國 立 交 通 大 學

統計學研究所

碩 士 論 文

考慮非中心卡方製程的製程平均發生偏移下之製程能力調整

Capability Adjustment for Non-Central Chi-Squared Processes with Mean Shift Consideration

研 究 生： 侯宏興

指導教授: 洪慧念 博士

彭文理 博士

中華民國九十九年六月

考慮非中心卡方製程的製程平均發生偏移下之製程能力調整

Capability Adjustment for Non-Central Chi-Squared Processes with Mean Shift Consideration

研 究 生：侯宏興　　Student : Hung-Hsing Hou

指導教授：洪慧念　教授　　Advisor : Dr. Hui-Nien Hung

彭文理　教授　　Dr. W. L. Pearn

國 立 交 通 大 學

統計學研究所

碩 士 論 文

A Thesis

Submitted to Institute of Statistics

College of Science

National Chiao Tung University

In partial Fullfillment of the Requirement

For the Degree of

Master

in

Statistics

June 2010

Hsinchu, Taiwan, Republic of China

中華民國九十九年六月

考慮非中心卡方製程的製程平均發生偏移下之製程能力調整

研究生：侯宏興　　　　指導教授：洪慧念 博士

彭文理 博士

國立交通大學統計學研究所碩士班

摘要

製程能力指標被用來衡量製程製造產品符合規格的能力，不僅是提供品質保證的工具，也是在品質改善方面的一個方針。不過，自從Motorola公司在1980年代提出6個標準差的觀念後，很多統計學家質疑提倡6個標準差的學者，為什麼在衡量製程能力時需要對製程平均做1.5倍的標準差調整。Bothe (2002) 針對這個問題，利用管制圖的機制來偵測製程平均發生偏移的情況，發現它隨著不同的抽樣個數可以有不同的調整量，可是Bothe 的研究是在常態分配的假設之下，事實上，非常態分配製程在業界是較常發生的。過去的研究也有針對了非常態分配（伽瑪、韋伯、對數常態分配）的調整。所以我們針對非常態非中心卡方方配做詳細的分析，導出在不同非常態分配下應調整的偏移量，並針對非常態分配適用的 C_{pk} 指標做調整。在本研究的最後，以實例來說明如何在非常態分配製程的情況下，在考慮製程平均發生變動的情況下，如何調整製程能力指標 C_{pk} 。

關鍵字：非常態、非中心卡方分配、製程偏移、製程能力指標

Capability Adjustment for Non-Central Chi-Squared Processes with Mean Shift Consideration

Student : Hung-Hsing Hou **Advisor : Dr. Hui-Nien Hung**
Dr. W. L. Pearn

Institute of Statistics
National Chiao Tung University

Abstract

Process capability indices have been proposed in the manufacturing industry to prove numerical measures on process reproduction capability, which are effective tools for quality assurance and guidance for process improvement. Motorola, Inc. introduced its Six Sigma quality initiative to the world in the 1980s. Some quality practitioners questioned why the Six Sigma advocates claim it is necessary to add a 1.5σ shift to the process mean when estimating process capability. Bothe (2002) provides a statistical reason for including such a shift in the process average that is base on the chart's subgroup size. Data in Bothe' study was assumed to be approximately normally distributed, but the process output is usually not from approximately normally. Some research is about the PCIs adjustment for process output has a non-normal distribution. This paper investigates the average run length of non-normal distribution, non-central chi-squared distribution, and calculate the mean shift adjustments and addresses this problem computing reliable estimates for capability index C_{pk} for non-central chi-squared process when the statistically adjustments is considered. For illustration purpose, an application example is presented.

Keyword : Process capability index , Dynamic C_{pk} , Mean shift , Non-central chi-squared distribution

誌　謝

轉眼間，兩年的碩士求學生涯就這樣畫下了句點。本篇論文能順利完成，首先要感謝我的指導教授：洪慧念教授與彭文理教授。謝謝老師們在研究過程中適時的給予建議並指引至正確的方向上，使我在這過程中獲益匪淺，沒有老師們的耐心指導與大力支持外，我可能無法順利完成；除了課業上的指導外，老師們也教導我了一些做事情的方法、態度等，讓我成長不少。同時，也要感謝口試委員提供的意見與指導，使本論文更加完整、充實。

再來要感謝身旁的同學們，謝謝你們在課業上的幫忙與生活上的協助，讓身處異地的我備感溫馨，能與你們當朋友真是我的福氣，我會好好的記得你們的！

最後，我要謝謝我最愛的家人，感謝你們一路的支持與陪伴，使我在求學的過程中無後顧之憂；特別要感謝我的媽媽，求學生涯若沒有你的關懷與付出，我將無法順利完成學業，媽媽辛苦了。

在此，將此篇論文獻給我的師長、家人和朋友們，致上我最誠摯的謝意。

侯 宏 興　謹至于

國 立 交 通 大 學 統 計 學 研 究 所

中 華 民 國 九十九 年 六 月

Contents

List of Tables

List of Figures

1 Introduction

Process capability indices (PCIs) which provide numerical measure of production characteristic to reflect the quality of product have been used in the manufacturing industry. Those indices have become popular as unit-less measures on process potential and performance. The most commonly used ones, C_p and C_{pk} discussed in Kane (1986), and more advanced indices C_{pm} and C_{pmk} developed by Chan et al. (1988) and Pearn et al. (1992). Many authors have promoted the use of various PCIs for evaluating a supplier's process capability. Based on analyzing the PCIs, a production department can trace and improve a poor process so that the quality level can be enhanced and the requirements of the customers can be satisfied. These PCIs have been defined explicitly as:

$$C_p = \frac{USL - LSL}{6\sigma}, \quad C_{pk} = \min\{\frac{USL-\mu}{3\sigma}, \frac{\mu - LSL}{3\sigma}\}, \quad C_{pm} = \frac{USL - LSL}{6\sqrt{\sigma^2 + (\mu - T)^2}},$$

$$C_{pmk} = \min\{\frac{USL-\mu}{3\sqrt{\sigma^2+(\mu-T)^2}}, \frac{\mu - LSL}{3\sqrt{\sigma^2+(\mu-T)^2}}\},$$

where USL is the upper specification limit, LSL is the lower specification limit, μ is the process mean, σ is the process standard deviation (overall process variation), and T is the target value. The index C_p considers the overall process variability relative to the manufacturing tolerance, reflecting product quality consistency. The index C_{pk} takes the magnitude of process variance as well as process departure from target value, and has been regarded as a yield-based index since it providing lower bounds on process yield. The index C_{pm} emphasizes on measuring the ability of the process to cluster around the target, which therefore reflects the degrees of process targeting (centering). Since the design of C_{pm} is based on the average process loss relative to the manufacturing tolerance, the index C_{pm} provides an upper bound on the average process loss, which has been alternatively called the Taguchi index. The index C_{pmk} is constructed by appropriately combining the yield-based index C_{pk} and the loss-based index C_{pm}, accounting for the process yield as well as the process loss.

Since Motorola, Inc. introduced its Six Sigma quality initiative in the 1980s, quality practitioners have questioned why the followers of this initiative have added a 1.5σ shift to the process mean when estimating process capability. The advocates of Six Sigma have claimed that such an adjustment is necessary, but they have offered only personal experiences and three dated empirical studies as justification for this claim (see Bender (1975); Evans (1975); Gilson (1951)). By examining the sensitivity of control charts to detect changes of various magnitudes, Bothe (2002) provided a statistically based reason for this claim. In his study, Bothe assumed that the process data is approximately normally distributed. However, non-normal processes occur frequently, in particular, in the semiconductor industry. Pyzdek (1992) mentioned that the distributions of certain chemical processes, such as zinc plating in a hot-dip galvanizing process, are very often skewed. Choi et al. (1996) presented an example of a skewed distribution in the "active area" shaping stage of the wafer's production processes. The abundance of outputs from skewed distributions, the stratification, tec., makes the normality assumption often unreasonable. The non-central chi-square distribution plays an important role in communications, for example in the analysis of mobile and wireless communication systems. It not only includes the important cases of a squared Rayleigh distribution and a squared Rice distribution, but also the generalizations to a sum of independent squared Gaussian random variables of identical variance with or without mean, i.e., a "squared MIMO Rayleigh" and "squared MIMO Rice" distribution. Therefore, a non-central chi-square process for data analysis has been chosen for this study. Moreover, if the capability indices based on the normal assumption concerning the data are used to deal with non-normal observations, the values of the capability indices may, in a majority of situations, be incorrect and quite likely misrepresent the actual product quality.

The control charts are commonly used in many industries for providing early warning for the shift in the process mean. If the control chart detects a process mean shift, then the process is not under control. However, for momentary process mean shifts, it may be beyond the control chart detection power. Consequently, the undetected shifts may result in overestimating process capability. If the process mean shifts are not detected, then unadjusted C_{pk} would overestimate the actual process yield. Bothe (2002) provided a statistical reason for considering such a shift in the process mean for normal processes. However, if the capability indices are based on the assumption of a normal distribution of data but are used to deal with non-normal observations, the values of the capability indices may, in the majority of situations, misrepresent actual product quality.

This paper is organized as follows. We first introduce the characteristic of non-central chi-squared distribution in Section 2. In Section 3, we examine Bothe's approach and finds that the detection power of the control chart is less than 0.5 when data comes from non-central chi-squared distribution. This shows that Bothe's adjustments are inadequate when we have non-central chi-squared processes. Therefore, we calculate the adjustments under various subgroup sizes (n) and non-central chi-square parameters λ with a fixed detection power of 0.5. Further, we provide the adjusted process capability formula to accommodate the undetected shifts when data is non-central chi-squared distribution. As a result, our adjustments provide significantly more accurate calculations of the capability of non-central chi-squared processes. In Section 4, we apply our method to asset of real data to illustrate the applicability of the process capability index. Finally, we conclude the paper with a brief summary in Section 5.

2 The Non-Central Chi-Squared Process

All of us know that the case of non-normal processes occurs frequently in practice, for example, in the semiconductor industry. Pyzdek (1992) pointed out the skewed distributions that are bounded on one side occur frequently in industry and gave several examples, such as a shearing process and a chemical dip process. The abundance of outputs from skewed distributions makes the normality assumption often unreasonable. The non-central chi-square distribution plays an important role in communications, for example in the analysis of mobile and wireless communication systems. It not only includes the important cases of a squared Rayleigh distribution and a squared Rice distribution, but also the generalizations to a sum of independent squared Gaussian random variables of identical variance with or without mean, i.e., a "squared MIMO Rayleigh" and "squared MIMO Rice" distribution. A non-central chi-squared distribution, with varied λ values, covers a wide class of non-normal applications. Therefore, a non-central chi-squared process for data analysis has been chosen for this study. The difference between normal and non-central chi-squared distributions is compared in Section 2.1. And the statistical property of non-central chi-squared distribution is discussed in Section 2.2.

2.1 The Non-Central Chi-Squared Distribution

In this section, we investigate the non-central chi-squared distribution to study the effect on the detection power of the control chart. Observations from the non-central chi-squared distribution are non-negative. The non-central chi-squared distribution can be denoted as $\chi^2_\nu(\lambda)$ with the probability density function given by Chou et al. (1984) as follows:

$$f(x)=\frac{\sqrt{2\pi}}{2^{\frac{-(\nu-1)}{2}}\cdot\Gamma(\frac{\nu-1}{2})}\cdot\int_0^x y^{\frac{\nu-3}{2}}\cdot\Phi'(\sqrt{y})\cdot\left[\Phi'(\sqrt{x-y}-\sqrt{\lambda})+\Phi'(-\sqrt{x-y}-\sqrt{\lambda})\right]\cdot\frac{1}{2\sqrt{x-y}}dy\ ,$$

where x>0, ν>0, λ>0, Φ(•) is the c.d.f. of N(0,1), and the mean and variance are given, respectively, by $(\nu+\lambda)$ and $2(\nu+2\lambda)$.

Denote the family of non-central chi-squared distributions with mean $(1+\lambda)$ and degree of freedom 1 by $\chi^2(\lambda)$. The non-central chi-squared distributions are skewed. To see how this distribution is different from the standard normal distribution in terms of skewness and kurtosis, Table 2-1 presents the values of skewness and kurtosis (which are defined as the third and fourth moments of the standardized distribution, respectively) of the non-central chi-squared distributions under study. The skewness and kurtosis of $\chi^2(\lambda)$ are $\sqrt{8}(1+3\lambda)/(1+2\lambda)^{3/2}$ and 3+$12(1+4\lambda)/(1+2\lambda)^2$ respectively. We can find in Table 2-1 when the λ decreases, the corresponding values of skewness and kurtosis will become large and far away from the values of the standard normal distribution. The result through these distributions, we can get some insights of the effects of non-normality in terms of skewness and kurtosis. Fig. 2-1 presents several non-central chi-squared distributions along with a normal distribution for the same mean and variance. In this study, we let λ= 0.1, 0.5, 1, 2, 3, 5, 10, 20 and 100, when ν=1. As can be seen from Fig. 2-1 a–f, as λ increases, the non-central chi-squared distribution appears more nearly normal distribution. In fact, we demonstrate this convergence property in Table 2-1, by calculating the skewness and kurtosis. It can be seen that as λ increases, the skewness and kurtosis of non-central chi-squared distribution are very close to those of normal distribution. Through these distributions, we wish to get some insights of the effects of non-normality on the detection power in terms of skewness and kurtosis in Section 2.

Table 2-1 Values of Skewness and Kurtosis of Various

Non-Central Chi-Squared Distribution $\chi^2(\lambda)$

Distribution	Skewness	Kurtosis
N(0,1)	0	3
$\chi^2(0)$	2.828427	15
$\chi^2(0.1)$	2.797155	14.666667
$\chi^2(0.5)$	2.500000	12
$\chi^2(1)$	2.177324	9.666667
$\chi^2(2)$	1.770875	7.32
$\chi^2(3)$	1.527207	6.183673
$\chi^2(5)$	1.240441	5.082645
$\chi^2(10)$	0.911125	4.115646
$\chi^2(20)$	0.657202	3.578227
$\chi^2(100)$	0.298757	3.119106

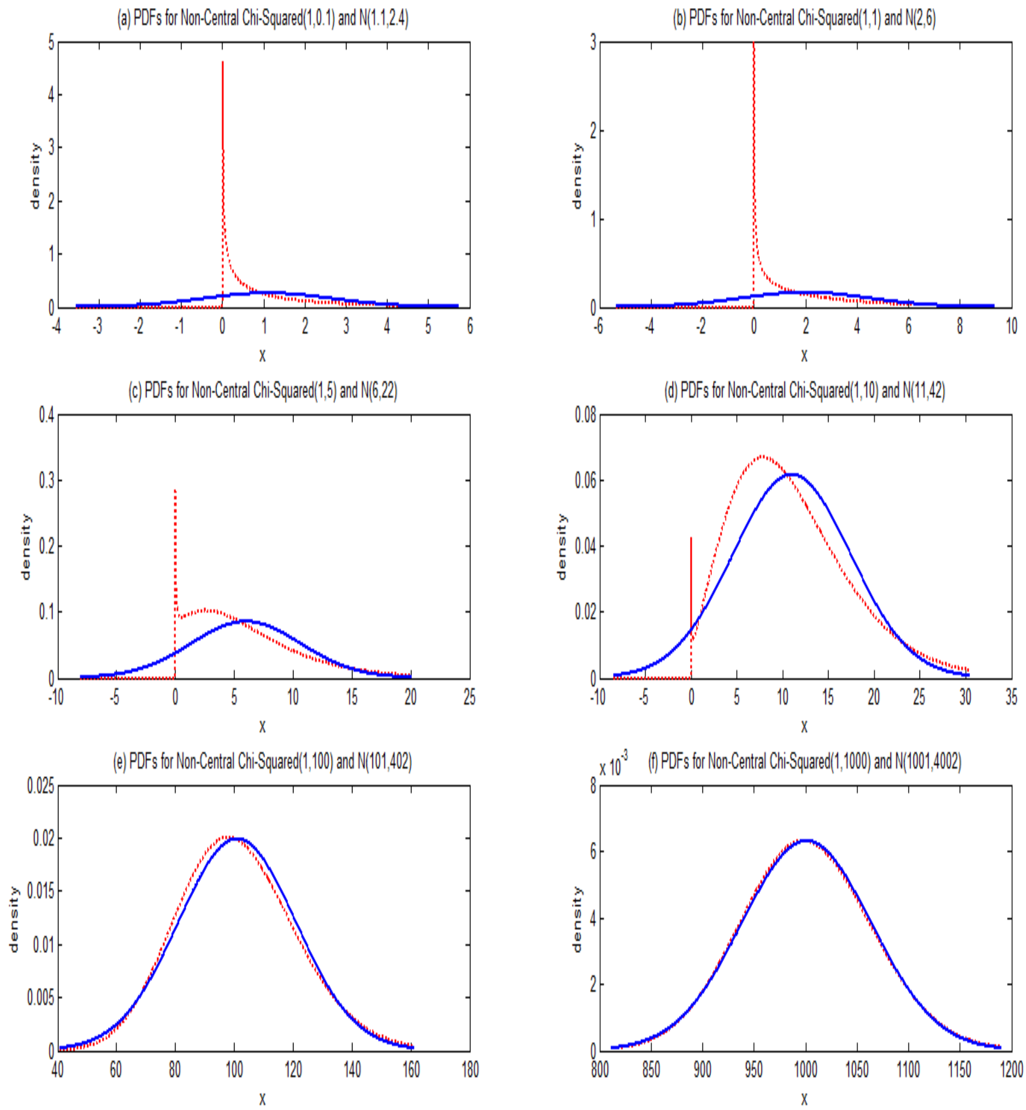

Fig. 2-1 (a) Probability density functions for $\chi^2(0.1)$ and N(1.1, 2.4). (b) Probability density functions for $\chi^2(1)$ and N(2, 6). (c) Probability density functions for $\chi^2(5)$ and N(6, 22). (d) Probability density functions for $\chi^2(10)$ and N (11, 42). (e) Probability density functions for $\chi^2(100)$ and N(102, 402). (f) Probability density functions for $\chi^2(1000)$ and N(1001, 4002).

2.2 Statistical Properties of Non-Central Squared Distribution

The non-central chi-squared distribution has a reproductive property: If X_1 and X_2 are independent random variables and each has a non-central chi-squared distribution with possible different values of ν_1, ν_2 of ν, and λ_1, λ_2 of λ, then X_1+X_2 also has a non-central chi-squared distribution, with $\nu=\nu_1+\nu_2$, and with $\lambda=\lambda_1+\lambda_2$. Applying this property, let X_1, X_2, …, X_n be a sequence of independent distribution of $\chi^2(\lambda)$ and then the distribution of $X_1+X_2+\ldots+X_n$ is $\chi_n^2(n\lambda)$. Using simply statistical technique, we can conclude that $\overline{X}_n \sim \chi_n^2(n\lambda)/n$.

The standard deviation of the $\overline{X}_n$ distribution, $\sigma_{\bar{x}}$, is calculated from its relationship to the distribution parameters and the subgroup size n as follows:

$$\sigma_{\overline{X}} = \sqrt{\frac{1}{n}(1+2\lambda)}$$

Let X_1, X_2, …, X_n be a sequence of independent distribution of $\chi^2(3)$ and we plot the probability density function of the average $\overline{X}n$ for subgroup size n=2(1)5 in Fig. 2-2 a–d. We can find that the variance of average $\overline{X}n$ will get smaller as subgroup size n increases. This situation means that the distribution of $\overline{X}n$ is more centralized when n>1. Also, Fig. 2-3 a-d presents several non-central chi-squared distributions of the average $\overline{X}n$ for subgroup size n=2(1)5 along with a normal distribution for the same mean and variance. As can be seen from Fig. 2-3 a–d as n increases, the non-central chi-squared distribution of the average $\overline{X}n$ appears more nearly normal distribution.

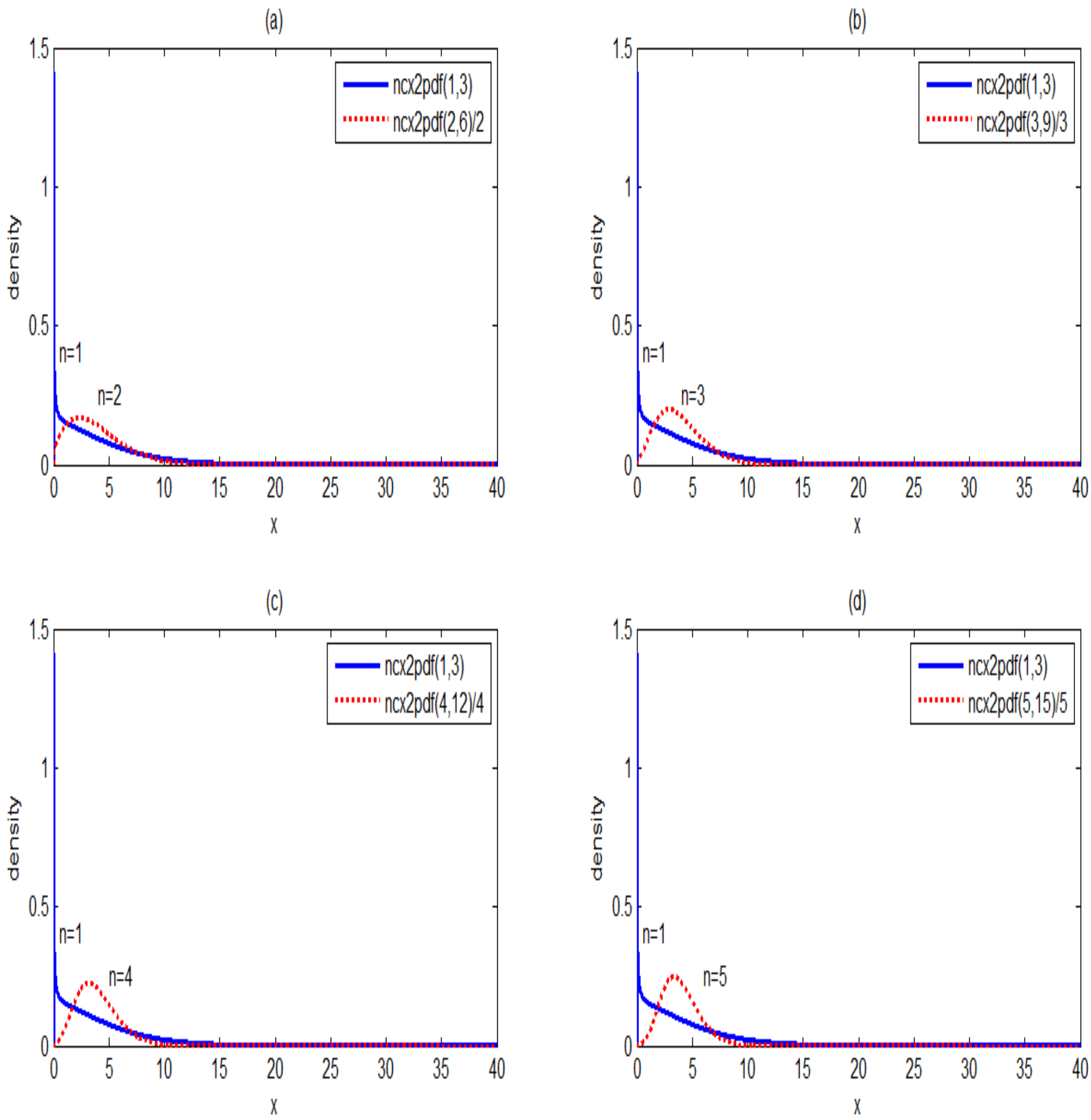

Fig. 2-2 (a) Probability density functions for $\chi^2(3)$ and $\overline{X}_n$ for n=2. (b) Probability density functions for $\chi^2(3)$ and $\overline{X}_n$ for n=3. (c) Probability density functions for $\chi^2(3)$ and $\overline{X}_n$ for n=4.(d) Probability density functions for $\chi^2(3)$ and $\overline{X}_n$ for n=5.

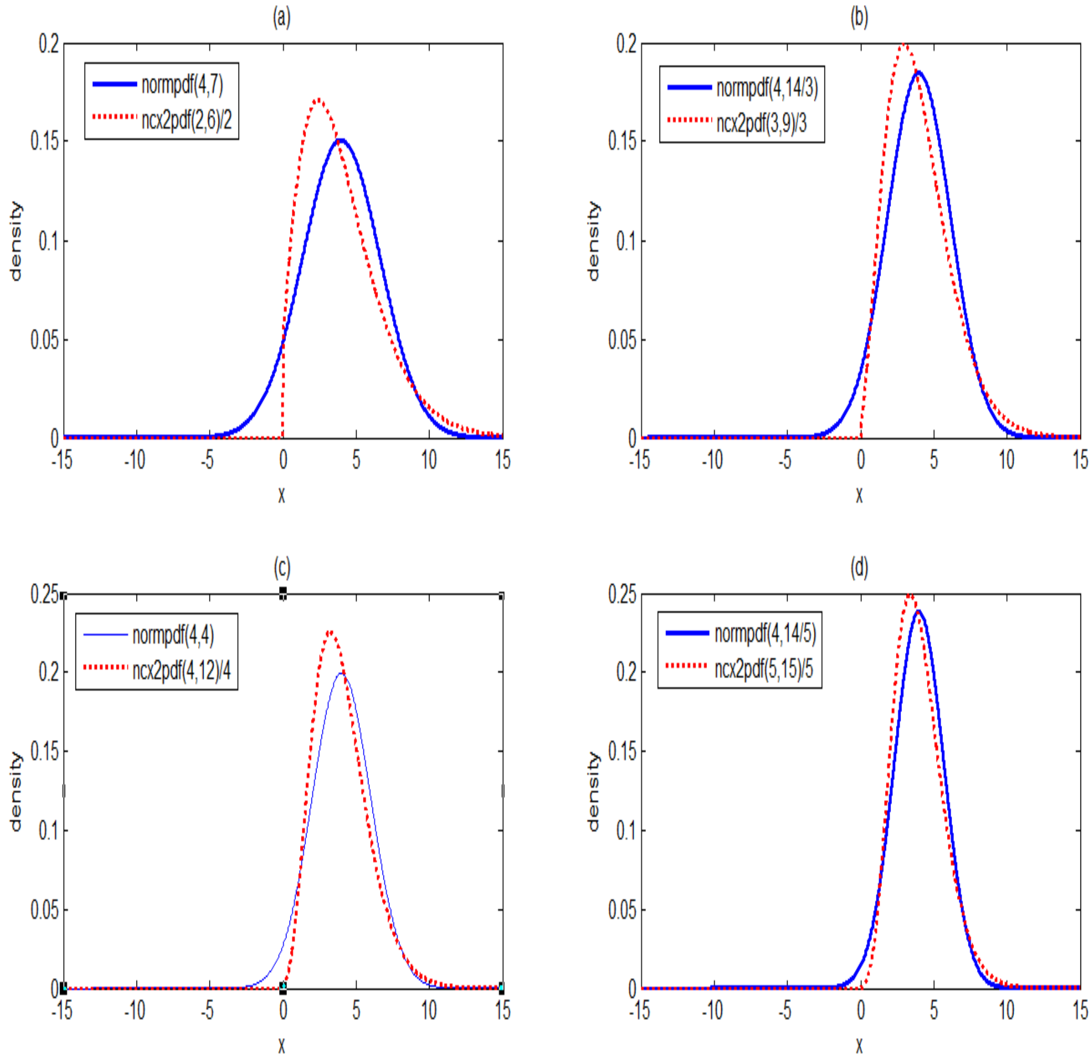

Fig. 2-3 (a) Probability density functions of $\overline{X}_n$ for n=2, $\chi^2(2,6)/2$, and N(4,7). (b) Probability density functions of $\overline{X}_n$ for n=3, $\chi^2(3,9)/3$, and N(4,14/3). (c) Probability density functions of $\overline{X}_n$ for n=4, $\chi^2(4,14)/4$, and N(4,4).(d) Probability density functions of $\overline{X}_n$ for n=5, $\chi^2(5,15)/5$,and N(4,14/5)

3 Process Mean Shift Investigation for Non-Central Chi-Squared Process

3.1 The Detection Power of Non-Central Chi-Squared Process Under Bothe's Adjustment

The major purpose of individuals control chart is assisting on identifying shifts and drifts in processes and it is easily to be implemented. But, some assumptions should be satisfied before control charts are used. The assumptions include that the process characteristics must follow normal distributions. Actually, non-normal processes occur frequently in practice. Due to above-mentioned statements, we replace the traditional, $\mu \pm 3\sigma$, to be the upper or lower control limits by the quantile of cumulative distribution function for different parameters of $\chi^2_\upsilon(\lambda)$ ($F_{0.00135,\nu,\lambda}$ and $F_{0.99865,\nu,\lambda}$) and detect the power of non-central chi-squared process under Bothe's capability adjustments.

Let $X_1, X_2, \ldots, X_n$ be a sequence observations of independent and identically distributed in $\chi^2(\lambda)$. Using the reproductive property of non-central chi-squared distribution, the mean of the observations is $\overline{X}n$ which is distributed in $\chi^2_n(n\lambda)/n$. Also, we can obtain that $\mu_{xi} = \mu_{\bar{x}} = 1+\lambda$, $\sigma_{xi} = \sqrt{2(1+2\lambda)}$ and $\sigma_{\bar{x}} = \sqrt{2(1+2\lambda)/n}$. Consequently, we derived the power of non-central chi-squared process as follows. Since the type II error β is

$$
\begin{aligned}
\beta &= P(LCL \le \overline{X}_n \le UCL \mid \mu_1 = \mu_0 + k\sigma_{X_i}) \\
&= P(F_{0.00135,n,n\lambda} - k\sigma_{X_i} \le \overline{X}_n - k\sigma_{X_i} \le F_{0.99865,n,n\lambda} - k\sigma_{X_i}) \\
&= \Phi_{(n,\lambda)}(F_{0.99865,n,n\lambda} - k\sigma_{X_i}) - \Phi_{(n,\lambda)}(F_{0.00135,n,n\lambda} - k\sigma_{X_i}),
\end{aligned}
$$

where *1-β* is the detection power of the process, $\Phi_{(n,\lambda)}(\cdot)$ is the cumulative distribution function of $\chi_n^2(n\lambda)/n$.The control limits *LCL* and *UCL* are calculated as $F_{0.00135,n,n\lambda}$ and $F_{0.99865,n,n\lambda}$, respectively. Table 3-1 presents the detection power under the alternative hypothesis test, the mean shift caused by the shift of location δ, when data comes from non-central chi-squared distribution with λ=0, 0.1, 0.5, 1(1)10, 20, 50, 100, and 700. Table 3-2 presents the detection power under the alternative hypothesis test, the mean shift caused by the shift of the parameter λ, when data comes from non-central chi-squared distribution with λ=0, 0.1, 0.5, 1(1)10, 20, 50, 100, and 700. The magnitude of shift in the second row on the left is Bothe's capability adjustments determined when data comes from normal distribution and the detection power is 0.5. From Table 3-1, we can find that the detection power is less than 0.5 when data comes from non-central chi-squared distribution under Bothe's capability adjustments. Our study shows that the detection power gets closer to 0.5 as λ increases, which is reasonable since the corresponding distributions get closer to the standard normal distribution. This is due to Bothe's (2002) approach is based on the normality assumption of the data and the detection power is 0.5. The skewness of $\chi^2(\lambda)$ is $\sqrt{8}(1+3\lambda)/(1+2\lambda)^{3/2}$. Therefore, as λ decreases the non-central chi-squared distribution is more skewed and the detection power is poorer. For example, when λ= 0.5 and the subgroup size n=2, the detection power is 0.037. It implies Bothe's adjustments are inadequate when we have skewed processes. Consequently, in our study, we determined the capability adjustment and calculation when process data comes from non-central chi-squared distribution.

Table 3-1 Detection power of various non-central chi-square processes under the shift of location

n	δ	$\chi^2(\lambda)$								
		λ=0	λ=0.1	λ=0.5	λ=1	λ=2	λ=3	λ=4	λ=5	λ=6
2	2.12	0.027	0.028	0.037	0.051	0.075	0.097	0.116	0.132	0.147
3	1.73	0.040	0.042	0.055	0.073	0.104	0.130	0.152	0.170	0.186
4	1.50	0.054	0.055	0.072	0.093	0.129	0.158	0.181	0.200	0.215
5	1.34	0.066	0.068	0.087	0.110	0.149	0.179	0.202	0.221	0.236
6	1.22	0.077	0.079	0.099	0.125	0.165	0.195	0.218	0.237	0.252
7	1.13	0.088	0.091	0.112	0.139	0.181	0.211	0.234	0.252	0.266
8	1.06	0.100	0.102	0.125	0.153	0.196	0.226	0.249	0.267	0.281
30	0.55	0.233	0.235	0.260	0.287	0.323	0.346	0.362	0.374	0.384

Table 3-1 Detection power of various non-central chi-square processes under the shift of location (**continued**)

n	δ	$\chi^2(\lambda)$								
		λ=7	λ=8	λ=9	λ=10	λ=20	λ=50	λ=100	λ=700	N(0,1)
2	2.12	0.160	0.172	0.182	0.192	0.256	0.332	0.377	0.452	0.5
3	1.73	0.200	0.212	0.222	0.232	0.293	0.360	0.398	0.460	0.5
4	1.50	0.229	0.241	0.251	0.260	0.317	0.379	0.412	0.466	0.5
5	1.34	0.250	0.261	0.271	0.280	0.333	0.389	0.420	0.468	0.5
6	1.22	0.264	0.275	0.285	0.293	0.344	0.395	0.424	0.468	0.5
7	1.13	0.279	0.289	0.298	0.306	0.354	0.403	0.429	0.470	0.5
8	1.06	0.293	0.303	0.312	0.320	0.366	0.412	0.437	0.475	0.5
30	0.55	0.391	0.398	0.403	0.408	0.434	0.459	0.472	0.493	0.5

Table 3-2 Detection power of various non-central chi-square processes under the shift of parameter

n	δ	$\chi^2(\lambda)$								
		λ=0	λ=0.1	λ=0.5	λ=1	λ=2	λ=3	λ=4	λ=5	λ=6
2	2.12	0.155	0.151	0.156	0.169	0.192	0.21	0.224	0.236	0.246
3	1.73	0.175	0.171	0.178	0.192	0.216	0.235	0.249	0.261	0.271
4	1.50	0.191	0.187	0.195	0.210	0.235	0.254	0.269	0.281	0.290
5	1.34	0.203	0.200	0.208	0.224	0.249	0.268	0.282	0.294	0.303
6	1.22	0.212	0.209	0.218	0.234	0.259	0.278	0.292	0.303	0.312
7	1.13	0.222	0.219	0.228	0.244	0.269	0.288	0.301	0.313	0.322
8	1.06	0.231	0.228	0.238	0.254	0.28	0.298	0.312	0.323	0.332
30	0.55	0.314	0.313	0.324	0.339	0.361	0.376	0.387	0.395	0.402

Table 3-2 Detection power of various non-central chi-square processes under the shift of parameter (**continued**)

n	δ	$\chi^2(\lambda)$								
		λ=7	λ=8	λ=9	λ=10	λ=20	λ=50	λ=100	λ=700	N(0,1)
2	2.12	0.254	0.262	0.269	0.275	0.315	0.365	0.396	0.455	0.5
3	1.73	0.280	0.288	0.294	0.300	0.339	0.384	0.412	0.462	0.5
4	1.50	0.299	0.306	0.313	0.318	0.355	0.397	0.423	0.468	0.5
5	1.34	0.312	0.319	0.325	0.331	0.366	0.405	0.429	0.470	0.5
6	1.22	0.320	0.327	0.333	0.338	0.372	0.409	0.431	0.471	0.5
7	1.13	0.330	0.336	0.342	0.347	0.380	0.415	0.436	0.472	0.5
8	1.06	0.339	0.346	0.352	0.357	0.388	0.423	0.443	0.476	0.5
30	0.55	0.407	0.412	0.416	0.42	0.441	0.462	0.474	0.493	0.5

3.2 The Modified Mean Adjustments for Non-Central Chi-Squared Process

The undetected mean shift adjustment cause by the shift of location δ in Table 3-3 is called AS_{50} which is the magnitude of shift we need to adjust based on designated detection power is 0.5 and process data comes from non-central chi-squared distribution. The undetected mean shift adjustment cause by the shift of parameter λ in Table 3-4 is called AS_{50} which is the magnitude of shift we need to adjust based on designated detection power is 0.5 and process data comes from non-central chi-squared distribution. Table 3-3 and Table 3-4 display the magnitude of adjustments AS_{50} based on the detection power is 0.5 and data comes from $\chi^2(\lambda)$ with various values of λ(=0.1, 0.5, 1(1)10, 20, 50, and 100) and n=2(1)30. For example, if we set λ=3 and n=5, then the adjustment from Table 3-3 is AS_{50}=1.79. We conclude that the adjustment $AS_{50}\sigma$ (=1.79σ) is required based on the detection power is 0.5 and data comes from $\chi^2(\lambda)$. It also shows from Table 3-3 that the adjustments AS_{50} get closer to Bothe's adjustments as λ increases (when n=2(1)10), which is reasonable since the corresponding distributions get closer to the standard normal distribution. However, we should notice that when λ is small (distribution is strongly skewed), the required adjustment in the capability index formula is much greater than those for normal processes. Using the adjusted process capability formula, the engineers can determine the actual process capability more accurately. Fig. 3-1 presents the power curves, these lines on the graph depict the probabilities of detecting a shift in μ for the commonly used subgroup size n=3, 4, 5 (expressed in σ units on the horizontal axis) when λ=3. All these lines are close to zero for small shifts in μ. It can be found that the power of the chart with all three curves eventually leveling off close to 100% as the size of the shifts in excess of 3.5σ. The dashed horizontal line drawn in Fig. 3-1 shows that there is a 50% probability of missing a 1.79σ shift in μ when n is 5, while μ must move by 2.472σ to have this same probability when n is only 3. The shift sizes that have a 50% probability of remaining undetected, called AS_{50} values are listed in Table 3-3 for subgroup sizes n=2(1)30. Momentary movements in μ smaller than $AS_{50}\sigma$ are more than likely to be missed by a control chart. Therefore our adjustment AS_{50} takes into account those shifts that are not detected by the control chart.

Table 3-3 AS_{50} values for several subgroup size n and various λ values under the shift of location

λ / n	0	0.1	0.5	1	2	3	4	5	6	N(0,1)
2	4.182	4.157	3.931	3.694	3.400	3.225	3.107	3.021	2.954	2.12
3	3.126	3.109	2.951	2.789	2.590	2.472	2.393	2.335	2.290	1.73
4	2.553	2.539	2.419	2.296	2.146	2.057	1.997	1.954	1.920	1.50
5	2.188	2.177	2.079	1.98	1.860	1.789	1.741	1.705	1.678	1.34
6	1.932	1.922	1.841	1.758	1.657	1.598	1.558	1.528	1.506	1.22
7	1.741	1.733	1.663	1.592	1.505	1.454	1.420	1.395	1.375	1.13
8	1.592	1.585	1.524	1.462	1.386	1.341	1.311	1.289	1.272	1.06
9	1.473	1.467	1.412	1.357	1.290	1.250	1.223	1.203	1.188	1.00
10	1.375	1.369	1.320	1.270	1.210	1.174	1.149	1.132	1.118	0.95
11	1.292	1.287	1.242	1.197	1.142	1.109	1.087	1.071	1.059	0.90
12	1.221	1.217	1.176	1.134	1.084	1.054	1.034	1.019	1.007	0.87
13	1.16	1.156	1.118	1.08	1.033	1.005	0.987	0.973	0.963	0.83
14	1.107	1.103	1.068	1.032	0.989	0.963	0.946	0.933	0.923	0.80
15	1.059	1.055	1.023	0.990	0.949	0.925	0.909	0.897	0.888	0.77
16	1.017	1.013	0.983	0.952	0.914	0.891	0.876	0.865	0.856	0.75
17	0.979	0.975	0.947	0.917	0.882	0.861	0.846	0.836	0.828	0.73
18	0.944	0.941	0.914	0.887	0.853	0.833	0.819	0.809	0.802	0.71
19	0.913	0.91	0.884	0.858	0.826	0.807	0.794	0.785	0.778	0.69
20	0.884	0.882	0.857	0.832	0.802	0.784	0.772	0.763	0.756	0.67
21	0.858	0.855	0.832	0.809	0.780	0.762	0.751	0.742	0.736	0.65
22	0.834	0.831	0.809	0.787	0.759	0.743	0.731	0.723	0.717	0.64
23	0.811	0.809	0.788	0.766	0.740	0.724	0.713	0.706	0.700	0.63
24	0.79	0.788	0.768	0.747	0.722	0.707	0.697	0.689	0.683	0.61
25	0.771	0.769	0.749	0.729	0.705	0.691	0.681	0.674	0.668	0.60
26	0.753	0.751	0.732	0.713	0.689	0.676	0.666	0.659	0.654	0.59
27	0.736	0.734	0.716	0.697	0.675	0.661	0.652	0.646	0.64	0.58
28	0.72	0.718	0.7	0.683	0.661	0.648	0.639	0.633	0.628	0.57
29	0.704	0.703	0.686	0.669	0.648	0.635	0.627	0.621	0.616	0.56
30	0.69	0.688	0.672	0.656	0.635	0.623	0.615	0.609	0.605	0.55

Table 3-3 AS_{50} values for several subgroup size n and various λ values under the shift of location (**continued**)

n \ λ	7	8	9	10	20	50	100	N(0,1)
2	2.900	2.856	2.818	2.786	2.604	2.432	2.343	2.12
3	2.254	2.224	2.199	2.177	2.055	1.940	1.880	1.73
4	1.893	1.870	1.851	1.835	1.743	1.656	1.611	1.50
5	1.656	1.638	1.623	1.610	1.536	1.467	1.431	1.34
6	1.487	1.472	1.460	1.449	1.387	1.329	1.299	1.22
7	1.359	1.346	1.336	1.326	1.273	1.223	1.198	1.13
8	1.258	1.247	1.237	1.229	1.183	1.139	1.116	1.06
9	1.176	1.166	1.157	1.150	1.108	1.070	1.050	1.00
10	1.107	1.098	1.090	1.084	1.046	1.011	0.993	0.95
11	1.049	1.040	1.033	1.027	0.993	0.962	0.945	0.90
12	0.998	0.991	0.984	0.979	0.947	0.918	0.903	0.87
13	0.954	0.947	0.941	0.936	0.907	0.880	0.866	0.83
14	0.915	0.909	0.903	0.898	0.872	0.847	0.834	0.80
15	0.880	0.874	0.869	0.865	0.840	0.816	0.804	0.77
16	0.849	0.844	0.839	0.835	0.811	0.789	0.778	0.75
17	0.821	0.816	0.811	0.807	0.785	0.765	0.754	0.73
18	0.795	0.790	0.786	0.782	0.761	0.742	0.732	0.71
19	0.772	0.767	0.763	0.759	0.740	0.721	0.712	0.69
20	0.750	0.746	0.742	0.739	0.720	0.702	0.693	0.67
21	0.730	0.726	0.722	0.719	0.701	0.685	0.676	0.65
22	0.712	0.708	0.704	0.701	0.684	0.668	0.660	0.64
23	0.695	0.691	0.687	0.684	0.668	0.653	0.645	0.63
24	0.679	0.675	0.672	0.669	0.653	0.639	0.631	0.61
25	0.664	0.660	0.657	0.654	0.639	0.625	0.618	0.60
26	0.650	0.646	0.643	0.640	0.626	0.613	0.606	0.59
27	0.636	0.633	0.630	0.628	0.614	0.601	0.594	0.58
28	0.624	0.621	0.618	0.615	0.602	0.589	0.583	0.57
29	0.612	0.609	0.606	0.604	0.591	0.579	0.572	0.56
30	0.601	0.598	0.595	0.593	0.580	0.569	0.563	0.55

Table 3-4 AS_{50} values for several subgroup size n and various λ values under the shift of parameter

n \ λ	0	0.1	0.5	1	2	3	4	5	6	N(0,1)
2	4.314	4.259	3.973	3.712	3.406	3.228	3.109	3.022	2.955	2.12
3	3.207	3.170	2.976	2.800	2.594	2.475	2.394	2.336	2.291	1.73
4	2.611	2.583	2.437	2.304	2.149	2.059	1.999	1.954	1.920	1.50
5	2.232	2.210	2.093	1.987	1.862	1.790	1.742	1.706	1.679	1.34
6	1.967	1.949	1.852	1.763	1.660	1.599	1.559	1.529	1.506	1.22
7	1.771	1.755	1.672	1.596	1.507	1.455	1.420	1.395	1.375	1.13
8	1.618	1.605	1.532	1.466	1.388	1.342	1.312	1.289	1.272	1.06
9	1.495	1.484	1.419	1.360	1.291	1.251	1.223	1.204	1.188	1.00
10	1.395	1.384	1.326	1.273	1.211	1.174	1.150	1.132	1.118	0.95
11	1.310	1.300	1.248	1.200	1.143	1.110	1.088	1.071	1.059	0.90
12	1.237	1.229	1.181	1.137	1.085	1.054	1.034	1.019	1.008	0.87
13	1.175	1.167	1.123	1.082	1.034	1.006	0.987	0.973	0.963	0.83
14	1.120	1.112	1.072	1.034	0.989	0.963	0.946	0.933	0.923	0.80
15	1.071	1.065	1.027	0.991	0.950	0.925	0.909	0.897	0.888	0.77
16	1.028	1.022	0.986	0.953	0.914	0.891	0.876	0.865	0.856	0.75
17	0.989	0.983	0.950	0.919	0.882	0.861	0.846	0.836	0.828	0.73
18	0.954	0.949	0.917	0.888	0.853	0.833	0.819	0.809	0.802	0.71
19	0.922	0.917	0.887	0.860	0.827	0.808	0.795	0.785	0.778	0.69
20	0.893	0.888	0.860	0.834	0.802	0.784	0.772	0.763	0.756	0.67
21	0.866	0.861	0.835	0.810	0.780	0.763	0.751	0.742	0.736	0.65
22	0.841	0.837	0.812	0.788	0.760	0.743	0.732	0.723	0.717	0.64
23	0.819	0.814	0.790	0.767	0.740	0.724	0.714	0.706	0.700	0.63
24	0.797	0.793	0.770	0.748	0.722	0.707	0.697	0.689	0.683	0.61
25	0.777	0.774	0.751	0.730	0.706	0.691	0.681	0.674	0.668	0.60
26	0.759	0.755	0.734	0.714	0.690	0.676	0.666	0.659	0.654	0.59
27	0.742	0.738	0.717	0.698	0.675	0.662	0.652	0.646	0.641	0.58
28	0.725	0.722	0.702	0.683	0.661	0.648	0.639	0.633	0.628	0.57
29	0.710	0.707	0.688	0.670	0.648	0.635	0.627	0.621	0.616	0.56
30	0.695	0.692	0.674	0.656	0.636	0.624	0.615	0.609	0.605	0.55

Table 3-4 AS_{50} values for several subgroup size n and various λ values under the shift of parameter (**continued**)

n \ λ	7	8	9	10	20	50	100	N(0,1)
2	2.901	2.856	2.819	2.786	2.604	2.432	2.343	2.12
3	2.254	2.224	2.199	2.177	2.055	1.940	1.880	1.73
4	1.893	1.870	1.851	1.835	1.743	1.656	1.611	1.50
5	1.657	1.639	1.623	1.610	1.536	1.467	1.431	1.34
6	1.488	1.473	1.460	1.449	1.387	1.329	1.299	1.22
7	1.360	1.347	1.336	1.326	1.273	1.223	1.198	1.13
8	1.258	1.247	1.237	1.229	1.183	1.139	1.116	1.06
9	1.176	1.166	1.157	1.150	1.108	1.070	1.050	1.00
10	1.107	1.098	1.090	1.084	1.046	1.011	0.993	0.95
11	1.049	1.040	1.033	1.027	0.993	0.962	0.945	0.90
12	0.998	0.991	0.984	0.979	0.947	0.918	0.903	0.87
13	0.954	0.947	0.941	0.936	0.907	0.880	0.866	0.83
14	0.915	0.909	0.903	0.898	0.872	0.847	0.834	0.80
15	0.881	0.874	0.869	0.865	0.840	0.816	0.804	0.77
16	0.849	0.844	0.839	0.835	0.811	0.789	0.778	0.75
17	0.821	0.816	0.811	0.807	0.785	0.765	0.754	0.73
18	0.795	0.790	0.786	0.782	0.761	0.742	0.732	0.71
19	0.772	0.767	0.763	0.760	0.740	0.721	0.712	0.69
20	0.750	0.746	0.742	0.739	0.720	0.702	0.693	0.67
21	0.730	0.726	0.722	0.719	0.701	0.685	0.676	0.65
22	0.712	0.708	0.704	0.701	0.684	0.668	0.660	0.64
23	0.695	0.691	0.687	0.684	0.668	0.653	0.645	0.63
24	0.679	0.675	0.672	0.669	0.653	0.639	0.631	0.61
25	0.664	0.660	0.657	0.654	0.639	0.625	0.618	0.60
26	0.650	0.646	0.643	0.641	0.626	0.613	0.606	0.59
27	0.636	0.633	0.630	0.628	0.614	0.601	0.594	0.58
28	0.624	0.621	0.618	0.615	0.602	0.589	0.583	0.57
29	0.612	0.609	0.606	0.604	0.591	0.579	0.572	0.56
30	0.601	0.598	0.595	0.593	0.580	0.569	0.563	0.55

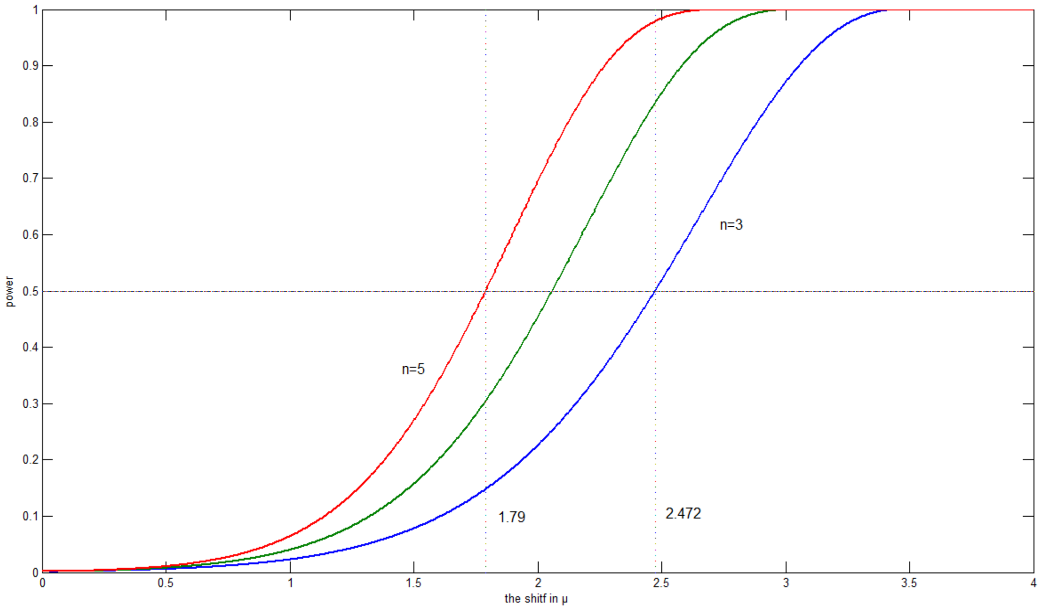

Fig. 3-1 Power curve for the commonly used subgroup size 3 , 4 , 5 when $\lambda = 3$

3.3 The Modified Estimator of Process Capability C_{pk}

3.3.1 C_{pk} in Non-Normal Case

The index C_{pk} has been viewed as a yield-based index since it provides bounds on the process yield for a normally distributed process with a fixed value of C_{pk}. This index C_{pk} is defined as:

$$C_{pk} = \min\{\frac{USL-\mu}{3\sigma}, \frac{\mu - LSL}{3\sigma}\} ,$$

where as above *USL* is the upper specification limit, *LSL* is the lower specification limit, μ is the process mean and σ is the process standard deviation. The proper use of process capability indices, which are statistical measures of process capability, is based on several assumptions. One of the most essential is that the process monitored is supposed to be stable and the output is approximately normally distributed. When the distribution of a process characteristic is non-normal, PCIs calculated using conventional methods could often lead to erroneous and misleading interpretation of the process's capability.

In the recent years, several approaches to problems of PCIs for the non-normal populations have been suggested. A widely accepted approach for PCI computation is to use the popular normal plot so that the normality assumption can be verified simultaneously. Analogous to the normal probability plot, where the "nature" process width is between the 0.135 percentile and 99.865 percentile, surrogate PCI values may be obtained via appropriately selected probability plots. Since the median is usually the preferable central value for a skewed distribution, the corresponding C_{pu} and C_{pl} are defined as:

$$C_{pu} = \frac{USL - median}{(upper\ 0.135\%\ \text{point}) - median} = \frac{USL - F_{0.5}}{F_{0.99865} - F_{0.5}}$$

$$C_{pl} = \frac{median - LSL}{median - (lower\ 0.135\%\ \text{point})} = \frac{F_{0.5} - LSL}{F_{0.5} - F_{0.00135}}$$

Then the index C_{pk} would be calculated as the minimum of C_{pu} and C_{pl}, namely:

$$C_{pk} = \min\{C_{pu}, C_{pl}\} = \min\{\frac{USL - F_{0.5}}{F_{0.99865} - F_{0.5}}, \frac{F_{0.5} - LSL}{F_{0.5} - F_{0.00135}}\}$$

so that the normality assumption can be verified simultaneously.

We can obtain more accurate measures of these percentile points ($F_{0.00135}$, $F_{0.5}$ and $F_{0.99865}$) under consideration in the non-normal case, if we are able to find a better distributional form for the data, which provides a very satisfactory fit. This involves modeling the process data with alternative probability plot models, such as the Weibull or gamma ones (see e.g. Dudewicz and Mishra, 1998;

Kotz and Lovelace, 1998). Nevertheless, an obvious disadvantage of probability plotting is that it is not a truly objective procedure. It is quite possible for two analysts to arrive at different conclusions using the same data. Accordingly, it is often desirable to supplement probability plots with goodness-of-fit tests, which possess more formal statistical foundations (see, e.g., Shapiro, 1995). Choosing proper distribution to fit the data is an important step in probability plotting. Sometimes one can use the available knowledge of the physical phenomenon or the past experience to suggest a choice of the distribution.

3.3.2 Adjustment of C_{pk}

Acknowledging that a process will experience shift in $F_{0.5}$ of various magnitudes and knowing that not all of these will be discovered, some allowance for them must be made when estimating outgoing quality so customers are not disappointed. Because shifts ranging in size from 0 to $AS_{50}\sigma$ are the ones likely to remain undetected (large moves should be caught by the chart), a conservative approach is to assume that every missed shift is as large as AS_{50}.

Considering the undetected process mean shift as large as $AS_{50}\sigma$, we use $F_{0.5}$ minus $AS_{50}\sigma$ to evaluate how well the process output meets the LSL and $F_{0.5}$ plus $AS_{50}\sigma$ for determining conformance to the USL when estimating the index C_{pk}. Incorporating both of these adjustments into the basic C_{pk} formula we obtained the "dynamic" C_{pk} index by making the following modifications:

$$C_{pk} = \min\{\frac{USL-(F_{0.5}+AS_{50}\sigma)}{F_{0.99865}-F_{0.5}}, \frac{(F_{0.5}-AS_{50}\sigma)-LSL}{F_{0.5}-F_{0.00135}}\}$$

$$= \min\{\frac{USL-F_{0.5}}{F_{0.99865}-F_{0.5}}-\frac{AS_{50}\sigma}{F_{0.99865}-F_{0.5}}, \frac{(F_{0.5}-LSL}{F_{0.5}-F_{0.00135}}-\frac{AS_{50}\sigma}{F_{0.5}-F_{0.00135}}\}$$

By considering an adjustment $AS_{50}\sigma$ in this assessment for undetected shifts in process median, the estimate of dynamic index C_{pk} will decrease and the expected total number of nonconforming parts will increase. It must be noticed that this nonconforming level assumes that undetected shifts are happening almost constantly and that everyone is equal to $AS_{50}\sigma$. From Table 3-3, the practitioners can find the AS_{50} to calculate the dynamic index C_{pk} for determining whether their process meets the preset capability requirement, and make reliable decisions to the process.

4 Application

Recently, due to the excellent current driving capability and microwave performances, heterojunction bipolar transistors (HBTs) have extensively employed on digital and analogy applications and are recognized as promising electronic devices for high frequency and high performance circuit applications, such as monolithic microwave integrated circuit and optoelectronic integrated circuit. For successful process control, process optimization, circuit design, and compact transistor modeling, there are several problems that must be overcome to realize practical high speed ICs. One of the problems is a current gain reduction associated with the scaling down of transistor size. Since the emitter dimension must be minimized for higher switching speed operation, elimination of the current gain reduction is very important for HBT designs. Also, Cutoff frequency and maximum oscillation frequency were changed with emitter dimension, and this was attributed to the variation of resistances and junction capacitances with emitter structure.

Therefore, we should address on one of the characteristics of HBTs, the emitter area. The upper and lower manufacturing specific limits are set to USL=45 um^2 and LSL=5 um^2, respectively. If the characteristic data does not fall within the tolerance (LSL, USL), the component of the emitter area is consider to nonconforming/defective, and will not be used to make the emitter area of that particular model.

As shown in Table 4-1, a part of historical data is collected. Fig. 4-2 displays the histogram, and Fig. 4-1 displays the normal probability plot of these historical data. From the Fig. 4-1 and Fig. 4-2 , it is evident to conclude the data collected from the factory are not normal distributed. The data analysis results justify that the process is significantly away from the normal distribution. By the goodness-of-fit tests, the historical data indicates that the process pretty approximates to be distributed as non-central chi-squared. The parameter ν and λ of non-central chi-squared distribution could be calculated from historical data giving ν=1 and λ=20. Therefore, it is approximate to use this approach and we can obtain more accurate measures of the three quantile: $F_{0.00135}$, $F_{0.5}$ (median) , and $F_{0.99865}$ for

$$\sigma = \sqrt{\frac{1}{n}(1+2\lambda)} = \sqrt{\frac{1}{10}(1+2\cdot 20)} = \sqrt{4.1} = 2.025$$

under consideration. Then "dynamic" C_{pk} index can be calculated as follows:

$$\begin{aligned} dynamic\ C_{pk} &= \min\{\frac{USL - F_{0.5} - AS_{50}\sigma)}{F_{0.99865} - F_{0.5}}, \frac{F_{0.5} - AS_{50}\sigma - LSL}{F_{0.5} - F_{0.00135}}\} \\ &= \min\{\frac{45 - \ 20 - 1.046\cdot 2.025}{55.832 - 20}, \frac{20 - 1.046\cdot 2.025 - 5}{20 - 2.167}\} \\ &\approx \min\{0.64, 0.72\} \\ &= 0.64 \end{aligned}$$

with AS_{50}=1.046 for n=10 from Table 3-3. Compared it to the value of the following conventional index:

$$\begin{aligned} C_{pk} &= \min\{C_{pu}, C_{pl}\} = \min\{\frac{USL - F_{0.5}}{F_{0.99865} - F_{0.5}}, \frac{F_{0.5} - LSL}{F_{0.5} - F_{0.00135}}\} \\ &= \min\{\frac{45 - 20}{55.832\text{-}20}, \frac{20\text{-}5}{20\text{-}2.167}\} \\ &\approx \min\{0.70, 0.84\} = 0.70 \end{aligned}$$

calculated by a traditional capability study (the shift of process mean is not considered), we can find that the value of the modified C_{pk} is much smaller. This result indicates if the process mean shifts that are not detected then unadjusted C_{pk} would overestimate the actual process yield which is not derisible. Our adjustment takes into account those shifts that are not detected so that the practitioner would be able to keep its quality promise for this process. As the adjusted process capability drops below the desired quality level, the practitioner should stop the process because the process does not meet his preset capability requirement. As the subgroup size n increases, the shift in process mean have a higher probability of detection. For example, if n=15, the AS_{50} would be 0.840 for $\chi_1^2(20)$ from Table 3-3, and then the "dynamic" C_{pk} index is

$$dynamic\ C_{pk} = \min\{\frac{USL - F_{0.5} - AS_{50}\sigma)}{F_{0.99865} - F_{0.5}}, \frac{F_{0.5} - AS_{50}\sigma - LSL}{F_{0.5} - F_{0.00135}}\}$$

$$= \min\{\frac{45 - \ 20 - 0.840 \cdot 1.653}{55.832 - 20}, \frac{20 - 0.840 \cdot 1.653 - 5}{20 - 2.167}\}$$

$$\approx \min\{0.66, 0.76\} = 0.66$$

Changing n from 10 to 15 increases the dynamic C_{pk} index from 0.64 to 0.66, and the total number of nonconforming parts would be reduced.

Table 4-1 The 100 observations are collected from the historical data

32.955	15.736	25.510	18.311	6.255	19.248	16.3	32.922	17.451	11.374
9.858	18.385	14.725	37.943	27.512	20.158	23.992	24.408	19.497	20.954
28.506	16.812	31.125	34.926	21.003	8.116	19.456	18.011	16.695	23.67
18.49	9.844	11.532	25.789	22.311	19.973	17.759	24.597	15.493	16.397
17.771	24.566	24.018	6.658	14.296	20.389	29.304	10.274	13.462	17.752
33.647	23.895	20.944	30.906	10.373	26.093	21.52	20.644	29.198	24.368
31.458	13.159	23.962	27.004	24.527	23.57	15.256	20.438	24.599	9.911
12.983	8.23	26.109	22.977	10.126	41.423	8.854	16.815	17.774	13.339
11.316	18.924	5.114	33.823	22.43	20.686	31.242	11.123	9.868	37.314
14.859	43.001	17.45	35.999	17.945	16.318	12.035	11.187	19.182	19.607

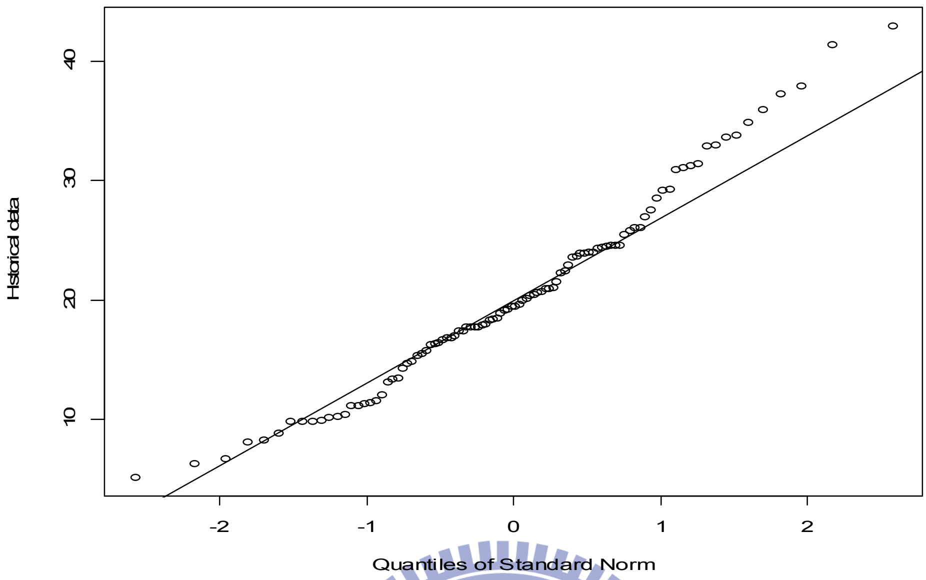

Fig. 4-1 Normal probability plot of the historical data

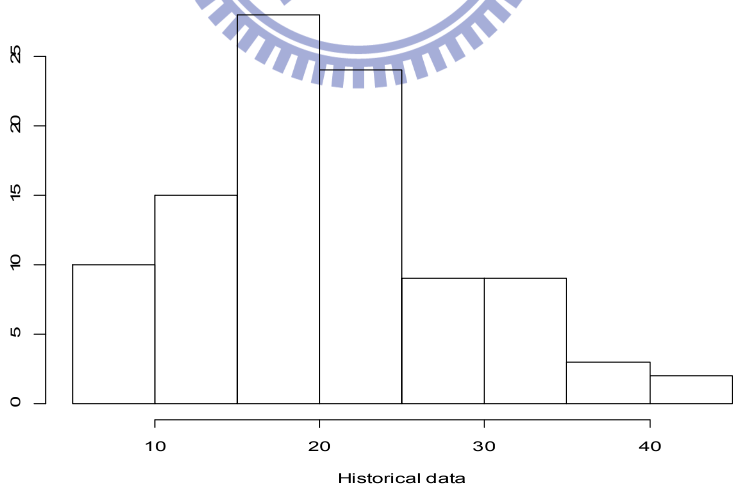

Fig. 4-2 Histogram plot of the historical data

5 Conclusions

In this paper, we considered the problem of how to determine the adjustments for process capability with mean shift when data follows the non-central chi-squared distribution. We first examined Bothe's approach and found the detection power is less than 0.5 when data comes from the non-central chi-squared distribution, showing that Bothe's adjustments are inadequate when we have non-central chi-squared processes. For non-central chi-squared processes, we calculated the adjustments for various sample sizes (n) and non-central chi-square parameter (ν, λ) with detection power fixed to 0.5. For small value of λ (distribution is strongly skewed), the required adjustment in the capability index formula is much greater than those for normal processes. Using the adjusted process capability formula, the engineers can determine the actual process capability more accurately. Tables are also provided for engineers/practitioners to use in their in-plant applications. A real-world semi-conductor production plant is investigated and presented to illustrate the applicability of the proposed approach.

Reference

[1] Bender, A. (1975). Statistical tolerancing as it relates to quality control and the designer. *Automotive Division Newsletter of ASQC.*

[2] Bothe, D.R. (2002). Statistical reason for the 1.5σ shift. *Quality Engineering* , 14 (3), 479–487.

[3] Chan, L.K., Cheng, S.W., Spiring, F. A. (1988). A new measure of process capability C_{pm}. *Journal of Quality Technology 20 (3), 162–175.*

[4] Choi, K.C., Nam, K.H., Park, D.H., (1996). Estimation of capability index based on bootstrap method. *Microelectronics Reliability* 36 (9), 1141–1153.

[5] Clements, J.A. (1989). Process capability calculations for non-normal distributions. *Quality* Progress, 95–100.

[6] Ding, J. (2004). A method of estimating the process capability index from the first four moments of non-normal data. *Quality and Reliability Engineering International* 20, 787–805.

[7] Dudewicz, E.J., Mishra, S.N. (1998). *Modern Mathematical Statistics.* John Wiley, New York.

[8] Evans, D.H. (1975). Statistical tolerancing: The state of the art, Part III: Shifts and drifts. *Journal of Quality Technology* 7 (2), 72–76.

[9] Franklin, L.A., Wasserman, G.S. (1992). Bootstrap lower confidence limits for capability indices. *Journal of Quality Technology* 24 (4), 196–210.

[10] Gilson, J.A. (1951). New Approach to Engineering Tolerances. Machinery Publishing Co., London.

[11] Kane, V.E. (1986). Process capability indices. *Journal of Quality Technology* 18 (1), 41–52.

[12] Kocherlakota, S., Kocherlakota, K., Kirmani, S.N.U.A. (1992). Process capability index under non-normality. *International Journal of Mathematical Statistics* 1 (2), 175–210.

[13] Kotz, S., Lovelace, C.R. (1998). *Process Capability Indices in Theory and Practice.* Arnold,

London, UK.

[14] Lucas, J.M. (1976). The design and use of cumulative sum quality control schemes. *Journal of Quality Technology* 8, 1–12.

[15] Lucas, J.M., Crosier, R.B. (2000). Fast initial response for CUSUM Quality-Control schemes: Give your CUSUM a head start. *Technometrics* 42 (1), 102–107.

[16] Luceno, A., Puig-Pey, J. (2002). Computing the run length probability distribution of CUSUM charts. *Journal of Quality Technology* 34(2), 209–215.

[17] Pal, S. (2005). Evaluation of non-normal process capability indices using generalized lambda distribution. *Quality Engineering* 17, 77–85.

[18] Pearn, W.L., Chen, K.S. (1997). Capability indices for non-normal distributions with an application in electrolytic capacitor manufacturing. *Microelectronics and Reliability* 37 (12), 1853–1858.

[19] Pearn, W.L., Kotz, S., Johnson, N.L. (1992). Distributional and inferential properties of process capability indices. *Journal of Quality Technology* 24 (4), 216–233.

[20] Polansky, A.M. (1998). A smooth nonparametric approach to process capability. *Quality and Reliability Engineering International* 14, 43–48.

[21] Pyzdek, T. (1992). Process capability analysis using personal computers. *Quality Engineering* 4 (3), 419–440.

[22] Ross, S. (2005). A First Course in Probability Theory, seventh ed. Academic Press, New York.

[23] Schilling, E.G., Nelson, P.R. (1976). The effect of non-normality on the control limits of charts. *Journal of Quality Technology* 8, 183–188.

[24] Shapiro, S.S. (1995). Goodness-of-fit tests. In: Balakrishnan, N., Basu, A.P. (Eds.), The Exponential Distribution: Theory Methods and Applications. Gordon and Breach, Langhorne, Pennsylvania, Chapter 13.

[25] Shore, H. (1998). A new approach to analyzing non-normal quality data with application to process capability analysis. *International Journal of Production Research* 36 (7), 1917–1933.

[26] Somerville, S.E., Montgomery, D.C. (1996). Process capability indices and non-normal distributions. *Quality Engineering* 9 (2), 305–316.