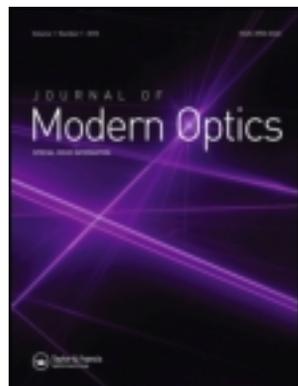


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Method of solving a triplet comprising a singlet and a cemented doublet with given primary aberrations

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Abstract. A simple algebraic algorithm is proposed as a computational tool for the thin-lens design of a triplet which consists of a singlet and a cemented doublet. The triplet is required to yield specified amounts of lens power, primary spherical aberration, central coma, longitudinal chromatic aberration and secondary spectrum. The three element powers of the triplet are first obtained by solving the simultaneous linear equations of the total lens power, longitudinal chromatic aberration and secondary spectrum. A quadratic equation is obtained by combining the equations of spherical aberration and central coma, and the lens shapes are then obtained by solving this quadratic equation. The solving process is purely algebraic and is therefore easy to calculate and guarantees that all the solutions can be found.

1. Introduction

Optical design problems in which the system will be designed from primary aberration theory normally begin with two stages. First, the first-order layout is given to determine the lens powers K and locations of the system components. Thus, for each component, the ray heights at the principal planes and the slope angles before and after refraction of both paraxial marginal and chief rays are also determined. Second, there is a thin-lens aberration design stage. During this stage a set of aberration targets is specified for each component. Then, according to the possibility of satisfying these aberrations, the component is replaced by a suitable thin lens which may be a singlet, doublet, triplet or even a complex assembly of many elements. The work at this stage is to solve the constructional parameters of the lens to meet the required power and aberration targets, and then to use the solutions as starting points for future thickening and optimization. The primary aberration targets of thin lenses are normally three in number: the Seidel coefficients S_1 , S_{2C} and C_L , representing spherical aberration, central coma and longitudinal chromatic aberration respectively. Central coma is defined as the coma value for the case of the stop located at the lens. There is no need for the targets of the other primary aberrations: astigmatism S_3 , field curvature S_4 , distortion S_5 and lateral chromatic aberration C_T , since they can be expressed as combinations of K , S_1 , S_{2C} and C_L by using the well known stop-shift formulae of aberration theory.

Hence, systematic thin lens design methods of various lens types are useful tools and have been the subject of research by several workers. For example, the

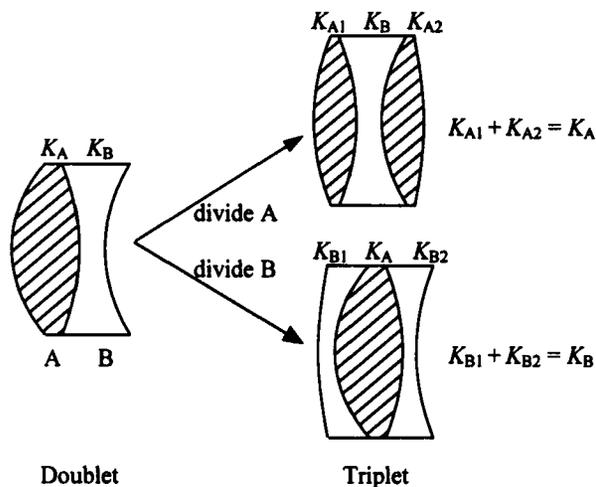


Figure 1. The cemented triplets with two glass types [2, 6] are obtained from dividing either the flint lens or the crown lens of the cemented doublets into two parts. The triplets have the same values of K and C_L as the doublet, they also produce the same kind of secondary spectrum. For two chosen glass types, they may have at most two thin-lens solutions which meet the targets of K , S_1 , S_{2C} and C_L .

thin-lens designs of various doublet types have been provided by Dreyfus *et al.* [1], Kingslake [2], Khan and Macdonald [3], Smith [4] and Chen *et al.* [5], among many others.

Triplets are also elementary and have been widely used in many optical systems. Cemented triplets which have four surfaces as constructional degrees of freedom can meet the targets K , S_1 , S_{2C} and C_L . Conrady [6] (see also [2]) has given a study of thin-lens design of cemented triplets which are produced by dividing one element of a cemented doublet and thus have the same glass type for the first and third elements (figure 1). The method has three steps. First, the powers K_A and K_B of the two elements of a cemented doublet are determined to meet the targets K and C_L . Second, the flint element of the cemented doublet is divided into two so that the total power is equal to the power of the original flint element, and the two components placed either side of the crown element to make a cemented triplet. The new triplet has the same K and C_L as the original doublet. Of course, the crown element could also be divided in this way. Third, in performing the Conrady's G sum analysis, the spherical aberration formula is a quadratic, while the central coma formula is linear. Hence there may be two solutions to the problem. Since the triplet is obtained from dividing a doublet, they produce the same amount of secondary spectrum C_{LP} (defined later). If the secondary spectrum is expected to be reduced, triplets with three different glass types are preferred because the secondary spectrum may be reduced by using two of the glass types to match the third glass type [7].

Recently, we have derived an algorithm for solving the structures of thin-lens cemented triplets which consist of three different glass types and have the given target values of K , S_1 , S_{2C} and C_L [8]. The formulae for the aberrations are combined with the power formula into a fifth-order polynomial equation in one variable, whose roots can all be solved by a combination of numerical and algebraic

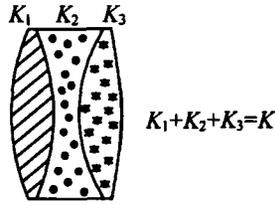


Figure 2. Cemented triplet with three glass types. The performance of the secondary spectrum is usually better than that of the triplet with two glass types as shown in Fig. 1. For three chosen glass types, they may have at most five thin-lens solutions which meet the targets of K , S_1 , S_{2C} and C_L [7].

methods. Compared with cemented triplets with two glass types, cemented triplets with three glass types may have at most five solutions to the problem rather than two solutions. Although glass choice is usually used to affect the secondary spectrum, glasses cannot be considered as continuous variables because the characteristics (refractive index, Abbe number and partial dispersion) on the glass map are discontinuous. Cemented triplets have only four surface curvatures as degrees of freedom; they cannot meet the five targets of K , S_1 , S_{2C} , C_L and C_{LP} at the same time; lenses with five surfaces are necessary for the task.

Consider the thin-lens triplet shown in figure 3; this consists of a singlet and a cemented doublet and is widely used as a component in many optical systems. The triplets use three different glasses and have five surfaces as constructional degrees of freedom which may be used to meet the specified amounts of K , S_1 , S_{2C} , C_L and C_{LP} . Also, they usually provide smoother surfaces than cemented triplets so that the higher-order aberrations can be reduced. The powers of the three elements can be easily calculated by solving the simultaneous linear equations for K , C_L and C_{LP} . However, to our best knowledge, no simple method has been proposed for solving the S_1 and S_{2C} targets of triplets consisting of a singlet and cemented doublet.

In this paper, we derive a simple algebraic algorithm to solve the triplets with targets not only for K , C_L and C_{LP} , but also for S_1 and S_{2C} . After the three element powers have been obtained by solving the equations for K , C_L and C_{LP} , the lens shapes are obtained by solving the simultaneous equations for S_1 and S_{2C} . By combining the aberration equations of Hopkins and Rao (see also [3]) for a cemented doublet with the equations for a singlet, the S_1 and S_{2C} equations for the triplet become quadratic and linear respectively. Hence the two equations can be combined to obtain a quadratic equation which can easily be solved and may have at most two solutions. The algorithm is purely algebraic and hence is easy to calculate, and all the solutions can be found.

2. Method

There are two triplet types, singlet leading and doublet leading, as shown in figure 3. Because the algorithms of solving both types are similar, we only propose the singlet-leading type. The notation is defined as follows.

- (1) K is the power which is the inverse of the focal length f .
- (2) h is the marginal ray height. A ray height is positive if it is above the axis.
- (3) \bar{h} is the principal ray height.

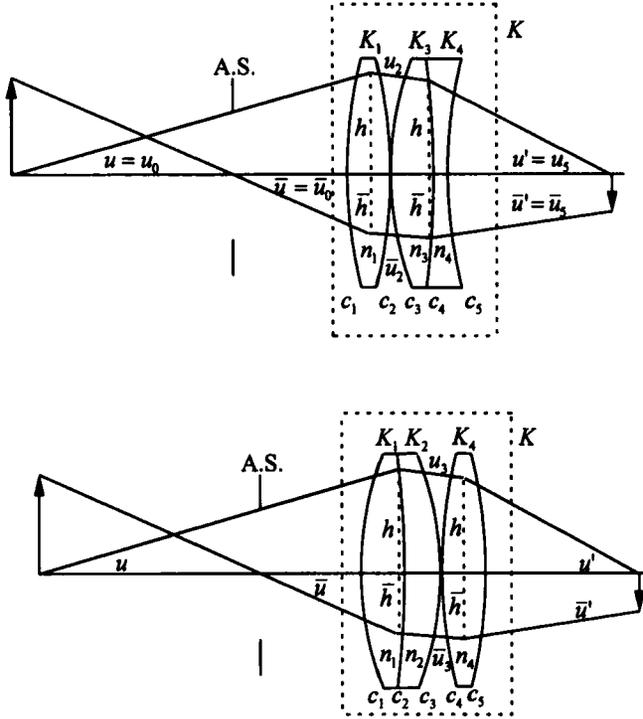


Figure 3. The scheme of the two thin-lens triplet types: singlet leading and doublet leading. These triplets have five surfaces as degrees of freedom to yield specified amounts of K , S_1 , S_{2C} , C_L , and C_{LP} . The surfaces of these triplet types are usually smoother than those of the cemented triplets as shown in figure 2. Because the algorithms for solving both types are similar, only the algorithm for solving the singlet-leading type is proposed. As a result, there are at most two thin-lens solutions for three chosen glass types.

- (4) u is the slope angle of the marginal ray. A ray angle is regarded as positive if a clockwise rotation of the ray brings it parallel to the optical axis.
- (5) \bar{u} is the slope angle of the principal ray.
- (6) c denotes the surface curvature.
- (7) n is the refractive index.
- (8) $K_i = (n_i - 1)(c_i - c_{i+1})$ is the power of element enclosed between the surfaces i and $i + 1$.
- (9) $A_i = n_{i-1}(hc_i + u_{i-1}) = n_i(hc_i + u_i)$ is the refraction invariant for surface i .
- (10) $H = \bar{h}u - h\bar{u}$ is the optical invariant.

2.1. Determining the element powers by solving the equations for K , C_L , and C_{LP}

For completeness, the algorithm for solving the element powers is first reviewed. Let λ_S , λ_M and λ_L be the short, middle and long wavelengths respectively over the band of interest, and n_S , n_M and n_L be the corresponding glass refractive indices. The equations for K , C_L , and C_{LP} of the triplet can be expressed as [2, 4, 10]:

$$K_1 + K_3 + K_4 = K, \tag{1}$$

$$h^2 \left(\frac{K_1}{V_1} + \frac{K_3}{V_3} + \frac{K_4}{V_4} \right) = C_L, \tag{2}$$

$$h^2 \left(\frac{P_1}{V_1} K_1 + \frac{P_3}{V_3} K_3 + \frac{P_4}{V_4} K_4 \right) = C_{LP}, \tag{3}$$

where

$$V = \frac{n_M - 1}{n_S - n_L}, \tag{4}$$

denotes the Abbe number; P is the partial dispersion ratio defined as

$$P = \frac{n_S - n_M}{n_S - n_L}. \tag{5}$$

Consider that

$$\frac{P}{V} = \frac{n_S - n_M}{n_S - n_L} \frac{n_S - n_L}{n_M - 1} = \frac{1}{(n_M - 1)/(n_S - n_M)}. \tag{6}$$

Comparing this with the definition of V shows that C_{LP} denotes the longitudinal chromatic aberration between λ_S and λ_M . If $C_L = C_{LP} = 0$, the foci of the three wavelengths are common and the triplet is apochromatic. It is usual to normalize the lens so that the corresponding normalized lens is of unit power and unit aperture [3, 9]. To achieve this, the following dimensionless normalized parameters are defined:

$$\tilde{K}_{1,3,4} = \frac{1}{K} K_{1,3,4},$$

$$\tilde{c}_i = \frac{1}{K} c_i, \quad i = 1, \dots, 5,$$

$$(\tilde{u}_i, \tilde{\bar{u}}_i) = \frac{1}{hK} (u_i, \bar{u}_i), \quad i = 1, \dots, 5,$$

$$\tilde{A}_i = \frac{1}{hK} A_i, \quad i = 1, \dots, 5, \tag{7}$$

$$\tilde{S}_1 = \frac{1}{h^4 K^3} S_1,$$

$$\tilde{S}_2 = \frac{1}{Hh^2 K^2} S_2,$$

$$(\tilde{C}_L, \tilde{C}_{LP}) = \frac{1}{h^2 K} (C_L, C_{LP}).$$

Equations (1)–(3) can now be rewritten as

$$\tilde{K}_1 + \tilde{K}_3 + \tilde{K}_4 = 1, \tag{8}$$

$$\frac{1}{V_1} \tilde{K}_1 + \frac{1}{V_3} \tilde{K}_3 + \frac{1}{V_4} \tilde{K}_4 = \tilde{C}_L, \tag{9}$$

$$\frac{P_1}{V_1} \tilde{K}_1 + \frac{P_3}{V_3} \tilde{K}_3 + \frac{P_4}{V_4} \tilde{K}_4 = \tilde{C}_{LP}. \tag{10}$$

These dimensionless normalized equations are independent of K and h and can be solved for the three powers by standard matrix algebra. Having determined $\tilde{K}_{1,3,4}$, the value of \tilde{u}_2 is calculated from

$$\tilde{u}_2 = \tilde{u} - \tilde{K}_1, \quad (11)$$

and the values of the conjugate factors of the singlet and the doublet are

$$Y_1 = \frac{u_2 + u}{u_2 - u} = \frac{\tilde{u}_2 + \tilde{u}}{\tilde{u}_2 - \tilde{u}}, \quad (12)$$

$$Y_D = \frac{u' + u_2}{u' - u_2} = \frac{\tilde{u}' + \tilde{u}_2}{\tilde{u}' - \tilde{u}_2}.$$

These values will be used in the following section.

2.2. Determining the curvatures by solving S_1 and S_{2C} formulae

The spherical aberration and central coma of the front singlet are [10]

$$(S_1)_1 = \frac{h^4 K_1^3}{4} [M_0(n_1)X_1^2 - M_1(n_1)X_1 Y_1 + M_2(n_1)Y_1^2 + M_3(n_1)], \quad (13)$$

$$(S_{2C})_1 = \frac{-Hh^2 K_1^2}{2} [M_4(n_1)X_1 - M_5(n_1)Y_1], \quad (14)$$

where $M_0(n)$ to $M_5(n)$ are functions of refractive index and are defined as follows:

$$M_0(n) = \frac{n+2}{n(n-1)^2},$$

$$M_1(n) = \frac{4(n+1)}{n(n-1)},$$

$$M_2(n) = \frac{3n+2}{n}, \quad (15)$$

$$M_3(n) = \left(\frac{n}{n-1}\right)^2,$$

$$M_4(n) = \frac{n+1}{n(n-1)},$$

$$M_5(n) = \frac{2n+1}{n},$$

and

$$X_1 = \frac{c_1 + c_2}{c_1 - c_2} = \frac{\tilde{c}_1 + \tilde{c}_2}{\tilde{c}_1 - \tilde{c}_2} \quad (16)$$

is the shape factor. The normalized forms of $(S_1)_1$ and $(S_{2C})_1$ are

$$(\tilde{S}_1)_1 = \frac{(S_1)_S}{h^4 K^3} = g_1 X_1^2 + g_2 X_1 + g_3, \quad (17)$$

$$(\tilde{S}_{2C})_1 = \frac{(S_{2C})_S}{Hh^2 K^2} = g_4 X_1 + g_5, \quad (18)$$

where the coefficients g_1 to g_5 are as follows:

$$\begin{aligned}
 g_1 &= \frac{\tilde{K}_1^3}{4} M_0(n_1), \\
 g_2 &= -\frac{\tilde{K}_1^3}{4} M_1(n_1) Y_1, \\
 g_3 &= \frac{\tilde{K}_1^3}{4} [M_2(n_1) Y_1^2 + M_3(n_1)], \\
 g_4 &= -\frac{\tilde{K}_1^2}{2} M_4(n_1), \\
 g_5 &= \frac{\tilde{K}_1^2}{2} M_5(n_1) Y_1.
 \end{aligned} \tag{19}$$

The Hopkins–Rao [9] expressions (see also [3]) for the spherical aberration and central coma of a cemented doublet are

$$\begin{aligned}
 \frac{(S_1)_D}{h^4 K_D^3} &= \left(N_0(n_3) \frac{1+\rho}{2} + N_0(n_4) \frac{1-\rho}{2} + 1 \right) \alpha^2 \\
 &+ \left[N_1(n_3) \left(\frac{1+\rho}{2} \right)^2 - N_1(n_4) \left(\frac{1-\rho}{2} \right)^2 + (\rho - Y_D) \right] \alpha \\
 &+ \left[N_2(n_3) \left(\frac{1+\rho}{2} \right)^3 + N_2(n_4) \left(\frac{1-\rho}{2} \right)^3 \right. \\
 &\left. - N_3(n_3) \left(\frac{1+\rho}{2} \right)^2 \frac{Y_D - 1}{2} + N_3(n_4) \left(\frac{1-\rho}{2} \right)^2 \frac{Y_D + 1}{2} \right], \tag{20}
 \end{aligned}$$

$$\begin{aligned}
 \frac{(S_2C)_D}{Hh^2 K_D^2} &= \left(N_4(n_3) \frac{1+\rho}{2} + N_4(n_4) \frac{1-\rho}{2} + 1 \right) \alpha \\
 &+ \left[N_5(n_3) \left(\frac{1+\rho}{2} \right)^2 - N_5(n_4) \left(\frac{1-\rho}{2} \right)^2 + \frac{\rho - Y_D}{2} \right], \tag{21}
 \end{aligned}$$

where $N_0(n)$ to $N_5(n)$ are functions of refractive index and are defined as follows:

$$\begin{aligned}
 N_0(n) &= \frac{2}{n}, \\
 N_1(n) &= \frac{3}{n-1}, \\
 N_2(n) &= \frac{n}{(n-1)^2}, \\
 N_3(n) &= \frac{n}{n-1}, \\
 N_4(n) &= \frac{1}{n}, \\
 N_5(n) &= \frac{1}{n-1}.
 \end{aligned} \tag{22}$$

The subscript D in equations (20) and (21) denotes ‘doublet’, and

$$\begin{aligned}
 K_D &= K_3 + K_4, \tilde{K}_D = \frac{K_D}{K} = \tilde{K}_3 + \tilde{K}_4, \\
 \rho &= \frac{K_3 - K_4}{K_D} = \frac{\tilde{K}_3 - \tilde{K}_4}{\tilde{K}_D}, \\
 \alpha &= \frac{1}{hK_D} A_4 = \frac{1}{\tilde{K}_D} \tilde{A}_4.
 \end{aligned}
 \tag{23}$$

K_D is the power of the doublet, ρ describes the power distribution, and

$$\tilde{A}_4 = \frac{1}{hK} A_4 = n_3(\tilde{c}_4 + \tilde{u}_3).
 \tag{24}$$

The normalized forms of $(S_1)_D$ and $(S_2)_D$ are

$$(\tilde{S}_1)_D = \frac{(S_1)_D}{h^4 K^3} = g_6 \tilde{A}_4^2 + g_7 \tilde{A}_4 + g_8,
 \tag{25}$$

$$(\tilde{S}_{2C})_D = \frac{(S_{2C})_D}{Hh^2 K^2} = g_9 \tilde{A}_4 + g_{10},
 \tag{26}$$

where

$$\begin{aligned}
 g_6 &= \tilde{K}_D \left(N_0(n_3) \frac{1+\rho}{2} + N_0(n_4) \frac{1-\rho}{2} + 1 \right), \\
 g_7 &= \tilde{K}_D^2 \left[N_1(n_3) \left(\frac{1+\rho}{2} \right)^2 - N_1(n_4) \left(\frac{1-\rho}{2} \right)^2 + (\rho - Y_D) \right], \\
 g_8 &= \tilde{K}_D^3 \left[N_2(n_3) \left(\frac{1+\rho}{2} \right)^3 + N_2(n_4) \left(\frac{1-\rho}{2} \right)^3 \right. \\
 &\quad \left. - N_3(n_3) \left(\frac{1+\rho}{2} \right)^2 \frac{Y_D - 1}{2} + N_3(n_4) \left(\frac{1-\rho}{2} \right)^2 \frac{Y_D + 1}{2} \right], \\
 g_9 &= -\tilde{K}_D \left(N_4(n_3) \frac{1+\rho}{2} + N_4(n_4) \frac{1-\rho}{2} + 1 \right), \\
 g_{10} &= -\tilde{K}_D^2 \left[N_5(n_3) \left(\frac{1+\rho}{2} \right)^2 - N_5(n_4) \left(\frac{1-\rho}{2} \right)^2 + \frac{\rho - Y_D}{2} \right].
 \end{aligned}
 \tag{27}$$

The aberration of the triplet is the sum of the aberrations of the singlet and doublet:

$$\tilde{S}_1 = (\tilde{S}_1)_1 + (\tilde{S}_1)_D,
 \tag{28}$$

$$\tilde{S}_{2C} = (\tilde{S}_{2C})_1 + (\tilde{S}_{2C})_D.$$

From equations (17), (18), (25) and (26), the above equations can be expressed as the following equations in X_1 and \tilde{A}_4 :

$$g_1 X_1^2 + g_2 X_1 + g_6 \tilde{A}_4^2 + g_7 \tilde{A}_4 + w_1 = 0,
 \tag{29}$$

$$g_4 X_1 + g_9 \tilde{A}_4 + w_2 = 0, \tag{30}$$

where

$$w_1 = g_3 + g_8 - \tilde{S}_1, \tag{31}$$

$$w_2 = g_5 + g_{10} - \tilde{S}_{2C}. \tag{32}$$

From equation (30), we have

$$X_1 = \frac{-g_9 \tilde{A}_4 - w_2}{g_4}. \tag{33}$$

Substituting into equation (29) gives

$$m_2 \tilde{A}_4^2 + m_1 \tilde{A}_4 + m_0 = 0, \tag{34}$$

where

$$m_2 = g_1 g_9^2 + g_4^2 g_6,$$

$$m_1 = 2g_1 g_9 w_2 - g_2 g_4 g_9 + g_4^2 g_7, \tag{35}$$

$$m_0 = g_1 w_2^2 - g_2 g_4 w_2 + g_4^2 w_1.$$

Since

$$\begin{aligned} \tilde{A}_3 &= \tilde{c}_3 + \tilde{u}_2 \\ &= n_3(\tilde{c}_3 + \tilde{u}_3), \end{aligned} \tag{36}$$

then we can have the value of \tilde{A}_3 from \tilde{A}_4 by

$$\tilde{A}_3 = \tilde{A}_4 + \frac{n_3}{n_3 - 1} \tilde{K}_3. \tag{37}$$

Finally, it is straightforward to obtain $(\tilde{c}_1, \tilde{c}_2)$ from (X_1, \tilde{K}_1) and $(\tilde{c}_3, \tilde{c}_4, \tilde{c}_5)$ from $(\tilde{A}_3, \tilde{K}_3, \tilde{K}_4)$ as follows:

$$\begin{aligned} \tilde{c}_1 &= \frac{(X_1 + 1)\tilde{K}_1}{2(n_1 - 1)}, \\ \tilde{c}_2 &= \frac{(X_1 - 1)\tilde{K}_1}{2(n_1 - 1)}, \\ \tilde{c}_3 &= \tilde{A}_3 - \tilde{u}_2, \\ \tilde{c}_4 &= \tilde{c}_3 - \frac{\tilde{K}_3}{n_3 - 1}, \\ \tilde{c}_5 &= \tilde{c}_4 - \frac{\tilde{K}_4}{n_4 - 1}. \end{aligned} \tag{38}$$

Thus the triplet is solved.

3. Example

As an example in the use of the algorithm, we give the thin-lens designs of a telescope objective system. The values for derived parameters in the paper are given sufficiently to enable a user to validate several intermediate steps in the calculation as well as the final result.

Table 1. Data for the glasses used in the example.

	Triplet			
	Glass 1 (PK51)	Glass 3 (FK51)	Glass 4 (LAK31)	Prism (BAK1)
n_M	1.528 55	1.486 56	1.696 73	1.572 50
n_S	1.533 33	1.490 56	1.705 31	1.579 44
n_L	1.526 46	1.484 80	1.692 96	1.569 48
V	76.930	84.439	56.397	57.526
P	0.695 71	0.694 34	0.694 91	0.697 3
$M_0(n)$	8.263 08	—	—	—
$M_1(n)$	12.518 8	—	—	—
$M_2(n)$	4.308 43	—	—	—
$M_3(n)$	8.363 45	—	—	—
$M_4(n)$	3.129 71	—	—	—
$M_5(n)$	2.65421	—	—	—
$N_0(n)$	—	1.345 39	1.178 74	—
$N_1(n)$	—	6.165 75	4.305 84	—
$N_2(n)$	—	6.279 29	3.495 31	—
$N_3(n)$	—	3.055 25	2.435 28	—
$N_4(n)$	—	0.672 694	0.589 37	—
$N_5(n)$	—	2.055 25	1.435 28	—

The telescope objective system has a focal length of 300 mm, a relative aperture of $F/3$, a field of view of $\pm 3^\circ$ and an image height of 15.72 mm (figure 4). The wavelengths over the band of interest are the hydrogen F line (0.4861 μm), the helium d line (0.5876 μm), and the hydrogen C line (0.6563 μm). The desirable primary aberrations for the system as a whole are zero spherical aberration and coma, and correction of both longitudinal chromatic aberration and secondary spectrum. The glasses PK51, FK51 and LAK31 are used for the triplet, and the glass BAK1 is used for the prism. These glasses are chosen from the Schott catalogue and their data are listed in table 1.

The prism induces the following aberrations [9]:

$$\begin{aligned}
 S_1 &= -\frac{n^2 - 1}{n^3} Du'^4 = -\frac{(1.5725)^2 - 1}{(1.5725)^3} 137(-0.1667)^4 = -0.040\,038, \\
 S_2 &= \frac{\bar{u}'}{u'} S_1 = \frac{0.0524}{-0.1667} S_1 = 0.012\,59, \\
 C_L &= -\frac{D}{V} \frac{n-1}{n^2} u'^2 = -\frac{137}{57.526} \frac{1.5725-1}{(1.5725)^2} (-0.1667)^2 = -0.015\,322\,3, \\
 C_{LP} &= -\frac{PD}{V} \frac{n-1}{n^2} u'^2 = -\frac{0.6973 \times 137}{57.525} \frac{1.5725-1}{(1.5725)^2} (-0.1667)^2 \\
 &= -0.010\,681\,1,
 \end{aligned} \tag{39}$$

where $D = 137$ mm is the thickness of the prism. Hence, negative values of the above aberrations are the targets of the front triplet. The values of S_2 and S_{2C} of the triplet are equal because the aperture stop is located at the lens. The aberration targets for the triplet are expressed in three forms in table 2.

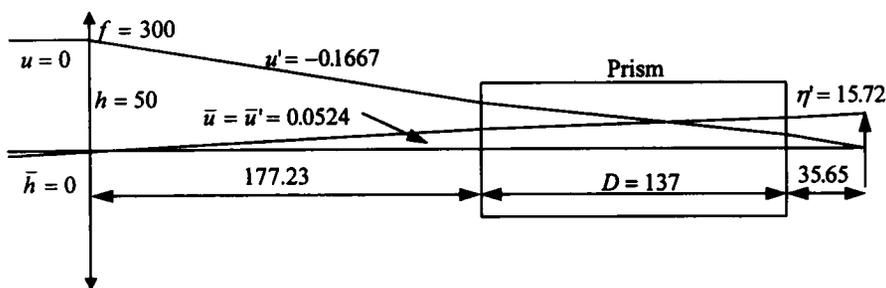


Figure 4. Gaussian design of the telescope objective system. The aberrations S_1 , S_2 , C_L , and C_{LP} produced by the prism are balanced by the triplet to make the whole system both aplanatic and achromatic.

Table 2. Aberration targets of the triplet in the example.

Aberration	Seidel coefficient	Wave front ^a (units of λ_d)	Normalized Seidel coefficient
Spherical	$S_1 = 0.040\ 038\ 1$	$\frac{S_1}{8\lambda_d} = 8.517\ 3$	$\tilde{S}_1 = \frac{S_1}{h^4 K^3} = 0.172\ 965$
Central coma	$S_{2C} = -0.012\ 59$	$\frac{S_{2C}}{2\lambda_d} = -10.713\ 1$	$\tilde{S}_{2C} = \frac{S_{2C}}{Hh^2 K^2} = 0.173\ 1$
Longitudinal chromatic	$C_L = 0.015\ 322\ 3$	$\frac{C_L}{2\lambda_d} = 13.038$	$\tilde{C}_L = \frac{C_L}{h^2 K} = 0.001\ 839\ 6$
Secondary chromatic	$C_{LP} = 0.010\ 684\ 1$	$\frac{C_{LP}}{2\lambda_d} = 9.091\ 3$	$\tilde{C}_{LP} = \frac{C_{LP}}{h^2 K} = 0.001\ 283$

^a $\lambda_d = 0.000\ 587\ 6\ \text{mm}$.

The solving process is shown in table 3. It can be seen that the calculation process is straightforward. There is a (+ + -) power distribution and two solutions. However, solution 2 is not realizable because the third and fifth surface radii are too small to satisfy the aperture diameter ($2h = 100\text{mm}$) requirement. The thin-lens design is now finished.

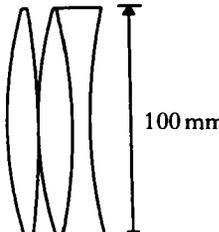
4. Discussion

A triplet comprising a singlet and a cemented doublet is an important lens type and is widely used as a component in many optical systems, for example telescope objective, telephoto, zoom and Petzval lens [7]. It has some advantages over cemented triplets. First, it is sufficient for satisfying five targets rather than four targets. Second, it usually offers smoother surfaces to reduce higher-order aberrations and therefore supports higher speed. Hence, a simple method for solving this triplet type is a useful tool.

In this paper, an algorithm is proposed for solving this triplet type with desired lens power, primary spherical aberration, coma, longitudinal chromatic aberration and secondary spectrum. One might consider obtaining the pre-design by a guess. This may work well for an experienced designer but it is usually difficult for a beginner. As a comparison, the proposed algorithm is a simple algebraic process

Table 3. Solving process for the example. The surface radii r_3 and r_5 of solution 2 are too small to satisfy the aperture diameter requirement.

Values		Note
$K = 0.00333, \bar{u} = 0, \bar{u}' = -1$		See equation (7)
$\bar{K}_1 = 1.41592, \bar{K}_3 = 1.56093, \bar{K}_4 = -1.97685$		By solving equations (8)–(10)
$\bar{u}_2 = -1.41592, \bar{K}_D = -0.41592, \rho = -8.50587$		See equations (11) and (23)
$Y_1 = 1, Y_D = -5.8086$		See equation (12)
$g_1 = 5.86407, g_2 = -8.88428, g_3 = 8.99288,$ $g_4 = -3.13728, g_5 = 2.66063, g_6 = -0.646057,$ $g_7 = -2.27073, g_8 = -4.14461, g_9 = 0.530989,$ $g_{10} = 0.834676$		See equations (19) and (27)
$w_1 = 4.6753, w_2 = 3.32218$		See equations (31) and (32)
$m_2 = -4.70544, m_1 = -16.4607, m_0 = 18.1405$		See equations (34) and (35)

Solution 1	Solution 2	Note
$\bar{A}_4 = -4.37867$	$\bar{A}_4 = 0.880453$	By solving equation (34)
$X_1 = 0.317841$	$X_1 = 1.20796$	See equation (33)
$\bar{A}_3 = 0.39035$	$\bar{A}_3 = 5.64947$	See equation (37)
$\bar{c}_1 = 1.76516$	$\bar{c}_1 = 2.95742$	See equation (38)
$\bar{c}_2 = -0.913709$	$\bar{c}_2 = 0.278544$	
$\bar{c}_3 = 1.80627$	$\bar{c}_3 = 7.06539$	
$\bar{c}_4 = -1.40182$	$\bar{c}_4 = 3.8573$	
$\bar{c}_5 = 1.43551$	$\bar{c}_5 = 6.69463$	
$r_1 = 169.96$	$r_1 = 101.44$	Non-normalized radius
$r_2 = -328.33$	$r_2 = 1077.03$	
$r_3 = 166.09$	$r_3 = 42.46$	
$r_4 = -214.01$	$r_4 = 77.77$	
$r_5 = 208.99$	$r_5 = 44.81$	
		Schematic diagram of the thin triplet
	Unreasonable solution	

that is quick and guarantees to find all the solutions. It is of particular use in setting up a preliminary system for subsequent optimization.

We have written a computer program and set constraints in it to restrict the acceptable surface curvatures so that only those solutions within the restriction will be adopted. One example of thin-lens design of an apochromatic telescope objective system has been demonstrated.

The algorithm for solving the kind of triplet which consists of a singlet and a cemented doublet with a finite air space between them and gives a required

distance of principal planes to adjust the length of optical system, for instance zoom lens and telephoto lens, and will give more freedom to find more desirable solutions, will be considered in the future.

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