# 國 立 交 通 大 學

## 統 計 學 研 究 所

## 碩 士 論 文

類別製造過程的多變量指數加權移動平均監控計劃

Multivariate Exponentially Weighted Moving Average Monitoring Scheme for a Categorical Manufacturing Process

研 究 生：洪鏡婷

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中 華 民 國 九 十 九 年 六 月

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## 摘　　要

本文是發展一個方法去監控不同類別的資料。首先介紹 Dirichlet-多項式模型，根據該模型提出了類別製造過程的多變量指數加權移動平均管制圖並研究其相關的特性。最後用模擬的資料來說明這個方法的實用性與適用性。

# Multivariate Exponentially Weighted Moving Average Monitoring Scheme for a Categorical Manufacturing Process

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# ABSTRACT

This paper is to develop a method to monitor the fractions of the tested items falling into different categories of pass/fail modes. The Dirichlet-compound multinomial model is first introduced and then a multivariate exponentially weighted moving average control chart is proposed for a categorical manufacturing process under the proposed model. Some relevant properties of the proposed control chart are also investigated. Finally, a simulation study is presented to show the usefulness and applicability of the proposed methodology.

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洪鏡婷　謹誌于

國立交通大學統計學研究所

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# 1 Introduction

## 1.1 Motivation

In the current international marketplace, quality improvement is a key point for maintaining competitive advantage. Statistical process control is an effective tool for achieving process stability and improving process capability through variation reduction. When a product item is tested, usually one has more information than just pass or fail. Often there are categories of failures. For instance, a product may have several categories of failure modes. Thus, in the paper, a multivariate exponentially weighted moving average monitoring scheme is proposed for a categorical manufacturing process under the Dirichlet-compound multinomial model and then some relevant properties of the proposed monitoring scheme are also investigated.

## 1.2 Literature Review

Statistical process control (SPC) refers to some statistical methods which are widely used to monitor and improve the quality and productivity of industrial processes and service operations. SPC primarily involves the implementation of control charts. The method of control charts is a graphical tool which is used to monitor processes in order to distinguish special significant causes of variation from general assignable causes of variation in processes. The Shewhart (Shewhart, 1931), cumulative sums (CUSUM) (Page, 1954), and exponentially weighted moving average (EWMA) (Roberts, 1959) control charts are widely used in practice. Standard control chart usage involves phase I and phase II applications, with two different and distinct objectives. In phase I, a set of process data is gathered and analyzed all at once in a retrospective analysis, constructing trial control limits to determine if the process has been in control over the period of time where the data were collected, and to see if reliable control limits can be established to monitor future production. In phase II, we use the control chart to monitor the process by comparing the sample statistic for each successive sample as it is drawn from the process to the control limits.

The Shewhart control chart (Shewhart, 1931) monitors the process observations directly. Suppose that $\{x_{tm}\colon\ t = 1, 2, ...$ and $m = 1, ..., n\}$ are independent univariate measurements, where $n$ is a known positive integer and $x_{t1}, ..., x_{tn}$ are identically distributed for each $t$. Set $\bar{x}_t \equiv \sum_{m=1}^{n} x_{tm}/n$ for $t = 1, 2, ....$ Let $\mu_0$ denote the known real-valued in-control process target and $\sigma_0$ the known positive in-control process standard deviation. When $\bar{x}_t \sim N(\mu_0, \sigma_0^2/n)$, the process is called in control at time $t$; otherwise, out of control at time $t$. Then the Shewhart control chart is based on the values

$$w_t = \bar{x}_t - \mu_0 \tag{1}$$

for $t = 1, 2, ....$ Shewhart (1931) proposed the stopping time of the Shewhart control chart as the first time $t$ such that

$$|w_t| > L\frac{\sigma_0}{\sqrt{n}}, \tag{2}$$

where $L = 3$. In practice, $L$ is often chosen to achieve a specified in-control average run length.

Page (1954) introduces the CUSUM chart as a sequential probability test. Suppose that $\{x_{tm}\colon\ t = 1, 2, ...$ and $m = 1, ..., n\}$ are independent univariate measurements, where $n$ is a known positive integer and $x_{t1}, ..., x_{tn}$ are identically distributed for each $t$. Set $\bar{x}_t \equiv \sum_{m=1}^{n} x_{tm}/n$ for $t = 1, 2, ....$ Let $\mu_0$ denote the known real-valued in-control process target and $\sigma_0$ the known positive in-control process standard deviation. When $\bar{x}_t \sim N(\mu_0, \sigma_0^2/n)$, the process is called in control at time $t$; otherwise, out of control at time $t$. The CUSUM algorithm assigns equal weights to past observations, and its tabular form consists of two quantities,

$$w_t^+ = \max\left[0, w_{t-1}^+ + (\bar{x}_t - \mu_0) - k\sigma_0/\sqrt{n}\right] \tag{3}$$

and

$$w_t^- = \min\left[0, w_{t-1}^- + (\bar{x}_t - \mu_0) + k\sigma_0/\sqrt{n}\right] \tag{4}$$

for $t = 1, 2, ...,$ where $w_0^+ = w_0^- = 0$ and $k$ is the reference value which is often chosen about halfway between the target $\mu_0$ and the out-of-control mean value $\mu_1$ of interest.

Page (1954) proposed the stopping time of the CUSUM control chart as the first time $t$ such that either $w_t^+$ or $w_t^-$ exceed the decision interval $H$, where $H$ is chosen to achieve a specified in-control average run length, e.g., $H = 5$ when $ARL_0 = 465$.

Roberts (1959) proposed a monitoring scheme which is based on the EWMA of the observations. The EWMA, originally called geometric moving average (GMA) in Roberts (1959), is briefly introduced as follows: Suppose that $\{x_{tm}$: $t = 1, 2, \ldots$ and $m = 1, \ldots, n\}$ are independent univariate measurements, where $n$ is a known positive integer and $x_{t1}, \ldots, x_{tn}$ are identically distributed for each $t$. Set $\bar{x}_t \equiv \sum_{m=1}^{n} x_{tm}/n$ for $t = 1, 2, \ldots$. Let $\mu_0$ denote the known in-control real-valued process target and $\sigma_0$ the known positive in-control process standard deviation. When $\bar{x}_t \sim N(\mu_0, \sigma_0^2/n)$, the process is called in control at time $t$; otherwise, out of control at time $t$. Then the EWMA control chart is based on the values

$$\begin{aligned} w_t &\equiv (1-\lambda)w_{t-1} + \lambda(\bar{x}_t - \mu_0) \\ &= \lambda(1-\lambda)^{t-1}(\bar{x}_1 - \mu_0) + \cdots + \lambda(1-\lambda)(\bar{x}_{t-1} - \mu_0) + \lambda(\bar{x}_t - \mu_0) \\ &= \lambda \sum_{i=0}^{t-1} (1-\lambda)^i (\bar{x}_{t-i} - \mu_0) \end{aligned} \tag{5}$$

for $t = 1, 2, \ldots$, where $w_0 \equiv 0$ and $\lambda$ is a specified value in $(0, 1]$. If $\bar{x}_t \sim N(\mu_0, \sigma_0^2/n)$, then $w_t \sim N(0, \sigma_t^2)$, where

$$\sigma_t = \sqrt{\frac{\lambda\left[1-(1-\lambda)^{2t}\right]}{n(2-\lambda)}}\,\sigma_0 \;\rightarrow\; \sqrt{\frac{\lambda}{n(2-\lambda)}}\,\sigma_0 \tag{6}$$

as $t \to \infty$. Roberts (1959) proposed the stopping time of the EWMA monitoring scheme as the first time $t$ such that

$$|w_t| > L\sigma_t, \tag{7}$$

where $L = 3$. In practice, $L$ is often chosen to achieve a specified in control average run length. It is the same as the Shewhart control chart when $\lambda = 1$, and nearly the same as the CUSUM control chart when $\lambda \to 0$. There have been numerous extensions and variations of the basic EWMA control chart.

Through modern technology that allows simultaneously monitoring all key quality characteristics during a manufacturing process, the monitored quality characteristics are usually dependent each other. This is especially true for quality characteristics related to safety, fault detection and diagnosis, quality control, and process control. Joint monitoring of quality characteristics ensures appropriate control of the overall process. Multivariate SPC techniques have recently been applied to novel fields such as environmental monitoring and detection of computer intrusion. The purpose of multivariate on-line techniques is to investigate whether quality characteristics are simultaneously in control or not. Versions of the multivariate Shewhart, CUSUM, and EWMA control charts have been proposed under the multivariate normality assumption.

To incorporate the recent historical information, Lowry *et al.* (1992) proposed a multivariate exponentially weighted moving average (MEWMA) control chart which is briefly introduced as follows: Suppose that $\{x_{tm}$: $t = 1, 2, ...$ and $m = 1, ..., n\}$ are independent $p$-variate measurements, where $n$ is a known positive integer; $x_{t1}, ..., x_{tn}$ are identically distributed for each $t$; and $p$ $(\geq 2)$ is a known positive integer. Set $\bar{x}_t \equiv \sum_{m=1}^{n} x_{tm}/n$ for $t = 1, 2, ....$ Let $\mu_0$ denote the known $p \times 1$ in-control process target vector in $(-\infty, \infty)^p$ and $\Sigma_0$ the known $p \times p$ positive definite in-control process covariance matrix. Let $N_p(\mu, \Sigma)$ denote the $p$-variate normal distribution with mean vector $\mu$ and covariance matrix $\Sigma$. When $\bar{x}_t \sim N_p(\mu_0, \Sigma_0/n)$, the process is called in control at time $t$; otherwise, out of control at time $t$. Lowry *et al.* (1992) proposed the MEWMA control chart as based on the $p \times 1$ vectors

$$\begin{aligned} w_t &\equiv (I_p - \Lambda)w_{t-1} + \Lambda(\bar{x}_t - \mu_0) \\ &= \Lambda(I_p - \Lambda)^{t-1}(\bar{x}_1 - \mu_0) + \cdots + \Lambda(I_p - \Lambda)(\bar{x}_{t-1} - \mu_0) + \Lambda(\bar{x}_t - \mu_0) \\ &= \Lambda \sum_{i=0}^{t-1} (I_p - \Lambda)^i (\bar{x}_{t-i} - \mu_0) \end{aligned} \tag{8}$$

for $t = 1, 2, ...,$ where $w_0 \equiv 0_{p\times 1}$, the $p \times 1$ vector $(0, ..., 0)^T$; $I_p$ denotes the identity matrix of order $p$; and $\Lambda$ is a specified diagonal matrix diag$\{\lambda_1, ..., \lambda_p\}$ with $\lambda_1, ..., \lambda_p \in (0, 1]$. Set $\Sigma_0 \equiv (\Sigma_{0jj'})_{j,j'=1,...,p}$. If $\bar{x}_t \sim N_p(\mu_0, \Sigma_0/n)$, then $w_t \sim N_p(0_{p\times 1}, \Sigma_t)$, where

$$\begin{aligned} \Sigma_t \equiv (\Sigma_{tjj'})_{j,j'=1,...,p} &= \frac{1}{n}\Lambda \left[ \sum_{i=0}^{t-1} (I_p - \Lambda)^i \Sigma_0 (I_p - \Lambda)^i \right] \Lambda \\ &\to \frac{1}{n}\Lambda \left[ \sum_{i=0}^{\infty} (I_p - \Lambda)^i \Sigma_0 (I_p - \Lambda)^i \right] \Lambda \end{aligned} \tag{9}$$

as $t \to \infty$ with

$$\Sigma_{tjj'} = \frac{\lambda_j \lambda_{j'} \left[1 - (1-\lambda_j)^t (1-\lambda_{j'})^t\right] \Sigma_{0jj'}}{n(\lambda_j + \lambda_{j'} - \lambda_j \lambda_{j'})} \to \frac{\lambda_j \lambda_{j'} \Sigma_{0jj'}}{n(\lambda_j + \lambda_{j'} - \lambda_j \lambda_{j'})} \tag{10}$$

as $t \to \infty$. In particular, when $\lambda_1 = ... = \lambda_p = \lambda$,

$$\Sigma_t = \frac{\lambda \left[1 - (1-\lambda)^{2t}\right]}{n(2-\lambda)} \Sigma_0 \;\to\; \frac{\lambda}{n(2-\lambda)} \Sigma_0 \tag{11}$$

as $t \to \infty$. Then the stopping time of the MEWMA monitoring scheme is the first time $t$ such that

$$w_t^T \Sigma_t^{-1} w_t \; (\equiv T_t^2) > h, \tag{12}$$

where $h$ is chosen to achieve a specified in-control average run length.

Consider a manufacturing process where each product units can be classified as one of $k+1$ disjoint categories for some fixed $k \in \{1, 2, ...\}$. When the outcome is recorded as one of two categories, e.g., {pass, fail}, the data are called binary. When the outcome is recorded as one of $k+1$ disjoint categories for some $k \in \{2, 3, ...\}$, the data are called polytomous, e.g., {pass, the first defect type, ..., the $k$th defect type}. See, e.g., McCullagh and Nelder (1989). Several researchers have investigated categorical data in different situations. Shiau *et al*. (2005) proposed the Dirichlet-compound multinomial empirical Bayes model to monitor the polytomous data. In the paper, we develop an MEWMA control chart for monitoring a manufacturing process under the Dirichlet-compound multinomial model.

## 1.3 Outline

The paper is organized as follows. In Section 2, the Dirichlet-compound multinomial model for a categorical manufacturing process is briefly introduced. In Section 3, a multivariate exponentially weighted moving average control chart is proposed and then some relevant properties of the proposed control chart are also investigated. In Section 4, a simulation study is presented to illustrate the proposed methodology. Finally, comparison and conclusions are given in Section 5.

# 2 Dirichlet-Compound Multinomial Model for a Categorical Manufacturing Process

Consider a manufacturing process which produces product units having $k$ different types of defects for some known positive integer $k$. In a product unit, let $p_{it}$ denote the probability of having the $i$th defect type at time $t$ for $i = 1, ..., k$. Then $1 - \sum_{i=1}^{k} p_{it}$ ($\equiv p_{0t}$) is the probability of having none of these $k$ defect types at time $t$. For $i = 1, ..., k$, let $x_{it}$ denote the number of tested product units having the $i$th defect type among $n_t$ randomly chosen tested product units at time $t$. Then $n_t - \sum_{i=1}^{k} x_{it}$ ($\equiv x_{0t}$) is the number of tested product units having none of these $k$ defect types at time $t$. Set $p_t \equiv (p_{0t}, p_{1t}, ..., p_{kt})^T$ and $x_t \equiv (x_{0t}, x_{1t}, ..., x_{kt})^T$. Then $p_t \in \mathcal{P}$ and $x_t \in \mathcal{X}_t$, where $\mathcal{P} \equiv \{p_t\colon\ p_{0t}, p_{1t}, ..., p_{kt} \in (0, 1)$ with $\sum_{i=0}^{k} p_{it} = 1\}$ and $\mathcal{X}_t \equiv \{x_t\colon\ x_{0t}, x_{1t}, ..., x_{kt} \in \{0, 1, ..., n_t\}$ with $\sum_{i=0}^{k} x_{it} = n_t\}$. Then the number of elements in $\mathcal{X}_t$ is $(n_t + k)!/(n_t!\,k!)$ ($\equiv |\mathcal{X}_t|$). Assume that $x_t$ given $p_t$ is distributed as either binomial$(n_t; p_t)$ for $k = 1$ or multinomial$(n_t; p_t)$ for $k \geq 2$. Then the conditional probability mass function (p.m.f.) of $x_t$ given $p_t$ is

$$f(x_t|p_t) = \frac{n_t!}{x_{0t}!\, x_{1t}! \cdots x_{kt}!}\, p_{0t}^{x_{0t}}\, p_{1t}^{x_{1t}} \cdots p_{kt}^{x_{kt}} \cdot 1_{\mathcal{X}_t}(x_t), \tag{13}$$

where $1_{\mathcal{X}_t}(x_t) = 1$ for $x_t \in \mathcal{X}_t$ and 0 otherwise.

Suppose that $p_t$ is distributed as either beta$(\alpha)$ for $k = 1$ or Dirichlet$(\alpha)$ for $k \geq 2$, where $\alpha$ ($\equiv (\alpha_0, \alpha_1, ..., \alpha_k)^T$) is the unknown $(k+1) \times 1$ parameter vector in the parameter space $(0, \infty)^{k+1}$. Set $\alpha_s \equiv \sum_{i=0}^{k} \alpha_i$. Then the probability density function (p.d.f.) of $p_t$ is

$$f(p_t; \alpha) = \frac{\Gamma(\alpha_s)}{\Gamma(\alpha_0)\,\Gamma(\alpha_1) \cdots \Gamma(\alpha_k)}\, p_{0t}^{\alpha_0 - 1}\, p_{1t}^{\alpha_1 - 1} \cdots p_{kt}^{\alpha_k - 1} \cdot 1_{\mathcal{P}}(p_t), \tag{14}$$

where $1_{\mathcal{P}}(p_t) = 1$ for $p_t \in \mathcal{P}$ and 0 otherwise. Then $p_t$ given $x_t$ is distributed as either beta$(\alpha_0 + x_{0t}, \alpha_1 + x_{1t})$ for $k = 1$ or Dirichlet$(\alpha_0 + x_{0t}, \alpha_1 + x_{1t}, ..., \alpha_k + x_{kt})$ for $k \geq 2$ with

$$E_\alpha(p_t|x_t) = \left( \frac{\alpha_0 + x_{0t}}{\alpha_s + n_t}, \frac{\alpha_1 + x_{1t}}{\alpha_s + n_t}, ..., \frac{\alpha_k + x_{kt}}{\alpha_s + n_t} \right)^T. \tag{15}$$

See, e.g., p. 217 of Johnson *et al.* (1995) for the parametric family of beta distributions and p. 488 of Kotz *et al.* (2000) for the parametric family of Dirichlet distributions.

Then the p.m.f. of $x_t$ is

$$\begin{aligned} f(x_t;\alpha) &= \frac{f(x_t,p_t;\alpha)}{f(p_t|x_t;\alpha)} = \frac{f(x_t|p_t)f(p_t;\alpha)}{f(p_t|x_t;\alpha)} \\ &= \exp\left[\sum_{j=0}^{n_t-1}\log\left(\frac{j+1}{\alpha_s+j}\right) - \sum_{i=0}^{k}\sum_{j=0}^{x_{it}-1}\log\left(\frac{j+1}{\alpha_i+j}\right)\right]\cdot 1_{\mathcal{X}_t}(x_t); \end{aligned} \tag{16}$$

see, e.g., pp. 80–81 of Johnson *et al.* (1997). Then, given $x_t$, the likelihood function for $\alpha$ is

$$L(\alpha;x_t) = \exp\left[\sum_{j=0}^{n_t-1}\log\left(\frac{j+1}{\alpha_s+j}\right) - \sum_{i=0}^{k}\sum_{j=0}^{x_{it}-1}\log\left(\frac{j+1}{\alpha_i+j}\right)\right], \tag{17}$$

the log-likelihood function for $\alpha$ is

$$l(\alpha;x_t) = \log\left[L(\alpha;x_t)\right] = \sum_{j=0}^{n_t-1}\log\left(\frac{j+1}{\alpha_s+j}\right) - \sum_{i=0}^{k}\sum_{j=0}^{x_{it}-1}\log\left(\frac{j+1}{\alpha_i+j}\right), \tag{18}$$

the score function for $\alpha$ is

$$\begin{aligned} &\frac{\partial l(\alpha;x_t)}{\partial\alpha} \\ &= \left(\sum_{j=0}^{x_{0t}-1}\frac{1}{\alpha_0+j}, \sum_{j=0}^{x_{1t}-1}\frac{1}{\alpha_1+j}, \ldots, \sum_{j=0}^{x_{kt}-1}\frac{1}{\alpha_k+j}\right)^T - \left(\sum_{j=0}^{n_t-1}\frac{1}{\alpha_s+j}\right)\cdot 1_{(k+1)\times 1} \\ &\equiv S(\alpha;x_t) \equiv \left(S_0(\alpha;x_t), S_1(\alpha;x_t), \ldots, S_k(\alpha;x_t)\right)^T, \end{aligned} \tag{19}$$

the observed Fisher information for $\alpha$ is

$$\begin{aligned} -\frac{\partial^2 l(\alpha;x_t)}{\partial\alpha\partial\alpha^T} &= \operatorname{diag}\left\{\sum_{j=0}^{x_{0t}-1}\frac{1}{(\alpha_0+j)^2}, \sum_{j=0}^{x_{1t}-1}\frac{1}{(\alpha_1+j)^2}, \ldots, \sum_{j=0}^{x_{kt}-1}\frac{1}{(\alpha_k+j)^2}\right\} \\ &\quad - \left[\sum_{j=0}^{n_t-1}\frac{1}{(\alpha_s+j)^2}\right]\cdot 1_{(k+1)\times 1}1_{(k+1)\times 1}^T \\ &\equiv J(\alpha;x_t) \equiv \left(J_{ii'}(\alpha;x_t)\right)_{i,i'=0,1,\ldots,k}, \end{aligned} \tag{20}$$

and the expected Fisher information for $\alpha$ is

$$Cov_\alpha\left(S(\alpha;x_t)\right) \equiv I_t(\alpha) \equiv \left(I_{tii'}(\alpha)\right)_{i,i'=0,1,\ldots,k}, \tag{21}$$

where $1_{(k+1)\times 1}$ denotes the $(k+1)\times 1$ vector $(1,\ldots,1)^T$. Notice that $E_\alpha(S(\alpha;x_t)) = 0_{(k+1)\times 1}$ and that $Cov_\alpha(S(\alpha;x_t)) = E_\alpha(J(\alpha;x_t))$, where $0_{(k+1)\times 1}$ denotes the $(k+1)\times 1$ vector

$(0, ..., 0)^T$. Then

$$I_{tii}(\alpha) = \sum_{x_t \in \mathcal{X}_t} \left( \sum_{j=0}^{x_{it}-1} \frac{1}{\alpha_i + j} - \sum_{j=0}^{n_t-1} \frac{1}{\alpha_s + j} \right)^2 f(x_t; \alpha)$$
$$= \sum_{x_t \in \mathcal{X}_t} \left[ \sum_{j=0}^{x_{it}-1} \frac{1}{(\alpha_i + j)^2} \right] f(x_t; \alpha) - \sum_{j=0}^{n_t-1} \frac{1}{(\alpha_s + j)^2} \tag{22}$$

for $i = 0, 1, ..., k$ and

$$I_{tii'}(\alpha) = -\sum_{j=0}^{n_t-1} \frac{1}{(\alpha_s + j)^2} \tag{23}$$

for $i, i' = 0, 1, ..., k$ with $i \neq i'$. Sometimes, $|\mathcal{X}_t|$ is very large at time $t$ in a manufacturing process, e.g., $|\mathcal{X}_t| = 82, 408, 626, 300$ when $n_t = 200$ and $k = 6$. In such situations, it will take too much time to evaluate $I_{tii}(\alpha)$s by equation (18). One possible approach to evaluate $I_{tii}(\alpha)$s is the Monte Carlo method as follows: First generate *i.i.d.* $(p_t^{(1)T}, x_t^{(1)T})^T, ..., (p_t^{(r)T}, x_t^{(r)T})^T$ such that $p_t^{(u)}$ is sampled from Dirichlet$(\alpha)$ and $x_t^{(u)}$ given $p_t^{(u)}$ is sampled from multinomial$(n_t; p_t^{(u)})$ for $u = 1, ..., r$, where $r$ is a large positive integer, e.g., $r = 100, 000$. Then $I_{tii}(\alpha)$ can be approximately evaluated by $\widehat{I_{tii}(\alpha)}$, where

$$\widehat{I_{tii}(\alpha)} \equiv \frac{1}{r} \sum_{u=1}^{r} \left( \sum_{j=0}^{x_{it}^{(u)}-1} \frac{1}{\alpha_i + j} - \sum_{j=0}^{n_t-1} \frac{1}{\alpha_s + j} \right)^2 \tag{24}$$

or

$$\frac{1}{r} \sum_{u=1}^{r} \sum_{j=0}^{x_{it}^{(u)}-1} \frac{1}{(\alpha_i + j)^2} - \sum_{j=0}^{n_t-1} \frac{1}{(\alpha_s + j)^2}. \tag{25}$$

# 3 Multivariate Exponentially Weighted Moving Average Monitoring Scheme

In this section, a multivariate exponentially weighted moving average control chart is proposed for monitoring a categorical manufacturing process under the Dirichlet-compound multinomial model as follows: Suppose that $\{(p_t^T, x_t^T)^T\colon\ t \geq 1\}$ are independent $(2k{+}2)\times 1$ random vectors, where both $p_t$ and $x_t$ are described in Section 2. Let $\alpha_0$ ($\in (0,\infty)^{k+1}$) denote the known $(k+1)\times 1$ in-control process parameter vector in phase I. The multivariate exponentially weighted moving average (MEWMA) control chart is based on the $(k+1)\times 1$ vectors

$$\begin{aligned} w_t &\equiv (I_{k+1}-R)w_{t-1} + R\,S(\alpha_0;x_t) \\ &= R(I_{k+1}-R)^{t-1}S(\alpha_0;x_1) + \cdots + R(I_{k+1}-R)S(\alpha_0;x_{t-1}) + R\,S(\alpha_0;x_t) \\ &= R\sum_{i=0}^{t-1}(I_{k+1}-R)^i S(\alpha_0;x_{t-i}) \end{aligned} \tag{26}$$

for $t = 1,2,...$, where $w_0 \equiv 0_{(k+1)\times 1}$; $I_{k+1}$ denotes the identity matrix of order $k+1$; $R$ is a specified $(k+1)\times(k+1)$ positive definite covariance matrix such that $I_{k+1}-R$ is non-negative definite; and $(I_{k+1}-R)^0 \equiv I_{k+1}$. It follows from the eigenvalue decomposition that $R = P\Lambda P^T$ where $P$ is an orthogonal matrix, i.e., $PP^T = I_{k+1}$, and $\Lambda$ is a diagonal matrix $\mathrm{diag}\{\lambda_0,\lambda_1,...,\lambda_k\}$ with $\lambda_0,\lambda_1,...,\lambda_k \in (0,1]$. Then

$$w_t = P\Lambda\sum_{i=0}^{t-1}(I_{k+1}-\Lambda)^i P^T S(\alpha_0;x_{t-i}) \tag{27}$$

for $t = 1,2,....$

When $p_1,...,p_t \sim \text{Dirichlet}(\alpha_0)$ for some $t = 1,2,...$, the following properties hold:

(i) $E_{\alpha_0}(w_t) = 0_{(k+1)\times 1}$.

(ii)

$$Cov_{\alpha_0}(w_t) = P\Lambda\left[\sum_{i=0}^{t-1}(I_{k+1}-\Lambda)^i P^T\, I_{t-i}(\alpha_0)P(I_{k+1}-\Lambda)^i\right]\Lambda P^T\ (\equiv \Sigma_t). \tag{28}$$

(iii) $E_{\alpha_0}(w_t^T\Sigma_t^{-1}w_t) = k+1$. Set

$$T_t^2 \equiv w_t^T\Sigma_t^{-1}w_t. \tag{29}$$

(iv)

$$(R^{-1}\Sigma_t R^{-1})^{-1/2}R^{-1}w_t \overset{a.s.}{\to} \left[\sum_{i=0}^{t-1} I_{t-i}(\alpha_0)\right]^{-1/2}\sum_{i=0}^{t-1} S(\alpha_0; x_{t-i}) \tag{30}$$

and

$$T_t^2 \overset{a.s.}{\to} \left[\sum_{i=0}^{t-1} S(\alpha_0; x_{t-i})\right]^T \left[\sum_{i=0}^{t-1} I_{t-i}(\alpha_0)\right]^{-1}\sum_{i=0}^{t-1} S(\alpha_0; x_{t-i}) \ (\equiv T_{t0}^2) \tag{31}$$

as $\max\{\lambda_0, \lambda_1, ..., \lambda_k\} \to 0$, where $T_{t0}^2$ is the score test statistic up to time $t$ for testing the null hypothesis $H_0$: $p_1, ..., p_t \sim$ Dirichlet$(\alpha_0)$ versus the alternative $H_1$: $p_1, ..., p_t \sim$ Dirichlet$(\alpha)$ for some $\alpha \neq \alpha_0$.

(v) If $\sup_{t\geq 1} n_t < \infty$, then $(R^{-1}\Sigma_t R^{-1})^{-1/2}R^{-1}w_t \overset{d}{\to} N_{k+1}(0_{(k+1)\times 1}, I_{k+1})$ and $T_t^2 \overset{d}{\to} \chi^2_{k+1}$ as $\max\{\lambda_0, \lambda_1, ..., \lambda_k\} \to 0$ and $t \to \infty$.

(vi) If $\Lambda = \lambda\, I_{k+1}$ for some $\lambda \in (0, 1]$, then

$$w_t = \lambda \sum_{i=0}^{t-1}(1-\lambda)^i S(\alpha_0; x_{t-i}), \tag{32}$$

$$T_t^2 = \left[\sum_{i=0}^{t-1}(1-\lambda)^i S(\alpha_0; x_{t-i})\right]^T \left[\sum_{i=0}^{t-1}(1-\lambda)^{2i} I_{t-i}(\alpha_0)\right]^{-1}\sum_{i=0}^{t-1}(1-\lambda)^i S(\alpha_0; x_{t-i}), \tag{33}$$

and

$$\Sigma_t = \lambda^2 \sum_{i=0}^{t-1}(1-\lambda)^{2i} I_{t-i}(\alpha_0) \tag{34}$$

for $t = 1, 2, ...,$ where $0^0 \equiv 1$. In particular, if $\Lambda = I_{k+1}$, then $w_t = S(\alpha_0; x_t)$, $T_t^2 = S^T(\alpha_0; x_t) I_t^{-1}(\alpha_0) S(\alpha_0; x_t)$ $(\equiv T_{t1}^2)$, and $\Sigma_t = I_t(\alpha_0)$ for $t = 1, 2, ...,$ where $T_{t1}^2$ is the score test statistic at time $t$ for testing the null hypothesis $H_0$: $p_t \sim$ Dirichlet$(\alpha_0)$ versus the alternative $H_1$: $p_t \sim$ Dirichlet$(\alpha)$ for some $\alpha \neq \alpha_0$.

(vii) If $n_1 = n_2 = ...$, then

$$\begin{aligned}
\Sigma_t &= P\Lambda \left[\sum_{i=0}^{t-1}(I_{k+1}-\Lambda)^i P^T\, I_1(\alpha_0) P (I_{k+1}-\Lambda)^i\right]\Lambda P^T \\
&\equiv P\Lambda \left[\sum_{i=0}^{t-1}(I_{k+1}-\Lambda)^i\, I_1^*(\alpha_0)(I_{k+1}-\Lambda)^i\right]\Lambda P^T \\
&= P\left(\frac{\lambda_j\lambda_{j'}\left[1-(1-\lambda_j)^t(1-\lambda_{j'})^t\right] I_{1jj'}^*(\alpha_0)}{\lambda_j+\lambda_{j'}-\lambda_j\lambda_{j'}}\right)_{j,j'=0,1,...,k} P^T \\
&\to P\left(\frac{\lambda_j\lambda_{j'} I_{1jj'}^*(\alpha_0)}{\lambda_j+\lambda_{j'}-\lambda_j\lambda_{j'}}\right)_{j,j'=0,1,...,k} P^T
\end{aligned} \tag{35}$$

as $t \to \infty$, where $P^T I_1(\alpha_0) P \equiv I_1^*(\alpha_0) \equiv (I_{1jj'}^*(\alpha_0))_{j,j'=0,1,\ldots,k}$.

(viii) If $n_1 = n_2 = \ldots$ and $\Lambda = \lambda\, I_{k+1}$ for some $\lambda \in (0, 1]$, then

$$\Sigma_t = \frac{\lambda\,[1 - (1-\lambda)^{2t}]}{2-\lambda}\, I_1(\alpha_0) \to \frac{\lambda}{2-\lambda}\, I_1(\alpha_0) \tag{36}$$

as $t \to \infty$. In particular, if $n_1 = n_2 = \ldots$ and $\Lambda = I_{k+1}$, then $\Sigma_t = I_1(\alpha_0)$ for $t = 1, 2, \ldots$, $w_1, w_2, \ldots$ are *i.i.d.*, and $T_1^2, T_2^2, \ldots$ are *i.i.d.*

Then the stopping time of the MEWMA monitoring scheme is the first time $t$ such that

$$T_t^2 > h, \tag{37}$$

where $h$ is chosen to achieve a specified in-control average run length, e.g., $1/[2\Phi(-3)] \approx 370.4$ with $\Phi(\cdot)$ denoting the cumulative distribution function (c.d.f.) of the standard normal distribution.

# 4 A Simulation Study

In the former information, it accumulate the data up to time $t$ when $t = 1, 2, ..., l$. Through the former information, this paper discusses the different $l$ for each $\lambda$. The $l$ is the past in-control data in phase I.

In order to study the performance of this quality control scheme, it compute the average run length. To evaluate the in-control average run length ($\equiv ARL_0$), it considers the special case where $k = 2$; $p_1, p_2, ...$ are sampled from Dirichlet$(\alpha_0)$ with $\alpha_0 = (85, 10, 5)^T$; $n_1 = n_2 = ... = 100$; $P = I_3$; and $\Lambda = \lambda I_3$ for $\lambda \in \{0.01, 0.05, 0.10, 0.15, 0.20, 0.30, 0.40, 0.50, 1\}$ ($\equiv S$).

For $\lambda \in S$ and $ARL_0 = 1/[2\Phi(-3)]$, The $h$ in equation (33) can be evaluated as follows:

Step 1: Generate *i.i.d.* $(p_1^T, x_1^T)^T, ..., (p_{370}^T, x_{370}^T)^T$ such that $p_t$ is sampled from Dirichlet$(\alpha_0)$ and $x_t$ is sampled from multinomial$(100; p_t)$ for $t = 1, 2, ..., 370$.

Step 2: To sort the $T_t^2$s such that $T_t^{2(1)}, T_t^{2(2)}, ..., T_t^{2(370)}$.

Step 3: Choose the maximum $T_t^{2(370)}$ ($\equiv T_t^{*2}$).

Repeat Steps 1–3 for 10,000 times independently. To sort the $T_t^{*2}$s such that $T_t^{*2(1)}, T_t^{*2(2)}, ..., T_t^{*2(10,000)}$. The initial value of $h_1$ is $T_t^{*2(5,000)}$. To compute the $ARL_0^{(1)}$ with $h_1$. If the $ARL_0^{(1)}$ is large than $1/[2\Phi(-3)]$, then $h_2$ is given such that $h_2 < h_1$, and compute the $ARL_0^{(2)}$. If the $ARL_0^{(1)}$ is smaller than $1/[2\Phi(-3)]$, then $h_2$ is given such that $h_2 > h_1$, and compute the $ARL_0^{(2)}$. Until it finds the $h$ such that $ARL_0 \approx 1/[2\Phi(-3)]$ for each $\lambda$; see Table 1.

To evaluate the out-of-control average run length ($\equiv ARL_1$), it considers the special case where $k = 2$; $p_1, ..., p_l$ are sampled from Dirichlet$(\alpha_0)$ and $p_{l+1}, p_{l+2}, ...$ are sampled from Dirichlet$(\alpha_1)$ for some $l \in \{0, 1, 2, ..., 500\}$ with $\alpha_0 = (85, 10, 5)^T$, $\alpha_1 \in \{(80, 12.5, 7.5)^T\ (\equiv \alpha_1^{(1)}),\ (75, 15, 10)^T\ (\equiv \alpha_1^{(2)}),\ (70, 20, 10)^T\ (\equiv \alpha_1^{(3)})\}$; $n_1 = n_2 = ... = 100$; $P = I_3$; and $\Lambda = \lambda I_3$ for $\lambda \in S$.

The $ARL_1$ can be evaluated as follows:

Step 1: Generate *i.i.d.* $(p_1^T, x_1^T)^T, ..., (p_l^T, x_l^T)^T$ such that $p_t$ is sampled from Dirichlet$(\alpha_0)$ and then $x_t$ is sampled from multinomial$(100; p_t)$ for $t = 1, 2, ..., l$.

Step 2: If $T_t^2 > h$ for some $t \in \{1, 2, ..., l\}$, then return to Step 1.

Step 3: Generate *i.i.d.* $(p_{l+1}^T, x_{l+1}^T)^T, ..., (p_{l+t^*}^T, x_{l+t^*}^T)^T$ such that $p_t$ is sampled from Dirichlet$(\alpha_1)$ and then $x_t$ is sampled from multinomial$(100; p_t)$ for $t = l+1, l+2, ..., l+t^*$, where $l + t^*$ is the first time $t$ such that $T_t^2 > h$.

Repeat Steps 1–3 for 100,000 times independently. Then the $ARL_1$ is approximated evaluated by the average of 100,000 $t^*$s for each $\lambda \in S$.

For $\lambda \in S$ and $ARL_0 = 1/[2\Phi(-3)]$, the $ARL_1$ is put in Tables 2–31. The proposed MEWMA monitoring scheme is also compared with the following monitoring scheme: the stopping time of the monitoring scheme is the first time $t$ such that $T_{t0}^2 > h_0$, where $T_{t0}^2$ is defined in equation (27) and $h_0$ is chosen to achieve $ARL_0 = 1/[2\Phi(-3)]$.

# 5 Comparison and Conclusions

In the paper, an MEWMA control chart is proposed for monitoring a manufacturing process in the Dirichlet-compound multinomial model. For each $\lambda \in S \cup \{0\}$ $(\equiv S')$, it is seen that $h$ increases as $\lambda$ increases; see Table 1.

For $0 \leq l \leq 4$ and $\lambda \in S'$, the $ARL_1$ increases as $\lambda$ increases; see Tables 2–6. Then $\lambda$ is chosen as 0.

For $l \geq 5$ and $\alpha_1 \in \{\alpha_1^{(1)}, \alpha_1^{(2)}, \alpha_1^{(3)}\}$, consider the following three cases:

**Case 1**: $\alpha_1 = \alpha_1^{(1)}$. When $5 \leq l \leq 20$, the MEWMA monitoring scheme with $\lambda = 0$ has the smallest $ARL_1$ for $\lambda \in S'$; see Tables 7–22. When $21 \leq l \leq 36$, the MEWMA monitoring scheme with $\lambda = 0.05$ has the smallest $ARL_1$ for $\lambda \in S'$; see Tables 23–24. When $38 \leq l \leq 500$, the MEWMA monitoring scheme with $\lambda = 0.1$ has the smallest $ARL_1$ for $\lambda \in S'$; see Tables 26–31.

**Case 2**: $\alpha_1 = \alpha_1^{(2)}$. When $5 \leq l \leq 11$, the MEWMA monitoring scheme with $\lambda = 0$ has the smallest $ARL_1$ for $\lambda \in S'$; see Tables 7–13. When $12 \leq l \leq 138$, the MEWMA monitoring scheme with $\lambda = 0.15$ has the smallest $ARL_1$ for $\lambda \in S'$; see Tables 14–27. When $140 \leq l \leq 500$, the MEWMA monitoring scheme with $\lambda = 0.2$ has the smallest $ARL_1$ for $\lambda \in S'$; see Tables 29–31.

**Case 3**: $\alpha_1 = \alpha_1^{(3)}$. When $5 \leq l \leq 8$, the MEWMA monitoring scheme with $\lambda = 0$ has the smallest $ARL_1$ for $\lambda \in S'$; see Tables 7–10. But when the $9 \leq l \leq 500$, the MEWMA monitoring scheme with $\lambda = 0.3$ has the smallest $ARL_1$ for $\lambda \in S'$; see Tables 11–31.

For these three cases, it is seen that when $\alpha_1$ is faraway from $\alpha_0$, e.g., $\alpha_1 = \alpha_1^{(3)}$, the $ARL_1$ is smaller. When $\alpha_1$ is close to $\alpha_0$, e.g., $\alpha_1 = \alpha_1^{(1)}$, the $ARL_1$ is bigger.

Compare $\lambda = 0.05$ and $0.01$:

**Case 1**: $\alpha_1 = \alpha_1^{(1)}$. When $0 \leq l \leq 15$, the $ARL_1$ for $\lambda = 0.01$ is smaller than taht for $\lambda = 0.05$, so $\lambda = 0.01$ is better than $\lambda = 0.05$; see Tables 2–17. When $16 \leq l \leq 500$, the $ARL_1$ for $\lambda = 0.05$ is smaller than taht for $\lambda = 0.01$, so $\lambda = 0.05$ is better than $\lambda = 0.01$; see Tables 18–31.

**Case 2**: $\alpha_1 = \alpha_1^{(2)}$. When $0 \leq l \leq 12$, the $ARL_1$ for $\lambda = 0.01$ is smaller than taht for $\lambda = 0.05$, so $\lambda = 0.01$ is better than $\lambda = 0.05$; see Tables 2–14. When $13 \leq l \leq 500$, the

$ARL_1$ for $\lambda = 0.05$ is smaller than taht for $\lambda = 0.01$, so $\lambda = 0.05$ is better than $\lambda = 0.01$; see Tables 14–31.

**Case 3**: $\alpha_1 = \alpha_1^{(3)}$. When $0 \leq l \leq 11$, the $ARL_1$ for $\lambda = 0.01$ is smaller than taht for $\lambda = 0.05$, so $\lambda = 0.01$ is better than $\lambda = 0.05$; see Tables 2–13. When $12 \leq l \leq 500$, the $ARL_1$ for $\lambda = 0.05$ is smaller than taht for $\lambda = 0.01$, so $\lambda = 0.05$ is better than $\lambda = 0.01$; see Tables 14–31.

So, when $l$ is large, the weight $\lambda = 0.05$ is better than $\lambda = 0.01$.

In general, the $\lambda$ of EWMA in the interval $0.05 \leq \lambda \leq 0.25$ works well in practice, with $\lambda = 0.05$, $\lambda = 0.10$, $\lambda = 0.20$ being popular choices. In this paper, it suggests a multivariate exponentially weighted moving average control chart for some relevant properties of the proposed control chart are also investigated in the Dirichlet-compound multinomial model for a categorical manufacturing process. It is found by simulation that the different $\alpha_1$ and $l$ have different weight such that the $ARL_1$ is smallest when $\lambda \in S'$. According to accumulating different in-control data up to time $l$ in phase I, $\lambda$ will be different. This can be taken as a reference.

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Table 1: $h$ for $\lambda \in S'$ and $\alpha_0 = (85, 10, 5)^T$.

| $\lambda$ | 0.00 | 0.01 | 0.05 | 0.10 | 0.15 | 0.20 | 0.30 | 0.40 | 0.50 | 1.00 |
|---|---|---|---|---|---|---|---|---|---|---|
| $h$ | 6.53 | 8.33 | 11.96 | 14.79 | 17.02 | 19.08 | 22.45 | 25.42 | 27.73 | 34.34 |

*** denotes the smallest $ARL_1$.**

Table 2: $ARL_1$ for $l = 0$, $\lambda \in S'$; $\alpha_1 \in \{\alpha_1^{(1)}, \alpha_1^{(2)}, \alpha_1^{(3)}\}$, where $\alpha_1^{(1)} \equiv (80, 12.5, 7.5)^T$, $\alpha_1^{(2)} \equiv (75, 15, 10)^T$, and $\alpha_1^{(3)} \equiv (70, 20, 10)^T$.

| $l = 0$ | $\lambda$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 0.00 | 0.01 | 0.05 | 0.10 | 0.15 | 0.20 | 0.30 | 0.40 | 0.50 | 1.00 |
| $(80, 12.5, 7.5)^T$ | 5.00* | 6.19 | 8.32 | 10.10 | 12.00 | 14.08 | 18.64 | 23.47 | 27.90 | 45.20 |
| $(75, 15, 10)^T$ | 1.94* | 2.19 | 2.62 | 2.96 | 3.22 | 3.49 | 3.96 | 4.52 | 5.13 | 8.80 |
| $(70, 20, 10)^T$ | 1.30* | 1.39 | 1.54 | 1.66 | 1.77 | 1.86 | 1.99 | 2.16 | 2.28 | 3.32 |

Table 3: $ARL_1$ for $l = 1$, $\lambda \in S'$; $\alpha_1 \in \{\alpha_1^{(1)}, \alpha_1^{(2)}, \alpha_1^{(3)}\}$.

| $l = 1$ | $\lambda$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 0.00 | 0.01 | 0.05 | 0.10 | 0.15 | 0.20 | 0.30 | 0.40 | 0.50 | 1.00 |
| $(80, 12.5, 7.5)^T$ | 6.00* | 7.20 | 9.29 | 10.94 | 12.71 | 14.76 | 19.13 | 23.75 | 28.03 | 45.2 |
| $(75, 15, 10)^T$ | 2.37* | 2.65 | 3.14 | 3.44 | 3.65 | 3.86 | 4.26 | 4.75 | 5.29 | 8.80 |
| $(70, 20, 10)^T$ | 1.53* | 1.64 | 1.84 | 1.96 | 2.05 | 2.11 | 2.21 | 2.31 | 2.41 | 3.32 |

Table 4: $ARL_1$ for $l=2$, $\lambda \in S'$; $\alpha_1 \in \{\alpha_1^{(1)}, \alpha_1^{(2)}, \alpha_1^{(3)}\}$.

| $l=2$ | $\lambda$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 0.00 | 0.01 | 0.05 | 0.10 | 0.15 | 0.20 | 0.30 | 0.40 | 0.50 | 1.00 |
| $(80, 12.5, 7.5)^T$ | 6.64* | 7.94 | 9.83 | 11.31 | 13.06 | 14.94 | 19.23 | 24.10 | 27.87 | 45.20 |
| $(75, 15, 10)^T$ | 2.64* | 2.96 | 3.44 | 3.69 | 3.86 | 4.04 | 4.38 | 4.82 | 5.03 | 8.80 |
| $(70, 20, 10)^T$ | 1.69* | 1.82 | 2.03 | 2.14 | 2.19 | 2.24 | 2.29 | 2.37 | 2.44 | 3.32 |

Table 5: $ARL_1$ for $l=3$, $\lambda \in S'$; $\alpha_1 \in \{\alpha_1^{(1)}, \alpha_1^{(2)}, \alpha_1^{(3)}\}$.

| $l=3$ | $\lambda$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 0.00 | 0.01 | 0.05 | 0.10 | 0.15 | 0.20 | 0.30 | 0.40 | 0.50 | 1.00 |
| $(80, 12.5, 7.5)^T$ | 7.17* | 8.36 | 10.19 | 11.69 | 13.24 | 15.05 | 19.38 | 24.26 | 28.03 | 45.20 |
| $(75, 15, 10)^T$ | 2.89* | 3.22 | 3.66 | 3.87 | 3.99 | 4.11 | 4.41 | 4.84 | 5.31 | 8.80 |
| $(70, 20, 10)^T$ | 1.82* | 1.97 | 2.17 | 2.25 | 2.28 | 2.30 | 2.33 | 2.37 | 2.44 | 3.32 |

Table 6: $ARL_1$ for $l=4$, $\lambda \in S'$; $\alpha_1 \in \{\alpha_1^{(1)}, \alpha_1^{(2)}, \alpha_1^{(3)}\}$.

| $l=4$ | $\lambda$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 0.00 | 0.01 | 0.05 | 0.10 | 0.15 | 0.20 | 0.30 | 0.40 | 0.50 | 1.00 |
| $(80, 12.5, 7.5)^T$ | 7.56* | 8.90 | 10.52 | 11.76 | 13.24 | 15.27 | 19.52 | 24.07 | 27.93 | 45.20 |
| $(75, 15, 10)^T$ | 3.09* | 3.42 | 3.84 | 4.01 | 4.08 | 4.18 | 4.43 | 4.85 | 5.32 | 8.80 |
| $(70, 20, 10)^T$ | 1.94* | 2.10 | 2.29 | 2.34 | 2.34 | 2.35 | 2.35 | 2.38 | 2.44 | 3.32 |

Table 7: $ARL_1$ for $l = 5$, $\lambda \in S'$; $\alpha_1 \in \{\alpha_1^{(1)}, \alpha_1^{(2)}, \alpha_1^{(3)}\}$.

| $l = 5$ | $\lambda$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 0.00 | 0.01 | 0.05 | 0.10 | 0.15 | 0.20 | 0.30 | 0.40 | 0.50 | 1.00 |
| $(80, 12.5, 7.5)^T$ | 8.01* | 9.24 | 10.85 | 11.94 | 13.58 | 15.18 | 19.57 | 24.12 | 28.19 | 45.20 |
| $(75, 15, 10)^T$ | 3.27* | 3.63 | 3.99 | 4.10 | 4.13 | 4.23 | 4.46 | 4.87 | 5.37 | 8.80 |
| $(70, 20, 10)^T$ | 2.05* | 2.21 | 2.38 | 2.41 | 2.40 | 2.37 | 2.35 | 2.38 | 2.45 | 3.32 |

Table 8: $ARL_1$ for $l = 6$, $\lambda \in S'$; $\alpha_1 \in \{\alpha_1^{(1)}, \alpha_1^{(2)}, \alpha_1^{(3)}\}$.

| $l = 6$ | $\lambda$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 0.00 | 0.01 | 0.05 | 0.10 | 0.15 | 0.20 | 0.30 | 0.40 | 0.50 | 1.00 |
| $(80, 12.5, 7.5)^T$ | 8.40* | 9.55 | 11.14 | 12.05 | 13.49 | 15.21 | 19.49 | 24.14 | 28.09 | 45.20 |
| $(75, 15, 10)^T$ | 3.45* | 3.78 | 4.12 | 4.16 | 4.18 | 4.22 | 4.44 | 4.85 | 5.34 | 8.80 |
| $(70, 20, 10)^T$ | 2.16* | 2.31 | 2.46 | 2.46 | 2.42 | 2.39 | 2.37 | 2.38 | 2.44 | 3.32 |

Table 9: $ARL_1$ for $l = 7$, $\lambda \in S'$; $\alpha_1 \in \{\alpha_1^{(1)}, \alpha_1^{(2)}, \alpha_1^{(3)}\}$.

| $l = 7$ | $\lambda$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 0.00 | 0.01 | 0.05 | 0.10 | 0.15 | 0.20 | 0.30 | 0.40 | 0.50 | 1.00 |
| $(80, 12.5, 7.5)^T$ | 8.80* | 9.95 | 11.23 | 12.07 | 13.46 | 15.33 | 19.37 | 24.16 | 28.44 | 45.20 |
| $(75, 15, 10)^T$ | 3.58* | 3.93 | 4.23 | 4.22 | 4.20 | 4.24 | 4.44 | 4.85 | 5.36 | 8.80 |
| $(70, 20, 10)^T$ | 2.23* | 2.40 | 2.53 | 2.50 | 2.43 | 2.40 | 2.37 | 2.38 | 2.44 | 3.32 |

Table 10: $ARL_1$ for $l = 8$, $\lambda \in S'$; $\alpha_1 \in \{\alpha_1^{(1)}, \alpha_1^{(2)}, \alpha_1^{(3)}\}$.

| $l = 8$ | $\lambda$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 0.00 | 0.01 | 0.05 | 0.10 | 0.15 | 0.20 | 0.30 | 0.40 | 0.50 | 1.00 |
| $(80, 12.5, 7.5)^T$ | 9.10* | 10.10 | 11.31 | 12.14 | 13.43 | 15.30 | 19.17 | 24.17 | 27.59 | 45.20 |
| $(75, 15, 10)^T$ | 3.75* | 4.09 | 4.33 | 4.27 | 4.22 | 4.27 | 4.44 | 4.85 | 5.37 | 8.80 |
| $(70, 20, 10)^T$ | 2.33* | 2.49 | 2.60 | 2.53 | 2.46 | 2.40 | 2.36 | 2.38 | 2.44 | 3.32 |

Table 11: $ARL_1$ for $l = 9$, $\lambda \in S'$; $\alpha_1 \in \{\alpha_1^{(1)}, \alpha_1^{(2)}, \alpha_1^{(3)}\}$.

| $l = 9$ | $\lambda$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 0.00 | 0.01 | 0.05 | 0.10 | 0.15 | 0.20 | 0.30 | 0.40 | 0.50 | 1.00 |
| $(80, 12.5, 7.5)^T$ | 9.40* | 10.53 | 11.51 | 12.39 | 13.58 | 15.28 | 19.64 | 24.01 | 27.79 | 45.20 |
| $(75, 15, 10)^T$ | 3.91* | 4.21 | 4.41 | 4.29 | 4.22 | 4.23 | 4.45 | 4.84 | 5.36 | 8.80 |
| $(70, 20, 10)^T$ | 2.40 | 2.57 | 2.65 | 2.56 | 2.48 | 2.40 | 2.36* | 2.38 | 2.45 | 3.32 |

Table 12: $ARL_1$ for $l = 10$, $\lambda \in S'$; $\alpha_1 \in \{\alpha_1^{(1)}, \alpha_1^{(2)}, \alpha_1^{(3)}\}$.

| $l = 10$ | $\lambda$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 0.00 | 0.01 | 0.05 | 0.10 | 0.15 | 0.20 | 0.30 | 0.40 | 0.50 | 1.00 |
| $(80, 12.5, 7.5)^T$ | 9.68* | 10.79 | 11.62 | 12.26 | 13.55 | 15.35 | 19.35 | 23.99 | 28.01 | 45.20 |
| $(75, 15, 10)^T$ | 4.01* | 4.35 | 4.48 | 4.34 | 4.23 | 4.25 | 4.46 | 4.86 | 5.34 | 8.80 |
| $(70, 20, 10)^T$ | 2.50 | 2.64 | 2.69 | 2.58 | 2.47 | 2.40 | 2.35* | 2.37 | 2.45 | 3.32 |

Table 13: $ARL_1$ for $l = 11$, $\lambda \in S'$; $\alpha_1 \in \{\alpha_1^{(1)}, \alpha_1^{(2)}, \alpha_1^{(3)}\}$.

| $l = 11$ | $\lambda$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 0.00 | 0.01 | 0.05 | 0.10 | 0.15 | 0.20 | 0.30 | 0.40 | 0.50 | 1.00 |
| $(80, 12.5, 7.5)^T$ | 10.03* | 11.03 | 11.79 | 12.31 | 13.57 | 15.33 | 19.55 | 24.17 | 28.23 | 45.20 |
| $(75, 15, 10)^T$ | 4.16* | 4.45 | 4.55 | 4.36 | 4.27 | 4.25 | 4.46 | 4.85 | 5.33 | 8.80 |
| $(70, 20, 10)^T$ | 2.55 | 2.71 | 2.75 | 2.59 | 2.48 | 2.41 | 2.36* | 2.38 | 2.44 | 3.32 |

Table 14: $ARL_1$ for $l = 12$, $\lambda \in S'$; $\alpha_1 \in \{\alpha_1^{(1)}, \alpha_1^{(2)}, \alpha_1^{(3)}\}$.

| $l = 12$ | $\lambda$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 0.00 | 0.01 | 0.05 | 0.10 | 0.15 | 0.20 | 0.30 | 0.40 | 0.50 | 1.00 |
| $(80, 12.5, 7.5)^T$ | 10.24* | 11.31 | 11.82 | 12.35 | 13.55 | 15.31 | 19.60 | 24.19 | 28.19 | 45.20 |
| $(75, 15, 10)^T$ | 4.27 | 4.57 | 4.59 | 4.38 | 4.24* | 4.26 | 4.44 | 4.85 | 5.35 | 8.80 |
| $(70, 20, 10)^T$ | 2.64 | 2.78 | 2.77 | 2.61 | 2.48 | 2.41 | 2.36* | 2.39 | 2.44 | 3.32 |

Table 15: $ARL_1$ for $l = 13$, $\lambda \in S'$; $\alpha_1 \in \{\alpha_1^{(1)}, \alpha_1^{(2)}, \alpha_1^{(3)}\}$.

| $l = 13$ | $\lambda$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 0.00 | 0.01 | 0.05 | 0.10 | 0.15 | 0.20 | 0.30 | 0.40 | 0.50 | 1.00 |
| $(80, 12.5, 7.5)^T$ | 10.53* | 11.54 | 11.93 | 12.41 | 13.56 | 15.29 | 19.32 | 24.23 | 28.40 | 45.20 |
| $(75, 15, 10)^T$ | 4.40 | 4.68 | 4.65 | 4.39 | 4.26* | 4.27 | 4.44 | 4.85 | 5.36 | 8.80 |
| $(70, 20, 10)^T$ | 2.70 | 2.85 | 2.80 | 2.61 | 2.47 | 2.42 | 2.35* | 2.38 | 2.44 | 3.32 |

Table 16: $ARL_1$ for $l = 14$, $\lambda \in S'$; $\alpha_1 \in \{\alpha_1^{(1)}, \alpha_1^{(2)}, \alpha_1^{(3)}\}$.

| $l = 14$ | $\lambda$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 0.00 | 0.01 | 0.05 | 0.10 | 0.15 | 0.20 | 0.30 | 0.40 | 0.50 | 1.00 |
| $(80, 12.5, 7.5)^T$ | 10.70* | 11.63 | 11.97 | 12.40 | 13.56 | 15.31 | 19.23 | 24.15 | 28.32 | 45.20 |
| $(75, 15, 10)^T$ | 4.48 | 4.77 | 4.71 | 4.41 | 4.27* | 4.28 | 4.43 | 4.86 | 5.34 | 8.80 |
| $(70, 20, 10)^T$ | 2.76 | 2.92 | 2.81 | 2.62 | 2.47 | 2.41 | 2.37* | 2.38 | 2.44 | 3.32 |

Table 17: $ARL_1$ for $l = 15$, $\lambda \in S'$; $\alpha_1 \in \{\alpha_1^{(1)}, \alpha_1^{(2)}, \alpha_1^{(3)}\}$.

| $l = 15$ | $\lambda$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 0.00 | 0.01 | 0.05 | 0.10 | 0.15 | 0.20 | 0.30 | 0.40 | 0.50 | 1.00 |
| $(80, 12.5, 7.5)^T$ | 10.96* | 11.81 | 12.03 | 12.39 | 13.57 | 15.33 | 19.37 | 24.17 | 28.28 | 45.20 |
| $(75, 15, 10)^T$ | 4.59 | 4.86 | 4.74 | 4.42 | 4.27* | 4.28 | 4.45 | 4.86 | 5.35 | 8.80 |
| $(70, 20, 10)^T$ | 2.82 | 2.97 | 2.84 | 2.62 | 2.48 | 2.41 | 2.35* | 2.38 | 2.43 | 3.32 |

Table 18: $ARL_1$ for $l = 16$, $\lambda \in S'$; $\alpha_1 \in \{\alpha_1^{(1)}, \alpha_1^{(2)}, \alpha_1^{(3)}\}$.

| $l = 16$ | $\lambda$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 0.00 | 0.01 | 0.05 | 0.10 | 0.15 | 0.20 | 0.30 | 0.40 | 0.50 | 1.00 |
| $(80, 12.5, 7.5)^T$ | 11.22* | 12.02 | 12.00 | 12.38 | 13.58 | 15.29 | 19.24 | 24.26 | 28.17 | 45.20 |
| $(75, 15, 10)^T$ | 4.68 | 4.95 | 4.77 | 4.42 | 4.26* | 4.27 | 4.44 | 4.87 | 5.35 | 8.80 |
| $(70, 20, 10)^T$ | 2.88 | 3.05 | 2.87 | 2.63 | 2.47 | 2.42 | 2.33* | 2.37 | 2.44 | 3.32 |

Table 19: $ARL_1$ for $l = 17$, $\lambda \in S'$; $\alpha_1 \in \{\alpha_1^{(1)}, \alpha_1^{(2)}, \alpha_1^{(3)}\}$.

| $l = 17$ | $\lambda$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 0.00 | 0.01 | 0.05 | 0.10 | 0.15 | 0.20 | 0.30 | 0.40 | 0.50 | 1.00 |
| $(80, 12.5, 7.5)^T$ | 11.50* | 12.37 | 12.16 | 12.38 | 13.56 | 15.31 | 19.30 | 24.03 | 28.25 | 45.20 |
| $(75, 15, 10)^T$ | 4.81 | 5.07 | 4.79 | 4.43 | 4.26* | 4.27 | 4.44 | 4.85 | 5.34 | 8.80 |
| $(70, 20, 10)^T$ | 2.95 | 3.08 | 2.90 | 2.64 | 2.48 | 2.42 | 2.34* | 2.38 | 2.44 | 3.32 |

Table 20: $ARL_1$ for $l = 18$, $\lambda \in S'$; $\alpha_1 \in \{\alpha_1^{(1)}, \alpha_1^{(2)}, \alpha_1^{(3)}\}$.

| $l = 18$ | $\lambda$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 0.00 | 0.01 | 0.05 | 0.10 | 0.15 | 0.20 | 0.30 | 0.40 | 0.50 | 1.00 |
| $(80, 12.5, 7.5)^T$ | 11.77* | 12.61 | 12.18 | 12.38 | 13.55 | 15.34 | 19.17 | 23.92 | 28.33 | 45.20 |
| $(75, 15, 10)^T$ | 4.91 | 5.13 | 4.82 | 4.43 | 4.19* | 4.26 | 4.45 | 4.85 | 5.35 | 8.80 |
| $(70, 20, 10)^T$ | 3.01 | 3.11 | 2.91 | 2.64 | 2.49 | 2.41 | 2.33* | 2.38 | 2.45 | 3.32 |

Table 21: $ARL_1$ for $l = 19$, $\lambda \in S'$; $\alpha_1 \in \{\alpha_1^{(1)}, \alpha_1^{(2)}, \alpha_1^{(3)}\}$.

| $l = 19$ | $\lambda$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 0.00 | 0.01 | 0.05 | 0.10 | 0.15 | 0.20 | 0.30 | 0.40 | 0.50 | 1.00 |
| $(80, 12.5, 7.5)^T$ | 12.05* | 12.64 | 12.22 | 12.52 | 13.56 | 15.32 | 19.23 | 24.29 | 28.39 | 45.20 |
| $(75, 15, 10)^T$ | 5.01 | 5.25 | 4.82 | 4.44 | 4.25* | 4.27 | 4.44 | 4.86 | 5.37 | 8.80 |
| $(70, 20, 10)^T$ | 3.07 | 3.15 | 2.93 | 2.63 | 2.48 | 2.42 | 2.36* | 2.39 | 2.45 | 3.32 |

Table 22: $ARL_1$ for $l = 20$, $\lambda \in S'$; $\alpha_1 \in \{\alpha_1^{(1)}, \alpha_1^{(2)}, \alpha_1^{(3)}\}$.

| $l = 20$ | $\lambda$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 0.00 | 0.01 | 0.05 | 0.10 | 0.15 | 0.20 | 0.30 | 0.40 | 0.50 | 1.00 |
| $(80, 12.5, 7.5)^T$ | 12.12* | 12.84 | 12.23 | 12.49 | 13.58 | 15.33 | 19.33 | 24.03 | 28.24 | 45.20 |
| $(75, 15, 10)^T$ | 5.10 | 5.26 | 4.90 | 4.45 | 4.25* | 4.28 | 4.44 | 4.85 | 5.36 | 8.80 |
| $(70, 20, 10)^T$ | 3.09 | 3.21 | 2.94 | 2.63 | 2.49 | 2.42 | 2.37* | 2.39 | 2.44 | 3.32 |

Table 23: $ARL_1$ for $l = 21$, $\lambda \in S'$; $\alpha_1 \in \{\alpha_1^{(1)}, \alpha_1^{(2)}, \alpha_1^{(3)}\}$.

| $l = 21$ | $\lambda$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 0.00 | 0.01 | 0.05 | 0.10 | 0.15 | 0.20 | 0.30 | 0.40 | 0.50 | 1.00 |
| $(80, 12.5, 7.5)^T$ | 12.42 | 13.01 | 12.27* | 12.43 | 13.57 | 15.29 | 19.20 | 23.94 | 28.13 | 45.20 |
| $(75, 15, 10)^T$ | 5.17 | 5.38 | 4.91 | 4.44 | 4.26* | 4.27 | 4.45 | 4.87 | 5.35 | 8.80 |
| $(70, 20, 10)^T$ | 3.20 | 3.28 | 2.95 | 2.64 | 2.50 | 2.43 | 2.36* | 2.37 | 2.44 | 3.32 |

Table 24: $ARL_1$ for $l = 36$, $\lambda \in S'$; $\alpha_1 \in \{\alpha_1^{(1)}, \alpha_1^{(2)}, \alpha_1^{(3)}\}$.

| $l = 36$ | $\lambda$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 0.00 | 0.01 | 0.05 | 0.10 | 0.15 | 0.20 | 0.30 | 0.40 | 0.50 | 1.00 |
| $(80, 12.5, 7.5)^T$ | 15.63 | 14.09 | 12.34* | 12.45 | 13.62 | 15.35 | 19.40 | 24.17 | 28.18 | 45.20 |
| $(75, 15, 10)^T$ | 5.98 | 6.08 | 4.95 | 4.44 | 4.26* | 4.30 | 4.44 | 4.86 | 5.34 | 8.80 |
| $(70, 20, 10)^T$ | 3.77 | 3.69 | 3.01 | 2.65 | 2.50 | 2.43 | 2.34* | 2.38 | 2.44 | 3.32 |

Table 25: $ARL_1$ for $l = 37$, $\lambda \in S'$; $\alpha_1 \in \{\alpha_1^{(1)}, \alpha_1^{(2)}, \alpha_1^{(3)}\}$.

| $l = 37$ | $\lambda$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 0.00 | 0.01 | 0.05 | 0.10 | 0.15 | 0.20 | 0.30 | 0.40 | 0.50 | 1.00 |
| $(80, 12.5, 7.5)^T$ | 15.76 | 14.21 | 12.46* | 12.46* | 13.65 | 15.37 | 19.43 | 24.19 | 28.20 | 45.20 |
| $(75, 15, 10)^T$ | 6.11 | 6.14 | 4.99 | 4.44 | 4.27* | 4.29 | 4.44 | 4.86 | 5.36 | 8.80 |
| $(70, 20, 10)^T$ | 3.82 | 3.73 | 3.03 | 2.66 | 2.50 | 2.43 | 2.35* | 2.38 | 2.44 | 3.32 |

Table 26: $ARL_1$ for $l = 38$, $\lambda \in S'$; $\alpha_1 \in \{\alpha_1^{(1)}, \alpha_1^{(2)}, \alpha_1^{(3)}\}$.

| $l = 38$ | $\lambda$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 0.00 | 0.01 | 0.05 | 0.10 | 0.15 | 0.20 | 0.30 | 0.40 | 0.50 | 1.00 |
| $(80, 12.5, 7.5)^T$ | 16.15 | 14.37 | 12.51 | 12.47* | 13.63 | 15.39 | 19.47 | 24.04 | 28.17 | 45.20 |
| $(75, 15, 10)^T$ | 6.24 | 6.19 | 5.00 | 4.44 | 4.27* | 4.31 | 4.44 | 4.87 | 5.33 | 8.80 |
| $(70, 20, 10)^T$ | 3.90 | 3.76 | 3.04 | 2.65 | 2.50 | 2.43 | 2.36* | 2.37 | 2.44 | 3.32 |

Table 27: $ARL_1$ for $l = 138$, $\lambda \in S'$; $\alpha_1 \in \{\alpha_1^{(1)}, \alpha_1^{(2)}, \alpha_1^{(3)}\}$.

| $l = 138$ | $\lambda$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 0.00 | 0.01 | 0.05 | 0.10 | 0.15 | 0.20 | 0.30 | 0.40 | 0.50 | 1.00 |
| $(80, 12.5, 7.5)^T$ | 26.84 | 18.16 | 12.63 | 12.39* | 13.60 | 15.30 | 19.42 | 23.94 | 28.19 | 45.20 |
| $(75, 15, 10)^T$ | 11.32 | 7.78 | 5.07 | 4.47 | 4.28* | 4.31 | 4.44 | 4.85 | 5.35 | 8.80 |
| $(70, 20, 10)^T$ | 6.71 | 4.67 | 3.07 | 2.65 | 2.49 | 2.41 | 2.35* | 2.38 | 2.45 | 3.32 |

Table 28: $ARL_1$ for $l = 139$, $\lambda \in S'$; $\alpha_1 \in \{\alpha_1^{(1)}, \alpha_1^{(2)}, \alpha_1^{(3)}\}$.

| $l = 139$ | $\lambda$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 0.00 | 0.01 | 0.05 | 0.10 | 0.15 | 0.20 | 0.30 | 0.40 | 0.50 | 1.00 |
| $(80, 12.5, 7.5)^T$ | 26.91 | 18.17 | 12.63 | 12.41* | 13.59 | 15.31 | 19.43 | 24.06 | 28.17 | 45.20 |
| $(75, 15, 10)^T$ | 11.36 | 7.79 | 5.07 | 4.46 | 4.26* | 4.26* | 4.45 | 4.86 | 5.36 | 8.80 |
| $(70, 20, 10)^T$ | 6.74 | 4.68 | 3.06 | 2.65 | 2.50 | 2.42 | 2.36* | 2.38 | 2.44 | 3.32 |

Table 29: $ARL_1$ for $l = 140$, $\lambda \in S'$; $\alpha_1 \in \{\alpha_1^{(1)}, \alpha_1^{(2)}, \alpha_1^{(3)}\}$.

| $l = 140$ | $\lambda$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 0.00 | 0.01 | 0.05 | 0.10 | 0.15 | 0.20 | 0.30 | 0.40 | 0.50 | 1.00 |
| $(80, 12.5, 7.5)^T$ | 27.04 | 18.18 | 12.64 | 12.42* | 13.58 | 15.31 | 19.47 | 24.07 | 28.17 | 45.20 |
| $(75, 15, 10)^T$ | 11.38 | 7.79 | 5.06 | 4.45 | 4.32 | 4.21* | 4.45 | 4.86 | 5.36 | 8.80 |
| $(70, 20, 10)^T$ | 6.75 | 4.68 | 3.05 | 2.66 | 2.48 | 2.42 | 2.37* | 2.39 | 2.44 | 3.32 |

Table 30: $ARL_1$ for $l = 450$, $\lambda \in S'$; $\alpha_1 \in \{\alpha_1^{(1)}, \alpha_1^{(2)}, \alpha_1^{(3)}\}$.

| $l = 450$ | $\lambda$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 0.00 | 0.01 | 0.05 | 0.10 | 0.15 | 0.20 | 0.30 | 0.40 | 0.50 | 1.00 |
| $(80, 12.5, 7.5)^T$ | 45.17 | 19.08 | 12.75 | 12.61* | 13.55 | 15.38 | 19.45 | 24.34 | 28.52 | 45.20 |
| $(75, 15, 10)^T$ | 19.36 | 8.34 | 5.11 | 4.54 | 4.47 | 4.32* | 4.46 | 5.09 | 5.39 | 8.80 |
| $(70, 20, 10)^T$ | 11.71 | 4.87 | 3.10 | 2.68 | 2.51 | 2.42 | 2.33* | 2.41 | 2.40 | 3.32 |

Table 31: $ARL_1$ for $l = 500$, $\lambda \in S'$; $\alpha_1 \in \{\alpha_1^{(1)}, \alpha_1^{(2)}, \alpha_1^{(3)}\}$.

| $l = 500$ | $\lambda$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 0.00 | 0.01 | 0.05 | 0.10 | 0.15 | 0.20 | 0.30 | 0.40 | 0.50 | 1.00 |
| $(80, 12.5, 7.5)^T$ | 48.04 | 19.30 | 12.80 | 12.67* | 13.57 | 15.42 | 19.44 | 24.38 | 28.49 | 45.20 |
| $(75, 15, 10)^T$ | 20.21 | 8.42 | 5.19 | 4.53 | 4.49 | 4.36* | 4.47 | 5.13 | 5.42 | 8.80 |
| $(70, 20, 10)^T$ | 12.09 | 4.89 | 3.11 | 2.66 | 2.52 | 2.40 | 2.31* | 2.43 | 2.36 | 3.32 |