allocated according to

$$b_k = \frac{b}{M} - log_2\sigma_{e_k}^2 + \frac{1}{M}log_2 E_0 \quad (5.6)$$

$$= \frac{b}{M} - log_2\left(N_0\sum_{i=1}^{M}\frac{|p_{k,i}|^2}{\lambda_i^2}\right) + \frac{1}{M}log_2\left(\prod_{t=1}^{M}N_0\sum_{i=1}^{M}\frac{|p_{t,i}|^2}{\lambda_i^2}\right). \quad (5.7)$$

The entry $p_{k,i}$ represents the unitary matrix for row k and column i. We define the coding gain Cg as $P_{direct}(b)$, the power needed for transmitting b bits when there is no bit allocation, over P(b). Without bit allocation, $b_k = b/M$, for $k = 1, 2, \ldots, M$; then

$$P_{direct}(b) = c2^{b/M}\sum_{k=1}^{M}\sigma_{e_k}^2. \quad (5.8)$$

The coding gain of bit allocation is

$$Cg = \frac{1}{M}\sum_{k=1}^{M}\sigma_{e_k}^2/\left(\prod_{k=1}^{M}\sigma_{e_k}^2\right)^{1/M} \geq 1. \quad (5.9)$$

The above inequality follows from the arithmetic mean over the geometric mean inequality.

From Fig. 5.1, we see that the optimum $\mathbf{P}$ is determined by the autocorrelation matrix $\mathbf{R_{ee}}$ of $\mathbf{e}$, which is as indicated in Fig. 5.1. Using Hadamard inequality, we know that $\prod_{k=1}^{M}\sigma_{e_k}^2 \geq det(\mathbf{R_{ee}}) = det(\mathbf{R_{qq}})$. We can use this result to minimize $\prod_{k=1}^{M}\sigma_{e_k}^2$. We choose $\mathbf{P}^H$ matrix to make $\mathbf{R_{ee}}$ is diagonal.

AWGN Channel

When the channel noise is a white noise, the autocorrelation matrix $\mathbf{R_{nn}} = \sigma_e^2\mathbf{I}$

is a diagonal matrix. The optimal $\mathbf{P}$ is simply $\mathbf{P}^H = I$. We get $[\prod_{k=1}^{M} \sigma_{e_k}^2]^{1/M} = [det\,(\mathbf{R_{ee}})]^{1/M} = \left(\prod_{k=1}^{M} \lambda_k^{-2}\right)^{1/M} N_0$. The minimized transmission total power represented as

$$P(b) = cM2^{b/M} \left(\prod_{k=1}^{M} \lambda_k^{-2} \right)^{1/M} N_0. \tag{5.10}$$

The transmission bits in k-th channel represented as

$$b_k = \frac{b}{M} - log_2 \left(\frac{N_0}{\lambda_k^2} \right) + \frac{1}{M} log_2 \left(\prod_{t=1}^{M} \frac{N_0}{\lambda_i^2} \right). \tag{5.11}$$

The coding gain is

$$Cg = \frac{1}{M} \sum_{k=1}^{M} 1/\lambda_k^2 / \left(\prod_{k=1}^{M} 1/\lambda_k^2 \right)^{1/M}. \tag{5.12}$$

5.1 Nonnegative Integer Bit Allocation

In the derivation of bit allocation in Sec. 5, we do not constraint b_k to be nonnegative integer. The solution in (5.7) may give bit allocation with fractions or negative numbers. To obtain nonnegative bit allocation, we will use greedy algorithm by allocating one bit at a time.

Assume we have bit budget b bits. For each bit to be allocated, we give the subchannel one bit such that the resulting total modulation symbols variance $\sum_{k=1}^{M} \sigma_{s_k}^2(b_k)$ is minimum. That is, each time we allocate one bit to the $k-th$ subchannel that has the smallest incremental energy $e_k(b_k)$ until all the bits are allocated. For example, a energy function $\sigma_{s_k}^2(b_k)$ for QAM modulation could be define as

$$\sigma_{s_k}^2(b_k) \simeq \frac{1}{3} \left[Q^{-1} \left(\frac{Pe}{2} \right) \right]^2 (2^{b_k} - 1)\sigma_{e_k}^2 = c(2^{b_k} - 1)\sigma_{e_k}^2. \tag{5.13}$$

where $c = \frac{1}{3} \left[Q^{-1} \left(\frac{Pe}{2} \right) \right]^2$, which depends on the given probability of symbol error P_e. The increment energy is then

$$e_k(b_k) = \sigma_{s_k}^2(b_k) - \sigma_{s_k}^2(b_k - 1) = c2^{b_k - 1}\sigma_{e_k}^2 = 2e_k(b_k - 1). \tag{5.14}$$

It may happen that some of the subchannel do not have any bit. Summarizing, we can describe the algorithm as follows:

1. set $b_k = 0$ and $e_k(0) = 0, \quad k = 1, 2, \ldots, M$.

2. compute $e_k(b_k + 1) = c2^{b_k}\sigma^2_{e_k}, \quad k = 1, 2, \ldots, M$.

3. find $m \leftarrow arg\left\{\min_{1\leq i\leq M}\left[e_i\left(b_i + 1\right)\right]\right\}$

4. $b_m = b_m + 1, \quad n = n + 1$.

5. if $n < b$, go to 3.

6. find $\sum_{k=1}^{M} \sigma^2_{s_k}(b_k), \quad k = 1, 2, \ldots, M$.

7. end.

From the optimal bit allocation algorithm, two characteristics can be observed here

- $\max_n \left[e_n\left(b_n\right)\right] \leq \min_m \left[e_m\left(b_m + 1\right)\right], \quad n = 1, \ldots, M, \; m = 1, \ldots, M.$
- $\sum\limits_{k=1}^{M} \sigma^2_{s_k} = \sum\limits_{k=1}^{M}\sum\limits_{i=1}^{b_k} e_k(i), \; where \; \sigma^2_{s_k} = \sum\limits_{i=1}^{b_k} e_k(i), \; for \; k = 1, \ldots, M.$

We will use this results in Sec. 5.2.3 to proof that PMBT with optimum bit allocation must have better performance than GMRT for flat fading channel.

5.2 Comparison of GMRT with PMBT

In this section, we will compare GMRT with PMBT. We will use ZF, MMSE receiver and optimum bit allocation case to compare GMRT. To start with the analysis, we will assume that the two systems have the same transmission rate. The PMBT system use 4-QAM modulation with M transmission antennae in place, the transmission rate is 2M bits per block. So GMRT use 2^{2M}-QAM modulation. In chapter 3, we know the overall SNR of GMRT β_{GMRT} given as

$$\beta_{GMRT} = \lambda_1^2 r \ , \quad where \;\; r = E_s/N_0. \tag{5.15}$$

We consider 2^{2M}-QAM modulation scheme, the corresponding bit error rate can be calculated by

$$P_e \simeq 2(1 - 2^{-M})Q(\sqrt{\frac{3}{2^{2M} - 1}\lambda_1 r}). \tag{5.16}$$

5.2.1 Zero forcing receiver without bit allocation

We consider the BER of the precoded MIMO transmission system with a zero forcing receiver for the case when there is no bit allocation. In Sec. 4.2, the overall SNR of PMBT is given as

$$\beta_{V^H W-ZF} = \frac{r}{\frac{1}{M}\sum\limits_{k=1}^{M}\frac{1}{\lambda_k^2}}. \tag{5.17}$$

We consider 4-QAM modulation scheme, the corresponding bit error rate can be calculated by

$$P_{V^H W-ZF} = Q(\sqrt{\beta_{V^H W-ZF}}). \tag{5.18}$$

Before this comparison, we will assume that the two systems have the same total transmission power. The total transmission power of PMBT system is M times of the symbol variance E_s. So the GMRT system uses M times of the symbol variance E_s to transmit symbol. Comparing (3.15) with (5.18), we observe that PMBT is better than GMRT if following condition is satisfied

$$\beta_{V^H W-ZF} - \frac{3M}{2^{2M}-1}\beta_{GMRT} \geq 0. \tag{5.19}$$

otherwise, if $\beta_{V^H W-ZF} - [3M/(2^{2M}-1)]\beta_{GMRT} \leq 0$, then GMRT has better performance than PMBT. The condition can be written as

$$\begin{aligned} \beta_{V^H W-ZF} - \alpha\beta_{GMRT} &= \left(\frac{1}{M}\sum_{k=1}^{M}\frac{1}{\lambda_k^2}\right)^{-1} r - \frac{3M}{2^{2M}-1}\lambda_1^2 r & (5.20)\\ &= c\left\{[\frac{(2^{2M}-4)}{3}]\left(\sum_{k=2}^{M}\frac{1}{\lambda_k^2}\right)^{-1} - \lambda_1^2\right\} r \geq 0. & (5.21) \end{aligned}$$

where $r = E_s/N_0$, $\alpha = 3M/(2^{2M}-1)$ and $c = \frac{3M\lambda_1^2}{2^{2M}-1}\prod_{k=1}^{M}\prod_{j\neq i}\lambda_j^2/\prod_{k=2}^{M}\prod_{j\neq i}\lambda_j^2$. We look at three specials cases: M=2, 3, and 4.

- M=2: $\begin{cases} 4\lambda_2^2 - \lambda_1^2 \geq 0 & P_{V^H W-ZF} \leq P_{GMRT} \\ 4\lambda_2^2 - \lambda_1^2 < 0 & P_{V^H W-ZF} > P_{GMRT} \end{cases}$
- M=3: $\begin{cases} 20\frac{\lambda_2^2\lambda_3^2}{\lambda_2^2+\lambda_3^2} - \lambda_1^2 \geq 0 & P_{V^H W-ZF} \leq P_{GMRT} \\ 20\frac{\lambda_2^2\lambda_3^2}{\lambda_2^2+\lambda_3^2} - \lambda_1^2 < 0 & P_{V^H W-ZF} > P_{GMRT} \end{cases}$

- M=4: $\begin{cases} 84\frac{\lambda_2^2\lambda_3^2\lambda_4^2}{\lambda_2^2\lambda_3^2+\lambda_3^2\lambda_4^2+\lambda_2^2\lambda_4^2} - \lambda_1^2 \geq 0 & P_{V^HW-ZF} \leq P_{GMRT} \\ 84\frac{\lambda_2^2\lambda_3^2\lambda_4^2}{\lambda_2^2\lambda_3^2+\lambda_3^2\lambda_4^2+\lambda_2^2\lambda_4^2} - \lambda_1^2 < 0 & P_{V^HW-ZF} > P_{GMRT} \end{cases}$

From above results, we know two curve of GMRT and ZF receiver case have no crossing point for all SNR.

5.2.2 MMSE Receiver without bit allocation

We consider the BER of the precoded MIMO transmission system with a minimum mean square error receiver for the case when there is no bit allocation. In the Sec. 4.3, the overall SNR of PMBT is given as

$$\beta_{V^HW-MMSE} = \frac{\sum_{k=1}^{M} \frac{\lambda_k^2 r}{1+\lambda_k^2 r}}{\sum_{k=1}^{M} \frac{1}{1+\lambda_k^2 r}}. \tag{5.22}$$

We consider 4-QAM modulation scheme, the corresponding bit error rate can be calculated by

$$P_{V^HW-MMSE} = Q(\sqrt{\beta_{V^HW-MMSE}}). \tag{5.23}$$

Before this comparison, we will assume that the two systems have the same total transmission power. The total transmission power of PMBT system is M times of the symbol variance E_s. So the GMRT system uses M times of the symbol variance E_s to transmit symbol. Comparing (3.15) with (5.23), we observe that PMBT is better than GMRT if following condition is satisfied

$$\beta_{V^HW-MMSE} - \frac{3M}{2^{2M}-1}\beta_{GMRT} \geq 0. \tag{5.24}$$

otherwise, if $\beta_{V^HW-MMSE} - [3M/(2^{2M}-1)]\beta_{GMRT} \leq 0$, then GMRT has better performance than PMBT. The condition $\zeta = \beta_{V^HW-MMSE} - [3M/(2^{2M}-1)]\beta_{GMRT}$ can be written as

$$\begin{aligned} \zeta &= c\left[\lambda_1^2 \prod_{k=2}^{M}\left(1+\lambda_k^2 r\right) + \cdots + \lambda_M^2 \prod_{k=1}^{M-1}\left(1+\lambda_k^2 r\right)\right] && (5.25) \\ &- c\alpha\lambda_1^2\left[\prod_{k=2}^{M}\left(1+\lambda_k^2 r\right) + \cdots + \prod_{k=1}^{M-1}\left(1+\lambda_k^2 r\right)\right] \geq 0. && (5.26) \end{aligned}$$

where $r = E_s/N_0$, $\alpha = [3M/(2^{2M}-1)]$, and $c = \prod_{k=1}^{M}(1+\lambda_k^2)$. Assume $r \gg 1$, we can approximate $\beta_{V^HW-MMSE} - \alpha\beta_{GMRT} \simeq \rho_1 r^{M-1} + \rho_2 r^{M-2}$. The ρ_1 and ρ_2 are represented as

- ρ_1: $\mu\left\{\left[\frac{\left(2^{2M}-1\right)}{3}-1\right]\left(\sum\limits_{i=2}^{M}\frac{1}{\lambda_i^2}\right)^{-1}-\lambda_1^2\right\}$.

- ρ_2: $\mu\left\{\left[\frac{(M-1)\left(2^{2M}-1\right)}{3M}-2\right]\left(\sum\limits_{i=2}^{M}\frac{1}{\lambda_i^2}\right)^{-1}-2\lambda_1^2+2\frac{\lambda_1^2}{\lambda_M^2}\left(\sum\limits_{i=2}^{M}\frac{1}{\lambda_i^2}\right)^{-1}\right\}$

where $\mu = c\alpha\lambda_1^2\left(\prod_{i=2}^{M}\prod_{j\neq i}\lambda_j^2\right)$. The $\beta_{W\_MMSE}-\alpha\beta_{GMRT}$ represented as

$$\rho_1 r^{M-1}+\rho_2 r^{M-2} = \mu r^{M-2}\left[\phi_1\left(\sum_{i=2}^{M}\frac{1}{\lambda_i^2}\right)^{-1}-\lambda_1^2\right]r \quad (5.27)$$

$$+\frac{\mu r^{M-2}}{\lambda_M^2}\left[\phi_2\left(\sum_{i=2}^{M}\frac{1}{\lambda_i^2}\right)^{-1}-2\lambda_1^2\right] \quad (5.28)$$

where $\phi_1=\left[\left(2^{2M}-4\right)/3\right]-1$, and $\phi_2=\left[(M-1)\left(2^{2M}-1\right)/3M\right]-2+2\lambda_1^2/\lambda_M^2$. We look at three cases for M=2, 3, and 4.

- M=2: $\begin{cases}(4\lambda_2^2-\lambda_1^2)r+0.5\geq 0 & P_{V^HW-MMSE}\leq P_{GMRT}\\ (4\lambda_2^2-\lambda_1^2)r+0.5<0 & P_{V^HW-MMSE}>P_{GMRT}\end{cases}$

- M=3: $\begin{cases}(20\frac{\lambda_2^2\lambda_3^2}{\lambda_2^2+\lambda_3^2}-\lambda_1^2)r+Z_3\geq 0 & P_{V^HW-MMSE}\leq P_{GMRT}\\ (20\frac{\lambda_2^2\lambda_3^2}{\lambda_2^2+\lambda_3^2}-\lambda_1^2)r+Z_3<0 & P_{V^HW-MMSE}>P_{GMRT}\end{cases}$

- M=4: $\begin{cases}(84\frac{\lambda_2^2\lambda_3^2\lambda_4^2}{\lambda_2^2\lambda_3^2+\lambda_3^2\lambda_4^2+\lambda_2^2\lambda_4^2}-\lambda_1^2)r+Z_4\geq 0 & P_{V^HW-MMSE}\leq P_{GMRT}\\ (84\frac{\lambda_2^2\lambda_3^2\lambda_4^2}{\lambda_2^2\lambda_3^2+\lambda_3^2\lambda_4^2+\lambda_2^2\lambda_4^2}-\lambda_1^2)r+Z_4<0 & P_{V^HW-MMSE}>P_{GMRT}\end{cases}$

where $z_3=\frac{2\left(7\lambda_2^2-\lambda_1^2\right)}{\lambda_2^2+\lambda_3^2}$, and $z_4=\left[\frac{(61.75+2\lambda_1^2/\lambda_4^2)\lambda_2^2\lambda_3^2}{\lambda_2^2\lambda_3^2+\lambda_3^2\lambda_4^2+\lambda_2^2\lambda_4^2}-\frac{2\lambda_1^2}{\lambda_4^2}\right]$. From above results, we know two curve of GMRT and MMSE receiver case may have a crossing point. The cdf of Z_3 and Z_4 are shown in Fig. 5.2. From the Fig. 5.2, we know that almost Z_3 and Z_4 are large or equal to zero. In this condition, we assume $Z_3\geq 0$ and $Z_4\geq 0$. In this assumption, we can find the crossing point $\tilde{r}$.

- M=2: If $4\lambda_2^2-\lambda_1^2<0$, the crossing point $\tilde{r}=-0.5/(4\lambda_2^2-\lambda_1^2)$.

- M=3: If $20\lambda_2^2\lambda_3^2/(\lambda_2^2+\lambda_3^2)-\lambda_1^2<0$, the crossing point give as
$$\Rightarrow \tilde{r}=\frac{-2(7\lambda_2^2-\lambda_1^2)}{[20\lambda_2^2\lambda_3^2-\lambda_1^2\left(\lambda_2^2+\lambda_3^2\right)]}.$$

- M=4: If $84\lambda_2^2\lambda_3^2\lambda_4^2/(\lambda_2^2\lambda_3^2+\lambda_3^2\lambda_4^2+\lambda_2^2\lambda_4^2)-\lambda_1^2<0$, the crossing point give as
$$\Rightarrow \tilde{r}=-\frac{61.75\lambda_2^2\lambda_3^2-2\lambda_1^2\left(\lambda_2^2+\lambda_3^2\right)}{[84\lambda_2^2\lambda_3^2\lambda_4^2-\lambda_1^2\left(\lambda_2^2\lambda_3^2+\lambda_3^2\lambda_4^2+\lambda_2^2\lambda_4^2\right)]}.$$

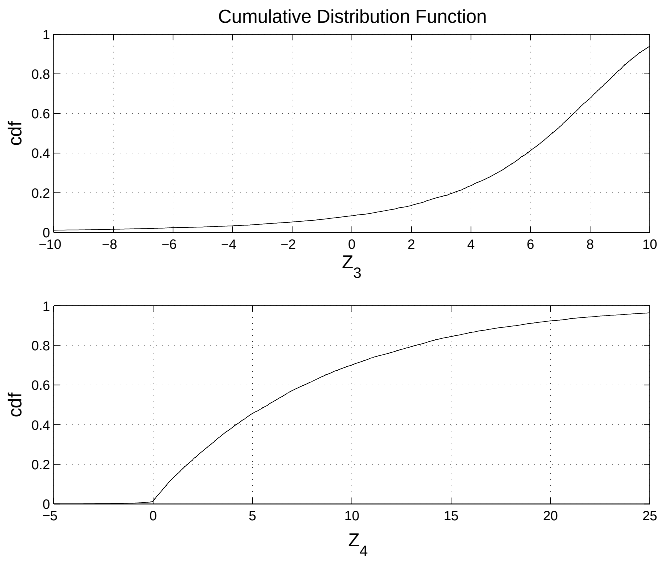

Figure 5.2: Cumulative Distribution Function for M=3, 4.

5.2.3 Zero forcing receiver with optimum bit allocation

In this section, we assume noise are white Gaussian noise and bit budget b bits. The modulation scheme is 2^b-QAM for GMRT, the corresponding BER can be calculated by

$$P_e \simeq 2(1-2^{-b/2})Q(\sqrt{\frac{3}{2^b-1}\lambda_1 r}). \tag{5.29}$$

Now we have P_e to find transmission power E_{GMRT}. From (3.16), we know that the total transmission power of GMRT system is

$$E_{GMRT} = \frac{N_0}{\lambda_1^2}\frac{2^b-1}{3}\left[Q^{-1}\left(\frac{P_e}{2(1-2^{-b/2})}\right)\right]^2 \simeq c(2^b-1)\frac{N_0}{\lambda_1^2}. \tag{5.30}$$

where $c = 1/3\left[Q^{-1}\left(\frac{P_e}{2}\right)\right]^2$. From Sec. 5.1, we know that E_{GMRT} change to represent as

$$E_{GMRT} \simeq c(2^b-1)\frac{N_0}{\lambda_1^2} = \sum_{i=1}^{b} e_1(i). \tag{5.31}$$

From Sec. 5.1, we also know that the total transmission power of PMBT is

$$E_{PMBT} = \sum_{k=1}^{M}\sigma_{s_k}^2(b_k) \simeq c\sum_{k=1}^{M}(2^{b_k}-1)\sigma_{e_k}^2 \tag{5.32}$$

$$= c\sum_{k=1}^{M}(2^{b_k}-1)\frac{N_0}{\lambda_k^2} = \sum_{k=1}^{M}\sum_{i=1}^{b_k} e_k(i). \tag{5.33}$$

Comparing (3.16) with (5.13), we observe that PMBT is better than GMRT if following condition is satisfied

$$E_{PMBT} - E_{GMRT} \leq 0. \tag{5.34}$$

otherwise, if $E_{PMBT} - E_{GMRT} > 0$, then GMRT has better performance than PMBT. The condition can be written as

$$E_{PMBT} - E_{GMRT} = \sum_{k=1}^{M}\sum_{i=1}^{b_k} e_k(i) - \sum_{i=1}^{b} e_1(i) \tag{5.35}$$

$$= \sum_{k=2}^{M}\sum_{i=1}^{b_k} e_k(i) - \sum_{i=b_1+1}^{b} e_1(i). \tag{5.36}$$

From (5.14), we know $e_k(b_k+1) = 2e_k(b_k)$ and $\max_n [e_n(b_n)] \leq \min_m [e_m(b_m+1)]$. Using this result, the condition change to represent as

$$E_{PMBT} - E_{GMRT} = \sum_{k=2}^{M} 2^{-b_k+1}\left(2^{b_k}-1\right) e_k(b_k) \tag{5.37}$$

$$- \left(2^{b-b_1}-1\right) e_1(b_1+1) \leq 0. \tag{5.38}$$

E_{PMBT} becomes the same as E_{GMRT} if we choose $b_1 = b$ and $b_2 = b_3 = \ldots = b_M = 0$. In optimal bit allocation, bit allocation is optimized to minimize E_{PMBT}. So the minimized E_{PMBT} is always smaller than E_{GMRT}. So E_{PMBT} with optimal bit allocation is always better than E_{GMRT}.

5.3 Complexity of PMBT and GMRT

In this section, we analyze the complexity of GMRT and PMBT and compare the complexity of GMRT and PMBT. We will discuss the complexity to divide into two parts. First, we can compute operational analysis to need some real multiplications and real additions for GMRT and PMBT. Second, we can compute design system to need some real multiplications and real additions for GMRT and PMBT.

- Operational analysis: We will divide system into three parts, which are transmitter, channel and receiver.

Generalized Maximum ratio Transmission

	real multiplications	real additions
Transmitter	$4M$	$3M$
Channel	$4M^2$	$M(5M-2)$
Receiver	$4M$	$6M-2$
Total	$4M(M+2)$	$5M^2+9M-2$

Precoded MIMO Block Transmission

	real multiplications	real additions
Transmitter	$4M^2$	$M(5M-2)$
Channel	$4M^2$	$M(5M-2)$
Receiver	$4M^2$	$M(5M-2)$
Total	$12M^2$	$15M^2-6M$

From these tables, these complexity of PMBT and GMRT are $O(M^2)$ and the complexity of PMBT is more than GMRT for $M \geq 2$.

- Design system : For GMRT system, we use the first column vector of $\mathbf{U}$ to maximize SNR. So we need to find the eigenvalues of hermitian matrix $\mathbf{HH}^H$ and the corresponding eigenvector matrix $\mathbf{U}$. For GMRT system, we will use $\mathbf{U}$ and $\mathbf{V}$ to find optimal solution. So we need to find the eigenvalues of channel matrix $\mathbf{H}$ and the corresponding unitary matrices $\mathbf{U}$ and $\mathbf{V}$.

  We can use singular value decomposition (SVD) to find eigenvalues of $\mathbf{HH}^H$ and $\mathbf{H}$ and the corresponding matrices $\mathbf{U}$ and $\mathbf{V}$. The SVD algorithm is usually computed by variant of QR decomposition. First, A is reduced to bidiagonal form by orthogonal transformations, then remaining off-diagonal entries are annihilated iteratively. So SVD has very high complexity and the complexity of SVD algorithm is $O(M^3)$. So the complexity of design system are more than operational analysis.

From above results, the complexity of design system is more than operational analysis and the complexity of PMBT is more than GMRT.

Chapter 6

Numerical Simulation

6.1 MRT Simulation

The 4-QAM scheme is used in the simulation for simplicity. A simple channel is adopted, where the fading channel coefficients are independent complex Gaussian random variables. It is also assumed that perfect knowledge of channel fading coefficients are available to both transmitter and receiver stations. Different pairs of (K,L) are selected as (2,1), (3,1), (4,1), (2,2), (3,2), (4,2). 10^4 different channels and 10^6 symbols are used in the simulation.

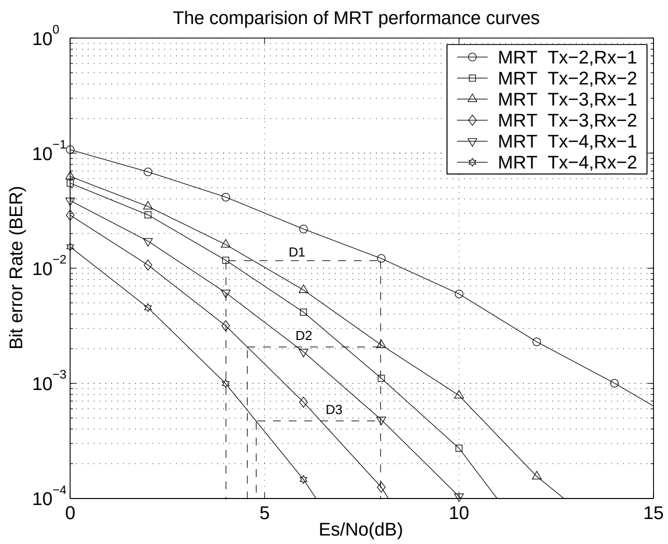

Figure 6.1: The comparison of MRT performance curves showing the effect of adding the second receiving antennae for K=2, K=3, and K=4.

The BER performance curves plotted in Fig. 6.1 show the results using one or two receiving antennae for K=2, K=3, and K=4. Two characteristics that are particularly associated with diversity can be observed here

1. The improvement becomes greater as SNR increase.
2. The incremental improvement becomes smaller as the diversity order increases.
   $\Rightarrow (i.e., D1 > D2 > D3)$.

In Fig. 6.2, the performance curves for different cases of the fourth-order diversity $(i.e., K \times L = 4)$ are given. Comparing the curve corresponding to $KL = 41$ with that corresponding to $KL = 22$, one may observe the 1-dB difference in SNR for the same BER.

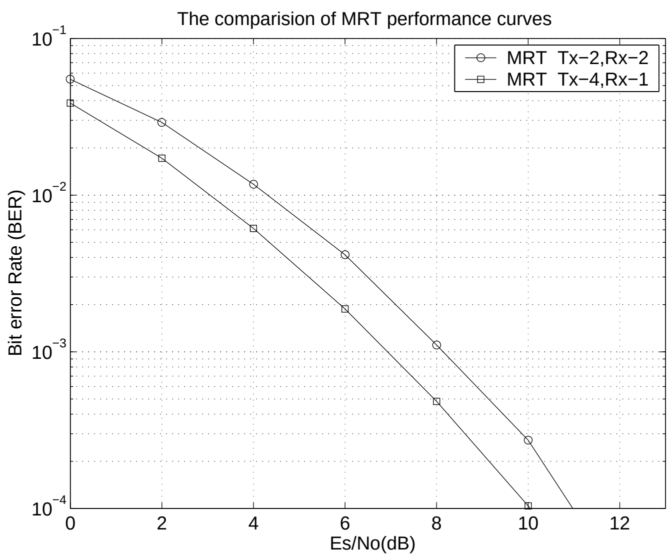

Figure 6.2: The comparison of MRT performance curves for the same diversity order with different receiver antennae number.

6.2 GMRT Simulation

In the Sec. 3, we prove that the generalized maximal SNR gain of GMRT is always greater than or equal to that presented by Lo under the normalized constraint condition on weighting vector for any given flat fading channel. In this section, we investigate it by simulation. 4-QAM scheme is used in the simulation for simplicity. A simple channel is adopted, where the fading channel coefficients are independent complex Gaussian random variables. It is also assumed that perfect knowledge of channel fading coefficients are available to both transmitter and receiver stations. Different pairs of (K,L) are selected as (2,1), (3,1), (4,1), (2,2), (3,2), (4,2). 10^4 different channels and 10^6 symbols are used in the simulation. Fig. 6.3 denotes that when the receiver antenna is single antenna, the two systems with different selections of weighting vectors have the same performance. The results in Fig. 6.4 indicate that for the same product diversity, the system performance by using more antennae at the transmitter will get better performance. Fig. 6.5 show that as the product diversity increases, the system performance will get better.

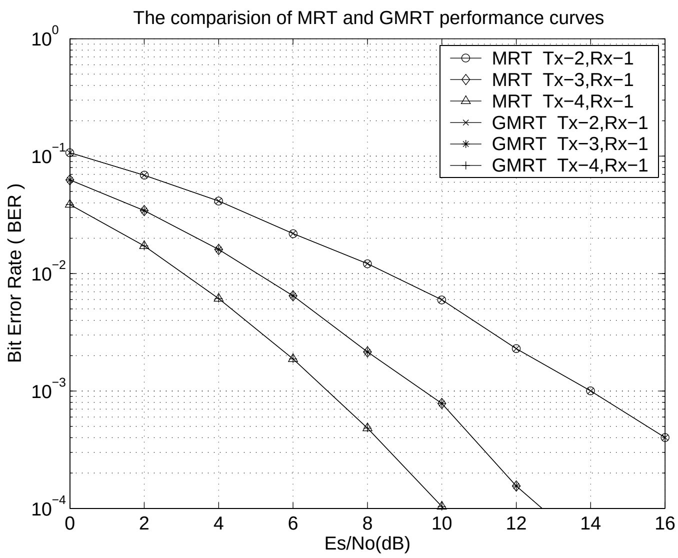

Figure 6.3: The comparison of MRT and GMRT performance curves showing the effect of one receiving antennae for K=2, K=3, and K=4.

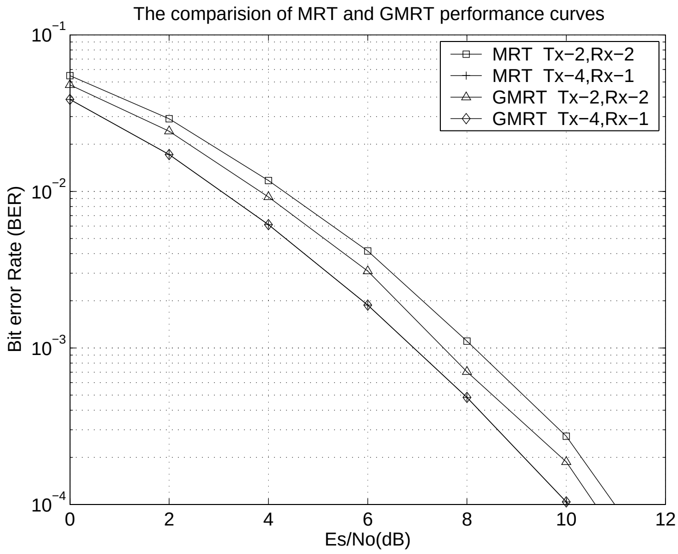

Figure 6.4: The comparison of MRT and GMRT performance curves for the same diversity order with different receiver antennae number.

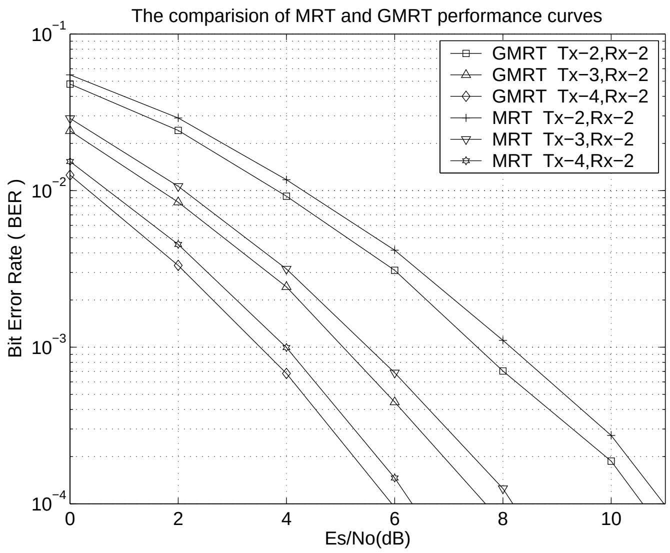

Figure 6.5: The comparison of MRT and GMRT performance curves for different product diversity.

6.3 PMBT Simulation

In the Section. 4, we analyze the BER performance of PMBT systems with ZF and MMSE receiver and derive the optimal bit allocation formula for a given target transmission rate. We want to use this results to compare with GMRT system at the same transmission rate.4-QAM modulation is used for precoded MIMO block transmission system. We use use 2^{2M}-QAM modulation for GMRT. In Fig. 6.6 - Fig. 6.11, we perform simulation for a single channel. In the Fig. 6.12 - Fig. 6.14, 10^4 Channel are used in the simulation. The fading channel coefficients are uncorrelated complex Gaussian random variable with mean 0, variance 1.

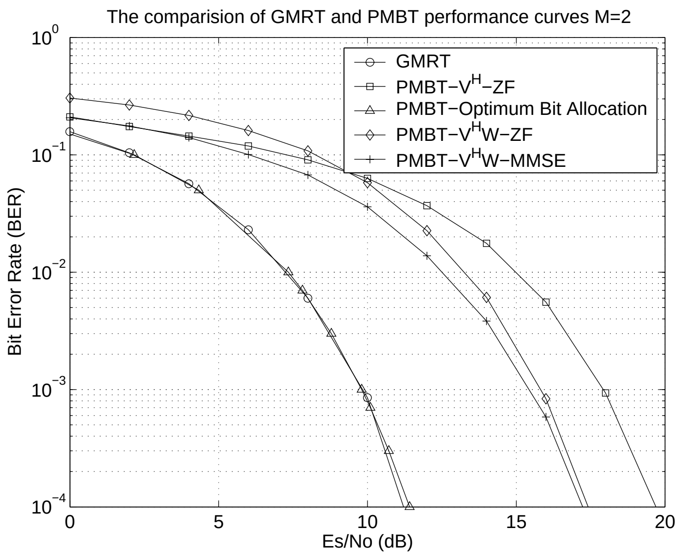

Figure 6.6: BER performances PMBT and GMRT for M=2, PMBT: 4-QAM, GMRT: 16-QAM, $\lambda_1^2 = 2.49$, $\lambda_2^2 = 0.13$.

In the Fig. 6.6, the eigenvalue of channel matrix are $\lambda_1^2 = 2.49$, $\lambda_2^2 = 0.13$ and we have $4\lambda_2^2 - \lambda_1^2 < 0$. Using (5.21), this results let me know GMRT have better performance than PMBT-V^HW-ZF for all SNR and GMRT and PMBT-V^HW-MMSE have no the crossing point for SNR, which value are larger or equal to 0(dB). In this simulation, we compute the value of r_0, $\overline{r}$, and r_1, respectively, as $0.79(dB)$, $10.84(dB)$, and $13.62(dB)$. Notice that the crossing of PMBT-V^H-ZF and PMBT-V^HW-ZF occurs around 10(dB), which is a value close to

$\overline{r} = 10.84(dB)$ than to $r_0 = 0.79(dB)$ or $r_1 = 13.62(dB)$. From the eigenvalue of channel matrix, we compute $b_0 = 4$, and $b_1 = 0$. This result let me know that GMRT and PMBT with optimal bit allocation have the same performance.

In the Fig. 6.7, the eigenvalue of channel matrix are $\lambda_1^2 = 1.22$, $\lambda_2^2 = 0.42$ and get $4\lambda_2^2 - \lambda_1^2 \geq 0$. From (5.21), we know PMBT-$V^H W$-ZF have better performance than GMRT for all SNR and GMRT and PMBT-$V^H W$-MMSE have no the crossing point. In this simulation, we compute the value of r_0, $\overline{r}$, and r_1, respectively, as $3.9(dB)$, $6.8(dB)$, and $8.53(dB)$. Notice that the crossing of PMBT-V^H-ZF and PMBT-$V^H W$-ZF occurs around 6(dB), which is a value close to $\overline{r} = 6.8(dB)$ than to $r_0 = 3.9(dB)$ or $r_1 = 8.53(dB)$. From the eigenvalue of channel matrix, we compute $b_1 = 3$, and $b_2 = 1$. As not all the bits are assigned to the same symbol, PMBT with optimal bit allocation have better performance than GMRT.

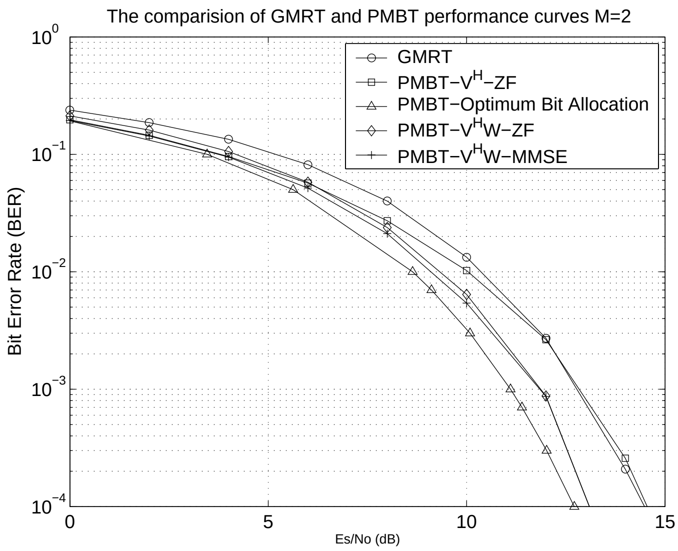

Figure 6.7: BER performances PMBT and GMRT for M=2, PMBT: 4-QAM, GMRT: 16-QAM, $\lambda_1^2 = 1.22$, $\lambda_2^2 = 0.42$.

In the Fig. 6.8, the eigenvalues of channel matrix are $\lambda_1^2 = 5.75$, $\lambda_2^2 = 2.53$, $\lambda_3^2 = 0.25$ and get $20\lambda_2^2\lambda_3^2/(\lambda_2^2 + \lambda_3^2) - \lambda_1^2 < 0$. From (5.21), we know GMRT have better performance than PMBT-$V^H W$-ZF for all SNR and the crossing of

GMRT and PMBT-$V^H W$-MMSE occurs around $8(dB)$, which is a value closer to $\tilde{r} = 8.5(dB)$. In this simulation, we compute the value of r_0, $\overline{r}$, and r_1, respectively, as $-2.82(dB), 6.6(dB)$, and $10.8(dB)$. Notice that the crossing of PMBT-V^H-ZF and PMBT-$V^H W$-ZF occurs around 6.2(dB), which is a value close to $\overline{r} = 6.6(dB)$ than to $r_0 = -2.82(dB)$ or $r_1 = 10.8(dB)$. From the eigenvalue of channel matrix, we compute $b_1 = 4$, $b_2 = 2$, and $b_3 = 0$. Therefore PMBT with optimal bit allocation have better performance than GMRT.

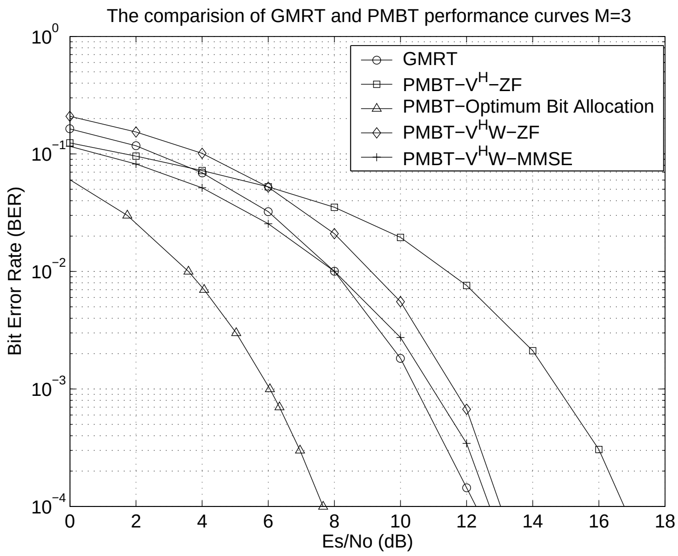

Figure 6.8: BER performances PMBT and GMRT for M=3, PMBT: 4-QAM, GMRT: 64-QAM, $\lambda_1^2 = 5.75$, $\lambda_2^2 = 2.53$, $\lambda_3^2 = 0.25$.

In the Fig. 6.9, the eigenvalue of channel matrix are $\lambda_1^2 = 3.62$, $\lambda_2^2 = 1.3$, $\lambda_3^2 = 0.7$ and we get $20\lambda_2^2\lambda_3^2/(\lambda_2^2 + \lambda_3^2) - \lambda_1^2 \geq 0$. From (5.21), we know PMBT-$V^H W$-ZF have better performance than GMRT for all SNR and GMRT and PMBT-$V^H W$-MMSE have no the crossing point. In this simulation, we compute the value of r_0, $\overline{r}$, and r_1, respectively, as $-0.82(dB), 3.93(dB)$, and $6.32(dB)$. Notice that the crossing of PMBT-V^H-ZF and PMBT-$V^H W$-ZF occurs around 4(dB), which is a value close to $\overline{r} = 3.93(dB)$ than to $r_0 = -0.82(dB)$ or $r_1 = 6.32(dB)$. From the eigenvalue of channel matrix, we compute $b_1 = 3$, $b_2 = 2$, and $b_3 = 1$. So we know that PMBT with optimal bit allocation have better performance than

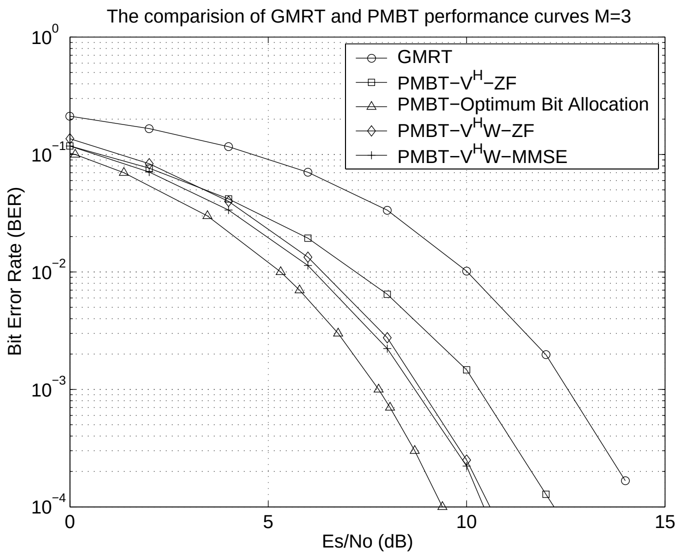

Figure 6.9: BER performances PMBT and GMRT for M=3, PMBT: 4-QAM, GMRT: 64-QAM, $\lambda_1^2 = 3.62,\ \lambda_2^2 = 1.3,\ \lambda_3^2 = 0.7$.

GMRT.

In the Fig. 6.10, the eigenvalue of channel matrix are $\lambda_1^2 = 6.94,\ \lambda_2^2 = 3.4,\ \lambda_3^2 = 0.1,\ \lambda_4^2 = 0.043$ and get $84\lambda_2^2\lambda_3^2\lambda_4^2/(\lambda_2^2\lambda_3^2 + \lambda_3^2\lambda_4^2 + \lambda_2^2\lambda_4^2) - \lambda_1^2 < 0$. From (5.21), we know GMRT have better performance than PMBT-V^HW-ZF for all SNR and the crossing of GMRT and PMBT-V^HW-MMSE occurs around $13(dB)$, which is a value closer to $\tilde{r} = 13.32(dB)$. In this simulation, we compute the value of $r_0,\ \overline{r}$, and r_1, respectively, as $-5.7(dB),\ 8.11(dB)$, and $13.31(dB)$. Notice that the crossing of PMBT-V^H-ZF and PMBT-V^HW-ZF occurs around 8(dB), which is a value close to $\overline{r} = 8.11(dB)$ than to $r_0 = -5.7(dB)$ or $r_1 = 13.31(dB)$. From the eigenvalue of channel matrix, we compute $b_1 = 4$, $b_2 = 3$, $b_3 = 1$, and $b_4 = 0$. So we know that PMBT with optimal bit allocation have better performance than GMRT.

In the Fig. 6.11, the eigenvalue of channel matrix are $\lambda_1^2 = 4.2,\ \lambda_2^2 = 2.2,\ \lambda_3^2 = 1.3,\ \lambda_4^2 = 0.16$ and get $84\lambda_2^2\lambda_3^2\lambda_4^2/(\lambda_2^2\lambda_3^2 + \lambda_3^2\lambda_4^2 + \lambda_2^2\lambda_4^2) - \lambda_1^2 \geq 0$. From (5.21), we know PMBT-V^HW-ZF have better performance than GMRT for all SNR and GMRT and PMBT-V^HW-MMSE have no the crossing point. In this simulation,

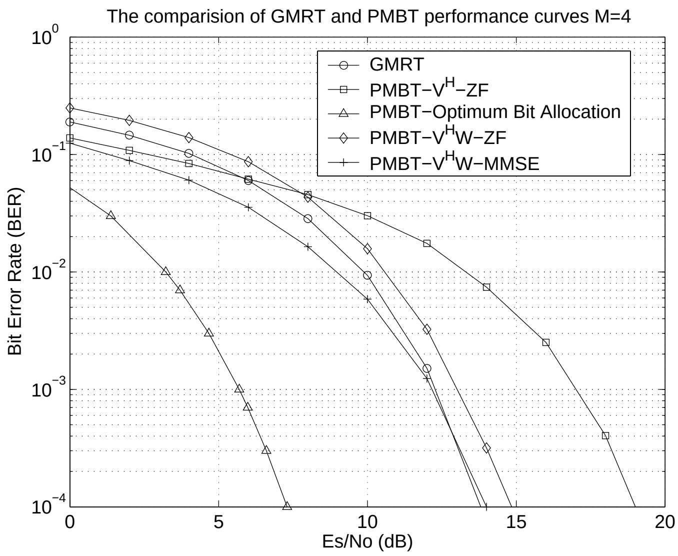

Figure 6.10: BER performances PMBT and GMRT for M=4, PMBT: 4-QAM, GMRT: 256-QAM, $\lambda_1^2 = 6.94,\ \lambda_2^2 = 3.4,\ \lambda_3^2 = 0.1,\ \lambda_4^2 = 0.043$.

we compute the value of $r_0,\ \overline{r}$, and r_1, respectively, as $-1.38(dB),\ 7.54(dB)$, and $12.63(dB)$. Notice that the crossing of PMBT-V^H-ZF and PMBT-V^HW-ZF occurs around 7(dB), which is a value close to $\overline{r} = 7.54(dB)$ than to $r_0 = -1.38(dB)$ or $r_1 = 12.63(dB)$. From the eigenvalue of channel matrix, we compute $b_1 = 3$, $b_2 = 3$, $b_3 = 2$, and $b_4 = 0$. So we know that PMBT with optimal bit allocation have better performance than GMRT.

In the Fig. 6.12, the BER of the MMSE receiver is lower than the zero-forcing receiver for all SNR. For the case of MMSE receiver, GMRT system better than PMBT system for all SNR. The PMBT with optimal bit allocation have better performance than GMRT.

In the Fig. 6.13 - Fig. 6.14, the BER of the MMSE receiver is lower than the zero-forcing receiver for all SNR. For the case of MMSE receiver, GMRT system better than PMBT system for high SNR. The PMBT with optimal bit allocation have better performance than GMRT.

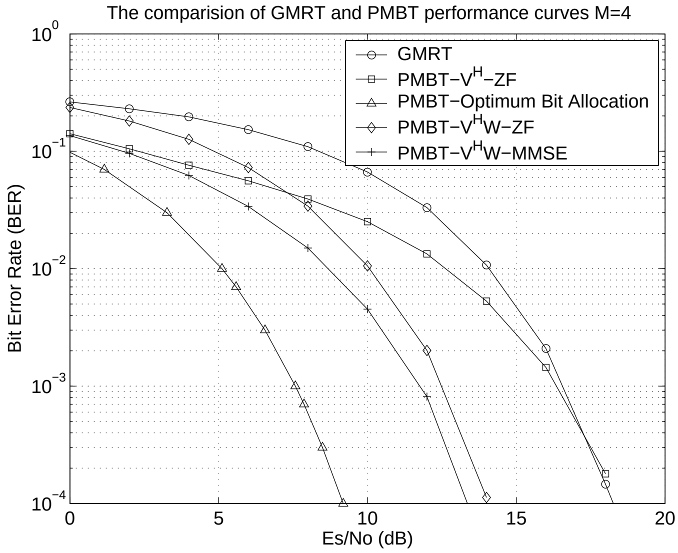

Figure 6.11: BER performances PMBT and GMRT for M=4, PMBT: 4-QAM, GMRT: 256-QAM, $\lambda_1^2 = 4.2,\ \lambda_2^2 = 2.2,\ \lambda_3^2 = 1.3,\ \lambda_4^2 = 0.16$.

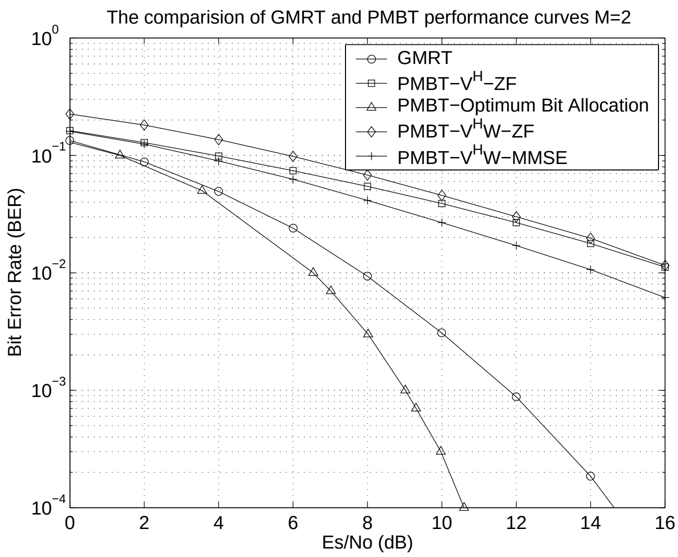

Figure 6.12: BER performances PMBT and GMRT for M=2, PMBT: 4-QAM, GMRT: 16-QAM, 10^4 Channel are used.

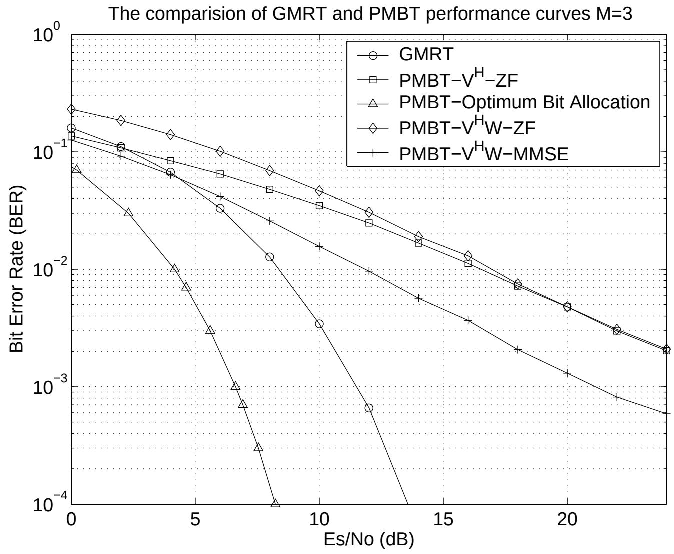

Figure 6.13: BER performances PMBT and GMRT for M=3, PMBT: 4-QAM, GMRT: 64-QAM, 10^4 Channel are used.

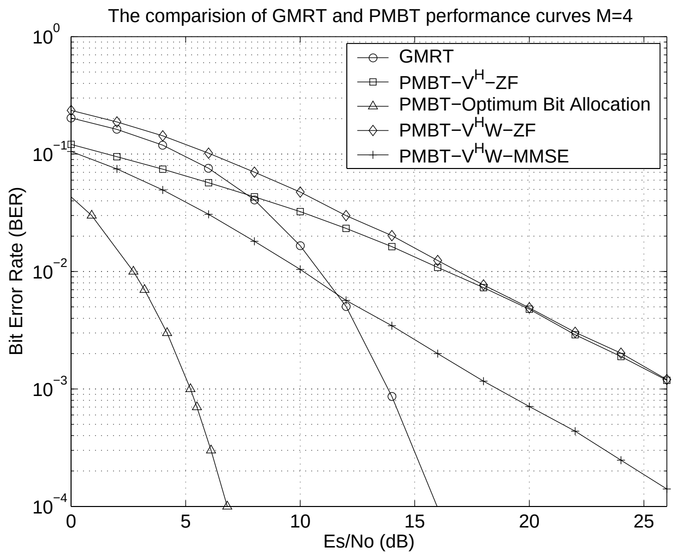

Figure 6.14: BER performances PMBT and GMRT for M=4, PMBT: 4-QAM, GMRT: 256-QAM, 10^4 Channel are used.

Chapter 7

Conclusion

In this thesis, we perform simulation of precoded MIMO block transmission (PMBT) with zero-forcing (ZF), minimum mean square error (MMSE) receivers and optimum bit allocation and also compare the performance with Generalized MRT (GMRT). In our experiments, we found that the PMBT with optimum bit allocation perform significantly better than GMRT. Furthermore, the PMBT with optimum bit allocation performs the same as the GMRT if we choose all bits fall into one subchannel, which has maximum eigenvalue of channel matrix. In the case of ZF and MMSE receiver, the relationship between PMBT and GMRT depend on the eigenvalues of channel matrix. If the eigenvalues of channel matrix close to each other, the PMBT with ZF and MMSE receivers will have the better performance. On the contrary, GMRT has the better performance at the large difference of eigenvalues values. This outcome shown that PMBT with optimum bit allocation always have better performance than GMRT for flat fading channel.

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