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WAVES GENERATED BY AN INCLINED-PLATE WAVE GENERATOR

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SUMMARY

This paper describes the characteristics of small-amplitude waves generated by a sinusoidally oscillating, inclined paddle-type wavemaker operating in a constant-depth channel. Two-dimensional, linearized potential flow is assumed. A semi-analytical method, the boundary collocation method, is used to establish the relationship between wave amplitude and paddle stroke. The numerical results are compared with the numerical results of the boundary integral equation method. It is found that the boundary collocation method is simpler and more flexible to implement and faster to compute. In addition, the numerical results are in reasonably good agreement with the laboratory experimental data. For the vertical wavemaker, which is a special case of the inclined wavemaker, an analytical series solution can be found. By using the boundary collocation method and the boundary integral equation method to solve the vertical wavemaker problem and comparing the results with the analytical series solution, it is found that the boundary collocation method yields a solution which is much more accurate than that from the boundary integral equation method. Finally, the relationships between wave amplitude and paddle stroke are established for different inclinations of the paddle-type wavemaker, based on the boundary collocation method.

KEY WORDS Wave Generator Boundary Collocation Method

INTRODUCTION

Numerous investigators have studied analytically the wave generating characteristics of vertical wave generators. Most of them have concentrated on piston-type wavemakers.[1–3] Fontanet[4] developed a general second-order theory for the waves generated by a sinusoidally moving wavemaker with horizontal as well as rotatory motion. However, his solution is very complicated and difficult to use in practice. Hyun[5] considered a paddle-type wavemaker of finite draft. The second-order theory of the paddle-type wavemaker as well as the piston-type wavemaker was presented by Flick and Guza.[6] Gilbert *et al.*[7] investigated the characteristics of waves generated by a moving wedge. Hammark[8] studied the waves generated by the motion of a section of the bottom of the ocean of constant depth. In addition, Tuck and Hwang[9] conducted a theoretical study of long waves generated by the impulsive motion of a continuously sloping beach. Unfortunately, none of the analytical methods used by the above investigators to solve the vertical wavemaker theory can be used to establish the inclined wavemaker theory. The first paper discussing the inclined wavemaker theory was presented by Raichlen and Lee[10] in 1978. They used the boundary integral equation method (BIEM) to study the characteristics of waves generated by an inclined paddle-type wavemaker.

The inclined paddle-type wavemaker discussed in this paper, which is exactly the same as that of Raichlen and Lee,[10] consists of a plate with one hinged edge attached to the bottom and

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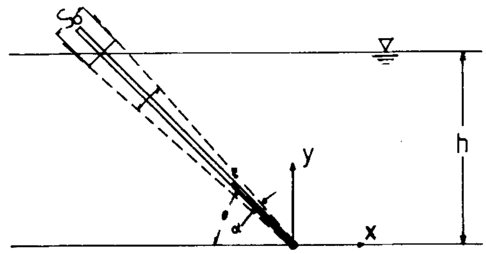

Figure 1. Definition sketch for theoretical analysis

mounted, in its rest position, at a given angle in a constant-depth wave tank; the plate is moved about this position with a periodic motion as shown in Figure 1. The inclined paddle-type wavemaker theory cannot be developed by the separation of variables that was used to develop the vertical wavemaker theory. Raichlen and Lee[10] used the BIEM to examine the waves generated by the inclined paddle-type wavemaker. The BIEM is a purely numerical method; in order to obtain accurate solutions, the basic requirement is that the length of each boundary element must be small. For small elements the number of node points must be increased: therefore more computer storage capacity and computer time are needed, and the round-off error will be increased. The other drawback of using the BIEM to solve the inclined wavemaker theory is that the treatment of the 'corner point' above the plate where the water depth goes to zero may induce instabilities in the numerical scheme and needs special consideration, which will reduce the accuracy.

The inclined wavemaker theory can be applied to investigate the waves generated by a landslide in a reservoir as well as the wave transmission characteristics of an offshore mobile breakwater constructed from a group of moored, partly sunk barges, each barge being moored in an inclined position with one end resting on the bottom and the other end near the free surface pointing in the direction of wave incidence.

FORMULATION OF THE PROBLEM

The usual assumptions of classical hydrodynamics are introduced, i.e. the fluid is inviscid, incompressible and of uniform density and the flow is irrotational. Therefore a velocity potential $\Phi(x, y, t)$ exists which must satisfy the Laplace equation

$$\Phi_{xx} + \Phi_{yy} = 0. \tag{1}$$

For the periodic motions of waves generated by the periodic oscillations of an inclined wavemaker, the velocity potential Φ can be expressed in separable form as

$$\Phi(x, y, t) = \mathrm{Re}[\phi(x, y)\mathrm{e}^{-\mathrm{i}\sigma t}], \tag{2}$$

where x is the horizontal direction and y the depthwise direction as shown in Figure 1, $\mathrm{i} = \sqrt{-1}$ and σ is the circular frequency defined as $2\pi/T$, with T the wave period. As a matter of convenience, the complex form will be used in what follows. Substituting equation (2) into equation (1) yields

$$\phi_{xx} + \phi_{yy} = 0. \tag{3}$$

Besides the Laplace equation, the potential function $\phi(x, y)$ needs to satisfy the following boundary conditions.

The bottom boundary condition (BBC):

$$\phi_y = 0 \quad \text{on} \quad y = 0. \tag{4}$$

The kinematic boundary condition (KBC) on the wavemaker:

$$\phi_{\mathbf{n}} = -i\sigma\xi\alpha. \tag{5}$$

The combined free surface boundary condition (CFSBC):

$$\phi_y - (\sigma^2/g)\phi = 0 \quad \text{on} \quad y = h. \tag{6}$$

The radiation condition:

$$\phi_x = ik\phi \quad \text{at far field.} \tag{7}$$

In the above equations, $\mathbf{n}$ is the unit vector normal to the wavemaker, ξ is the distance from the hinge point to the position (x, y) at the paddle surface, α is the maximum angular displacement of the paddle and k is the wave number.

Transforming the equation for the KBC on the wavemaker to the x–y co-ordinate system gives

$$\phi_x + \phi_y \cot\theta = -i\sigma\frac{\alpha}{\sin^2\theta}y \quad \text{on} \quad x = -y\cot\theta. \tag{8}$$

The above equations govern the boundary value problem for waves generated by an inclined paddle-type wavemaker.

SOLUTION BY THE BOUNDARY COLLOCATION METHOD

If we ignore the KBC on the wavemaker for the time being equations (3), (4), (6) and (7) are the equations governing waves propagating in the positive x-direction in a uniform-depth channel. These have the solution[11]

$$\phi = A_0 \cosh(k_0 y) e^{ik_0 x} + \sum_{n=1}^{\infty} A_n \cos(k_n y) e^{-k_n x}, \tag{9}$$

with the dispersion relations

$$\sigma^2 = gk_0 \tanh(k_0 h), \tag{10}$$

$$\sigma^2 = -gk_n \tan(k_n h), \quad n = 1, 2, \ldots, \tag{11}$$

where k_0 is the wave number of the progressive wave and k_n are the wave numbers of the standing waves. A_0 and A_n are unknown coefficients which must be determined by using the KBC on the wavemaker. Substituting equation (9) into equation (8), the KBC on the wavemaker, yields

$$-A_0 k_0 [i\cosh(k_0 y) + \sinh(k_0 y)\cot\theta] e^{-ik_0 y\cot\theta} + \sum_{n=1}^{\infty} A_n k_n [\cos(k_n y) + \sin(k_n y)\cot\theta] e^{k_n y\cot\theta} = i\frac{\alpha}{\sin^2\theta}y. \tag{12}$$

Unfortunately this does not form a Sturm–Liouville problem, except for $\theta = \pi/2$, and A_0 and A_n cannot be determined by using the orthogonal functions. In the following an approximate method, the boundary collocation method (BCM), will be introduced to determine the coefficients A_0 and A_n.

In the standard BCM[12] one proposes functions which identically satisfy the governing equation and approximately satisfy the remaining boundary conditions; it is a purely numerical method. The BCM defined in this paper is somewhat different in that it is a semi-analytical method, since equation (9) is an analytical solution for the governing Laplace equation and the three boundary conditions (the BBC, the CFSBC and the radiation condition). In order to solve this problem completely, the BCM is introduced to estimate the coefficients A_0 and A_n approximately. Therefore the BCM is called a semi-analytical method.

To estimate A_0 and A_n, equation (12) is modified to

$$-\bar{A}_0 k_0 h\{\mathrm{i}\cosh[k_0 h(y/h)] + \sinh[k_0 h(y/h)]\cot\theta\} \mathrm{e}^{-\mathrm{i}k_0 h(y/h)\cot\theta} + \sum_{n=1}^{\infty} \bar{A}_n k_n h\{\cos[k_n h(y/h)] + \sin[k_n h(y/h)]\cot\theta\} \mathrm{e}^{k_n h(y/h)\cot\theta} = \frac{\mathrm{i}}{2\sin\theta}\frac{y}{h}, \tag{13}$$

where

$$\bar{A}_0 = A_0/\sigma S_0 h, \qquad \bar{A}_n = A_n/\sigma S_0 h.$$

The BCM requires that $\bar{A}_0$ and $\bar{A}_n$ satisfy the KBC on the wavemaker approximately. In order to do this, the inclined paddle is divided into $M-1$ elements as shown in Figure 2. Each element has two node points; therefore there will be M node points on the paddle. Equation (13) must be satisfied at all node points. If M approaches infinity, the solution is an exact one; if M is a finite number, only an approximate solution can be obtained.

The discrete form of equation (13) is

$$-\bar{A}_0 k_0 h[\mathrm{i}\cosh[k_0 h(y/h)_m] + \sinh[k_0 h(y/h)_m]\cot\theta\} \mathrm{e}^{-\mathrm{i}k_0 h(y/h)_m\cot\theta} + \sum_{n=1}^{\infty} \bar{A}_n k_n h\{\cos[k_n h(y/h)_m] + \sin[k_n h(y/h)_m]\cot\theta\} \mathrm{e}^{k_n h(y/h)_m\cot\theta} = \frac{\mathrm{i}}{2\sin\theta}\left(\frac{y}{h}\right)_m. \tag{14}$$

If five standing waves are included in this evaluation, equation (14) can be expressed in the matrix form

$$[B_{mn}]_{M\times N}[\bar{\bar{A}}_n]_{N\times 1} = [D_m]_{M\times 1}, \tag{15}$$

where $N = 6$ (one progressive wave and five standing waves) and

$$B_{m1} = -k_0 h\{\mathrm{i}\cosh[k_0 h(y/h)_m] + \sinh[k_0 h(y/h)_m]\cot\theta\} \mathrm{e}^{-\mathrm{i}k_0 h(y/h)_m\cot\theta},$$

$$B_{mn} = k_{n-1} h\{\cos[k_{n-1} h(y/h)_m] + \sin[k_{n-1} h(y/h)_m]\cot\theta\} \mathrm{e}^{k_{n-1} h(y/h)_m\cot\theta}, \quad n \neq 1,$$

$$\bar{\bar{A}}_1 = \bar{A}_0, \qquad \bar{\bar{A}}_n = \bar{A}_{n-1}, \quad n \neq 1, \qquad D_m = \frac{\mathrm{i}}{2\sin\theta}\left(\frac{y}{h}\right)_m.$$

Equation (15) can be solved by the least-squares method, i.e.

$$[\bar{\bar{B}}_{mn}]_{N\times N}[\bar{\bar{A}}_n]_{N\times 1} = [\bar{\bar{D}}_n]_{N\times 1}, \tag{16}$$

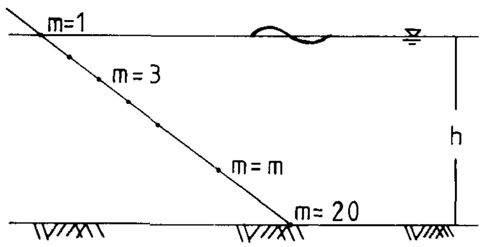

Figure 2. Elements and node points on paddle

where

$$\bar{\bar{\mathbf{B}}} = \mathbf{B}^{\mathrm{T}}\mathbf{B}, \qquad \bar{\bar{\mathbf{D}}} = \mathbf{B}^{\mathrm{T}}\mathbf{D}$$

and $\mathbf{B}^{\mathrm{T}}$ is the Hermitian transpose of matrix $\mathbf{B}$.

The standing waves will decay exponentially with distance; therefore at far field the wave potential function is

$$\Phi = A_0 \cosh(k_0 y) e^{i(k_0 x - \sigma t)}. \tag{17}$$

A_0 is estimated from equation (16). From the dynamic free surface boundary condition,[11] the wave profile can be expressed as

$$\eta = \left.\frac{\Phi_t}{g}\right|_{y=h} = \frac{H}{2} e^{i(k_0 x - \sigma t)}, \tag{18}$$

where

$$\frac{H}{2} = \left| \frac{-i\sigma A_0}{g} \cosh(k_0 h) \right|.$$

Therefore the ratio of wave height to paddle stroke is

$$\frac{H}{S_0} = \left| -i2\bar{A}_1 \sinh(k_0 h) \right|. \tag{19}$$

When $\theta = 90°$, by using the orthogonal properties, the analytical series solution can be obtained and the ratio of wave height to paddle stroke is

$$\frac{H}{S_0} = 4\left(\frac{\sinh(k_0 h)}{k_0 h}\right) \frac{k_0 h \sinh(k_0 h) - \cosh(k_0 h) + 1}{\sinh(2k_0 h) + 2k_0 h}, \tag{20}$$

which can be found in several references (e.g. Biesel and Suquet[1]).

IMPLEMENTATION AND NUMERICAL RESULTS

In this section the numerical results from the BCM, obtained by solving equation (16) with $N = 6$ and $M = 20$, and from the BIEM are presented and compared. These solutions are also compared with the experimental data.

The theoretical series solution, equation (20), and the numerical results computed by the BCM and the BIEM[10] for $\theta = 90°$ are presented in Figure 3. The ordinate is the ratio of the wave height to the total stroke of the wavemaker and the abscissa is the ratio of the depth to the wave length in the constant-depth region. The stroke is the maximum excursion of the paddle measured perpendicular to the plate at the plate/water surface intersection with the generator in its at-rest position. The wave length used is determined from small-amplitude water wave theory for the given depth in the constant-depth region and the measured wave period. Figure 3 shows that the numerical solution using the BCM is very close to the analytical solution and is somewhat better than that of the BIEM when $\theta = 90°$. The BIEM tends to overestimate the wave height at low frequency and high frequency but underestimate the wave height at intermediate frequency. The other curves in Figure 3 are a comparison of the numerical solutions of the BCM and the BIEM for $\theta = 45°$. The comparisons for $\theta = 33{\cdot}85°$ and $34{\cdot}5°$ are shown in Figure 4, while Figure 5 shows the comparisons of the numerical results from the BCM and the BIEM with the experimental data for $\theta = 45°$ and $34{\cdot}5°$. It is seen that there are some discrepancies between the numerical solutions and the measured data, possibly as a result of energy dissipation and energy

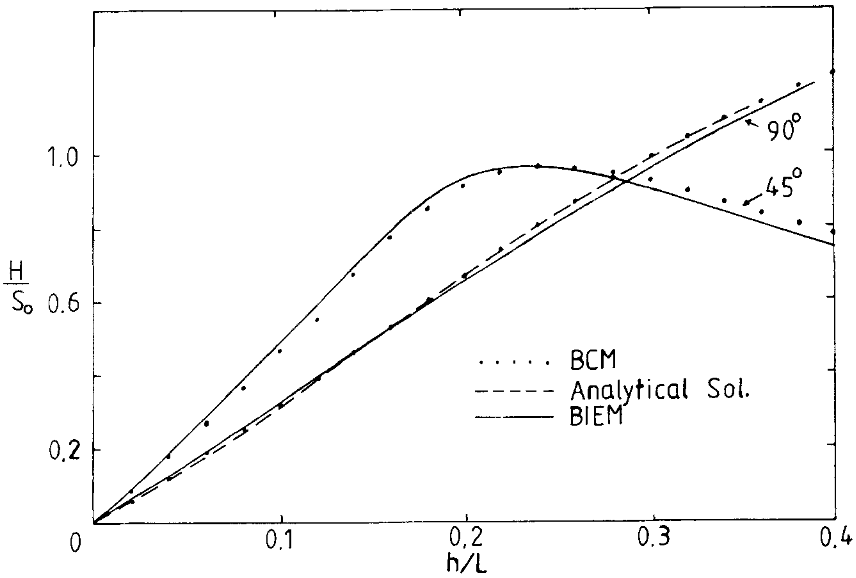

Figure 3. Waves generated by an oscillating inclined wave generator with $\theta = 90°$ and 45°

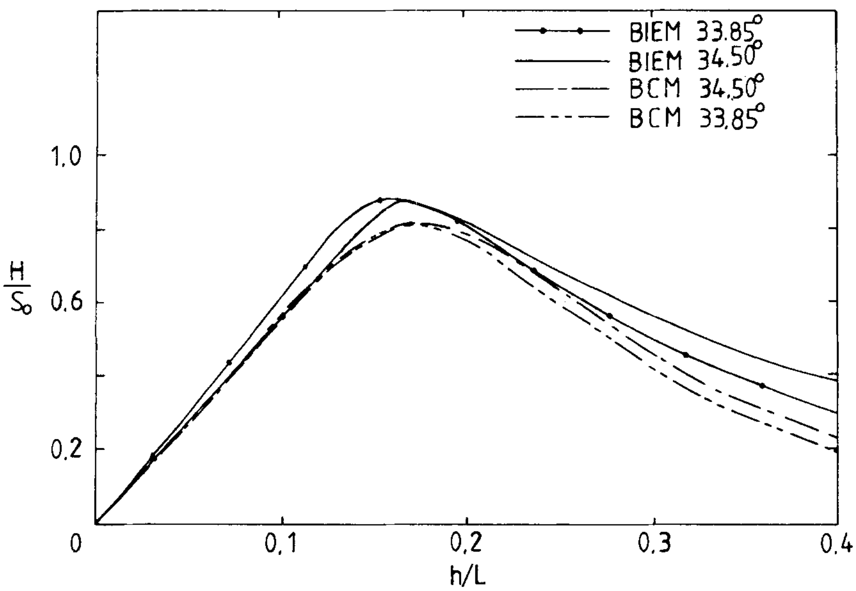

Figure 4. Waves generated by an oscillating inclined wave generator with $\theta = 33{\cdot}85°$ and 34·5°

leakage around the sides of the wave generator. The discrepancies are more serious for higher waves, because the energy dissipation and energy leakage are proportional to the wave height.[3] But solutions solved using the BIEM and the BCM at least have the same trend as the experimental results. For θ larger than 90°, as in Figure 6, both methods obtain solutions which are reasonably close in the region of shallow water, but very different solutions are found in the

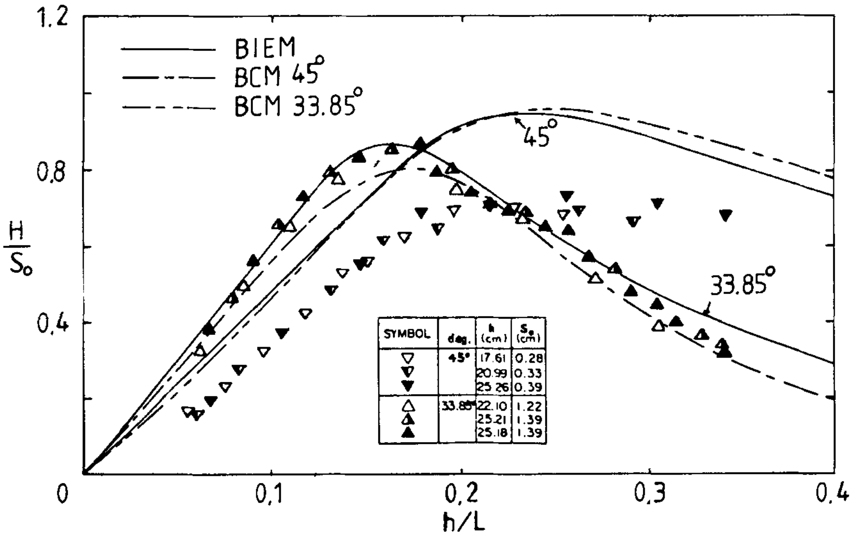

Figure 5. A comparison of experimental and numerical results for waves generated by an oscillating inclined wave generator

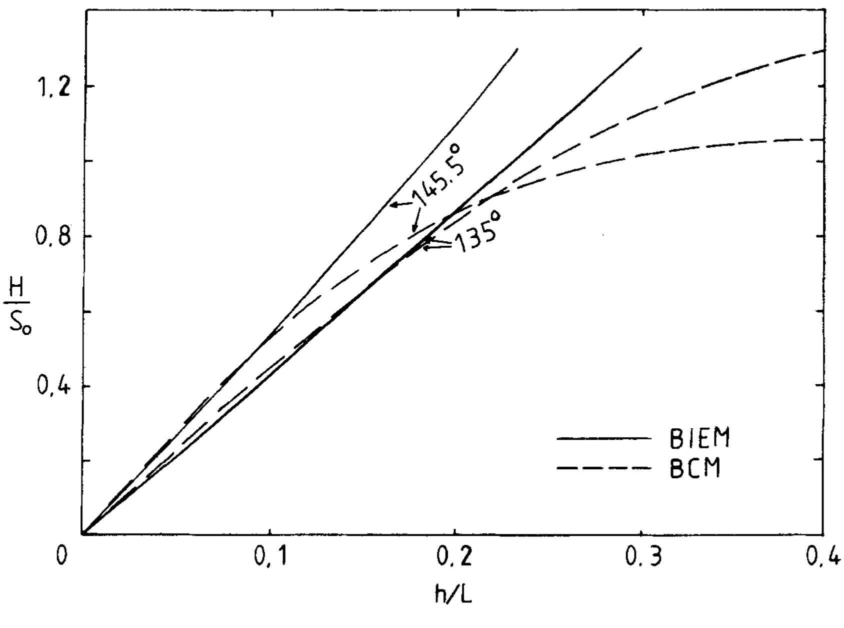

Figure 6. Waves generated by an oscillating inclined wave generator with $\theta = 135°$ and $145{\cdot}5°$

region of nearly deep water. In the nearly deep water region, solutions solved by BIEM tend to diverge but solutions by BCM tend to converge. Since the value H/S_0 must be bounded for any ratio of depth to wave length the BIEM solutions are in doubt. The divergent solutions of BIEM may be due to the inappropriate treatment of the 'corner point'[10] or due to the inappropriate artificial position of the radiative boundary. Finally, the numerical solutions for different inclinations are solved by using BCM and are shown in Figures 7 and 8.

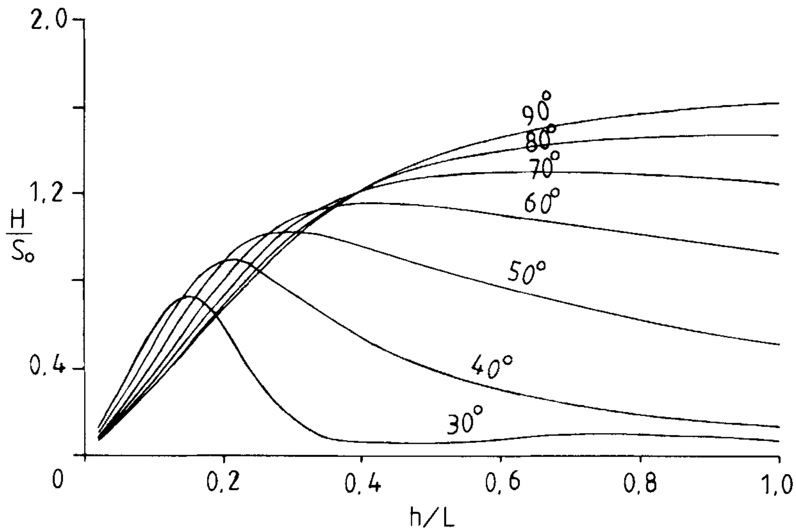

Figure 7. Waves generated by an oscillating inclined wave generator with inclinations less than 90°

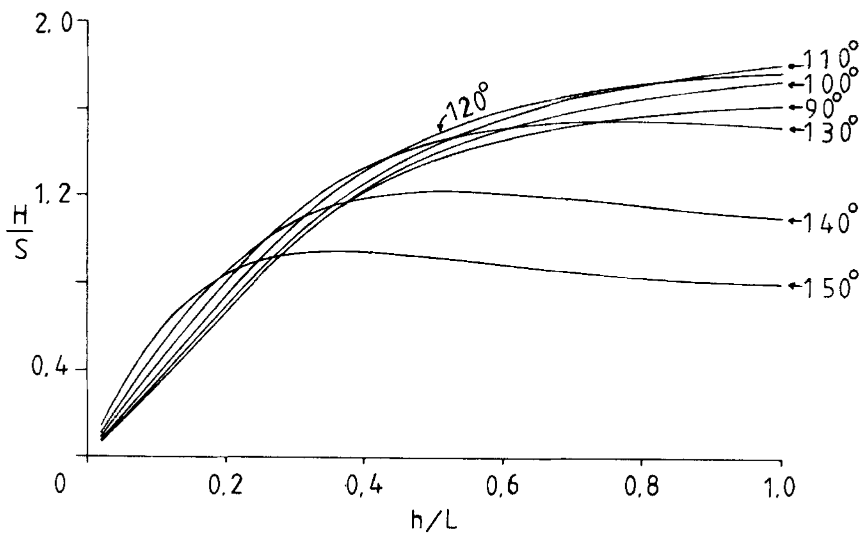

Figure 8. Waves generated by an oscillating inclined wave generator with inclinations greater than 90°

CONCLUSIONS

The BCM, modified to become a semi-analytical method, has been developed to examine the inclined wavemaker theory. In general, this method is much simpler than the BIEM and take less computational time. Also, the BCM provides a reasonable means of estimating the wave characteristics generated by an inclined wavemaker. The numerical solutions computed by the BCM are shown to be more accurate than those computed by the BIEM.

NOMENCLATURE

g	gravitational acceleration
h	water depth
H	wave height
k	wave number
k_0	wave number of progressive wave
k_n	wave number of standing waves
L	wave length
S	paddle stroke
S_0	maximum paddle stroke
T	wave period
α	maximum angular displacement of paddle
η	water surface elevation
θ	inclination of wavemaker
ξ	distance from hinge to position (x, y) on paddle
σ	radial frequency
Φ	velocity potential

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