國 立 交 通 大 學

應用數學系

碩 士 論 文

信用違約交換價差與信用評等之關係

The Relationship between Credit Default Swap Spread and Credit Rating

研 究 生：郭閔豪

指導老師：許元春 王克陸 教授

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研 究 生：郭閔豪 Student：Minhao Kuo

指導教授：許元春 Advisor：Yuanchung Sheu

王克陸 Kehluh Wang

國 立 交 通 大 學

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碩 士 論 文

A Thesis

Submitted to Department of Applied Mathematics

College of Science,

National Chiao Tung University

in Partial Fulfillment of the Requirements

for the Degree of

Master

In

Applied Mathematics

July 2011

Hsinchu, Taiwan, Republic of China

中 華 民 國 一 百 年 七 月

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學生：郭閔豪　　　　　　　　　　　　　　　指導老師：許元春 王克陸 教授

國立交通大學應用數學系(研究所)碩士班

摘　要

在這篇論文中，在報酬是以涉及一家實體違約為條件並且沒有對手違約情況下，我們將比較兩種分別由Hull & White及Duan所提出之方法，來評價信用違約交換合約。而且在評價信用違約交換合約時，我們將考慮到系統風險，然後我們將提供三個當信用評等被改變時信用違約交換合約價格差異變化的例子。在本文的三個例子中我們會發現由Duan所提出方法將會比Hull & White的提出之方法來的好。

關鍵詞：信用違約交換、系統風險、違約距離、等級體系模型

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Student: Minhao Kuo

Advisor: Yuanchung Sheu

Kehluh Wang

Department (Institute) of Applied Mathematics

National Chiao Tung University

Abstract

This paper compares two methodologies, which were developed by Hull & White (2000) and Duan (2010) respectively, for valuating credit default swap when the payoff is contingent on default by a single reference entity and there is no counter party risk. Furthermore, we take the systemic risk into account for valuating credit default swap and then we give three examples of variation of credit default swap spread when the credit rating had been changed by using this two methodologies. In our examples, the methodology developed by Duan (2010) is much better than Hull and White (2000).

Keywords : Credit default swaps, systemic risk, distance to default, hierarchical model

誌　謝

第一次接觸到財務方面是在升碩一那年，在許元春老師在國家理論中心所開的隨機過程，那時的我心想，原來以前所學的數學知識可以應用在財務上，因此許元春老師引領著我進入了這個領域。

這篇論文的完成，要感謝所有授我知識的老師，尤其是我的指導教授許元春老師及王克陸老師，不僅僅啟發我在財務上的興趣以及傳授我課業上所需要的知識，更因老師的平易近人個性，讓我們在生活各方面能與老師共同分享。

首要感謝我的女朋友秋妤，不僅常常陪我留在研究室而且時常在我為論文苦惱時給予我適時的鼓勵，在這段艱辛的研究之路帶給我無比的快樂及信心。最後要感謝在碩士生生涯這兩年間與我共同學習共同歡樂的定國、冠緯、意萱、劭芃、聲華、權益，還有一同修課的淑娟，是你們豐富了我的人生。

最後我要感謝我的家人，沒有你們的支持，在這條道路上我無法走得安穩，我衷心感念。

郭閔豪

謹誌于交通大學

2011年7月

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1 Introduction

Credit default swap (CDS) is a contract that provides insurance against the risk of a default by particular company. The company is known as the reference entity and a default by the company is known as credit event. The CDS buyer needs to make periodic payments to the seller until the maturity of CDS or until a credit event occurs. Many papers focus on the valuation of CDS, such as Duffie (1999) or Hull and White (2000).

In 2008, the financial crisis (often called credit crunch) had resulted in the collapse of large financial institutions, such as Lehman Brothers, the bailout of banks by national governments, such as American International Group (AIG), and downturns in the stock market around the world. In this paper, we take the risk of failure in the financial institutions into account because it affected whether a firm default or not even clustered default. Therefore, the risk made a certain impact on the valuation of CDS. As similar as Bartram, *et al* (2007), we consider the failure in the financial system as the systemic risk.

In 2008 financial crisis, the credit rating models used by the key rating agencies had been seriously questioned. This paper presents two different methodologies to compare the result of before and after credit rating had been downgraded or upgraded. The first of our methodologies is developed by Hull and White (2000), which used bonds of the reference entity or the same risk as the reference entity. The second method is the reduced form model, developed by Duan (2010).

2 The valuation

We introduce a reduced form type pricing model developed by Hull and White (2000). Here we assume that there is no counter party default risk and default events, Treasury interest rates, and recovery rates are mutually independent. Consider the valuation of a plain vanilla credit default swap with a $1 notional principal and maturity T.

For a CDS buyer, if a default occurs at time t ($t \leq T$), the present value of the payments is $w[u(t) + e(t)]$, where w is total payments per year, $u(t)$ is present value of payments at the rate of $1 per year on payment dates between time zero and time t, and $e(t)$ is present value of an accrual payment at time t equal to t mines the payment date immediately preceding time t. If there is no default prior to time T, the present value of the payments is $wu(T)$. The expected present value of the payments is, therefore:

$$w \int_0^T q(t)[u(t) + e(t)]dt + w\pi u(T) \quad (1)$$

where $q(t)$ denotes the risk-neutral default probability density at time t and π is the risk-neutral probability of no credit default event during the life of the swap.

Following Hull and White (2000), we assume that the claim made in the event of a default equals the face value of bond plus accrued interest. Usually, the payoff from a CDS in the event of a default at time t is the face value of the reference obligation minus its market value just after time t. Using the claim amount assumption, the market value of the reference obligation just after default is the recovery rate times the sum of its face value and accrued interest. This means that the expected payoff from the CDS is

$$1 - [1 + A(t)]\hat{R} = 1 - \hat{R} - A(t)\hat{R} \tag{2}$$

where $A(t)$ denotes accrued interest on the reference obligation at time t as a percent of face value and $\hat{R}$ denotes expected recovery rate on the reference obligation in a risk-neutral world[1]. Thus, the present value of the expected payoff from the CDS is

$$\int_0^T [1 - \hat{R} - A(t)\hat{R}]q(t)v(t)dt \tag{3}$$

where $v(t)$ denotes present value of \$1 received at time t.

The value of the credit default swap to the buyer is the present value of the expected payoff minus the present value of the payment made by the buyer or

$$\int_0^T [1 - \hat{R} - A(t)\hat{R}]q(t)v(t)dt - w\int_0^T q(t)[u(t) + e(t)]dt - w\pi u(T) \tag{4}$$

The CDS spread, s, is the value of w that makes this expression zero:

$$s = \frac{\int_0^T [1 - \hat{R} - A(t)\hat{R}]q(t)v(t)dt}{\int_0^T q(t)[u(t) + e(t)]dt + \pi u(T)} \tag{5}$$

[1]It is probably reasonable to assume that there is no systemic risk in recovery rates so that expected recovery rates observed in the real world are also expected recovery rates in the risk-neutral world. This allows the expected recovery rate to be estimated from historical data. It is same as Hull and White (2000)

The variable s is referred to as the *credit default swap spread* or CDS *spread*. It is the total of the payments per year, as a percent of the notional principal, for a newly issued credit default swap.

3 Estimating default probability density

Define $q(t)\Delta t$ as the probability of default between time t and $t+\Delta t$ as seen at time zero. The variable $q(t)$ is not the same as the hazard(default intensity) rate. The hazard rate, $\lambda(t)$, is defined so that $\lambda(t)\Delta t$ is the probability of default between time t and $t + \Delta t$ as seen at time t assuming no default between time zero and time t. The variables $q(t)$ and $\lambda(t)$ are related by

$$q(t) = \lambda(t)e^{-\int_0^t \lambda(s)ds} \tag{6}$$

We will present two different methods to estimate default probability density. The first method is developed by Hull and White (2000), and it is express results in terns of $q(t)$ rather than $\lambda(t)$.

3.1 Hull and White (2000)

3.1.1 A general analysis assuming defaults at discrete times

We assume that we have chosen a set of N bonds that are either issued by the reference entity or issued by another corporation that is considered to have the same risk of default as the reference entity.[2] We assume that defaults can

[2]By the same risk of default we mean that the probability of default in any future time interval, as seen today, is the same.

happen on any of the bond maturity dates. Later we generalize the analysis to allow defaults to occurs on any date. Suppose that the maturity of the ith bond is t_i with $t_1 < t_2 < \cdots < t_N$. Here we follow Hull and White (2000) that we assume interest rates are constant, recovery rate are known, and claim amounts are known.[3]

Because interest rates are deterministic, the price at time t of the no-default value of the jth bond is $F_j(t)$, where $F_j(t)$ is the forward price of the jth bond for a forward contract maturing at time t assuming the bond is default-free ($t < t_j$). If there is a default at time t , the bondholder makes a recovery at rate $\hat{R}$[4] on a claim of $C_j(t)$. It follow that the present value of the lose, α_{ij} from a default on the jth bond at time t_i is

$$v(t_i)[F_j(t_i) - \hat{R}C_j(t_i)] \tag{7}$$

where $v(t_i)$ denotes present value of \$1 received at time t_i with certainty. There is a risk-neutral probability, p_i of default at time t_i which incurs the loss α_{ij}. The total present value of the loss on the jth bond is, therefore,

[3]It can be shown that, for either of these two assumptions, if default event, interest rates, and recovery rates are mutually independent, the following equations (7) and (8) are still true for stochastic interest rate, uncertain recovery rate , and uncertain default probability providing the recovery rate is set equal to its expected value in a risk-neutral world.

[4]The recovery rates can in theory vary according to the bond and the default time. We assume, for ease of exposition, that all the bonds have the same seniority in the event of default by reference obligation and that the expected recovery rate is independent of time.

given by

$$G_j - B_j = \sum_{i=1}^{j} p_i \alpha_{ij} \tag{8}$$

where B_j denotes the price of the jth bond today and G_j denotes the price of the jth bond if there were no probability of default (that is, the price of a treasury bond promising the same flows as the jth bond). This equation allows the p's to be determined inductively

$$p_j = \frac{G_j - B_j - \sum_{i=1}^{j-1} p_i \alpha_{ij}}{\alpha_{ij}} \tag{9}$$

3.1.2 Extension to situation where defaults can happen at any time

The analysis used to derive equation (9) assumes that default can take place only on bond maturity dates. We now extend it to allow defaults at any time. We assume that $q(t)$ is constant and equal to q_i for $t_{i-1} < t < t_i$. Setting

$$\beta_{ij} = \int_{t_{i-1}}^{t_i} v(t)[F_j(t) - \hat{R}C_j(t)]dt \tag{10}$$

a similar analysis to that used in deriving equation (9) gives

$$q_j = \frac{G_j - B_j - \sum_{i=1}^{j-1} q_i \beta_{ij}}{\beta_{jj}} \tag{11}$$

The parameters β_{ij} can be estimated using standard procedures, such as $Simpson's\ rule$, for evaluating a definite integral.

3.2 Duan (2010)

3.2.1 The hierarchical intensity model

For firm (i,j) at time t, which is the jth member of the ith group where $i = 1, \ldots, K$ and $j = 1, \ldots n_i$, we assume its default is following a process

$$dM_{ijt} = \alpha_{ijt} dN_{ct} + \beta_{ijt} dN_{it} + dN_{ijt} \tag{12}$$

where N_{ct}, N_{it}, and N_{ijt} are Poisson processes with intensities λ_{ct}, λ_{it}, and λ_{ijt}, respectively. Moreover, λ_{ct}, λ_{it}, and λ_{ijt} are independent for all i's and j's. α_{ijt} and β_{ijt} are Bernoulli random variables taking value of 1 with probabilities p_{ijt} and q_{ijt} (0 with probabilities $1 - p_{ijt}$ and $1 - q_{ijt}$). We assume α_{ijt} and β_{ijt} are independent across different firms. The Poisson process N_{ct} is a common process shared by all firms, By the additively of independent Poisson processes, the equation (12) can be reduced to

$$dM_{ijt} \overset{d}{=} \alpha^*_{ijt} dN^*_{ijt} \tag{13}$$

where $\overset{d}{=}$ stands for distributional equivalence, N^*_{ijt} is a Poisson process with intensity $\lambda_{ct} + \lambda_{it} + \lambda_{ijt}$, and α^*_{ijt} is a Bernoulli random variable taking value of 1 with a probability p^*_{ijt} (0 with a probability $1 - p^*_{ijt}$). It is clear that M_{ijt} is a Poisson process with intensity $p^*_{ijt}(\lambda_{ct} + \lambda_{it} + \lambda_{ijt})$. If we look at a firm individually, the hierarchical intensity model is equivalent to the Duffie, *et al* (2007) model. Note that

$$p^*_{ijt} = \frac{\lambda_{ct}}{\lambda_{ct} + \lambda_{it} + \lambda_{ijt}} p_{ijt} + \frac{\lambda_{it}}{\lambda_{ct} + \lambda_{it} + \lambda_{ijt}} q_{ijt} + \frac{\lambda_{ijt}}{\lambda_{ct} + \lambda_{it} + \lambda_{ijt}} \tag{14}$$

Following Duan (2010), we let the Poisson intensities be the functions of some common state variables X_t, group-specific state variables Y_{it} and firm-specific factors Z_{ijt}. Thus we have

$$\lambda_{ct} = F(X_{t_-}) \tag{15}$$

$$\lambda_{it} = G(X_{t_-}, Y_{it_-}) \tag{16}$$

$$\lambda_{ijt} = H(X_{t_-}, Y_{it_-}, Z_{ijt_-}) \tag{17}$$

$$p_{ijt} = P(X_{t_-}, Y_{it_-}, Z_{ijt_-}) \tag{18}$$

$$q_{ijt} = Q(X_{t_-}, Y_{it_-}, Z_{ijt_-}) \tag{19}$$

where $i = 1, \ldots, K$, $j = 1, \ldots, n_i$, and t_- denotes the left time. F, G, and H must be non-negative functions. P and Q must be bounded 0 and 1. In practice, one can only observe discretely sampled data, and t_- means using the data available at time $t - \Delta t$.

3.2.2 Maximum likelihood estimator

We just need to estimate default intensities, so our the log-likelihood function is a special case of Duan (2010) the log-likelihood function. Following Duan (2010), we also assume φ are the parameters governing F, G, H, P, and Q functions. Let D_T be the data set related to X_t, Y_{it}, and Z_{ijt} from time 1 to time T and I_t be a matrix with rows respecting different groups and the column dimension equals the maximum number of firms in groups. This matrix corresponding the status of all firms. Prior to default for a firm, its corresponding entry in I_t is assigned to 0 otherwise it switches to 1. In order to reflect the time at which different firms enter the sample, we also use V, a matrix matching the dimension of I_t, to capture these entry time. Thus our log-likelihood function is

$$\mathcal{L}(\varphi; D_t, I_t, V) = \sum_{t=2}^{T} ln(A_t(\varphi; D_t, I_t, V)) \tag{20}$$

where

$$\begin{aligned}
&A_t(\varphi; D_t, I_t, V) \\
=&e^{-\lambda_{c(t-1)}\Delta t} \prod_{i=1}^{K} (e^{-\lambda_{i(t-1)}\Delta t} \prod_{j=1}^{n_i} C_{ijt}^{(1)} + (1 - e^{-\lambda_{i(t-1)}\Delta t}) \prod_{j=1}^{n_i} C_{ijt}^{(2)}) \\
&+ (1 - e^{-\lambda_{c(t-1)}\Delta t}) \prod_{i=1}^{K} (e^{-\lambda_{i(t-1)}\Delta t} \prod_{j=1}^{n_i} C_{ijt}^{(3)} + (1 - e^{-\lambda_{i(t-1)}\Delta t}) \prod_{j=1}^{n_i} C_{ijt}^{(4)})
\end{aligned}$$

$$\begin{aligned}
C_{ijt}^{(1)} =& 1_{\{V(i,j)>t-1\}} + 1_{\{V(i,j)\leq t-1\}}[1_{\{I_{t-1}(i,j)\neq 0\}} + 1_{\{I_{t-1}(i,j)=0\}} 1_{\{I_t(i,j)=0\}} e^{-\lambda_{ij(t-1)}\Delta t} \\
&+ 1_{\{I_{t-1}(i,j)=0\}} 1_{\{I_t(i,j)=1\}} (1 - e^{-\lambda_{ij(t-1)}\Delta t})] \\
C_{ijt}^{(2)} =& 1_{\{V(i,j)>t-1\}} + 1_{\{V(i,j)\leq t-1\}}\{1_{\{I_{t-1}(i,j)\neq 0\}} \\
&+ 1_{\{I_{t-1}(i,j)=0\}} 1_{\{I_t(i,j)=0\}} (1 - q_{ij(t-1)}) e^{-\lambda_{ij(t-1)}\Delta t} \\
&+ 1_{\{I_{t-1}(i,j)=0\}} 1_{\{I_t(i,j)=1\}} [q_{ij(t-1)} + (1 - e^{-\lambda_{ij(t-1)}\Delta t}) \\
&- q_{ij(t-1)}(1 - e^{-\lambda_{ij(t-1)}\Delta t})]\} \\
C_{ijt}^{(3)} =& 1_{\{V(i,j)>t-1\}} + 1_{\{V(i,j)\leq t-1\}}\{1_{\{I_{t-1}(i,j)\neq 0\}} \\
&+ 1_{\{I_{t-1}(i,j)=0\}} 1_{\{I_t(i,j)=0\}} (1 - p_{ij(t-1)}) e^{-\lambda_{ij(t-1)}\Delta t} \\
&+ 1_{\{I_{t-1}(i,j)=0\}} 1_{\{I_t(i,j)=1\}} [p_{ij(t-1)} + (1 - e^{-\lambda_{ij(t-1)}\Delta t}) \\
&- p_{ij(t-1)}(1 - e^{-\lambda_{ij(t-1)}\Delta t})]\}
\end{aligned}$$

$$
\begin{aligned}
C^{(4)}_{ijt} =& 1_{\{V(i,j)>t-1\}} + 1_{\{V(i,j)\le t-1\}}\{1_{\{I_{t-1}(i,j)\neq 0\}} \\
&+ 1_{\{I_{t-1}(i,j)=0\}}1_{\{I_t(i,j)=0\}}(1-q_{ij(t-1)})(1-p_{ij(t-1)})e^{-\lambda_{ij(t-1)}\Delta t} \\
&+ 1_{\{I_{t-1}(i,j)=0\}}1_{\{I_t(i,j)=1\}}[p_{ij(t-1)} + q_{ij(t-1)} + (1-e^{-\lambda_{ij(t-1)}\Delta t}) \\
&- p_{ij(t-1)}(1-e^{-\lambda_{ij(t-1)}\Delta t}) - q_{ij(t-1)}(1-e^{-\lambda_{ij(t-1)}\Delta t}) \\
&+ p_{ij(t-1)}q_{ij(t-1)}(1-e^{-\lambda_{ij(t-1)}\Delta t})]\}
\end{aligned}
$$

In order to implement the model, one must specify the intensity functions. In this paper, we let $F(x_1,\ldots,x_n) = e^{a_0+a_1x_1+\cdots+a_nx_n}$, since we know that from Duan (2010) it will make the log-likelihood function great than $F(x_1,\ldots,x_n) = ln(1+e^{a_0+a_1x_1+\cdots+a_nx_n})$. Similarly, the functions G and H are in the same form but allow for different coefficients. The default probability function corresponding to the common shock is the same as Duan (2010), $p(x_1,\ldots,x_n) = \frac{1}{1+e^{-b_0-b_1x_1-\cdots-b_nx_n}}$. The default probability functions corresponding to the group-specific shock are similarly specified. Needless to say, the coefficients can be different.

4 Empirical analysis

4.1 Data

We get the quotes for these corporation bonds from the Datastream and these benchmark government bonds from the Wall Street Journal's market data. The recovery rates of corporate bonds are from Moody's investor's service and are shown in Table 3. In our paper, we set the recovery rate of all companies, $\hat{R}$, to 0.492 because we just compare the relative values of the CDS spread from different methodologies.[5] Moreover, the date frequency is daily with the accounting data from the Compustat quarterly and annual datebase and the stock market data (stock prices, shares outstanding, and market index return) are from the CRSP daily file.

We follow Duffie, *et al* (2007) to use four variables for firm-specific intensity functions: trailing one-year S&P 500 index return, three-month treasury bill rate, firm's trailing one-year return, and firm's distance-to-default in accordance with Merton's model. Merton's model is typically implemented with a KMV assumption on the debt maturity and size. Moreover, we follow Duan (2010) to use the variable for common shock intensity functions: average financial distance-to default. The details about the distance-to-default for financial firms are seen at Appendix A. We select the large one hundred financial institutions in the United States as the financial system, which is similar as Bartram, *et al* (2007). The variable for default probability function is firm's distance-to-default.

[5]For example, from Hamilton, *et al* (2003), we found that the following relationship provides a good fit to the data: Recovery rate=59.1-8.356×Default rate; the recovery rate is the average recovery rate on senior unsecured bonds in a year measured as a percentage and the default rate is the corporate default rate in the year measured as a percentage

4.2 Empirical result

First, we consider the valuation of credit default swaps on Ashland Inc. at the close of trading on July 13, 2000 and on September 18, 2008.[6] We assume that the time to maturity is 5 years, because it is most popular. The common shock intensities in 2000 and 2008 are shown at Figure 1 and 2.

Table 1: CDS spread for Ashland Inc. at July 13, 2000

Maturity (years)	Hull and White	Firm-specific	Common Shock & Firm-specific
5	209	69	417

Table 2: CDS spread for Ashland Inc. at Sep 18, 2008

Maturity (years)	Hull and White	Firm-specific	Common Shock & Firm-specific
5	236	59	411

[6] We chose the same company as Hull and White (2000) in the different the close of trading dates. The day, July 13, 2000 is same as Hull and White (2000), and the other day, September 18, 2008, is the last day of trading day after Lehman Brothers bankruptcy. The quotes at September 18, 2008 for corporation bonds and government bonds are listed in Table 4.

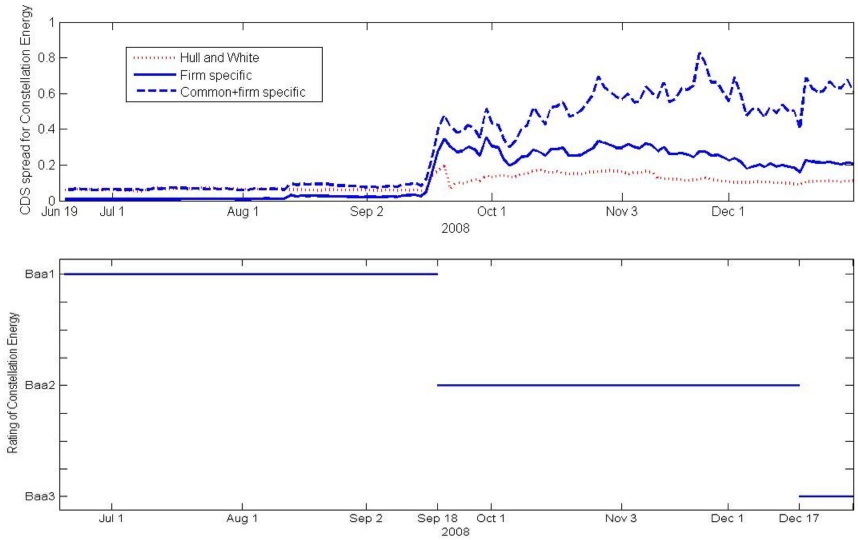

Figure 1: Constellation Energy had been downgraded to Baa2 from Baa1 in Sep 18, 2008 and then had been downgraded to Baa3 from Baa2 in Dec 17, 2008.

Finally, we consider the variation of CDS spreads in the situations of the credit rating of firms being downgraded or upgraded. From Moody's investor's service, we select three companies. Two of three companies we selected had been downgraded and the other had been upgraded. From Figure 1, we find that the CDS spread of Constellation Energy had increased before the credit rating had be downgraded no matter what methods we used.

But the following case, which is showed in Figure 2 , we see that the variation of CDS spreads were different when the credit rating had been downgraded. Using the method which is developed by Hull and White (2000), the spread did not increase when the rating had been downgraded in Nov 14, 2008.

Next, we choose a company, AmeriGas Partners, L.P., and it had been upgraded in July 15, 2008. From Figure 3, we find that they did not have big difference when it's rating was changed.

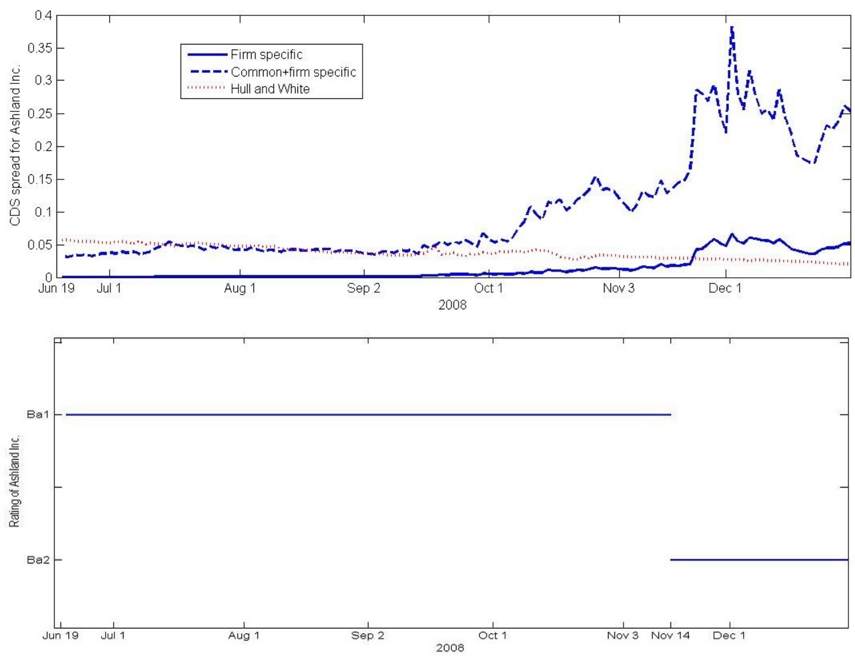

Figure 2: Ashland Inc. had been downgraded to Ba2 from Ba1 in Nov 14, 2008.

In this three case, we see that CDS spread was increasing after the collapse of Lehman Brother and the bailout of AIG in September 2008. If we take the common factor into account then it is fast to catch the current risky economic environment because we know that systemic risk can't be diversified away.

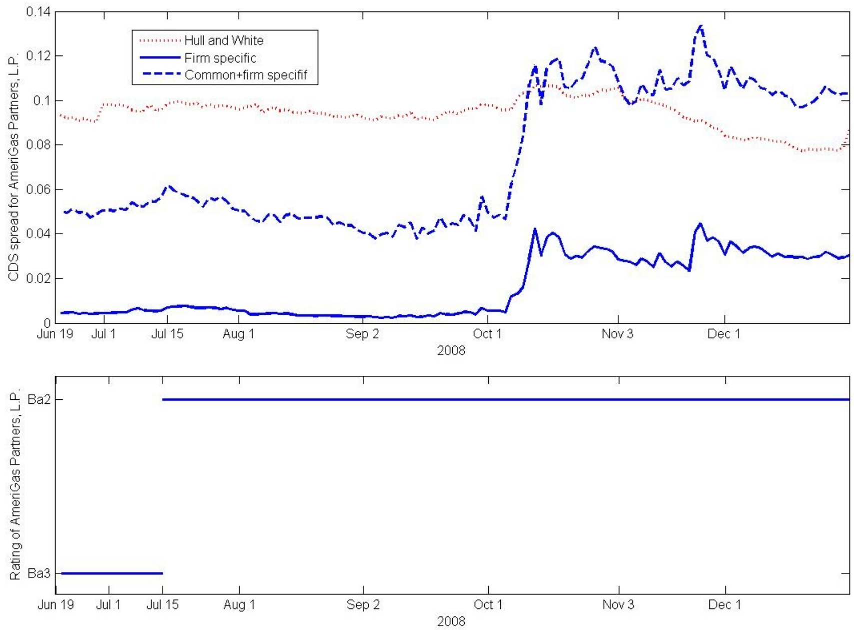

Figure 3: AmeriGas Partners, L.P. had been upgraded to Ba2 from Ba3 in July 15, 2008

Using the reduced form model, Duffie, *et al* (2007) or Duan (2010), it is more fast to reflect the the valuation of CDS spread. One of reasons is that corporate bonds are relatively illiquid than stocks. Some studies such as Ericsson and Renault (2000), show that a safe valuation of credit risk requires to take into account macroeconomic and financial factors as an explanation of some trend, credit quality factors and liquidity factors.

5 Conclusion

This paper offers two methodologies to calculate the CDS spread and compares the results. We also provide empirical implementation for three companies and take the systemic risk in account for the valuation of CDS spread.

In 2008 credit risk, the counter party risk is important to the valuation of the credit derivatives. But in this article, we assume that there is no counter party risk . Thus our future work is to deal this risk and take it into account.

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Appendix

A. Estimating distance-to-default (DTD)

As descried in Crosbie and Bohn (2002), the KMV assumption sets firm's debt maturity T to one year and size at time t, D_t, to the sum of short-term debt and a half of long-term debt. But we know that the assumption about debt has problem for financial firms. In order to deal with financial firms, we follow Duan (2010) to add a fraction, δ, of firm's other liabilities L_t (total liabilities minus short-term and long-term debts). In Merton (1974), the firm asset value V_t follows a geometric Brownian motion

$$dV_t = \mu V_t dt + \sigma V_t dW_t$$

where μ and σ measure the firm's mean rate of asset growth and asset volatility, respectively. Therefore, the equity value at time $t \leq T$ by the Black-Scholes option pricing formula becomes

$$E_t = V_t N(d_t) - e^{-r(T-t)}(D_t + \delta L_t)N(d_t - \sqrt{T-t}) \tag{21}$$

where r is the risk-free rate, $N(\cdot)$ is the cumulative distribution function of standard normal random variable, and

$$d_t = \frac{ln(\frac{V_t}{D_t+\delta L_t}) + (r + \frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{(}T-t)} \tag{22}$$

According to Merton (1974), the distance to default is

$$DTD_t = \frac{ln(\frac{V_t}{D_t+\delta L_t}) + (\mu - \frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{(}T-t)} \tag{23}$$

In order to estimate unknown parameters, the mean rate of asset growth μ, the asset volatility σ, and the fraction δ, we apply the maximum likelihood estimation method developed by Duan (1994,2000). For financial firms, we follow Duan (2011) to divide the model's implied asset value by book asset value so that the pure scaling effect will not distort the parameter values in the time series estimation. Thus the log-likelihood function is

$$\mathcal{L}(\mu,\sigma,\delta) = -\frac{n-1}{2}ln(2\pi) - \frac{1}{2}\sum_{t=2}^{n} ln(\sigma^2) - \sum_{t=2}^{n} ln(\frac{\hat{V}_t(\sigma,\delta)}{A_t}) \quad (24)$$

$$- \sum_{t=2}^{n} ln(N(\hat{d}_t(\sigma,\delta))) - \sum_{t=2}^{n} \frac{1}{2\sigma^2}\left[ln(\frac{\hat{V}_t(\sigma,\delta)}{\hat{V}_{t-1}(\sigma,\delta)}\frac{A_{t-1}}{A_t}) - (\mu - \frac{\sigma^2}{2})\right]^2 \quad (25)$$

where n is the total number of equity values in the time series sample, $\hat{V}_t$ is the model's implied asset value solved using equation (21), $\hat{d}_t$ is computed using equation (22) with $\hat{V}_t$, A_t is the book value. To avoid the " look-aheadbias ", we also follow Duan (2011) to employ a rolling window method to estimate DTD. More specifically, at the end of each day, we estimate DTD for each firm using its daily market values of equity capitalization in the preceding year.

Table 3: Recovery rates on corporate bonds from Moody's investor's service(1987-2010)

	Emergence Year	Default Year
Class	1987-2010(%)	1987-2010(%)
Senior Secured	63.5	63.5
Senior Unsecured	49.2	49.2
Senior Subordinated	29.4	29.4
Subordinated	29.3	29.3
Junior Subordinated	18.4	18.4

Table 4: Quotes for unsecured bonds issued by Ashland Inc. and for benchmark government bonds at close of trading on Sep 18, 2008

Unsecured Bonds Issued By Ashland Inc.

Maturity Date	Coupon % per annum	Quoted Price	Quoted Yield
May 1, 2009	6.86	101.967	3.5395
Nov 15, 2012	8.8	117.0732	4.2599
Apr 1, 2015	8.38	118.1944	5.0682

Benchmark Government Bonds

Maturity Date	Coupon % per annum	Quoted Price	Quoted Yield
Mar 19, 2009	Bill	0.605	0.615
Jun 04, 2009	Bill	1.228	1.252
Nov 15, 2012	4	106.781	2.2832
Feb 15, 2015	4	107.312	2.7493
May 15, 2015	4.125	108.156	2.7763

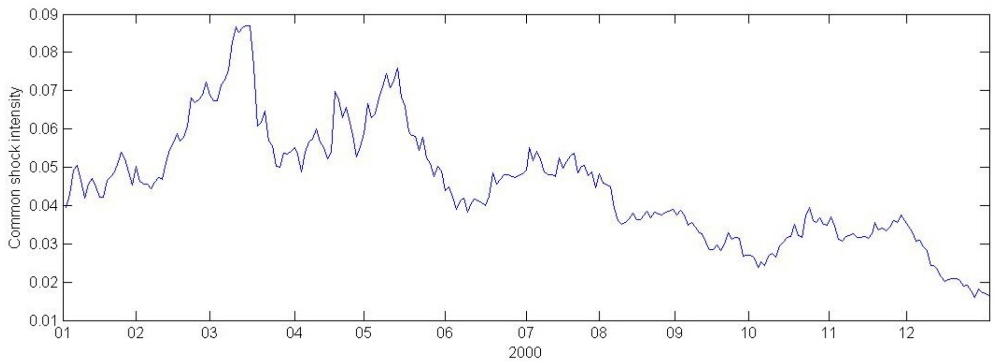

Figure 4: 2000 Common shock intensity

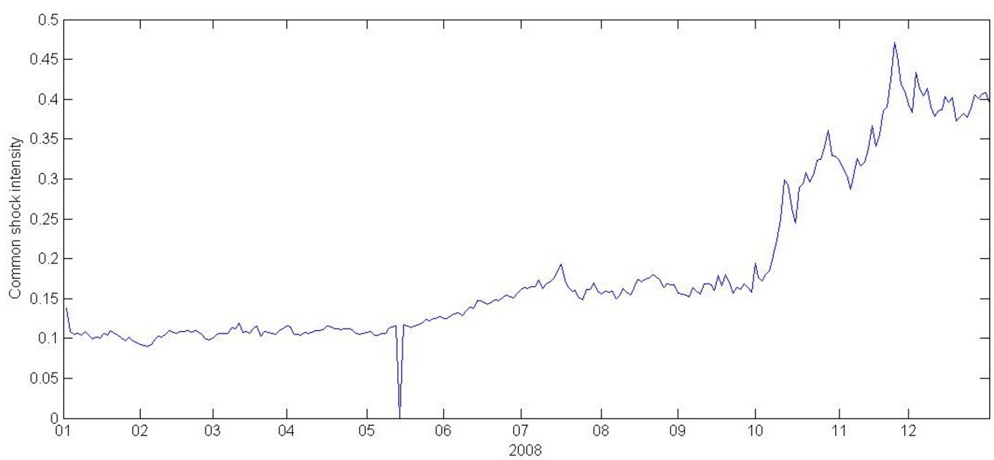

Figure 5: 2008 Common shock intensity